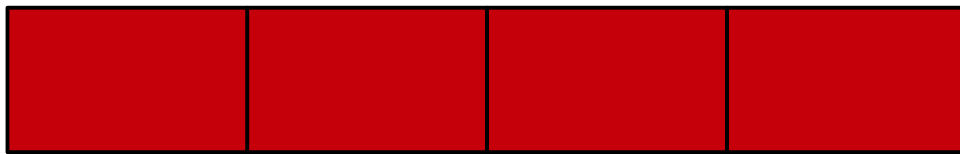
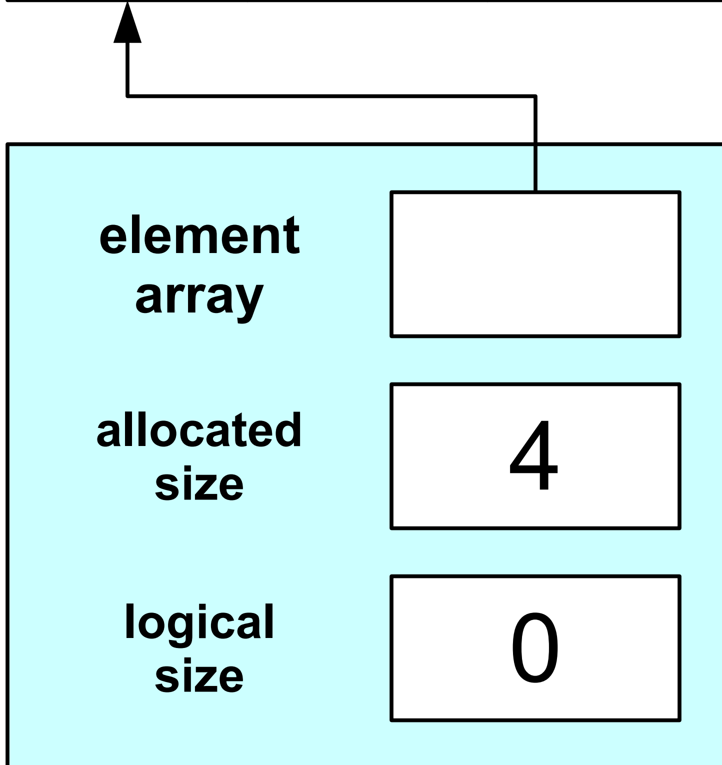
Implementing Abstractions

Part Two

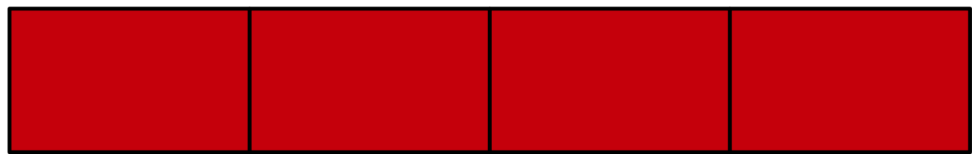
Previously, on CS106B…

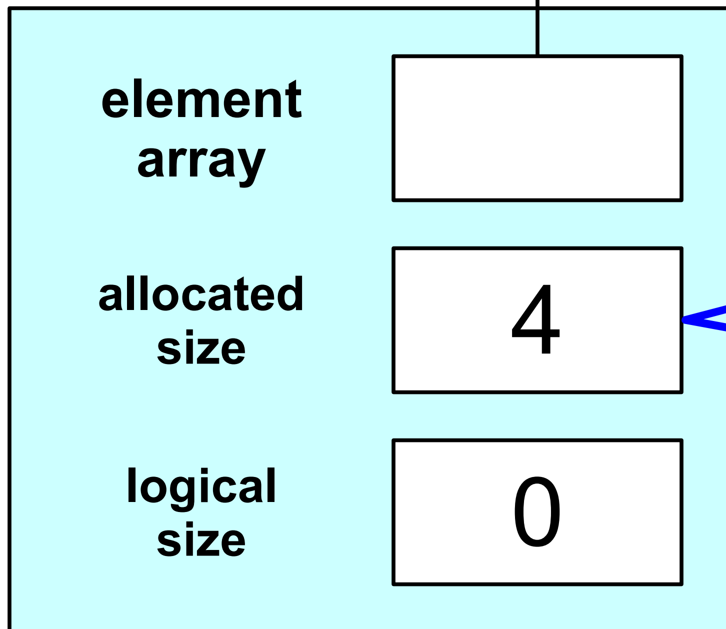
A Bounded Stack





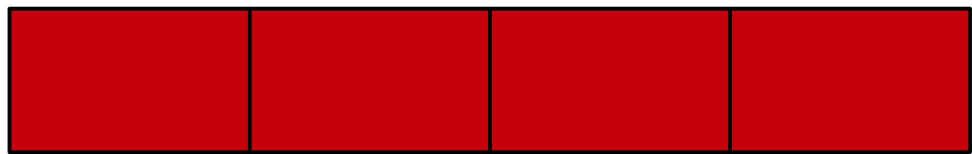
A Bounded Stack

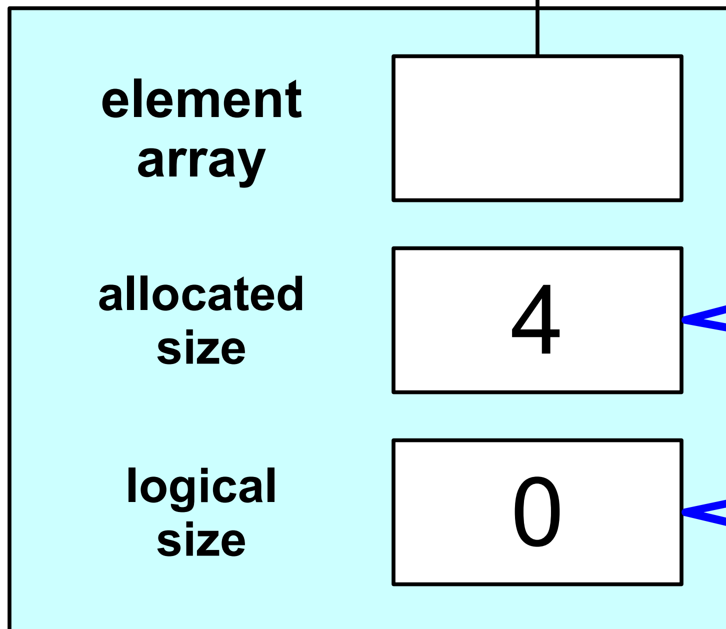




The stack's ***allocated size*** is the number of slots in the array. Remember – arrays in C++ cannot grow or shrink.

A Bounded Stack

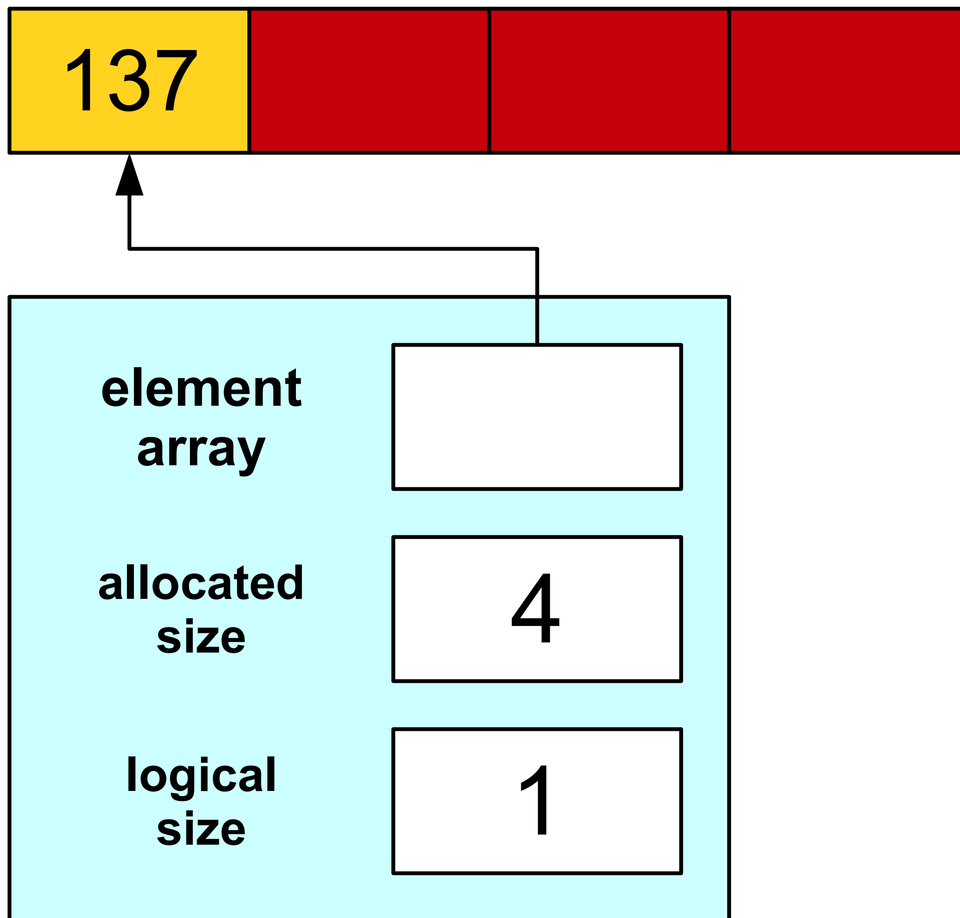




The stack's ***allocated size*** is the number of slots in the array. Remember – arrays in C++ cannot grow or shrink.

The stack's ***logical size*** is the number of elements actually stored in the stack. This lets us track how much space we're actually using.

A Bounded Stack



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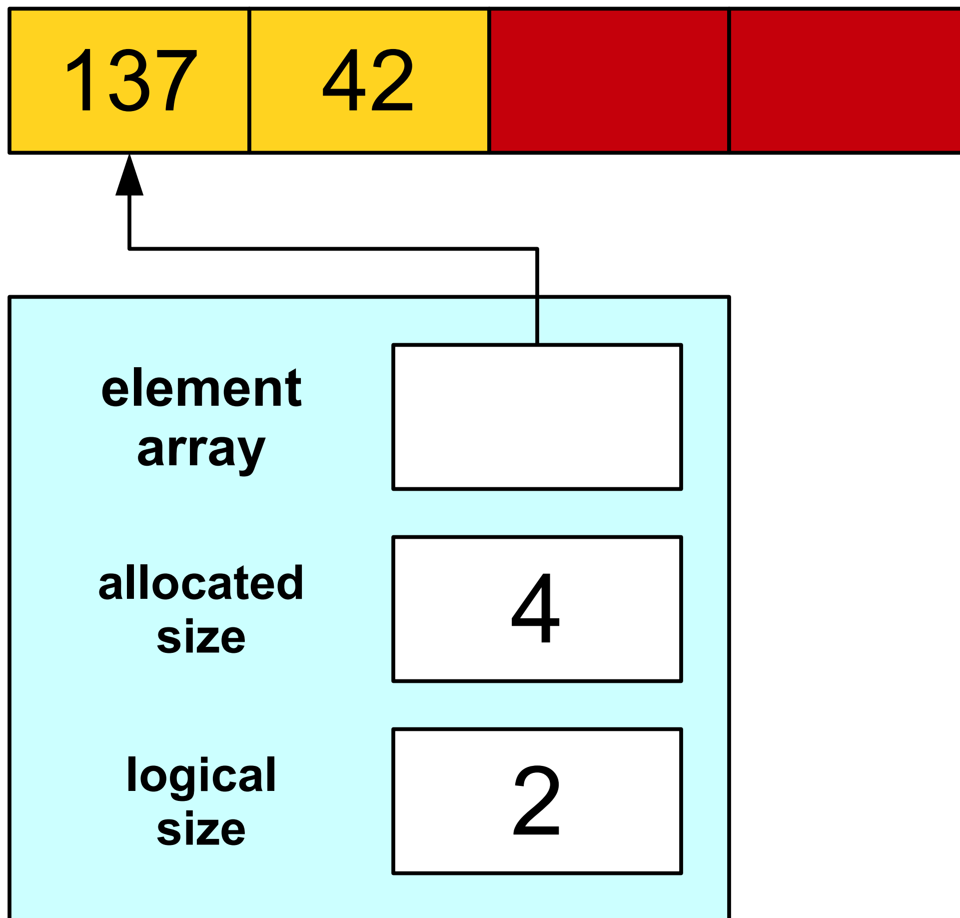

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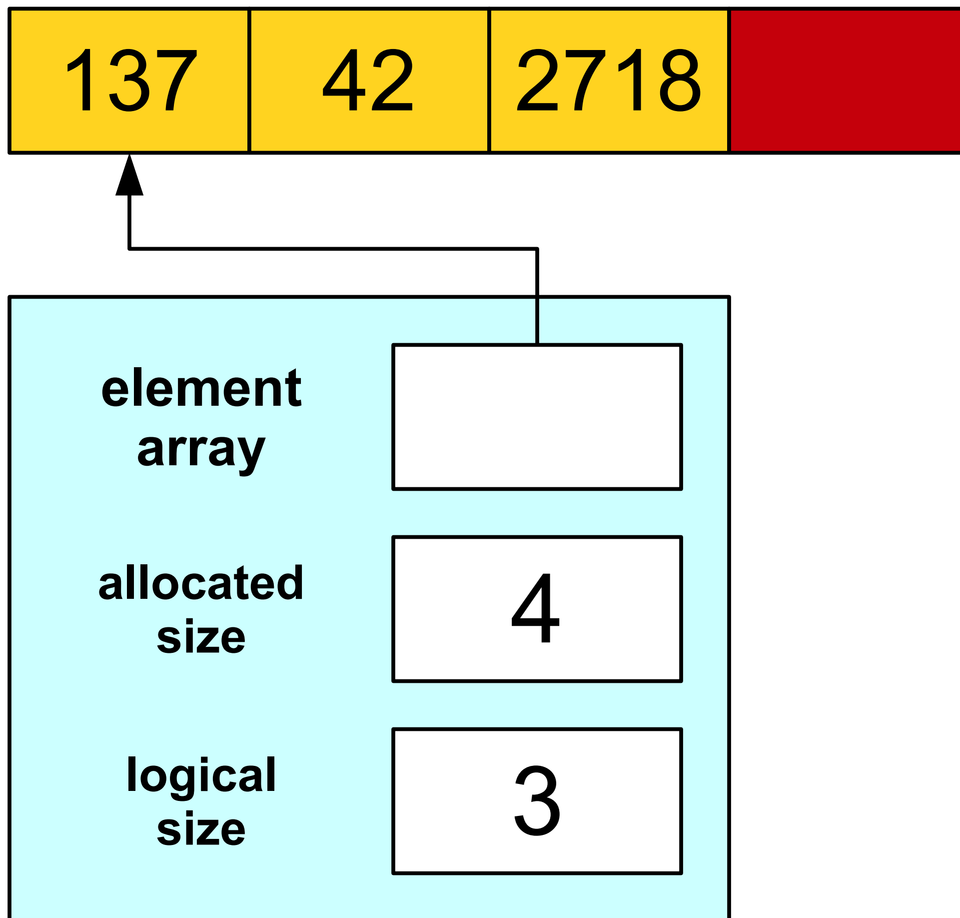

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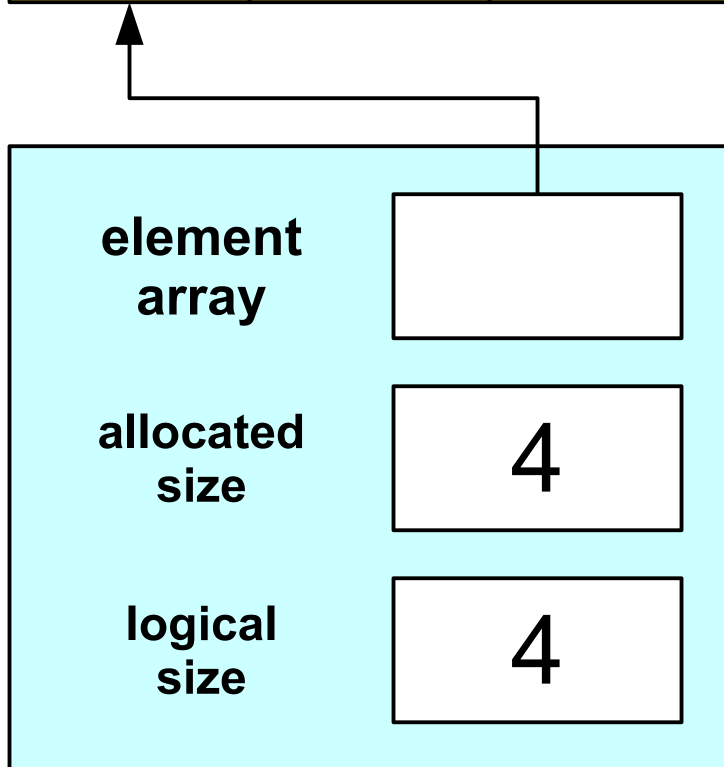

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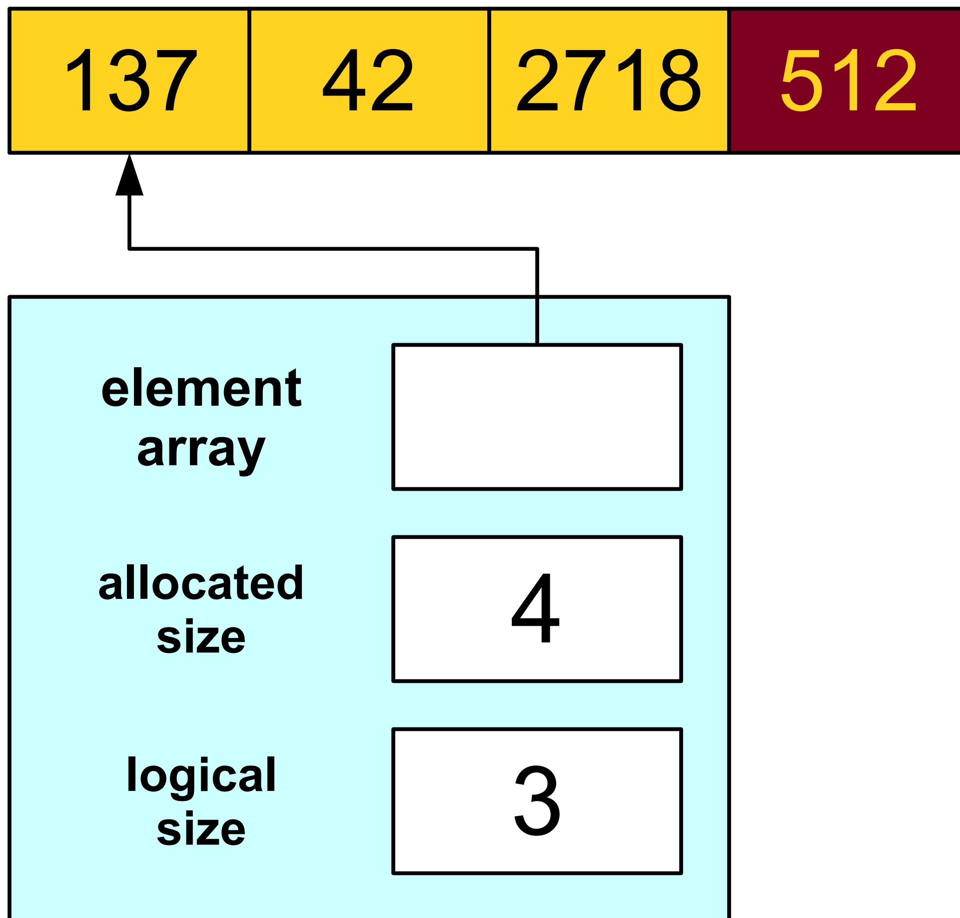

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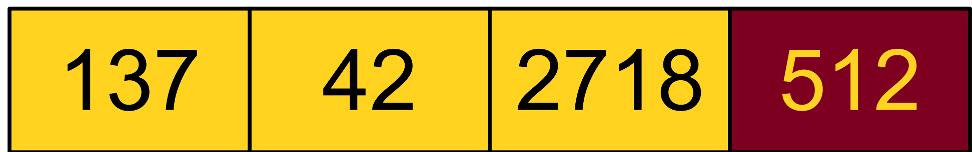

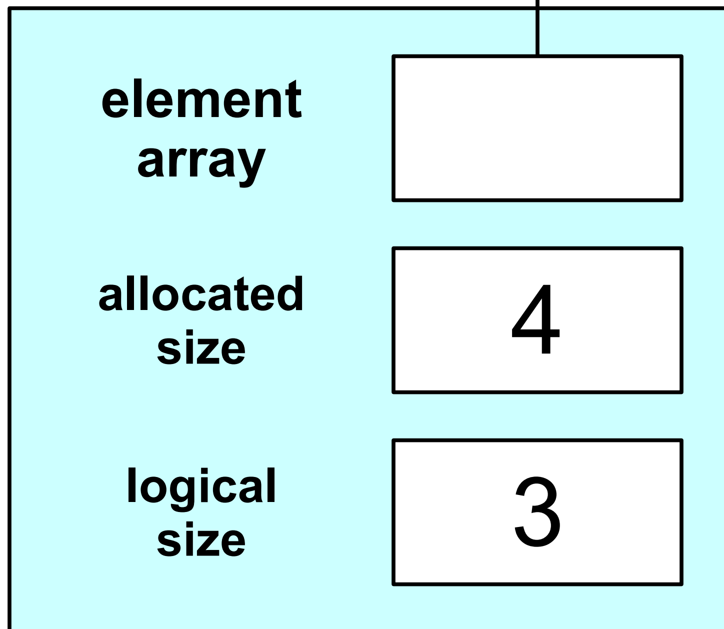

Arrays cannot grow or shrink, so this older value is still technically there in the array. We're just going to pretend it isn't.

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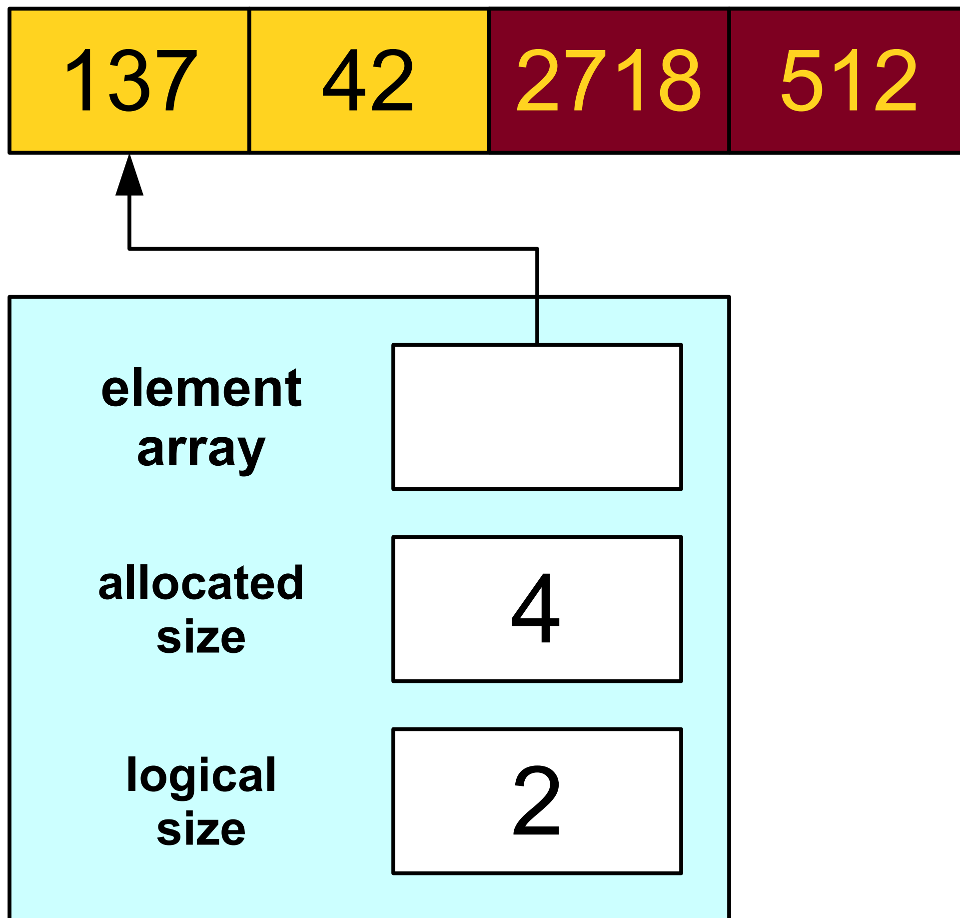

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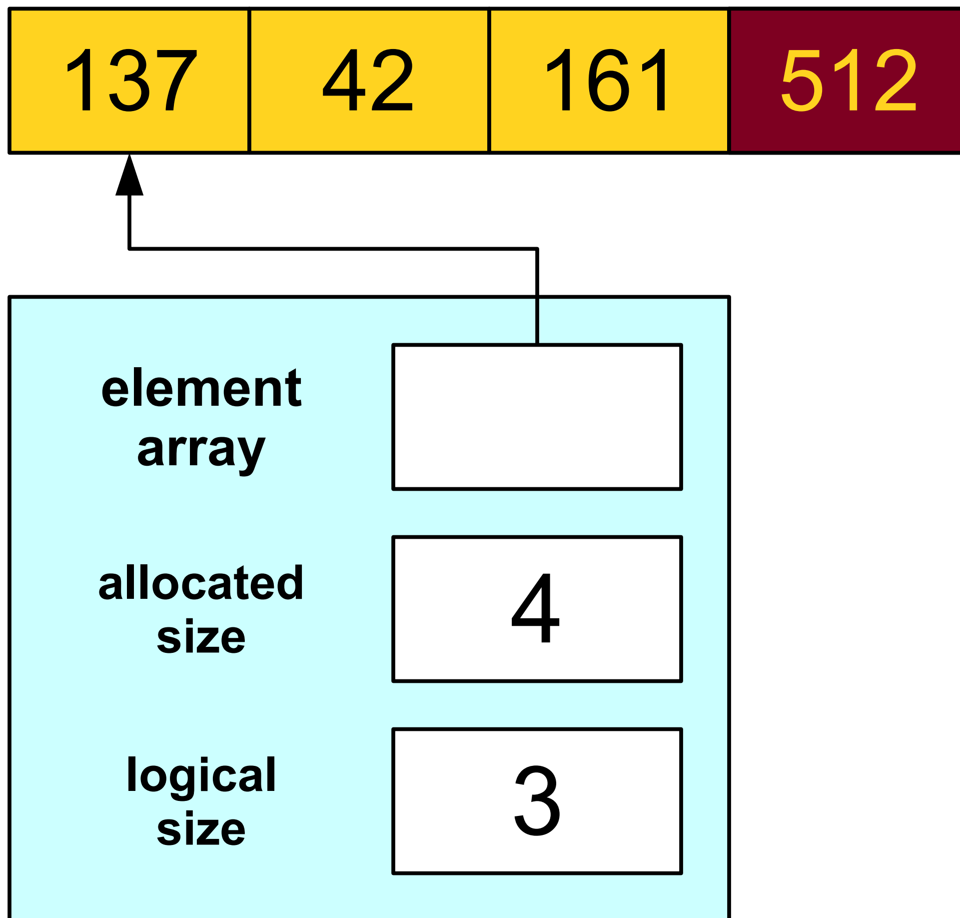

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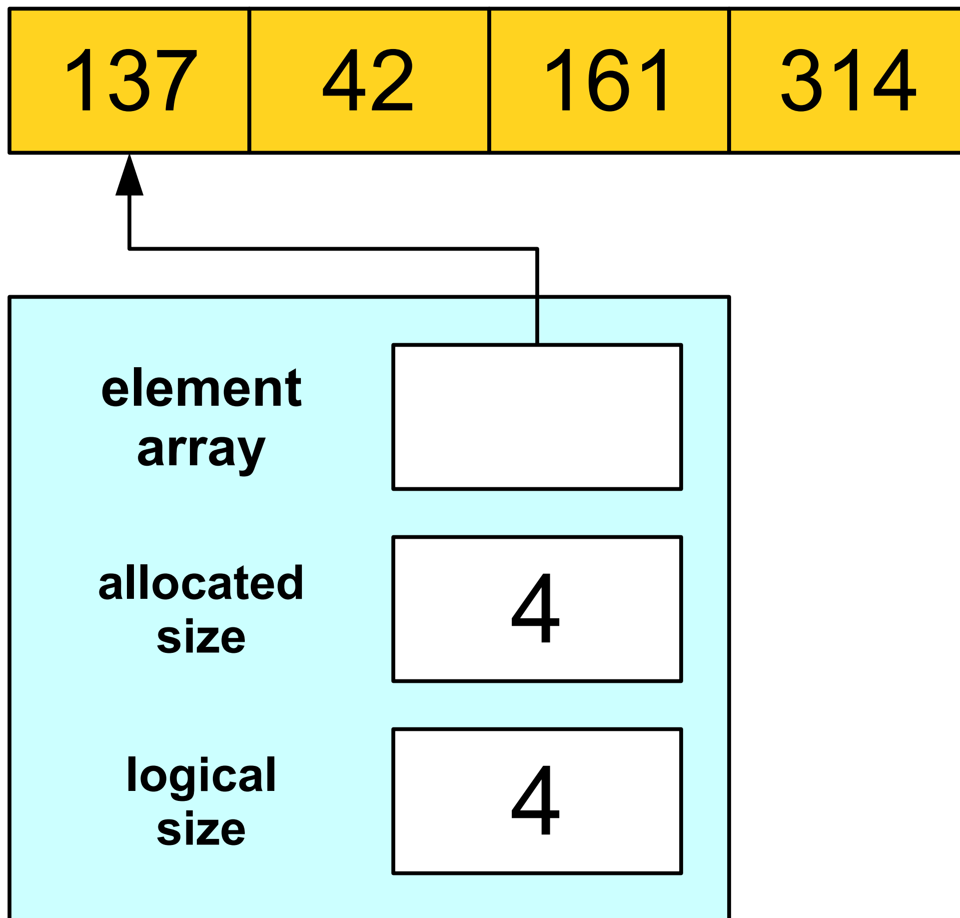
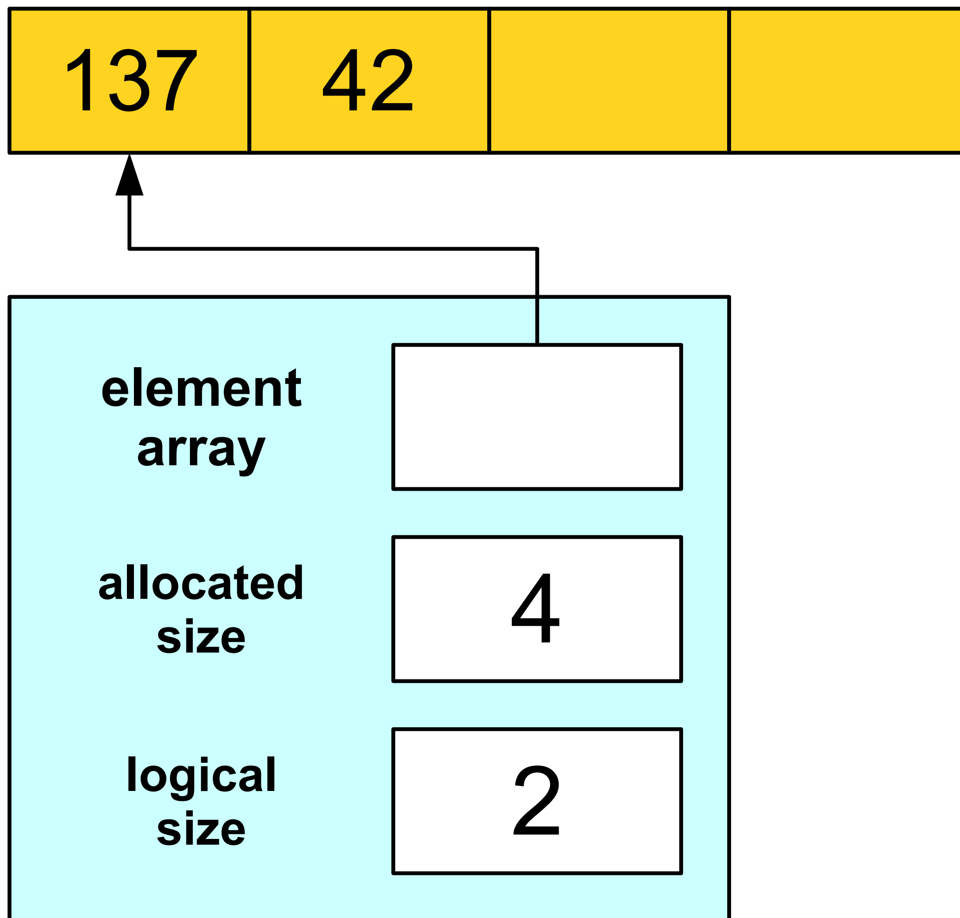


New Stuff!

Running out of Space

- Our current implementation very quickly runs out of space to store elements.
- What should we do when this happens?

An Initial Idea



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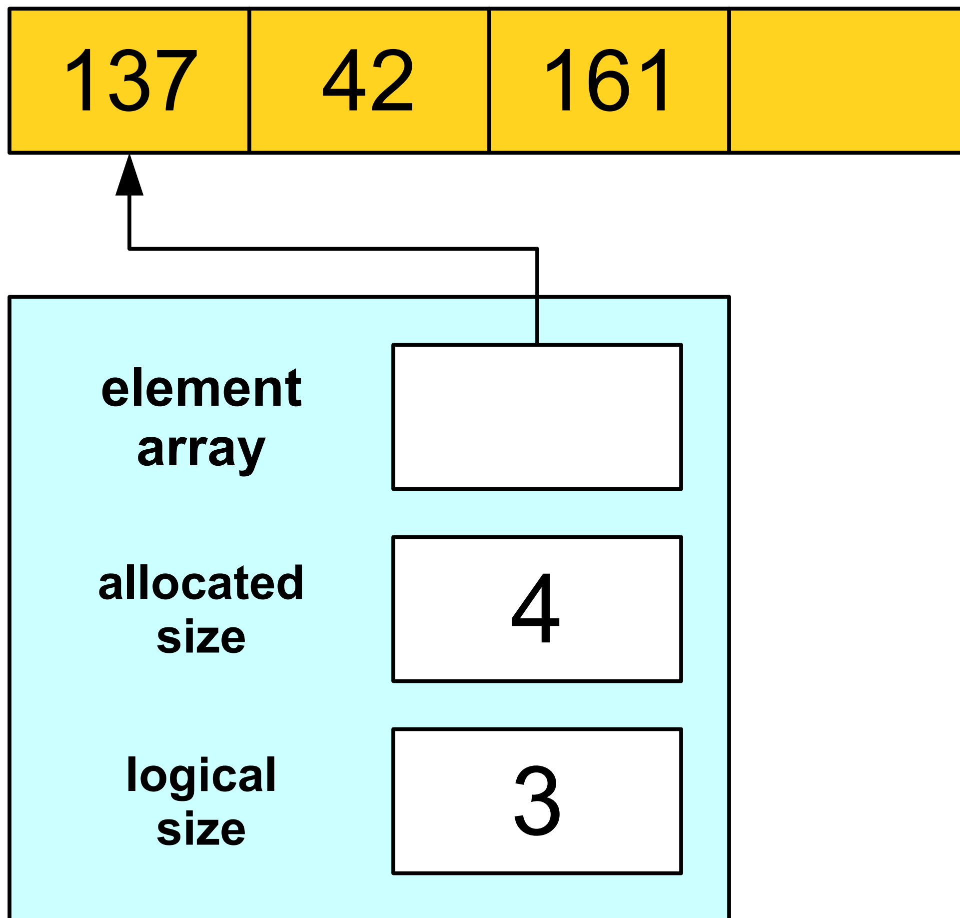

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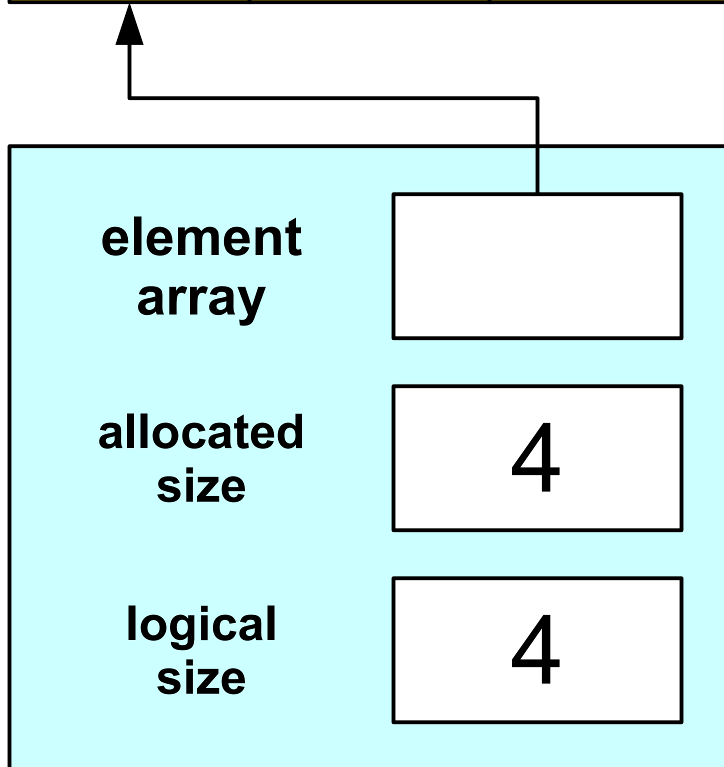

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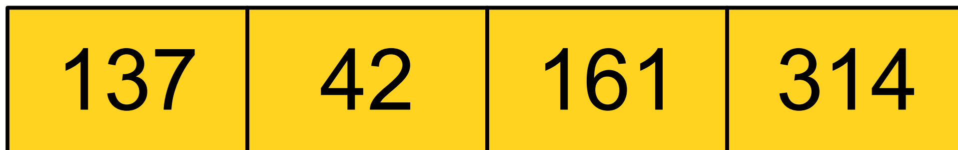

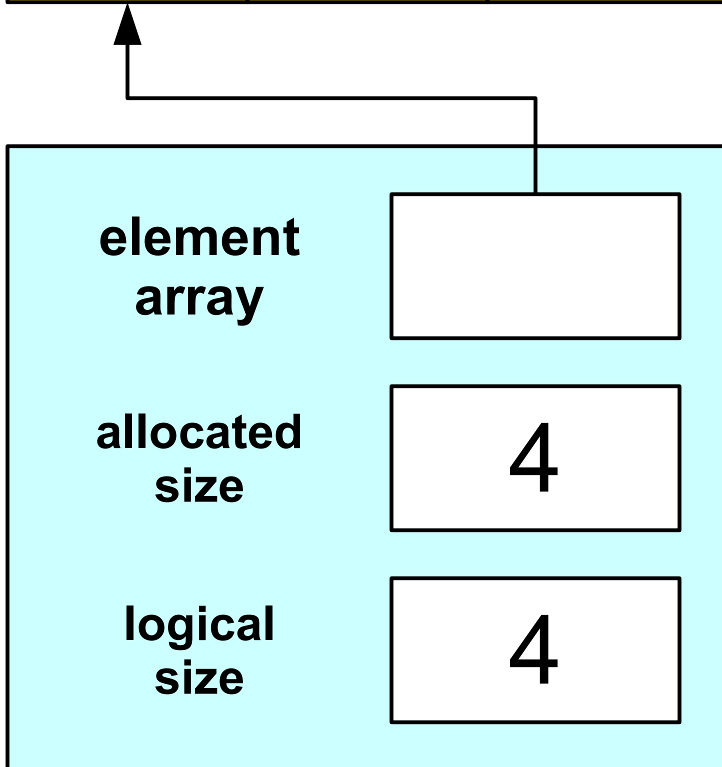

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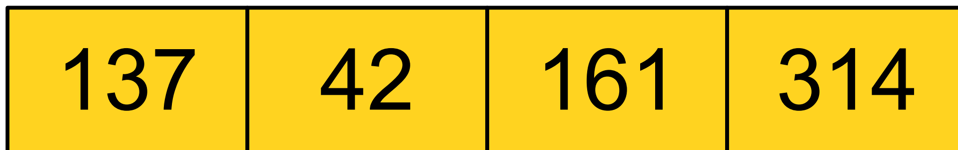

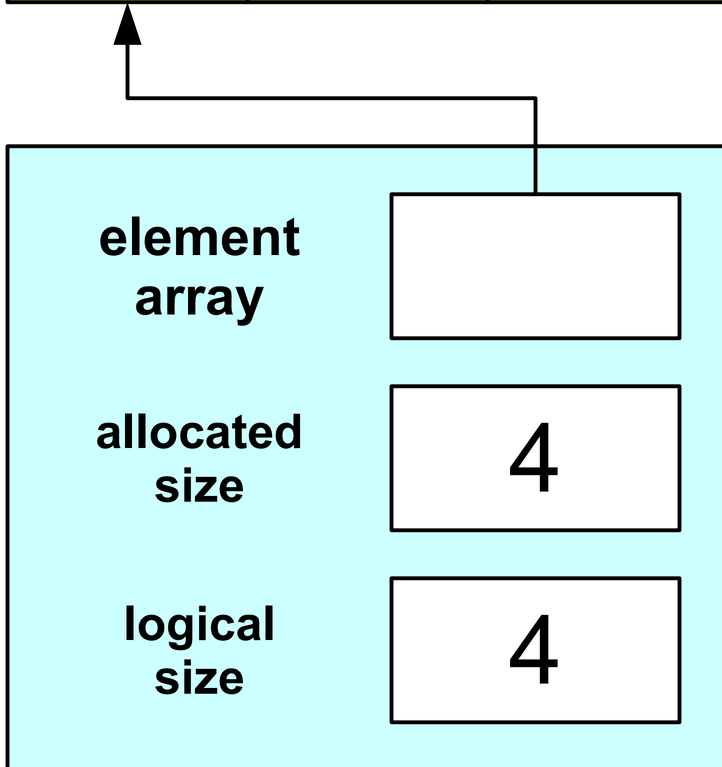

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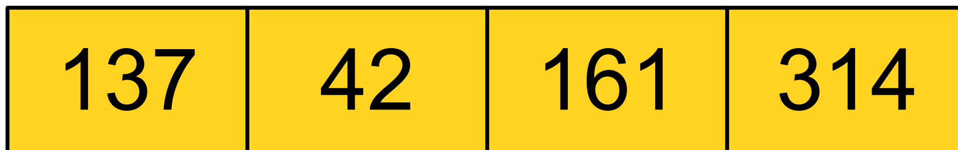

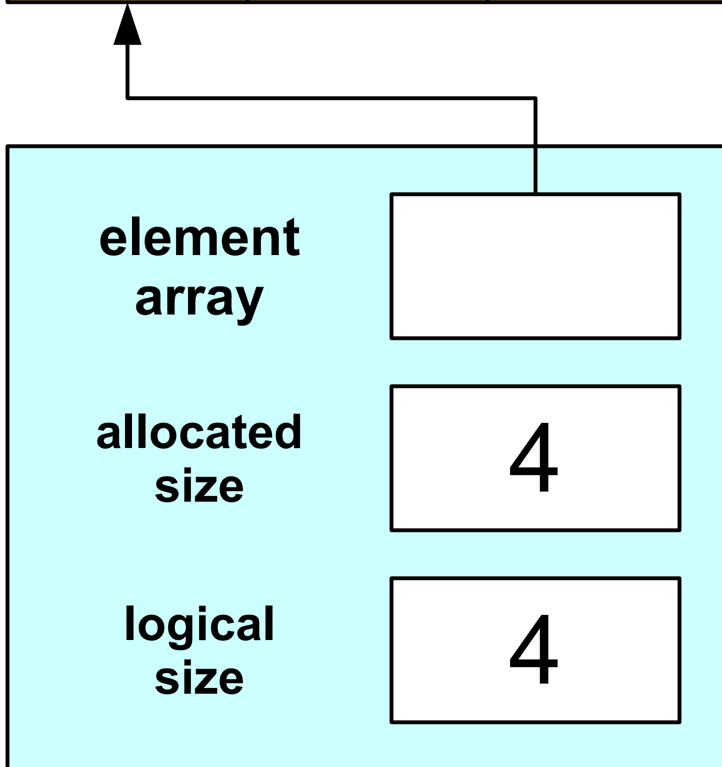

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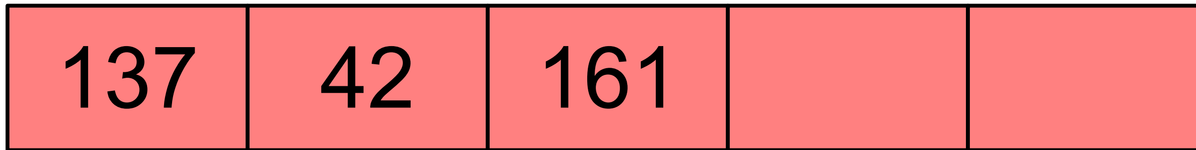

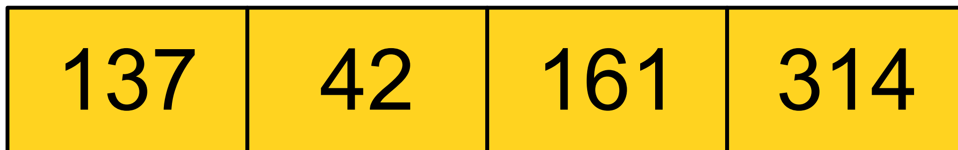

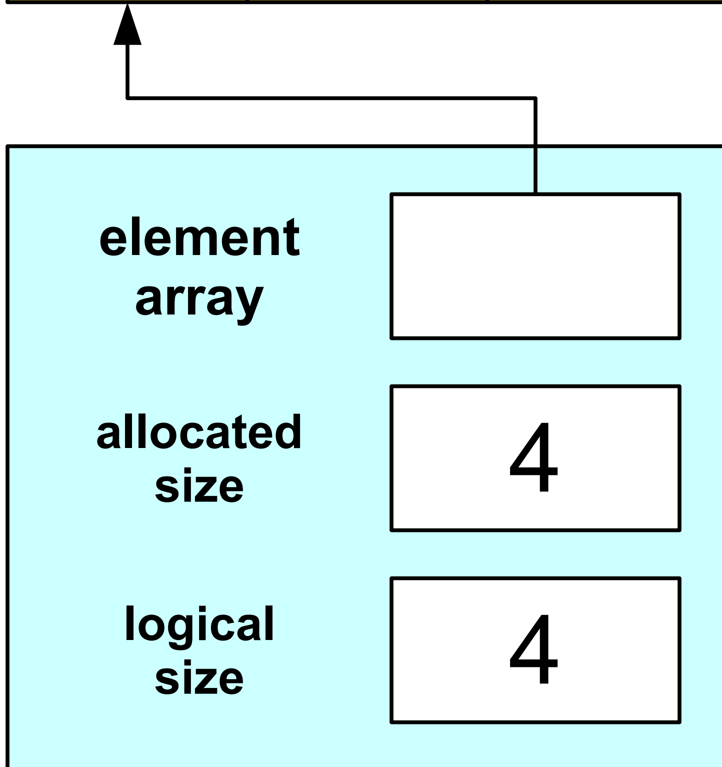

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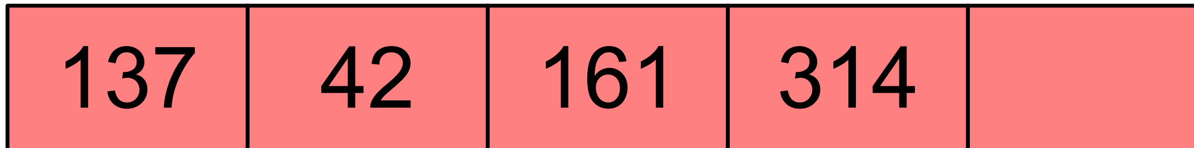

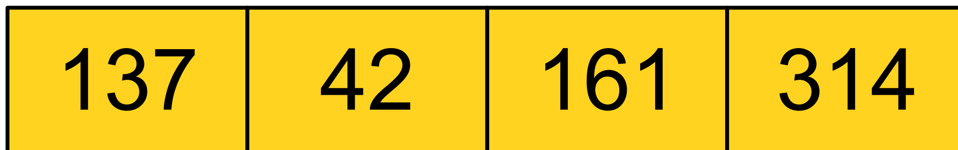

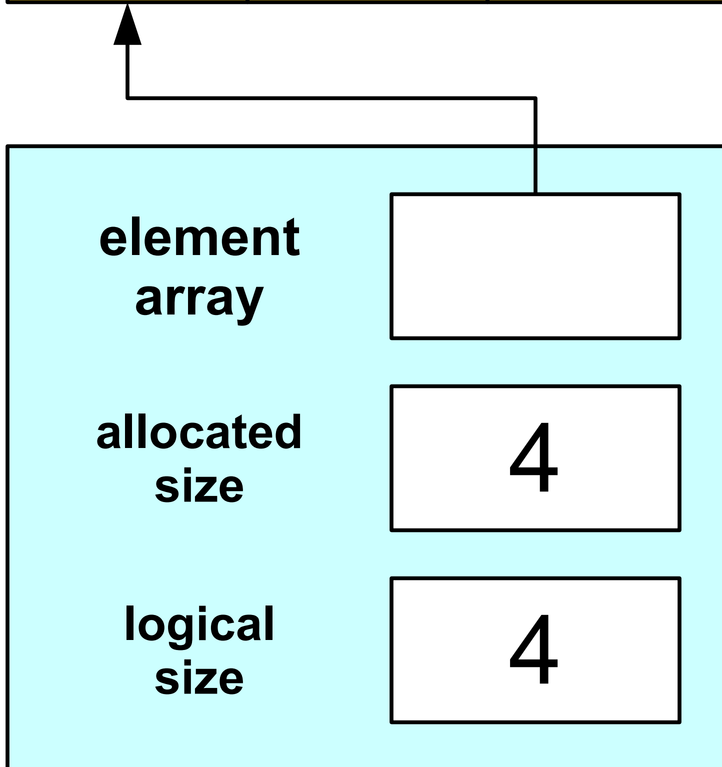

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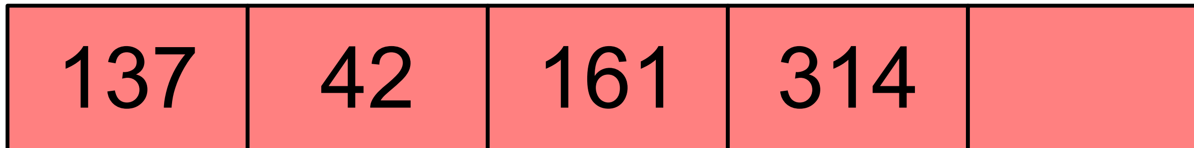

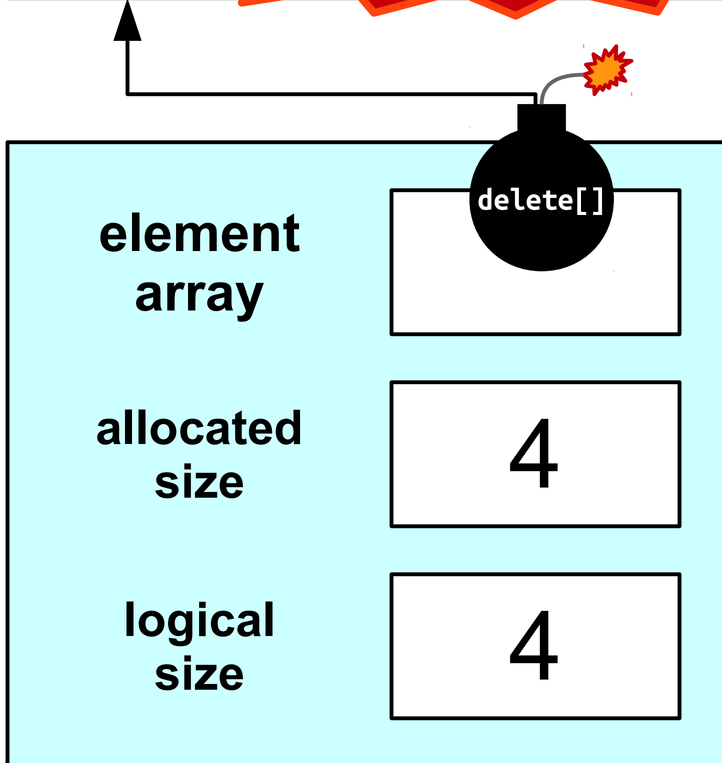

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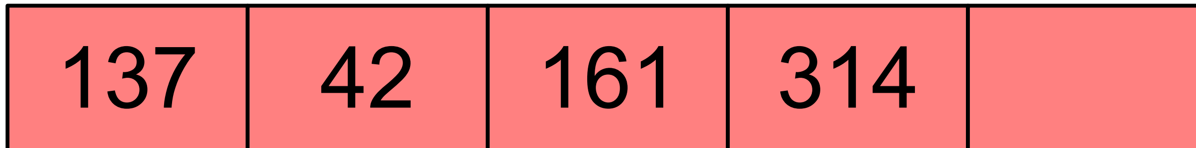

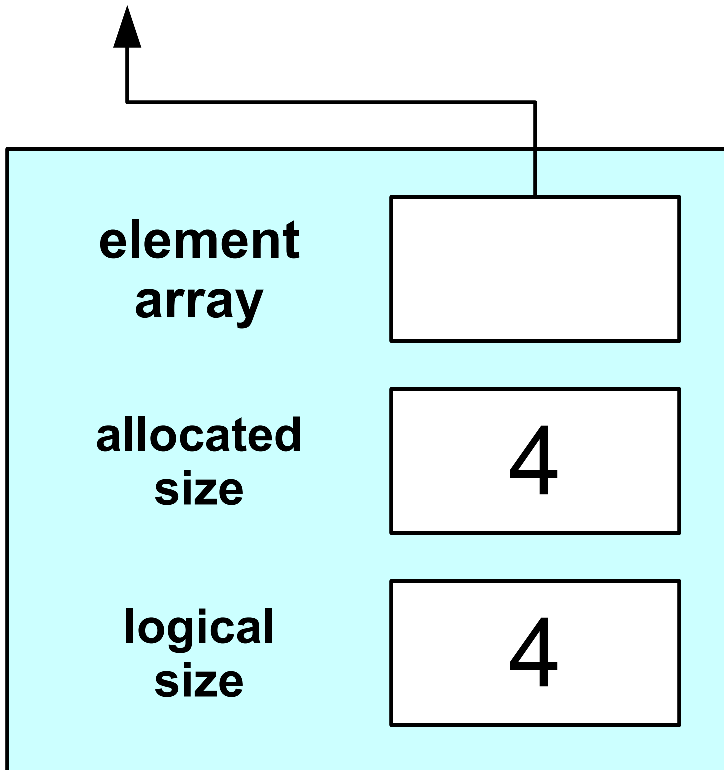

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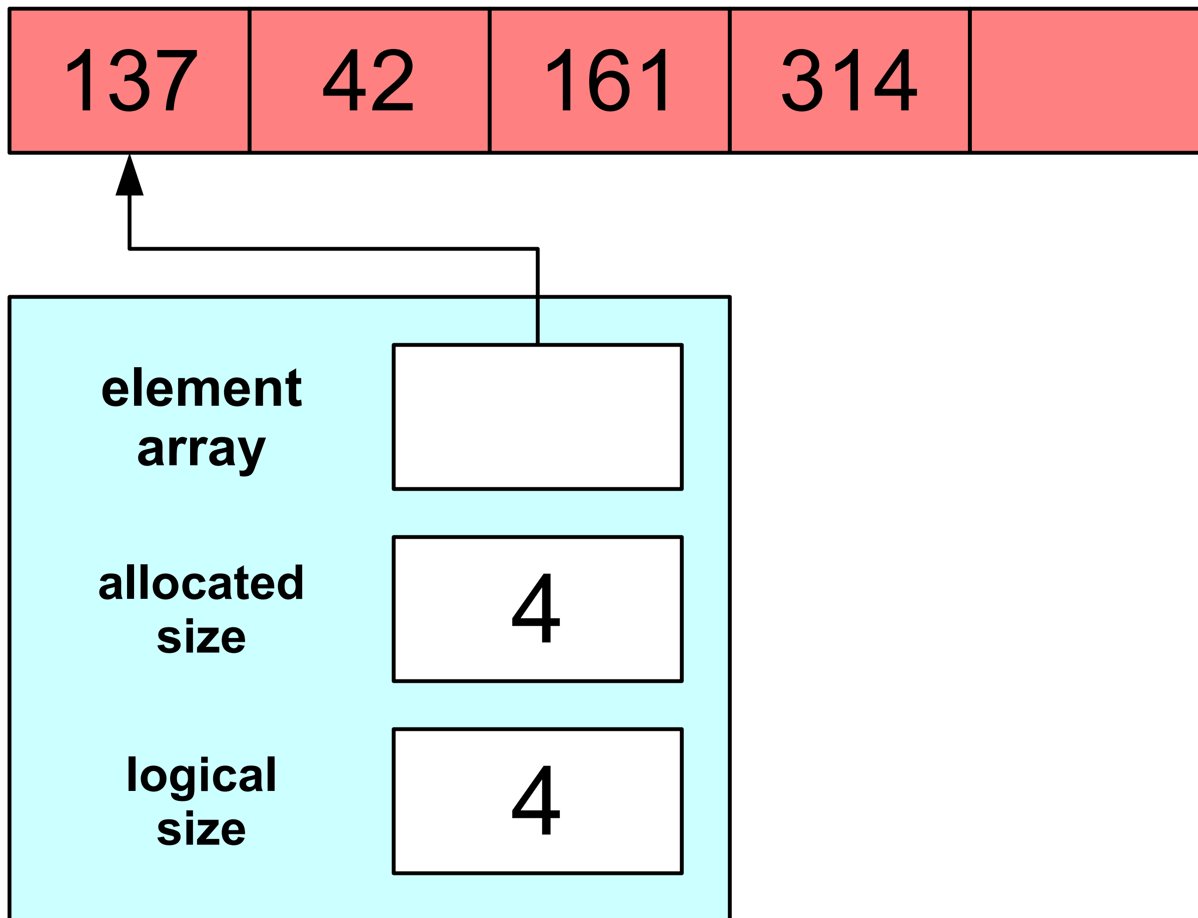

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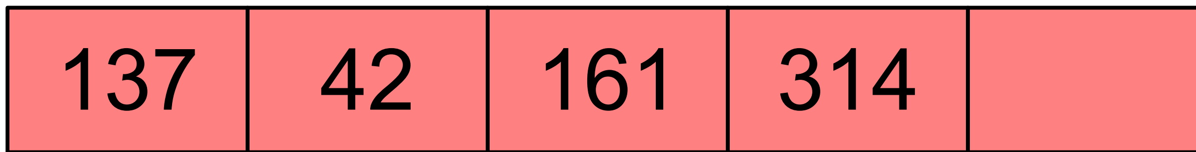

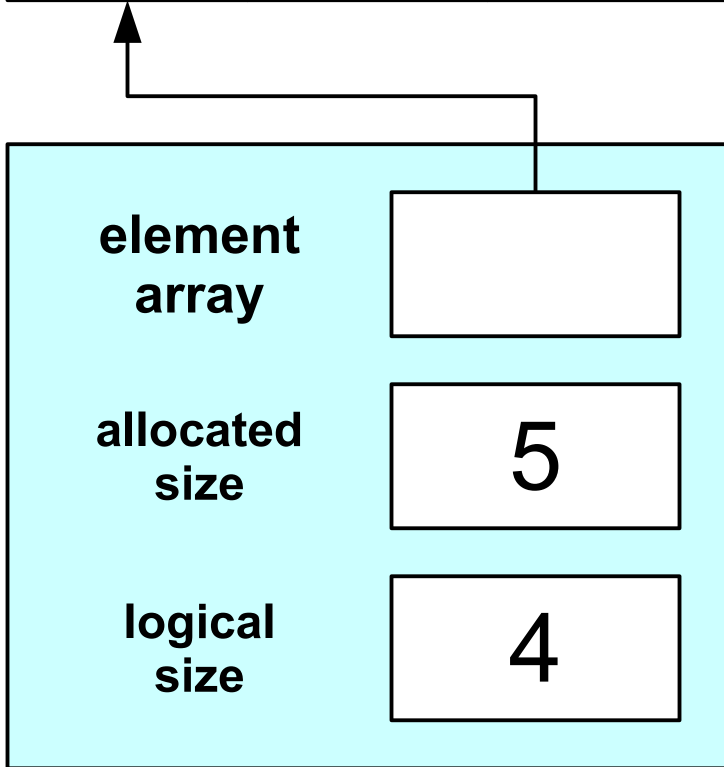

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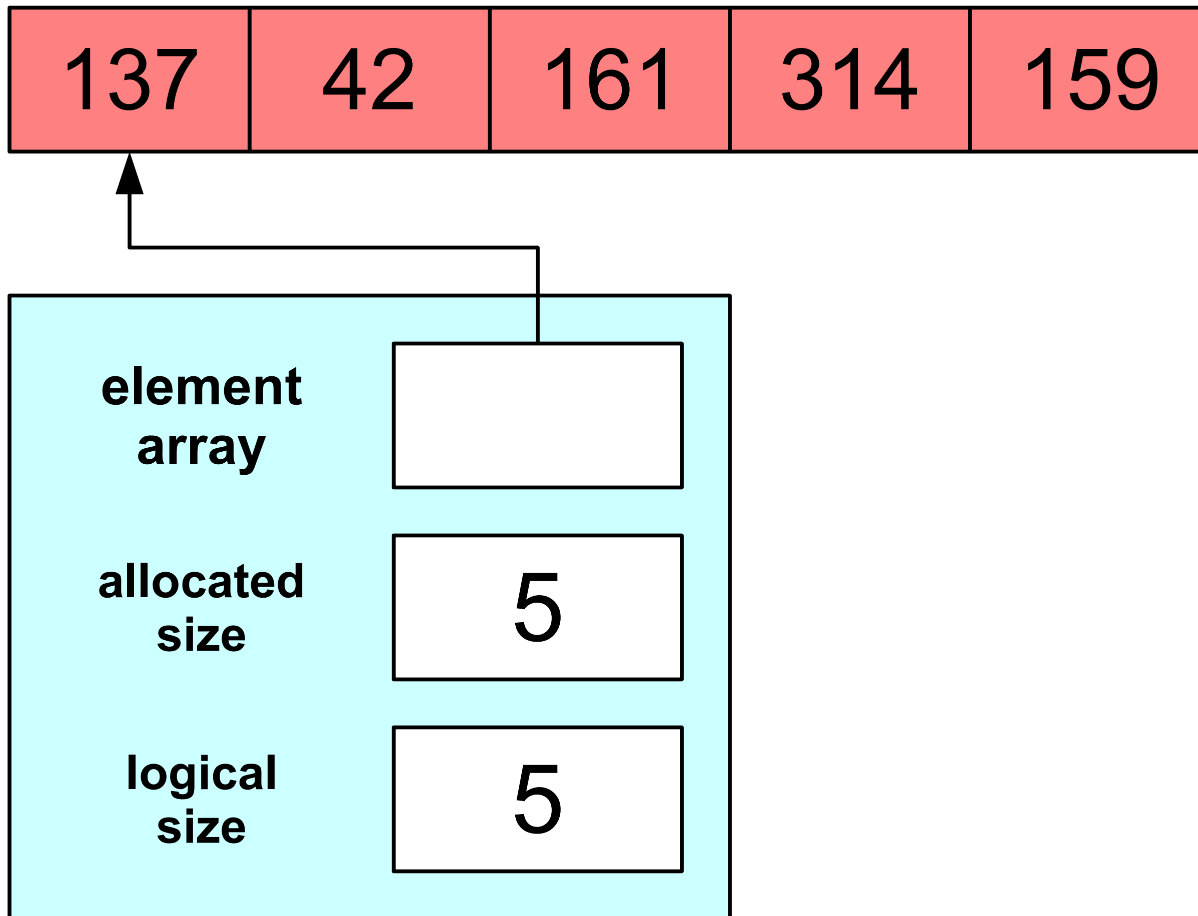

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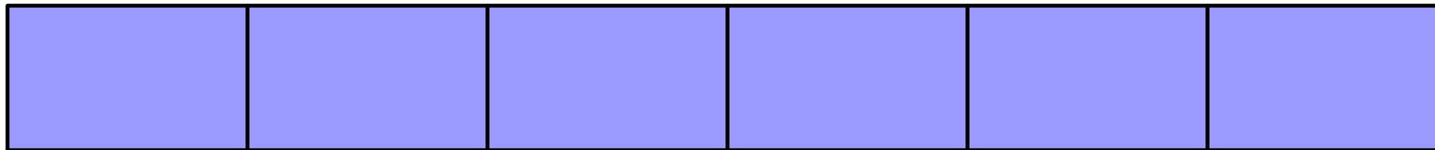

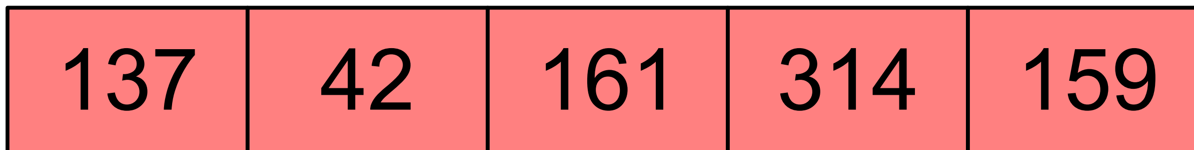

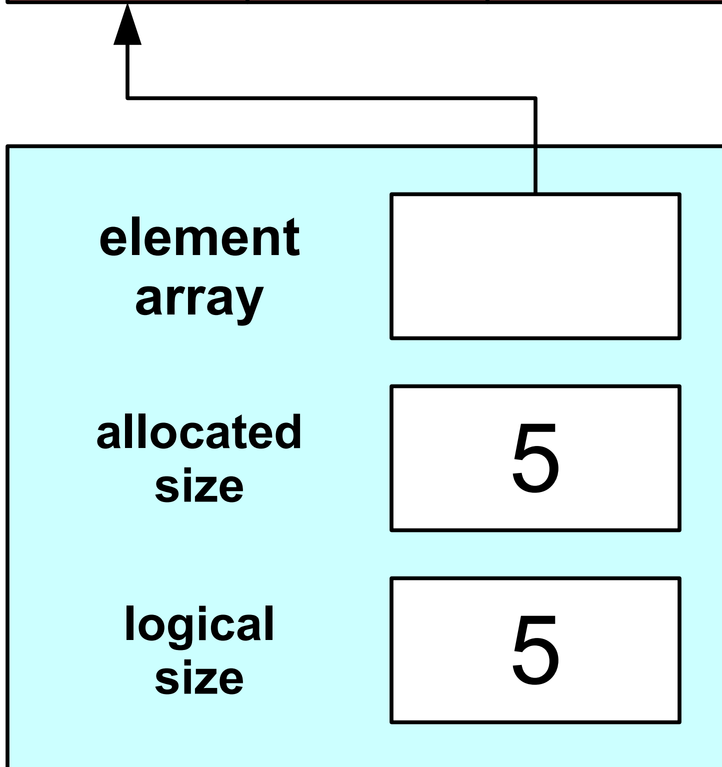

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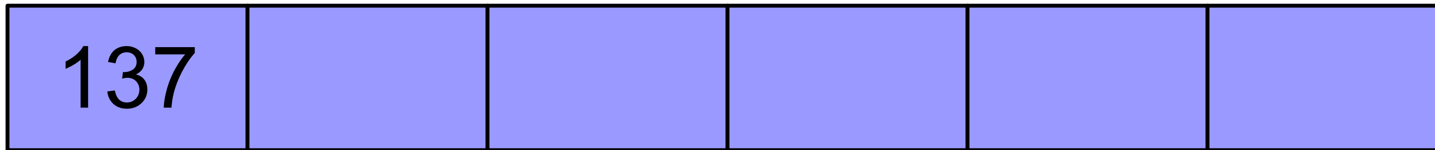

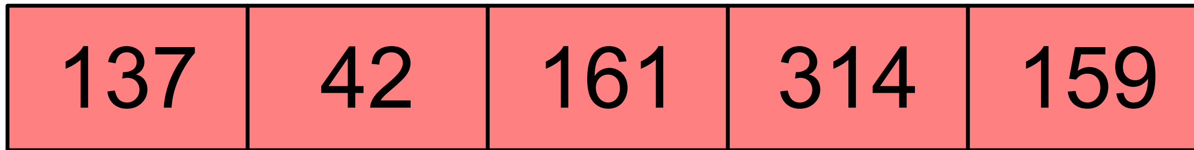

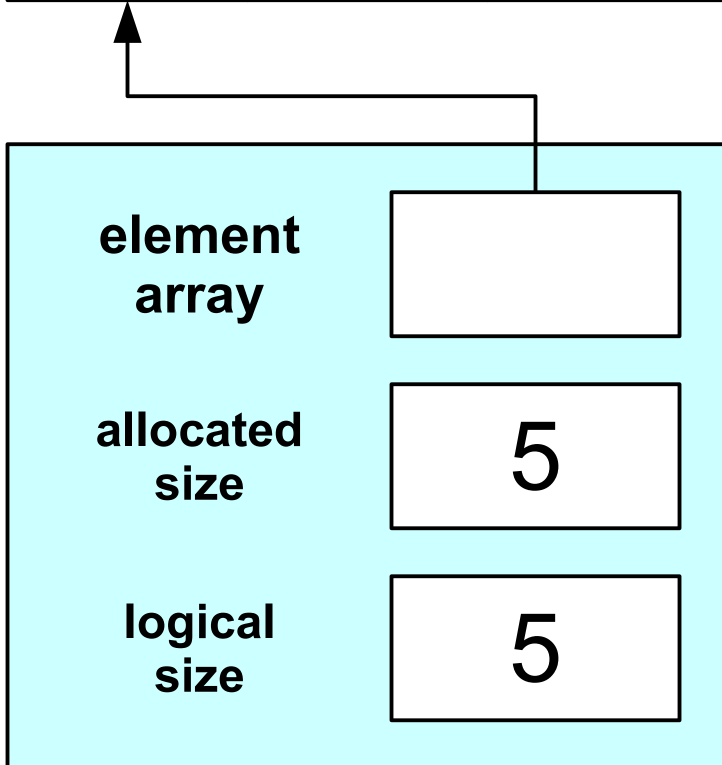

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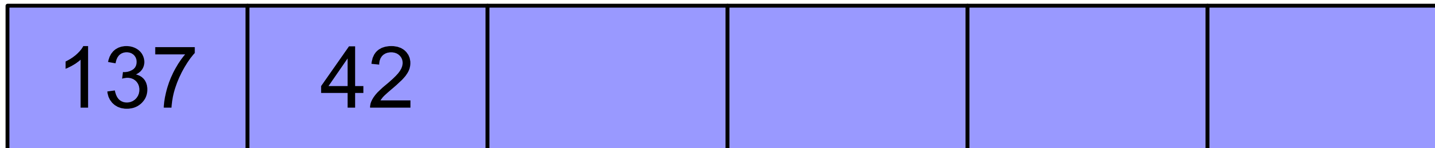

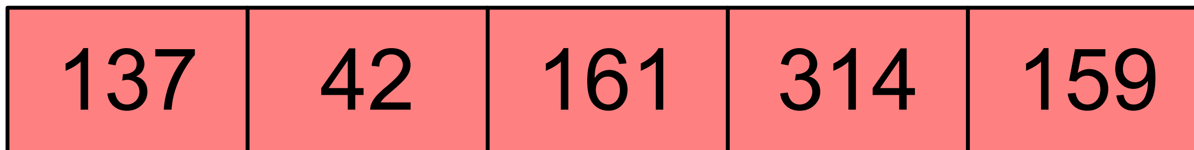

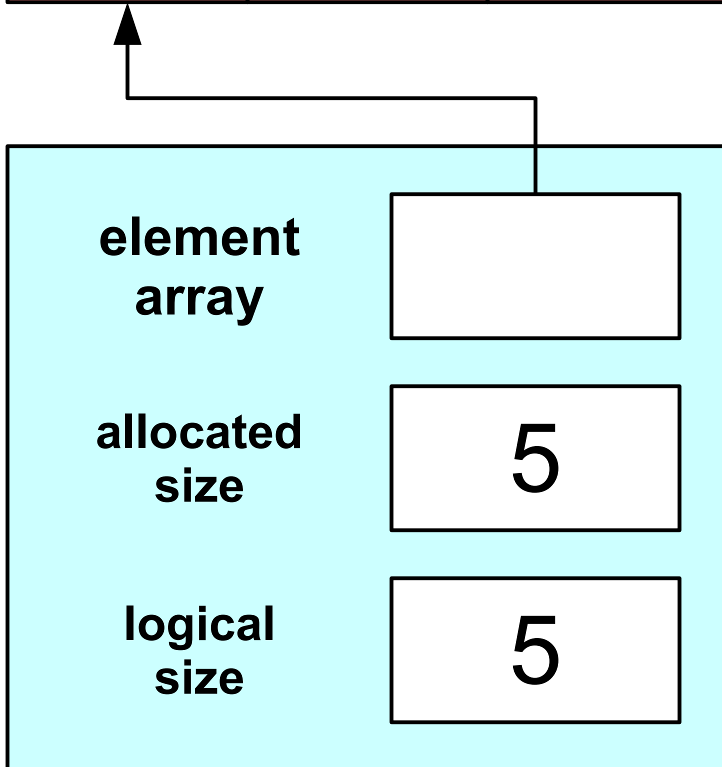

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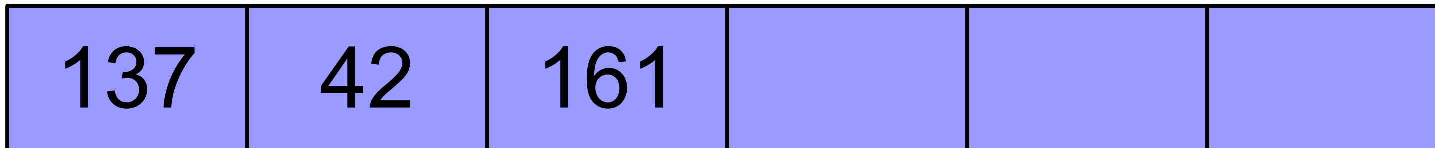

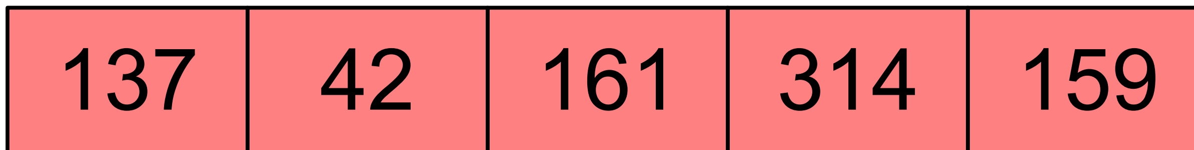

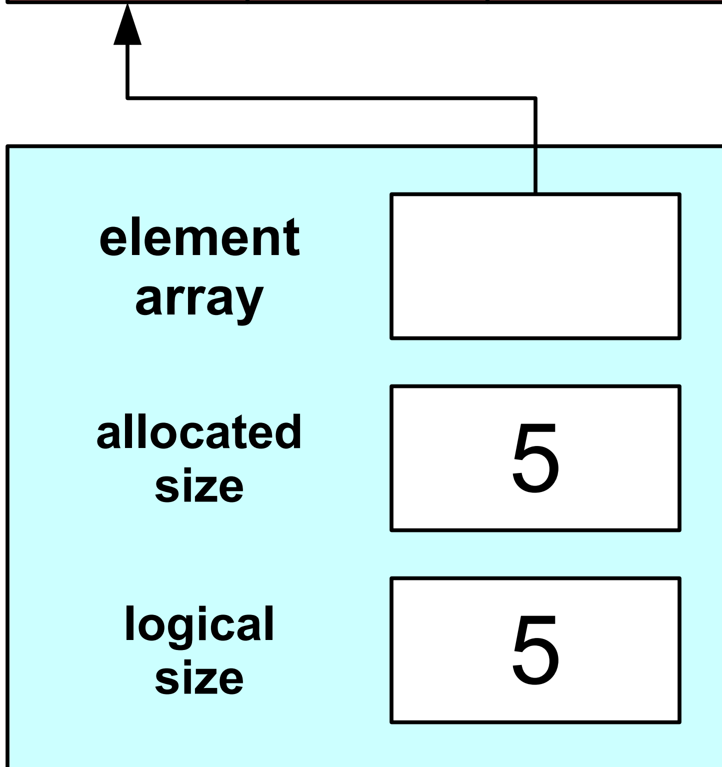

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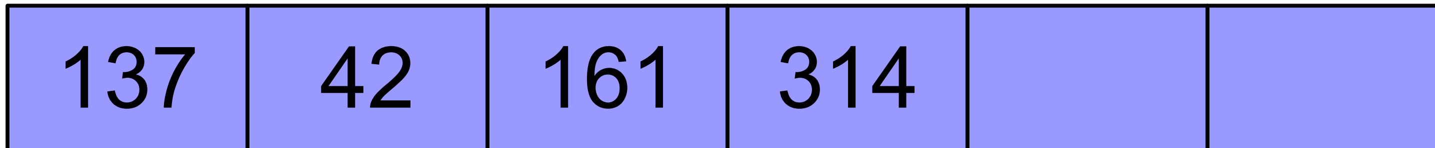

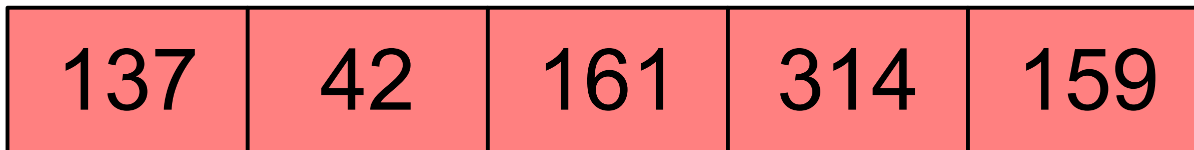

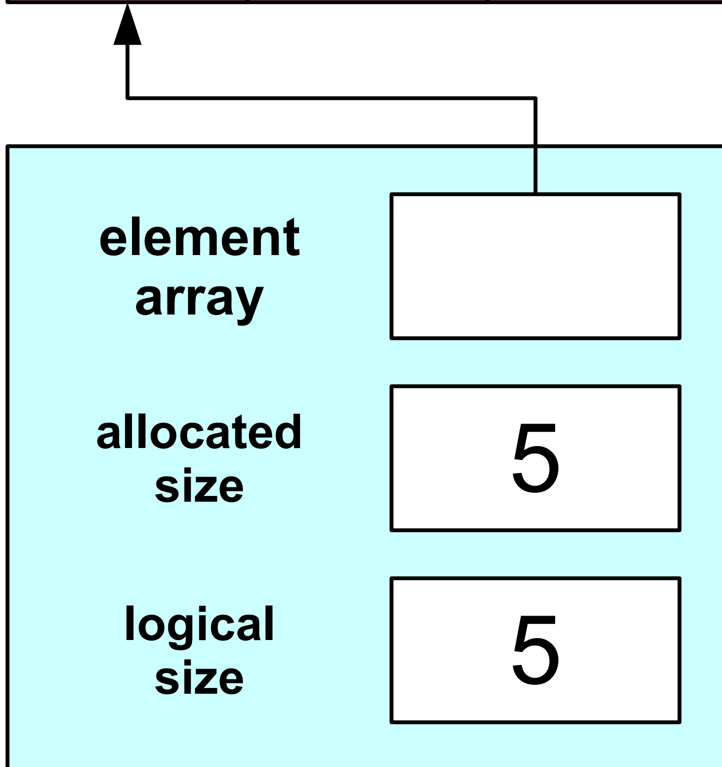

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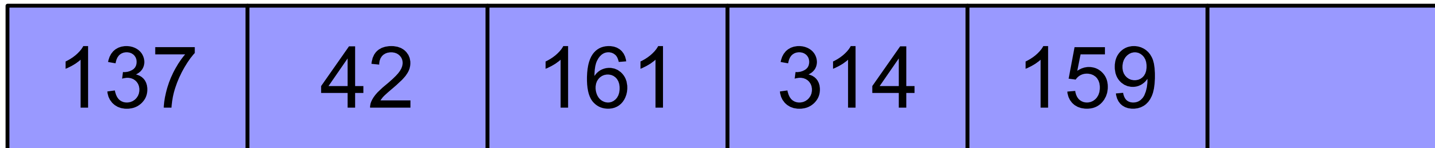

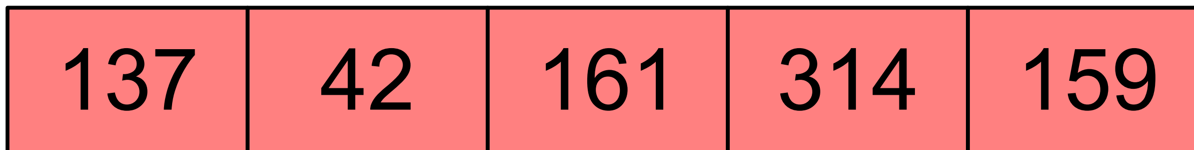

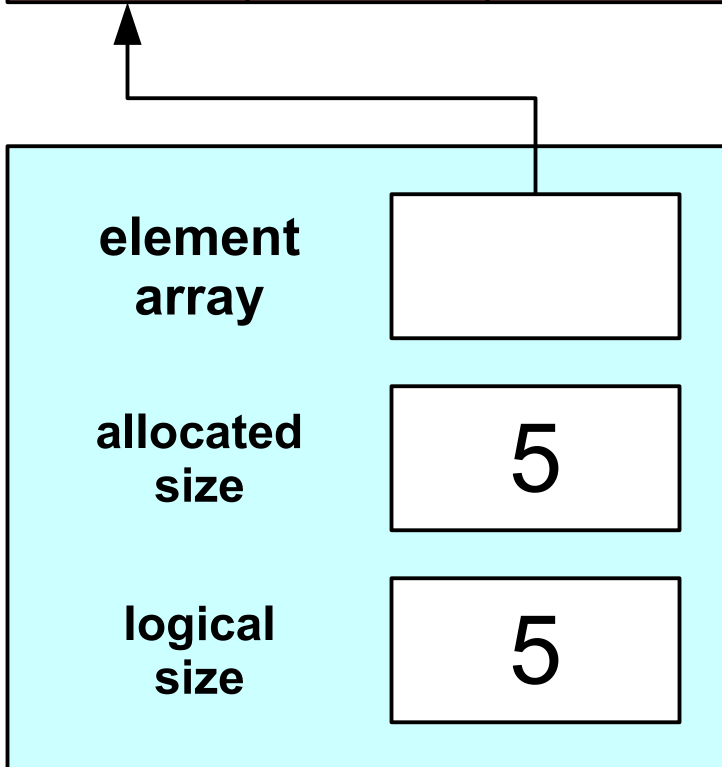

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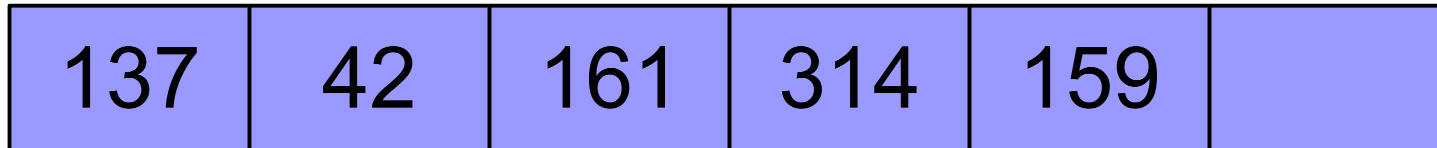

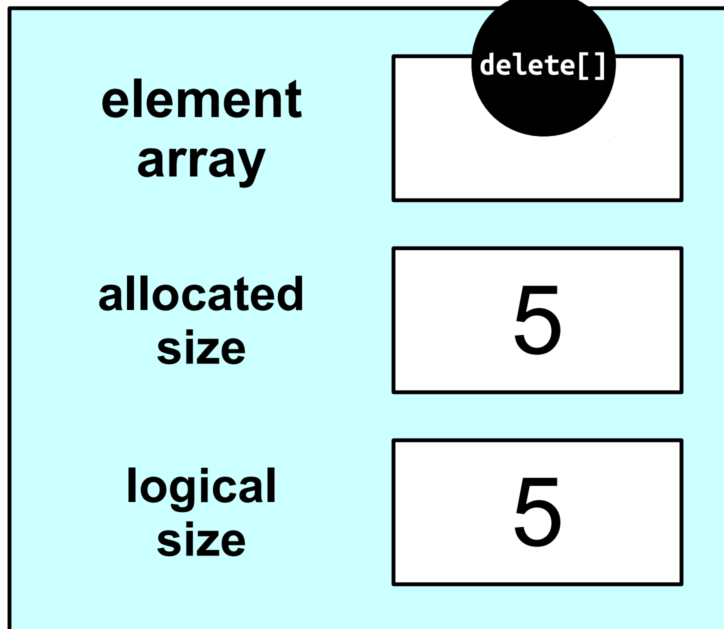

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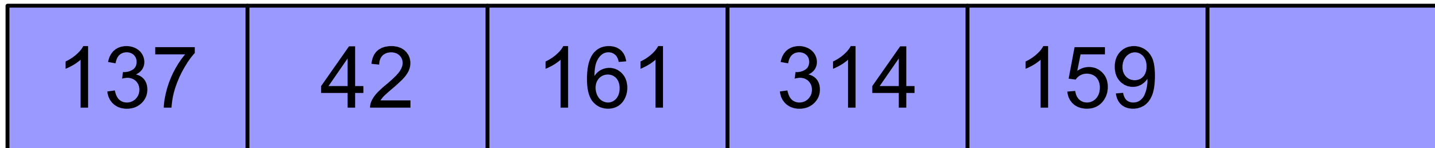

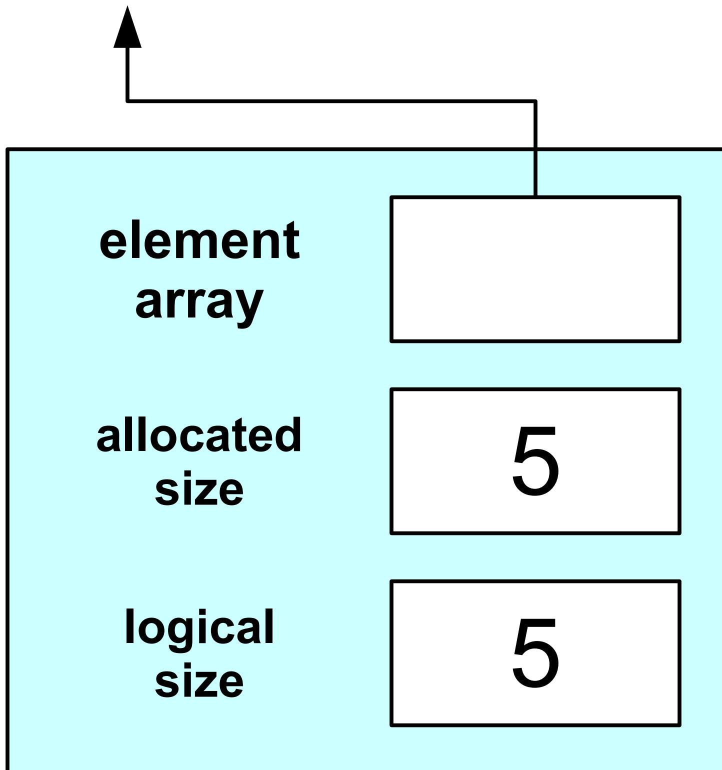

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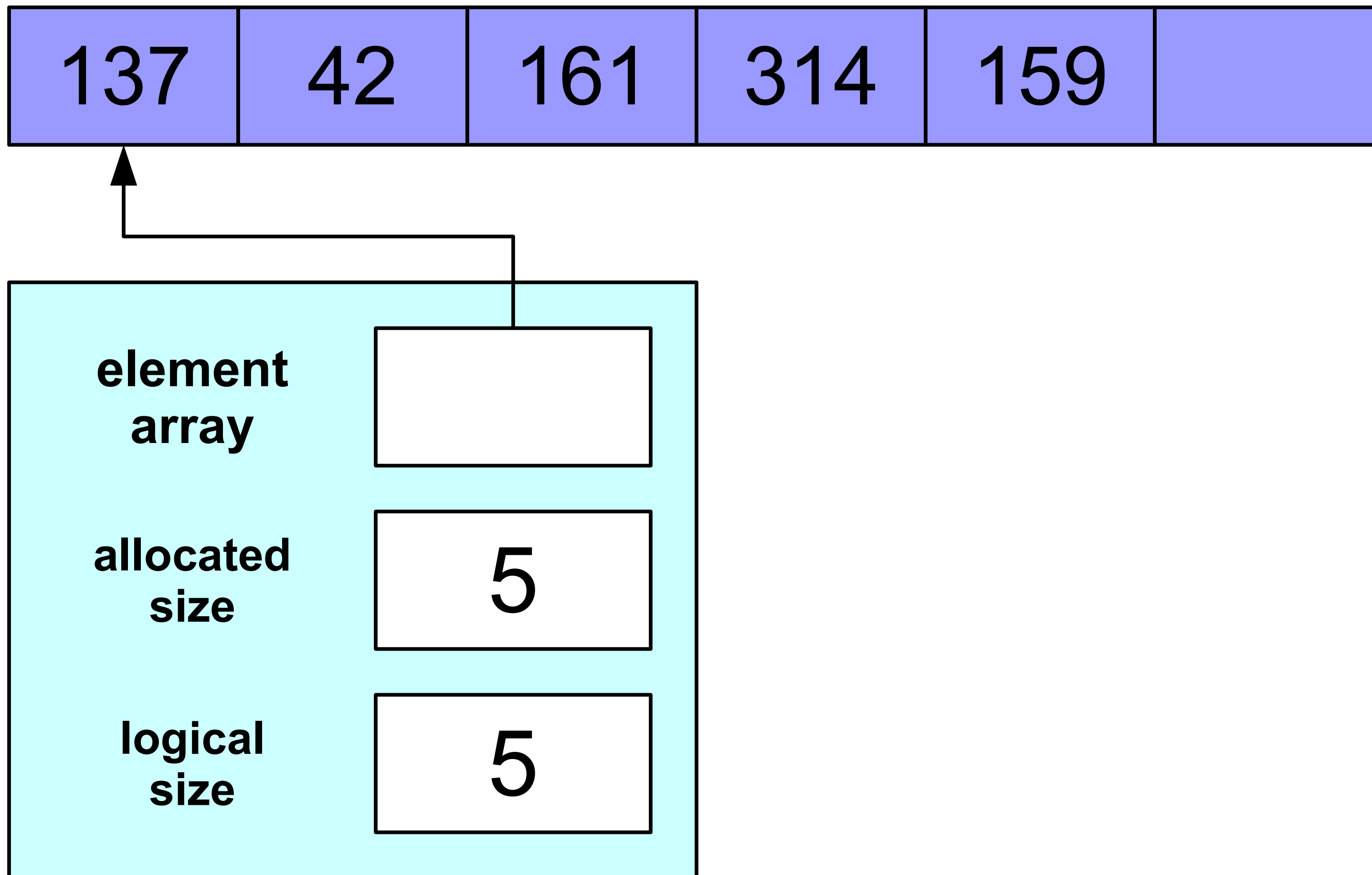

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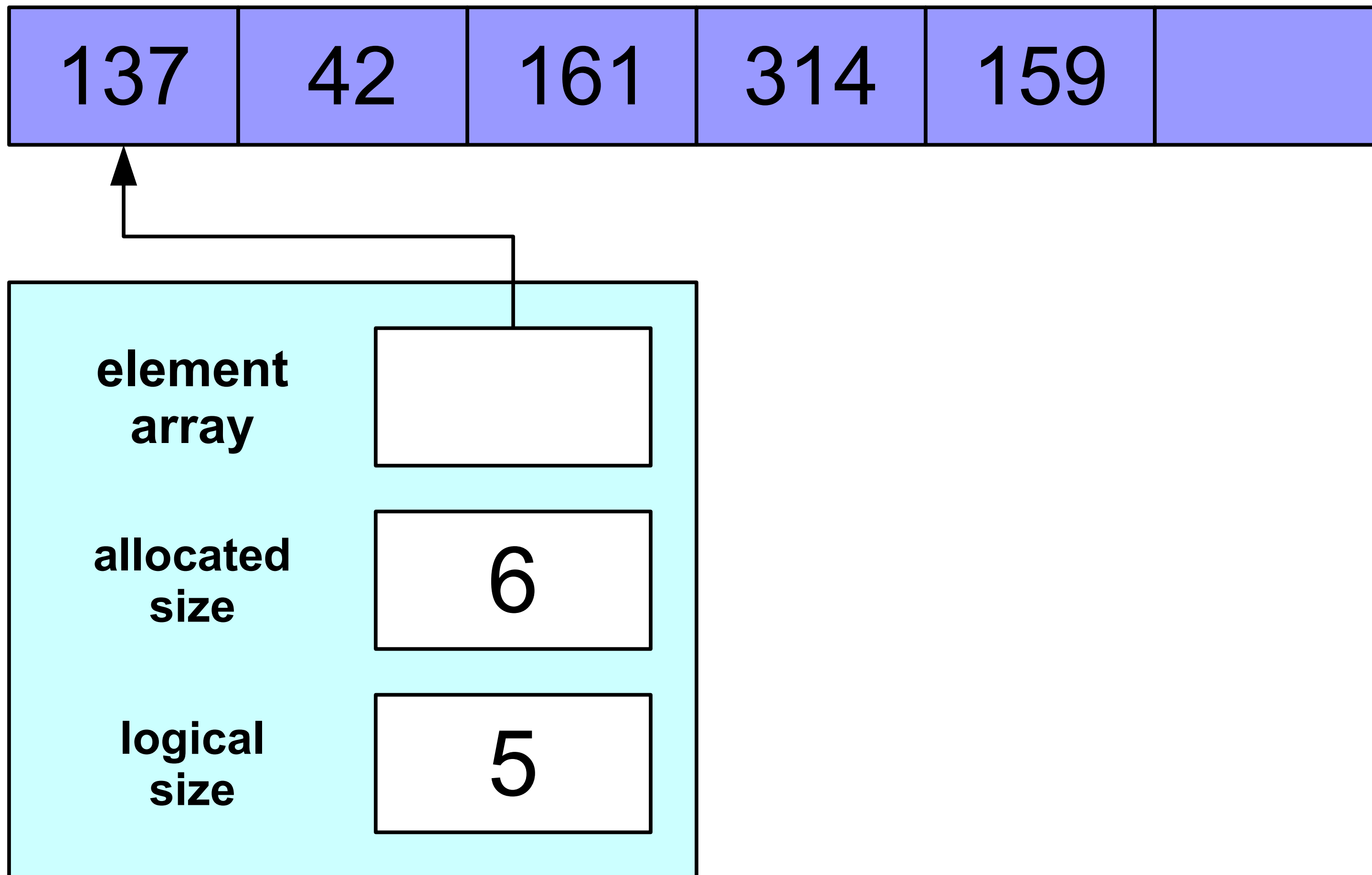

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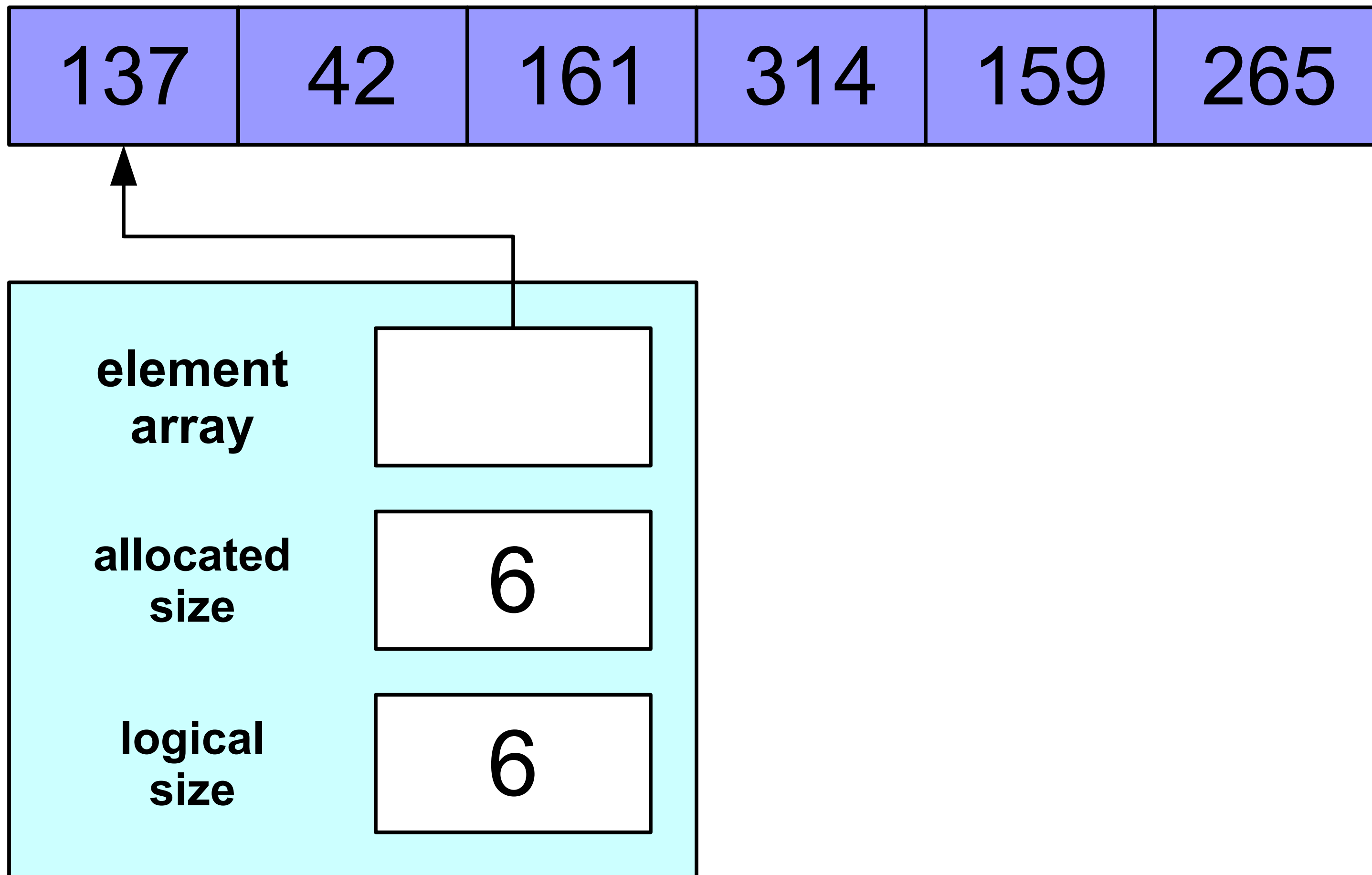

Ready… set… grow!

```cpp
class OurStack {
public:
    OurStack();
    ~OurStack();

    void push(int value);
    int  pop();
    int  peek() const;

    int  size() const;
    bool isEmpty() const;

private:


    int* elems;
    int  allocatedSize;
    int  logicalSize;
};
```

```
class OurStack {
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    void grow();

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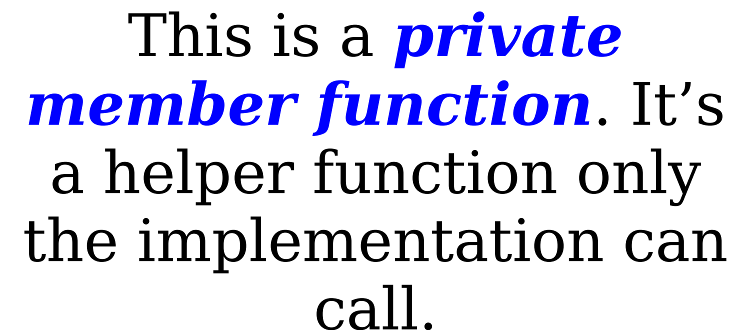

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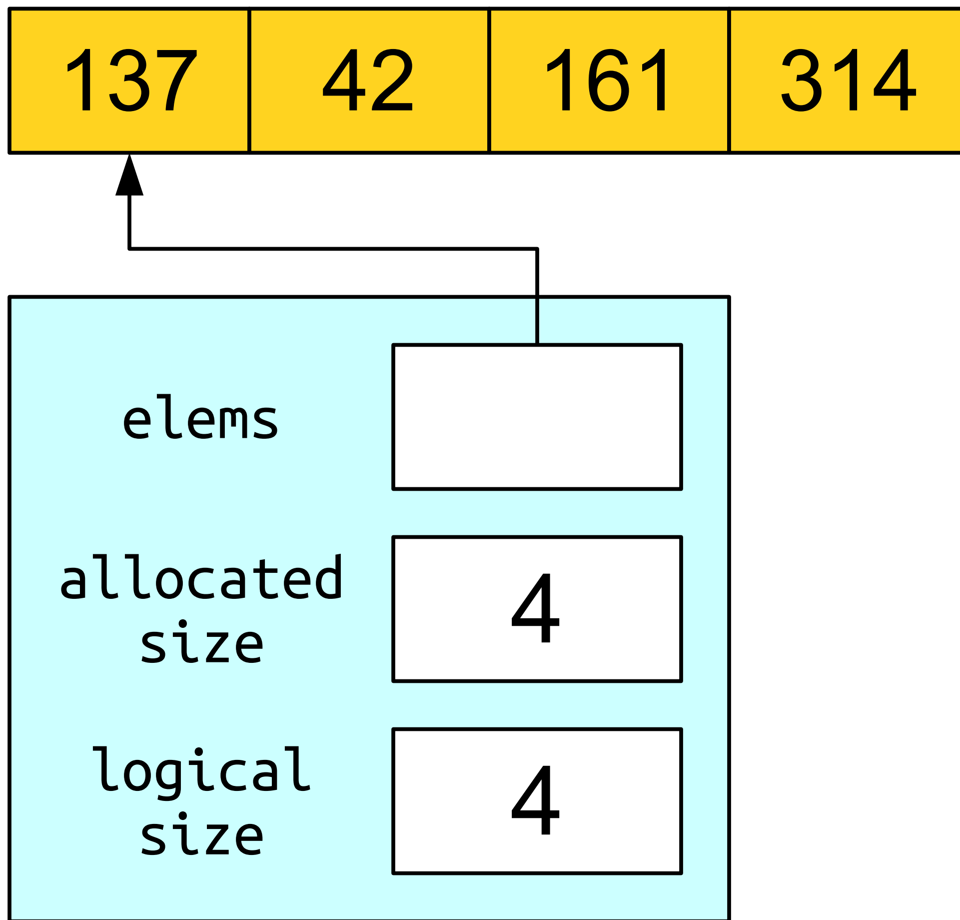

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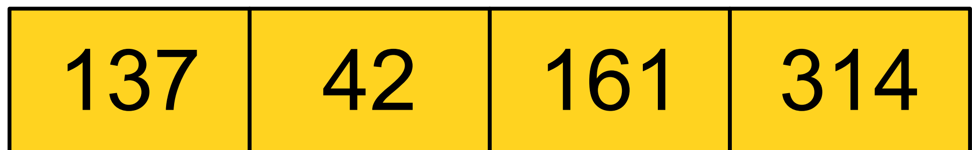

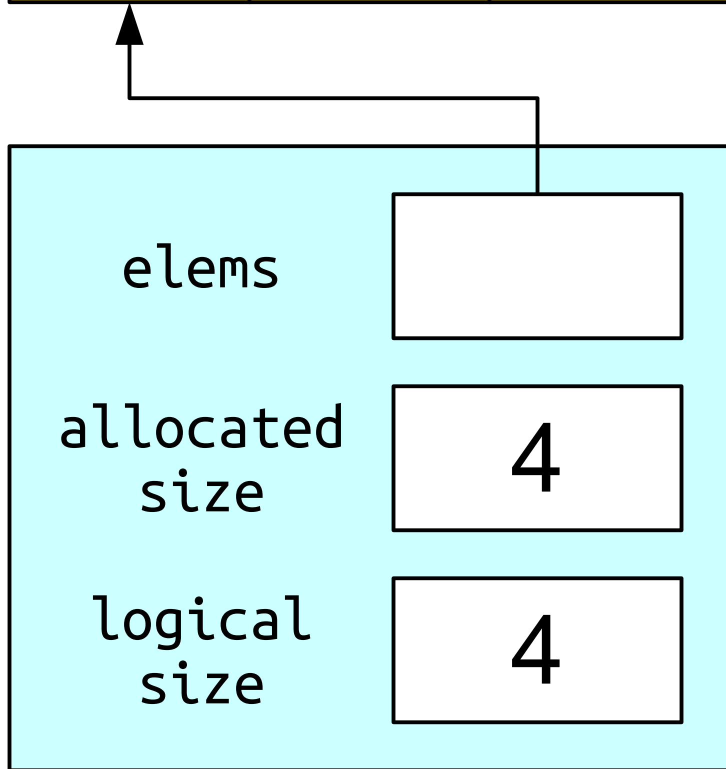


```
void OurStack::grow() {
    allocatedSize++;



}
```

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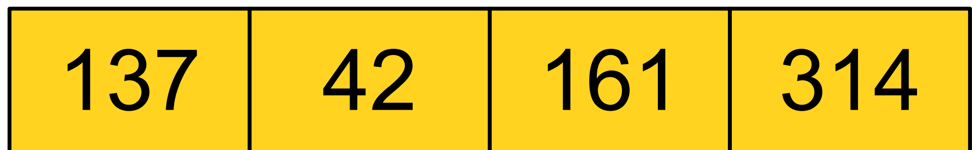

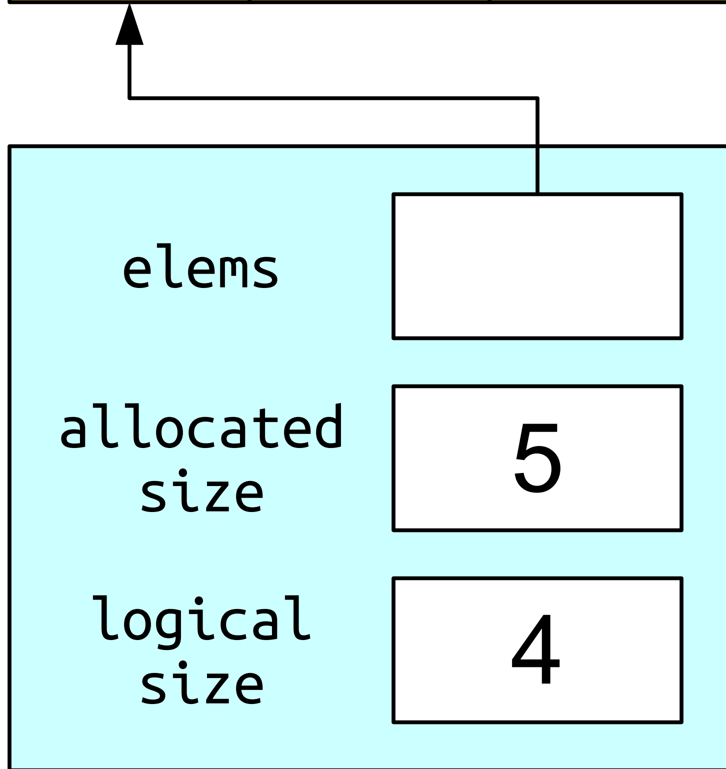


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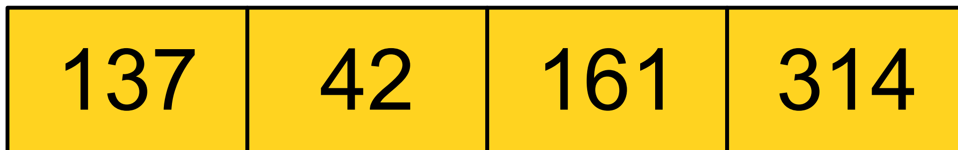
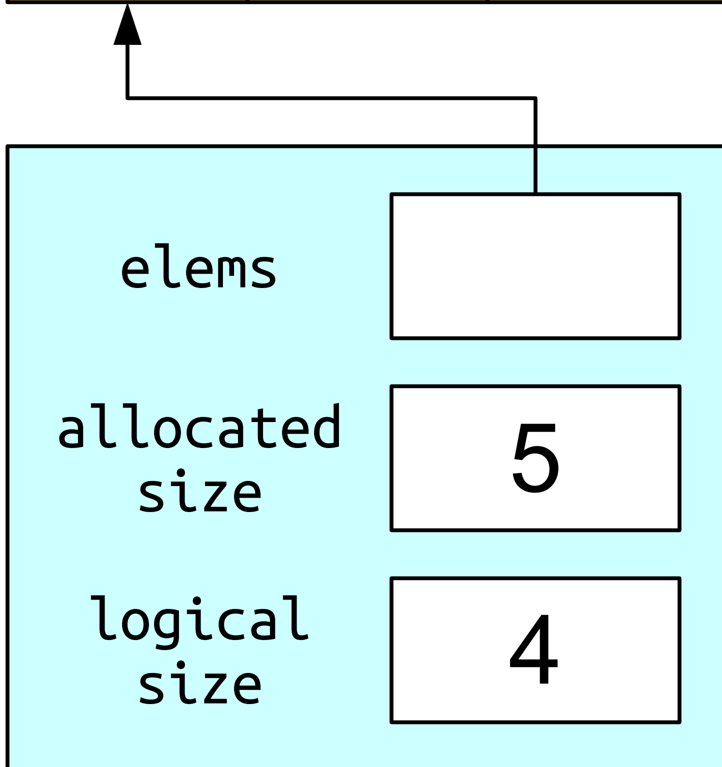
An Initial Idea



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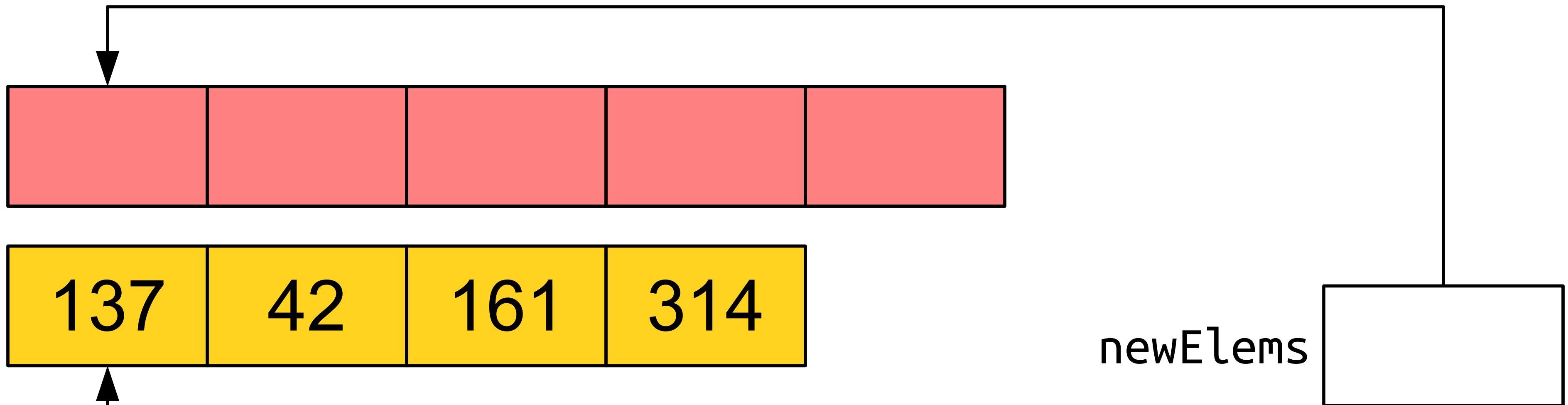

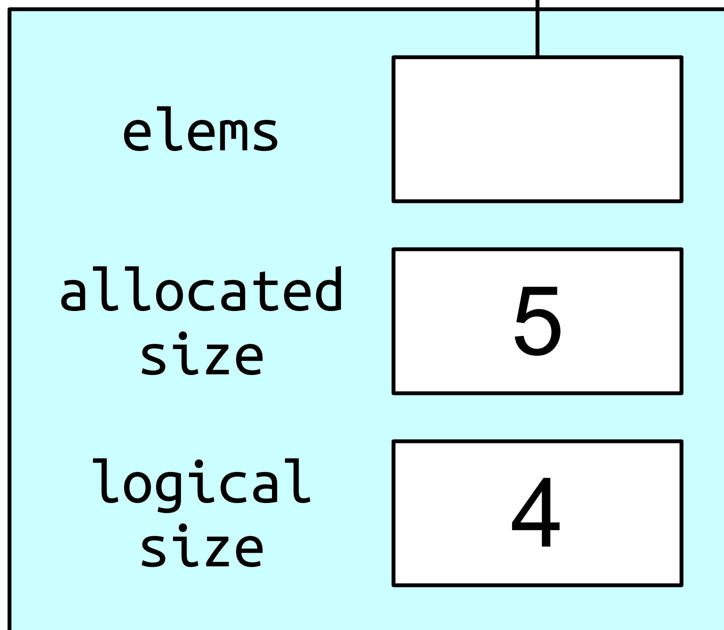


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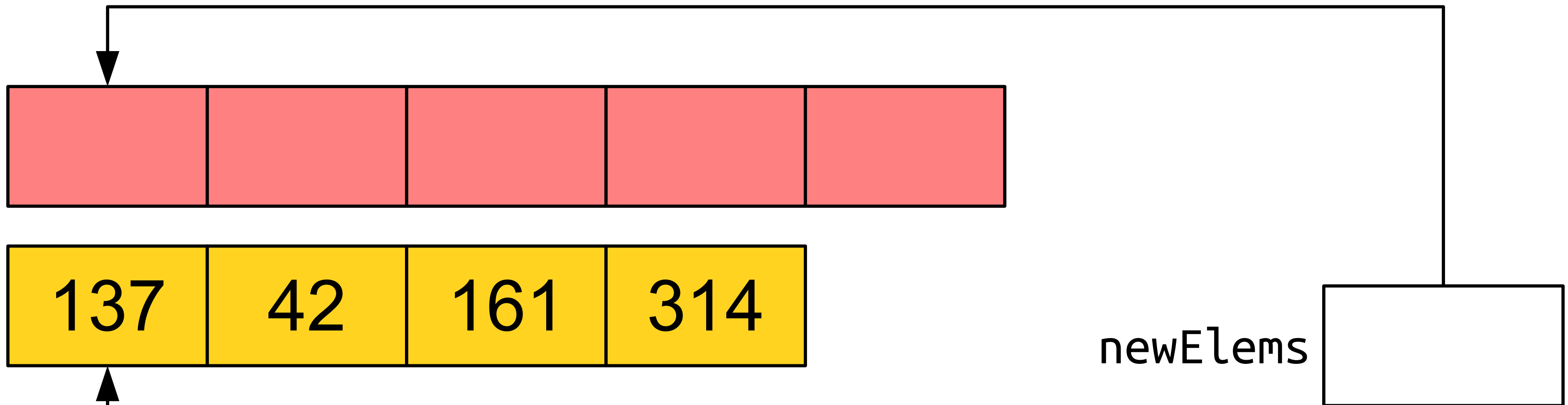

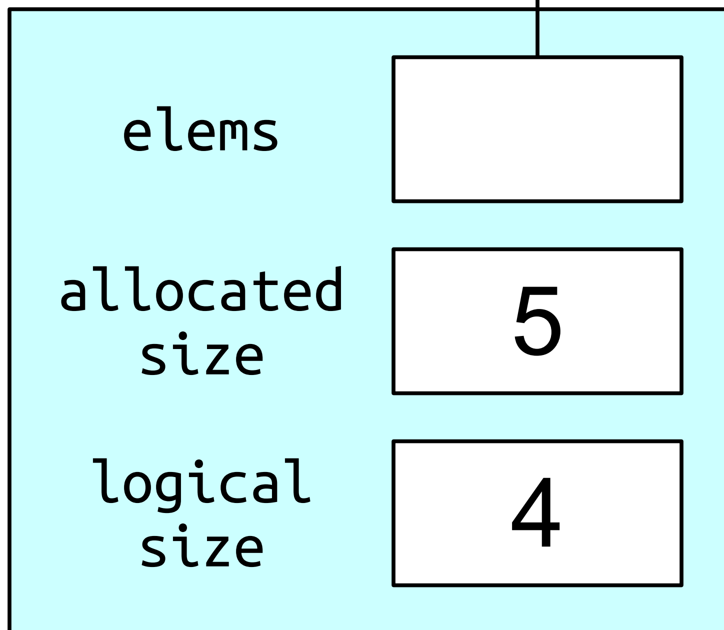


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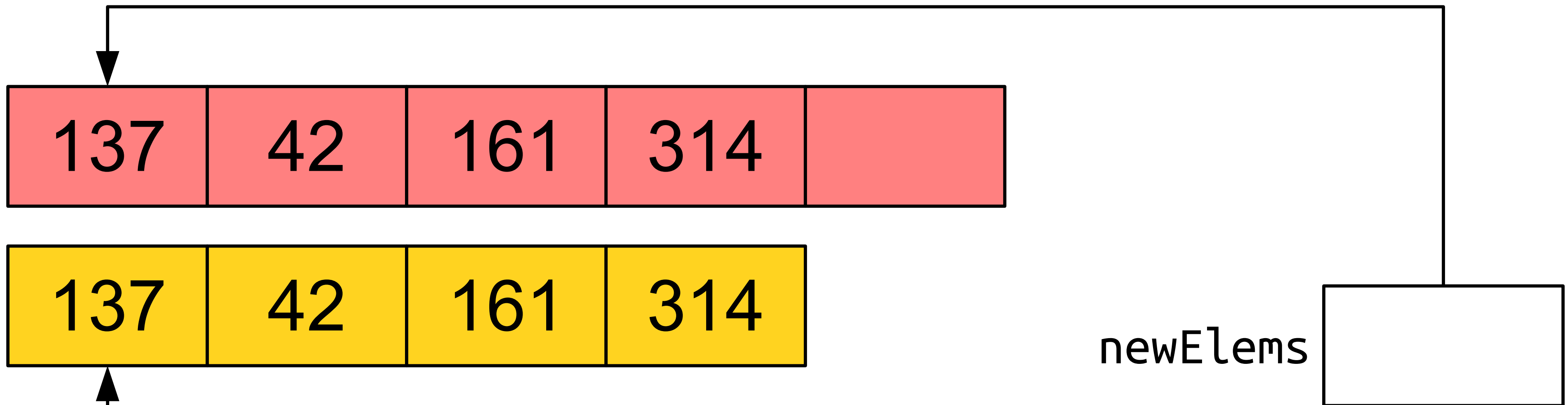

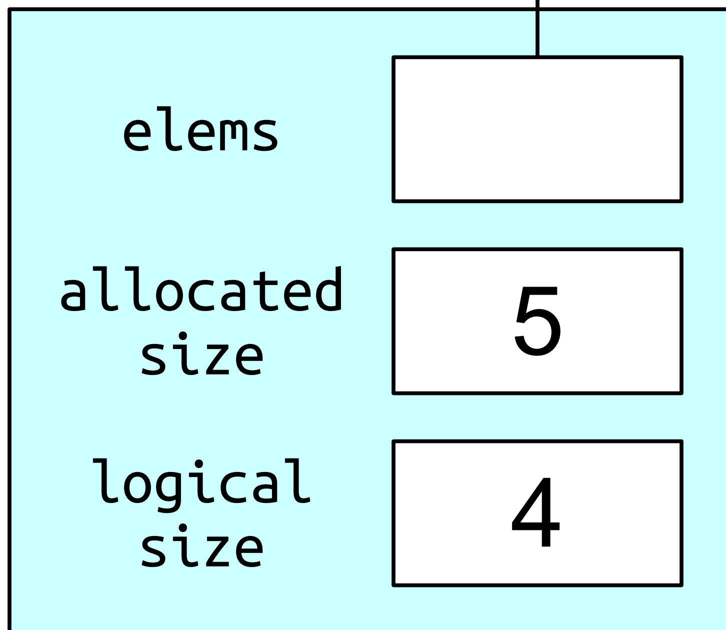


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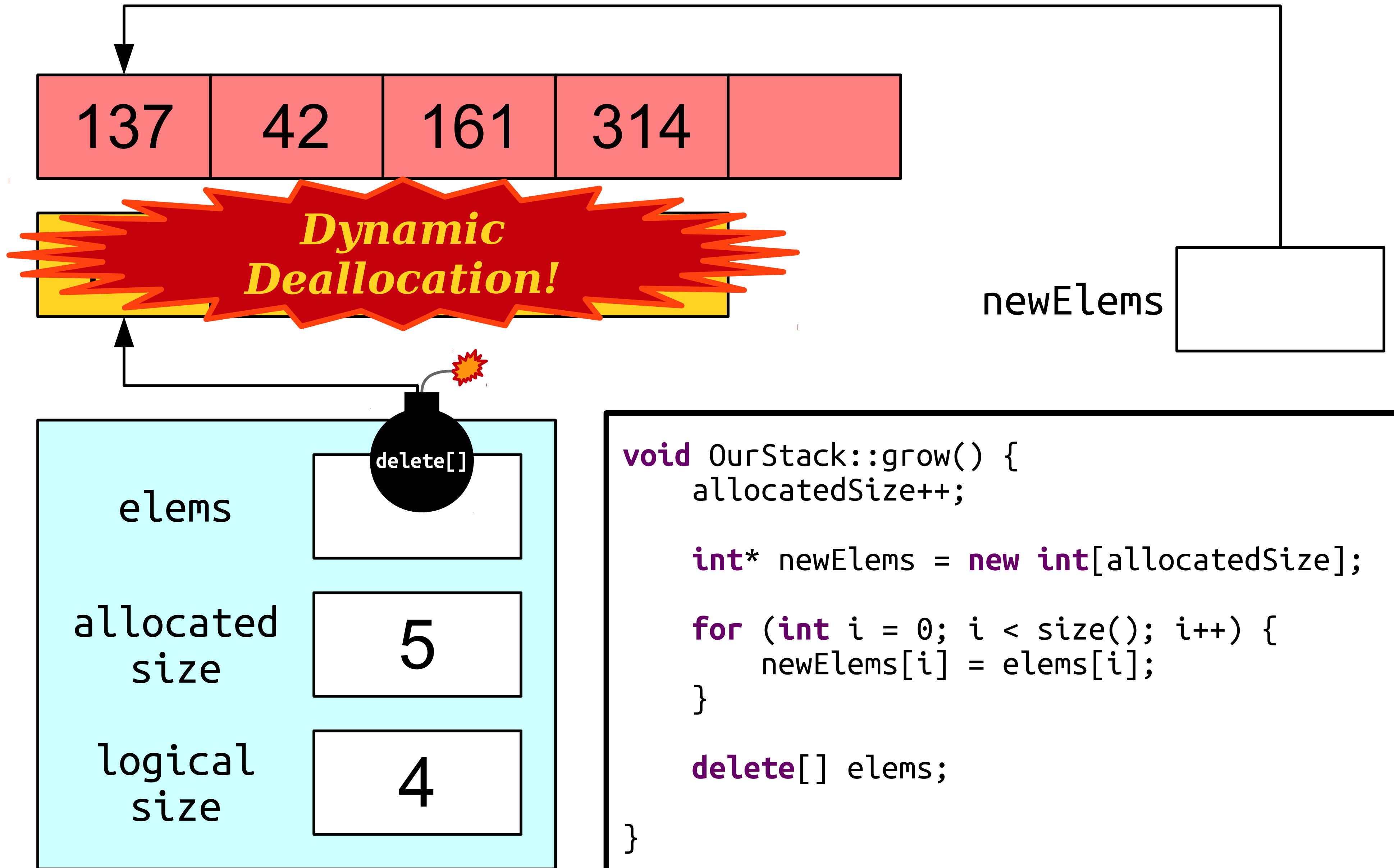

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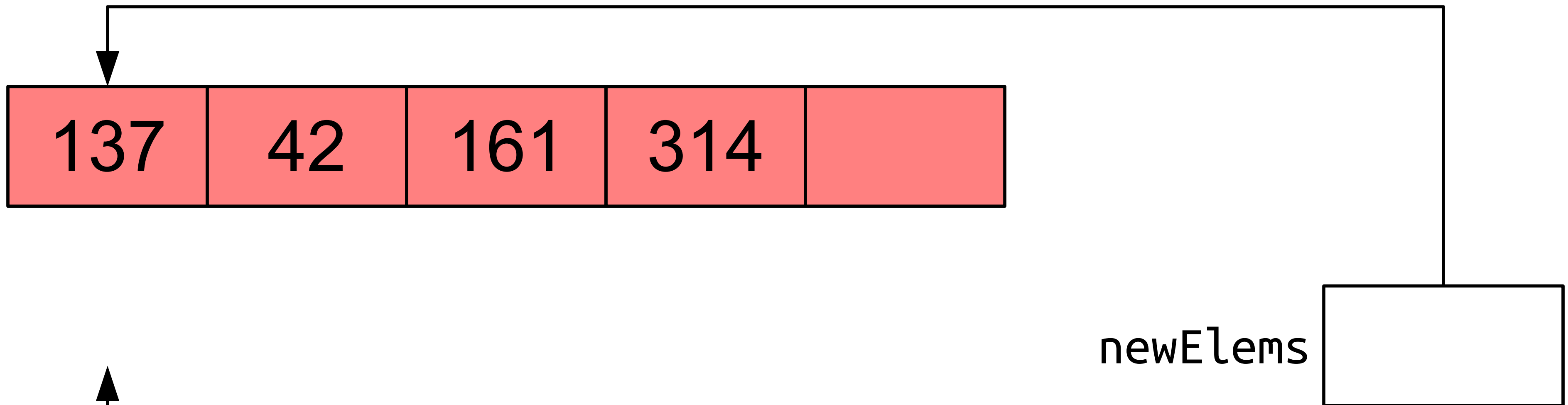

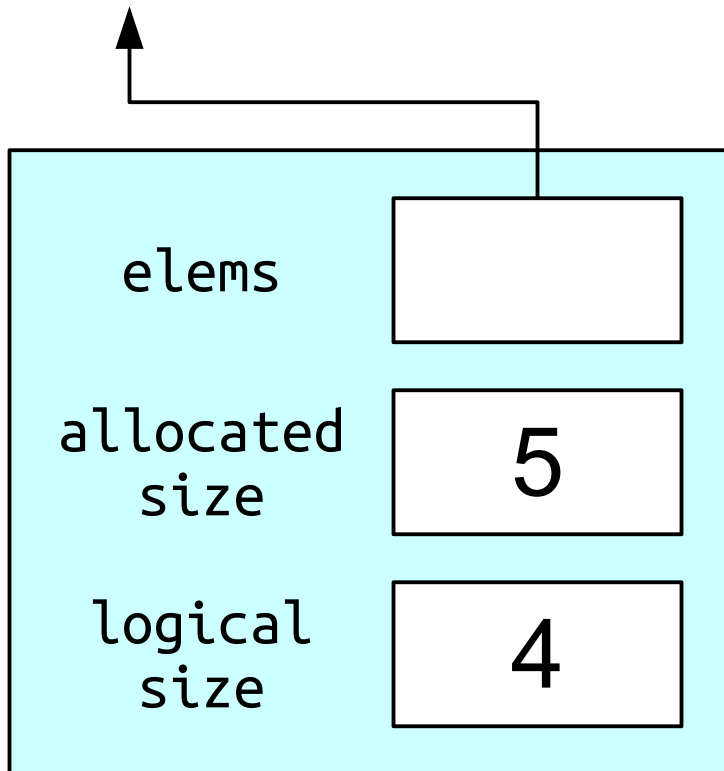


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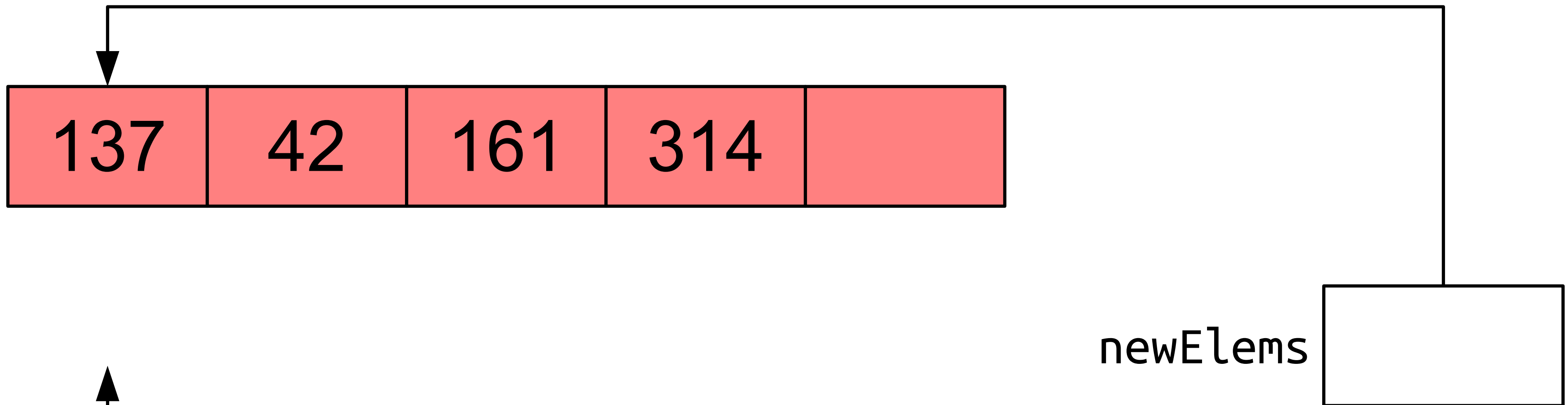

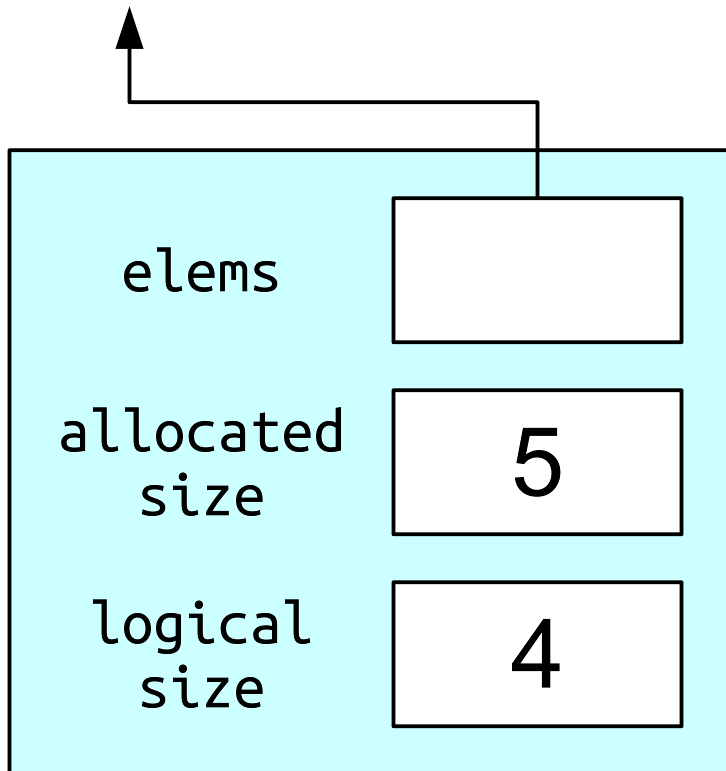


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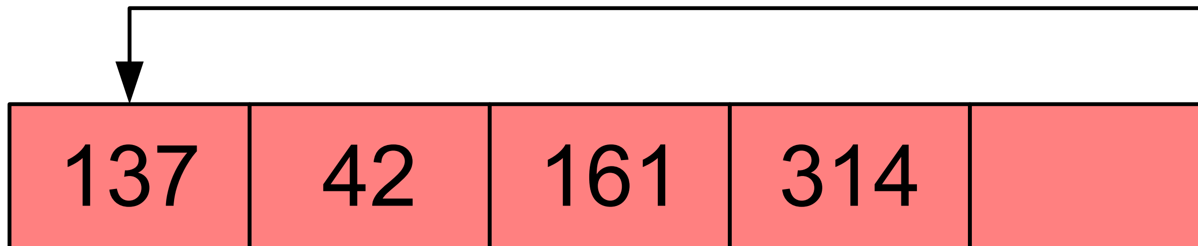

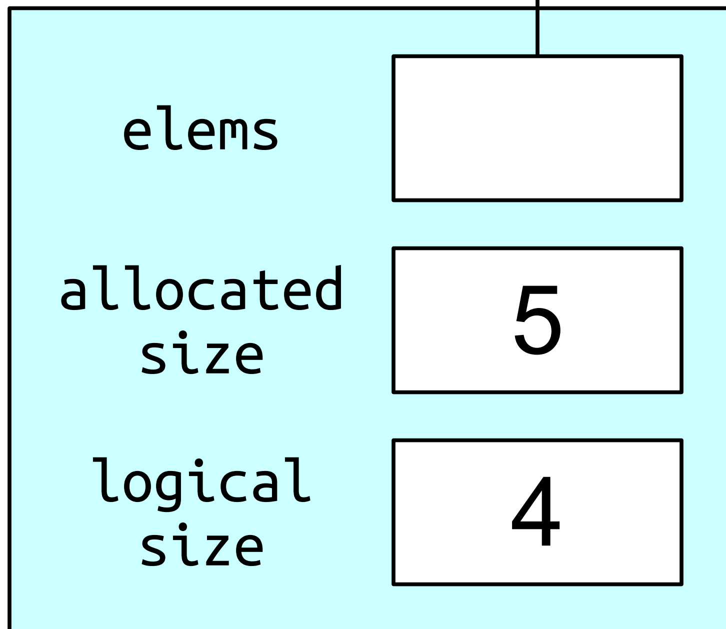


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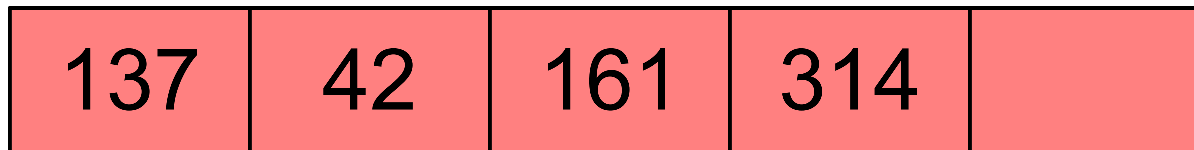
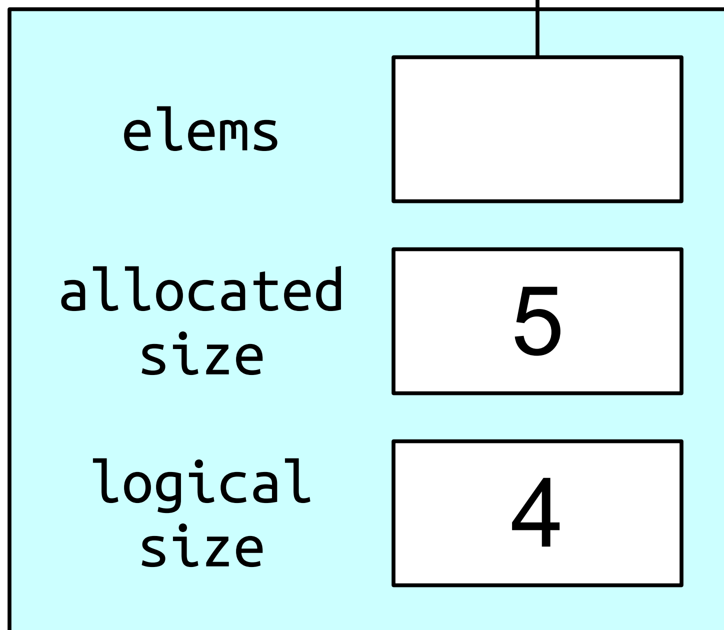
If a and b are pointers, then

```
a = b;
```

makes a point where b is pointing.

An Initial Idea



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Analyzing Our Approach

- We now have a working solution, but is it an *efficient* solution?
- Let's analyze the big-O complexity of the five operations. As usual, let *n* denote the number of items in the stack when the operation is performed.
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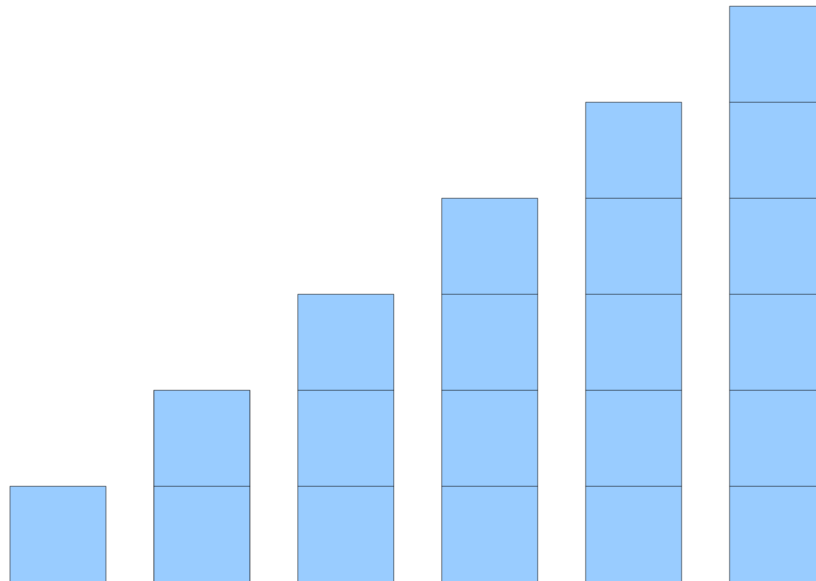

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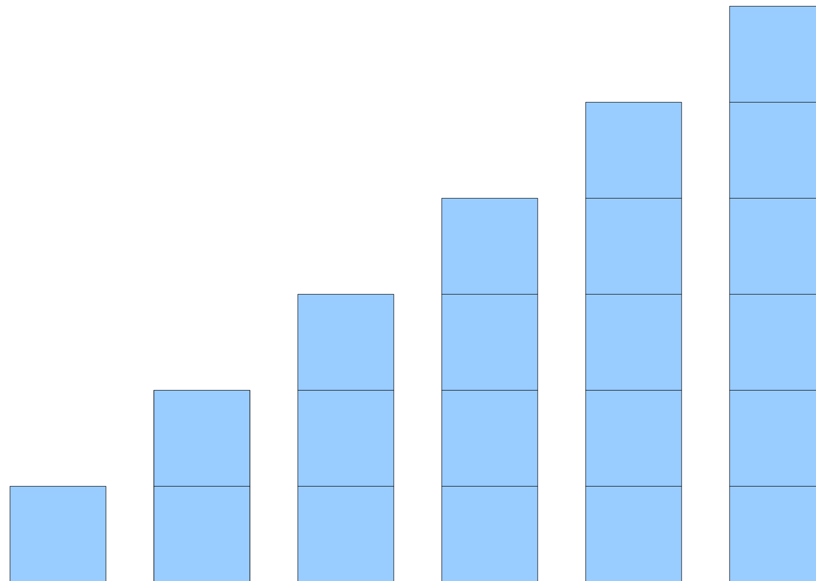

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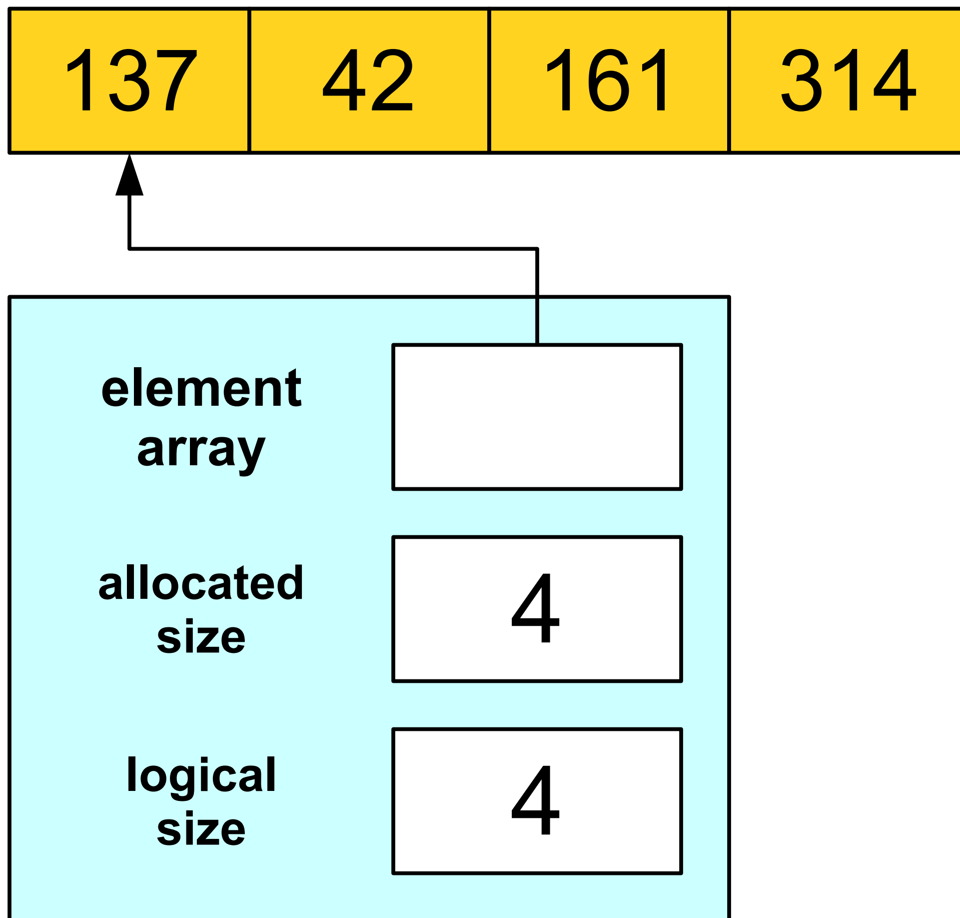
Validating Our Model

Speeding up the Stack

Key Idea: ***Plan for the Future***

A Better Idea



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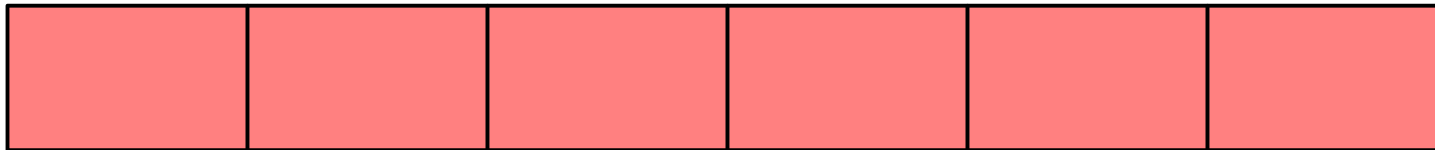

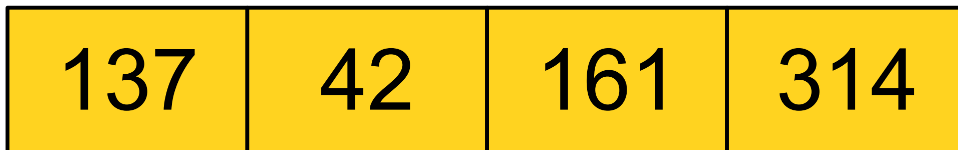

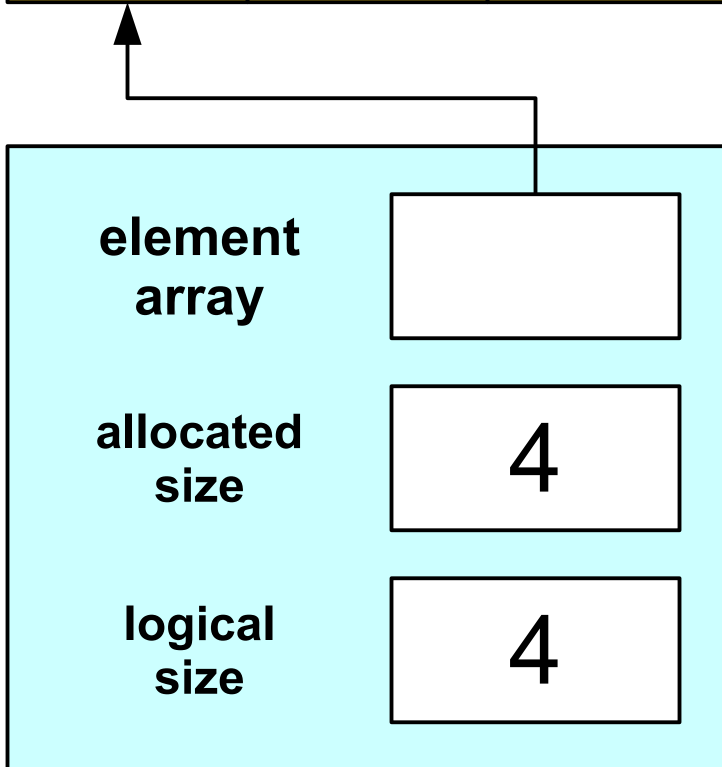

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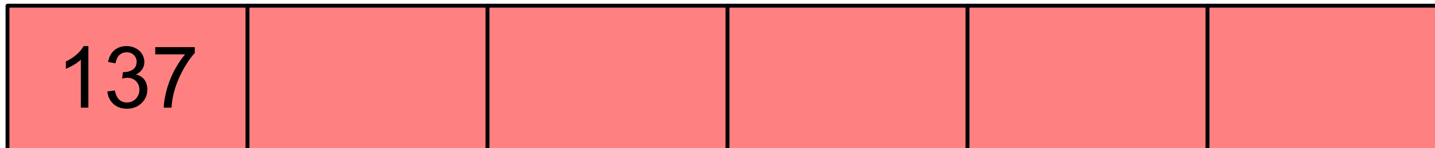

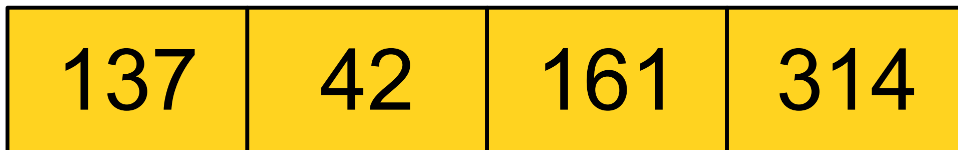

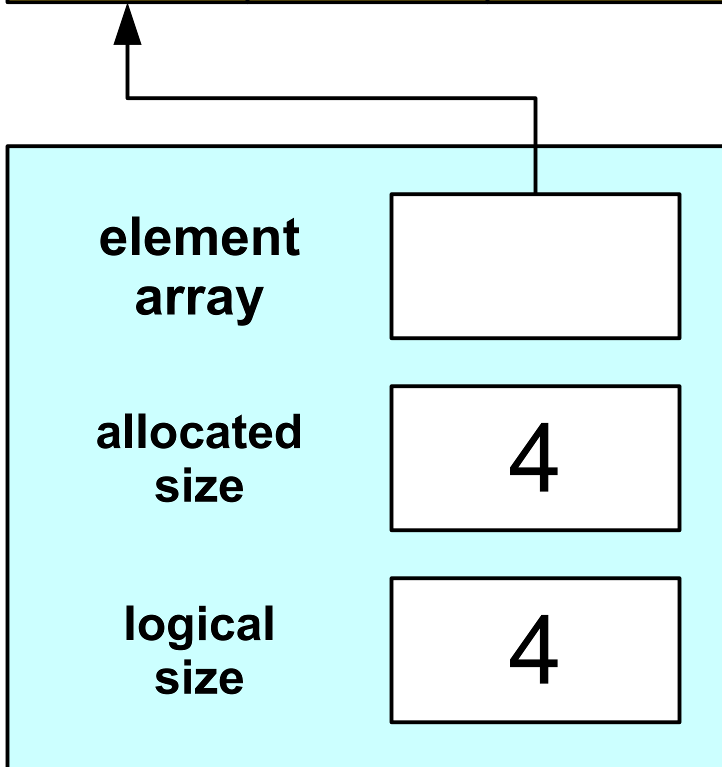

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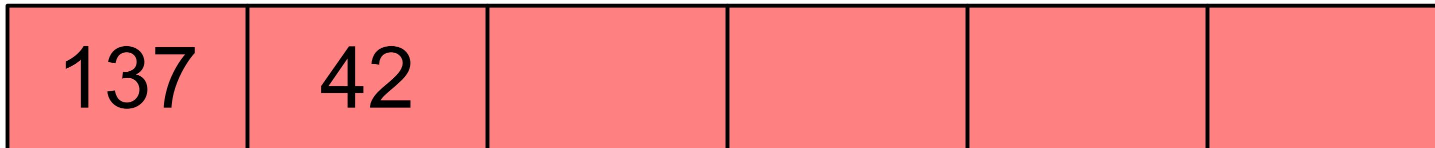

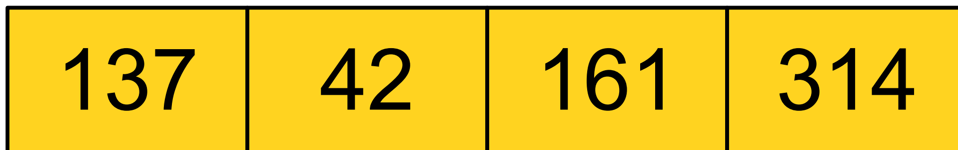

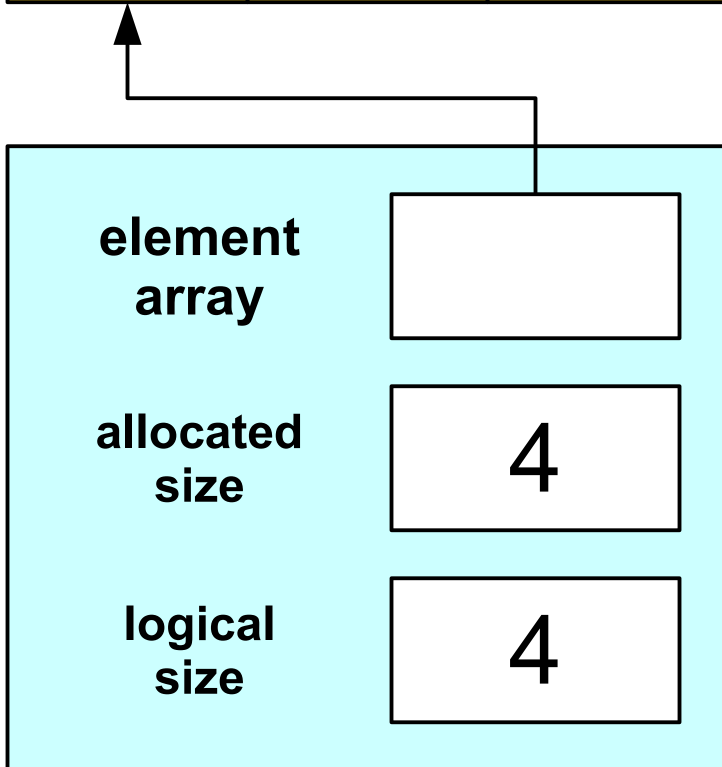

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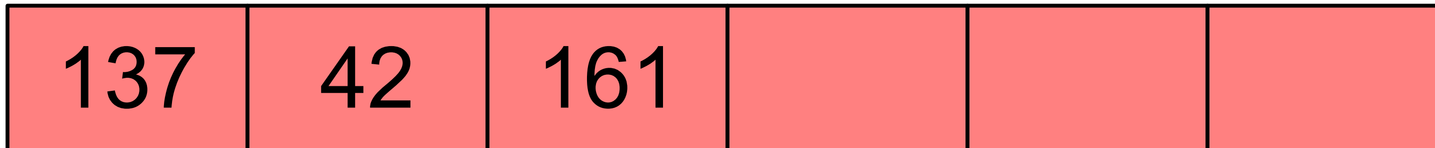

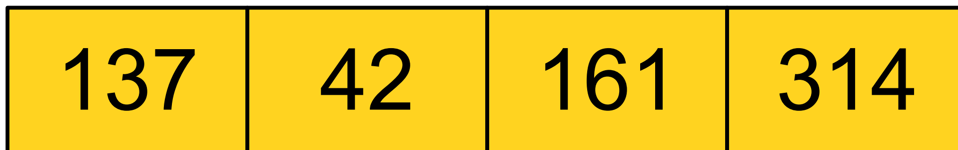

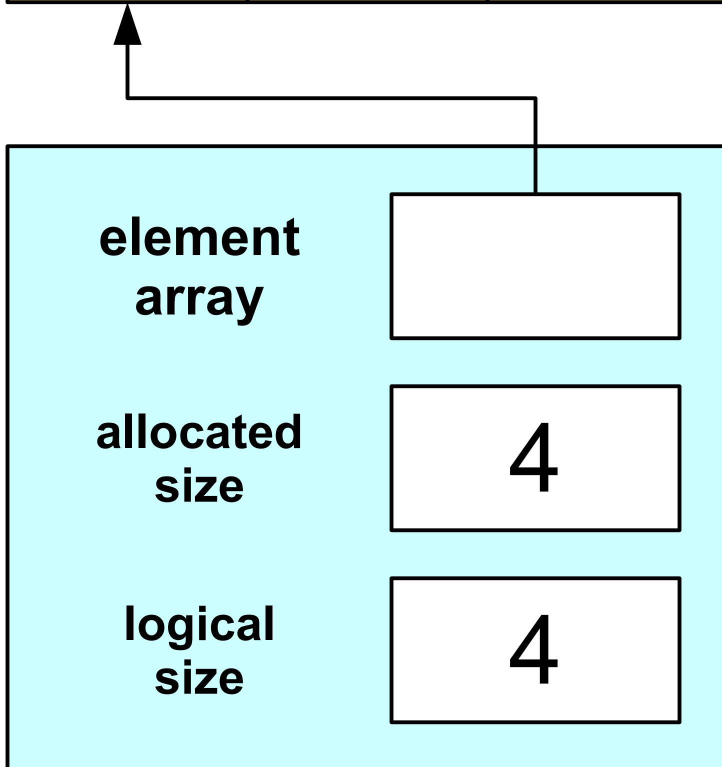

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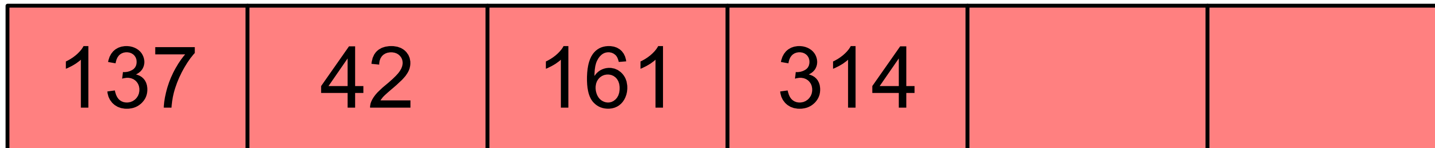

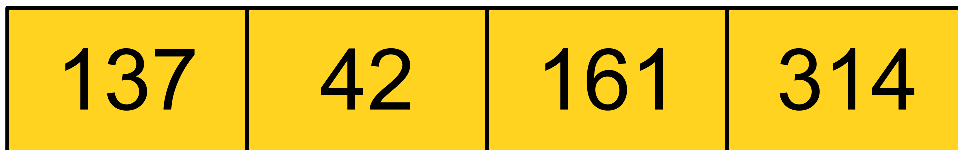

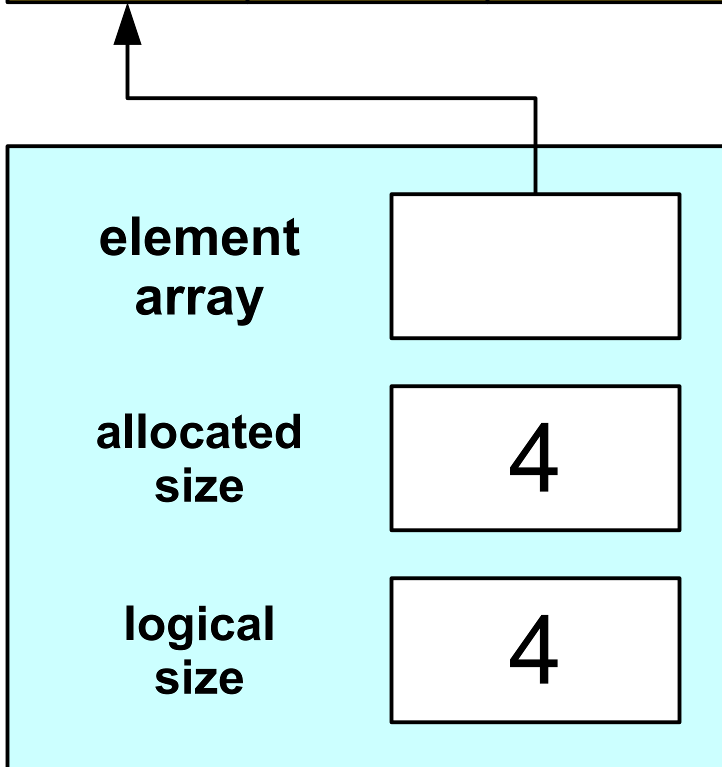

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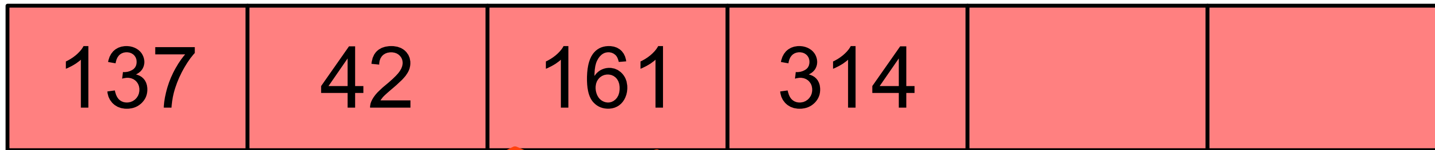

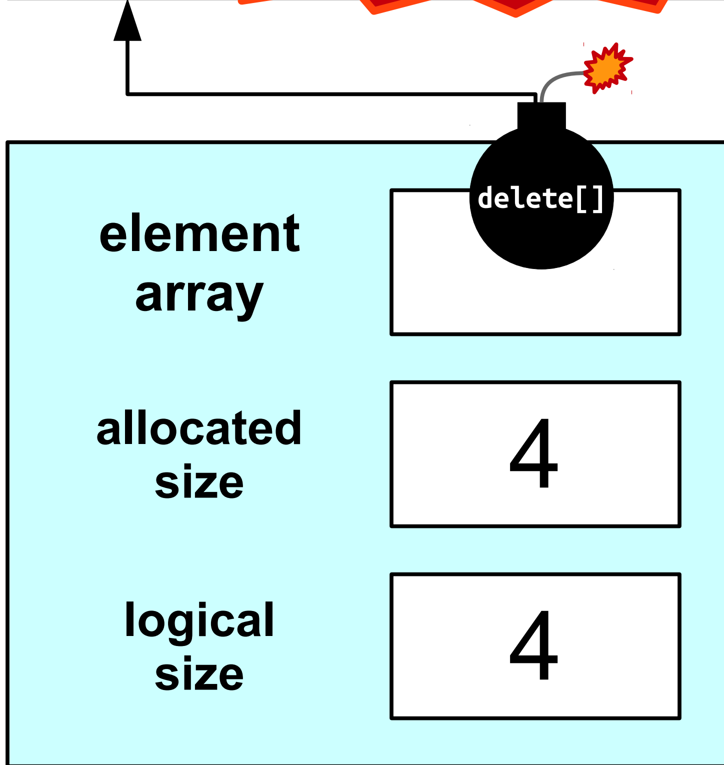

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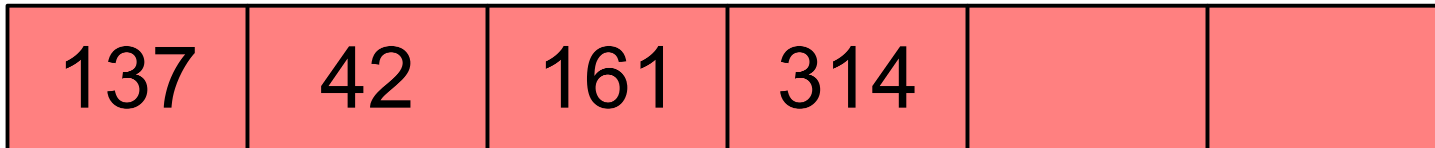

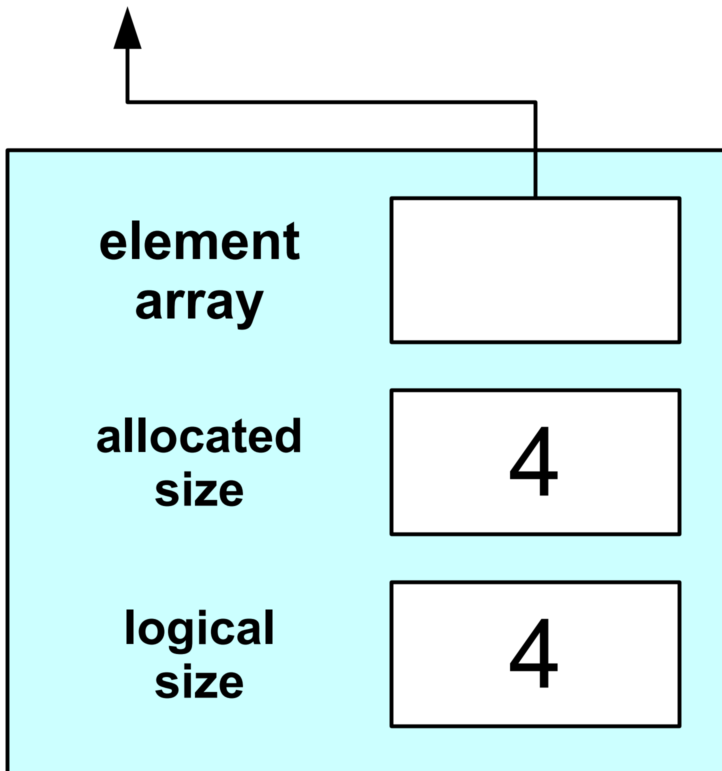

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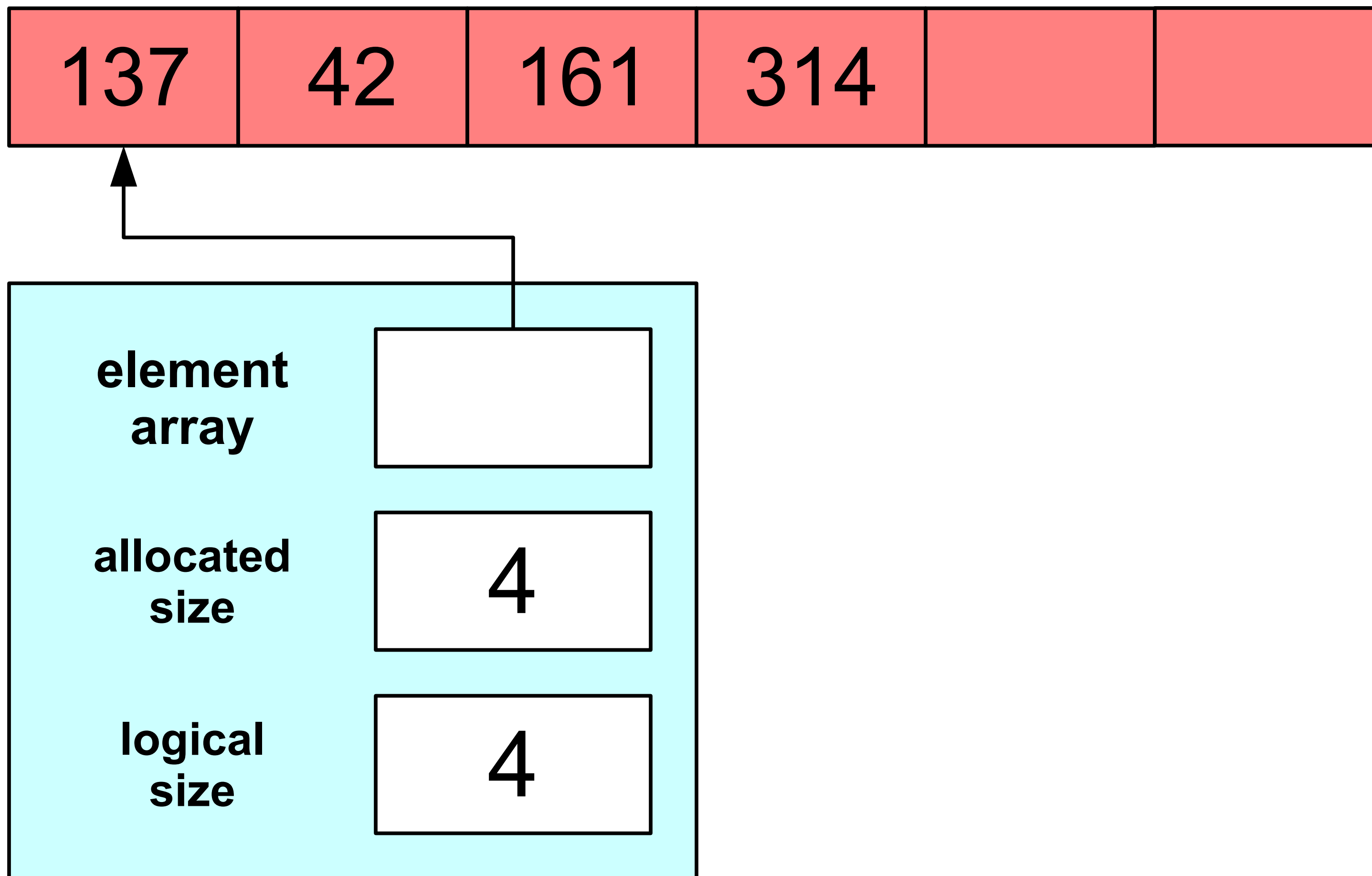

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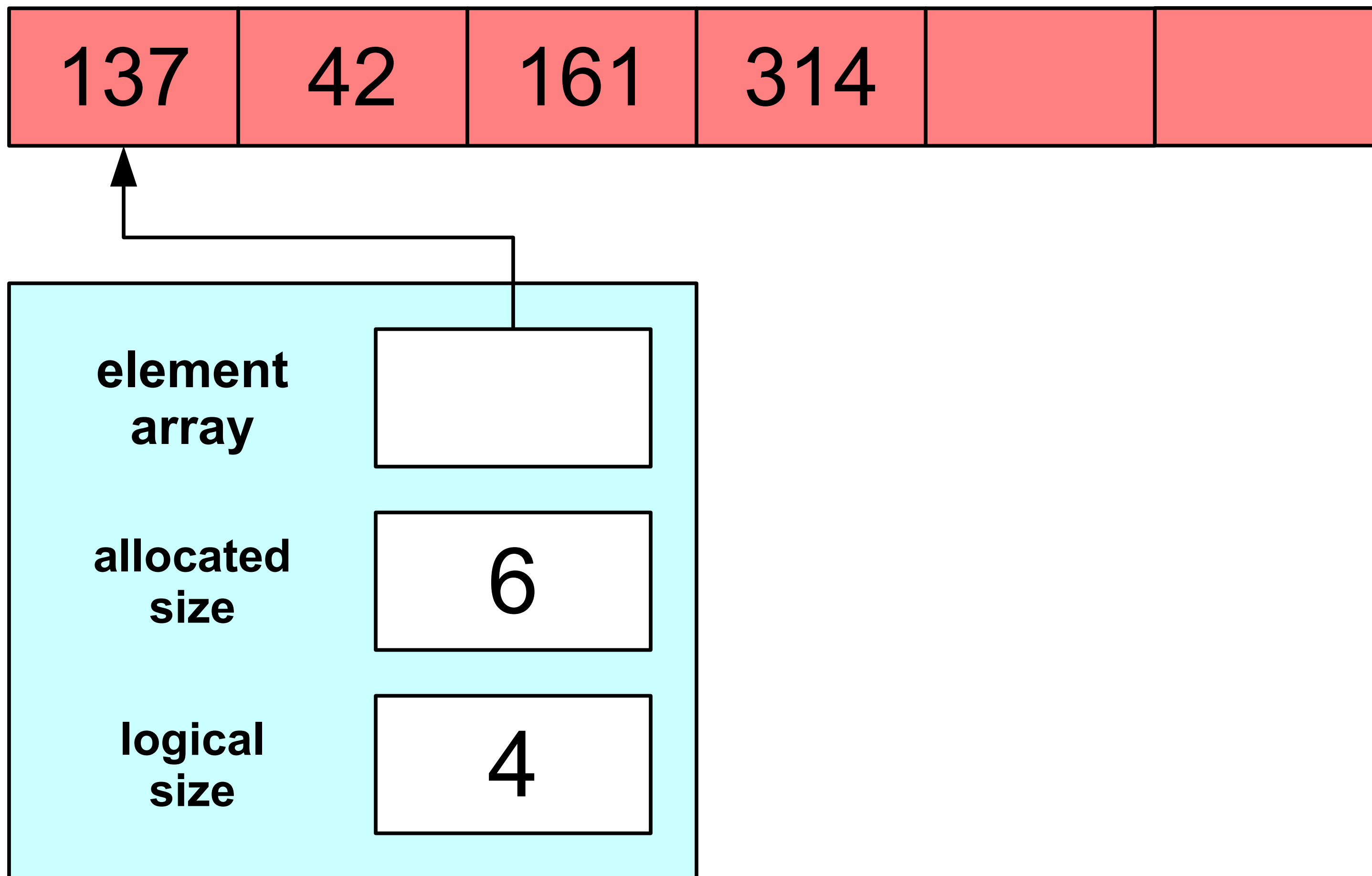

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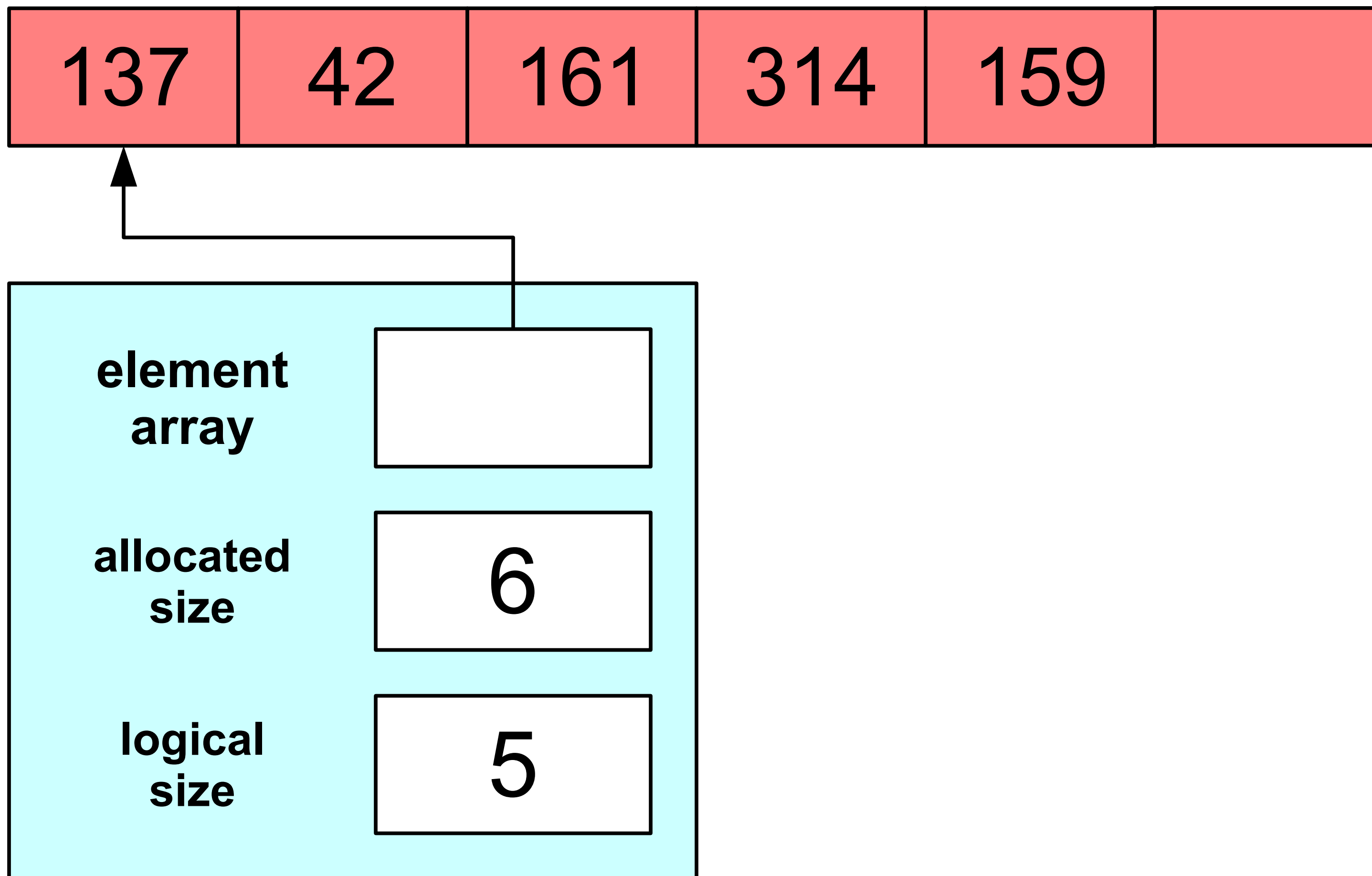

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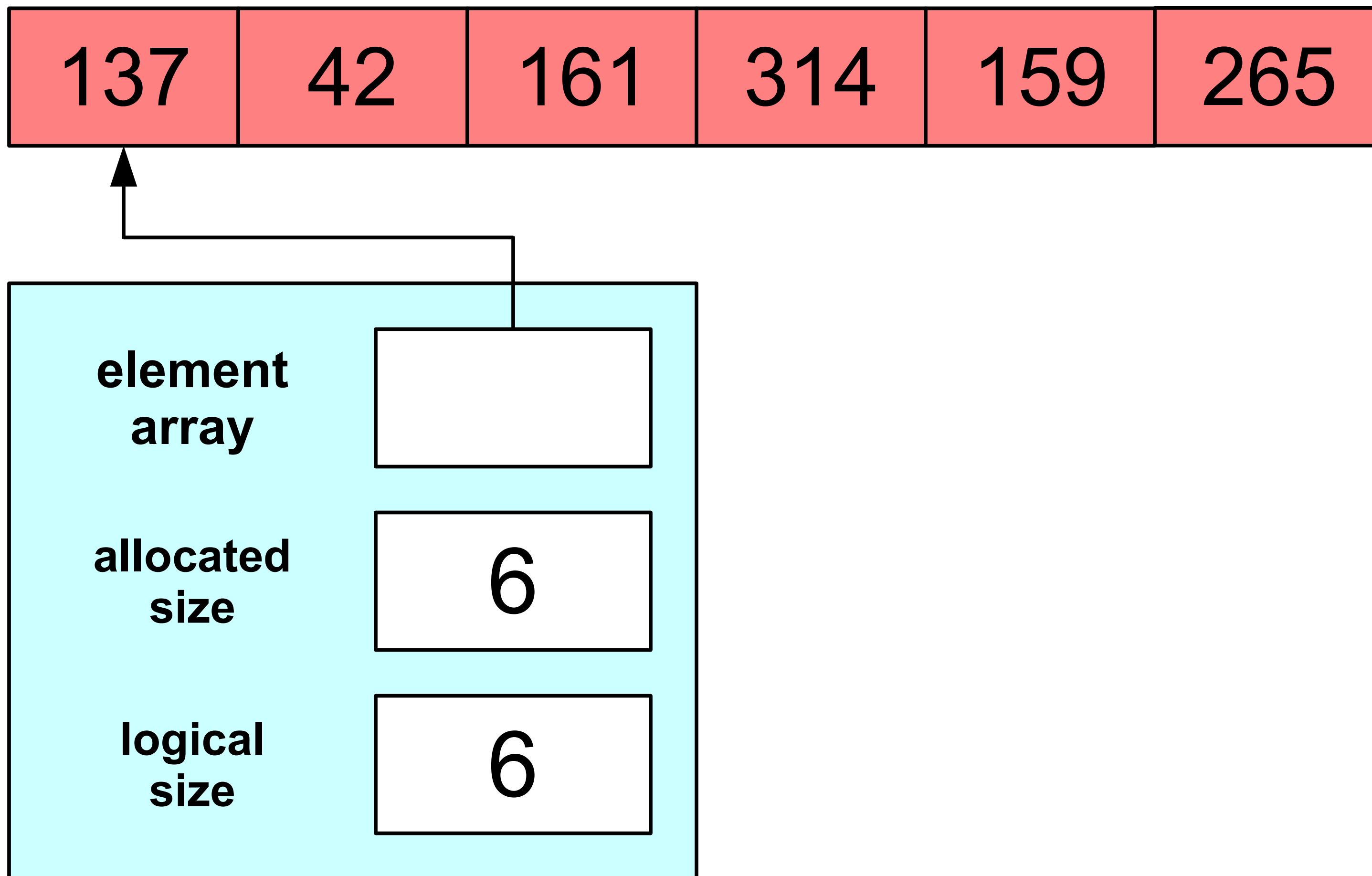
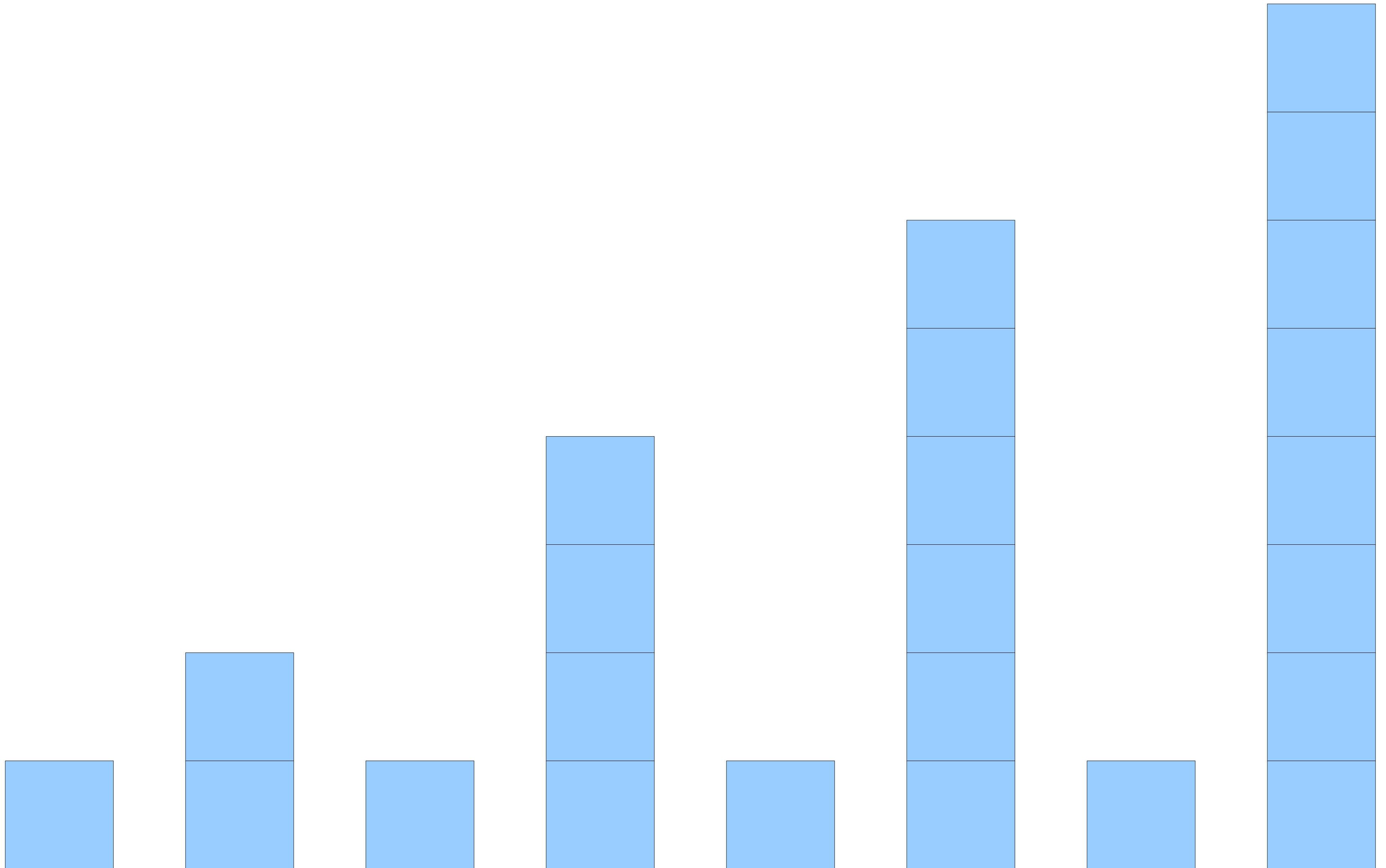


What Just Happened?

- Half of our pushes are now “easy” pushes, and half of our pushes are now “hard” pushes.
- Hard pushes still take time $O(n)$.
- Easy pushes only take time $O(1)$.
- Worst-case is still $O(n)$.
- What about the average case?

Analyzing the Work



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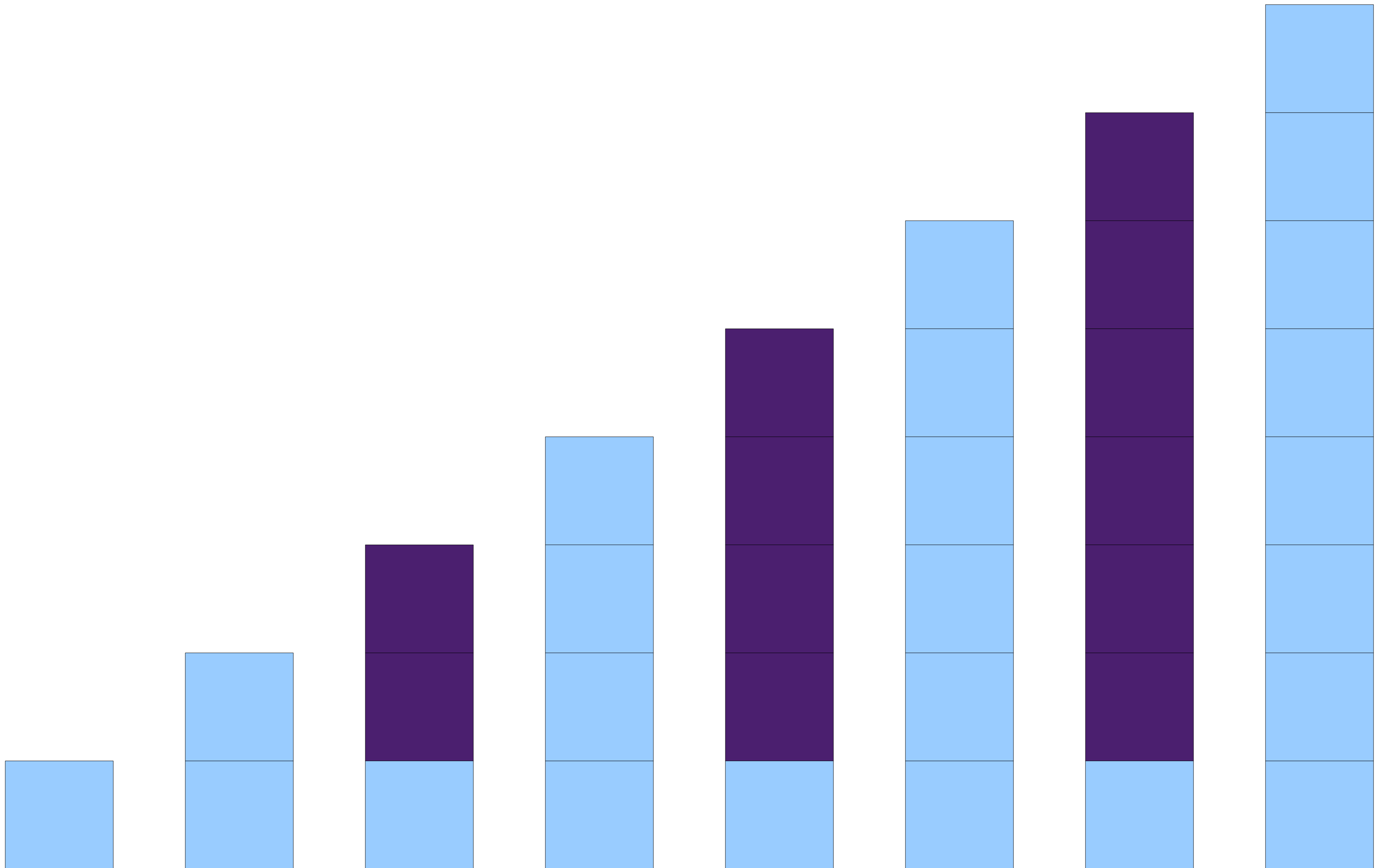

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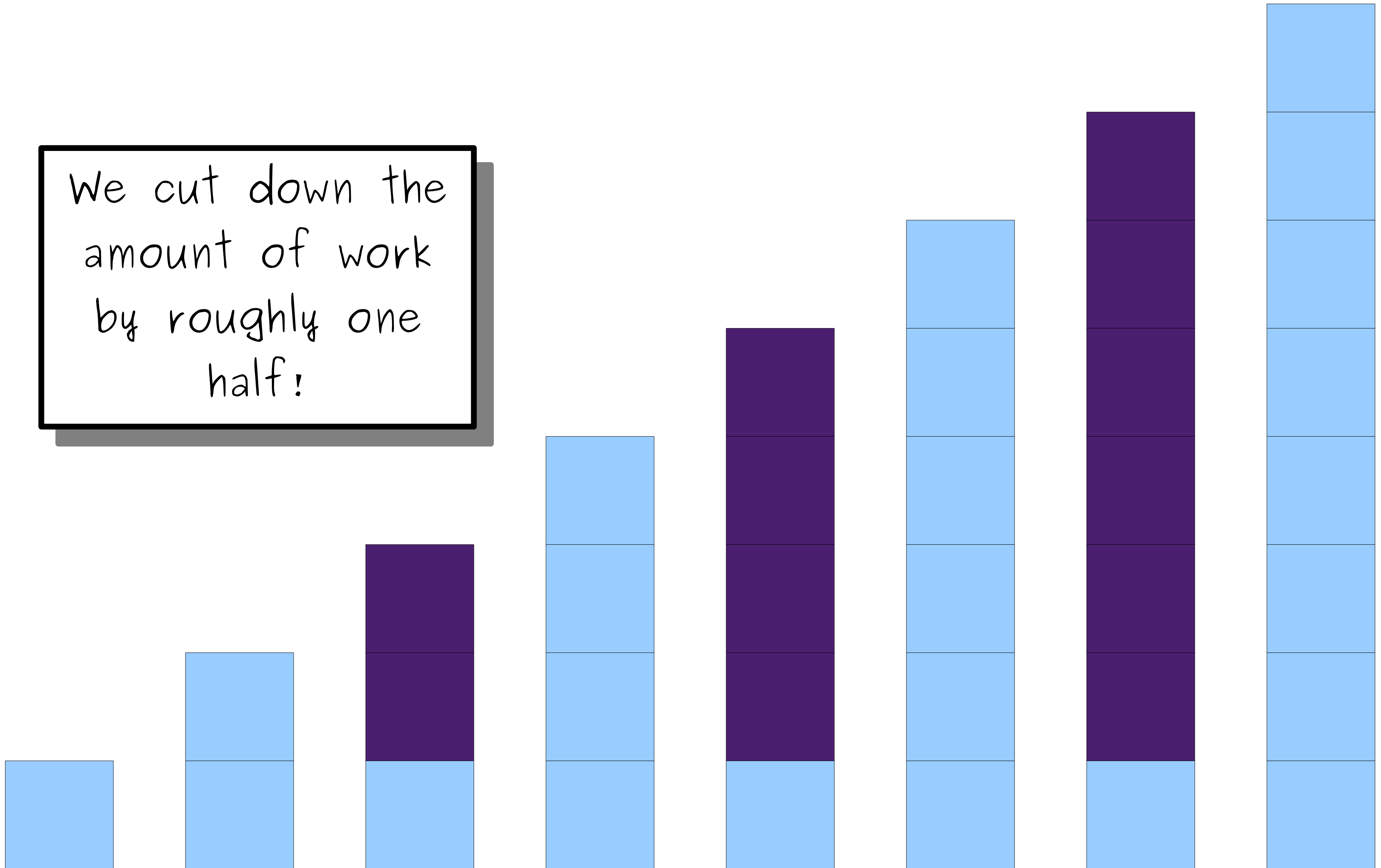

A Different Analysis

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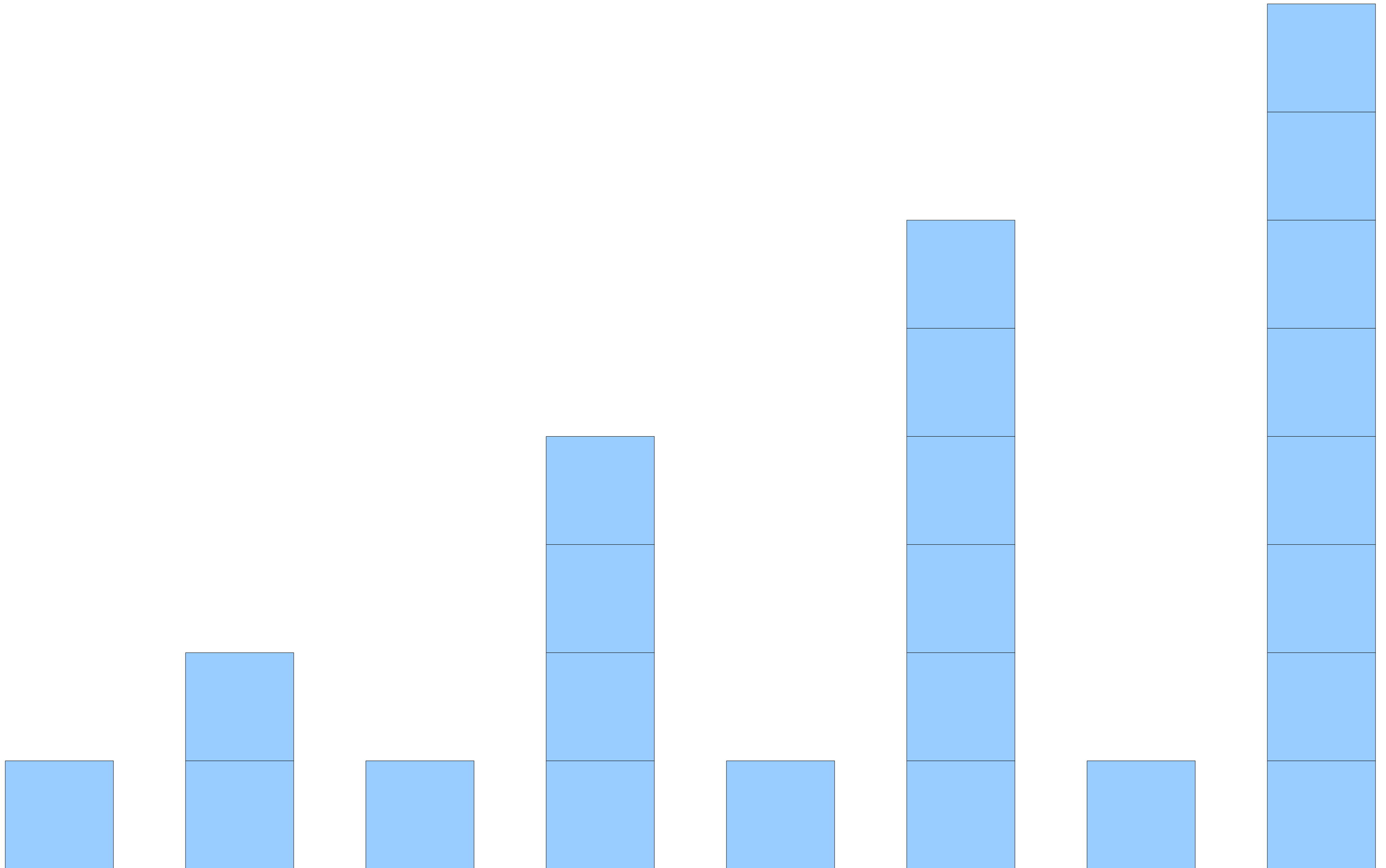

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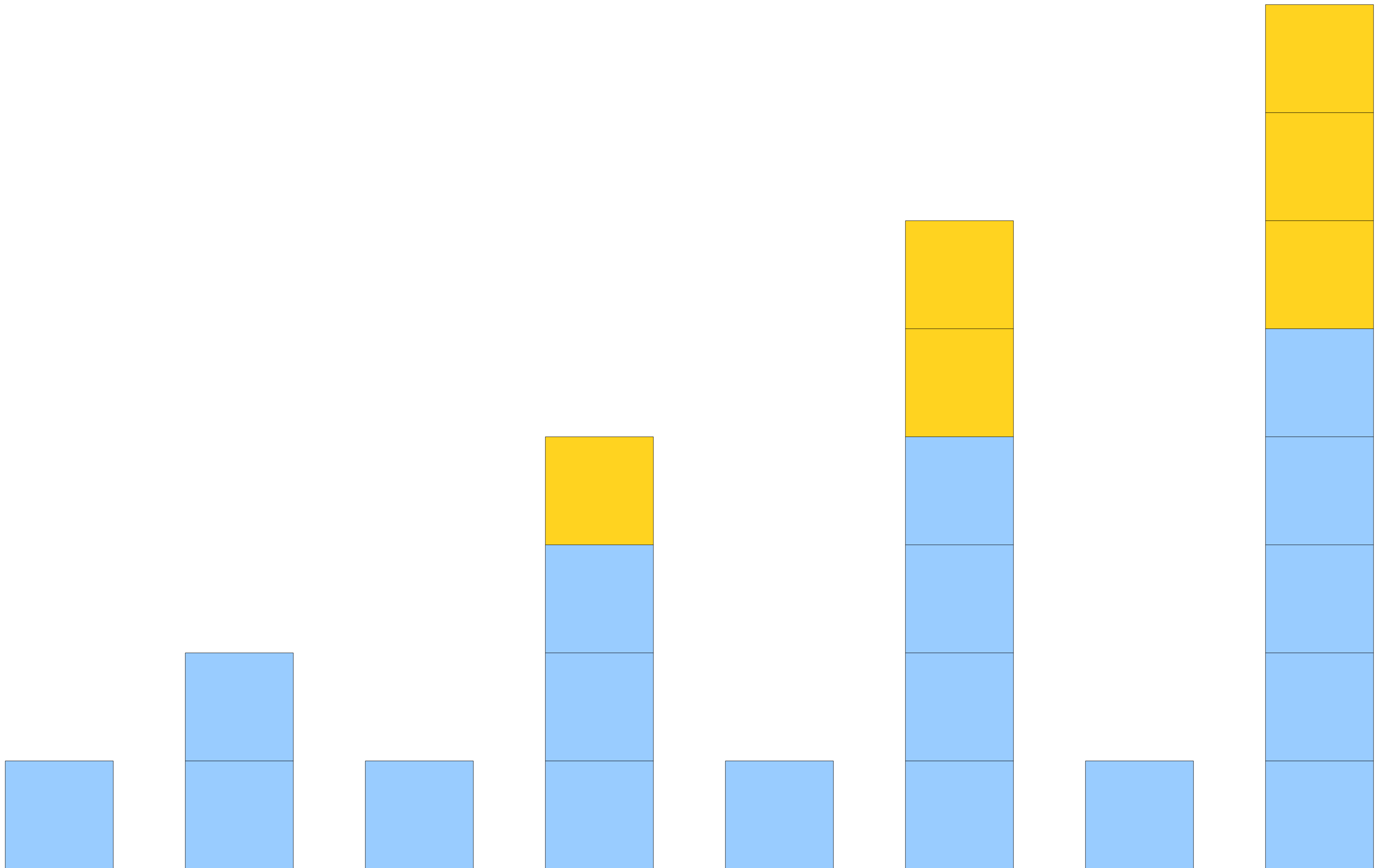

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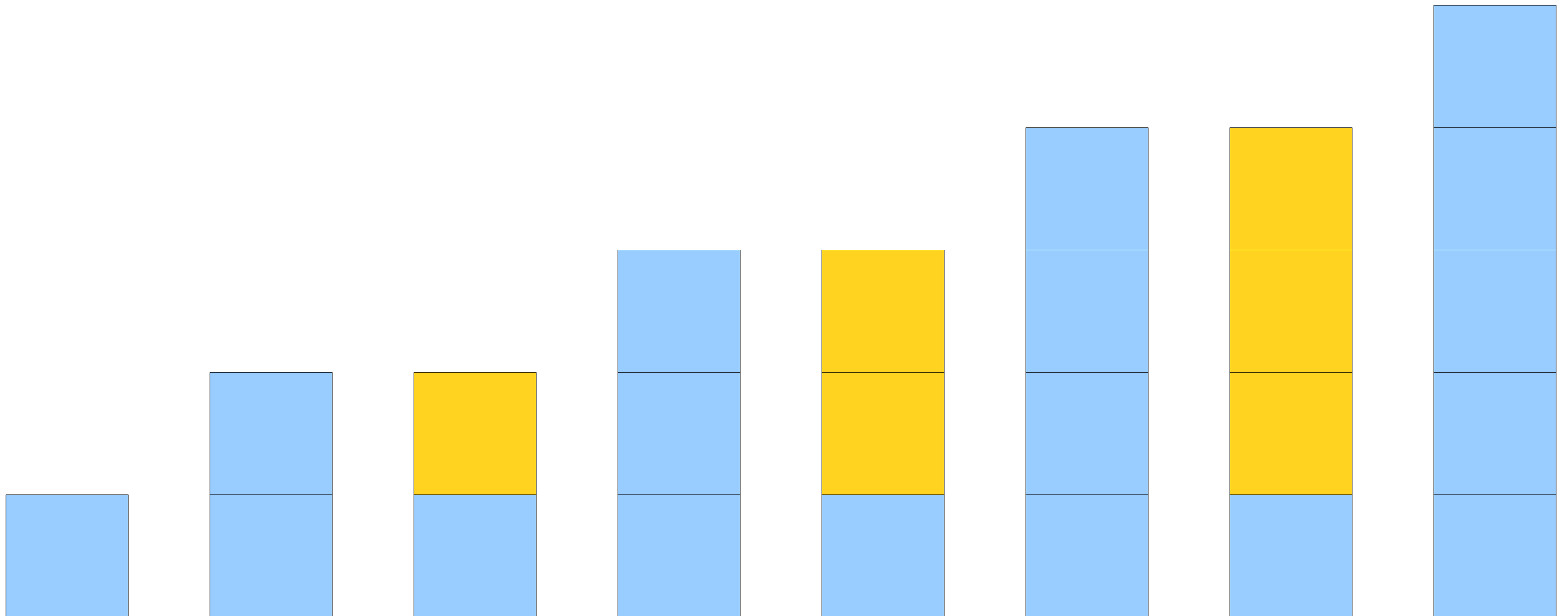

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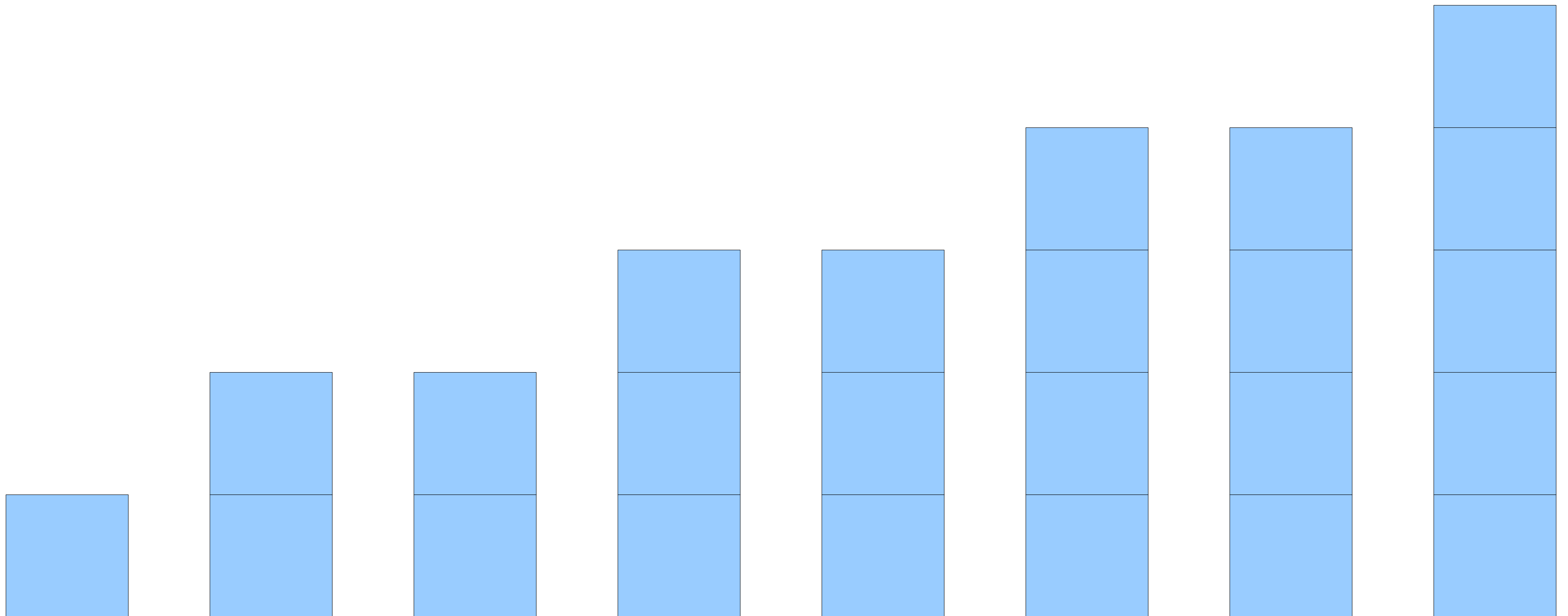

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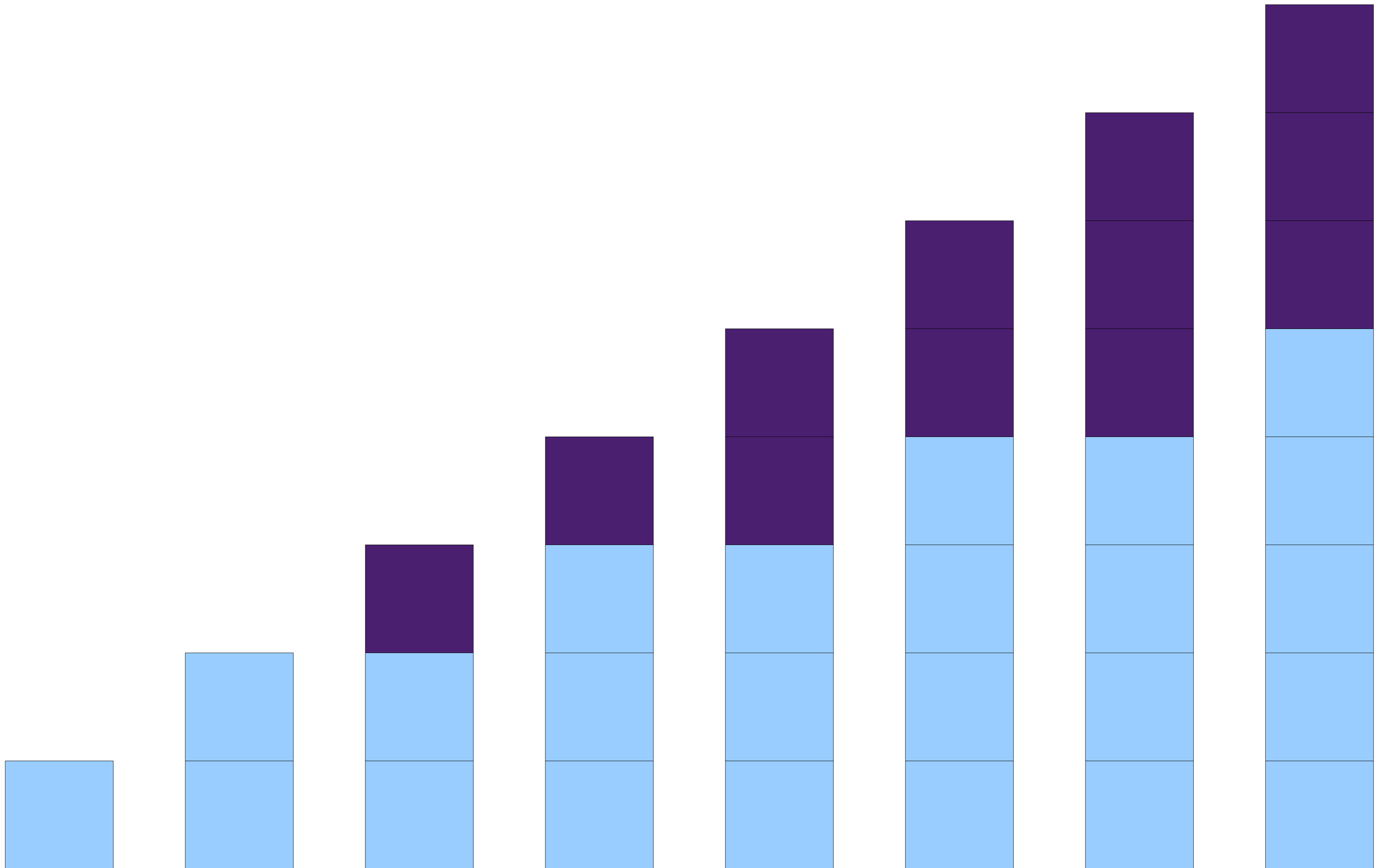

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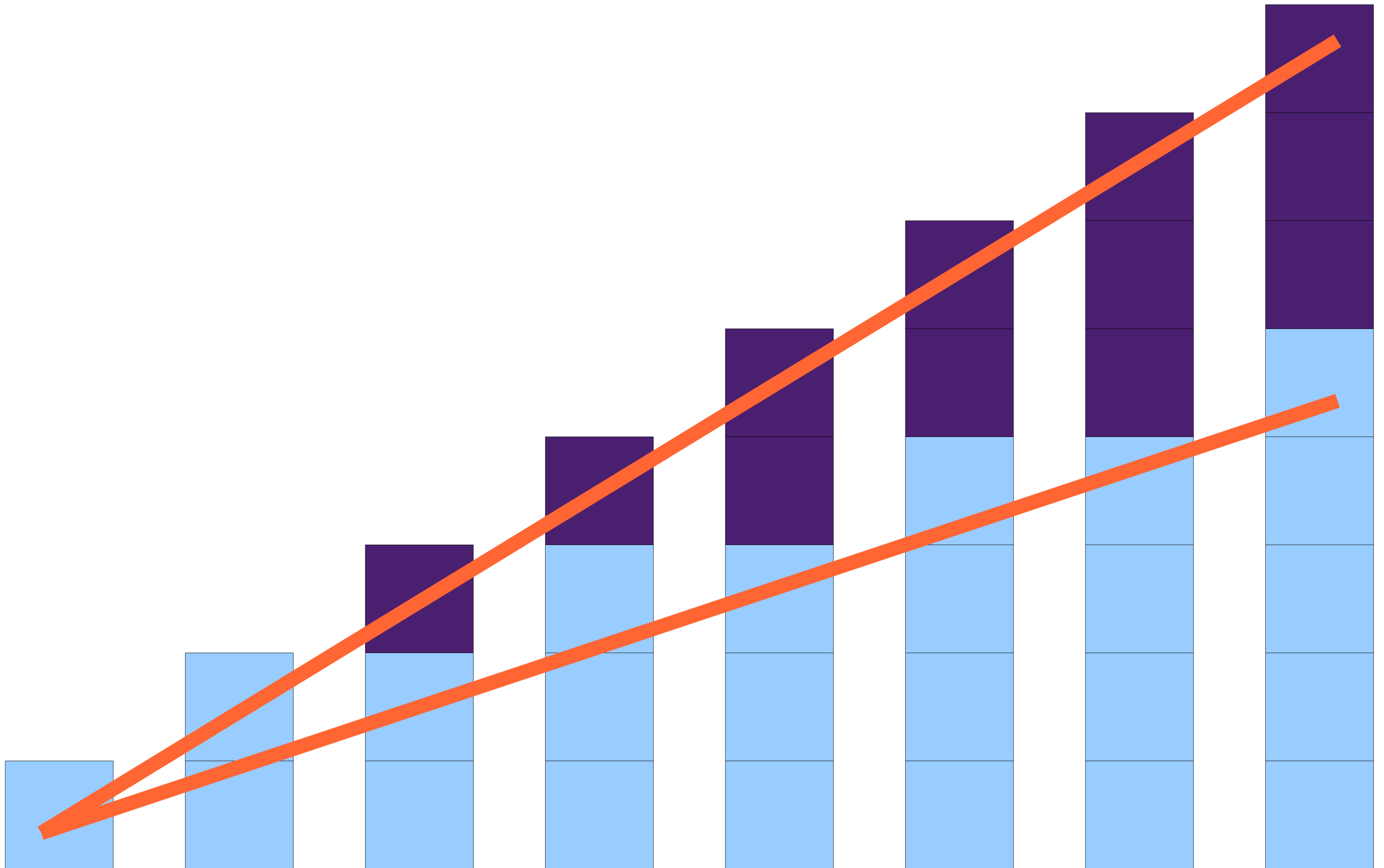

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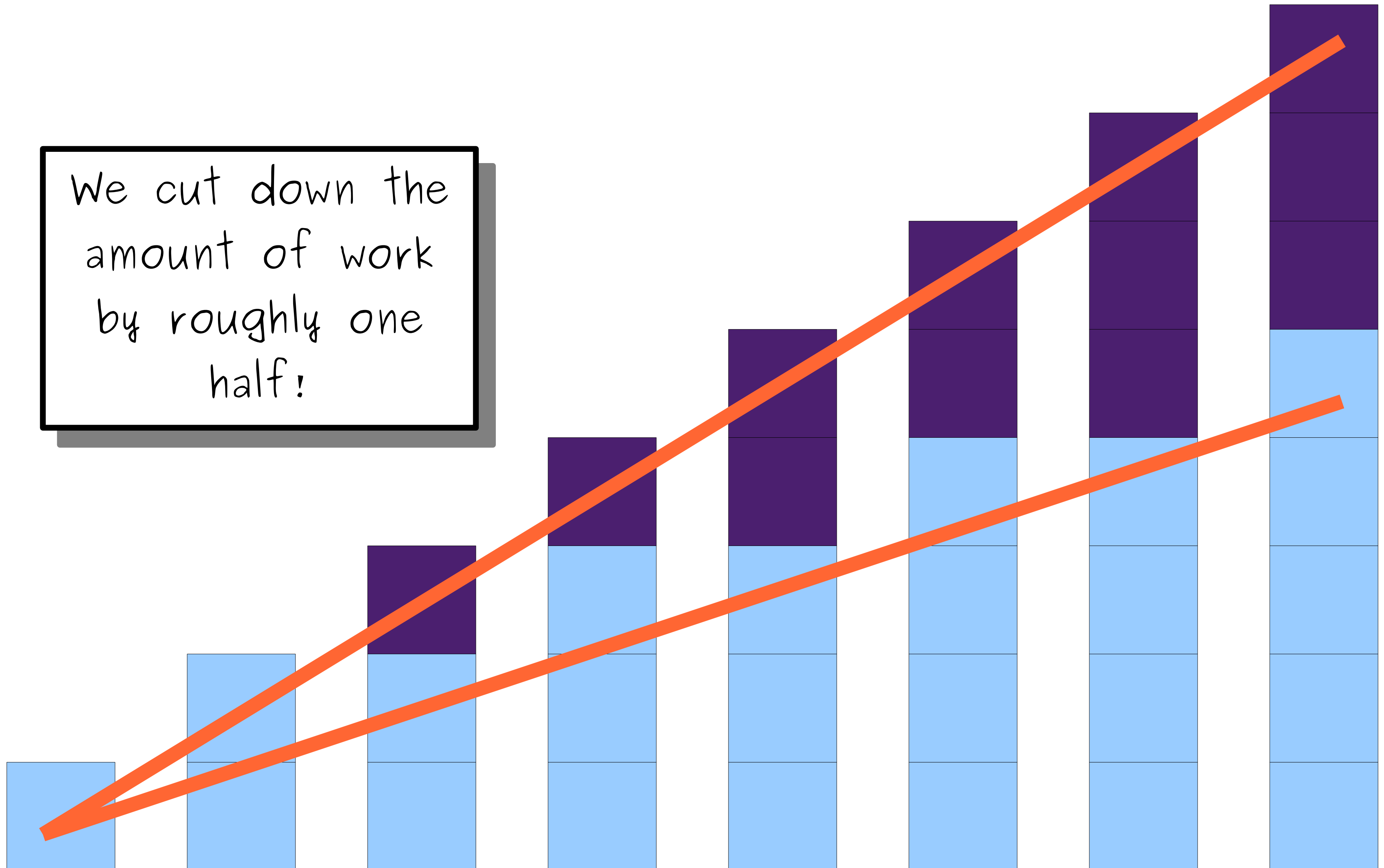
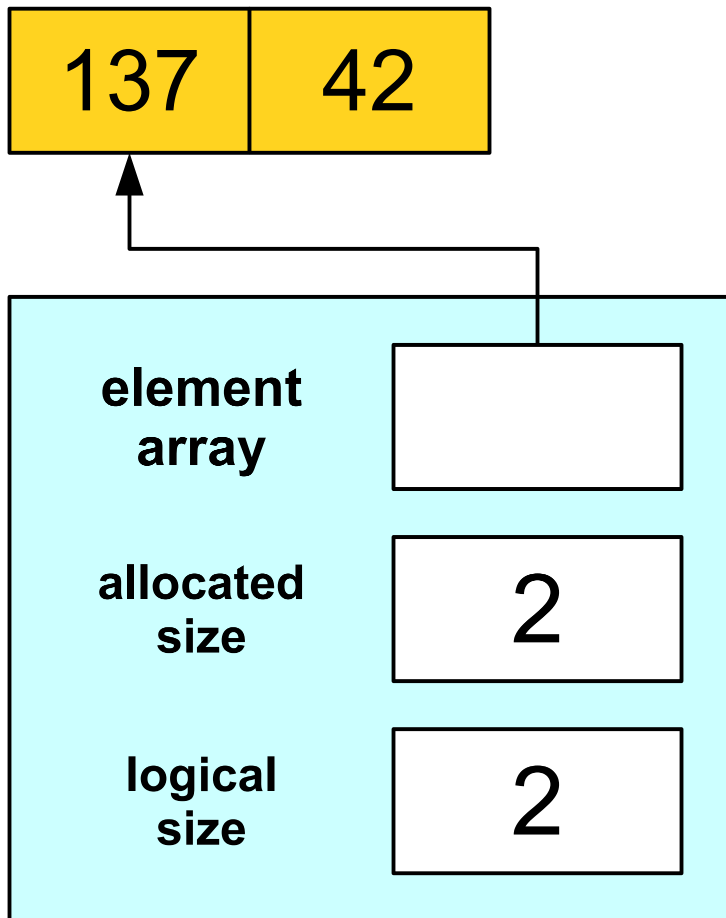


How does it stack up?

A Much Better Idea



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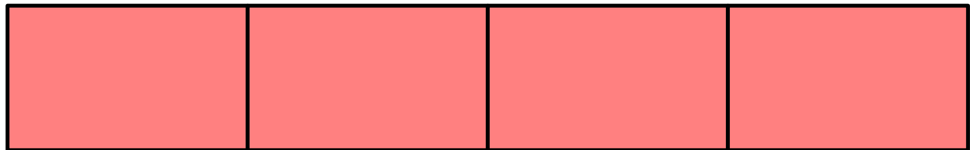

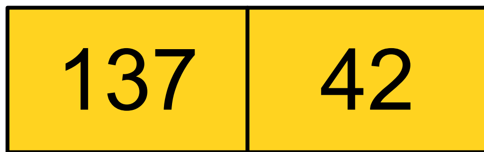

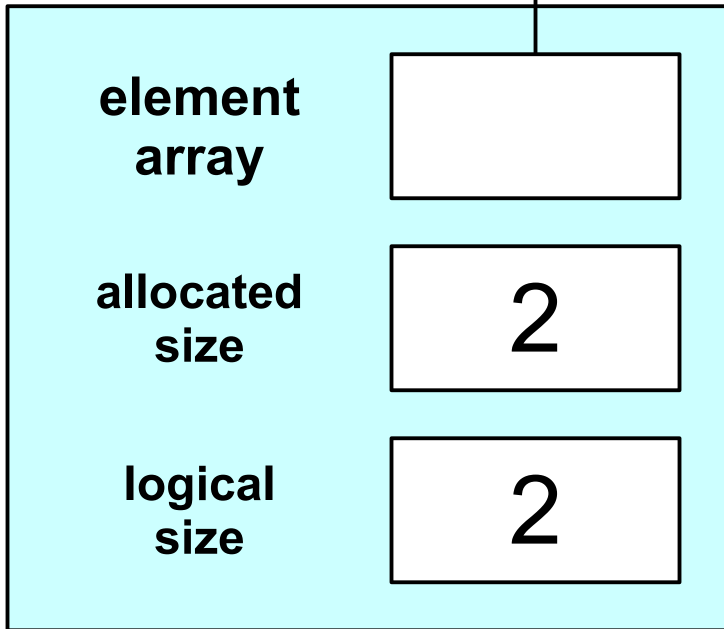

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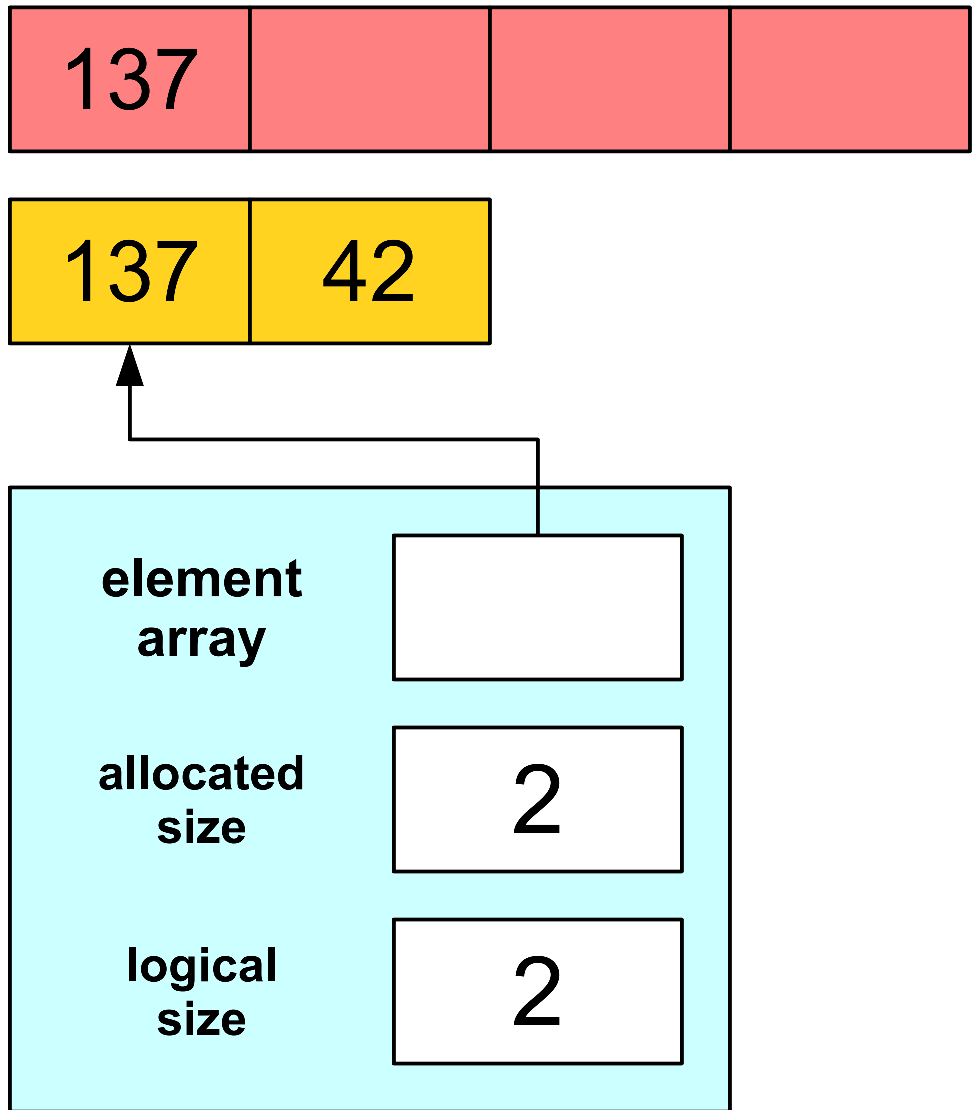

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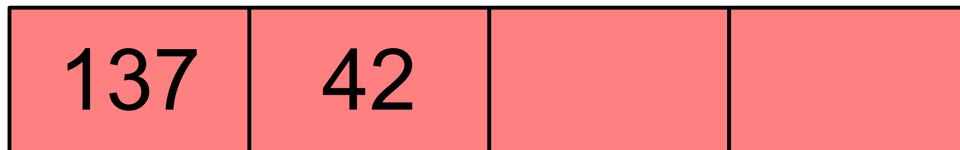

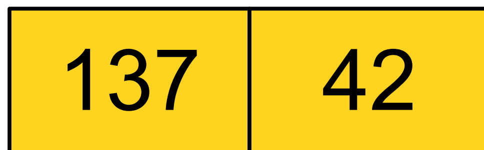

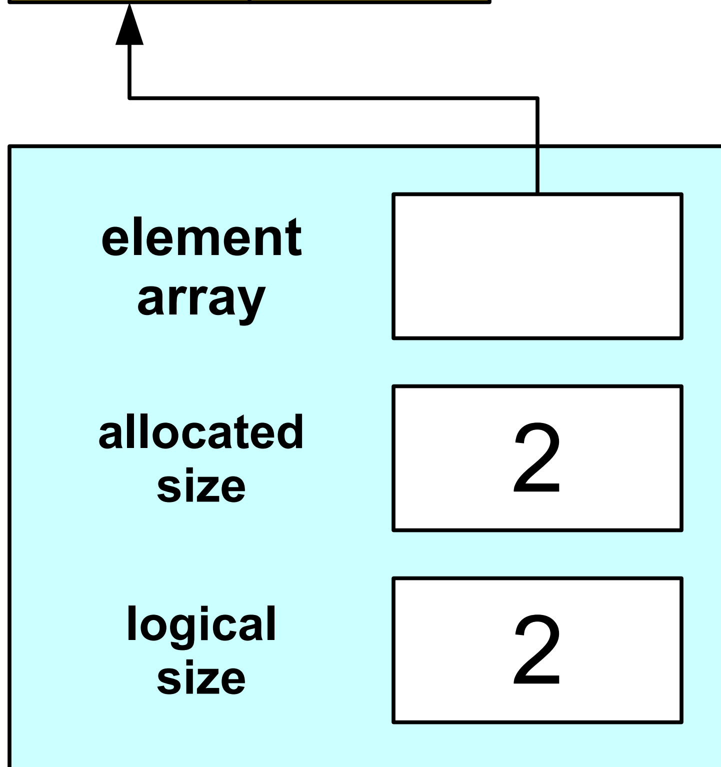

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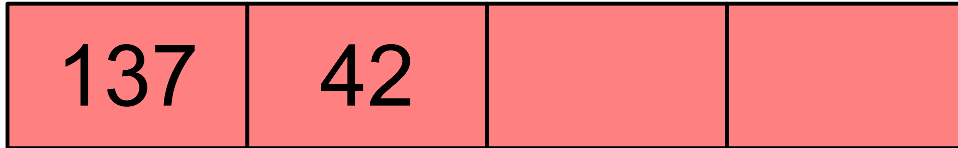

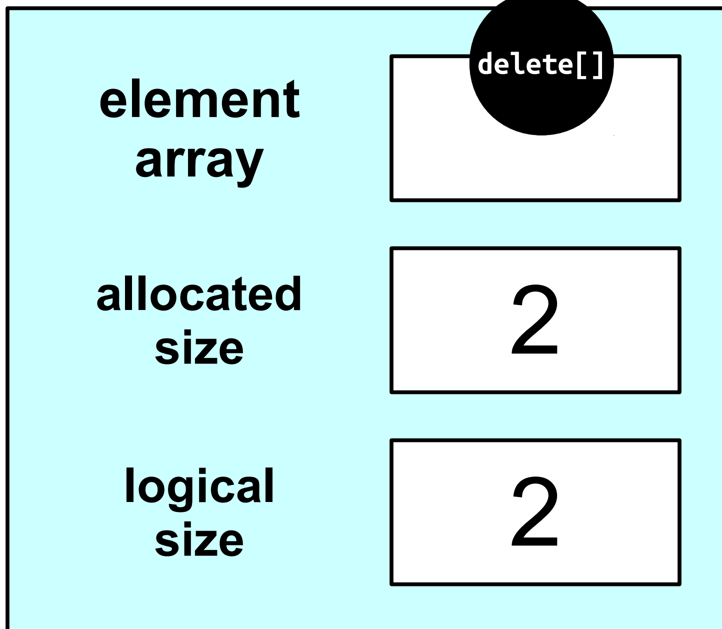

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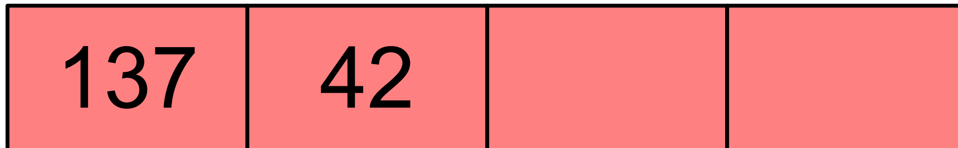

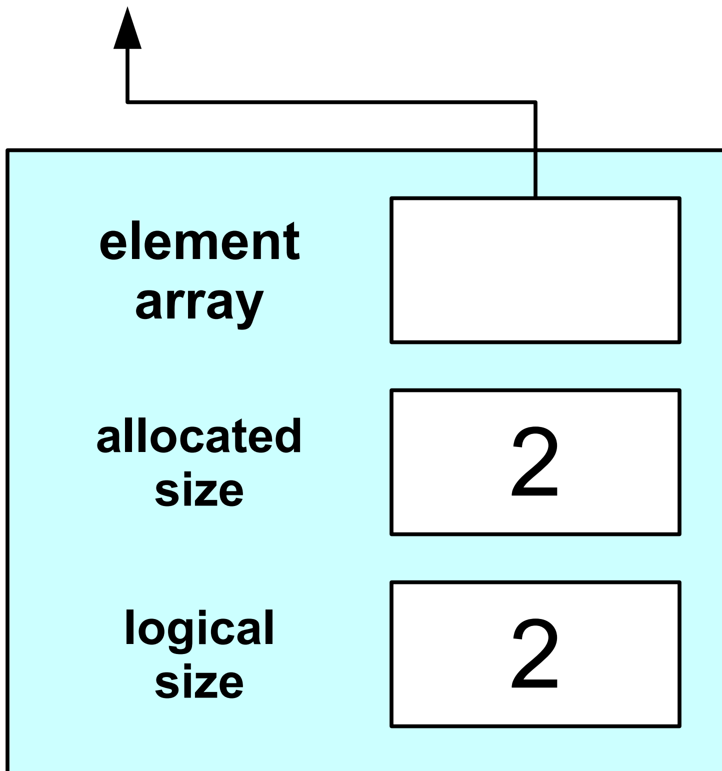

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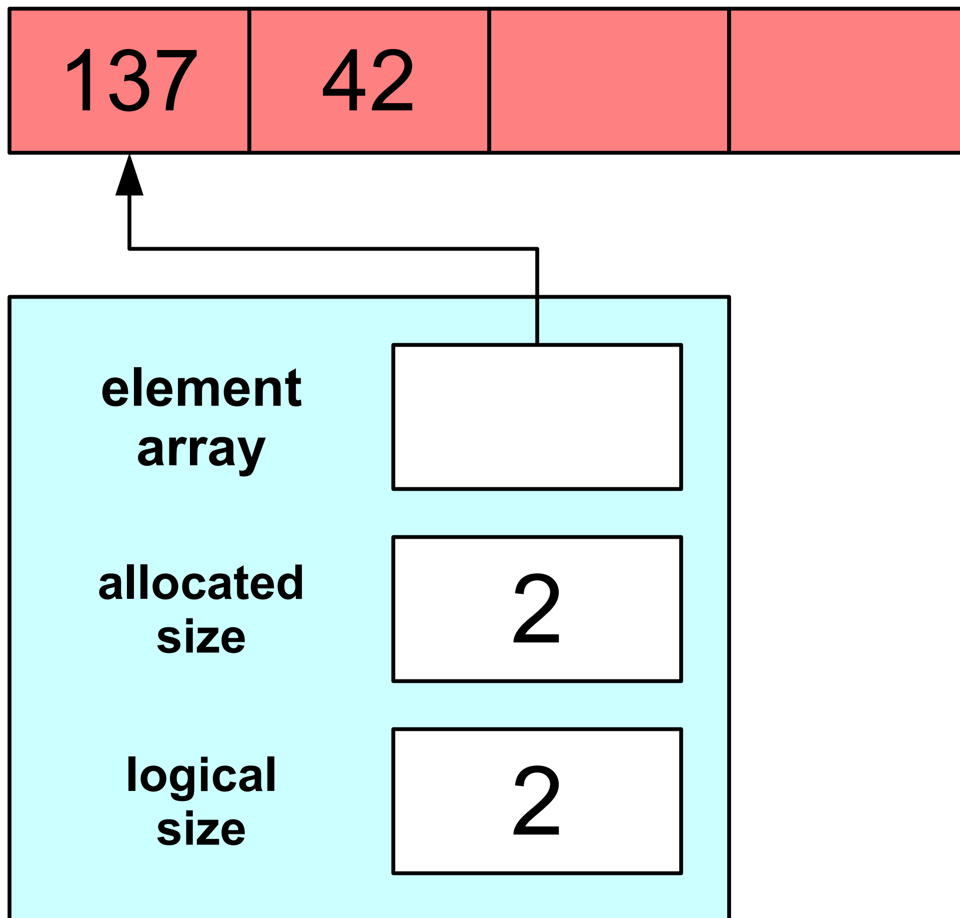

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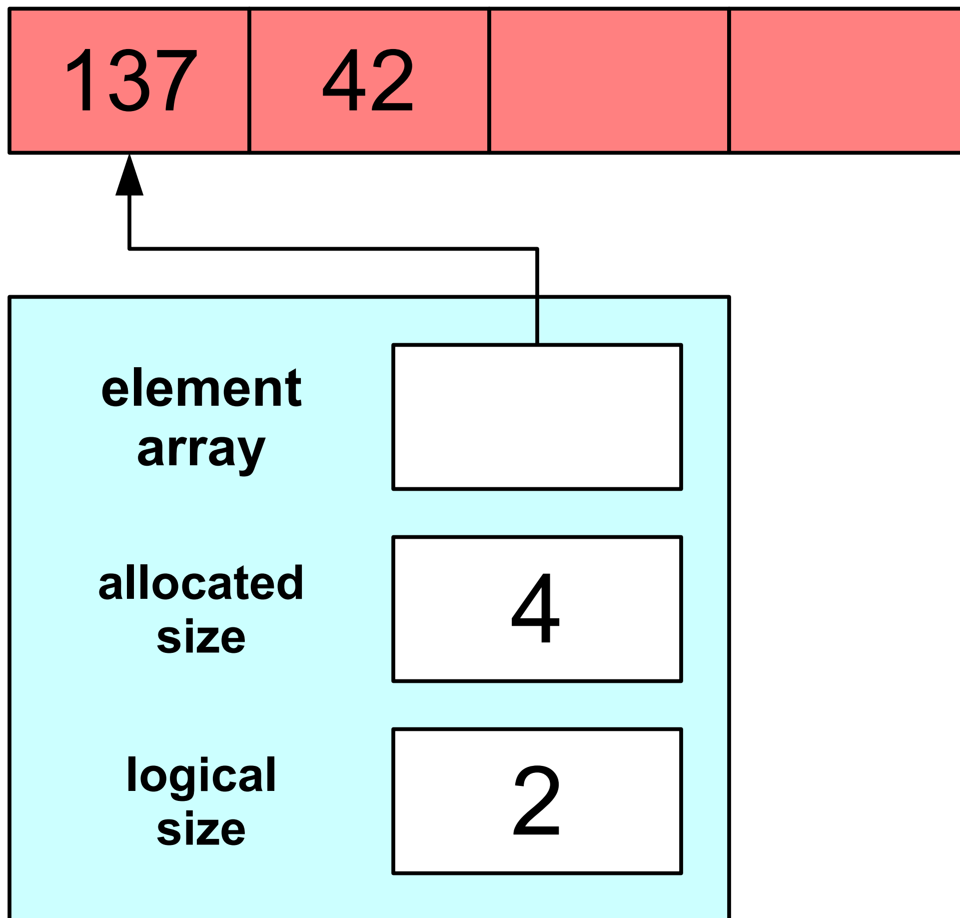

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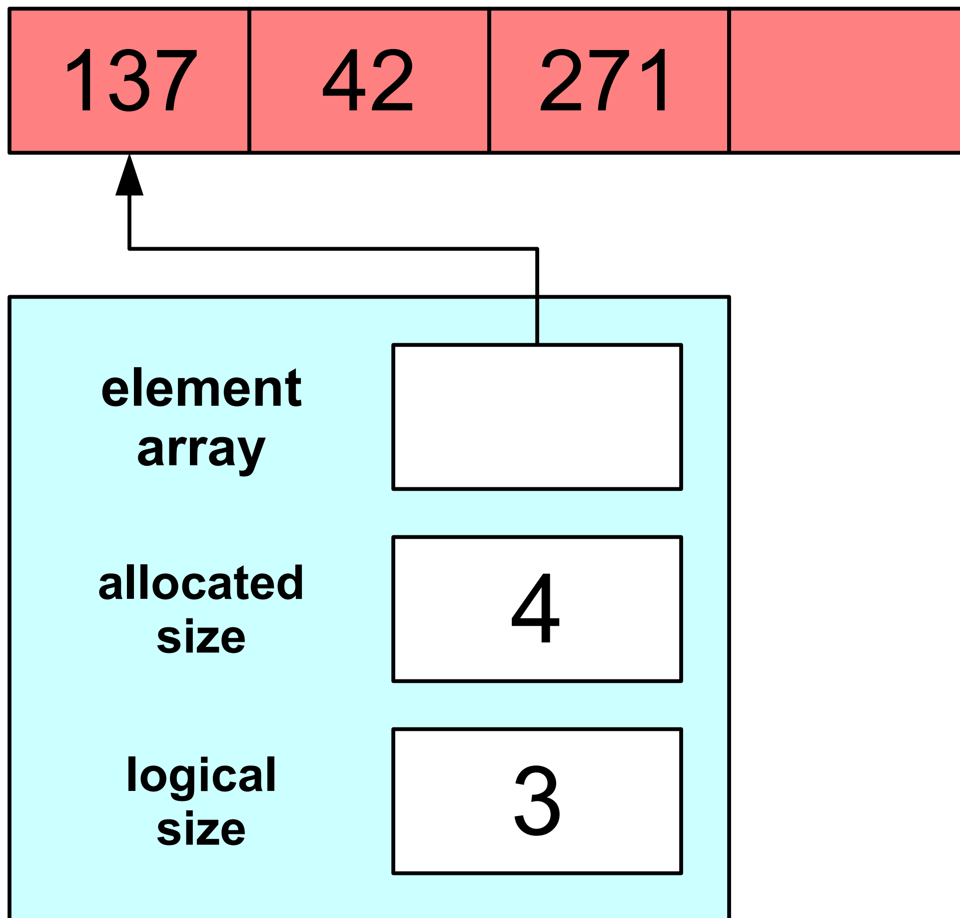

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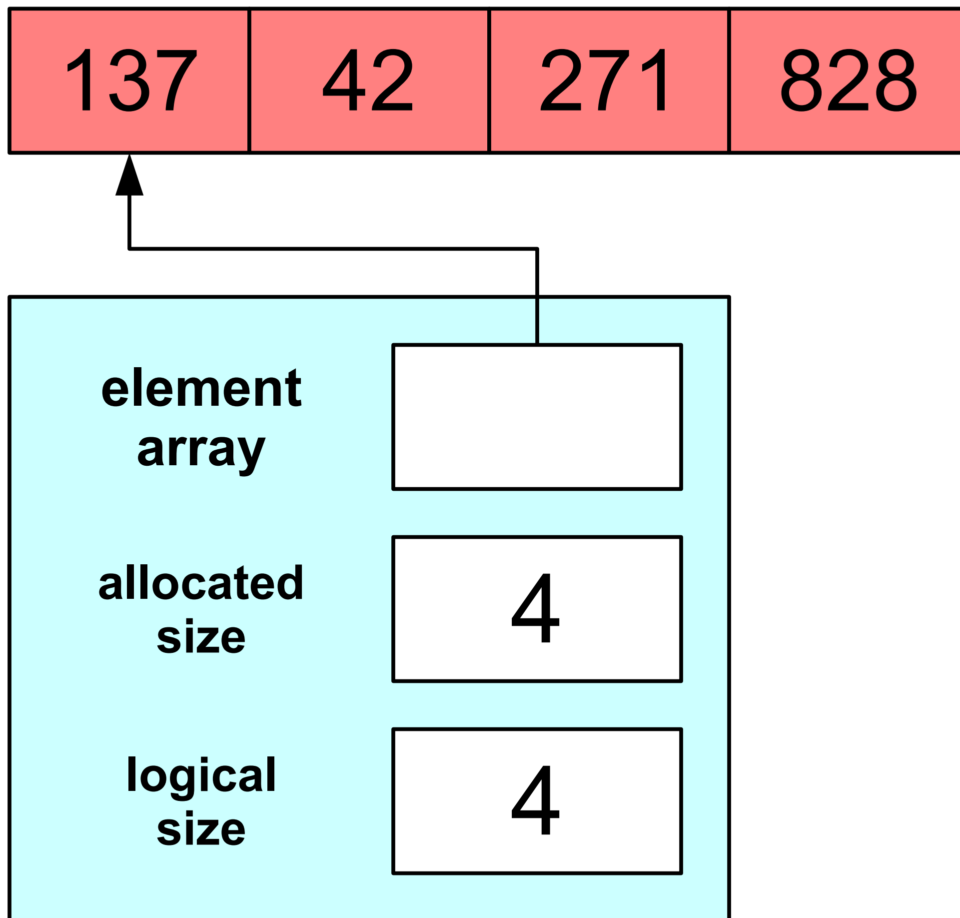

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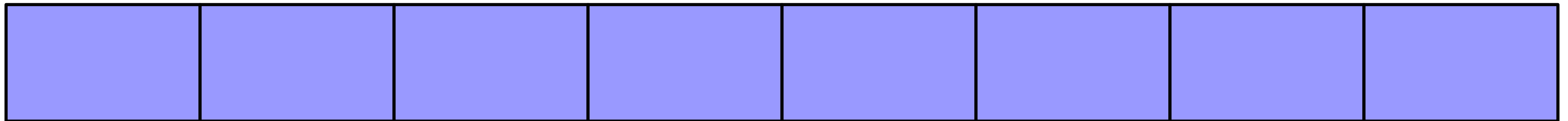

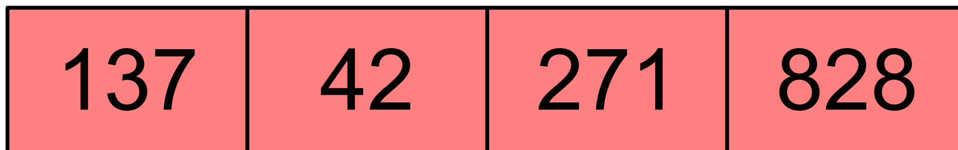

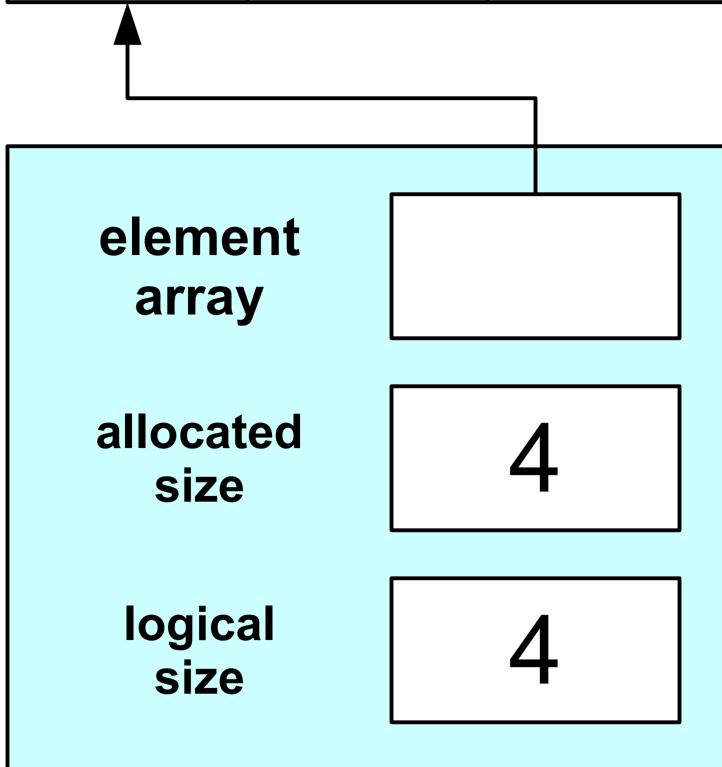

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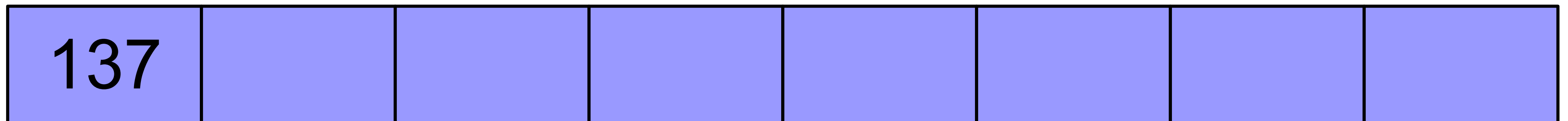

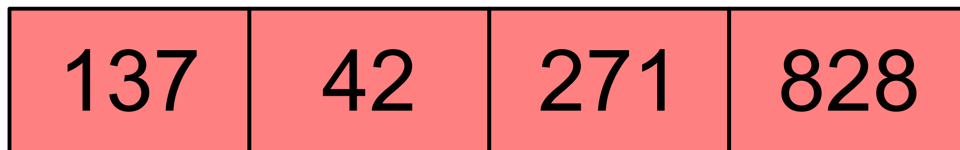

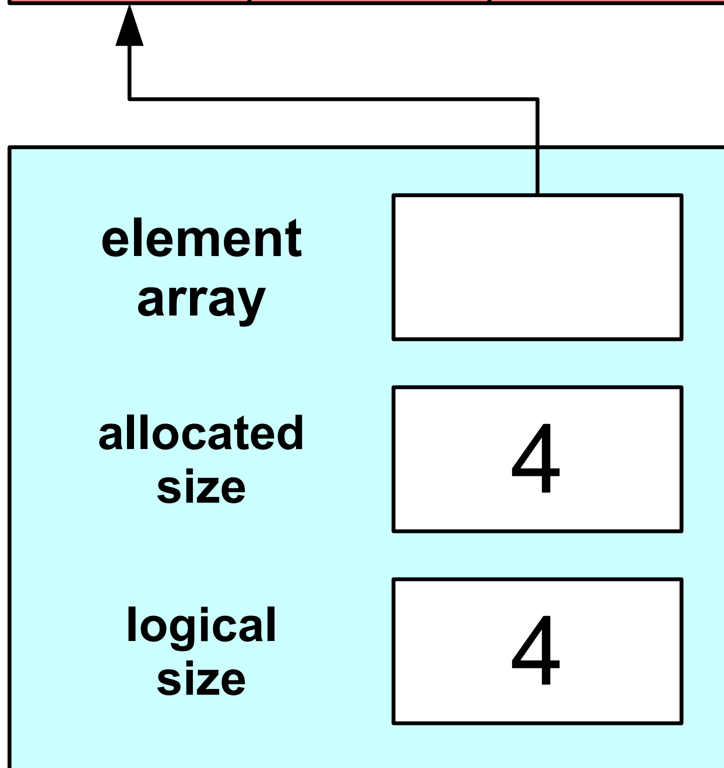

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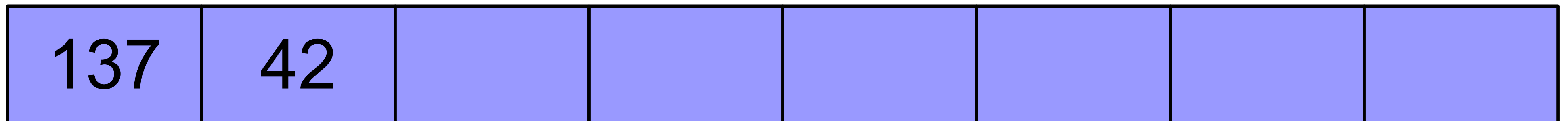

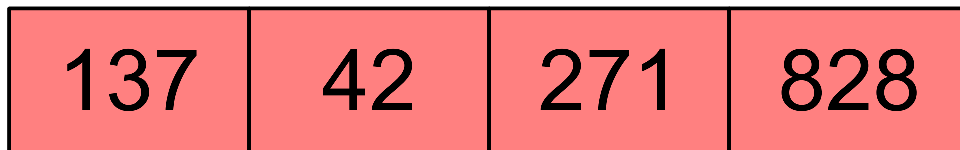

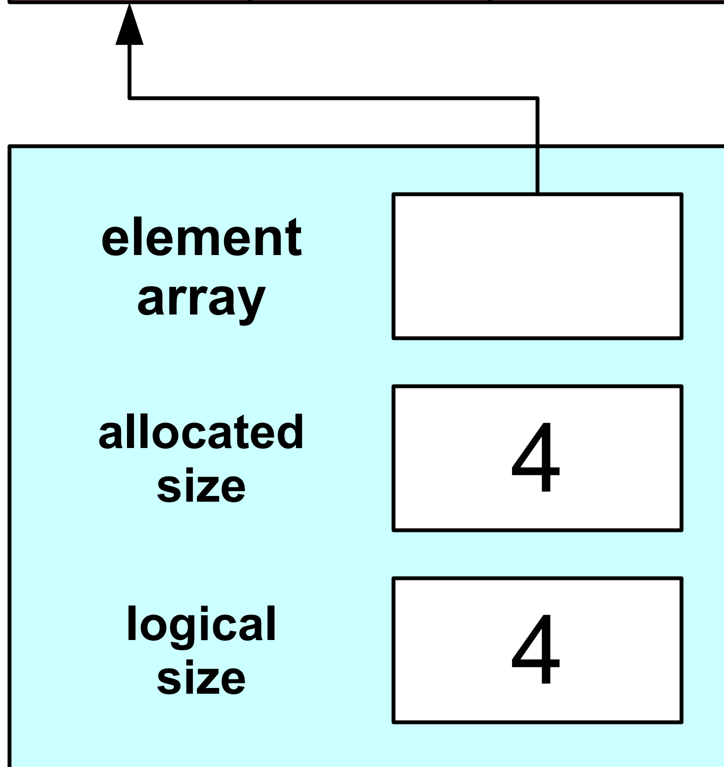

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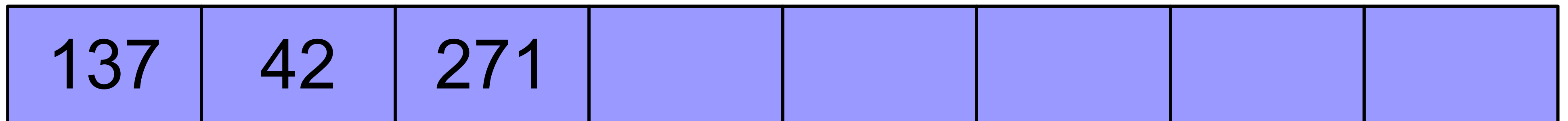

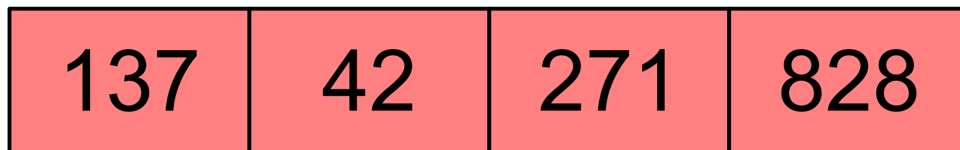

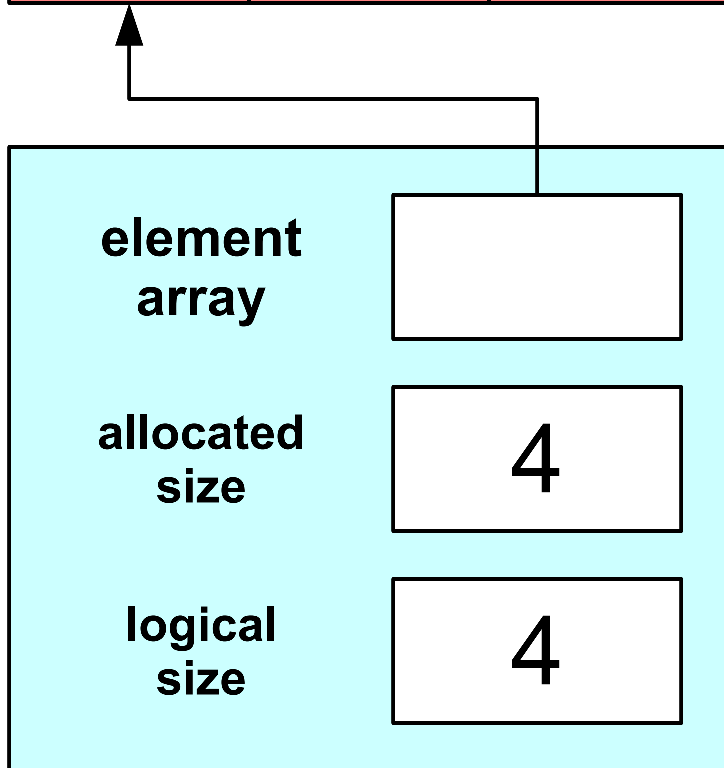

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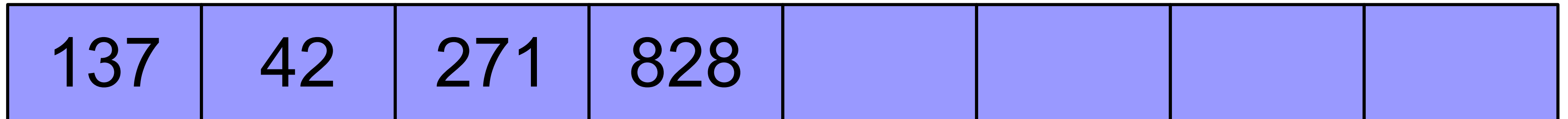

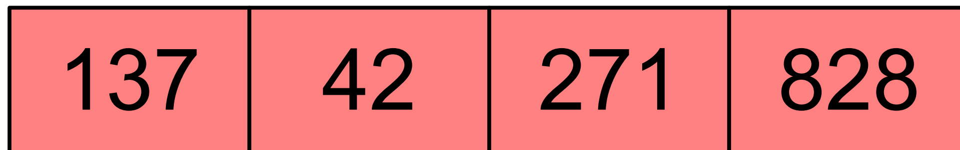

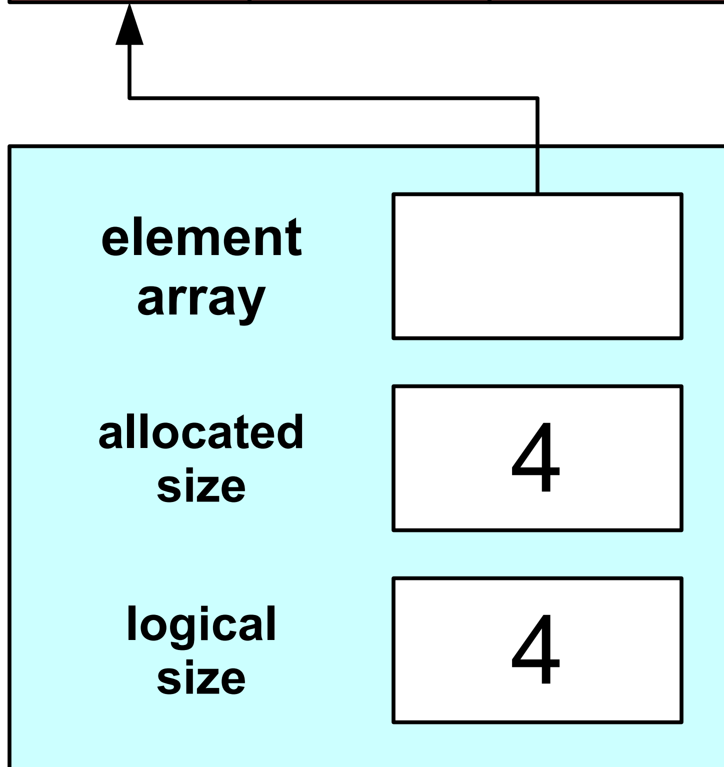

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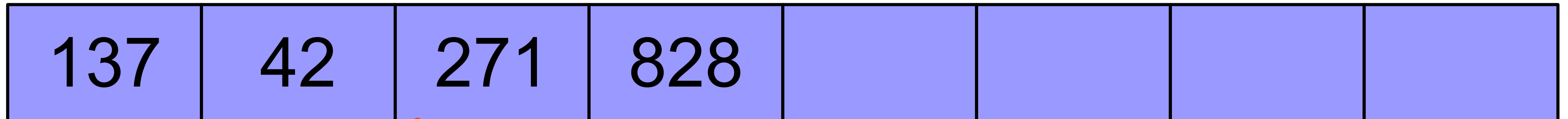

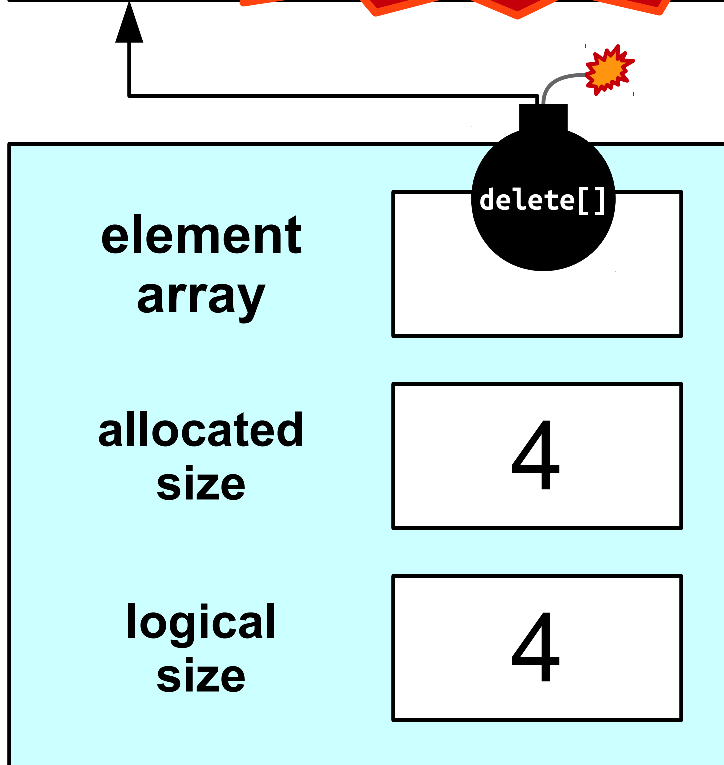

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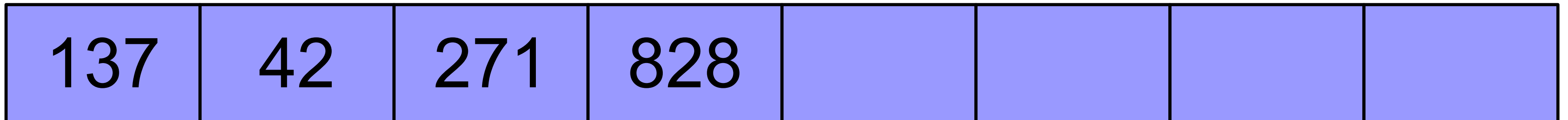

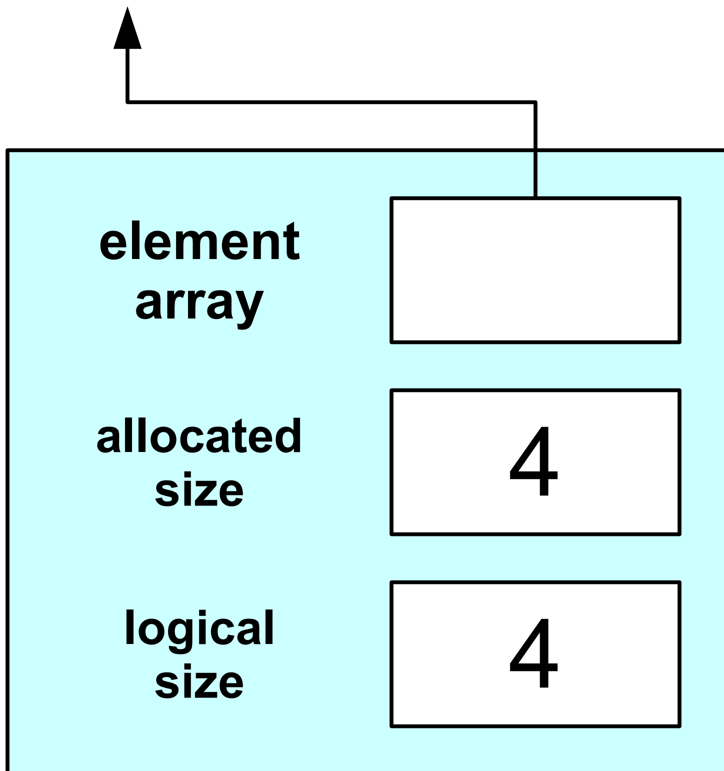

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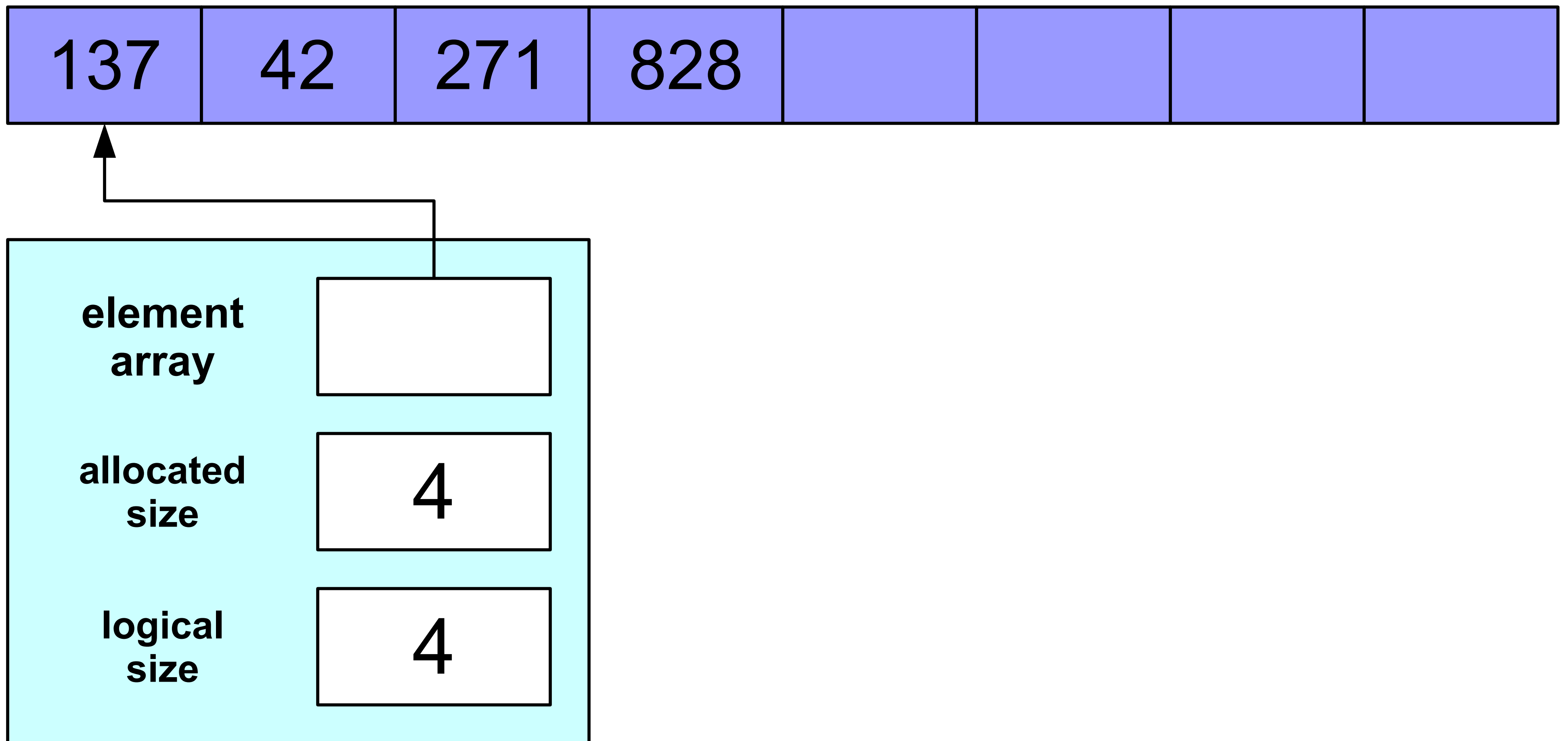

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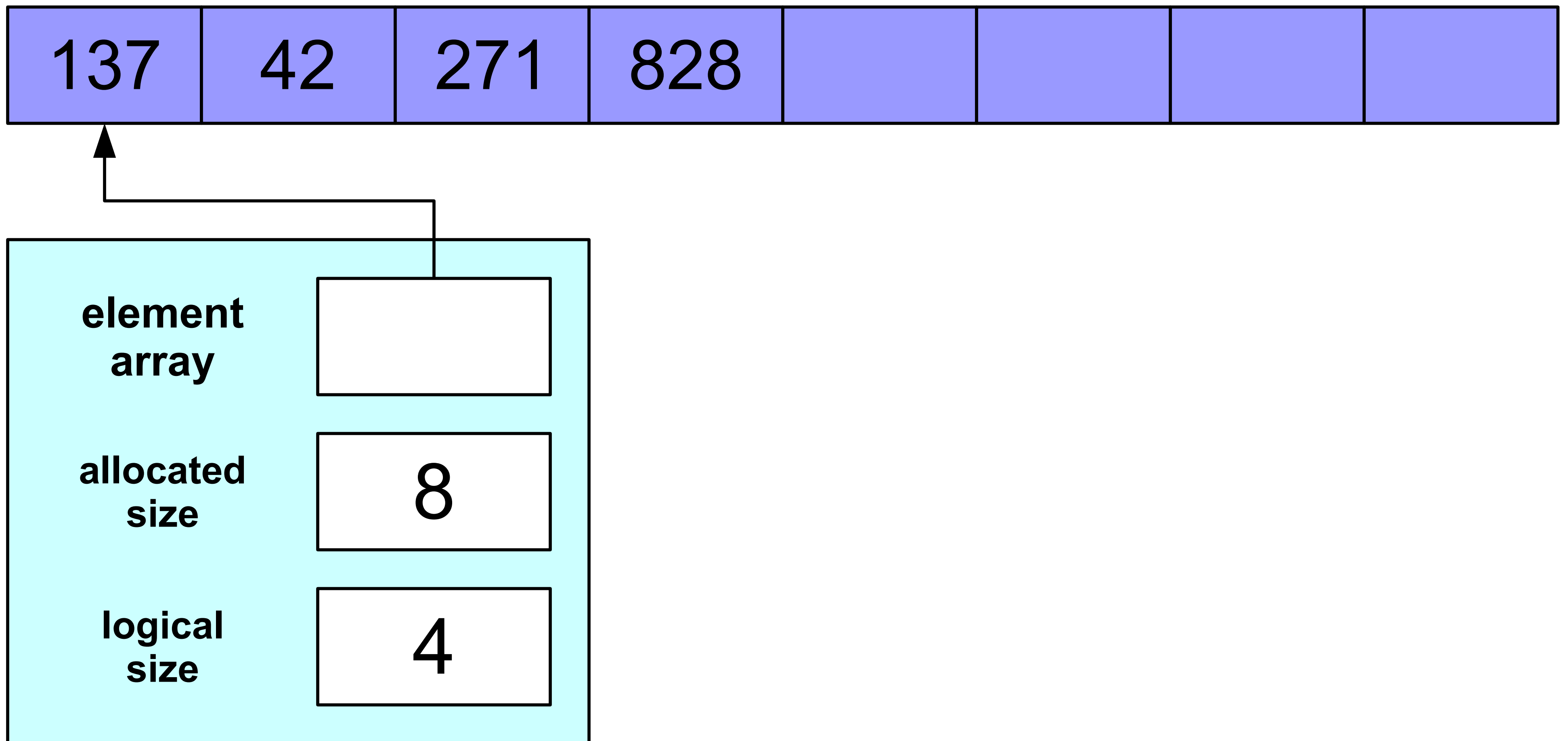

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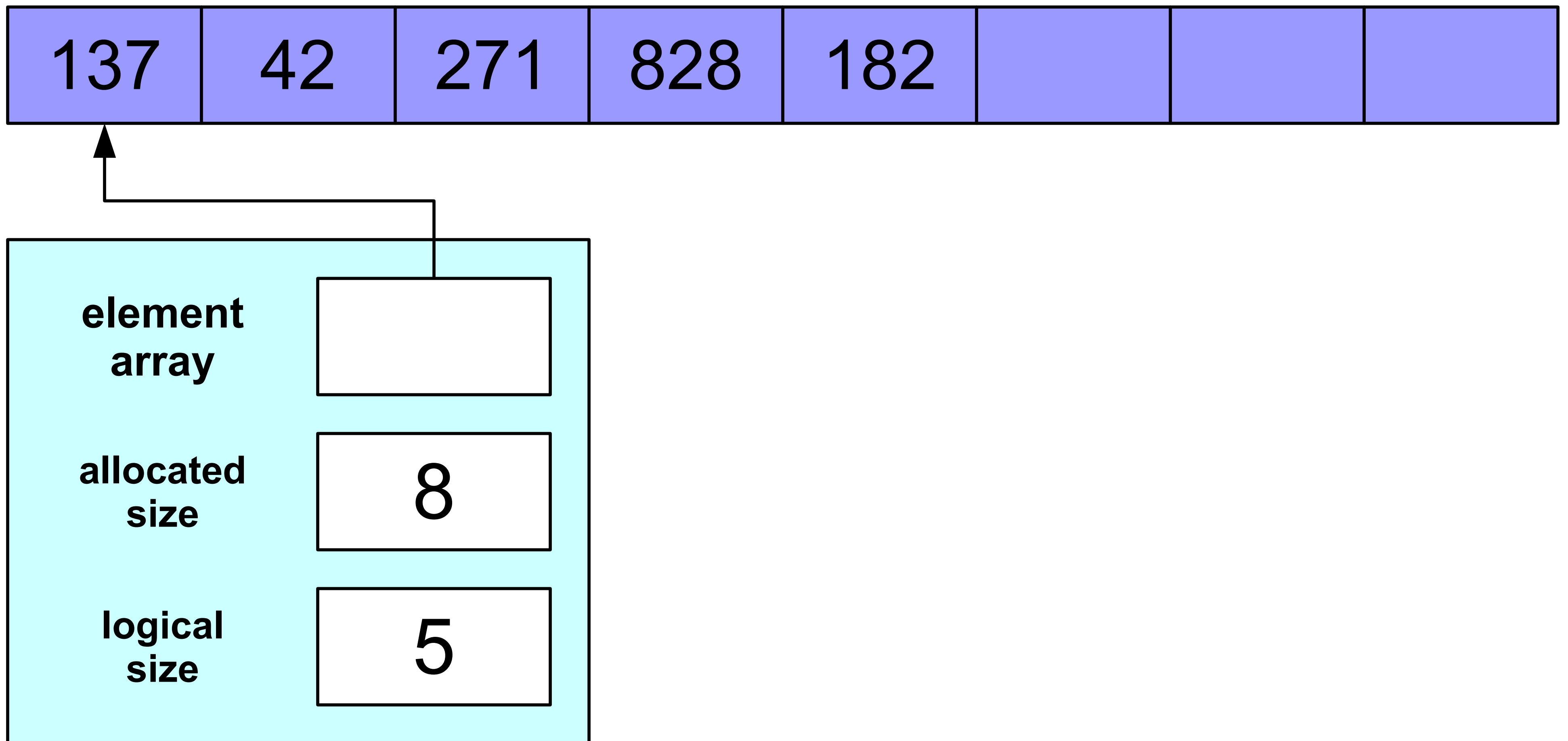

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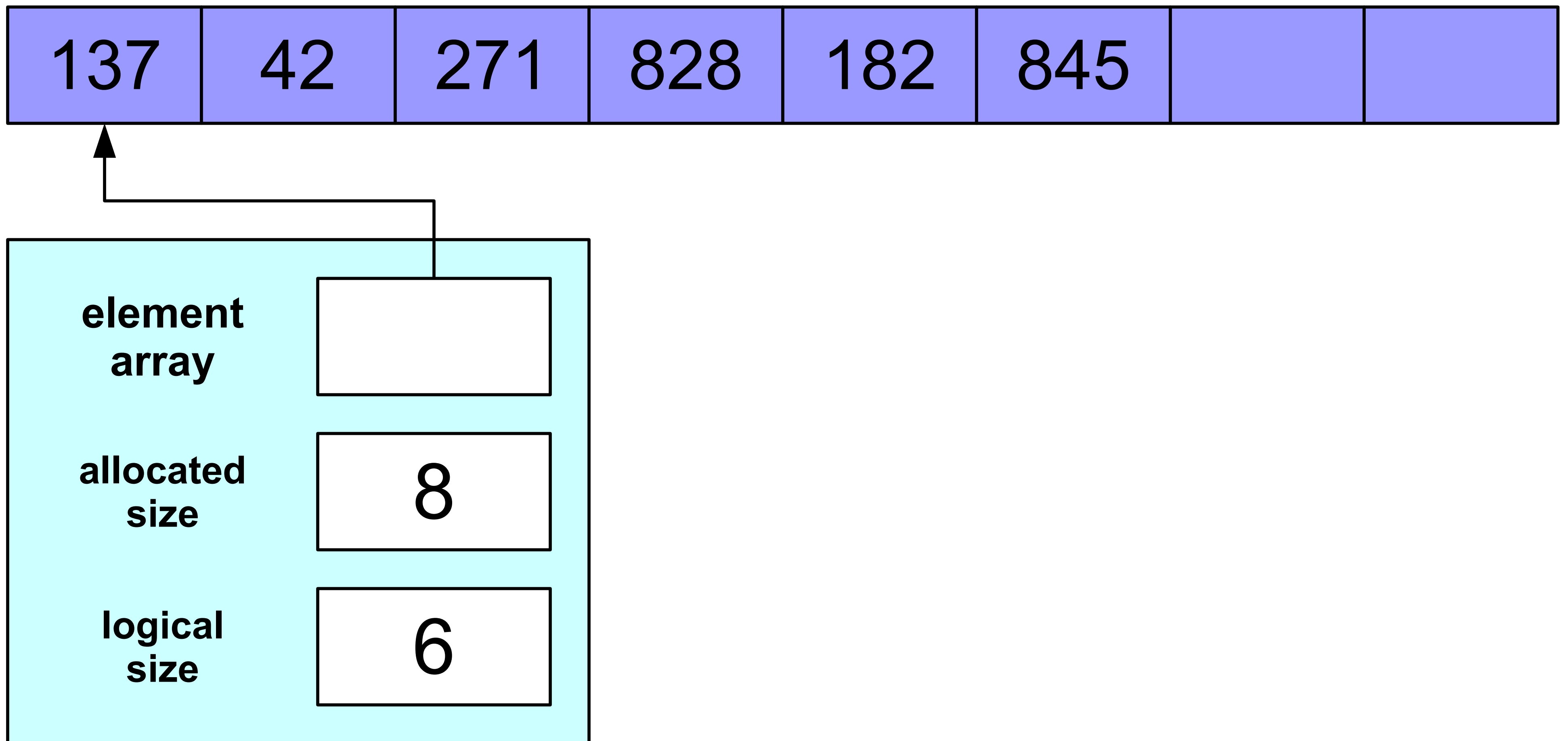

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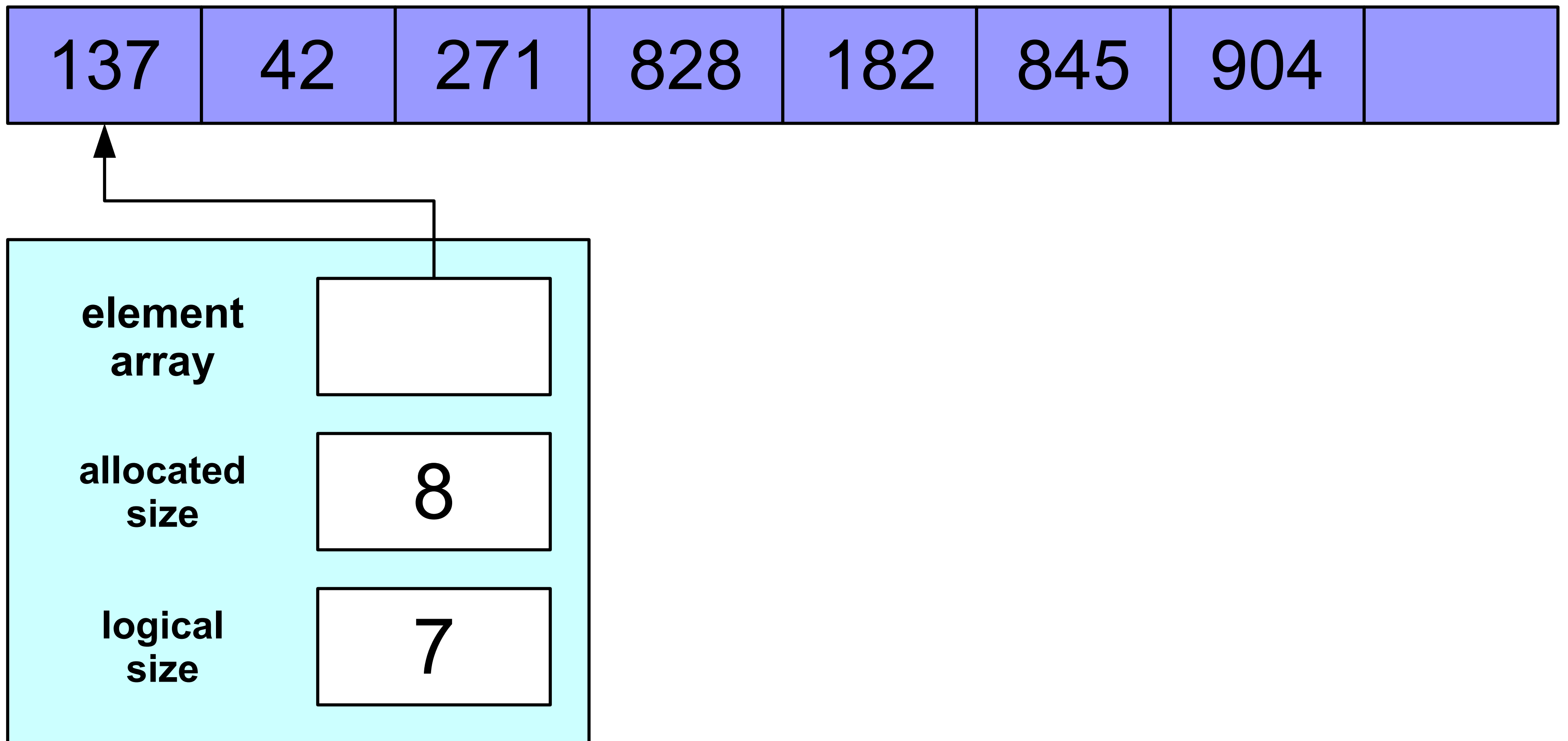

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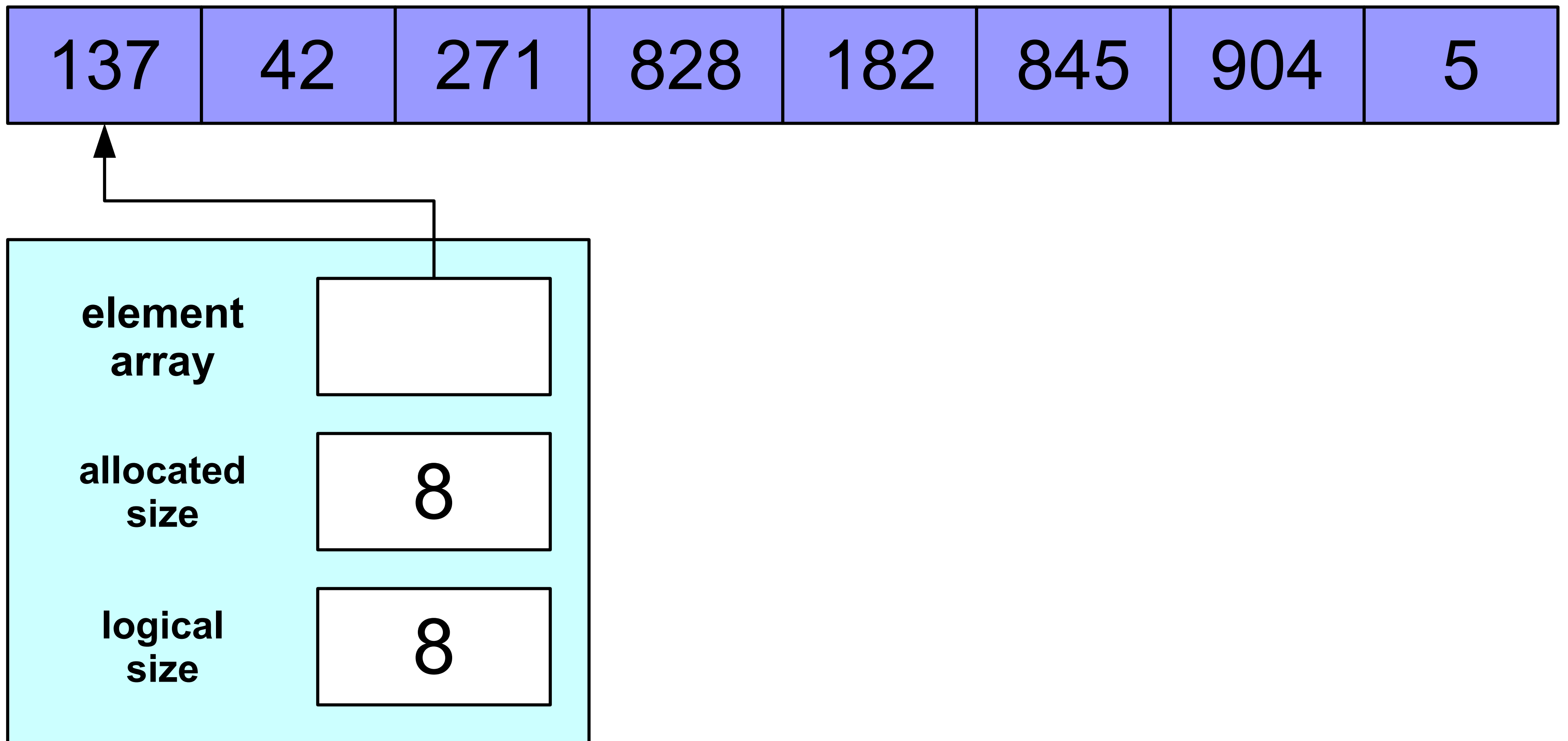
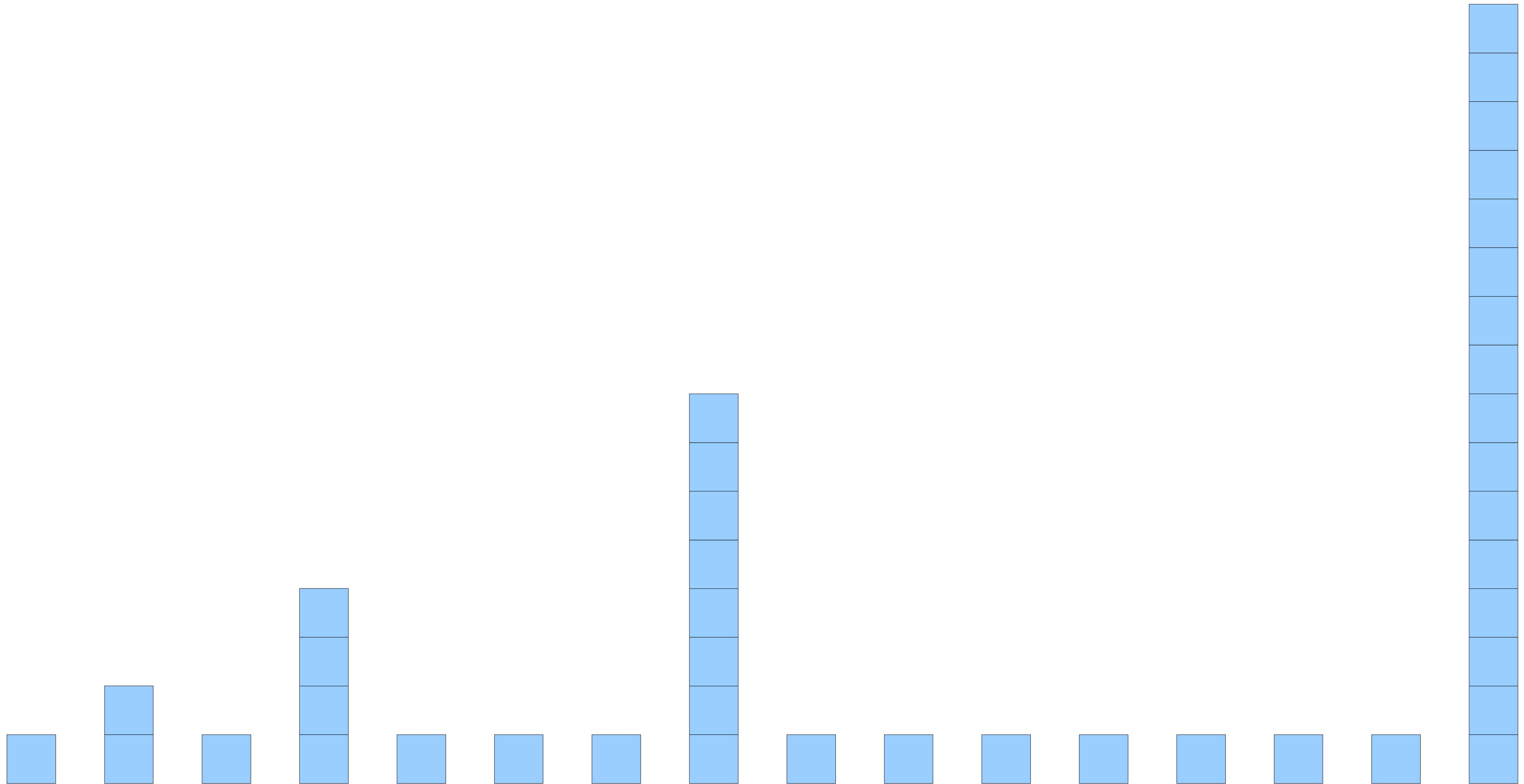


Let's Give it a Try!

How do we analyze this?

Spreading the Work



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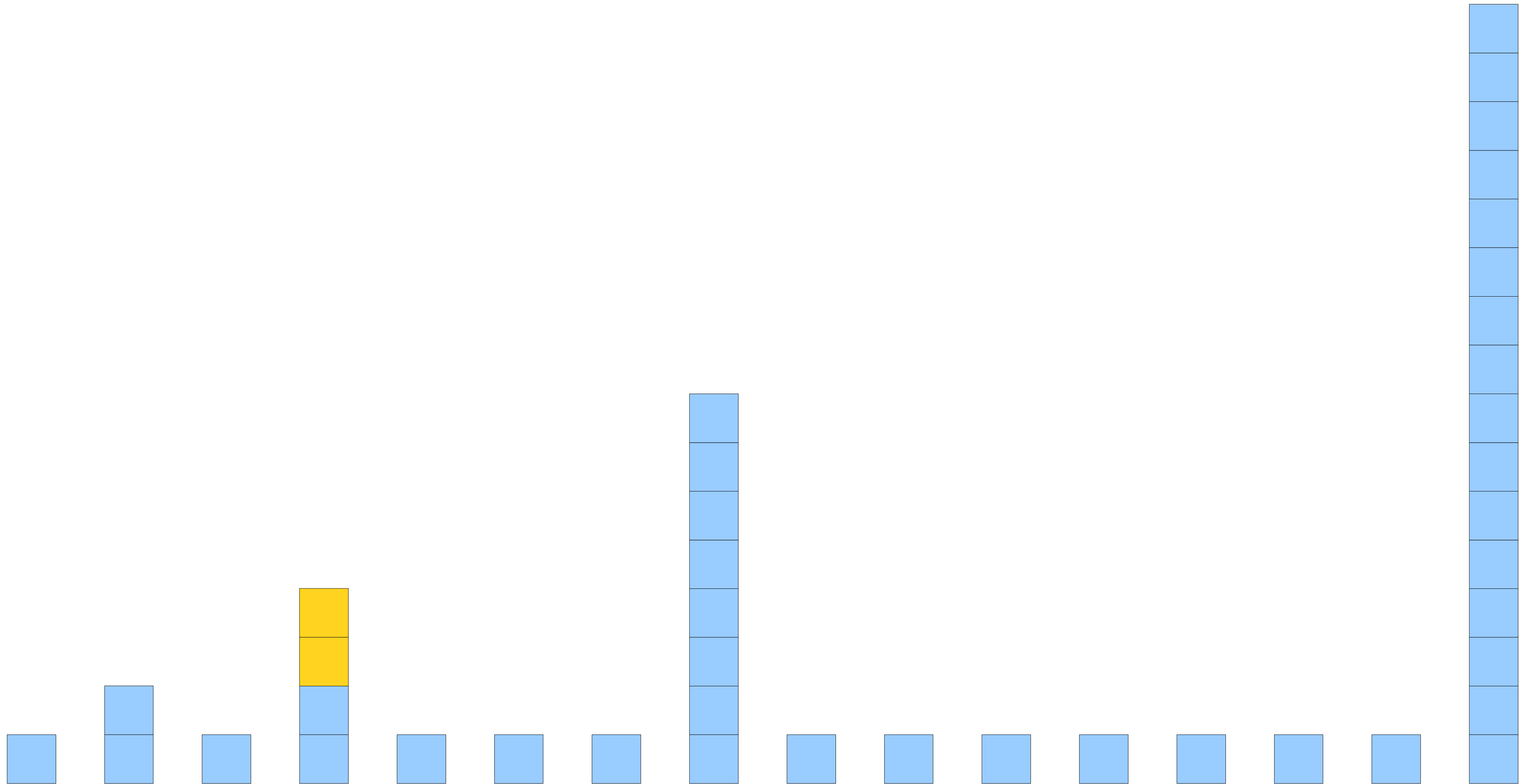

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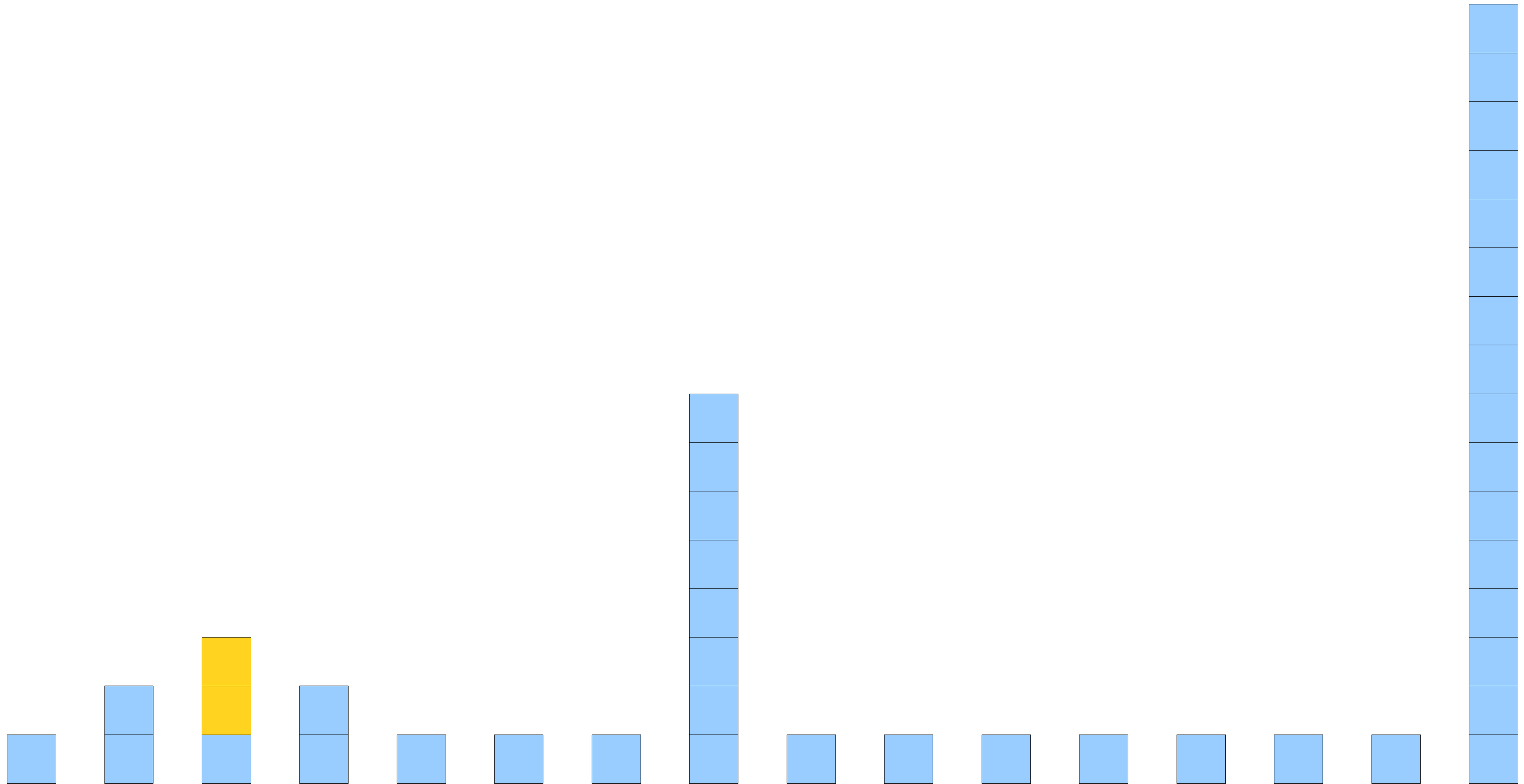

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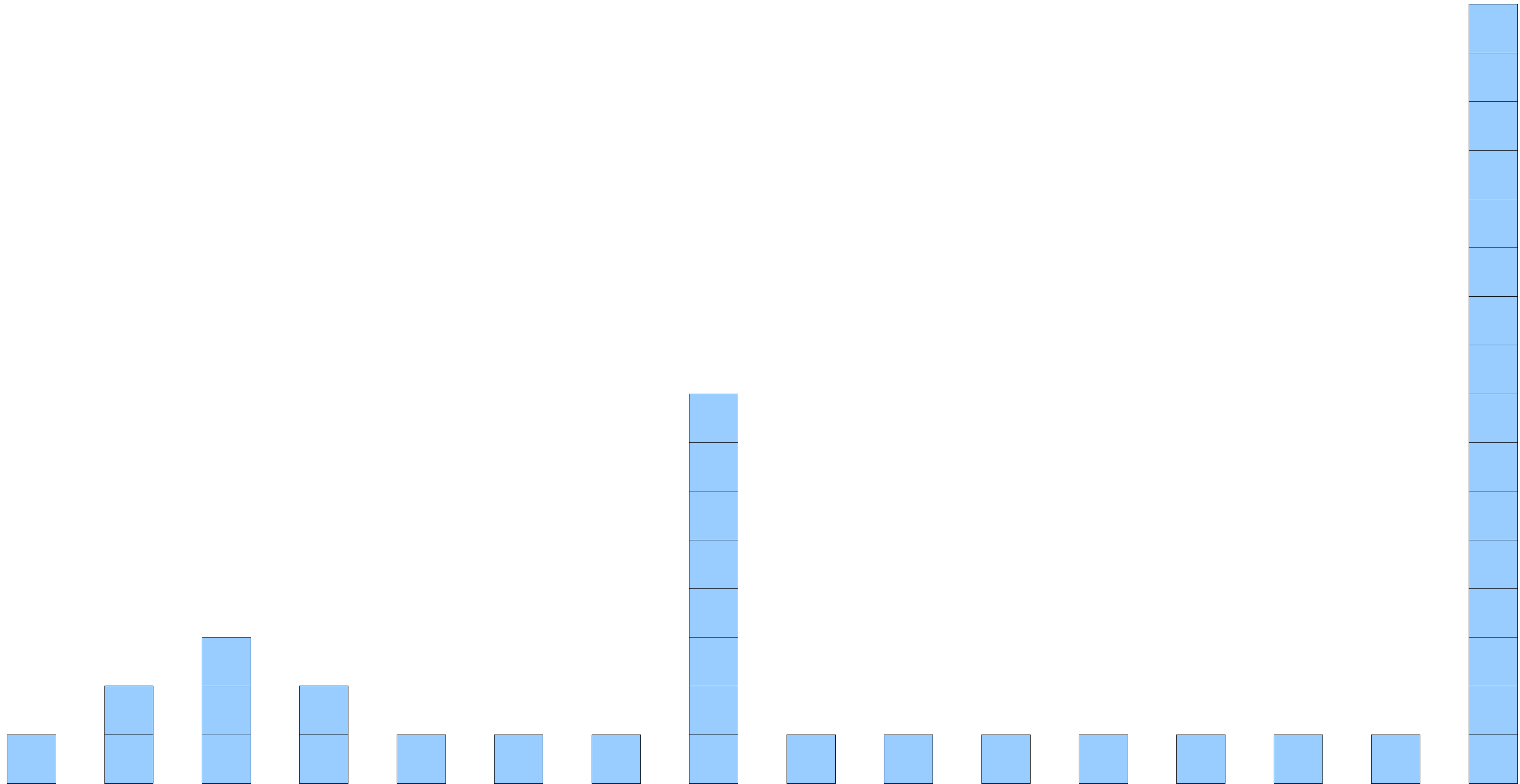

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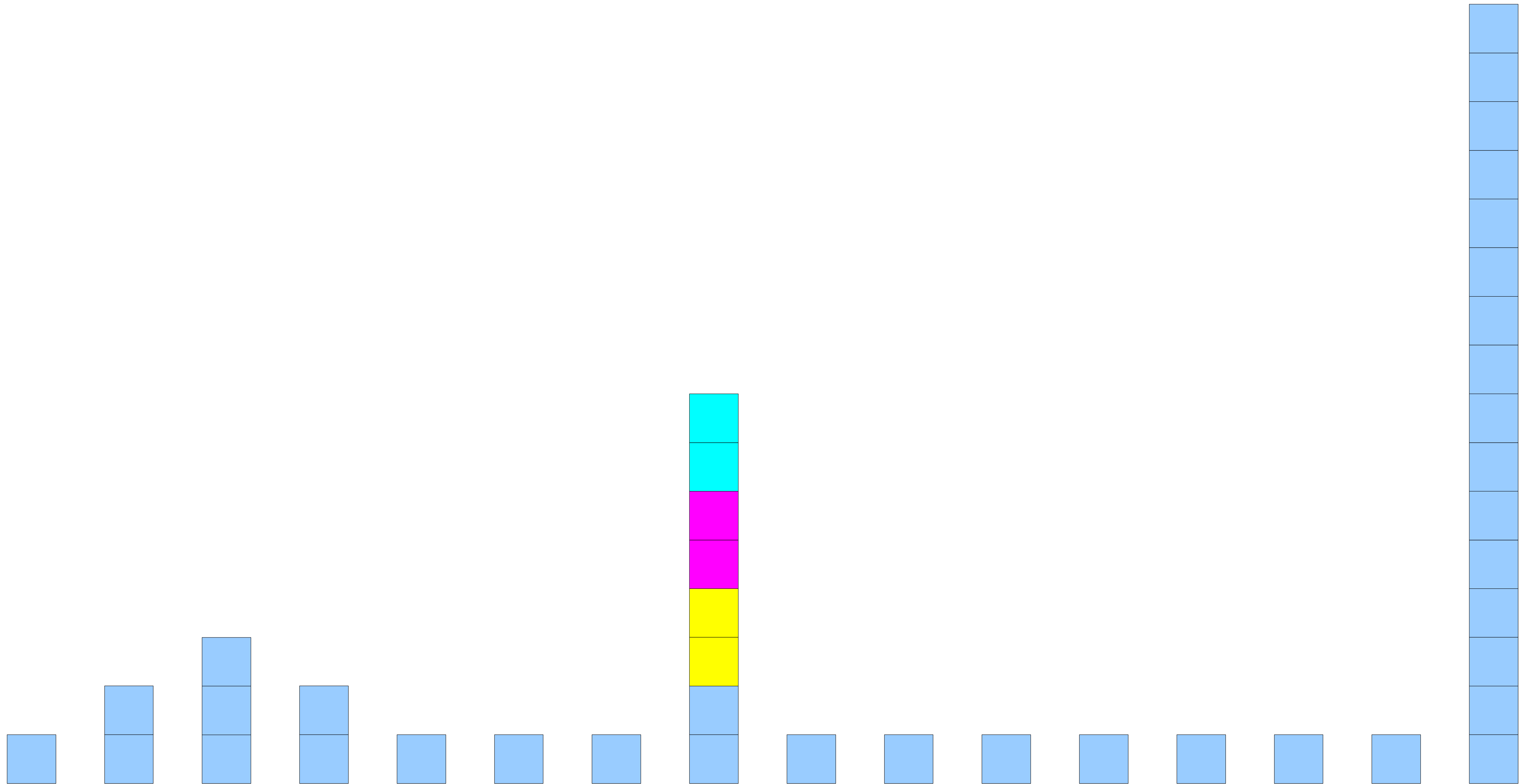

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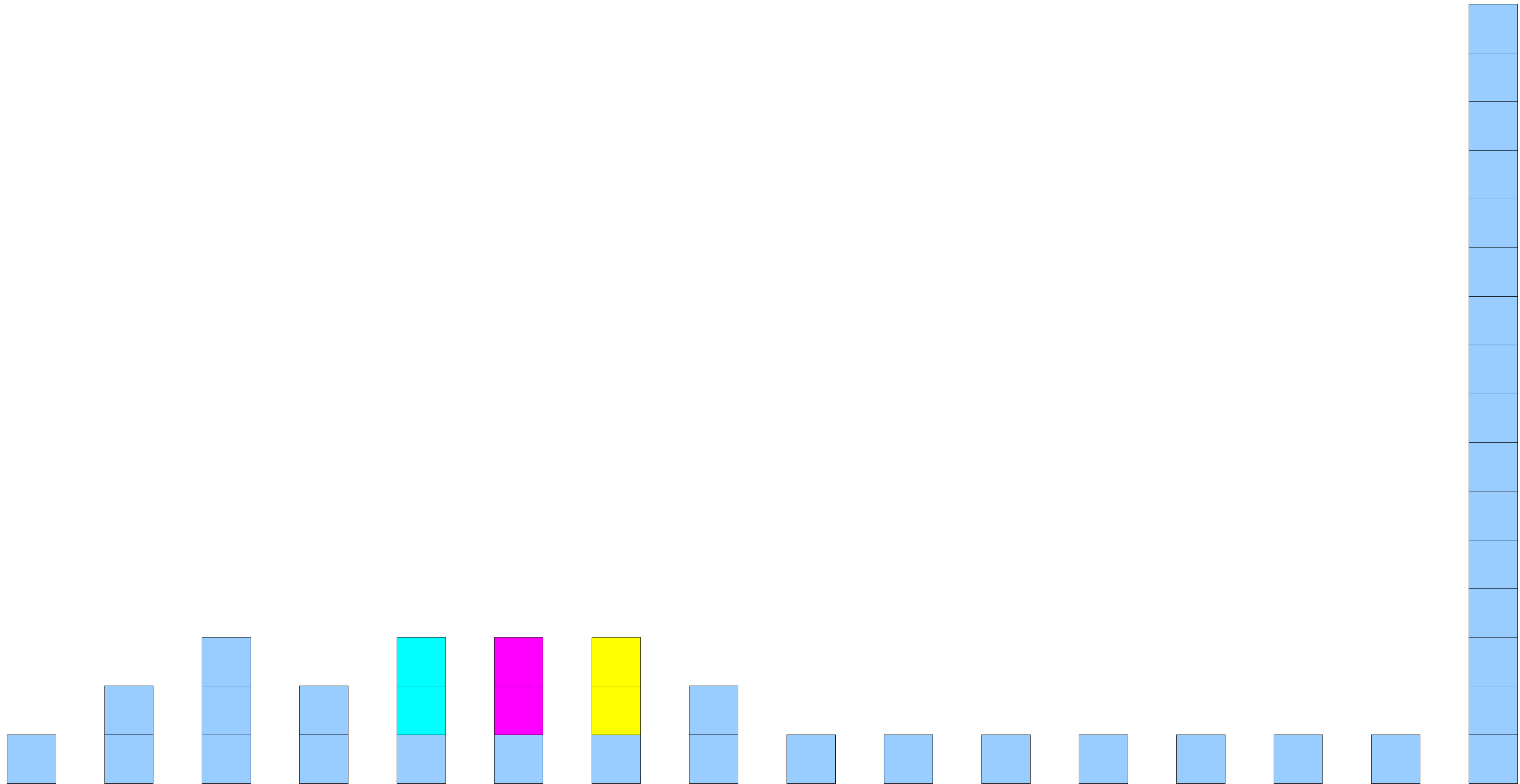

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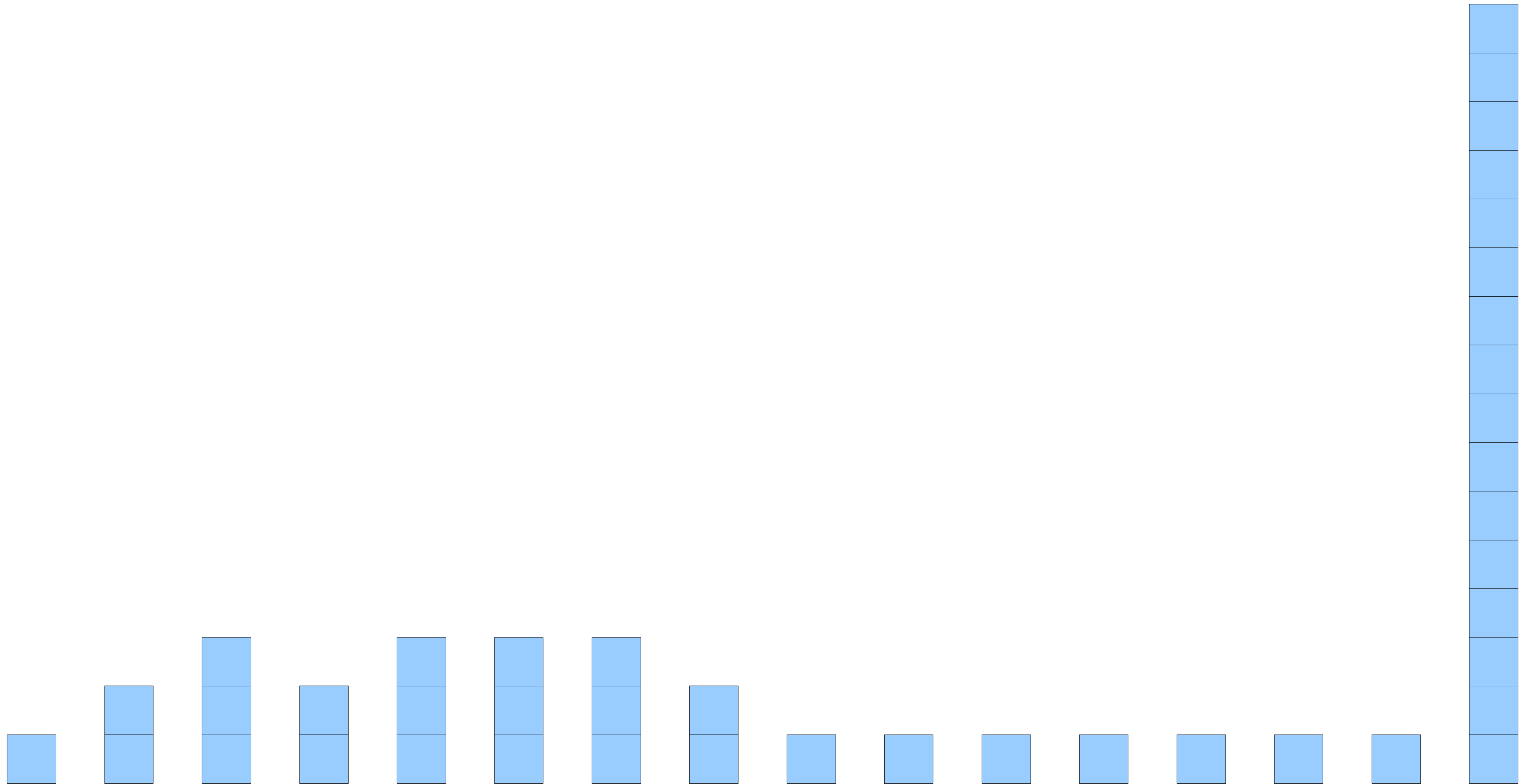

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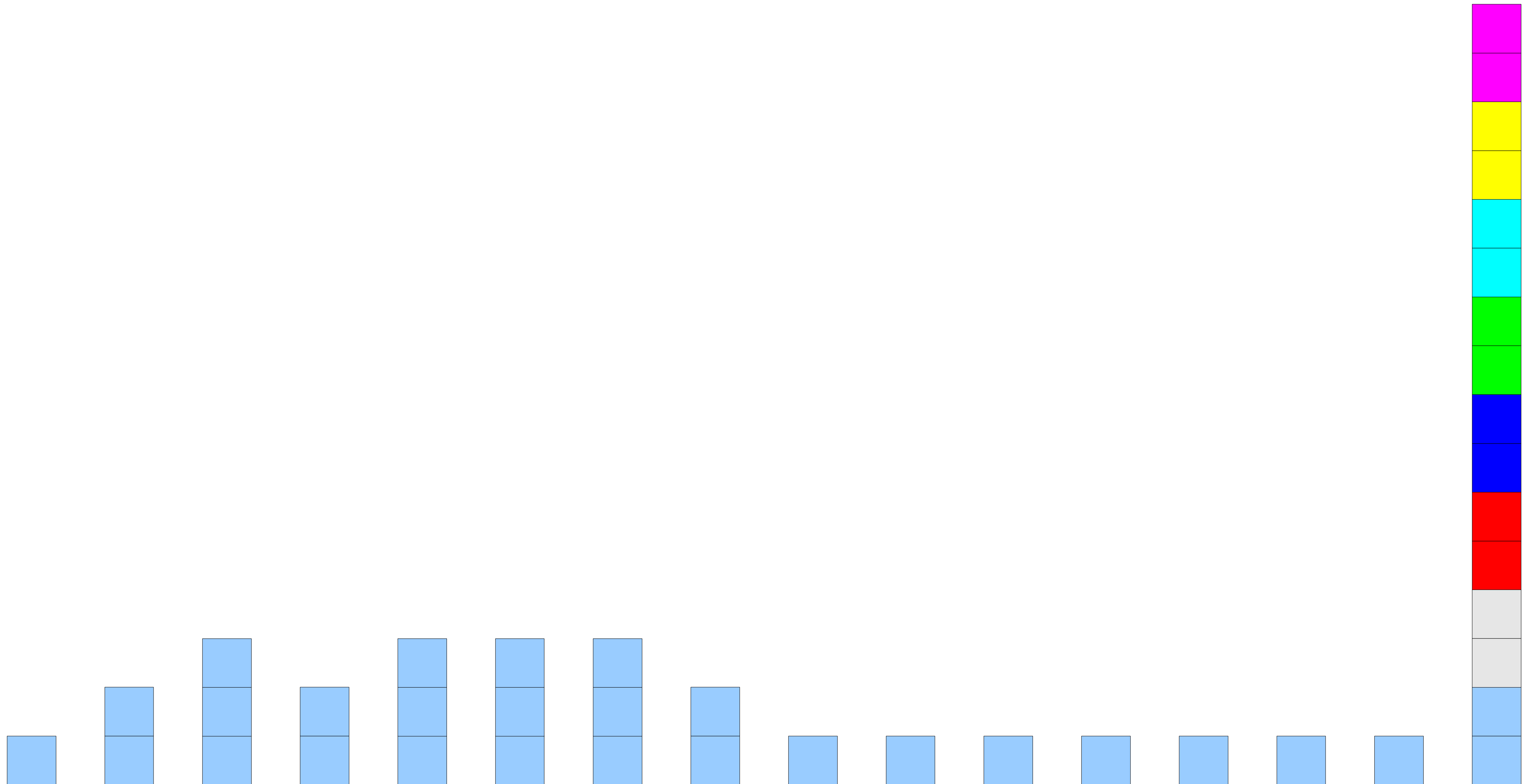

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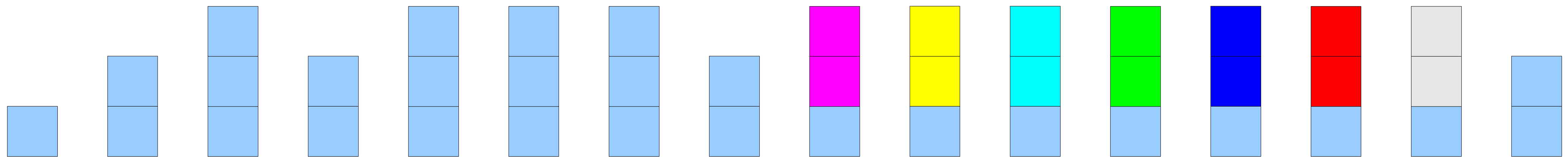

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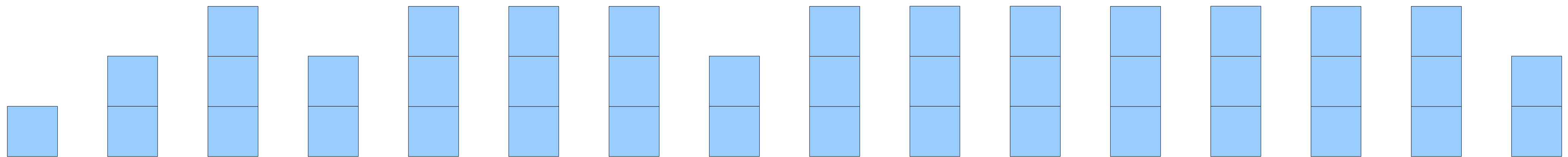
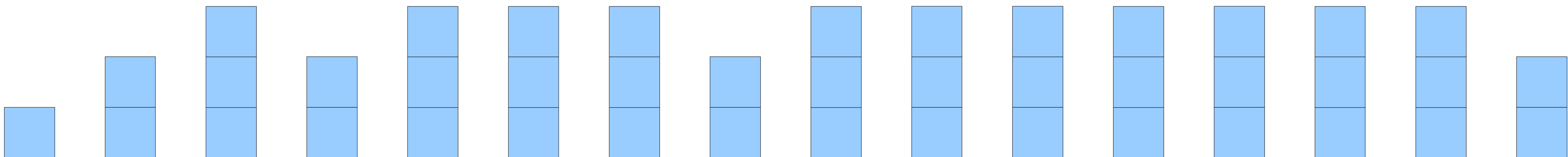


Spreading the Work

On average, we do just 3 units of work!

This is O(1) work on average!



Sharing the Burden

- We still have "heavy" pushes taking time O(n) and "light" pushes taking time O(1).
- Heavy pushes become so rare that the ***average*** time for a push is O(1).
- Cost of n pushes:
  - 1 + 1 + 1 + ... + 1 = **O(*n*)**.
- Cost of n pops:
  - 1 + 1 + 1 + ... + 1 = **O(*n*)**.
- Total work done: **O(*n*)**.
- Can we confirm this?

Amortized Analysis

- The analysis we have just done is called an ***amortized analysis***.
- We reason about the total work done, not the work done per operation.
- In an amortized sense, our implementation of the stack is extremely fast!
- This is one of the most common approaches to implementing `Stack`.

Your Action Items

- ***Keep working on Assignment 5***
  - Haven't started yet? Not a problem! You've got time if you make slow and steady progress from here on out.
  - Need help? Stop by the LaIR!

Next Time

- ***Hash Functions***
  - A magical and wonderful gift from the world of mathematics.
- ***Hash Tables***
  - How do we implement `HashMap` and `HashSet`?