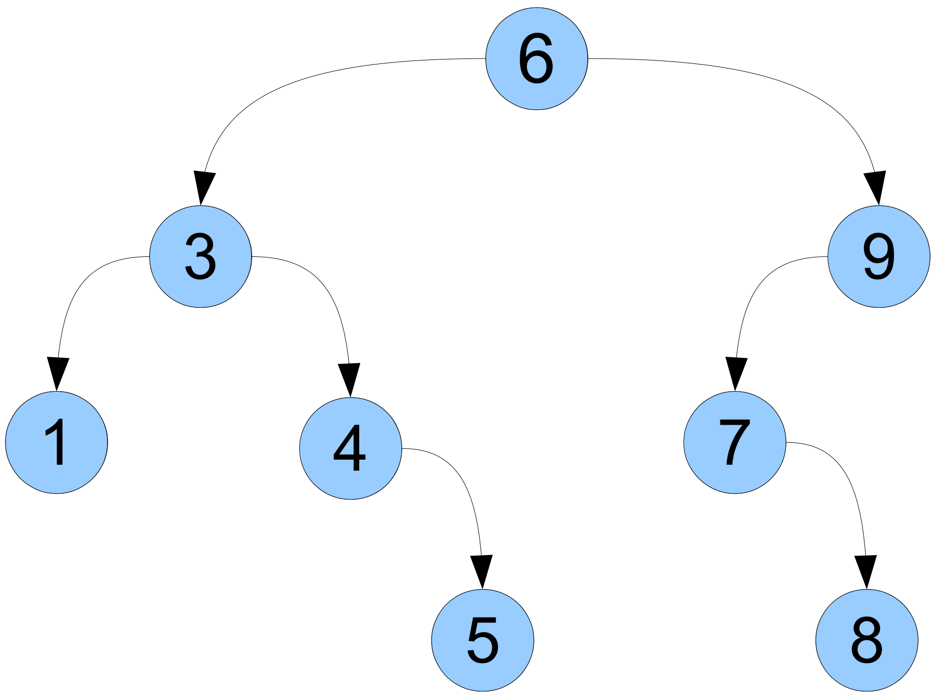
Binary Search Trees

Part Two

Recap from Last Time

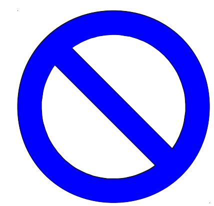
Binary Search Trees

- The data structure we have just seen is called a ***binary search tree*** (or ***BST***).
- The tree consists of a number of ***nodes***, each of which stores a value and has zero, one, or two ***children***.
- All values in a node's left subtree are ***smaller*** than the node's value, and all values in a node's right subtree are ***greater*** than the node's value.

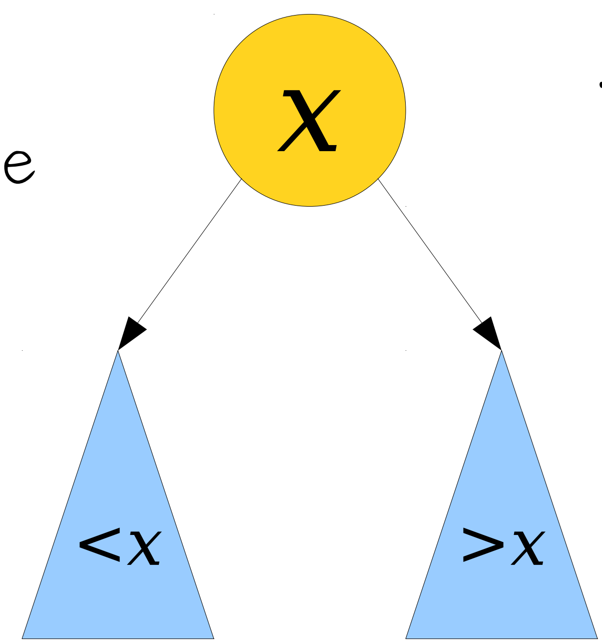


A Binary Search Tree Is Either…

an empty tree, represented by **nullptr**, or…



… a single node, whose left subtree is a BST of smaller values …



… and whose right subtree is a BST of larger values.

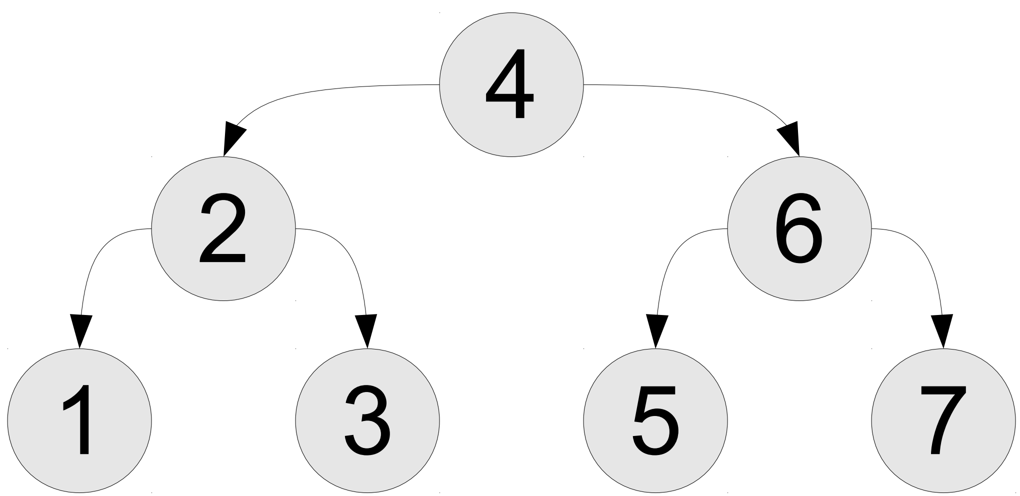
New Stuff!

Getting Rid of Trees

http://www.tigersheds.com/garden-resources/image.axd?picture=2010%2F6%2Fdeforestation1.jpg

Freeing a Tree

- Once we're done with a tree, we need to free all of its nodes.
- As with a linked list, we have to be careful not to use any nodes after freeing them.



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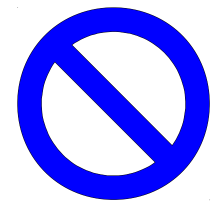

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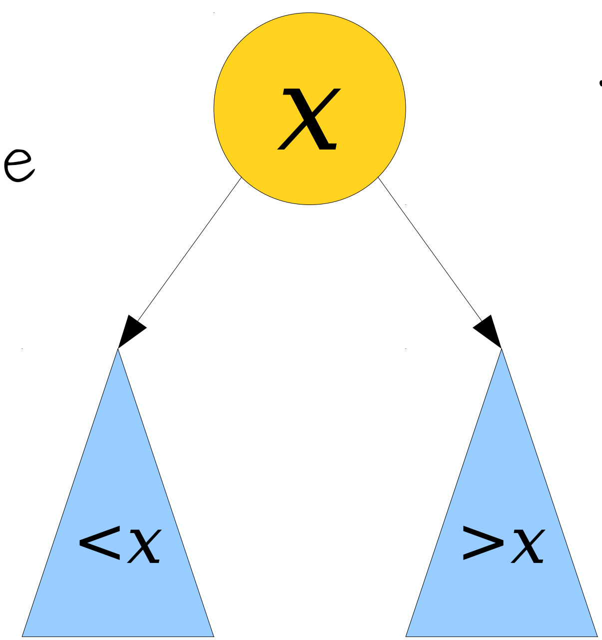

… and whose right
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of larger values.

Which Options Work?

```
void deleteTree(Node* root) {
  if (root == nullptr) return;

  delete root;
  deleteTree(root->left);
  deleteTree(root->right);
}
```

```
void deleteTree(Node* root) {
  if (root == nullptr) return;

  delete root;
  deleteTree(root->right);
  deleteTree(root->left);
}
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```
void deleteTree(Node* root) {
  if (root == nullptr) return;

  deleteTree(root->left);
  delete root;
  deleteTree(root->right);
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  delete root;
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}
```

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void deleteTree(Node* root) {
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  deleteTree(root->left);
  deleteTree(root->right);
  delete root;
}
```

```
void deleteTree(Node* root) {
  if (root == nullptr) return;

  deleteTree(root->right);
  deleteTree(root->left);
  delete root;
}
```

Postorder Traversals

- The particular recursive pattern we just saw is called a ***postorder traversal*** of a binary tree.
- Specifically:
  - Recursively visit all the nodes in the left subtree.
  - Recursively visit all the nodes in the right subtree.
  - Visit the node itself.

Tree Efficiency

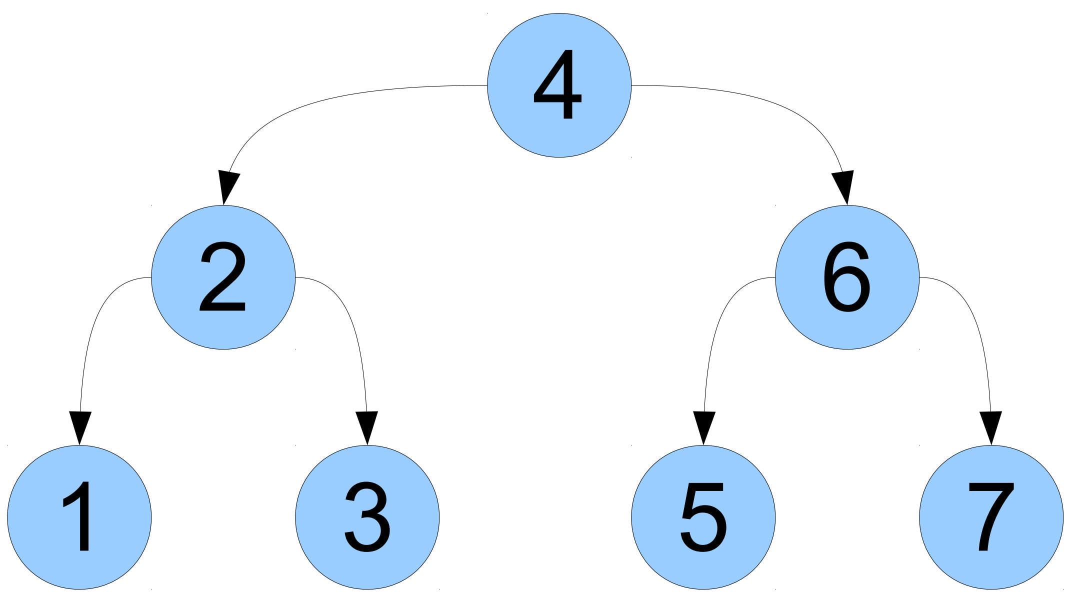
How fast are BST lookups?

How fast are BST insertions?

Insertion Order Matters

- You can have multiple BSTs holding the same elements
- Here's the BST we get by inserting these elements in this order:

4, 2, 1, 3, 6, 5, 7



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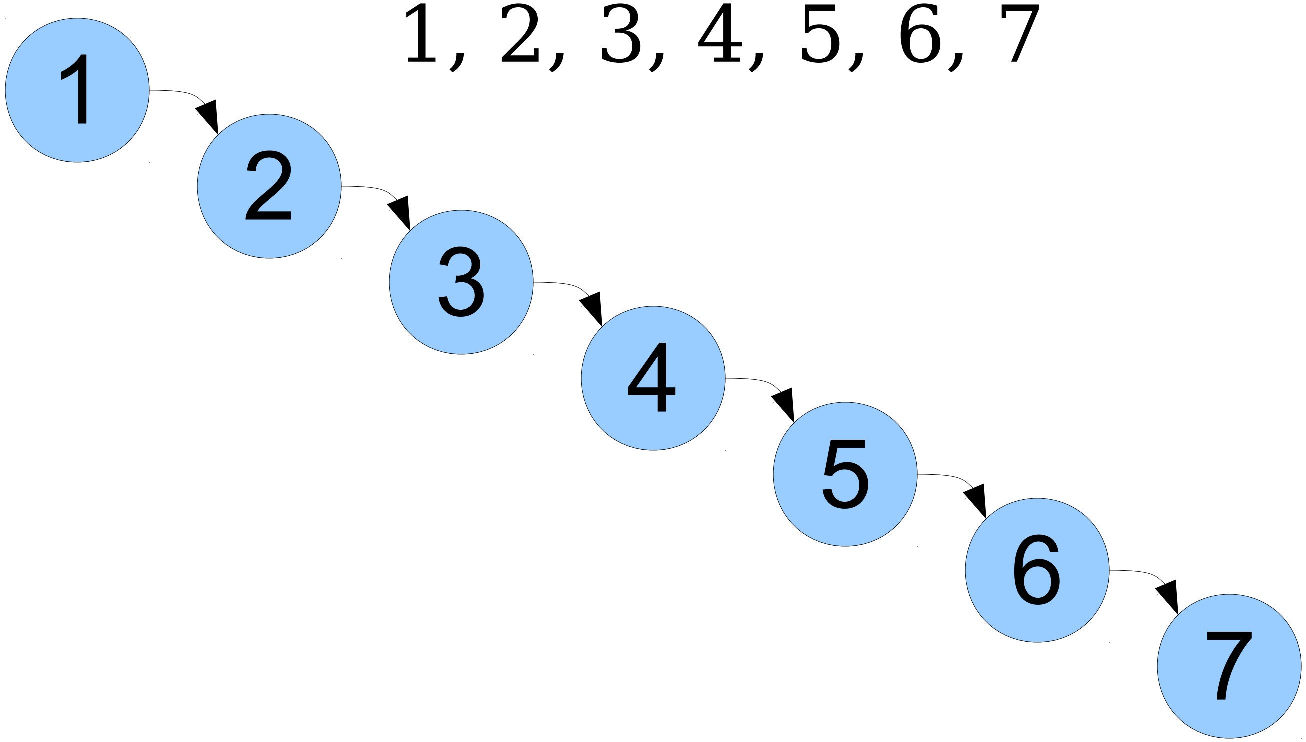
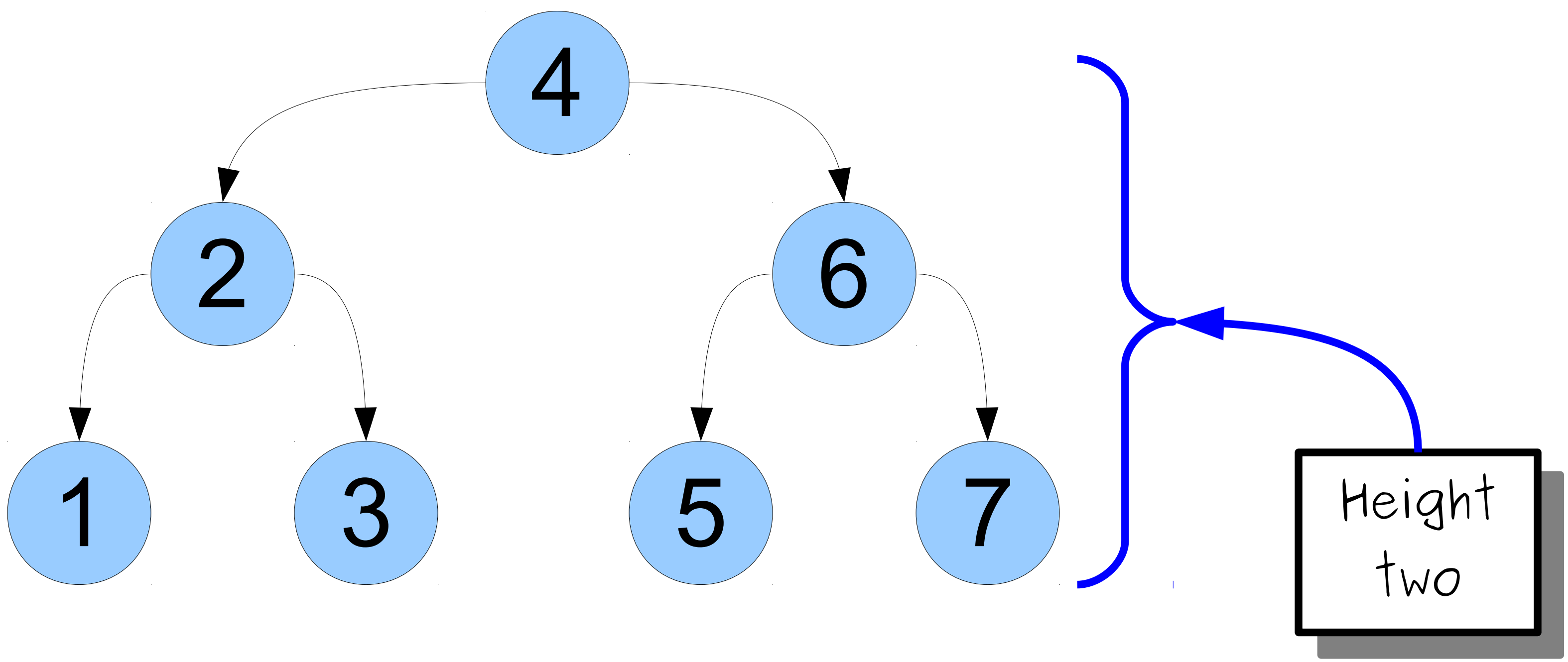


Tree Terminology

- The ***height*** of a tree is the number of nodes in the longest path from the root to a leaf.



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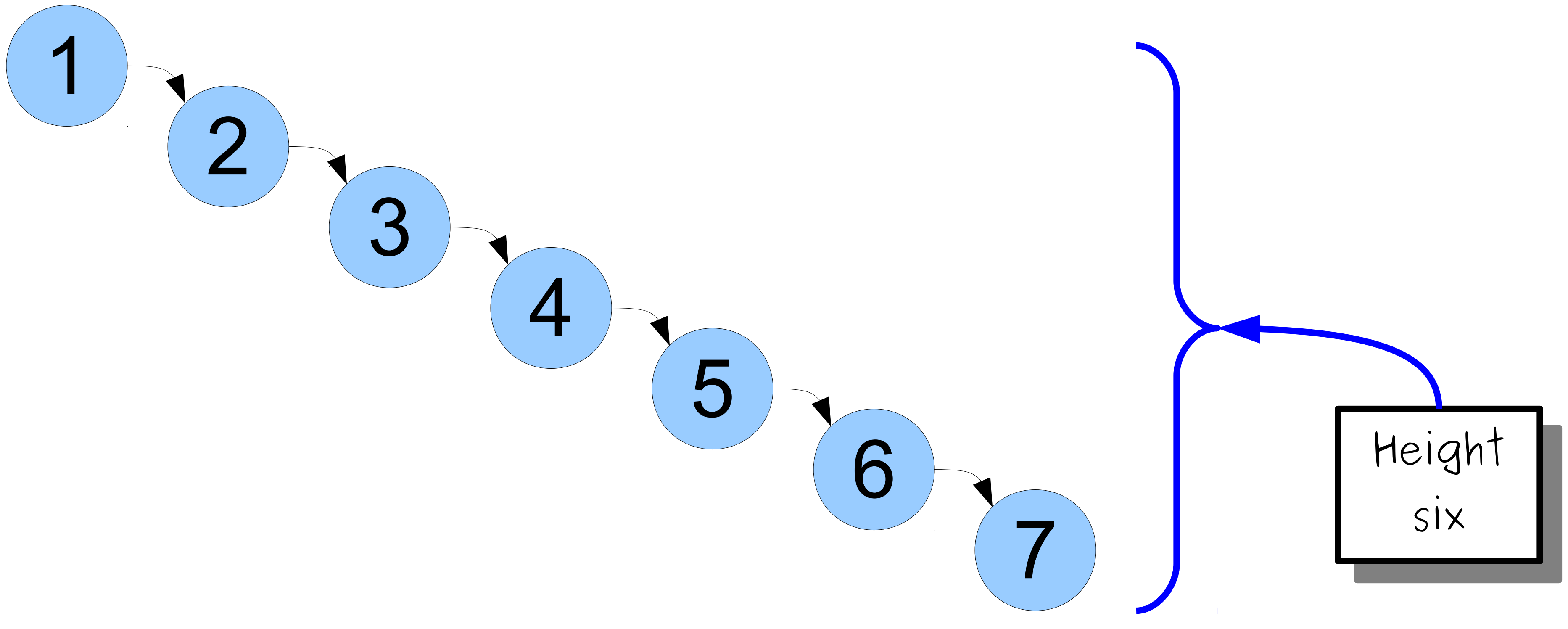

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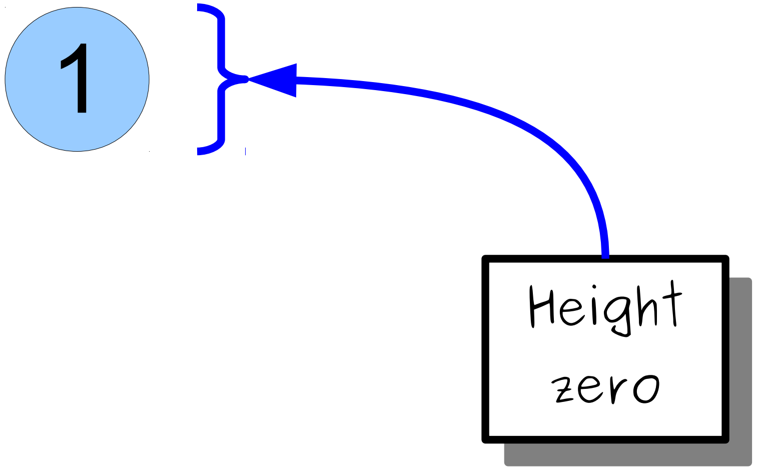
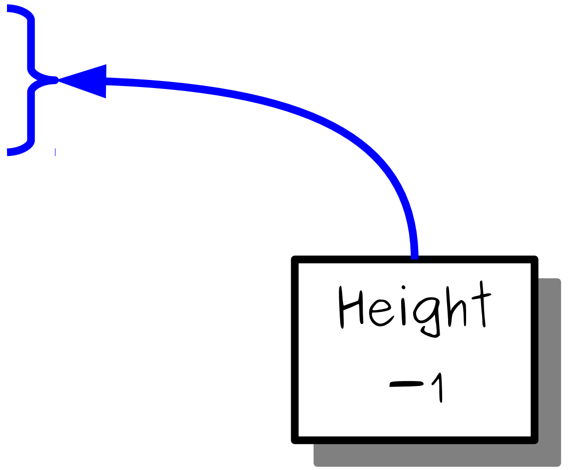


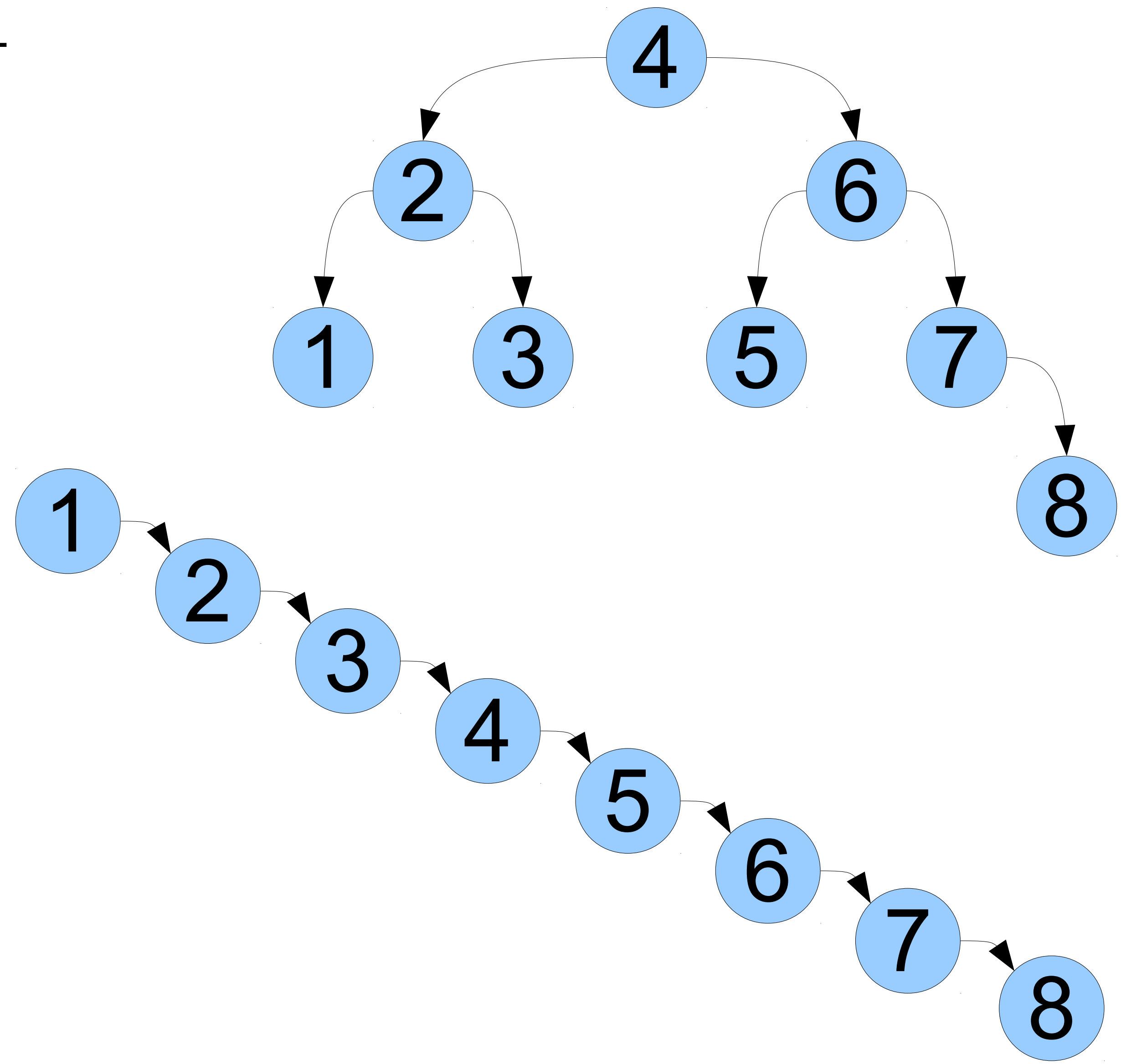
Tree Terminology

- The ***height*** of a tree is the number of nodes in the longest path from the root to a leaf.
- By convention, an empty tree has height -1.



Efficiency Questions

- The time to add an element to a BST (or look up an element in a BST) depends on the height of the tree.
- The runtime is **O(*h*)**, where *h* is the height of the tree.



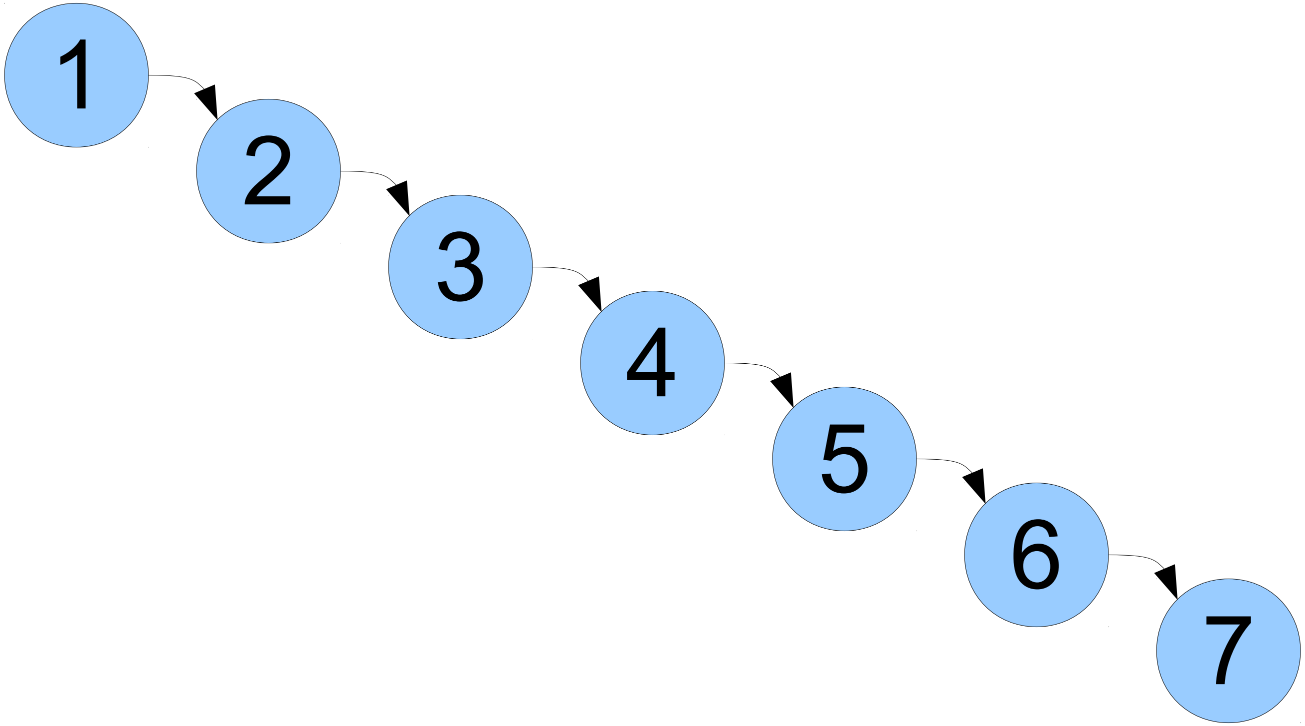
The cost of an insertion or lookup on a BST is O(h), where h is the height of the tree.

Is there a connection between h, the tree height, and n, the number of nodes?

Balanced Trees

Tree Heights

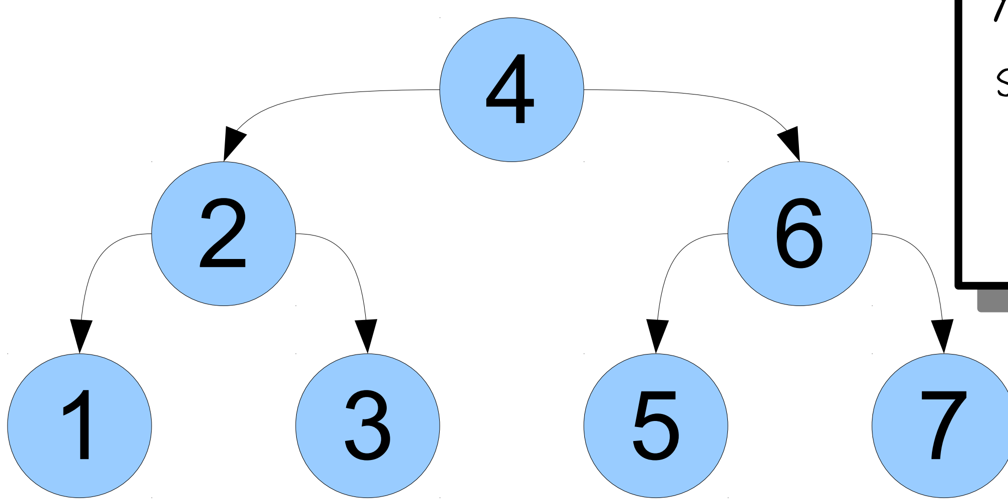
- What are the maximum and minimum heights of a tree with n nodes?
- Maximum height: all nodes in a chain. Height is $O(n)$.



Tree Heights

- What are the maximum and minimum heights of a tree with n nodes?
- Maximum height: all nodes in a chain. Height is $O(n)$.
- Minimum height: tree is as complete as possible. Height is $O(\log n)$.

You can only double something $O(\log n)$ times before it exceeds n.

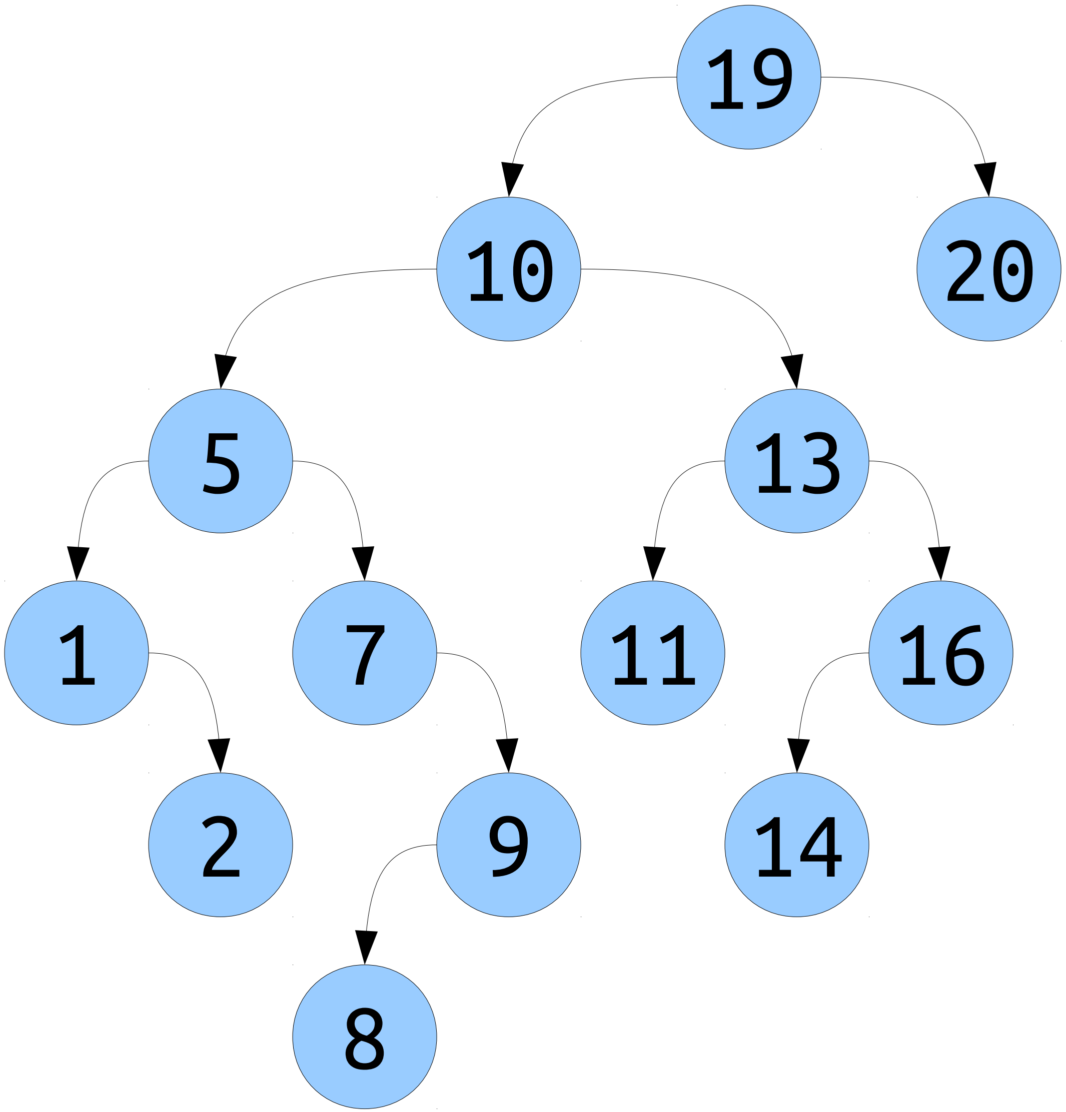


Balanced Trees

- A binary search tree is called ***balanced*** if its height is O(log n), where n is the number of nodes in the tree.
- Balanced trees are extremely efficient:
  - Lookups take time O(log n).
  - Insertions take time O(log n).
  - Deletions take time O(log n).
- ***Question:*** How do you balance a tree?

Balanced Trees

- ***Theorem:*** If you start with an empty tree and add in random values, then, with high probability, the tree is balanced.
- ***Proof:*** Take CS161!
- ***Takeaway:*** If you're adding elements to a BST and their values are actually random, then your tree is likely to be balanced.



Balanced Trees

- A ***self-balancing tree*** is a BST that reshapes itself on insertions and deletions to stay balanced.
- There are many strategies for doing this. They're beautiful. They're clever. And they're beyond the scope of CS106B.
- Some suggested topics to read up on, if you're curious:
  - Red/black trees (take CS161 or CS166!)
  - AVL trees (covered in the textbook)
  - Splay trees (trees that reshape on lookups)
  - Scapegoat trees (yes, that's what they're called)
  - Treaps (half binary heap, half binary search tree!)

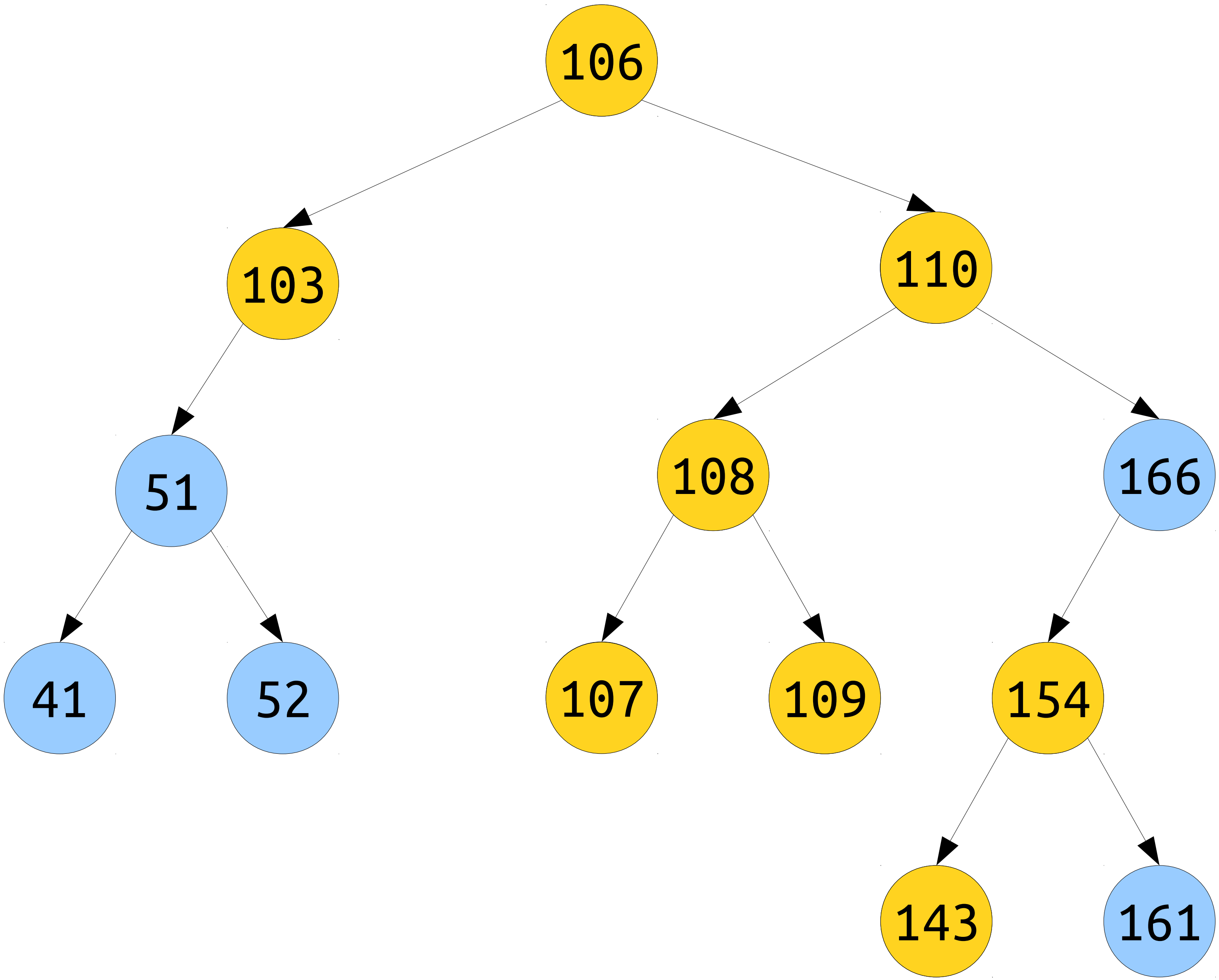
Balanced Trees

- If you're given a collection of values to put in a BST, and they're already sorted, you can construct a perfectly-balanced tree from them.
- Things to think about:
  - Which element would you put up at the root?
  - What would the children of that element be?
- These are great questions to think through.

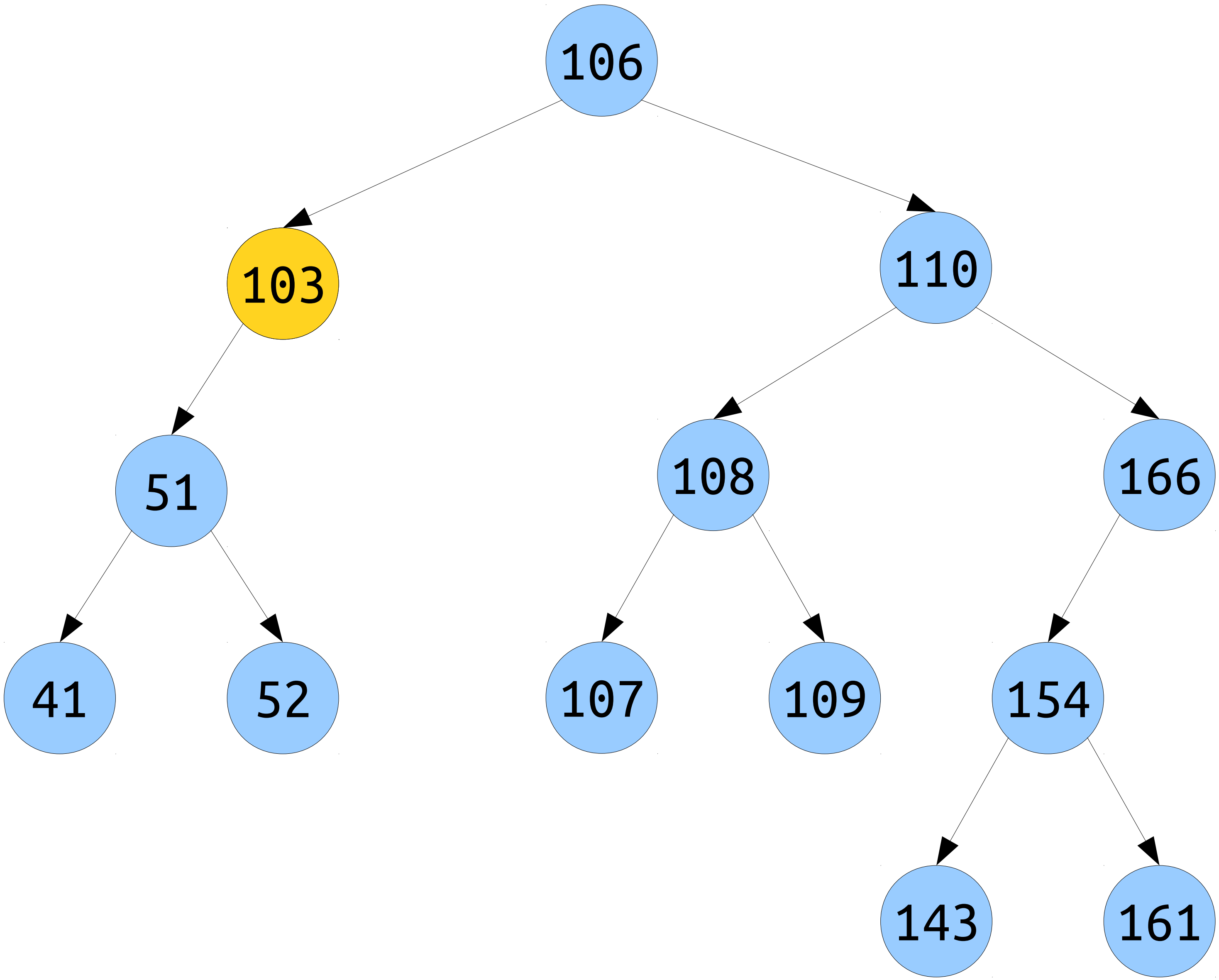
Range Searches

Range Searches

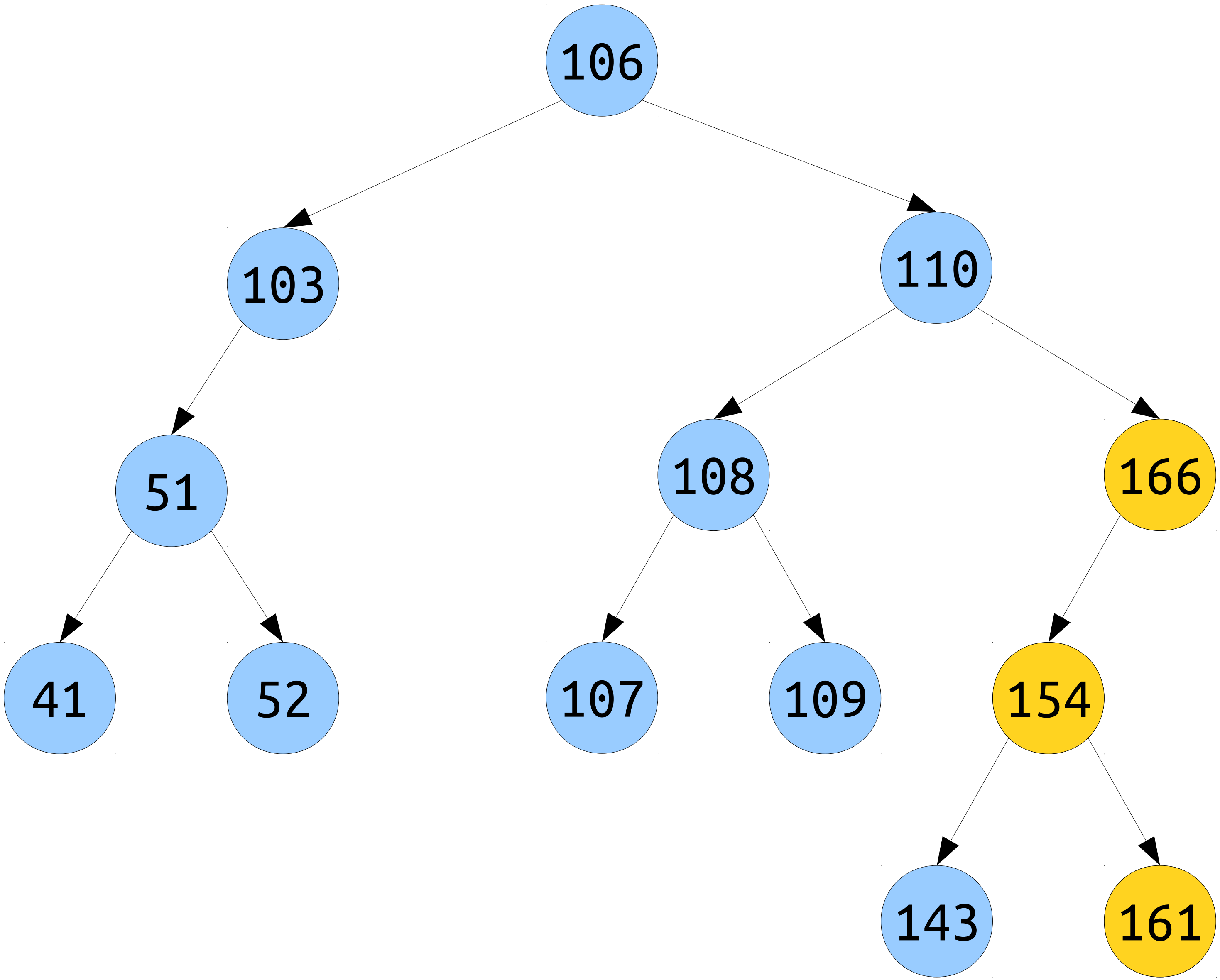
- We can use BSTs to do ***range searches***, in which we find all values in the BST within some range.
- For example:
  - If the values in the BST are dates, we can find all events that occurred within some time window.
  - If the values in the BST are number of diagnostic scans ordered, we can find all doctors who order a disproportionate number of scans.



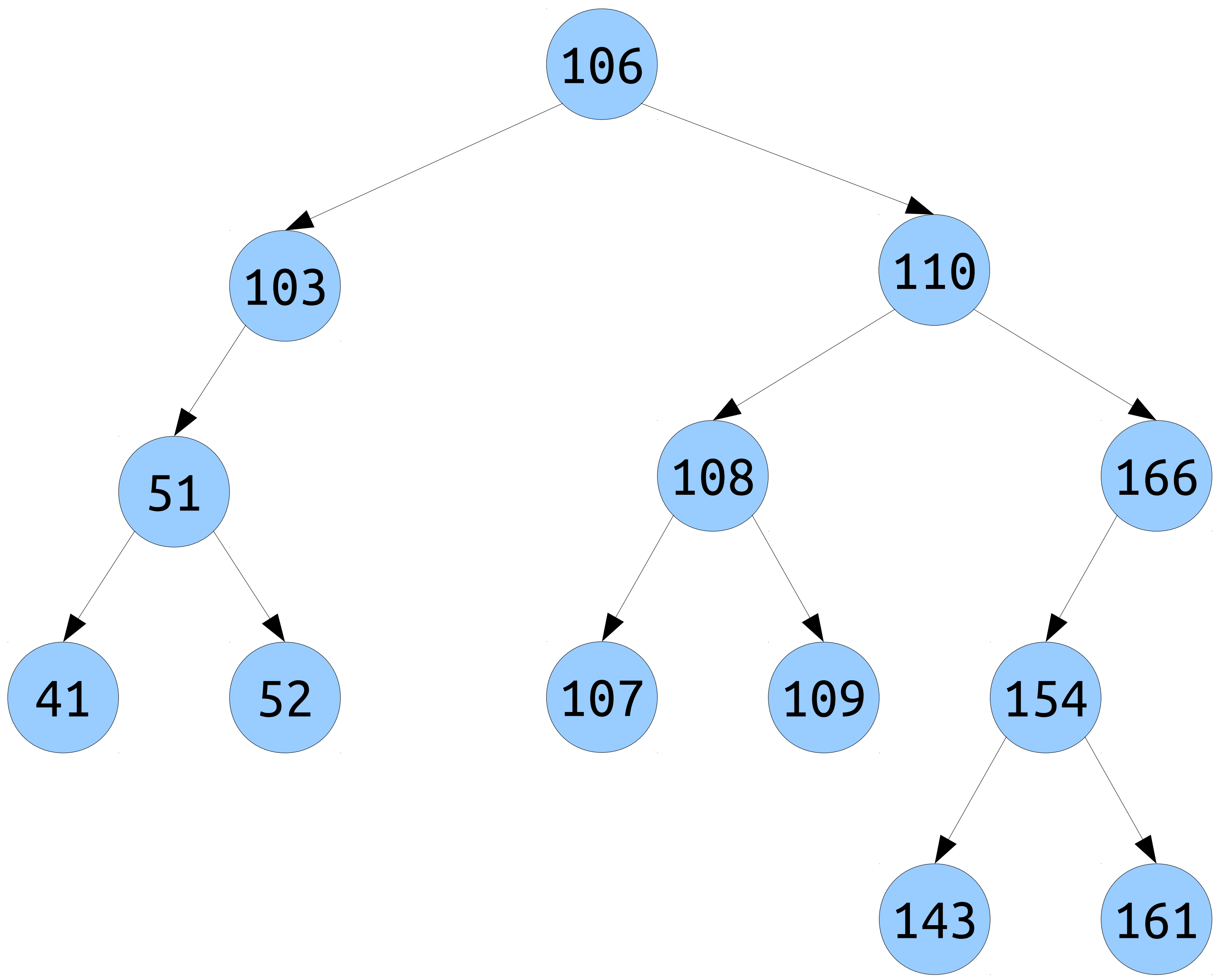
Find all elements in this tree in the range **[103, 154]**.



Find all elements in this tree in the range **[99, 105]**.



Find all elements in this tree in the range **[150, 170]**.



Find all elements in this tree in the range **[137, 138]**.

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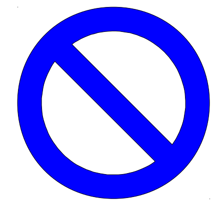

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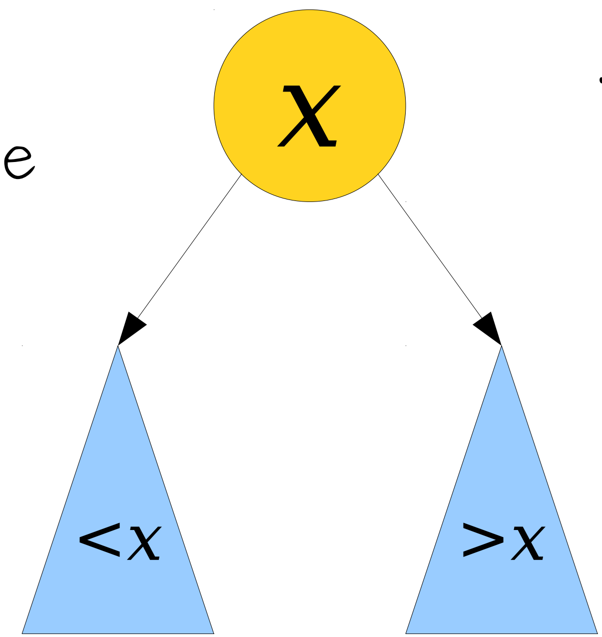

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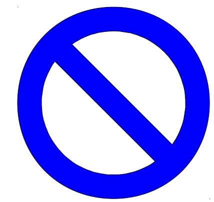

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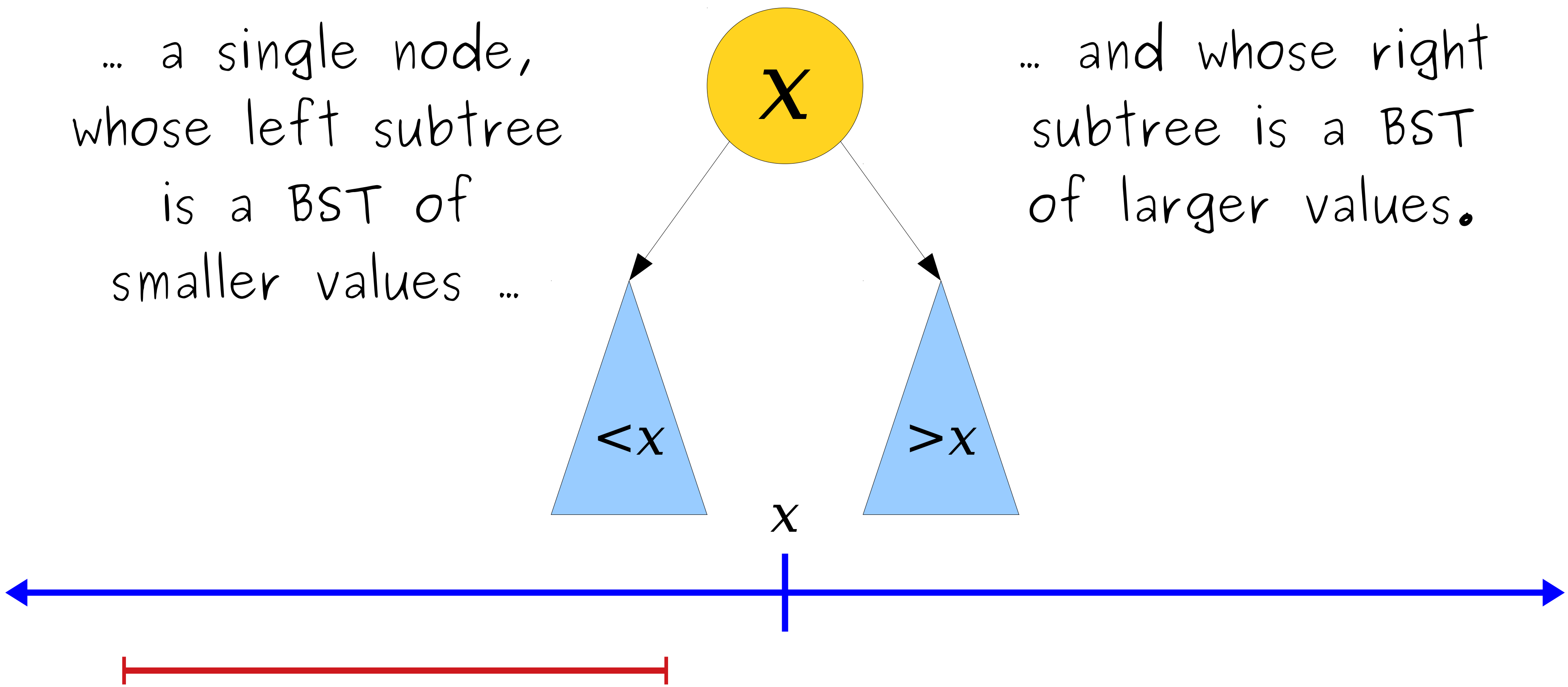

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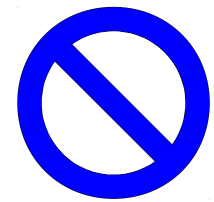

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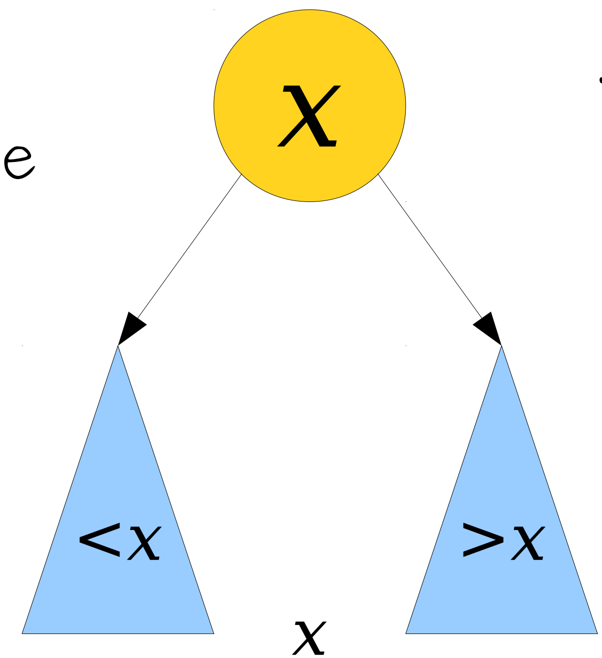

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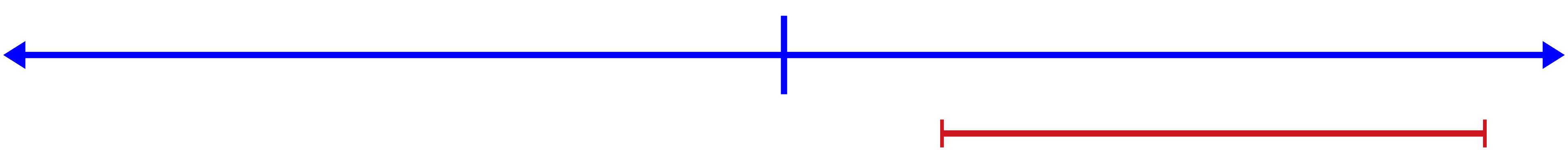

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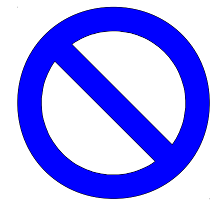

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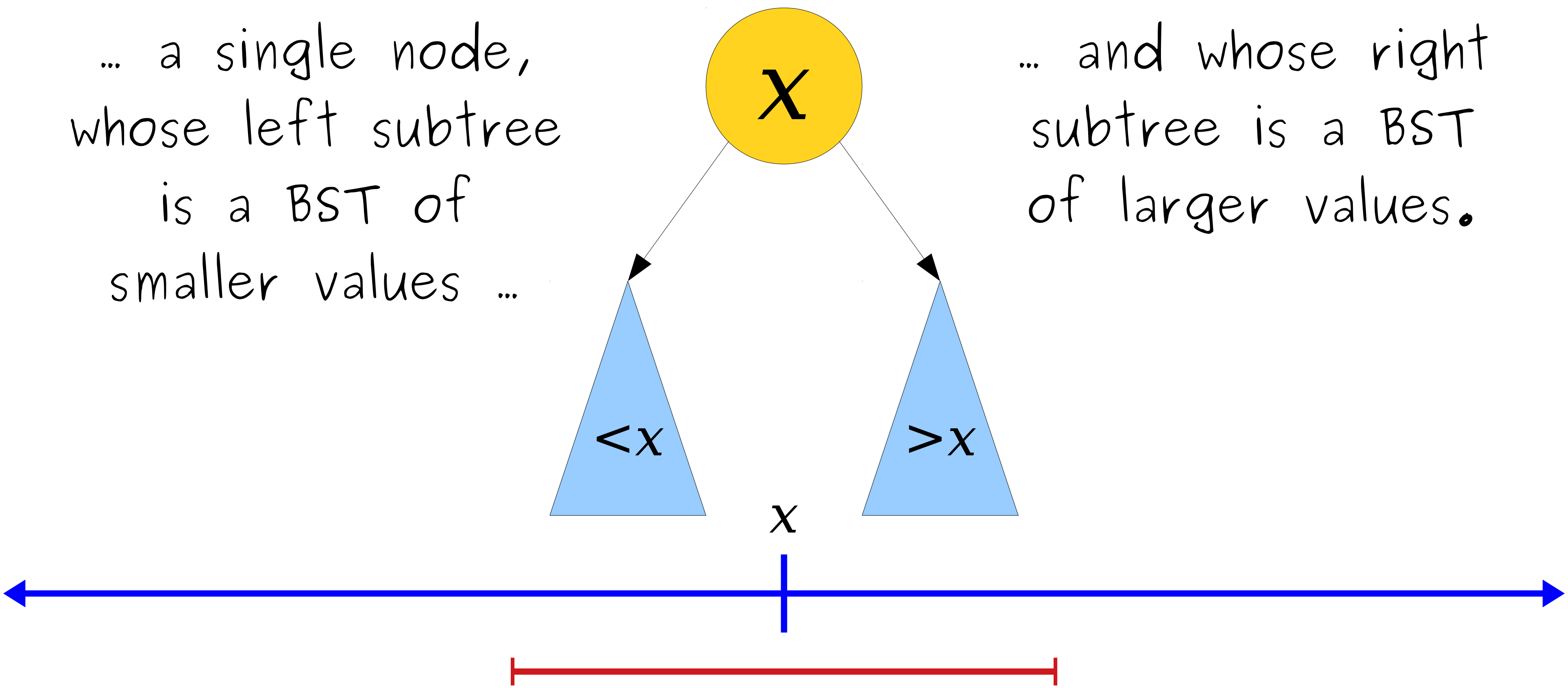

… and whose right subtree is a BST of larger values.

Range Searches

- A hybrid between an inorder traversal and a regular BST lookup!
- The idea:
    - If the node is in the range being searched, add it to the result.
    - Recursively explore each subtree that could potentially overlap with the range.
- ***Fun fact:*** The runtime of a range search is $O(h + z)$, where h is the height of the tree and z is the number of items in the range. Come chat with me after class if you're curious why this is!

To Summarize:

A Binary Search Tree Is Either…

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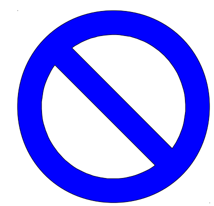

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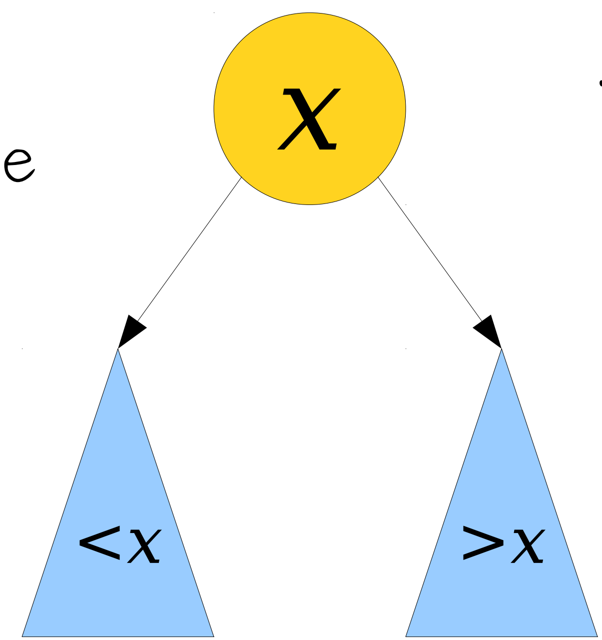
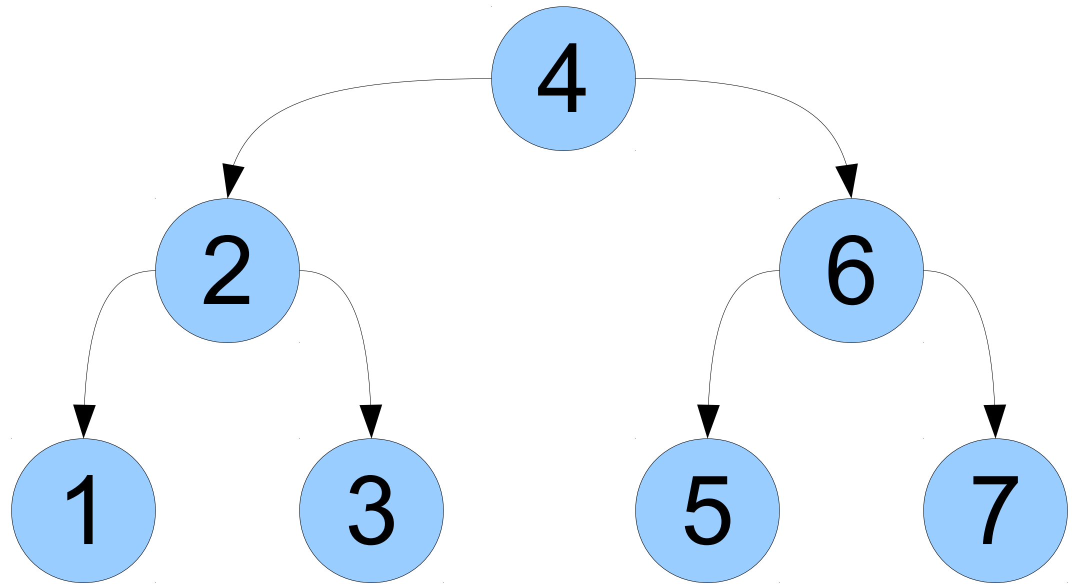


… and whose right
subtree is a BST
of larger values.

```
struct Node {
    Type value;
    Node* left;  // Smaller values
    Node* right; // Bigger values
};
```

```
bool contains(Node* root, const string& key) {
    if (root == nullptr) return false;
    else if (key == root->value) return true;
    else if (key <  root->value) return contains(root->left,  key);
    else return contains(root->right, key);
}

void insert(Node*& root, const string& key) {
    if (root == nullptr) {
        root = new Node;
        node->value = key;
        node->left = node->right = nullptr;
    } else if (key < root->value) {
        insert(root->left, key);
    } else if (key > root->value) {
        insert(root->right, key);
    } else {
        // Already here!
    }
}
```


4
2
6
1
3
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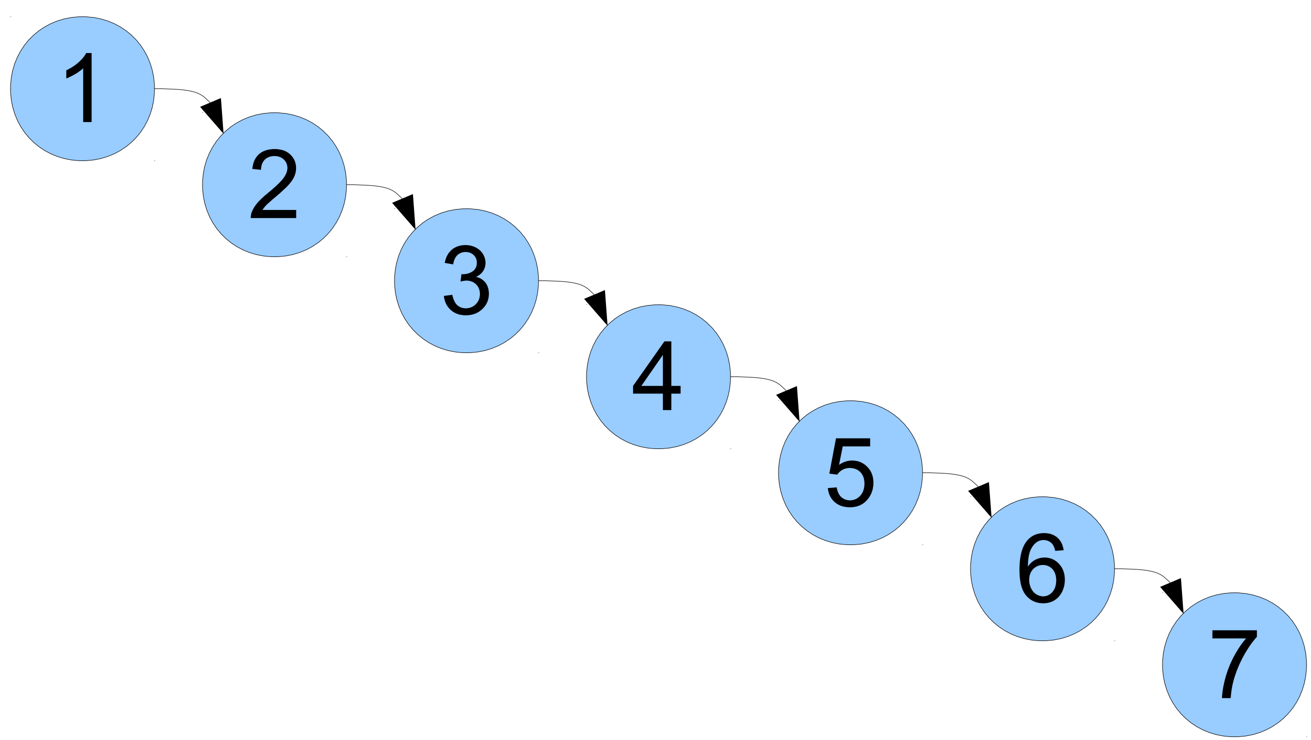
7


1
2
3
4
5
6
7

```cpp
void printTree(Node* root) {
    if (root == nullptr) return;

    printTree(root->left);
    cout << root->value << endl;
    printTree(root->right);
}

void deleteTree(Node* root) {
    if (root == nullptr) return;

    deleteTree(root->left);
    deleteTree(root->right);
    delete root;
}
```

```
void printInRange(Node* tree, const string& low, const string& high) {
    if (tree == nullptr) return;

    if (high < tree->value) {
        printInRange(tree->left, low, high);
    } else if (low > tree->value) {
        printInRange(tree->right, low, high);
    } else {
        printInRange(tree->left, low, high);
        cout << tree->value << endl;
        printInRange(tree->right, low, high);
    }
}
```

Your Action Items

- ***Read Chapter 16.1 – 16.2.***
  - All about BSTs!
- ***Finish Assignment 7.***
  - You can use late periods here if you'd like, but be careful about doing so since you can't use them on the next assignment.

Next Time

- ***Other Binary Trees***
  - BSTs are wonderful, but other tree structures with similar shapes exist.
- ***Huffman Coding***
  - Using fewer bits to send a message.