Beyond Data Structures

It's All Bits and Bytes

- Digital data is stored as sequences of 0s and 1s.
  - Usually encoded by magnetic orientation on small (10nm!) metal particles or by trapping electrons in small gates.
- A single 0 or 1 is called a ***bit***.
- A group of eight bits is called a ***byte***.

`00000000`, `00000001`, `00000010`,
`00000011`, `00000100`, `00000101`, ...

- There are $2^8 = 256$ different bytes.
  - ***Great practice:*** Write a function to list all of them!

Representing Text

- We think of strings as being made of characters representing letters, numbers, emojis, etc.
- Computers require everything to be written as zeros and ones.
- To bridge the gap, we need to agree on some way of representing characters as sequences of bits.
- ***Idea:*** Assign each character a sequence of bits called a ***code***.

ASCII

- Early (American) computers needed some standard way to send output to their (physical!) printers.
- Since there were fewer than 256 different characters to print (1960's America!), each character was assigned a one-byte value.
- This initial code was called ***ASCII***. Surprisingly, it's still around, though in a modified form (more on that later).
- For example, the letter `A` is represented by the byte `01000001` (65). You can still see this in C++:

```
cout << int('A') << endl; // Prints 65
```

01001000010001010100000101000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

0100100001000101010000001000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H 01000101010000000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H 01000101010000000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E 0100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E 0100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	code
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H **E** **A** 01000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E A 01000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E A D

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

An Observation

- In ASCII, every character has exactly the same number of bits in it.
- Any message with n characters will use up exactly $8n$ bits.
  - Space for `CS106BLECTURE`: 104 bits.
  - Space for `COPYRIGHTABLE`: 104 bits.
- ***Question:*** Can we reduce the number of bits needed to encode text?

KIRK'S DIKDIK

A Different Encoding

- ASCII uses one byte per character. There are 256 possible bytes.
- If we're specifically writing the string **`KIRK'S DIKDIK`**, which has only seven different characters, using full bytes is wasteful.
- Here's a three-bit encoding we can use to represent the letters in **`KIRK'S DIKDIK`**.
- This uses 37.5% as much space as what ASCII uses.

character	*code*
K	000
I	001
R	010
'	011
S	100
␣	101
D	110

000	001	010	000	011	100	101	110	001	000	110	001	000
K	I	R	K	'	S	␣	D	I	K	D	I	K

Where We're Going

- Storing data using the ASCII encoding is portable across systems, but is not ideal in terms of space usage.
- Building custom codes for specific strings might let us save space.
- ***Idea:*** Use this approach to build a ***compression algorithm*** to reduce the amount of space needed to store text.

The Key Idea

- If we can find a way to

  give all characters a bit pattern,

  that both the sender and receiver know about, and

  that can be decoded uniquely,

  then we can represent the same piece of text in multiple different ways.
- ***Goal:*** Find a way to do this that uses *less space* than the standard ASCII representation.

Exploiting Redundancy

- Not all letters have the same frequency in **KIRK'S DIKDIK**.
- Here's the frequencies of each letter.
- So far, we've given each letter codes of the same length.
- ***Key Question:*** Can we give shorter encodings to more common characters?

character	*frequency*
K	4
I	3
D	2
R	1
'	1
S	1
␣	1

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

01010101110000100010

0	1	01	0	10	11	100	00	1	0	00	1	0
K	I	R	K	'	S	␣	D	I	K	D	I	K

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111000010010

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111000001000010

0	1	01	0	10	11	100	00	1	0	00	1	0
K	I	R	K	'	S	␣	D	I	K	D	I	K

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

01010101110000100010

01	01	01	01	1	10	0	00	10	0	0	10
R	R	R	R	I	'	K	D	'	K	K	'

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

01010101110

01	01	01	01	1	10	0	00	10	0	0	10
R	R	R	R	I	'	K	D	'	K	K	'

The Problem

- If we use a different number of bits for each letter, we can't necessarily uniquely determine the boundaries between letters.
- We need an encoding that makes it possible to determine where one character stops and the next starts.
- Is this possible? If so, how?

Prefix Codes

- A ***prefix code*** is an encoding system in which no code is a prefix of another code.
- Here's a sample prefix code for the letters in **`KIRK'S DIKDIK`**.

character	*code*
`K`	`10`
`I`	`01`
`D`	`111`
`R`	`001`
`'`	`000`
`S`	`1101`
`␣`	`1100`

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

`1001001100001101110011101101110110`

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

<u>10</u>0100110000110111001110110111 0110

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

<u>10</u>01001100001101110011101101110110

10
K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

`1001001100001101110011101101110110`

10
K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10**0**100110000110111001110110110110

10	
K	

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10**01**001100001101110011101101110110

10
K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10**01**001100001101110011101101110110

10	01
K	I

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

`1001001100001101110011101101110110`

10	01
K	I

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**0**0110000110111001110110110110

10	01
K	I

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**00**110000110111001110110111011 0

10	01
K	I

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**001**100001101110011101101110110

10	01
K	I

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**001**100001101110011101101110110

10	01	001
K	I	R

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

`1001001100001101110011101101110110`

10	01	001
K	I	R

Prefix Codes

- Using this prefix code, we can represent **`KIRK'S DIKDIK`** as the sequence

  `1001001100001101110011101101110110`

- This uses just 34 bits, compared to our initial 104. Wow!
- But where did this code come from? How could you come up with codes like this for other strings?

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

---

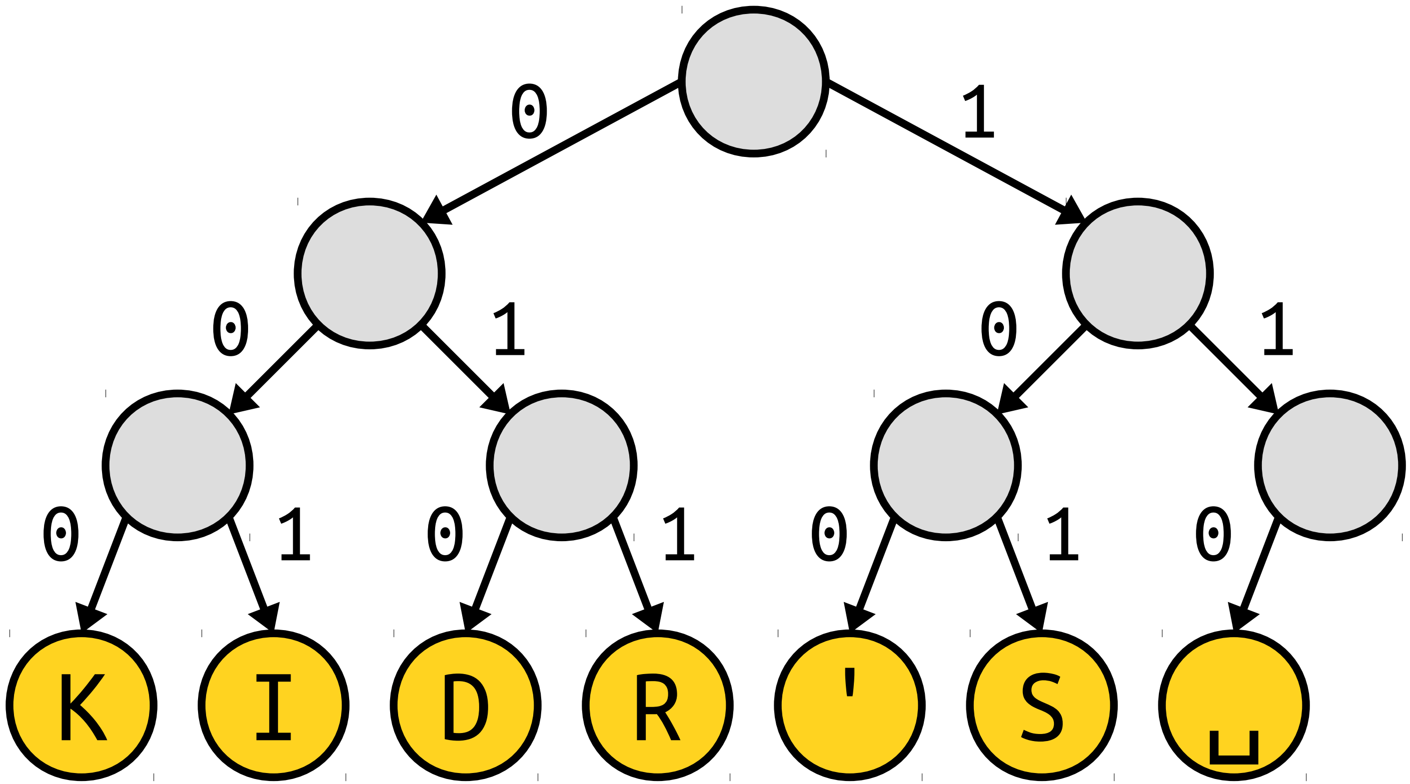
character	*code*
K	1111110
I	111110
D	11110
R	1110
'	110
S	10
␣	0

1111110111110111011111101101001111011111011111101111011111101111110

How do you find a “good” prefix code?

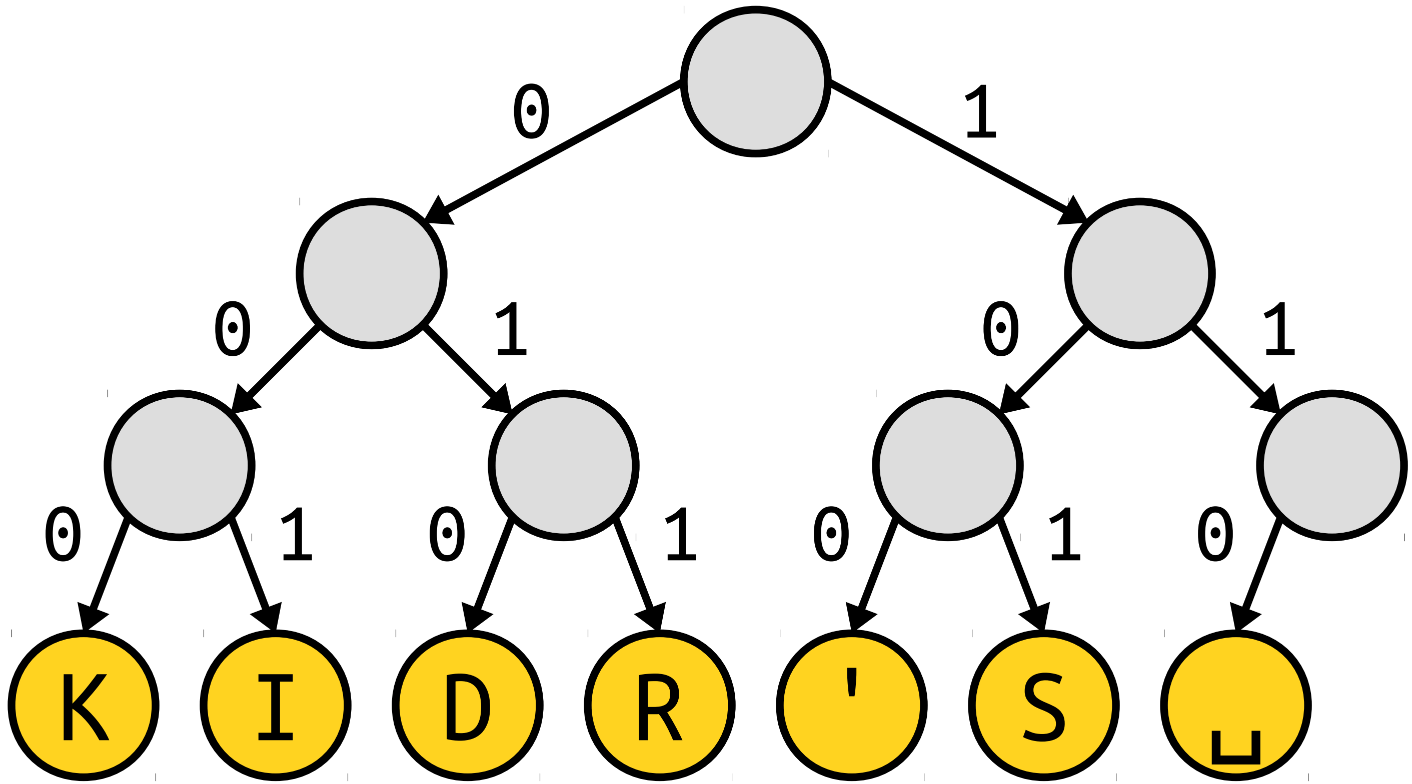
The Main Insight

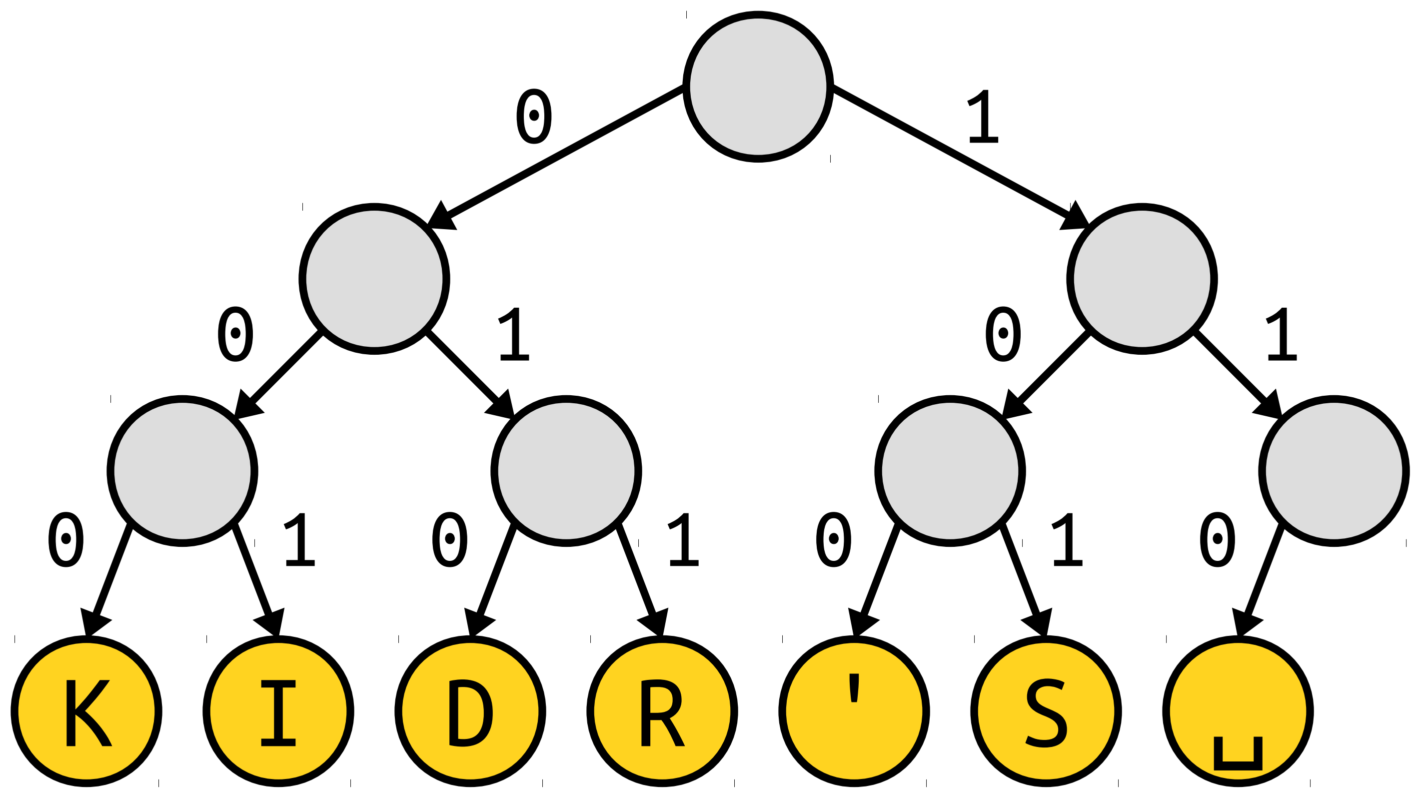
character	*code*
K	000
I	001
D	010
R	011
'	100
S	101
	110



This special type of binary tree is called a ***coding tree***.

character	*code*
K	000
I	001
D	010
R	011
'	100
S	101
	110

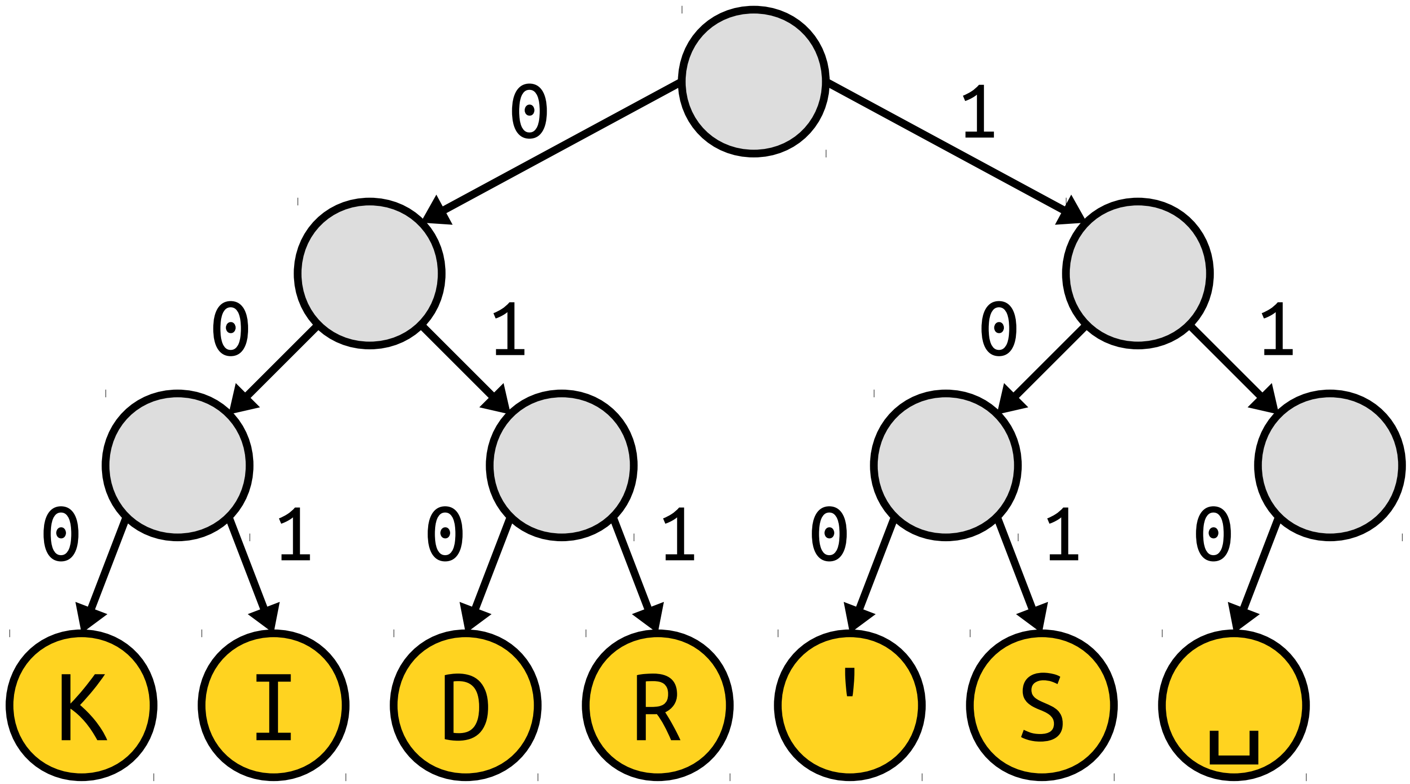




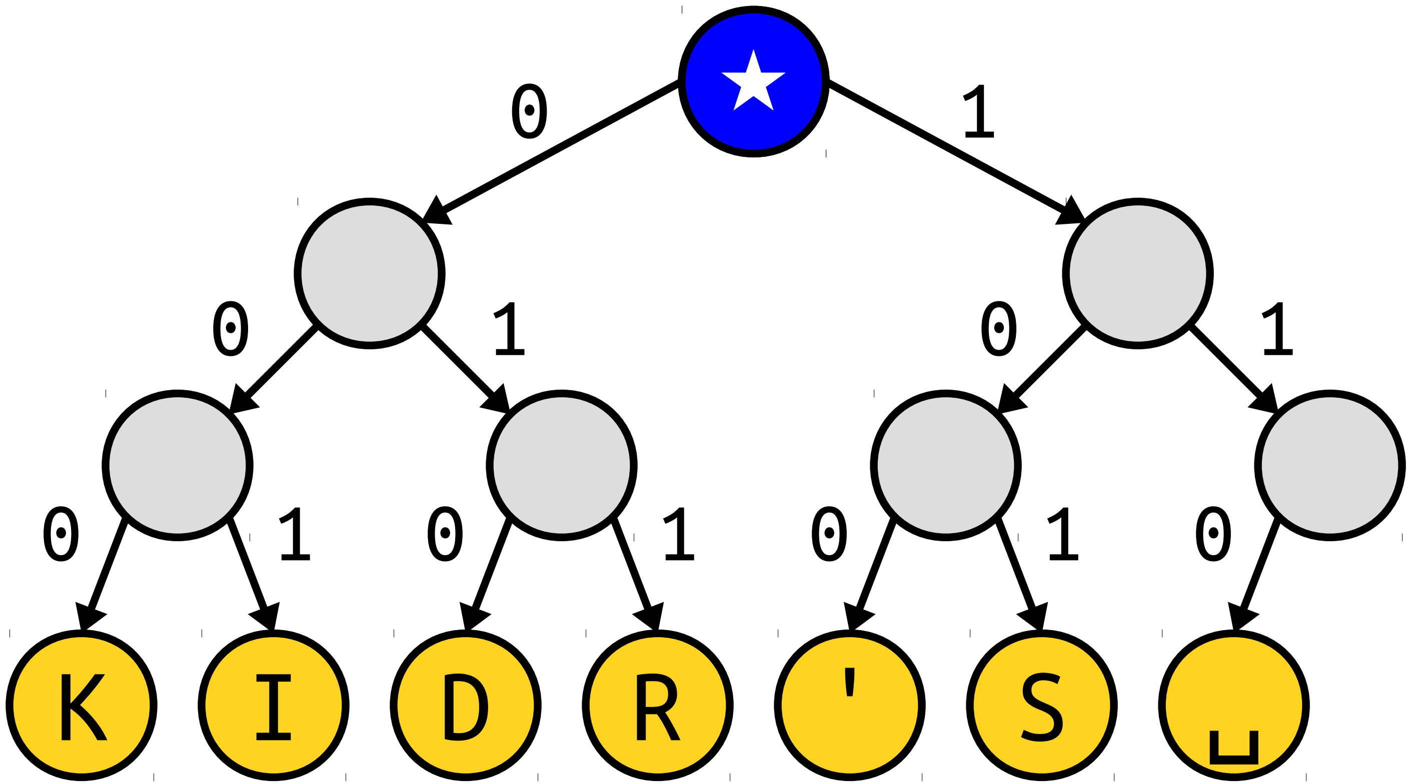
0
1
0
1
0
1
0
1
0
1
0
1
0
K
I
D
R
'
S
␣

101000001

What is the title of this slide?



101000001



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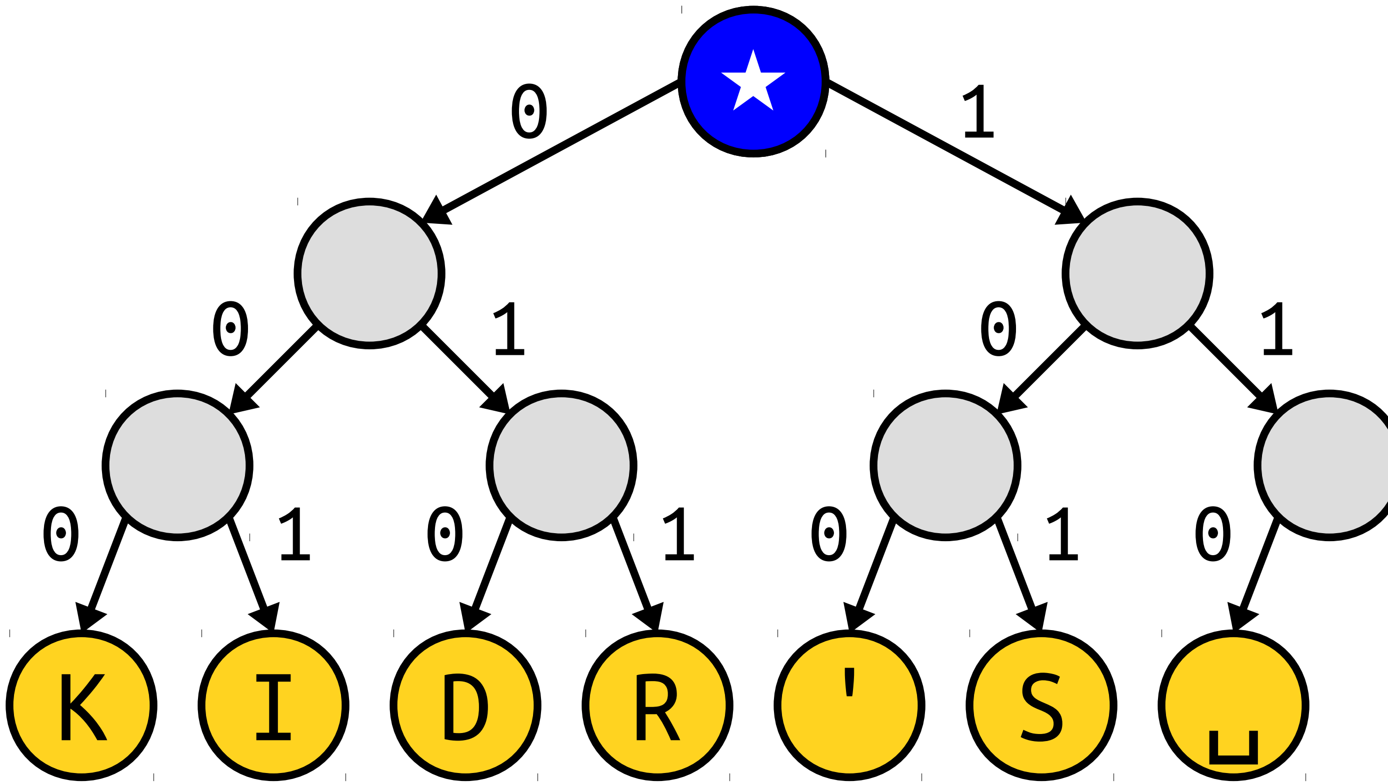
101000001

What is the title of this slide?



101000001

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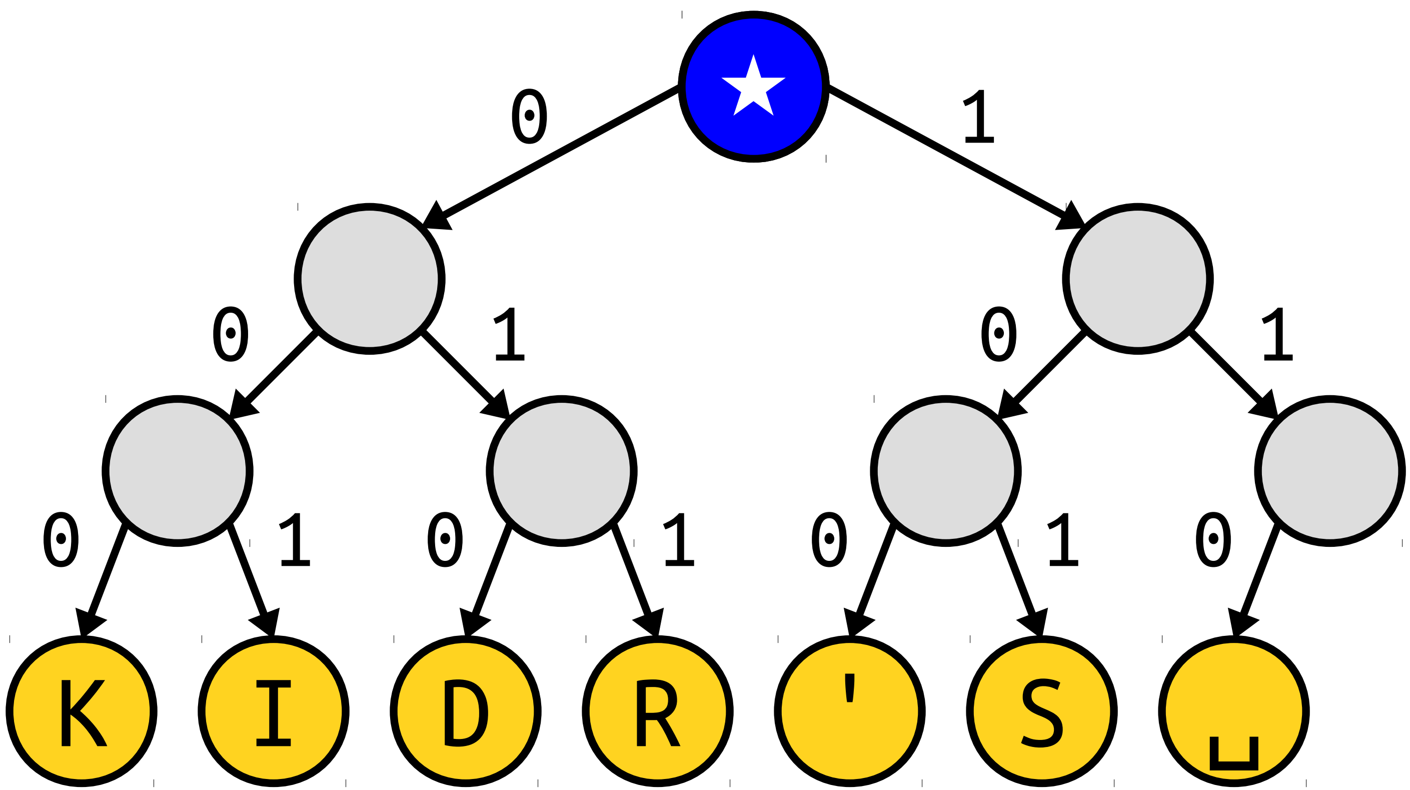

101000001

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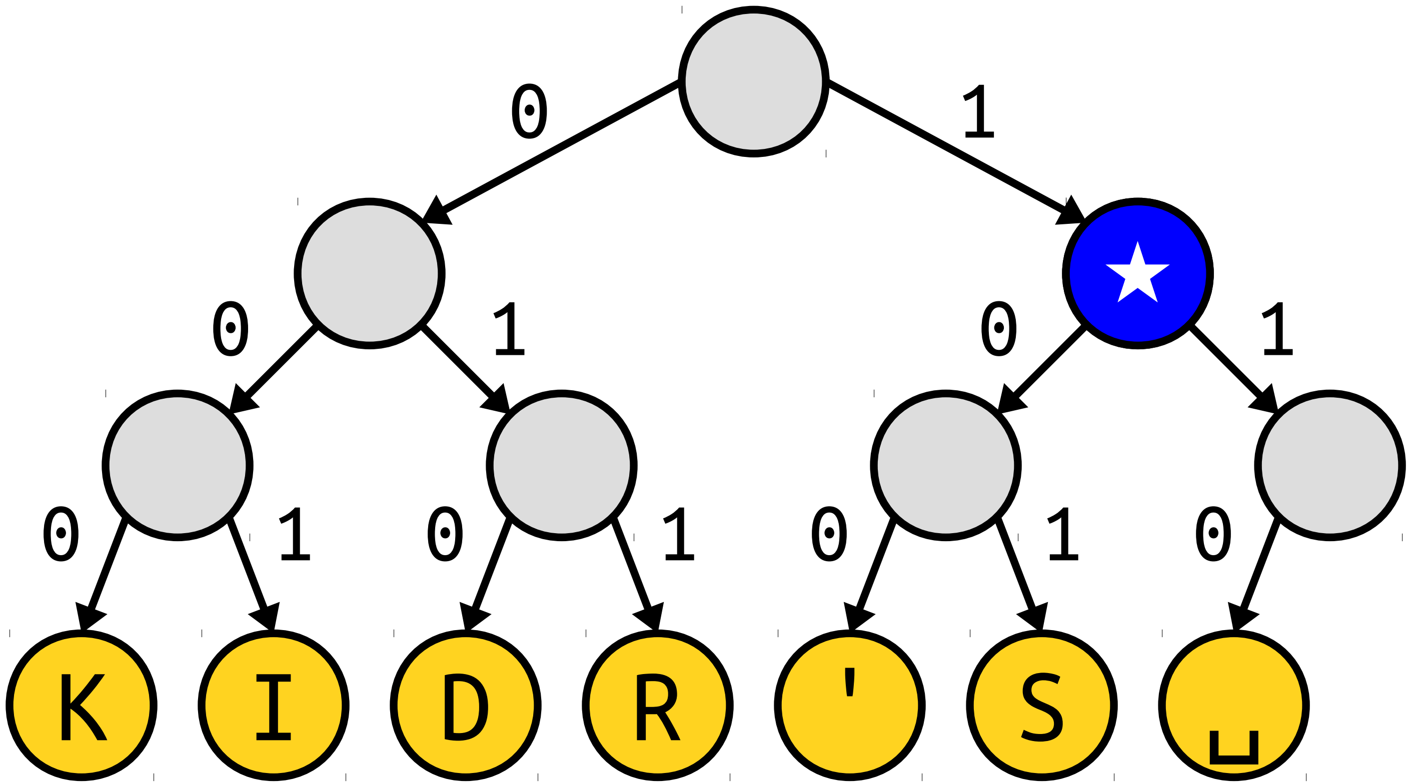
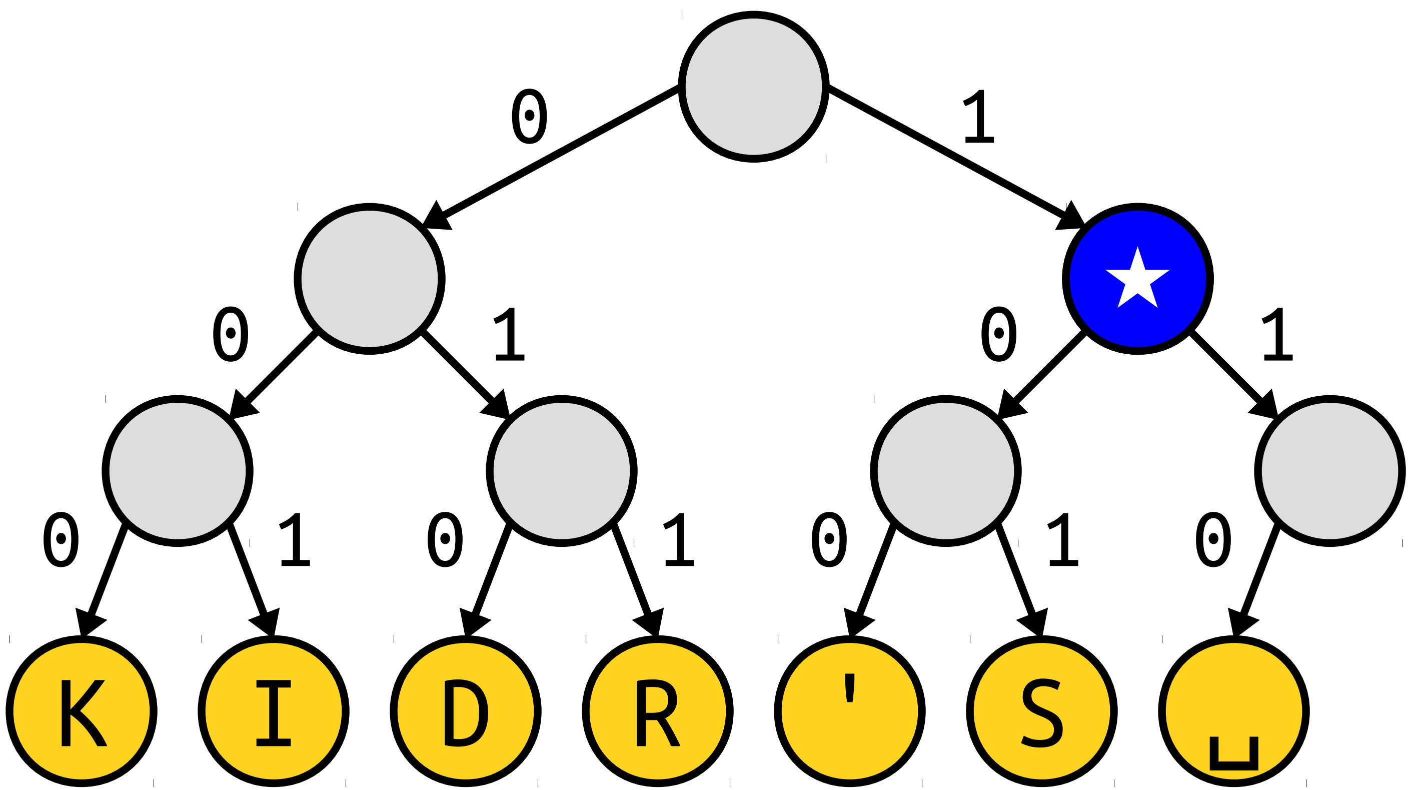


101000001



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10100000001

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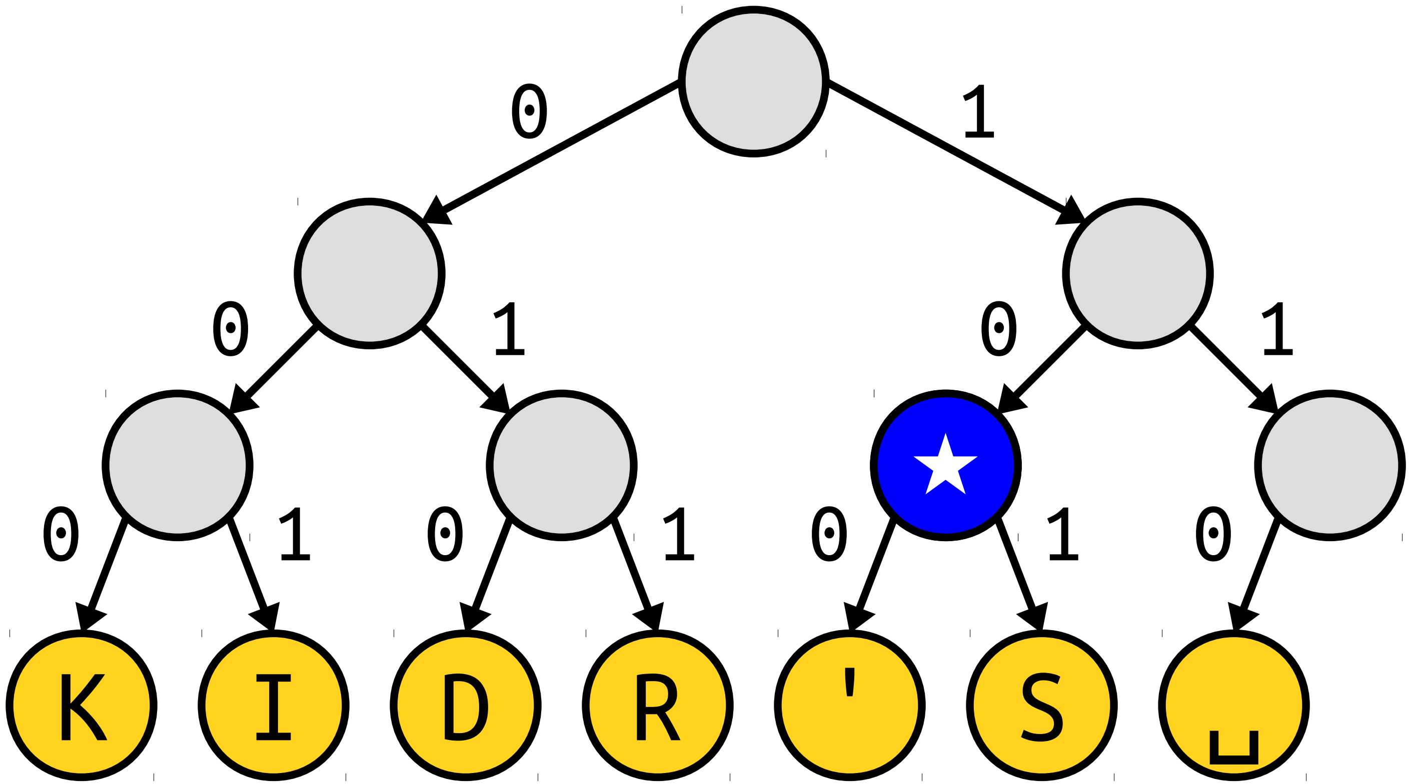

101000001

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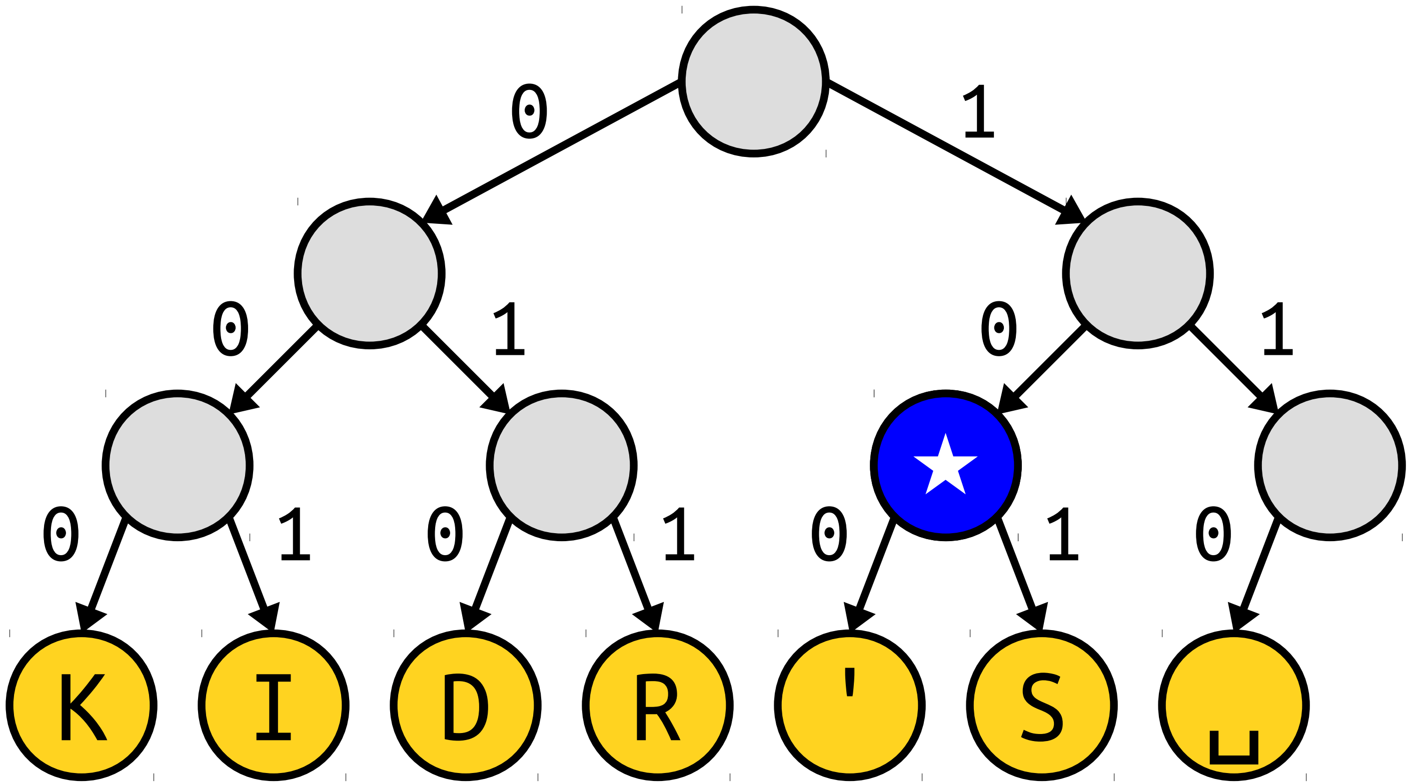
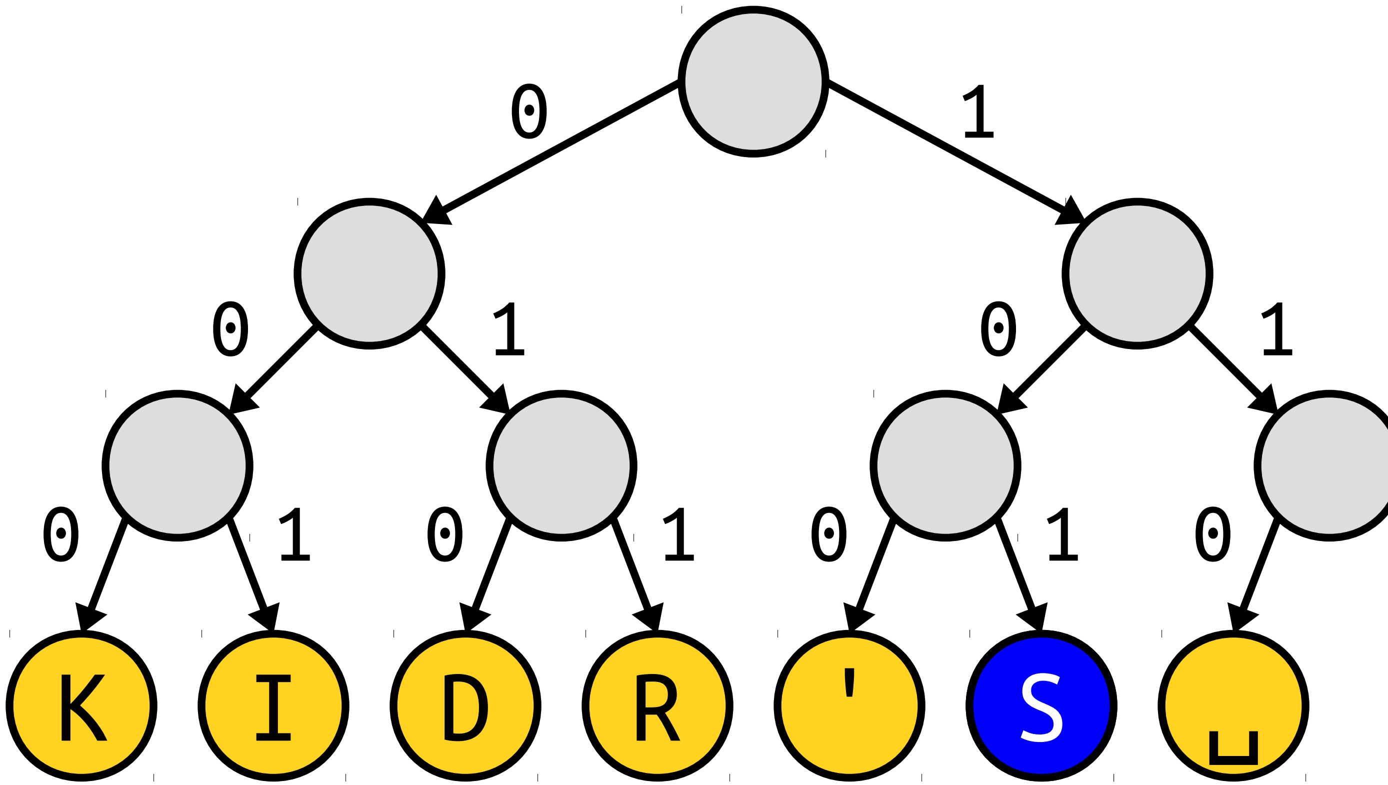


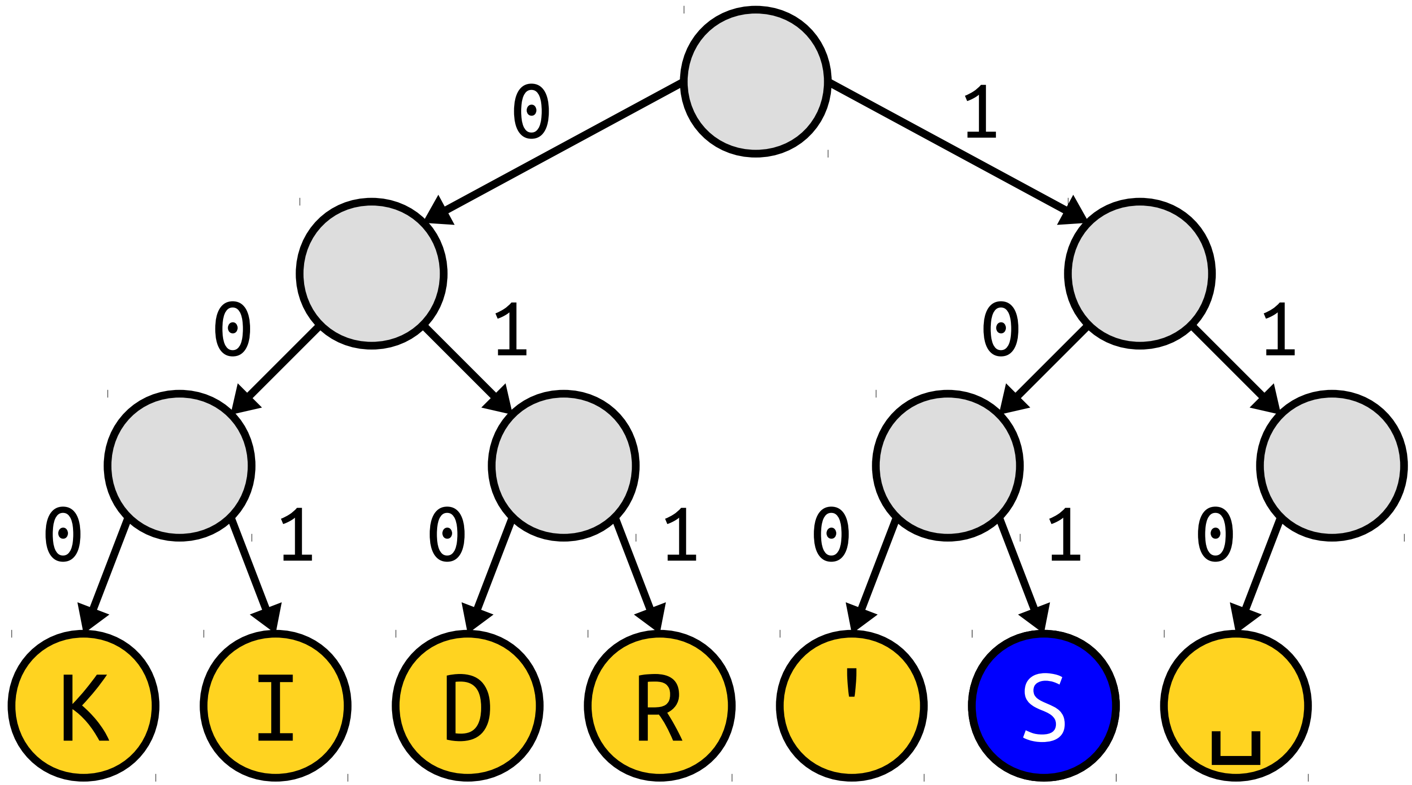
101000001



What is the title of this slide?

S 000001

What is the title of this slide?



S 000001

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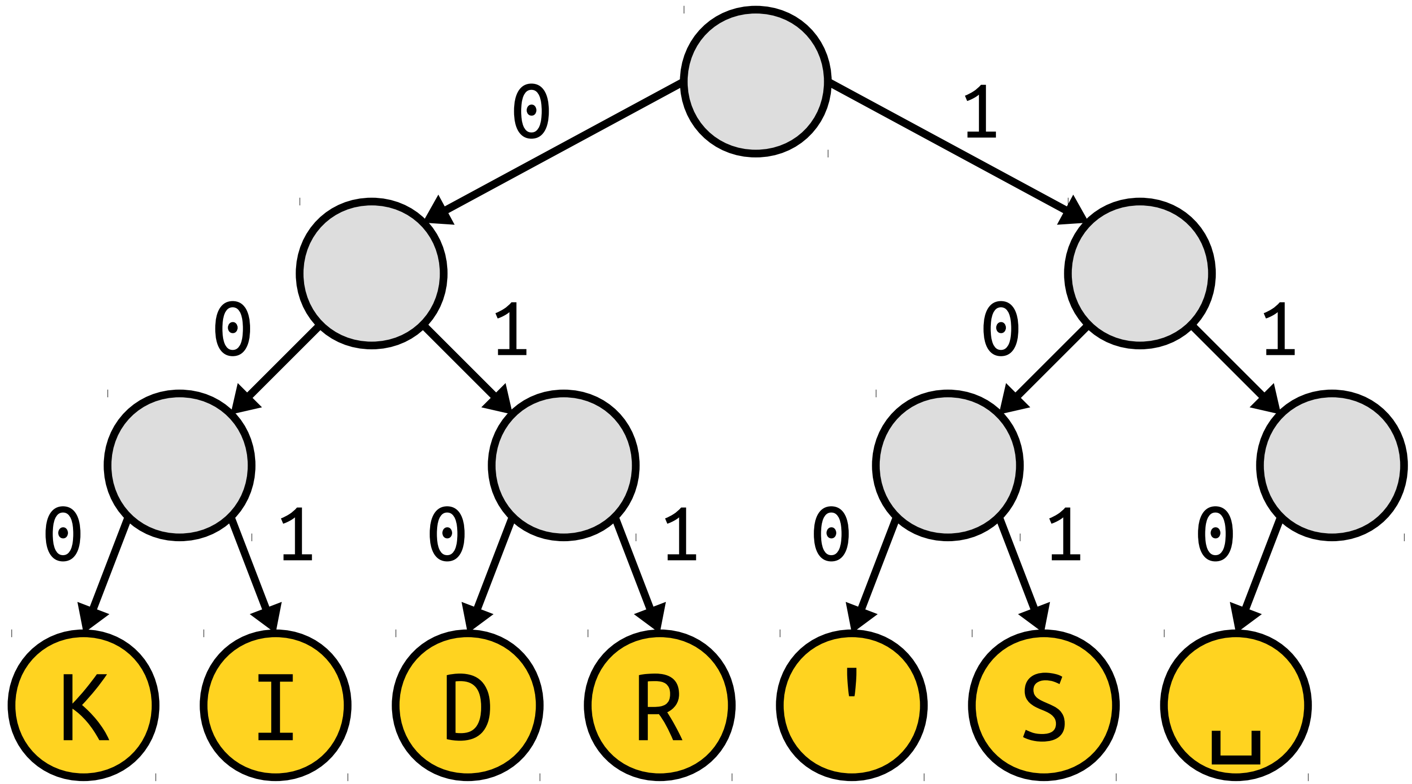

S 000001

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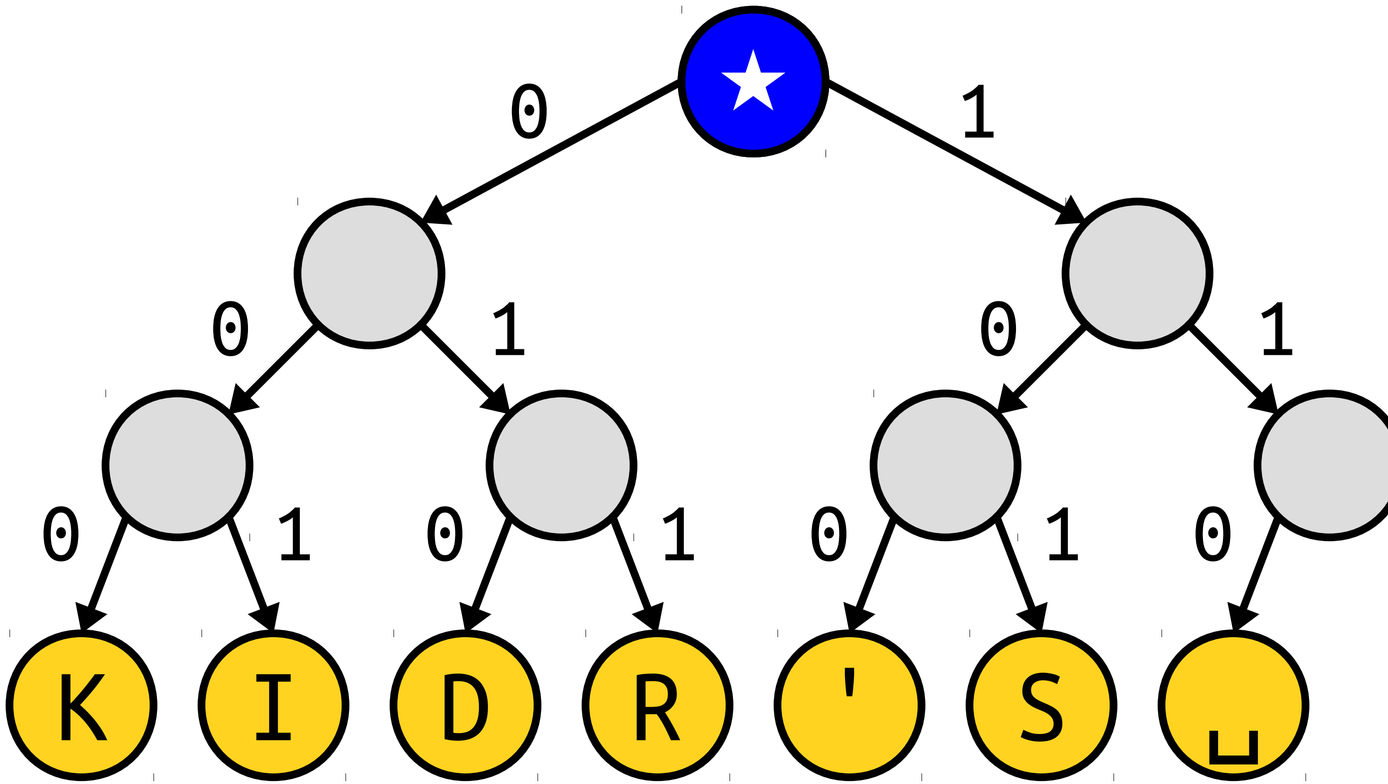

S 000001

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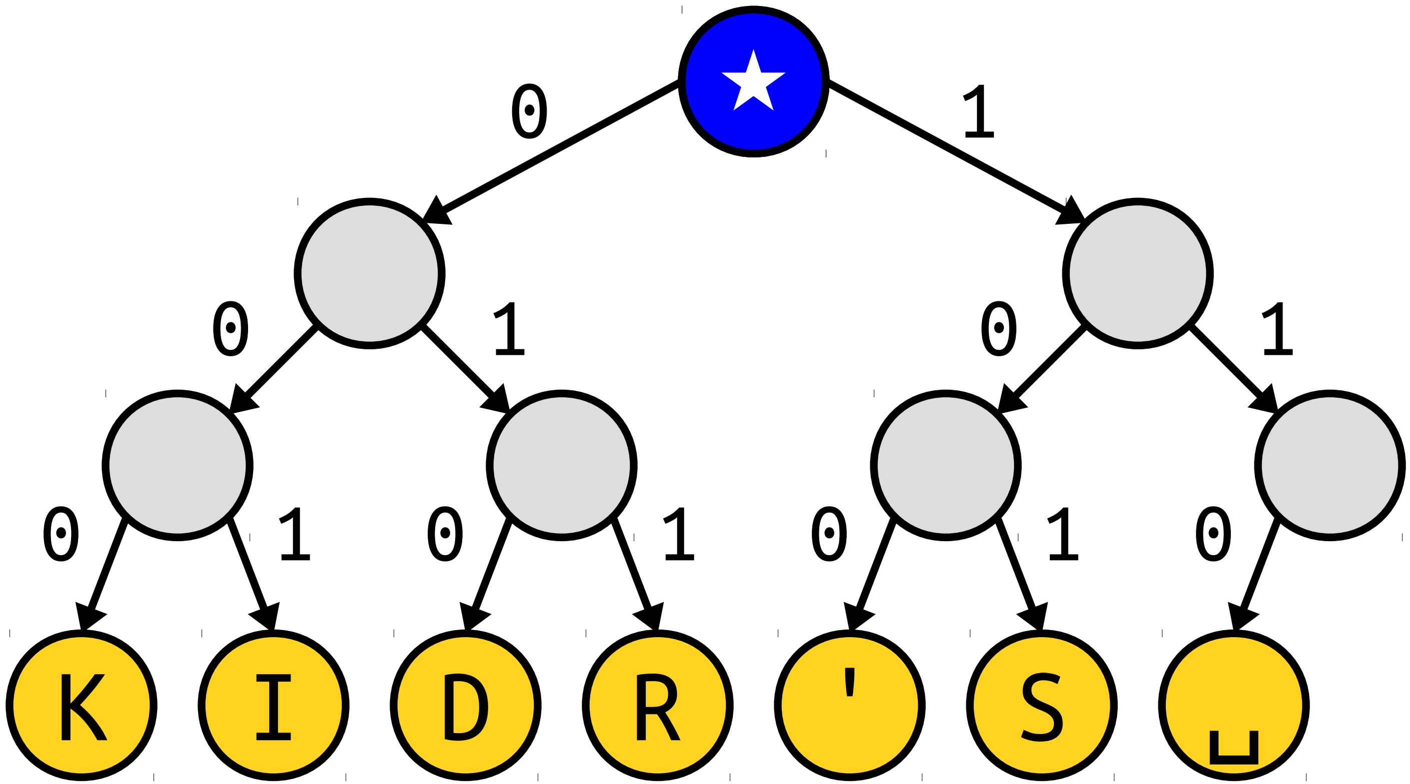

S 000001

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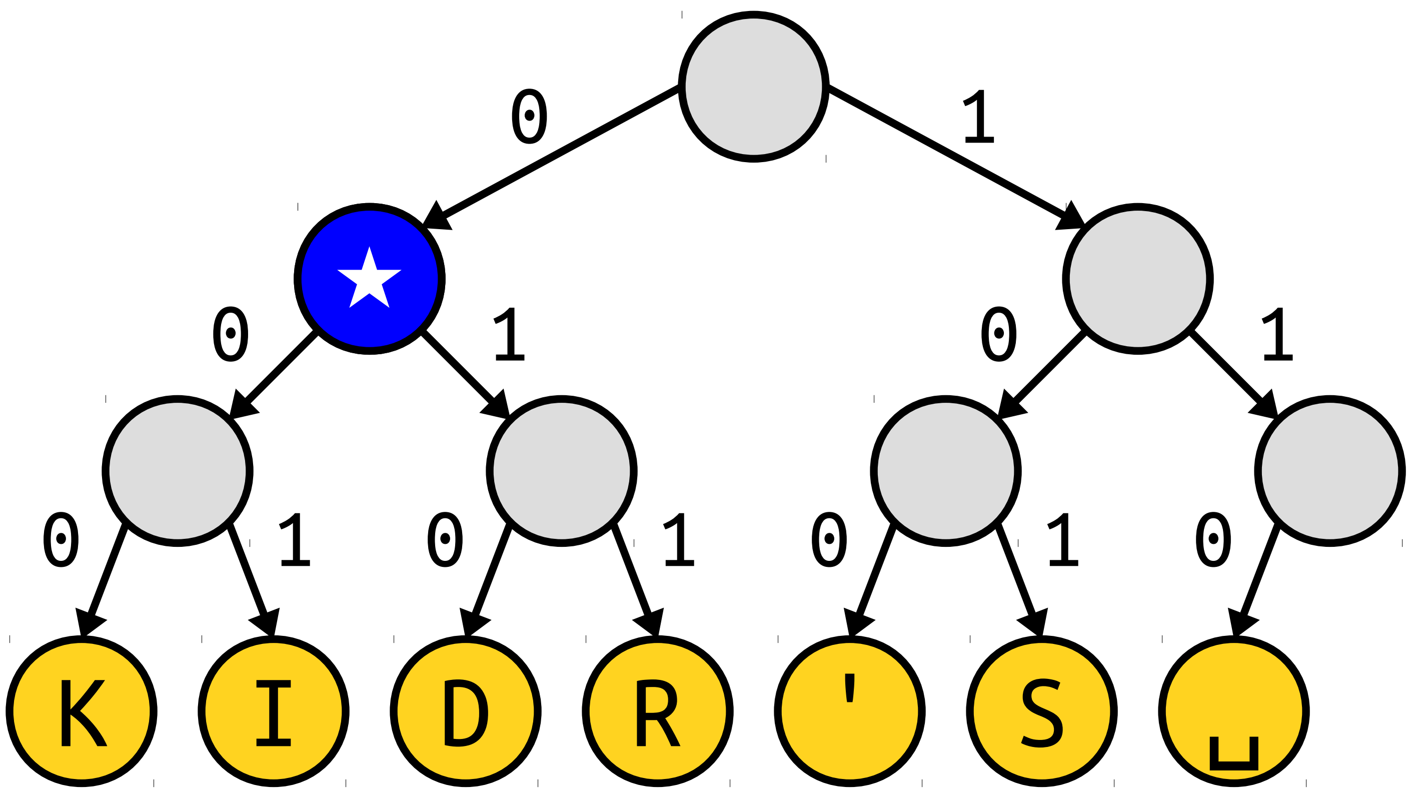
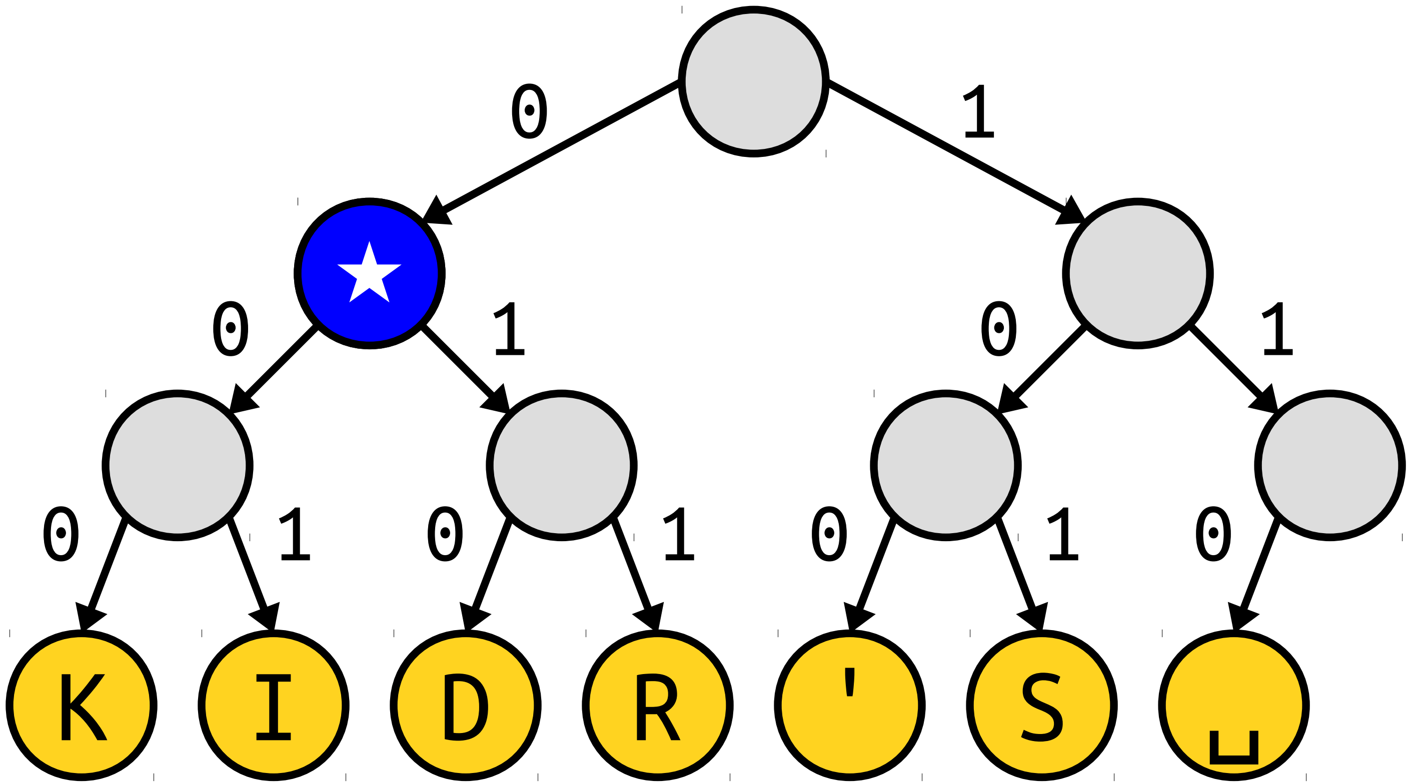


S 000001



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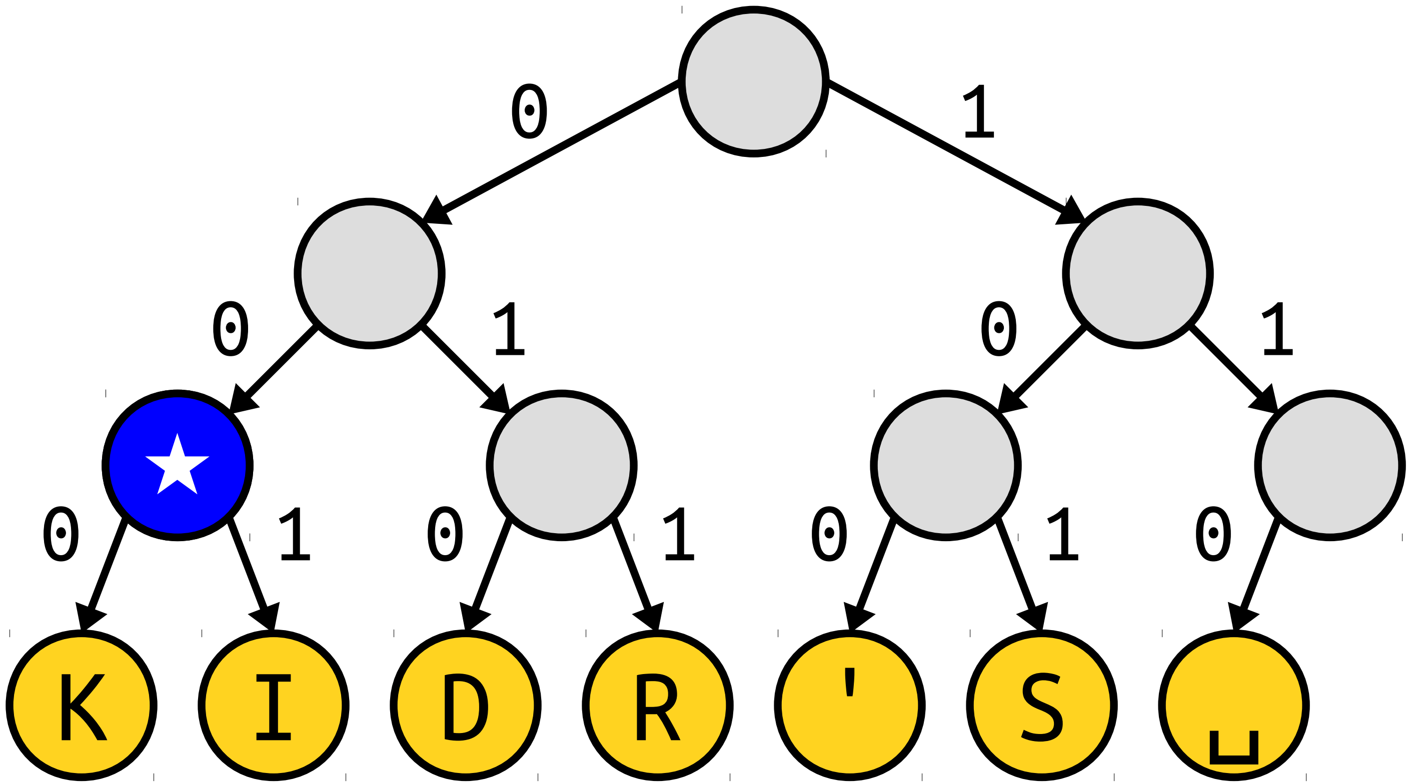
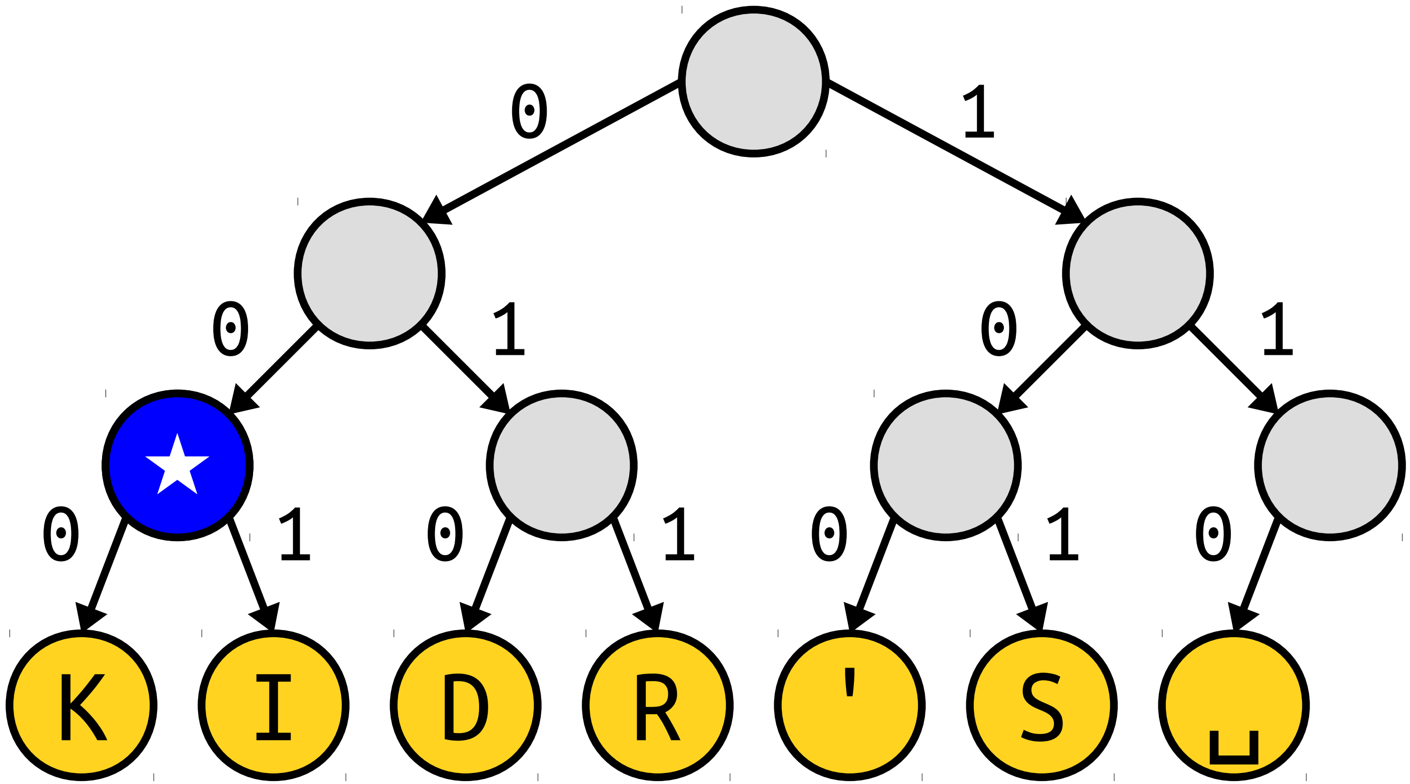
S 000001

What is the title of this slide?



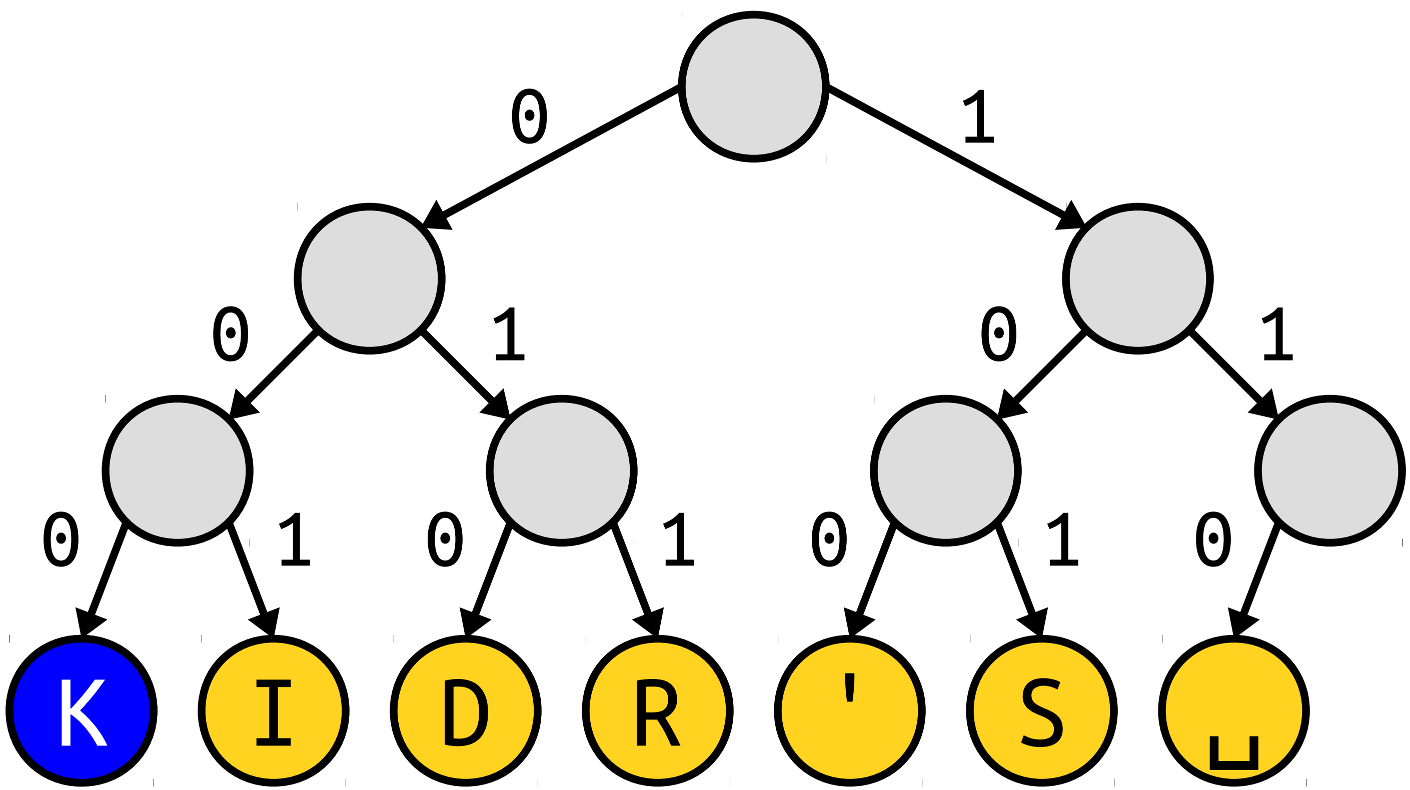
S 000001



What is the title of this slide?

S 000001

What is the title of this slide?



S K 001

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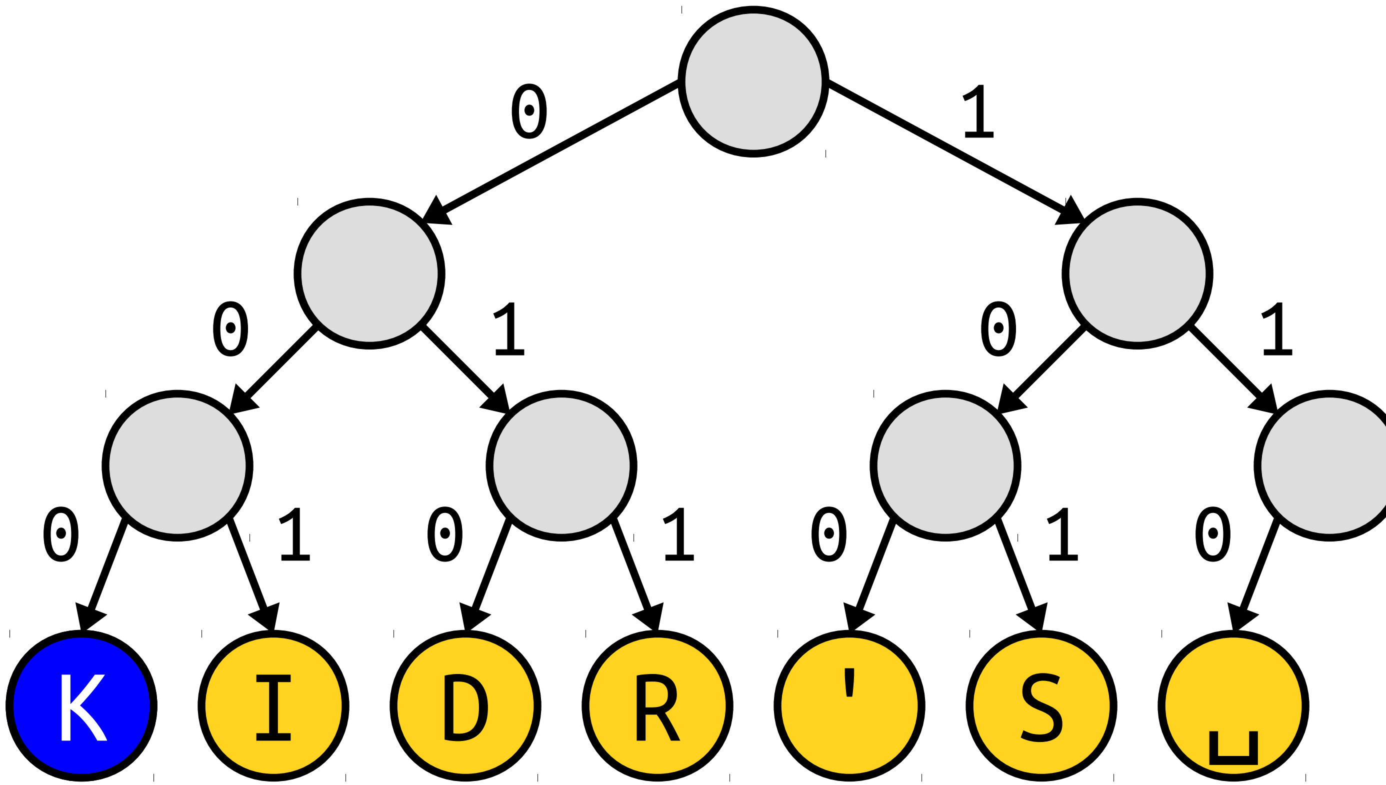

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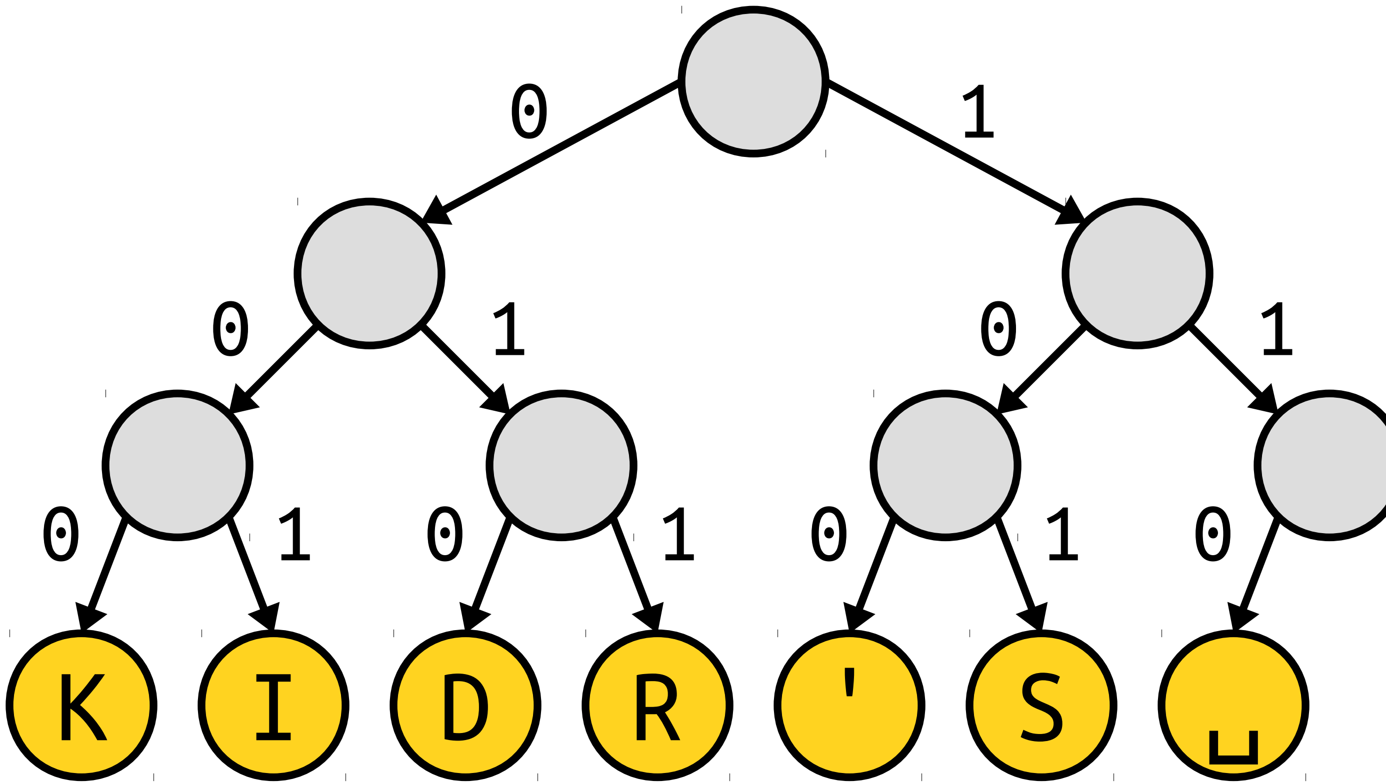

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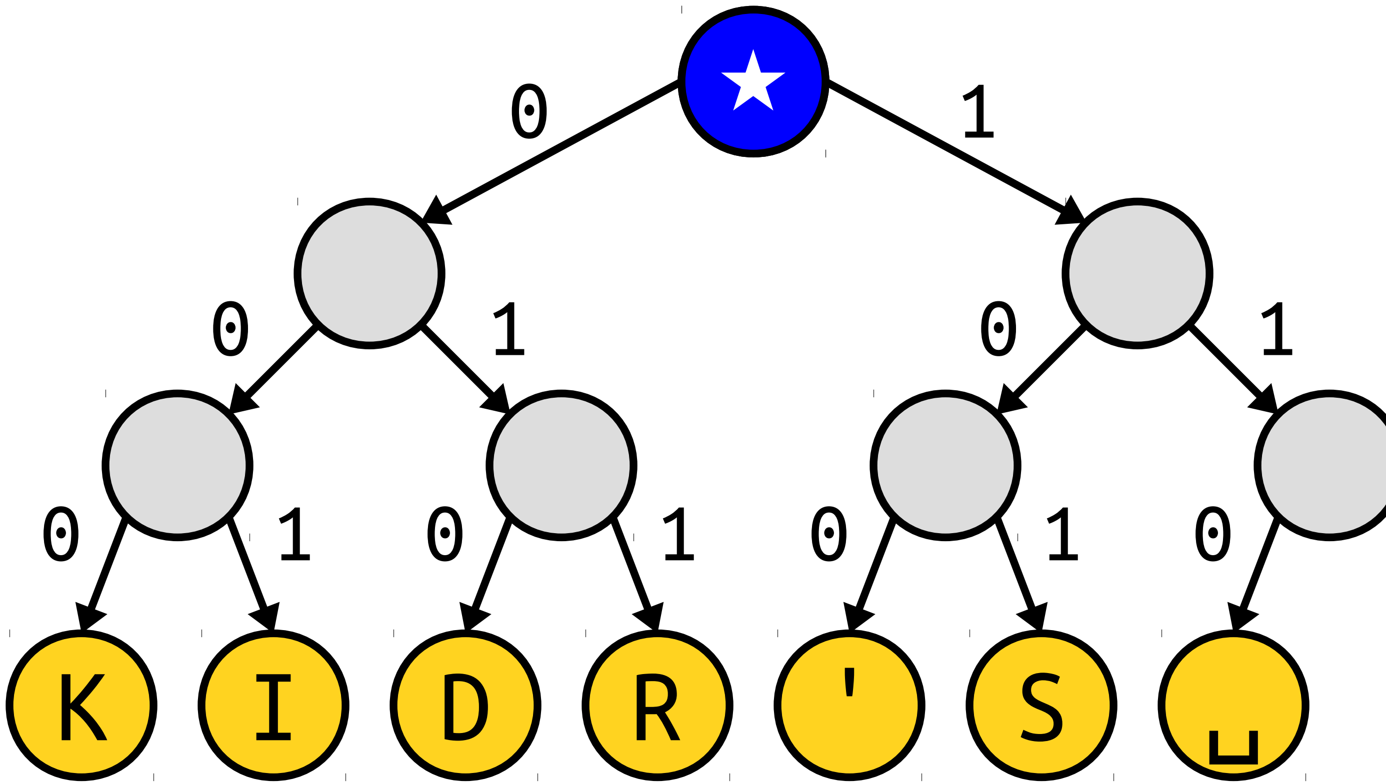

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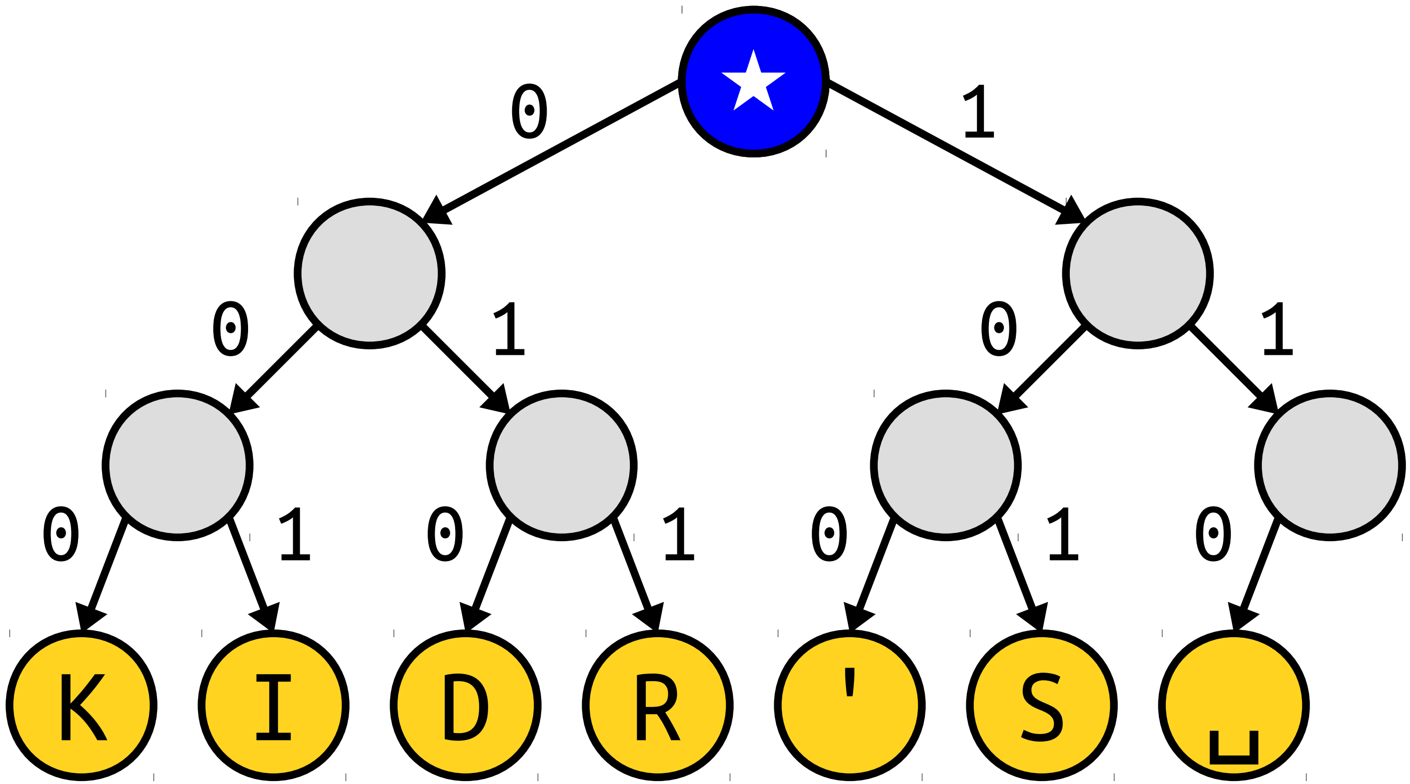
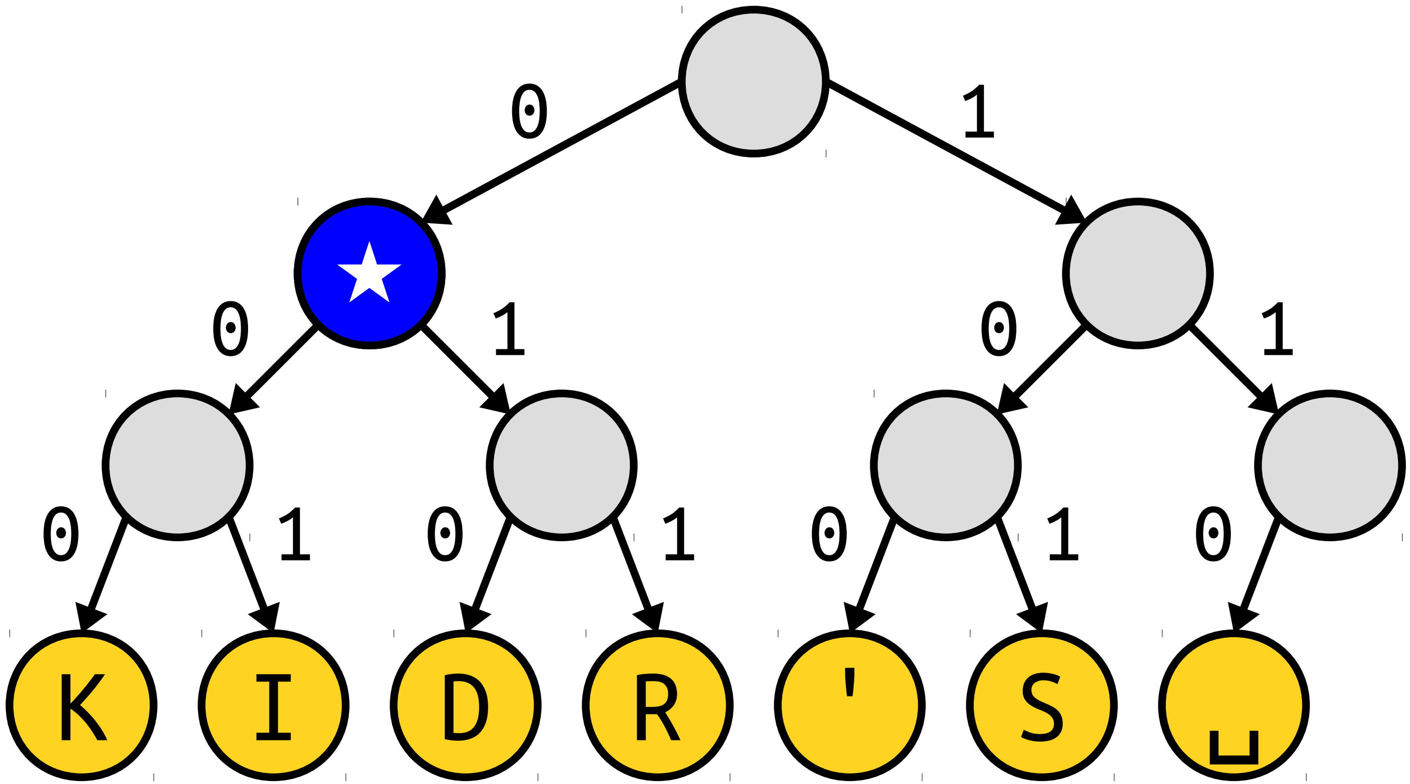


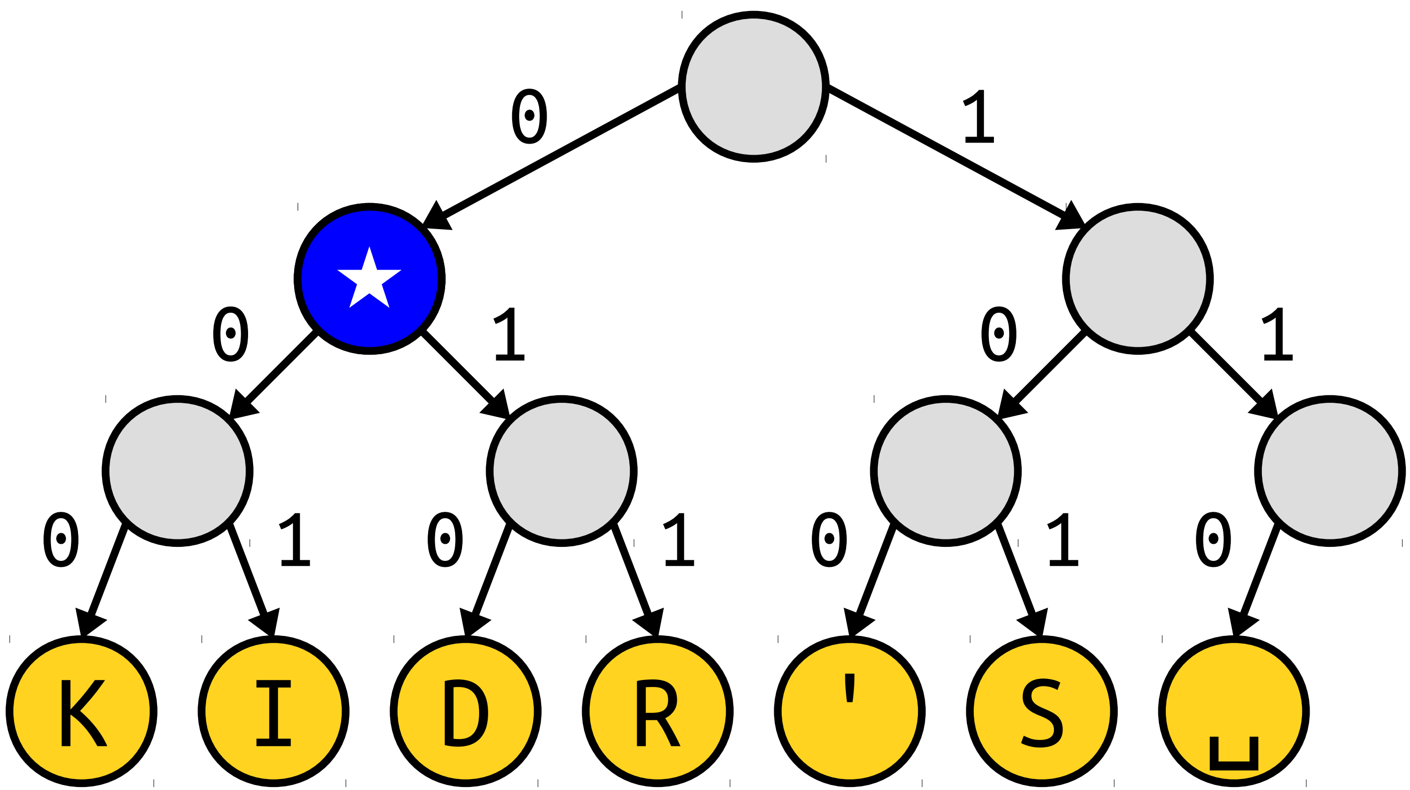
S K 001



What is the title of this slide?

S K 001

What is the title of this slide?



S K 001

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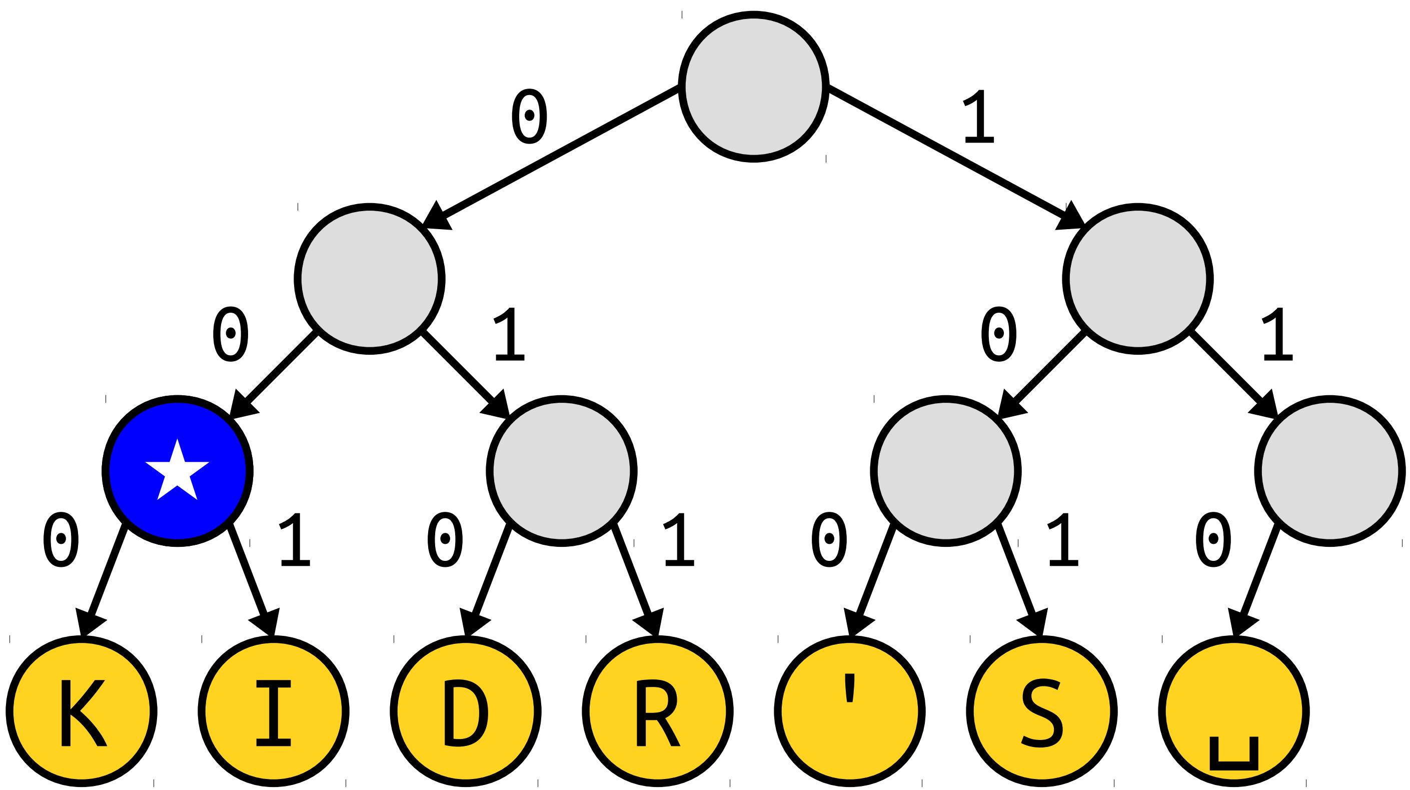
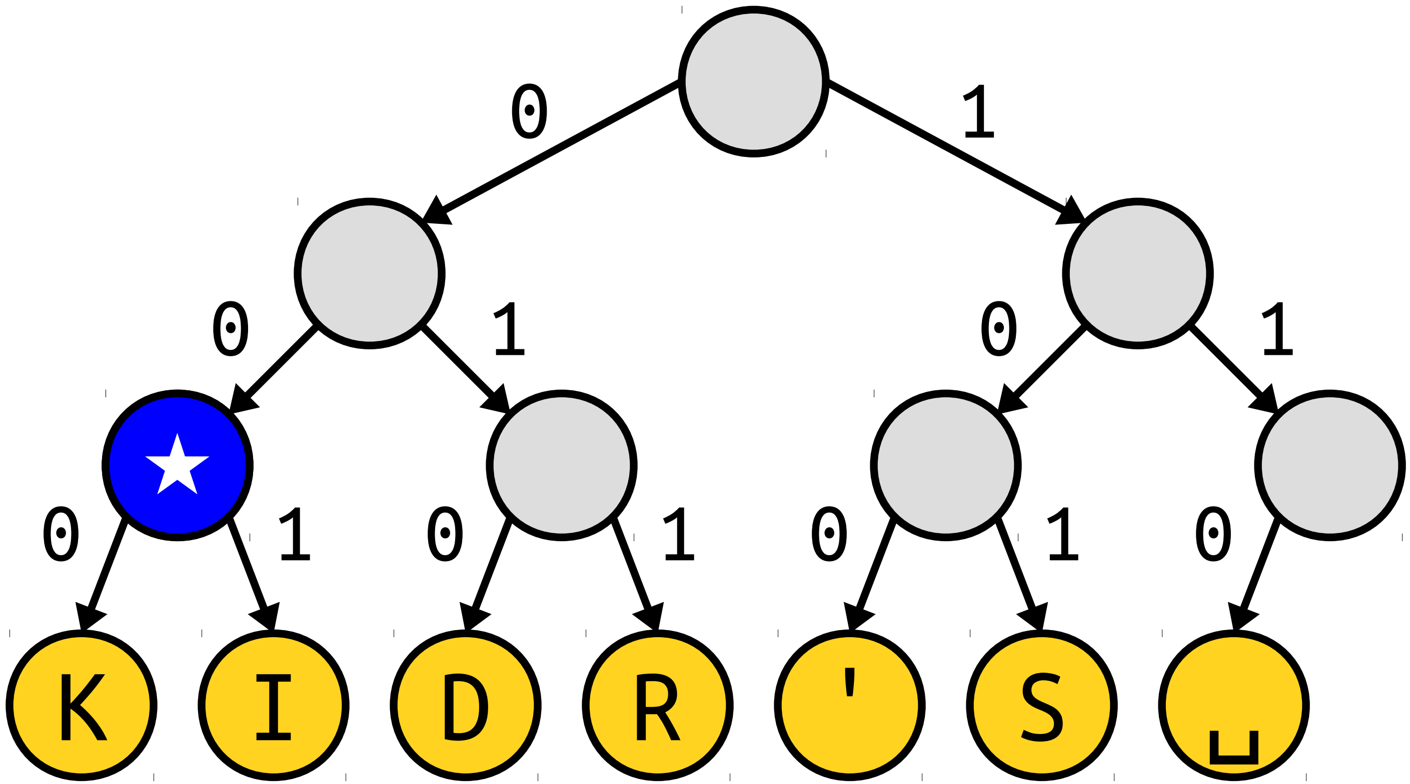
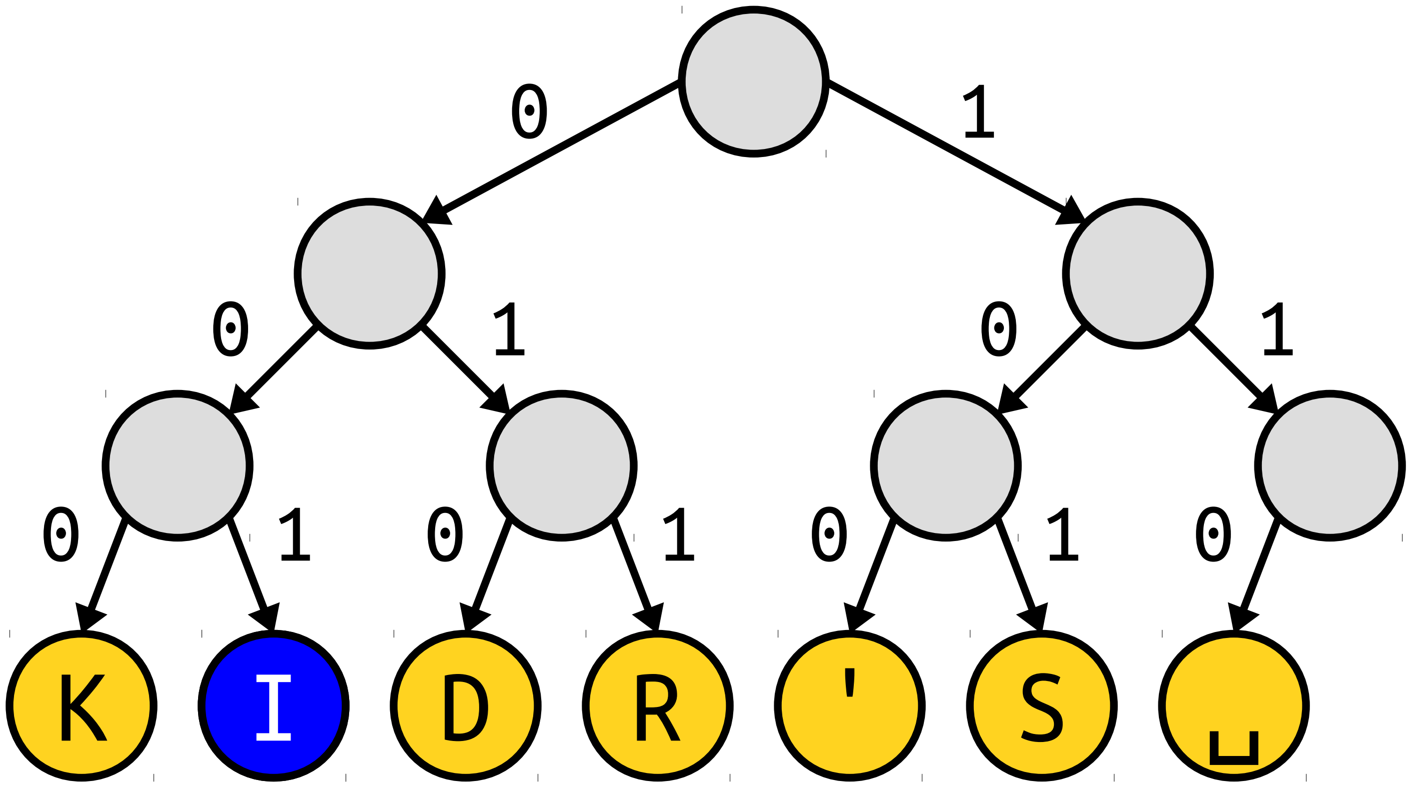


S K 001



What is the title of this slide?

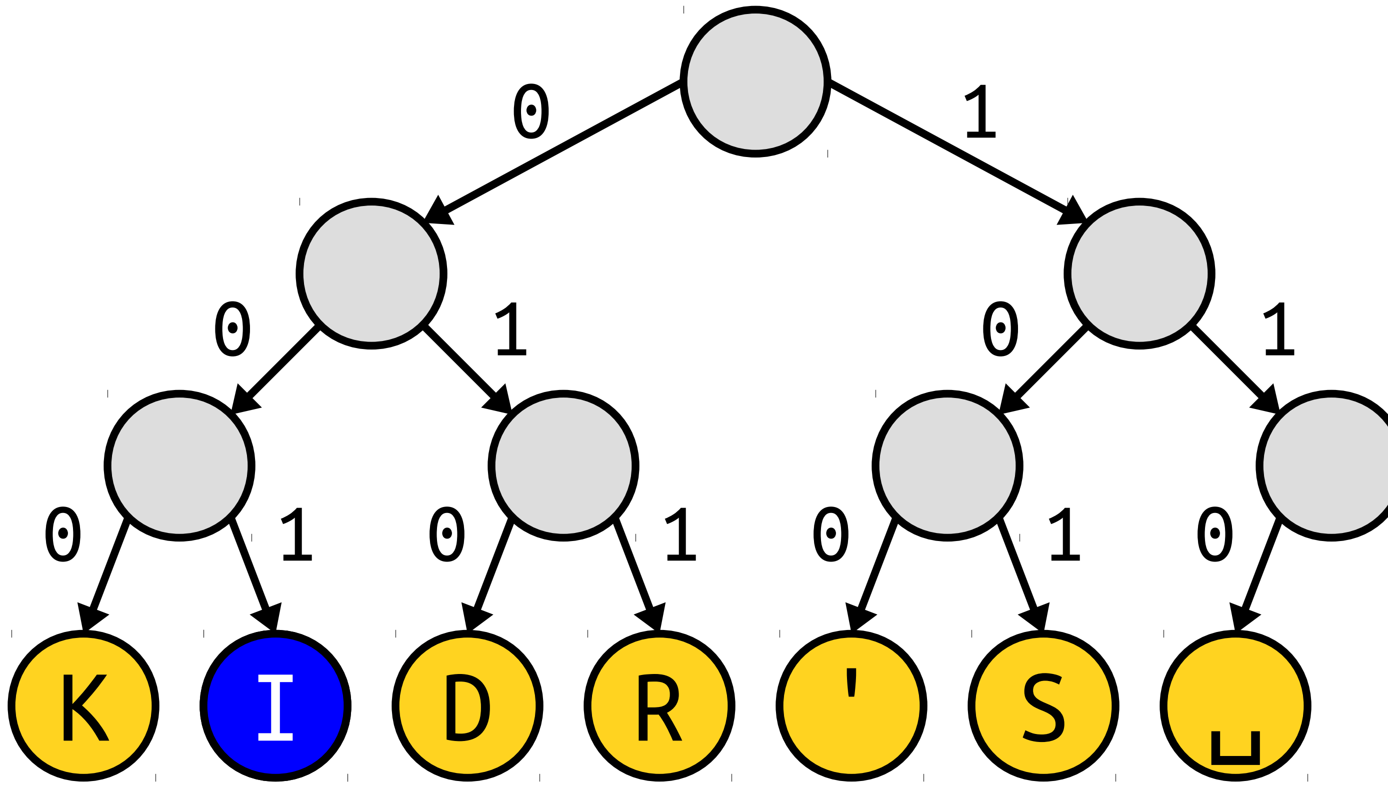
S K 001



What is the title of this slide?

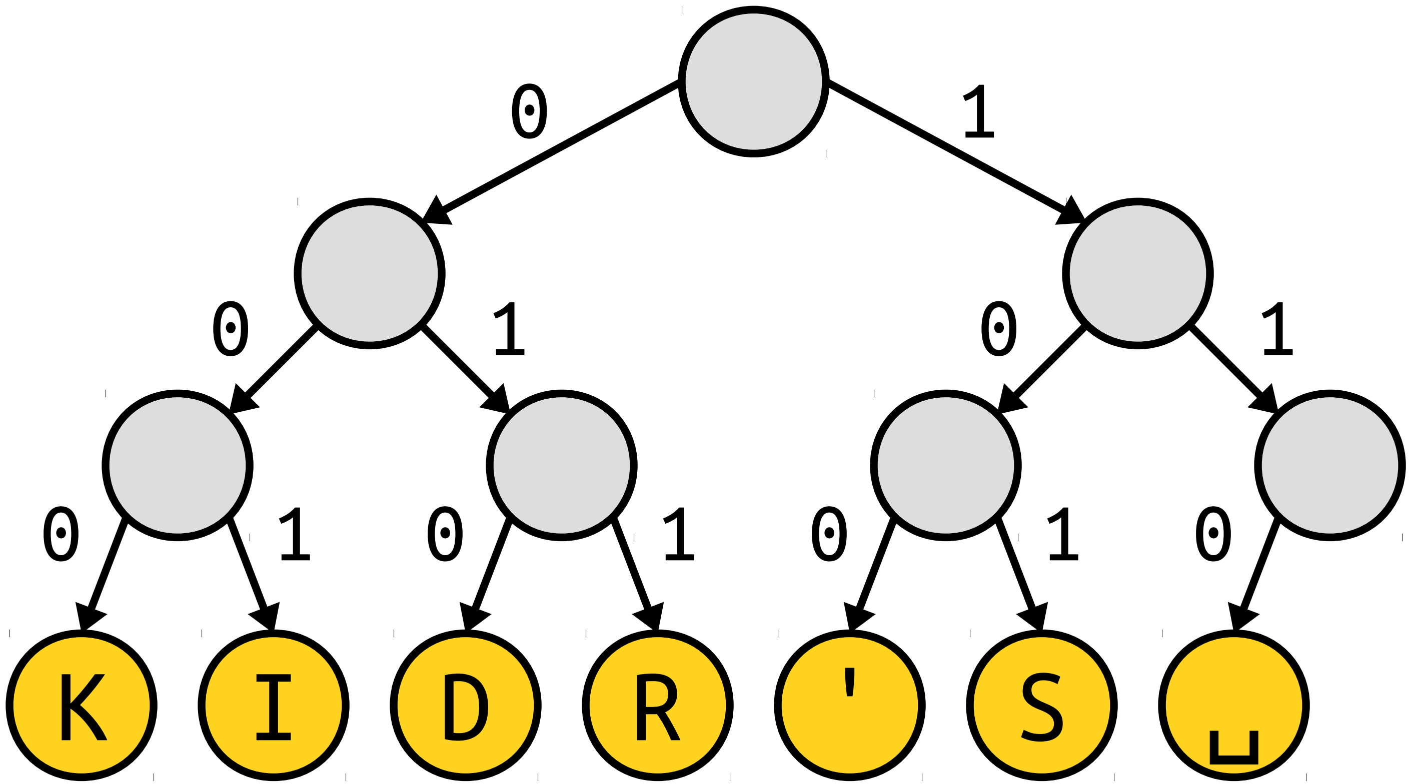
S K I

What is the title of this slide?



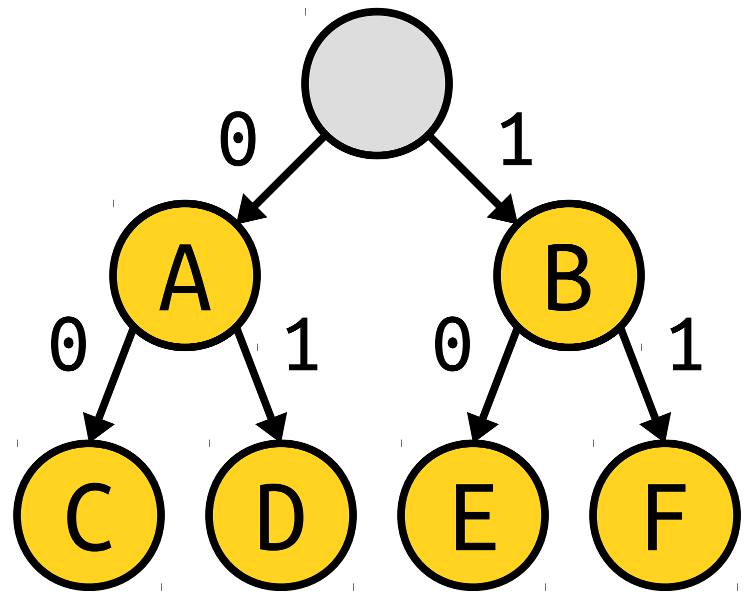
S K I

What is the title of this slide?



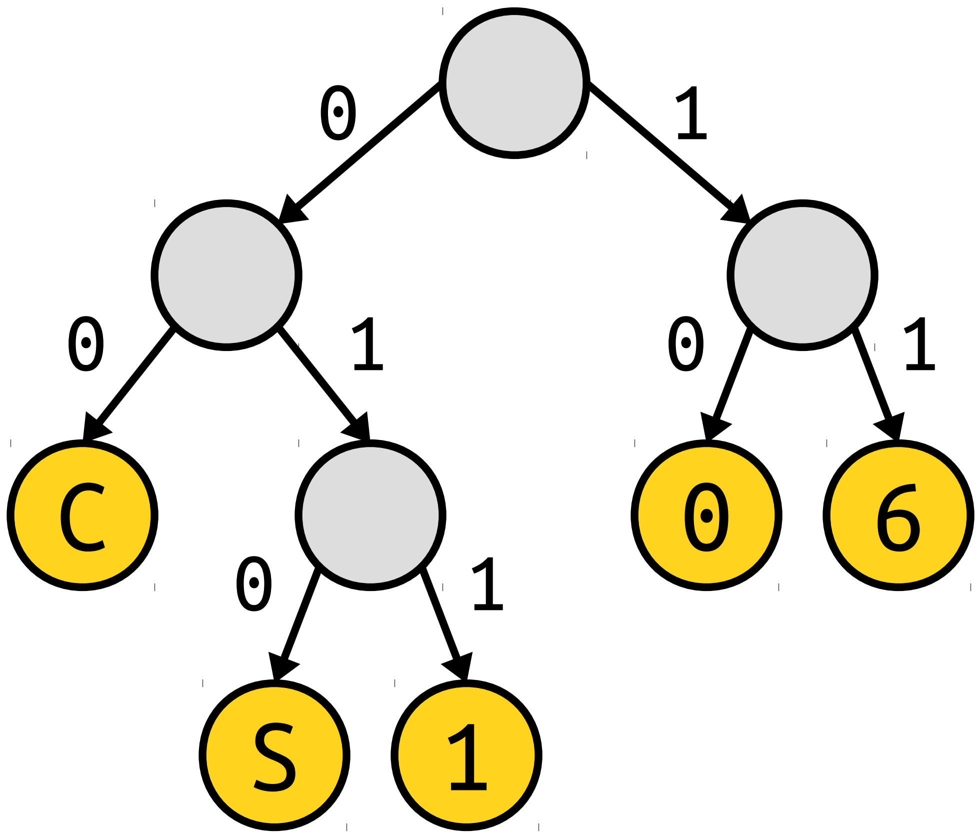
Coding Trees

- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?
- ***Answer:*** It doesn't give a prefix code. The code for `A` is a prefix for the codes for `C` and `D`.



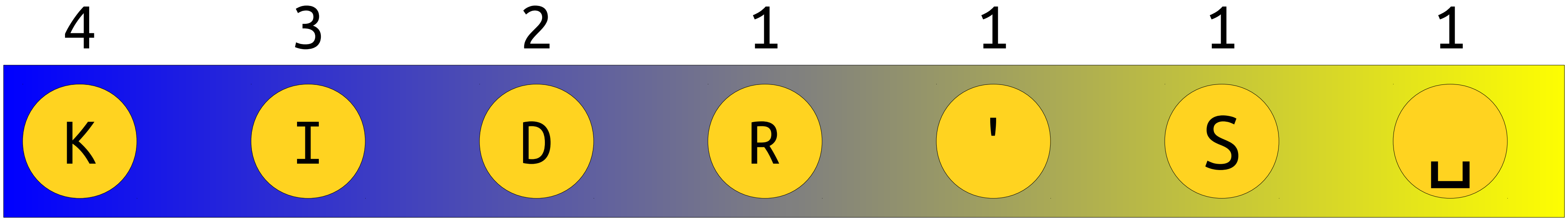
Coding Trees

- A coding tree is valid if all the letters are stored at the ***leaves***, with internal nodes just doing the routing.
- ***Goal:*** Find the best coding tree for a string.

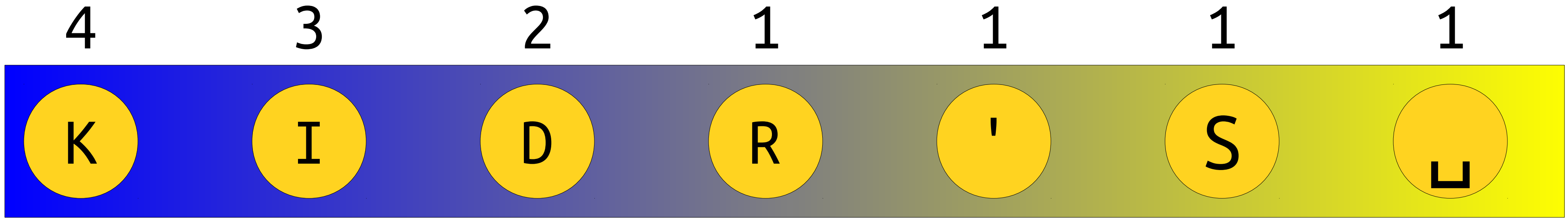


How do we find the best binary tree with this property?

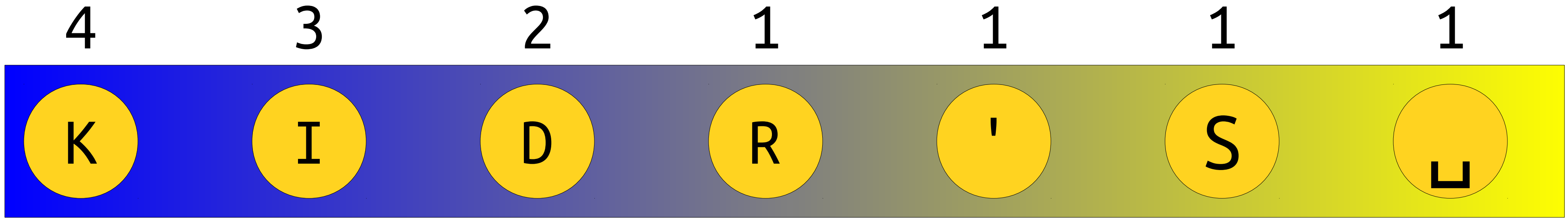
Huffman Coding

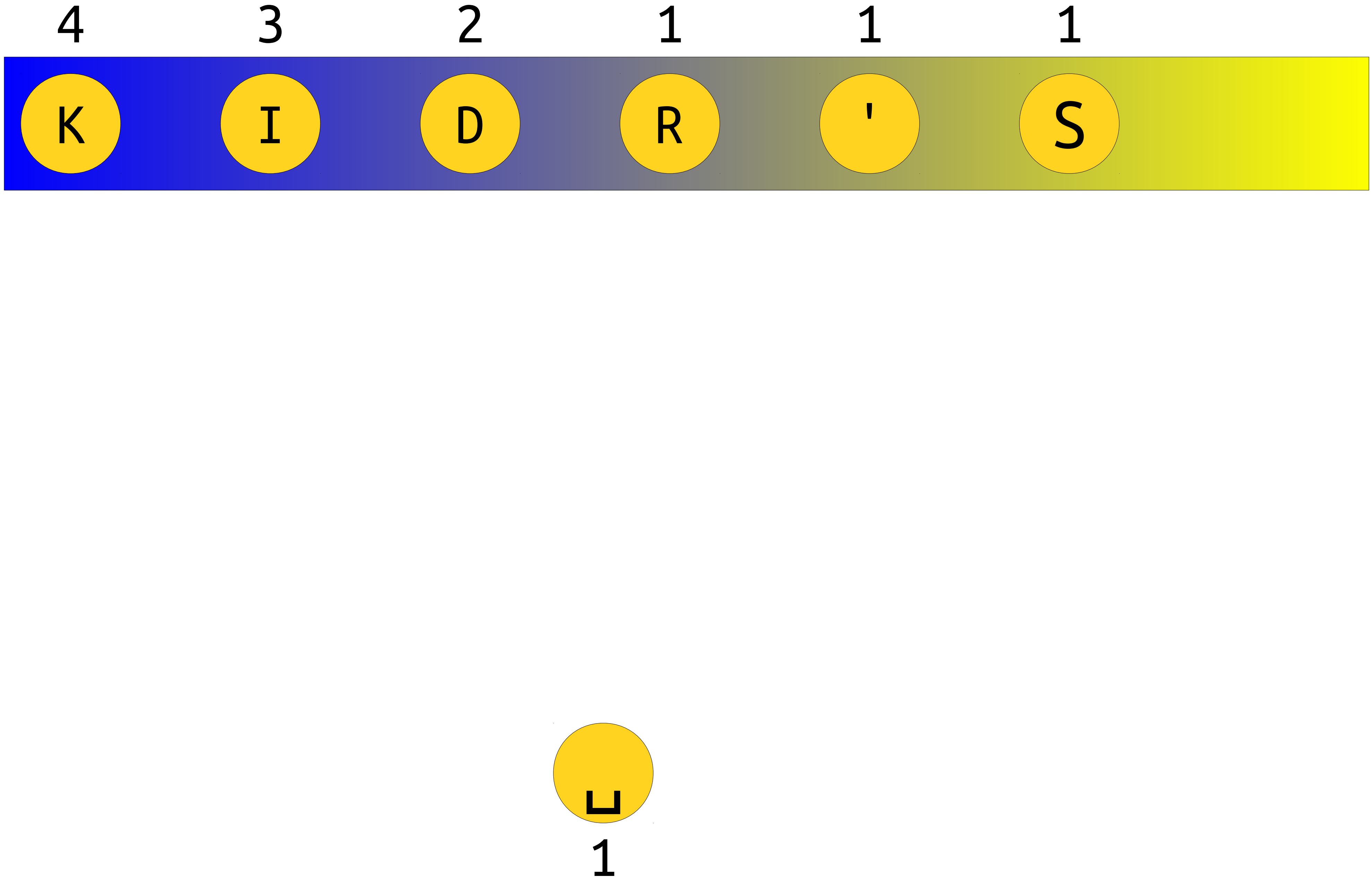


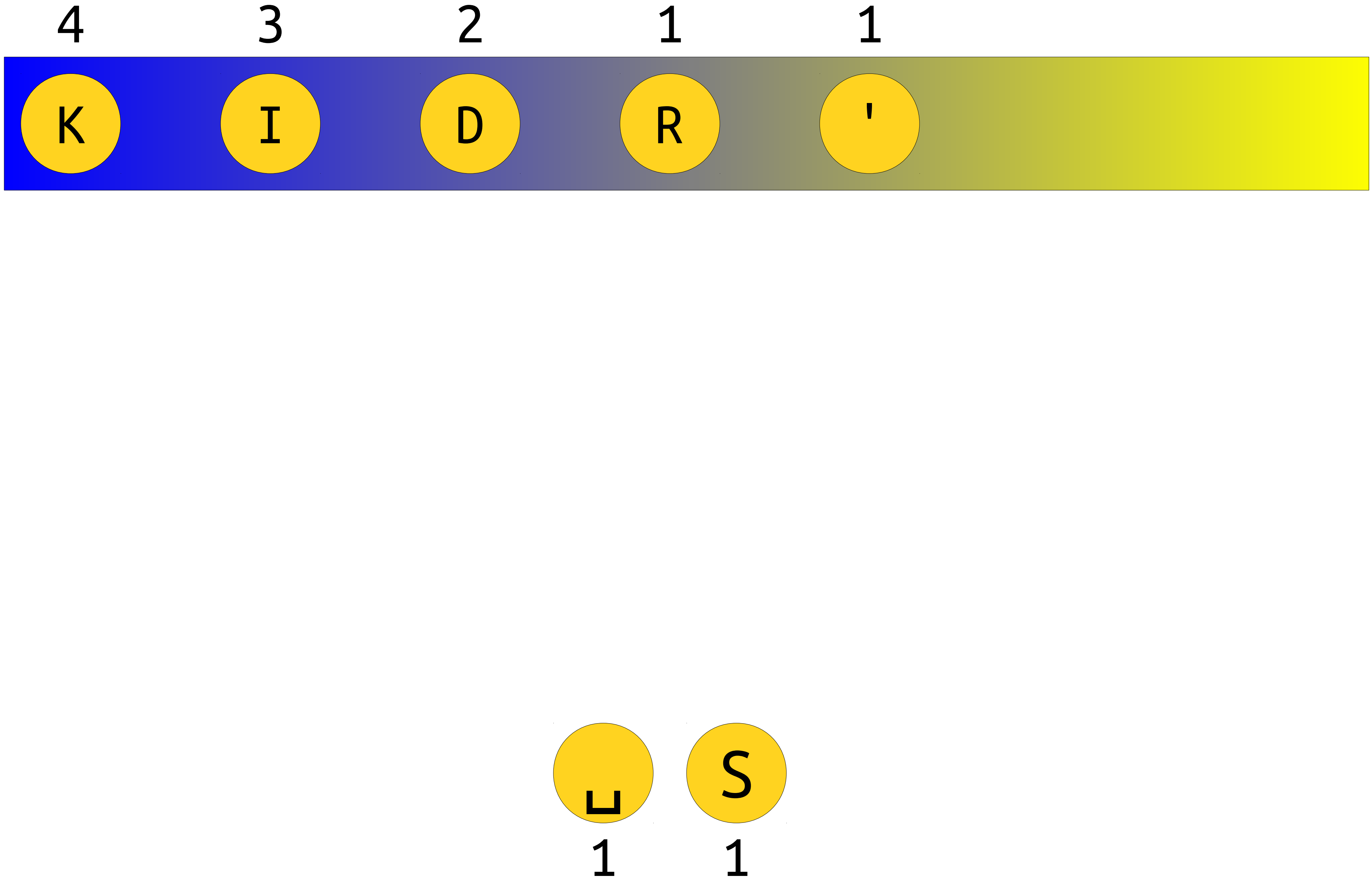
character	*frequency*
K	4
I	3
D	2
R	1
'	1
S	1
	1



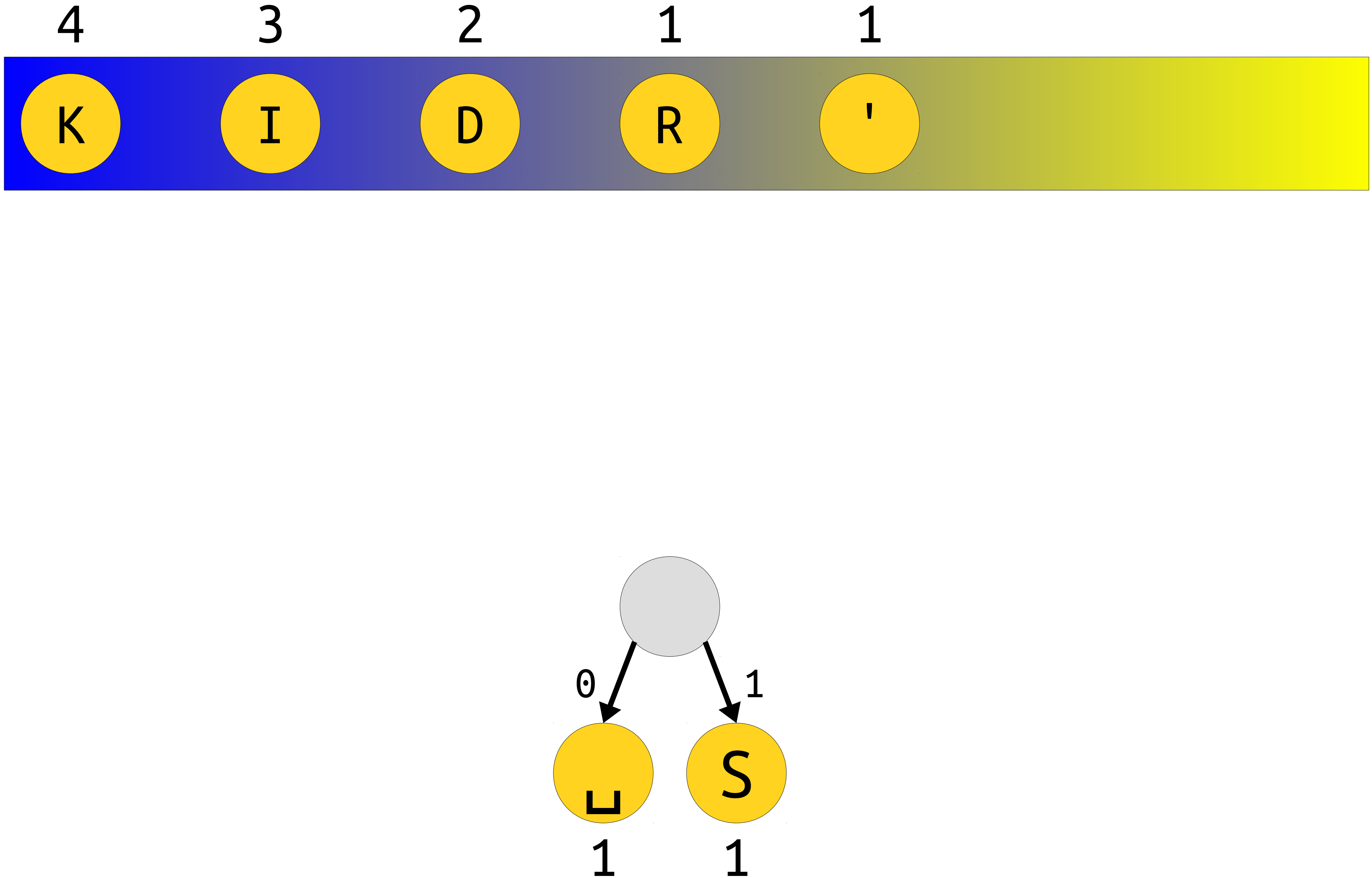
Right now, we have all the leaves of the tree. We now need to build the tree around them.


4 3 2 1 1 1 1
K I D R ' S ␣


4
K
3
I
2
D
1
R
1
'
1
S
␣
1



4
3
2
1
1
K
I
D
R
'
␣
S
1
1


4
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K
I
D
R
'
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1
␣
S
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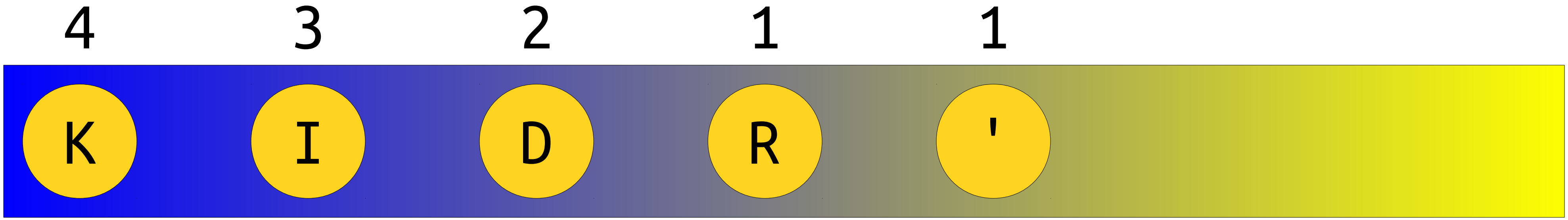
4
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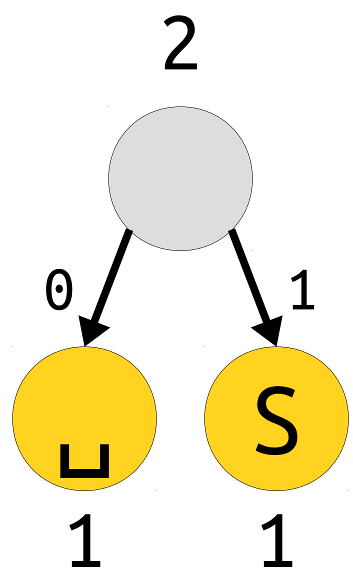
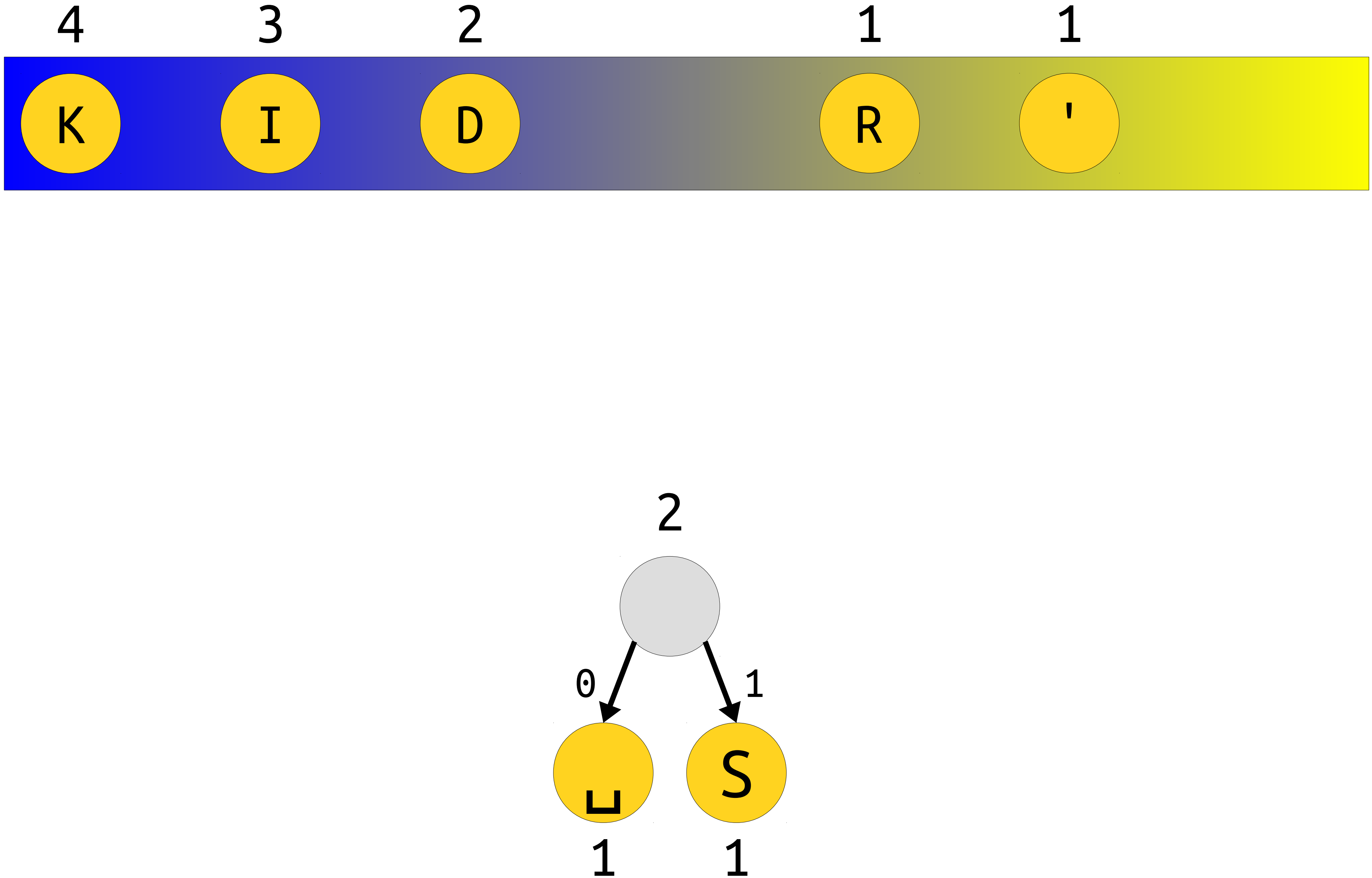
'


2
0
1
␣
S
1
1



4
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K
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D
R
'
2
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S
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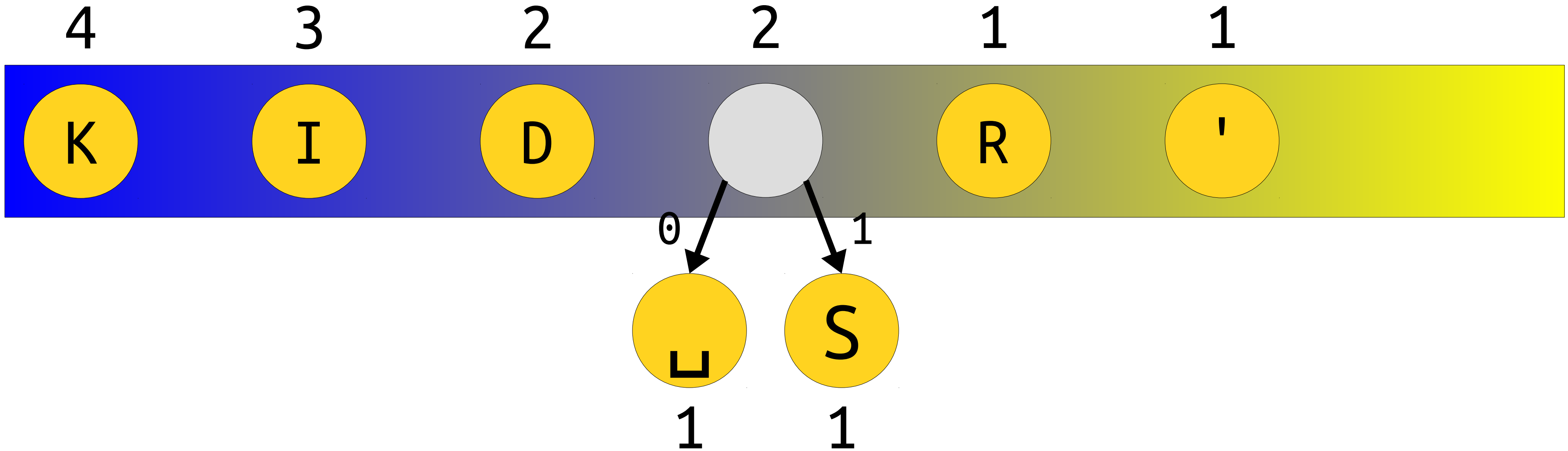

4
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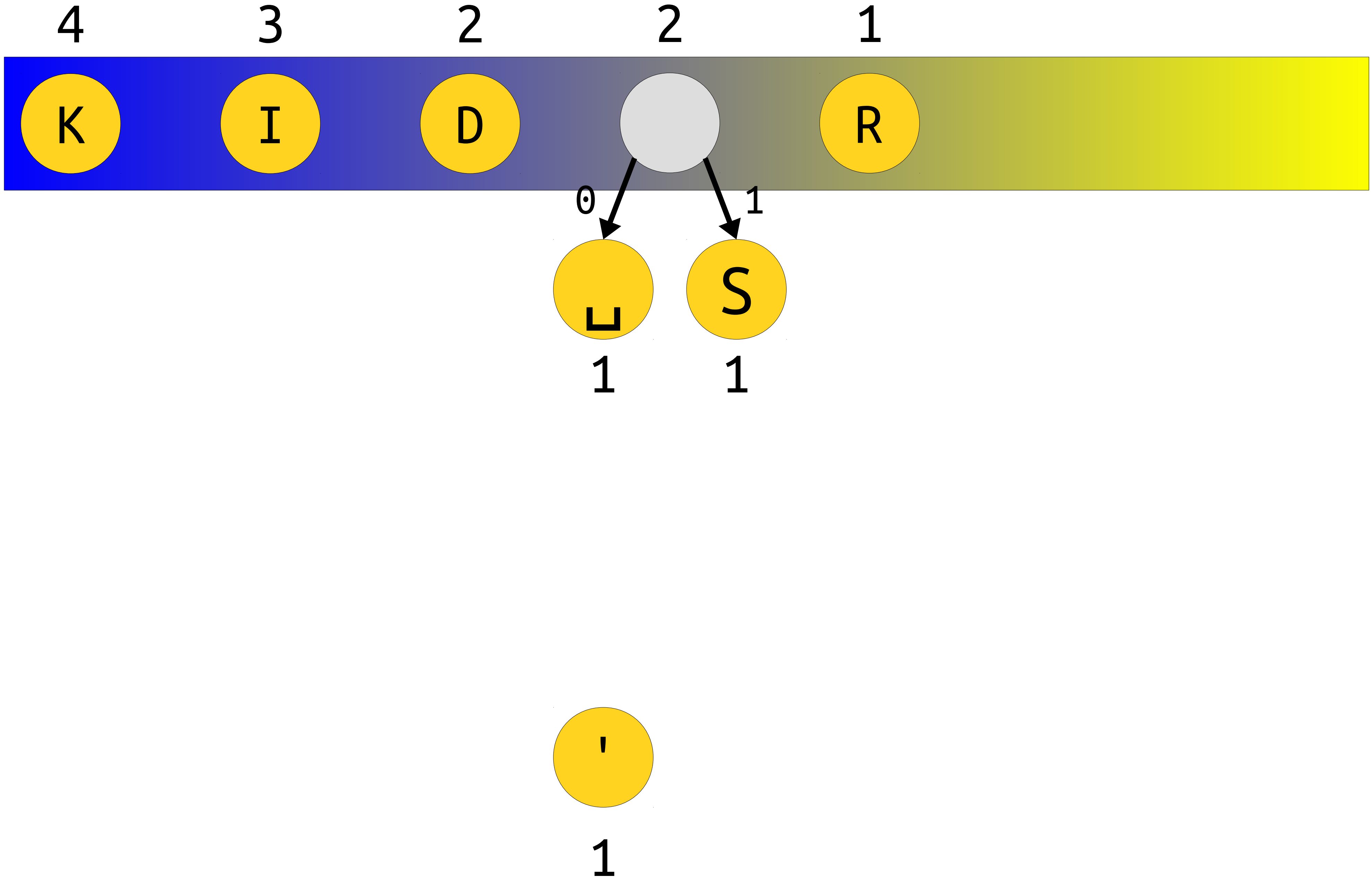
4
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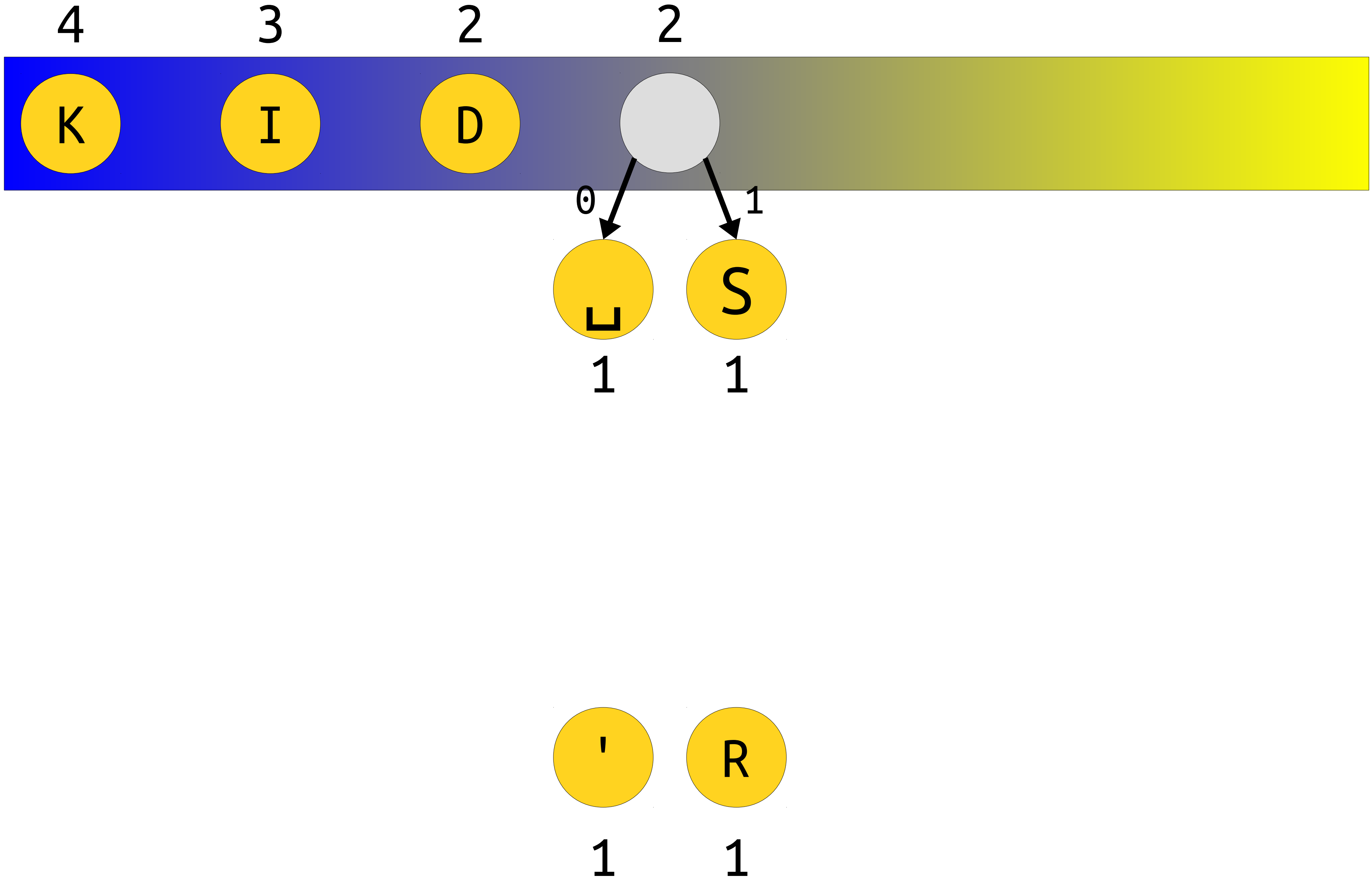
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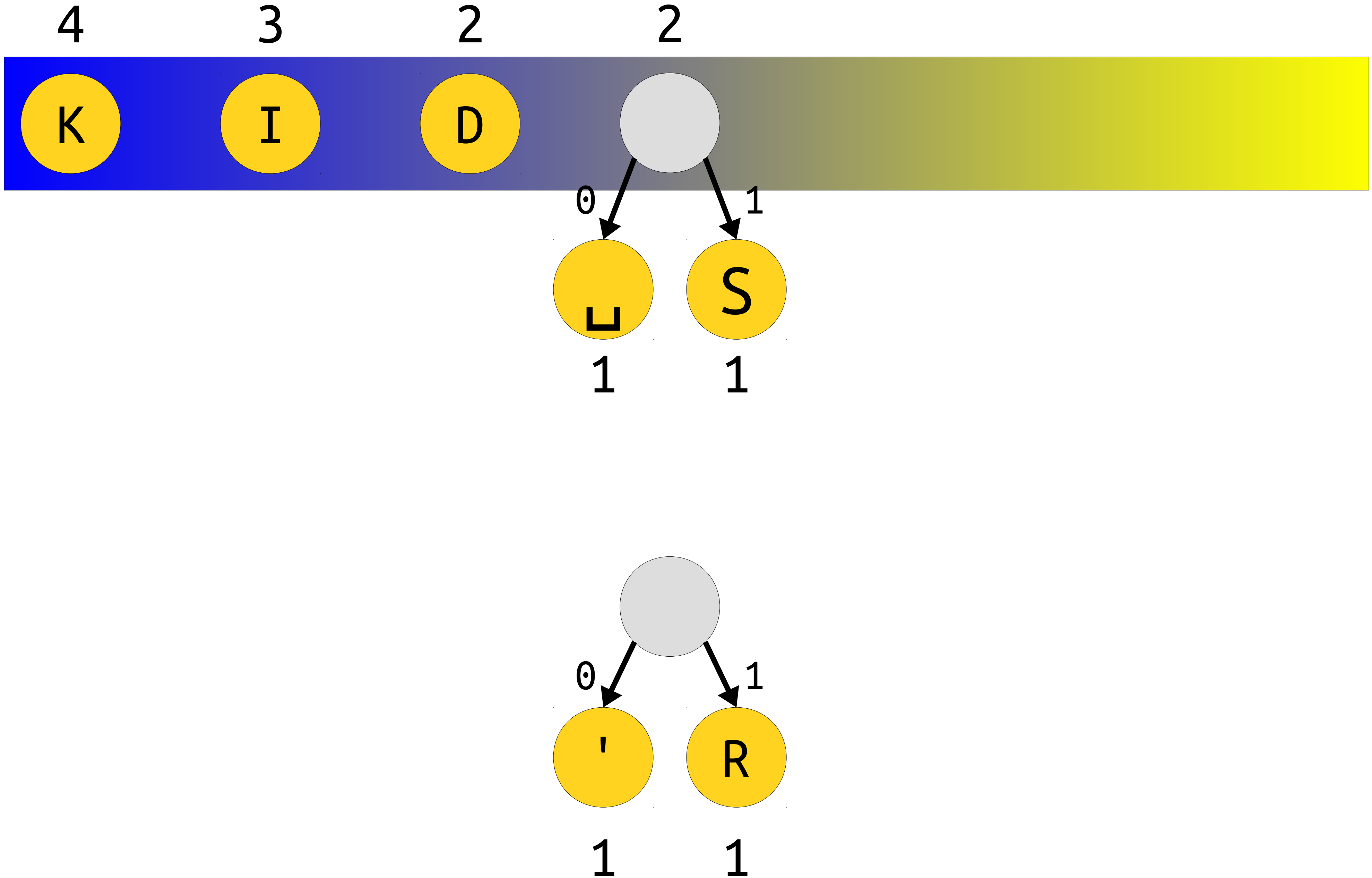

4
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␣
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1
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R
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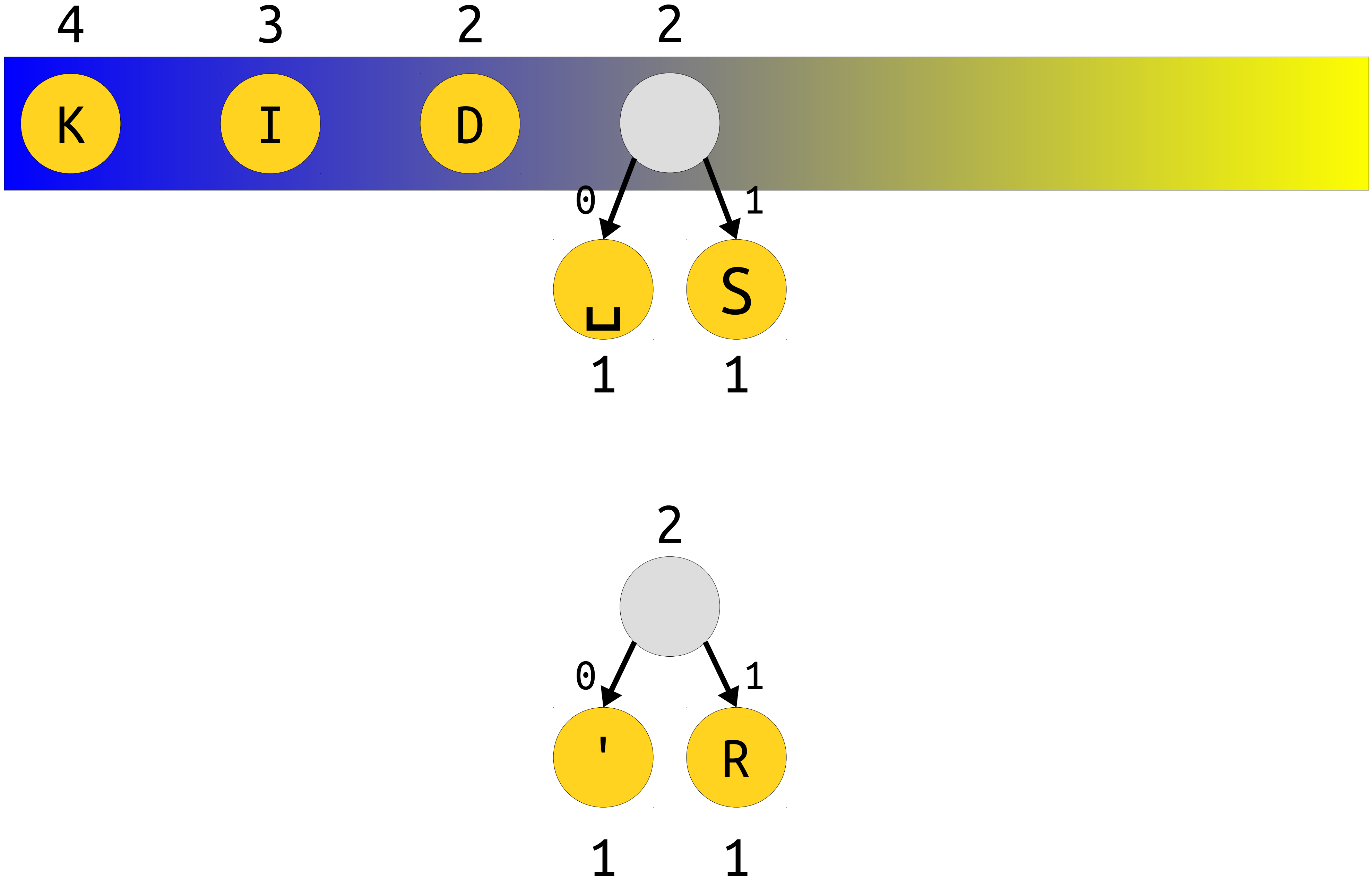
4
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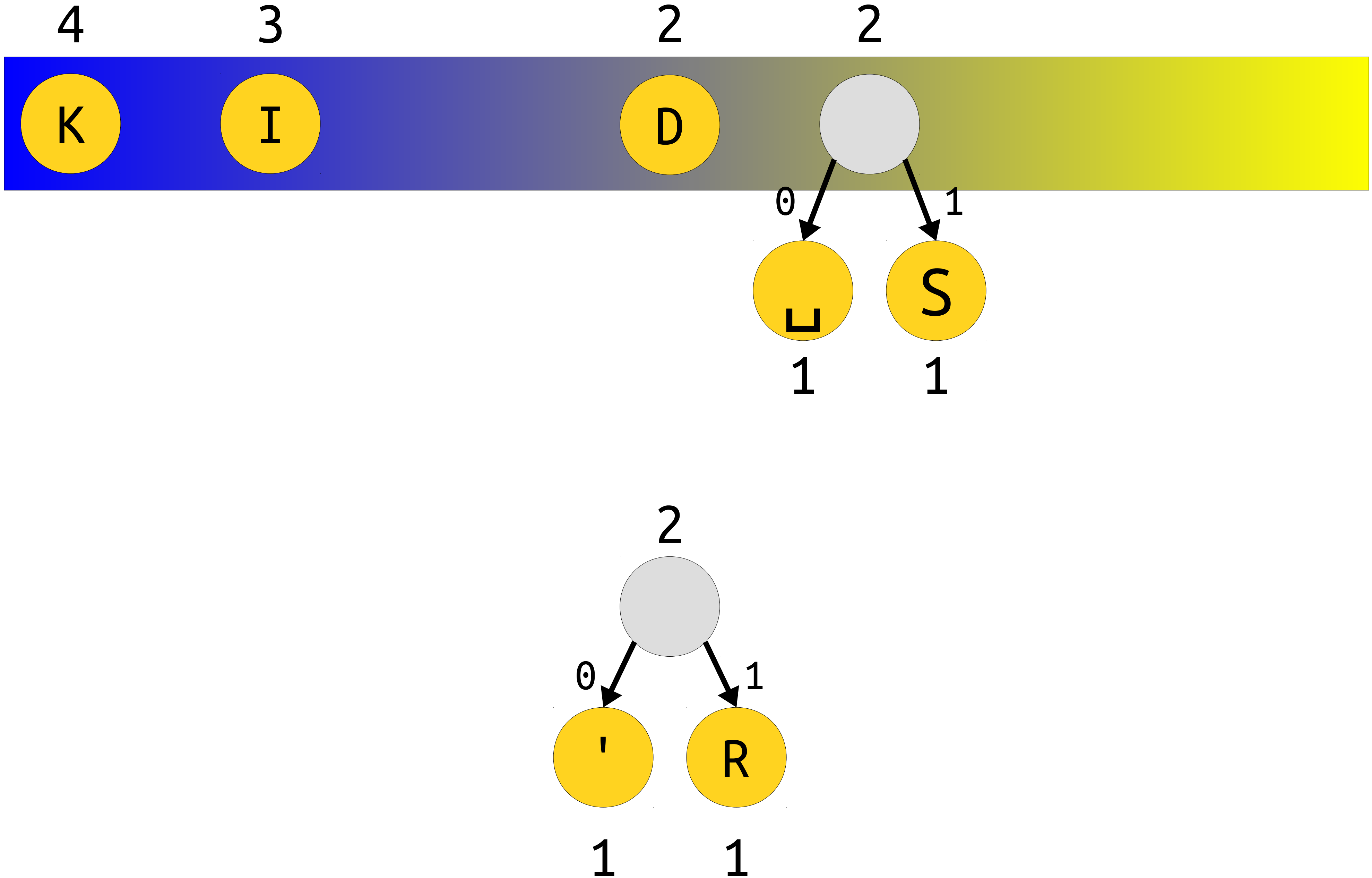
4
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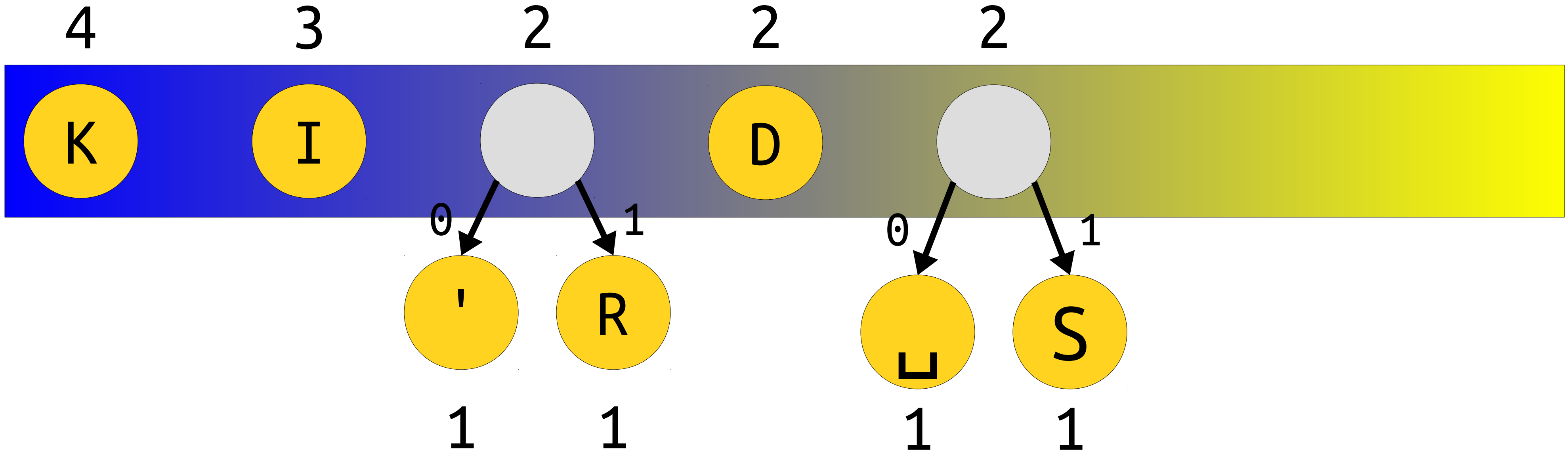
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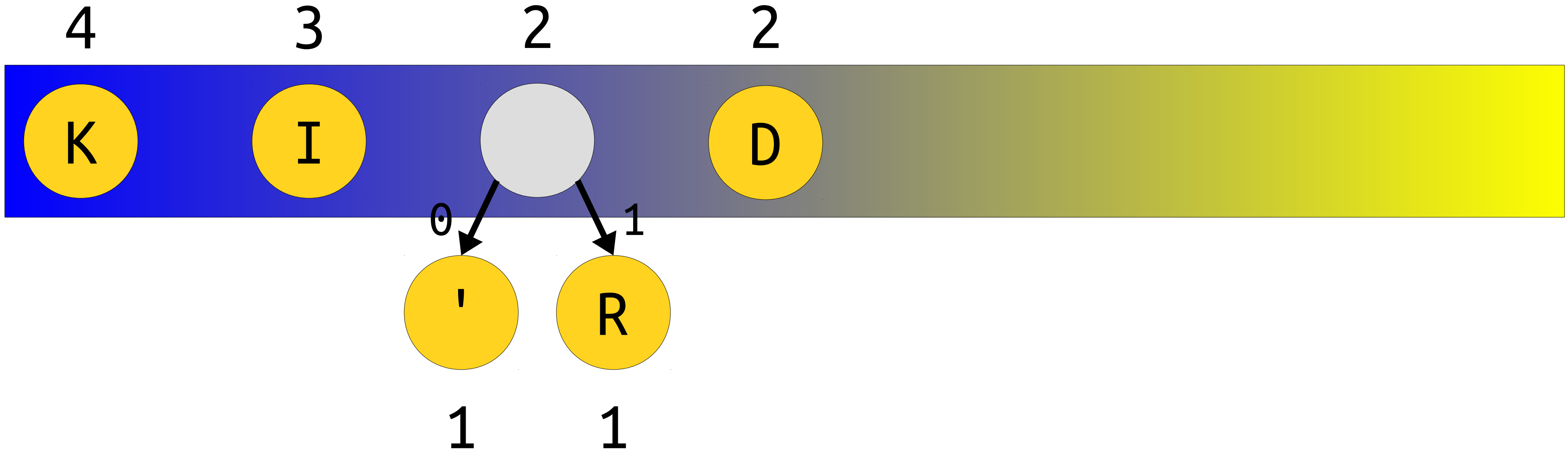
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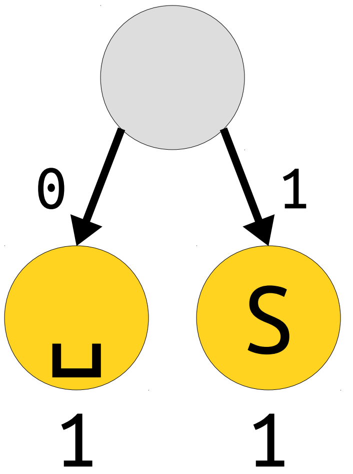
0
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␣
S
1
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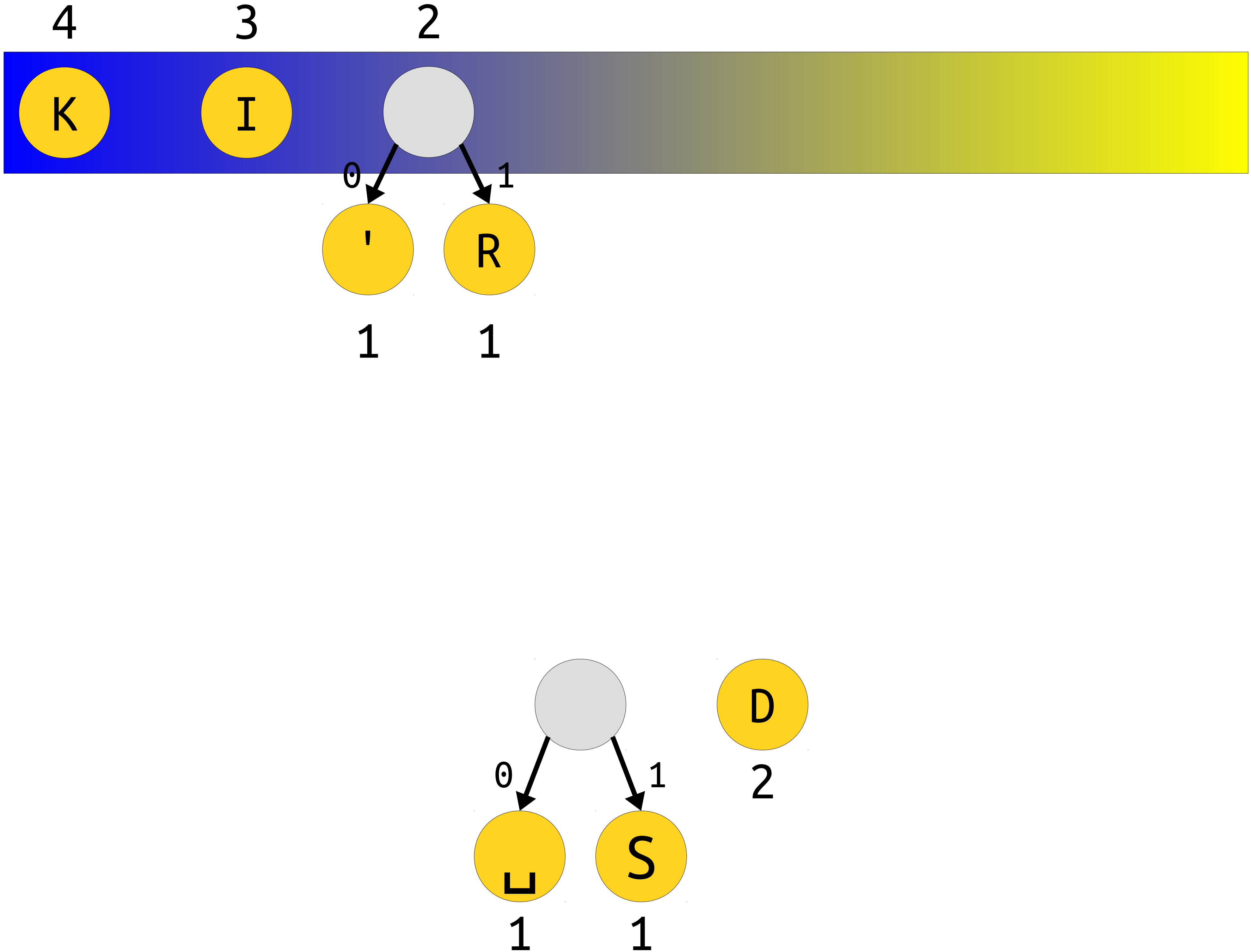

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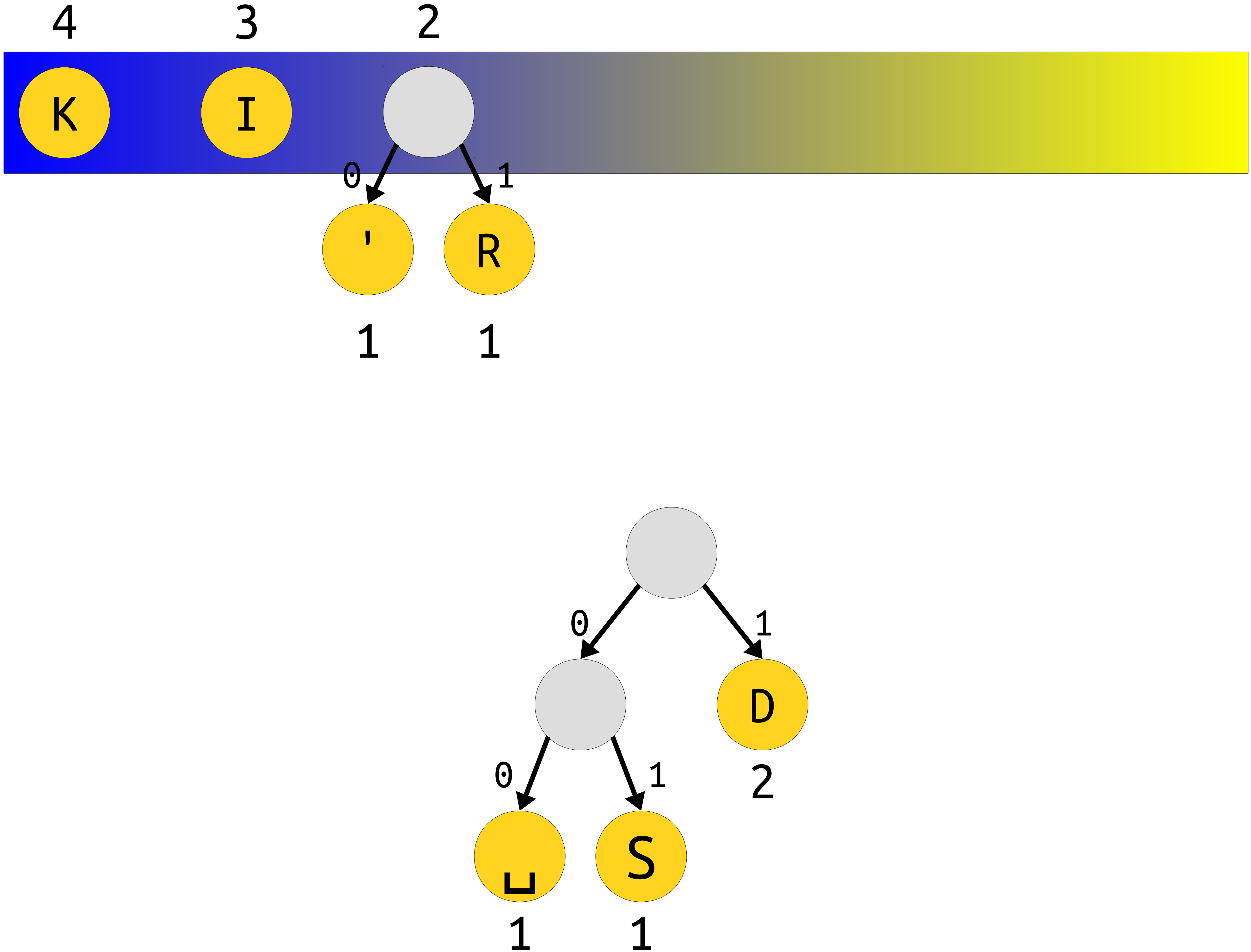
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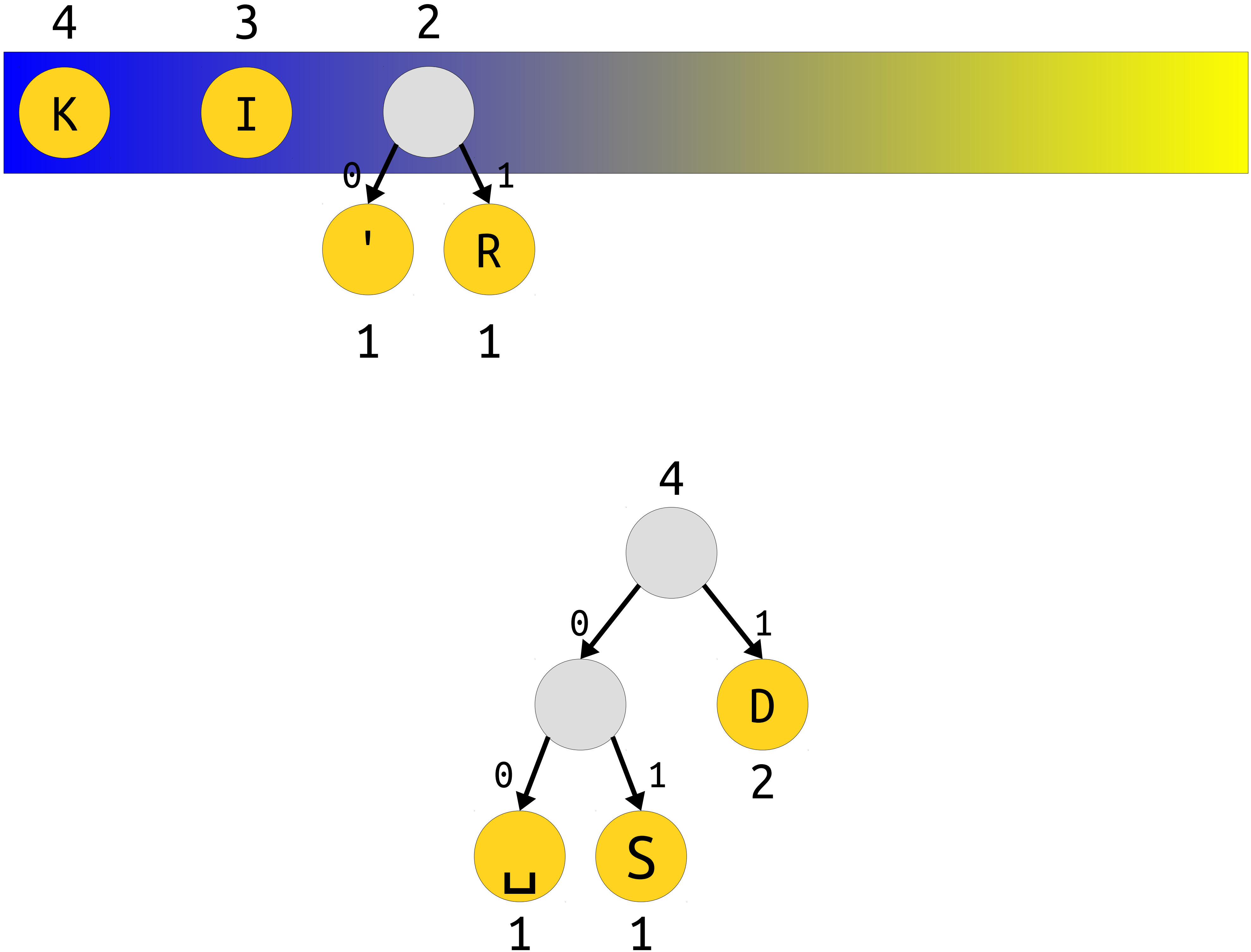

4
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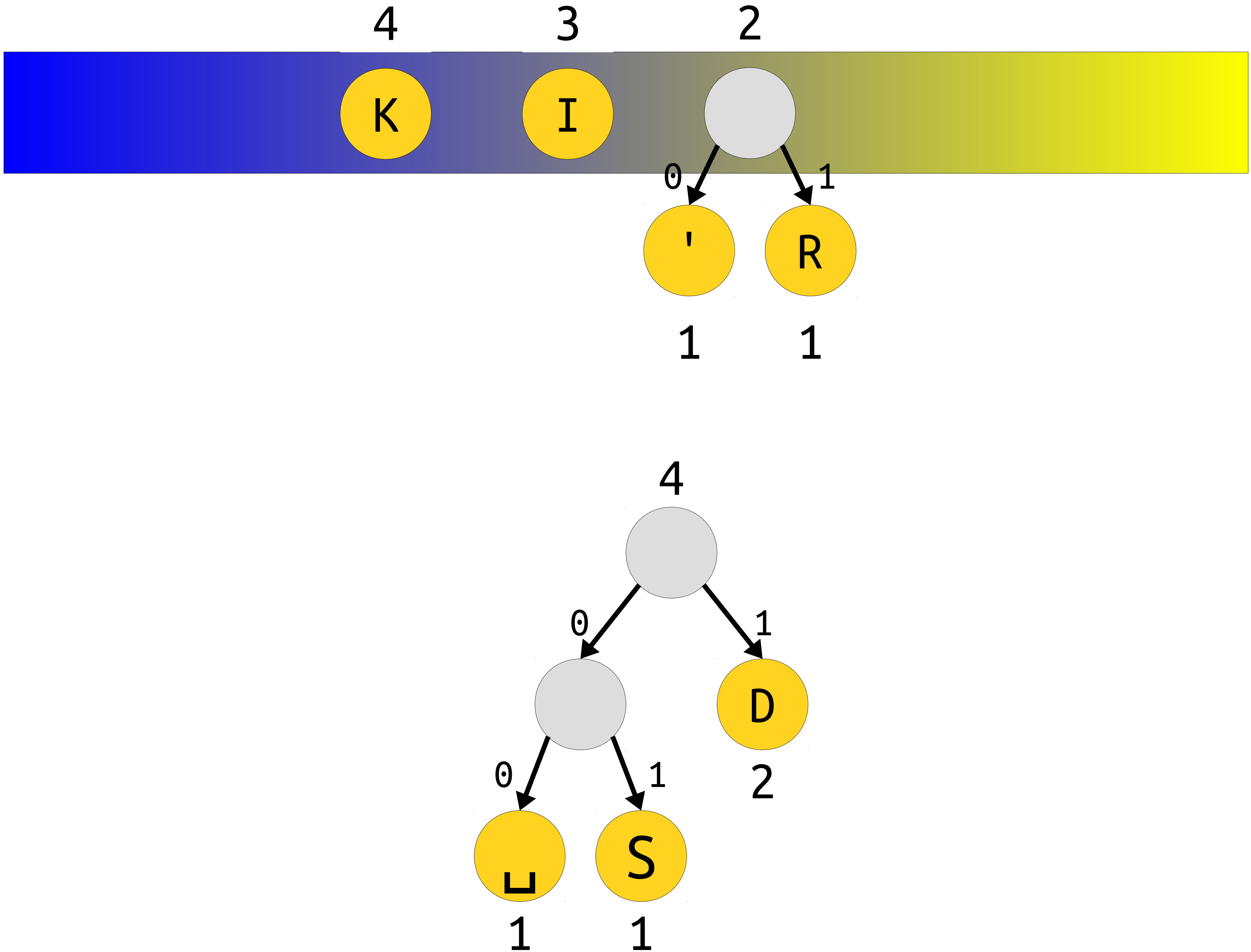
4
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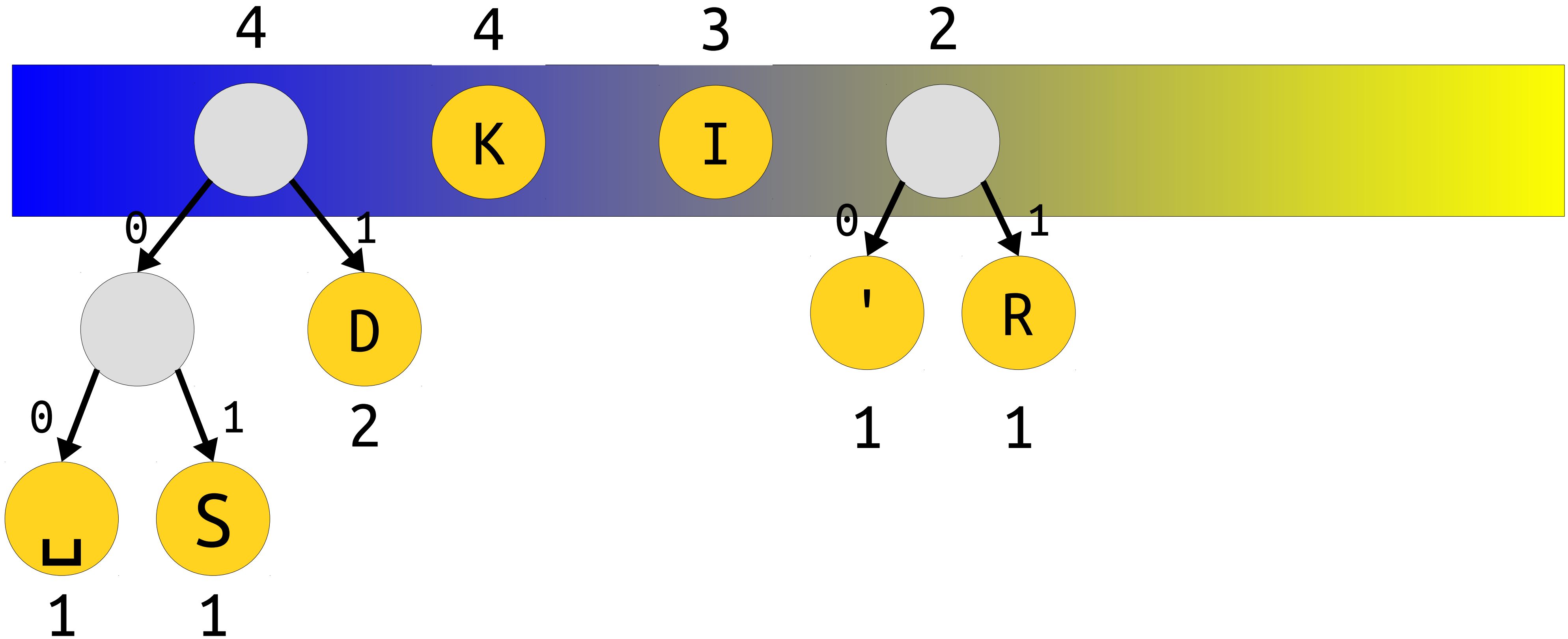
4
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R
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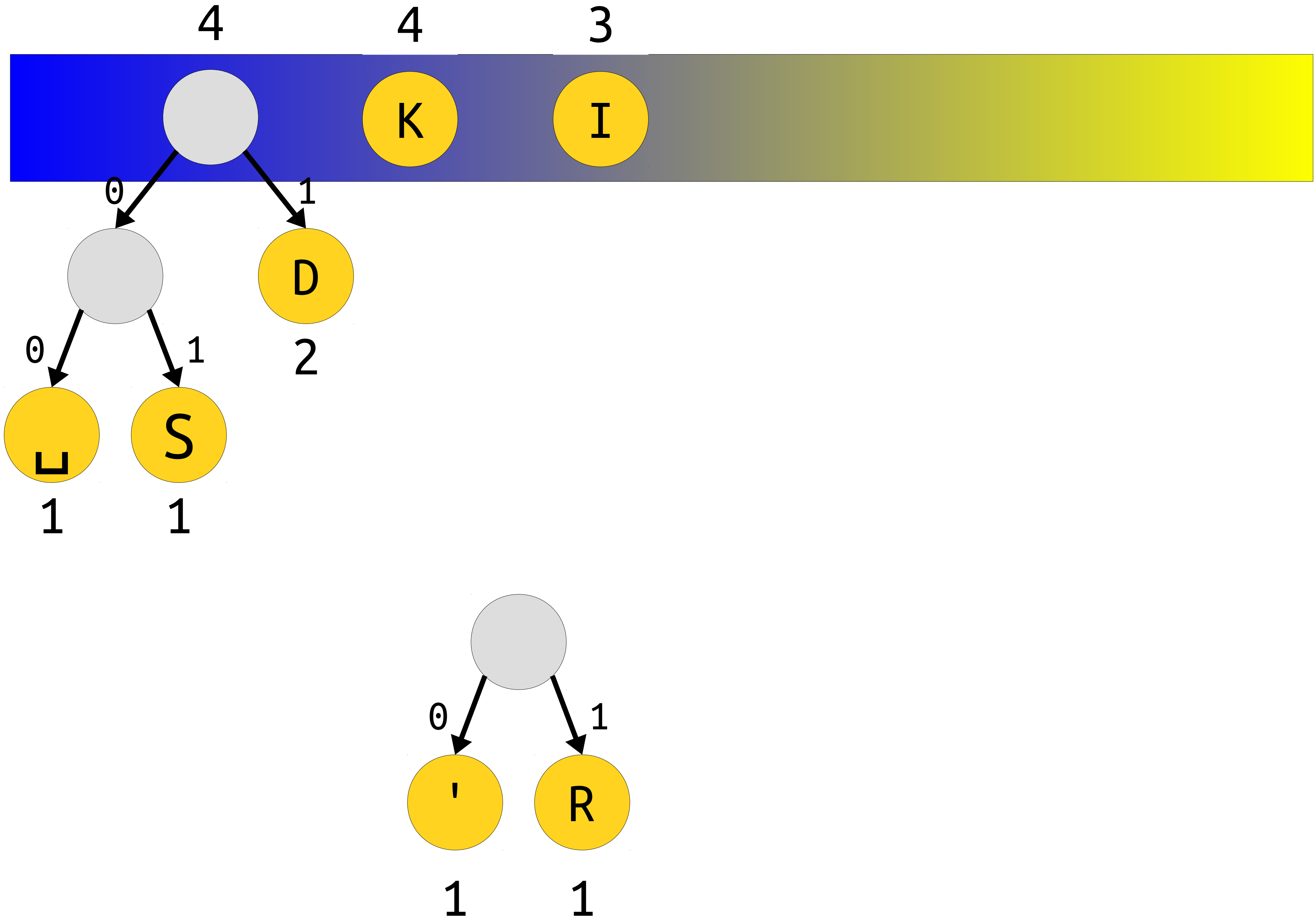

4
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S
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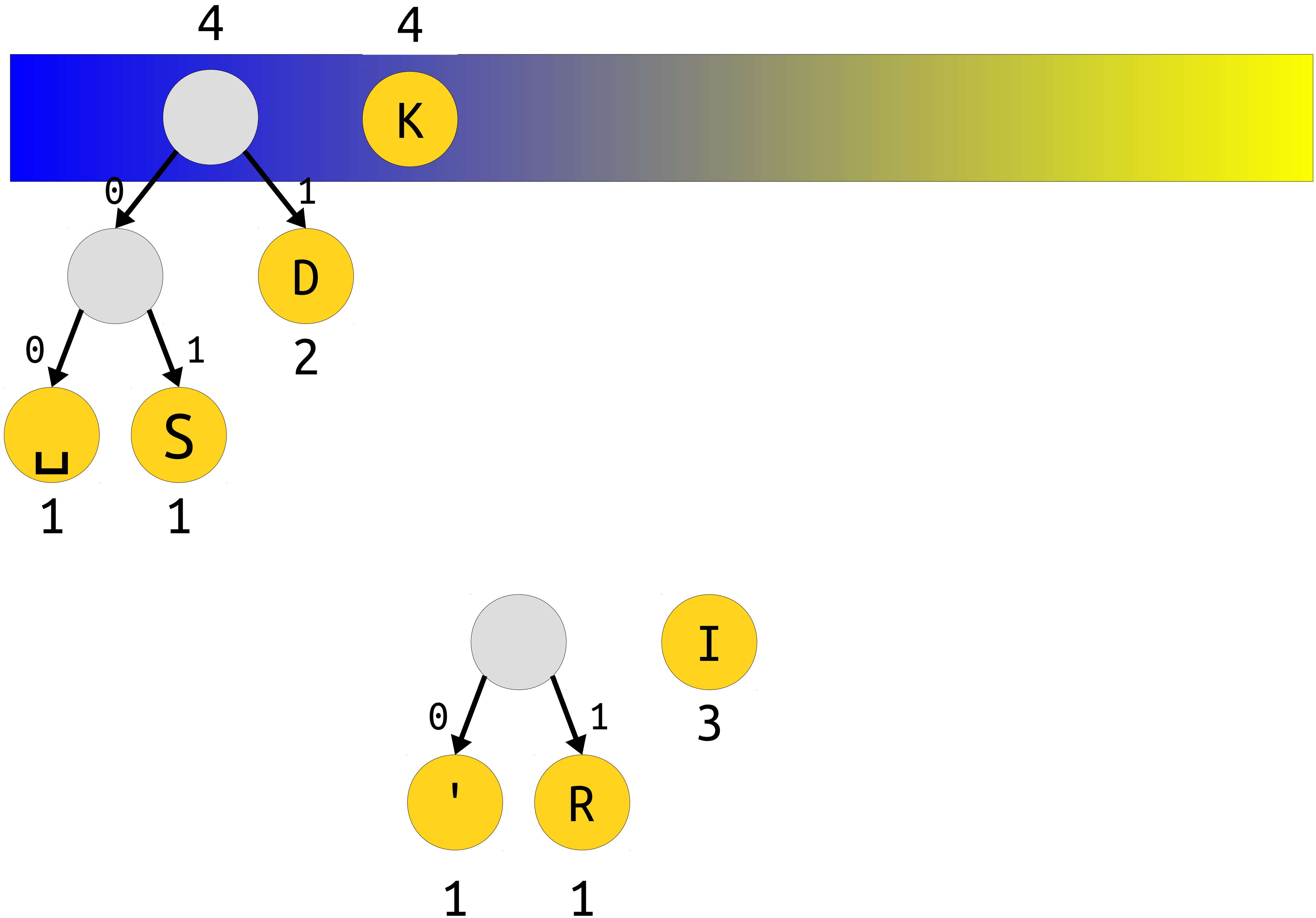
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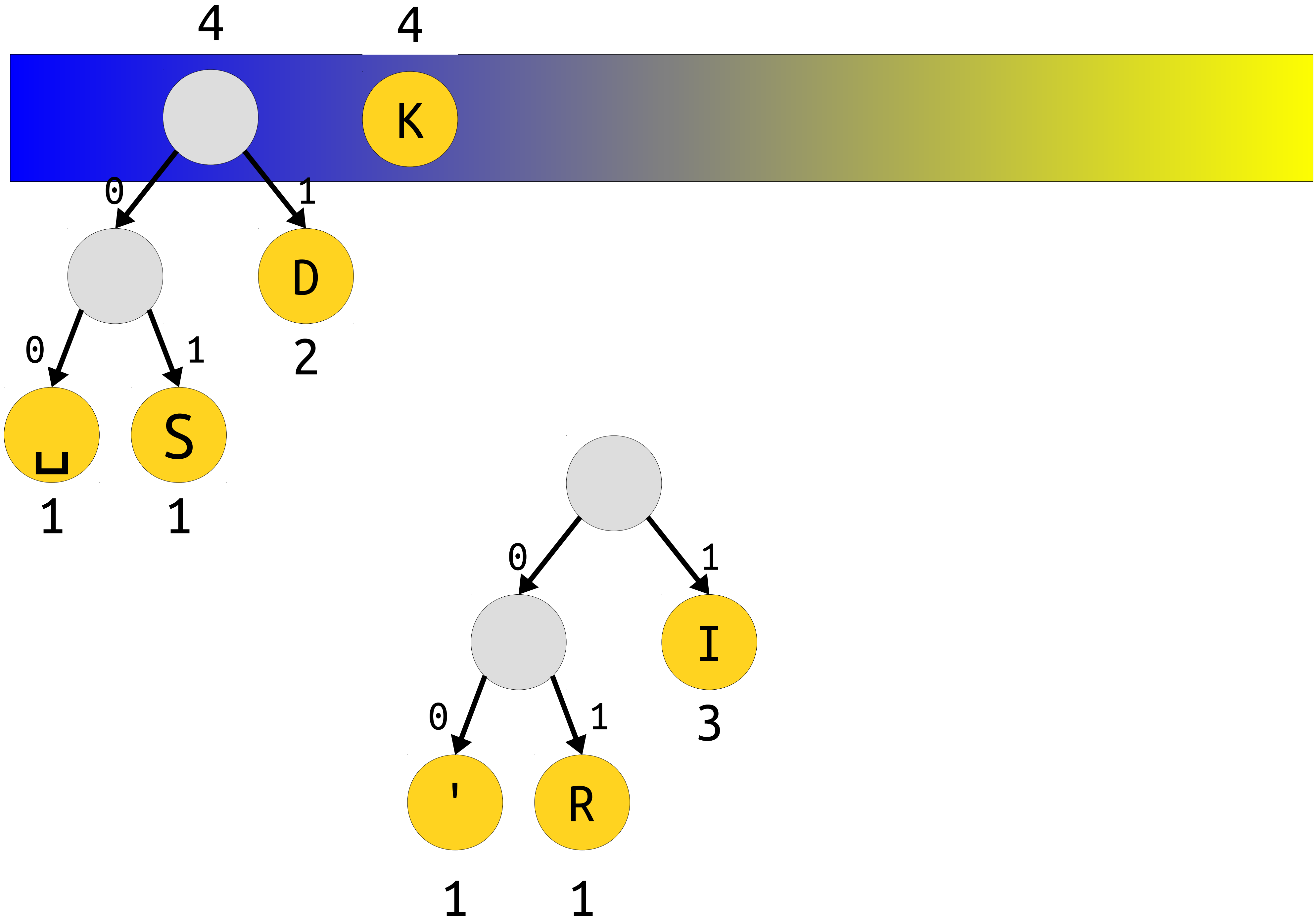

4
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K
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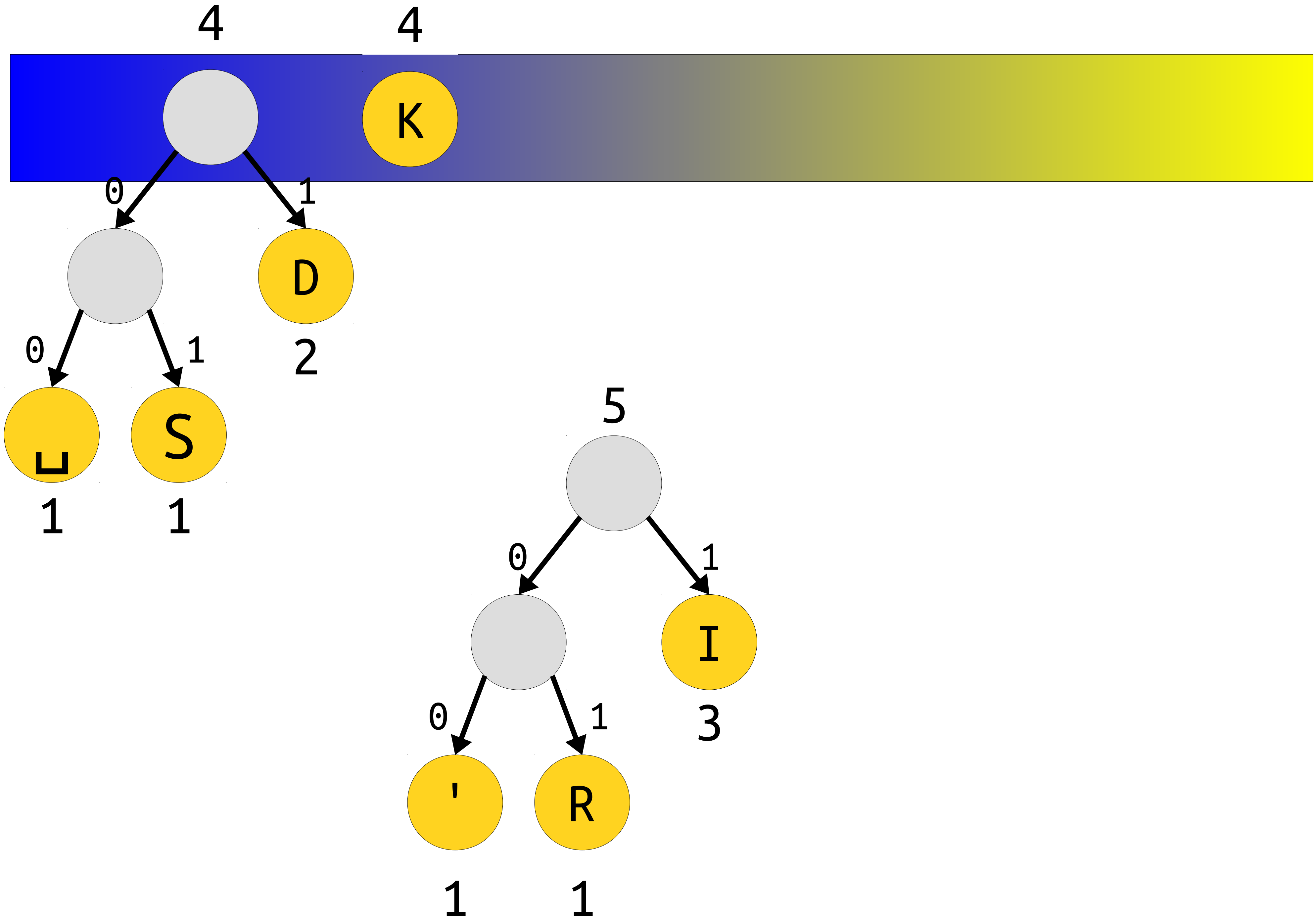

4
4
K
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D
2
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␣
S
1
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5
0
1
I
3
0
1
'
R
1
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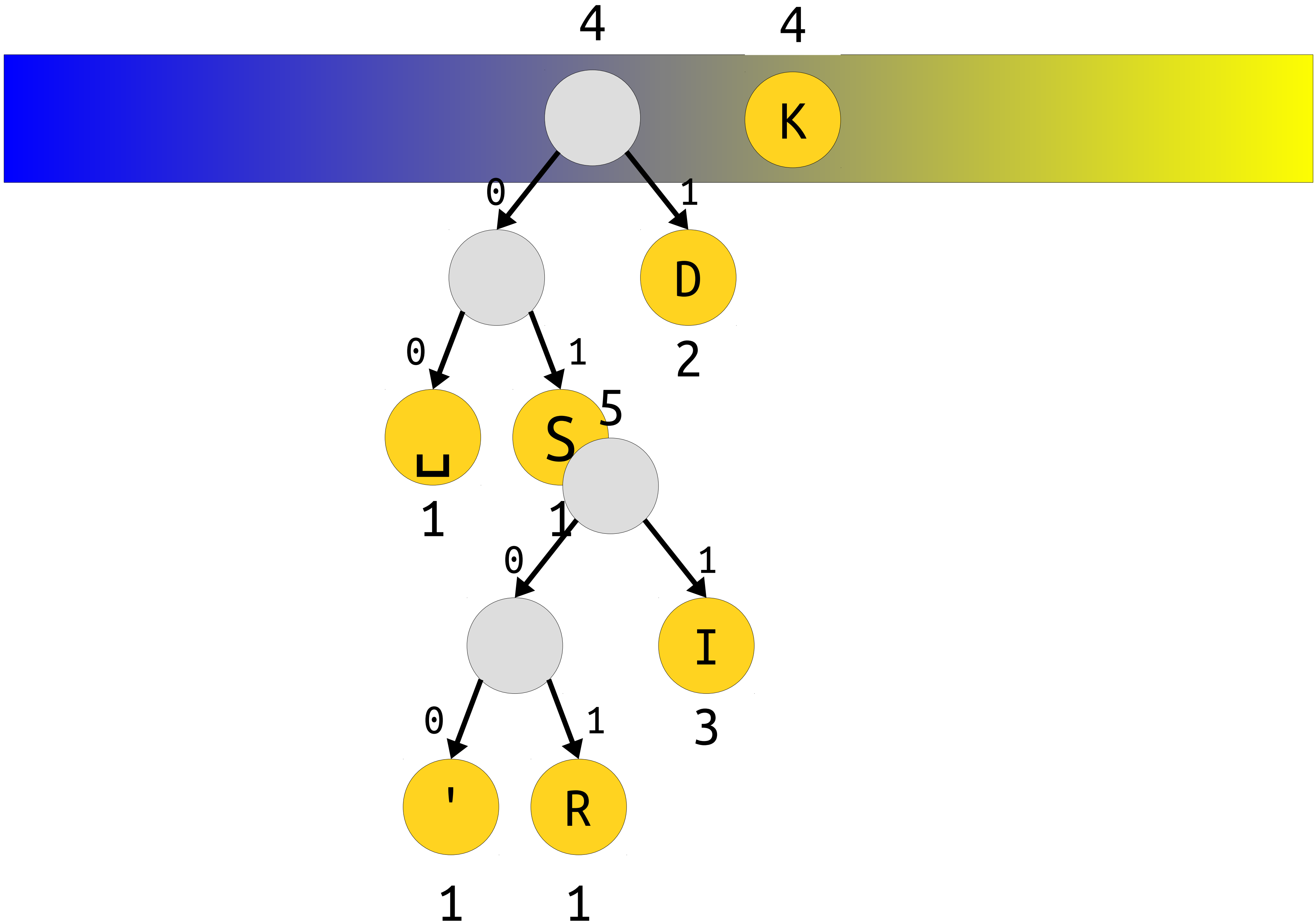
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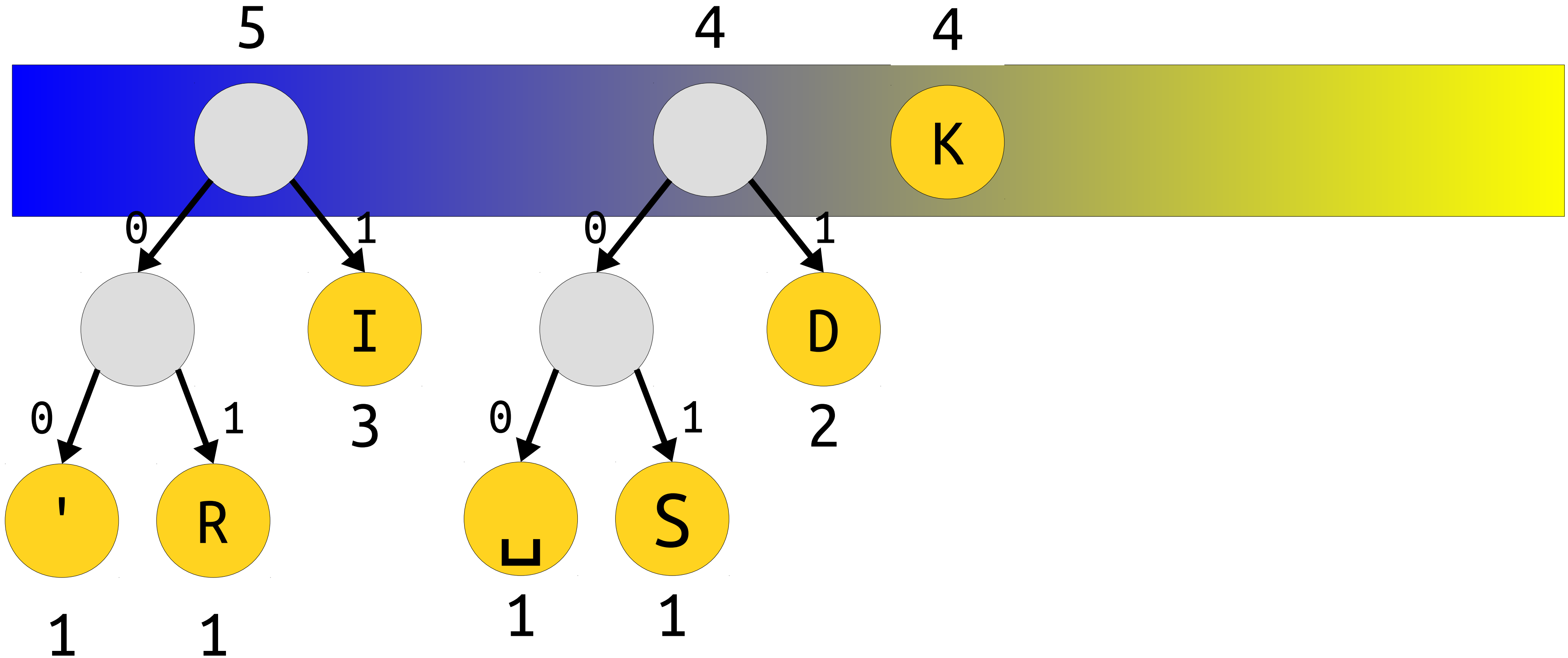
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D
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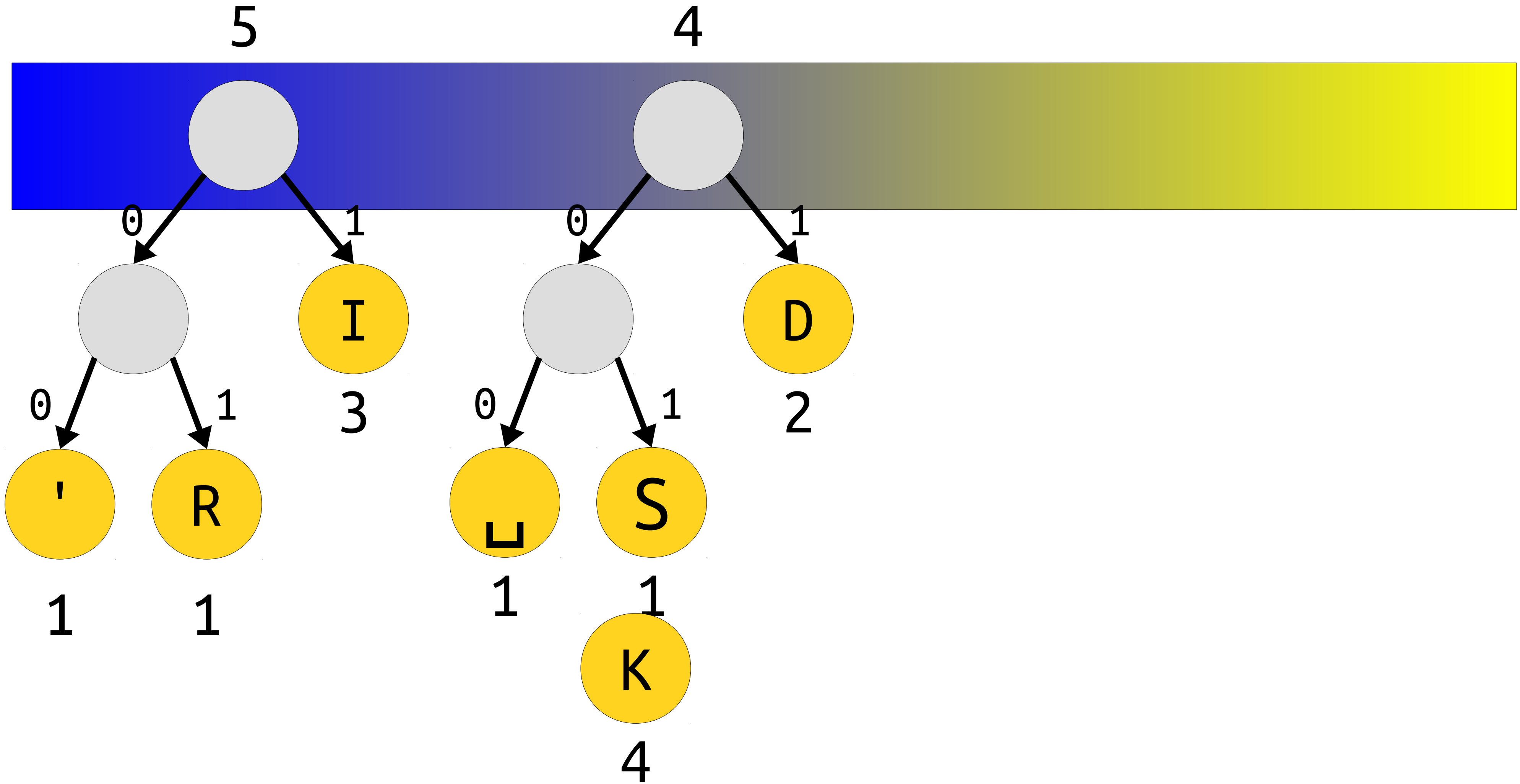
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K
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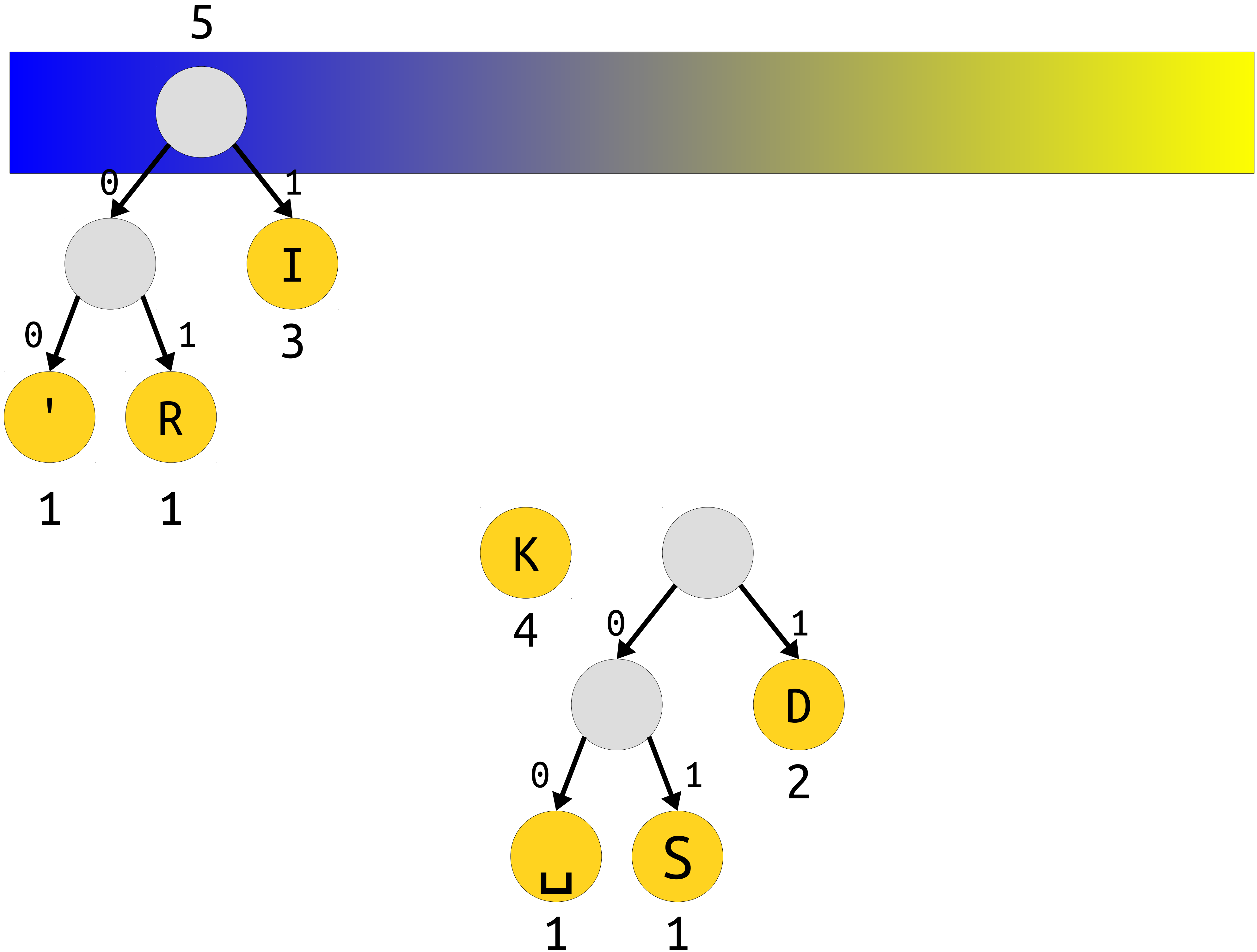
5
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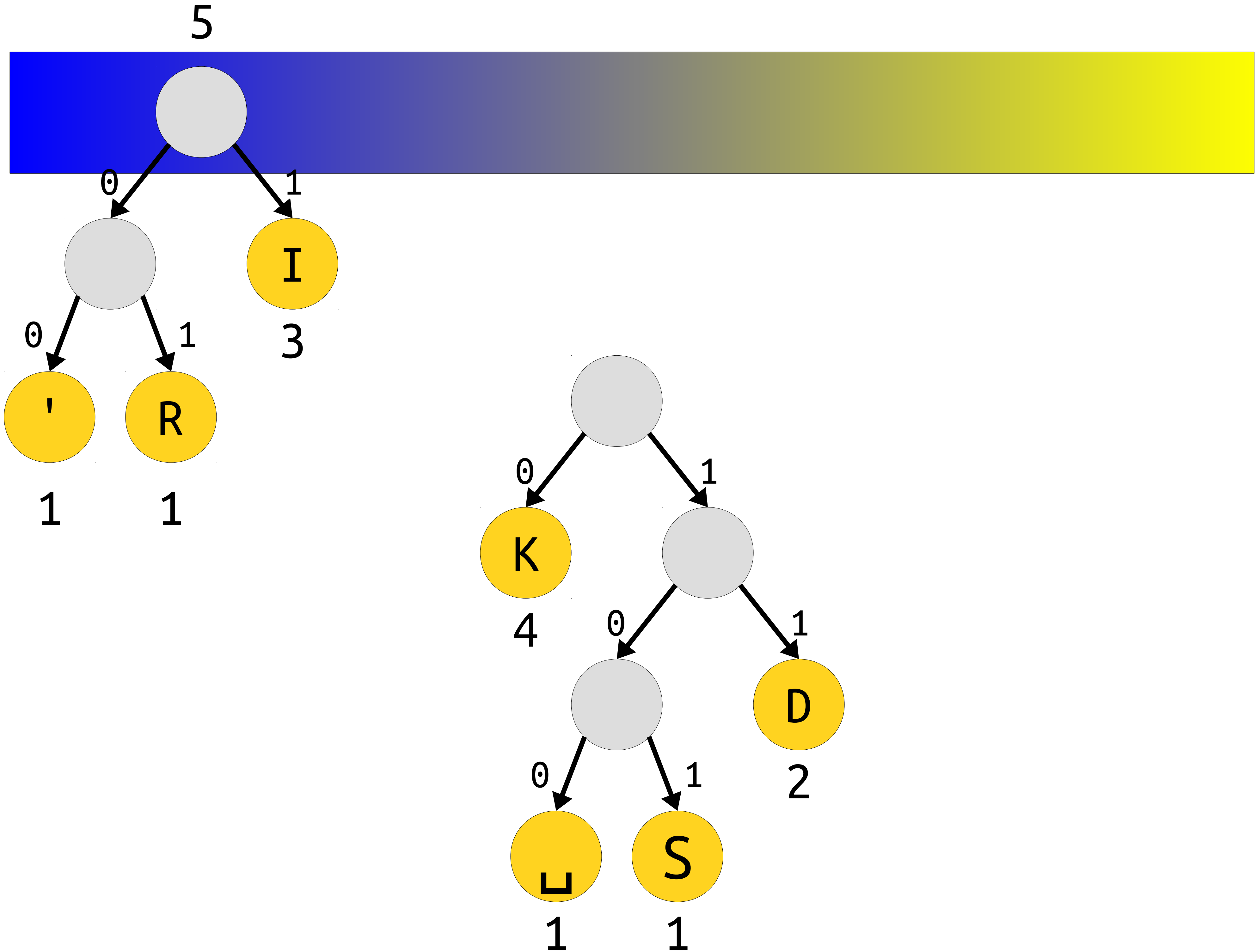
5
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D
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␣
S
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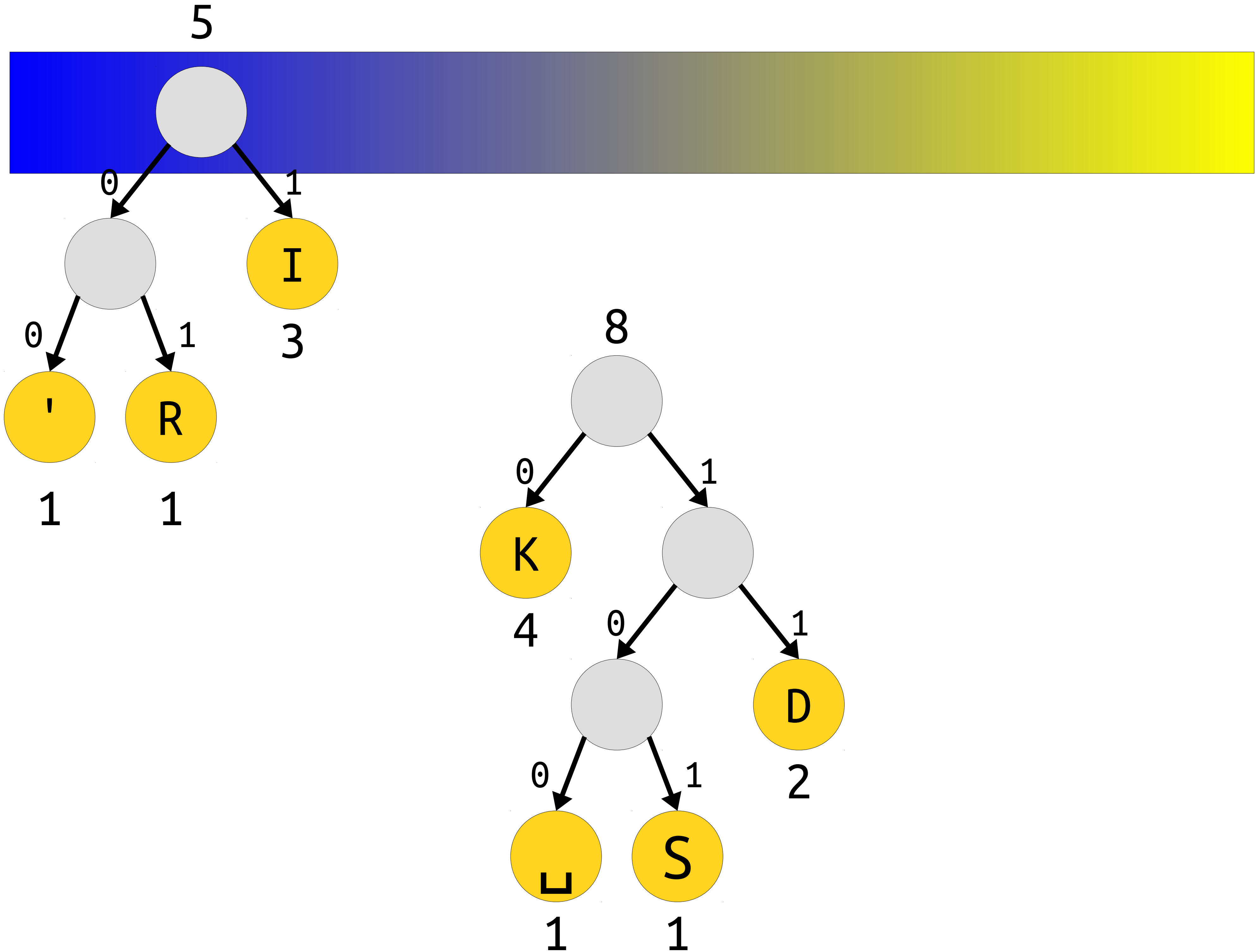

5
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R
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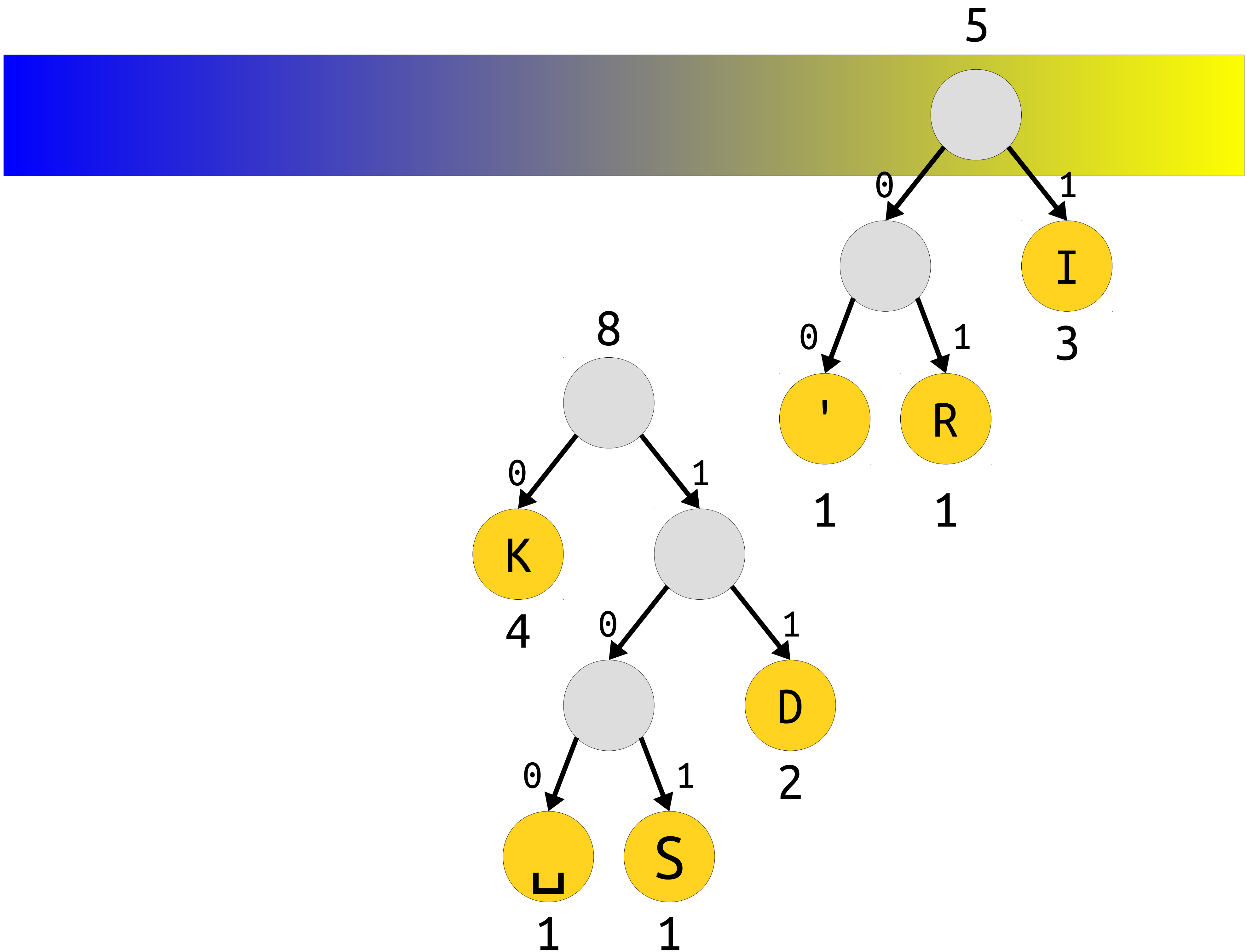
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8
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K
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D
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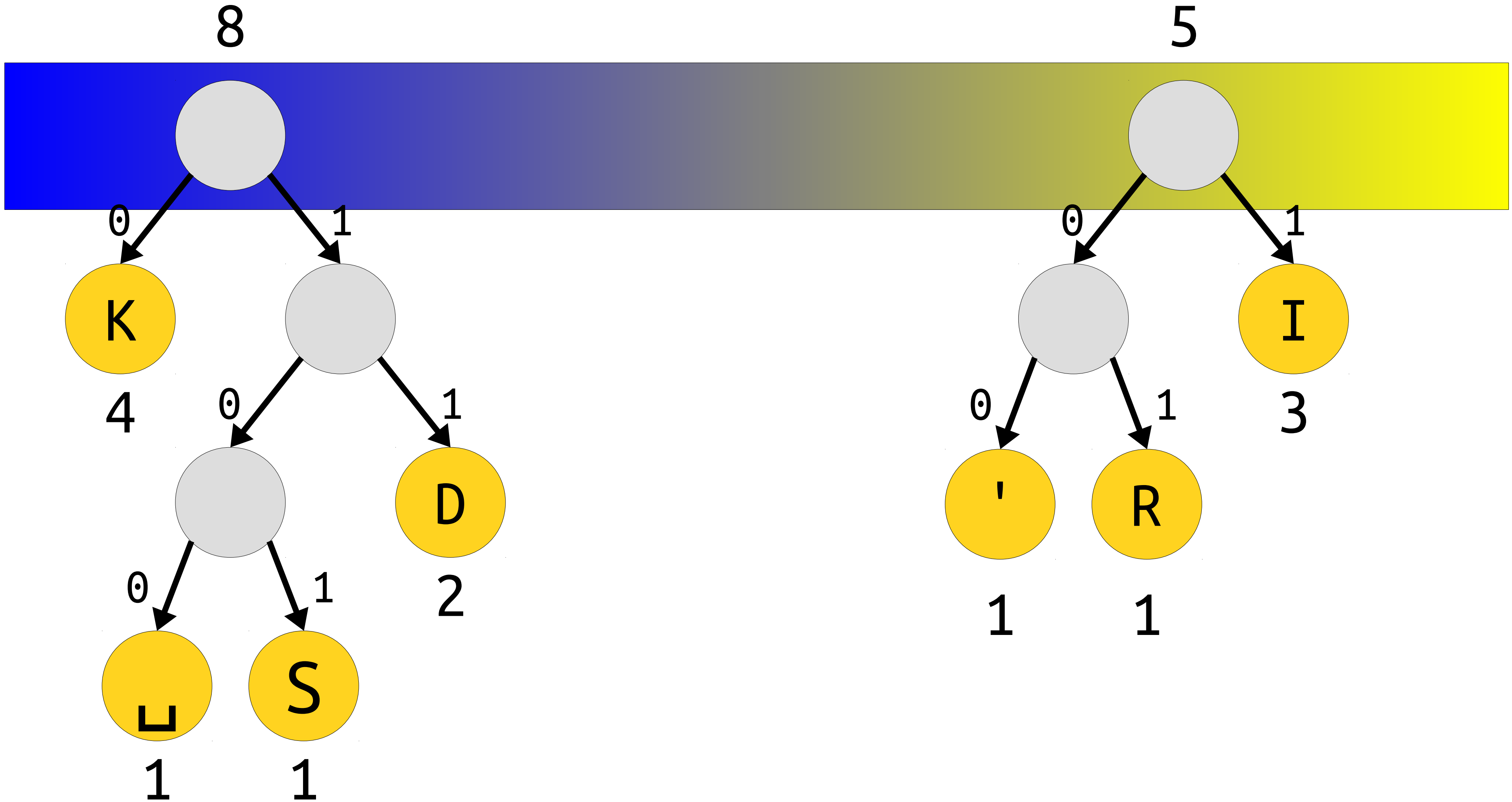
␣
S
1
1


8
5
0
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0
1
K
I
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D
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R
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␣
S
1
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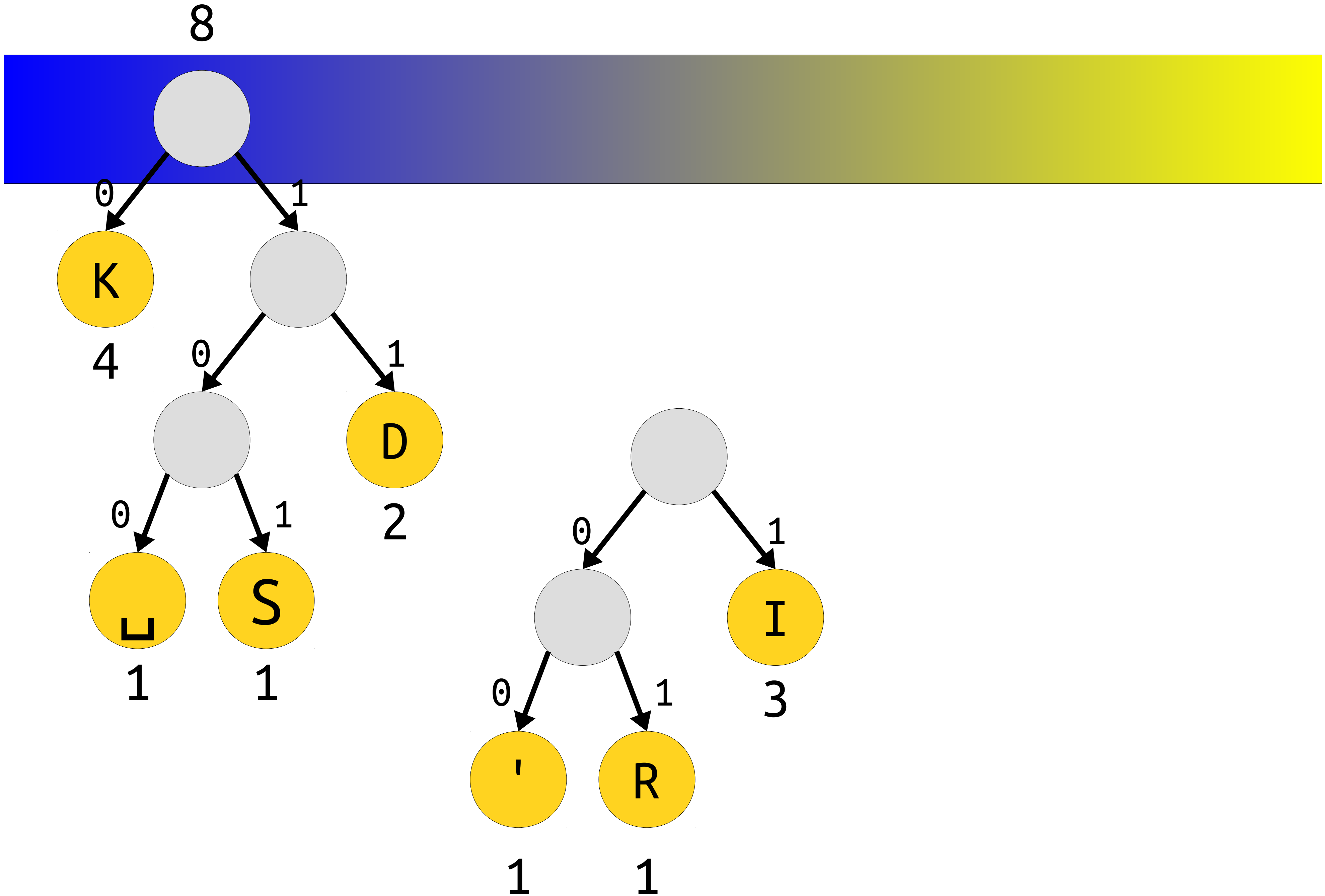

8
0
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K
4
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1
D
2
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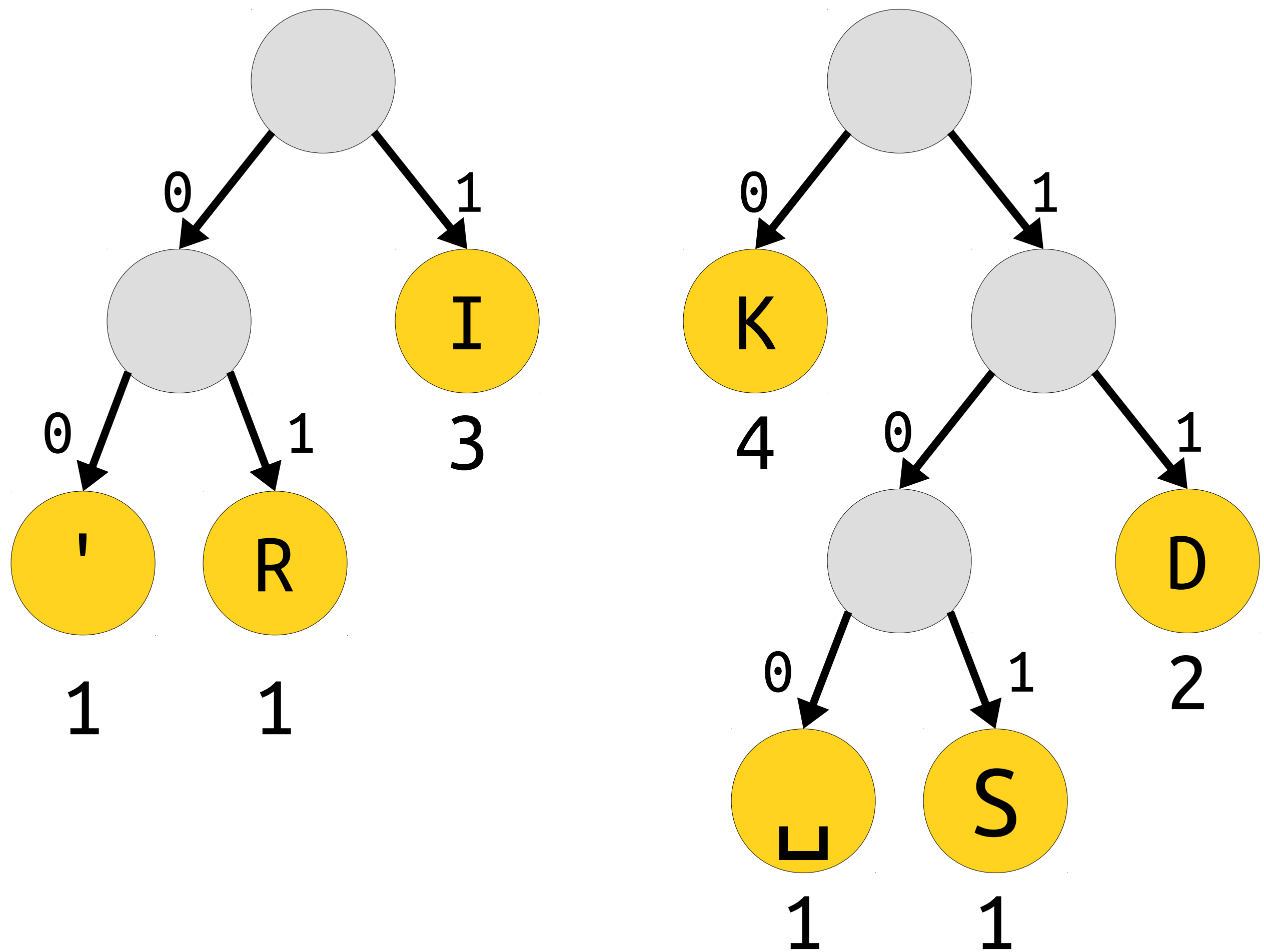
1
␣
1
S
1
0
1
I
3
0
1
'
1
R
1



0
1
I
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'
R
1
1
0
1
K
4
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1
D
2
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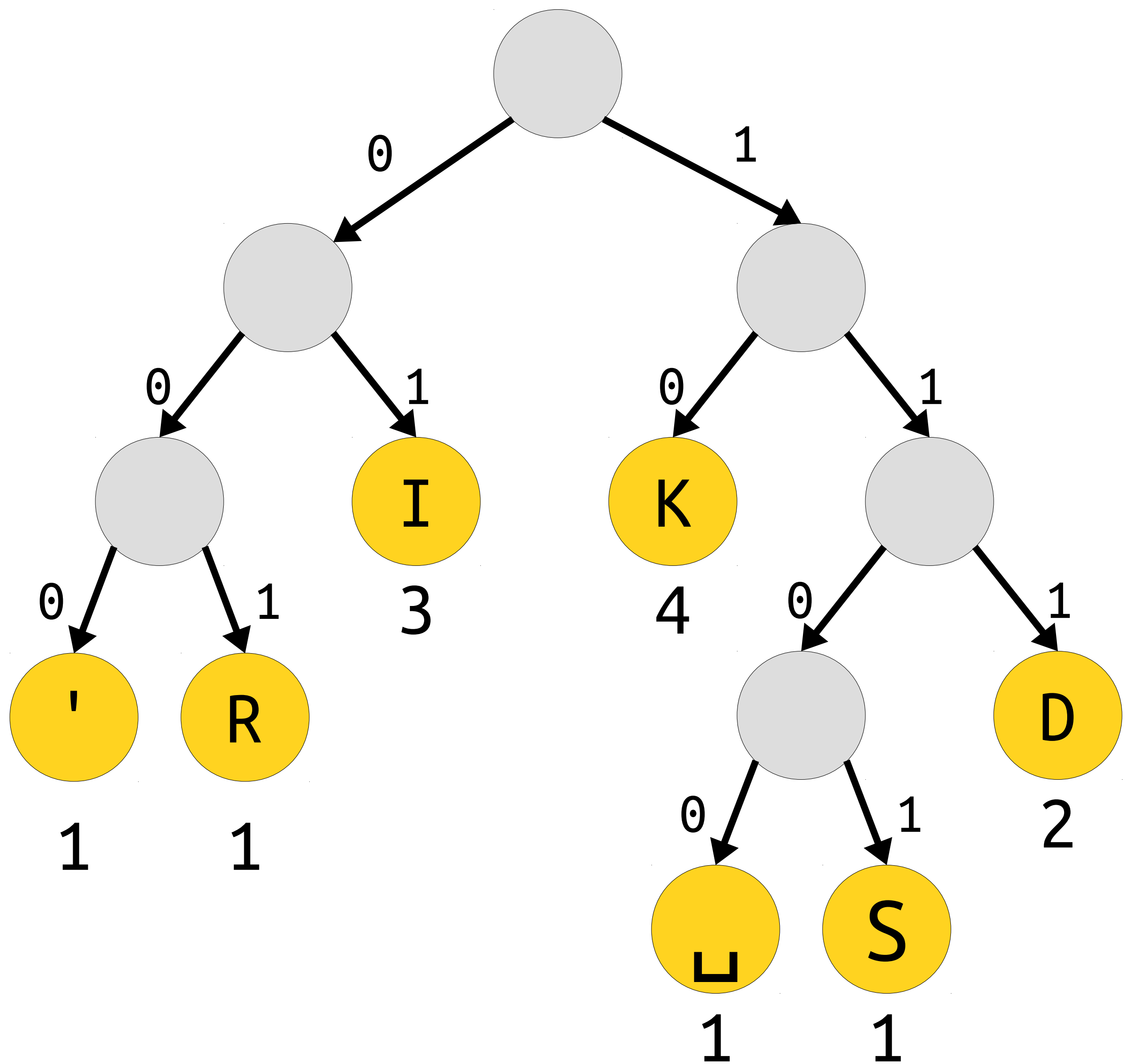
␣
S
1
1


0
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K
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D
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1
␣
S
1
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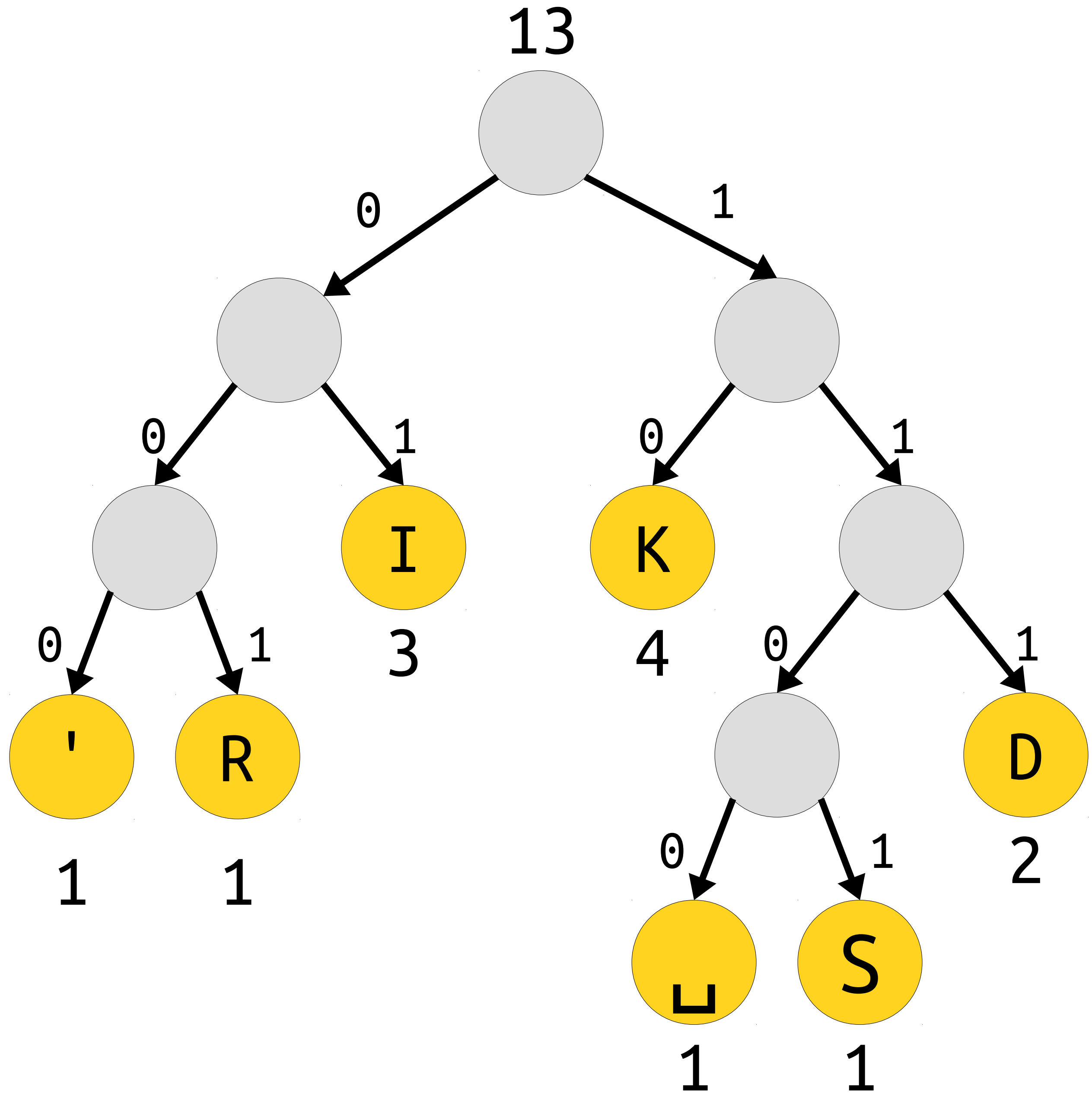
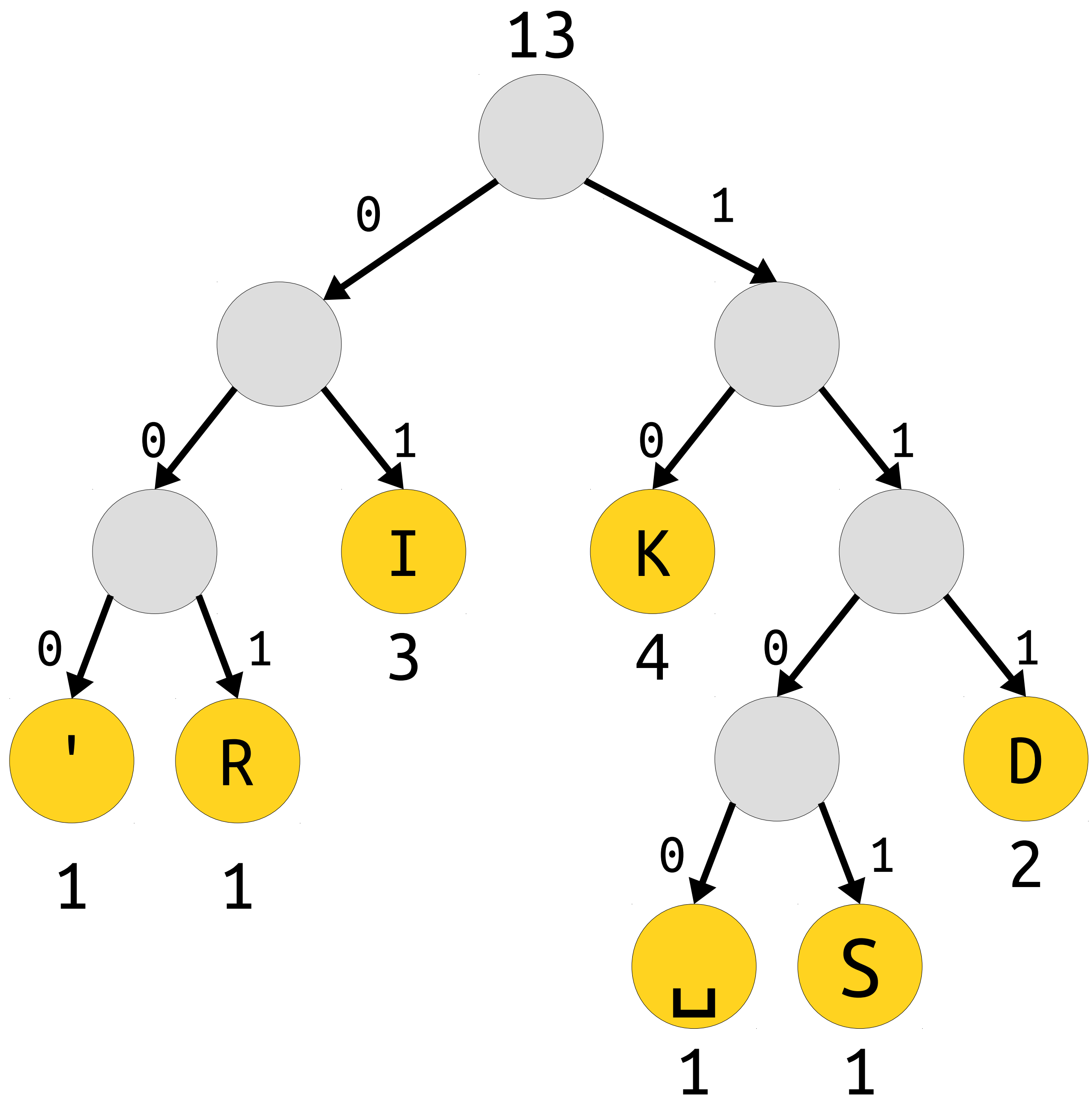

13
0
1
0
1
0
1
I
K
3
4
0
1
0
1
'
R
D
1
1
2
0
1
␣
S
1
1


13
0
1
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1
I
K
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3
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D
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0
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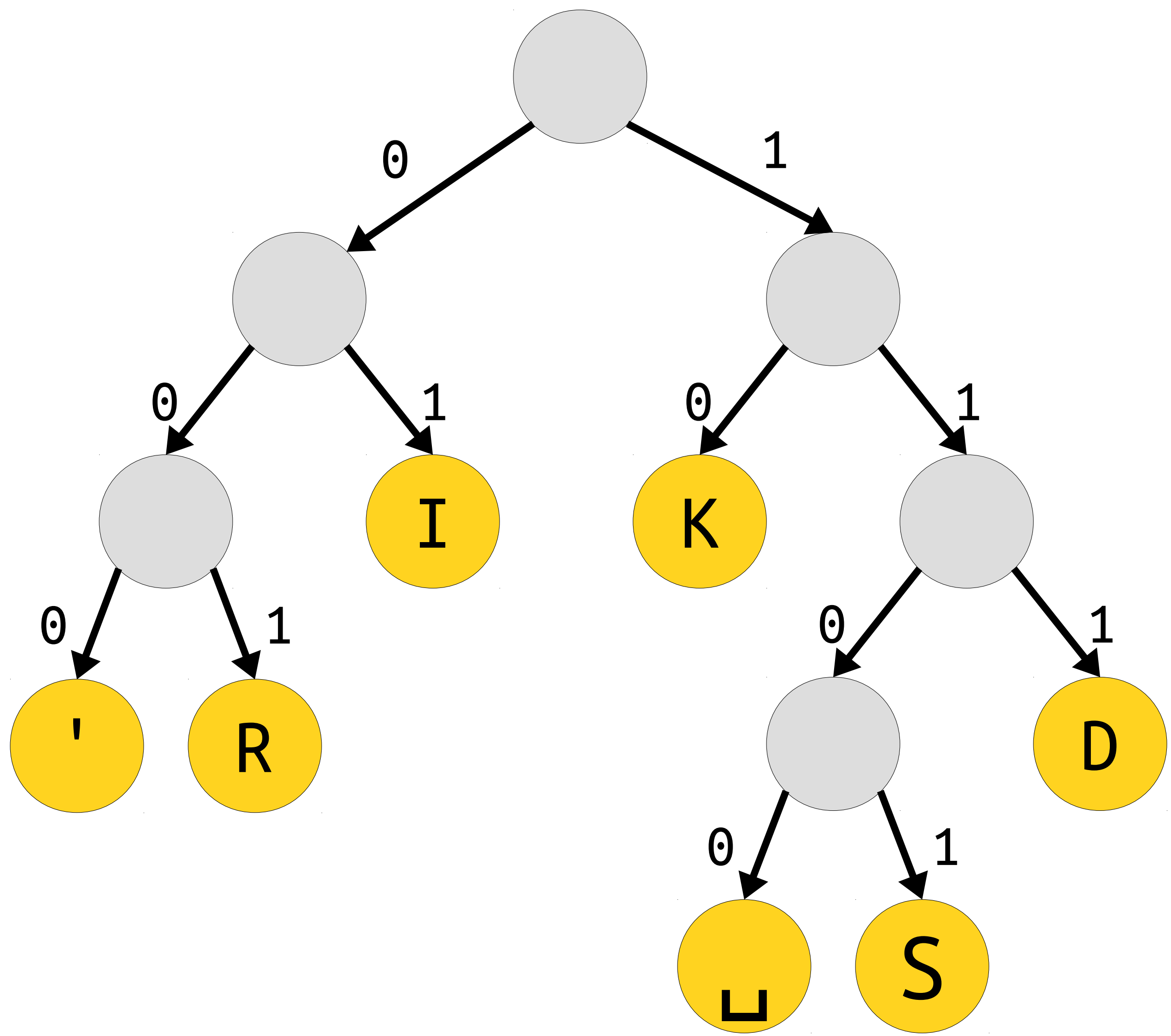
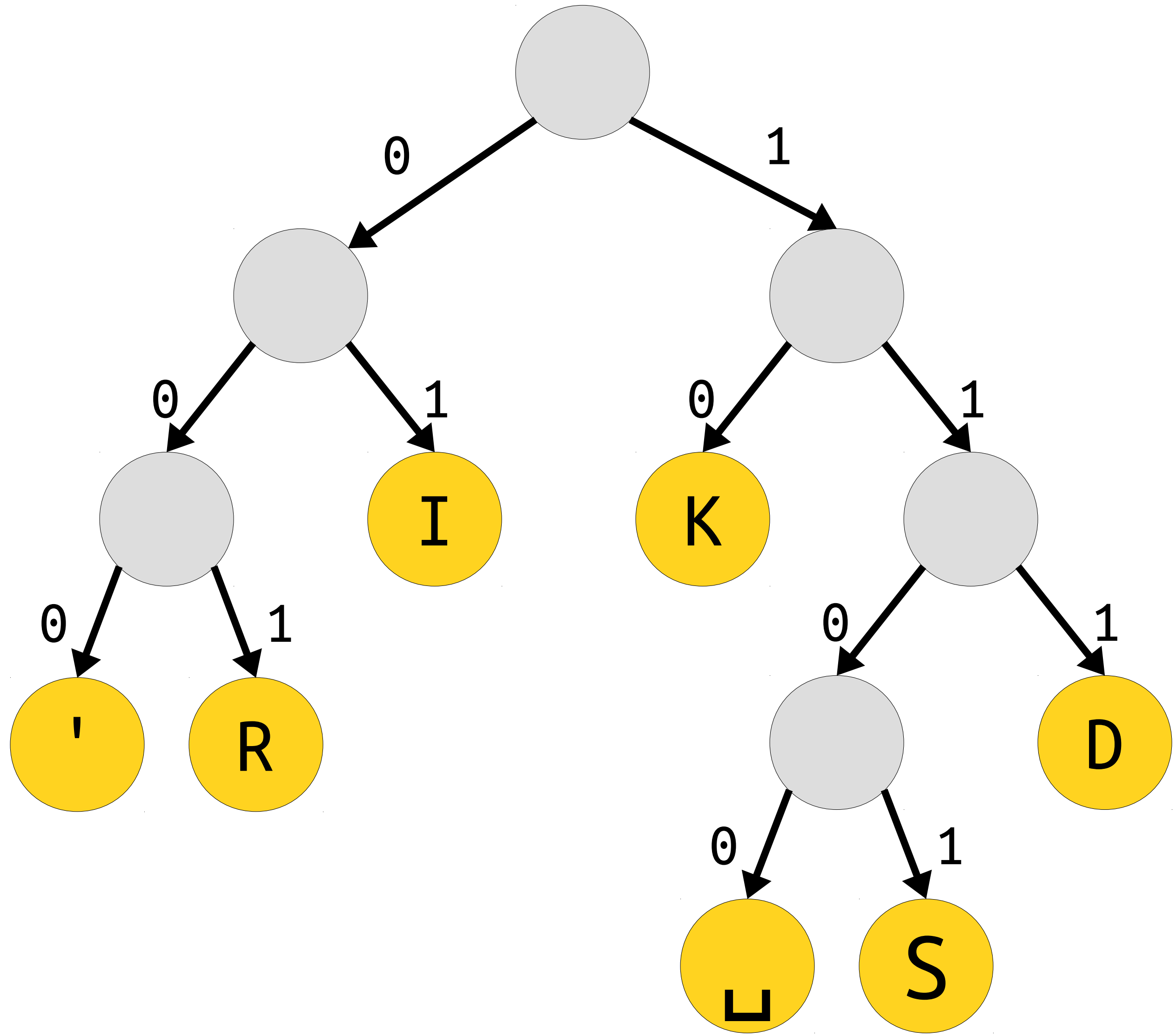
␣
S
1
1



0
1
0
1
0
1
I
K
0
1
0
1
'
R
D
0
1
␣
S

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100



★ Huffman Coding ★

- Create a priority queue that holds partial trees.
- Create one leaf node per distinct character in the input string. The weight of that leaf is the frequency of the character. Add each to the priority queue.
- While there are two or more trees in the priority queue:
  - Dequeue the two lowest-priority trees.
  - Combine them together to form a new tree whose weight is the sum of the weights of the two trees.
  - Add that tree back to the priority queue.

An Important Detail

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix Codes

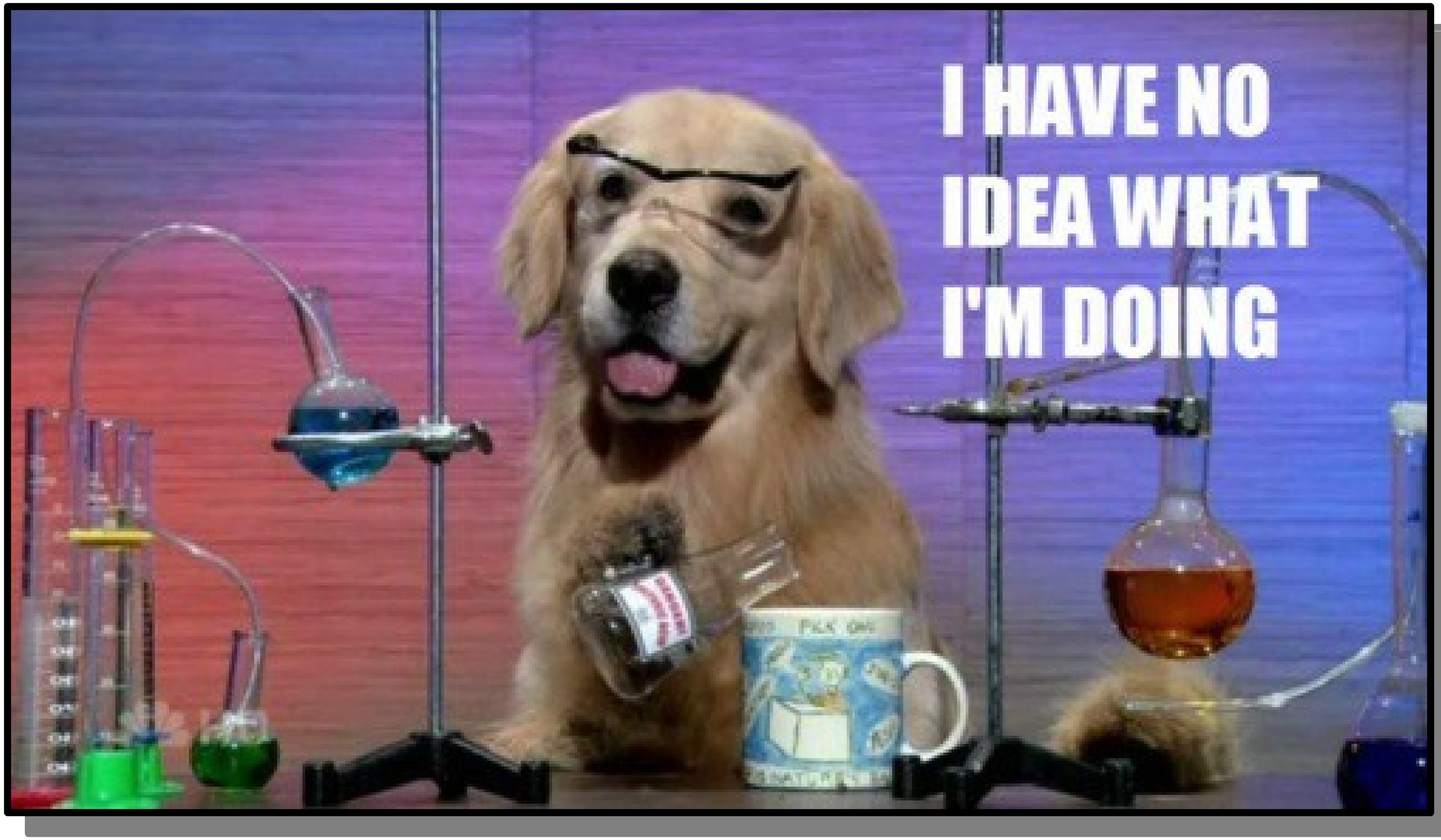
character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

Prefix Codes

1001001100001101110011101101110110

Prefix Codes



110110

Transmitting the Tree

- In order to decompress the text, we have to remember what encoding we used!
- ***Idea:*** Prefix the compressed data with a header containing information to rebuild the tree.

Encoded Tree	`11011100101110111100010011010101110…`

- This might increase the total file size!
- ***Theorem***: There is no compression algorithm that can always compress all inputs.
  - ***Proof:*** Take CS103!

Summary of Huffman Encoding

- Prefix-free encodings can be modeled as binary trees.
- Huffman encoding uses a greedy algorithm to construct encodings.
- We need to send the encoding table with the compressed message.

More to Explore

- ***UTF-8 and Unicode***
  - A variable-length encoding that has since replaced ASCII.
- ***Kolmogorov Complexity***
  - What's the theoretical limit to compression techniques?
- ***Adaptive Coding Techniques***
  - Can you change your encoding system as you go?
- ***Shannon Entropy***
  - A mathematical bound on Huffman coding.
- ***Binary Tries***
  - Other applications of trees like these!

Your Action Items

- ***Start Assignment 8.***
  - You have plenty of time to finish this one if you begin early.
  - Please don’t wait until the last minute – no late submissions will be accepted.

Next Time

- ***Graphs***
  - Representing networks of all sorts.
- ***Graph Searches***
  - A new perspective on some earlier ideas.