

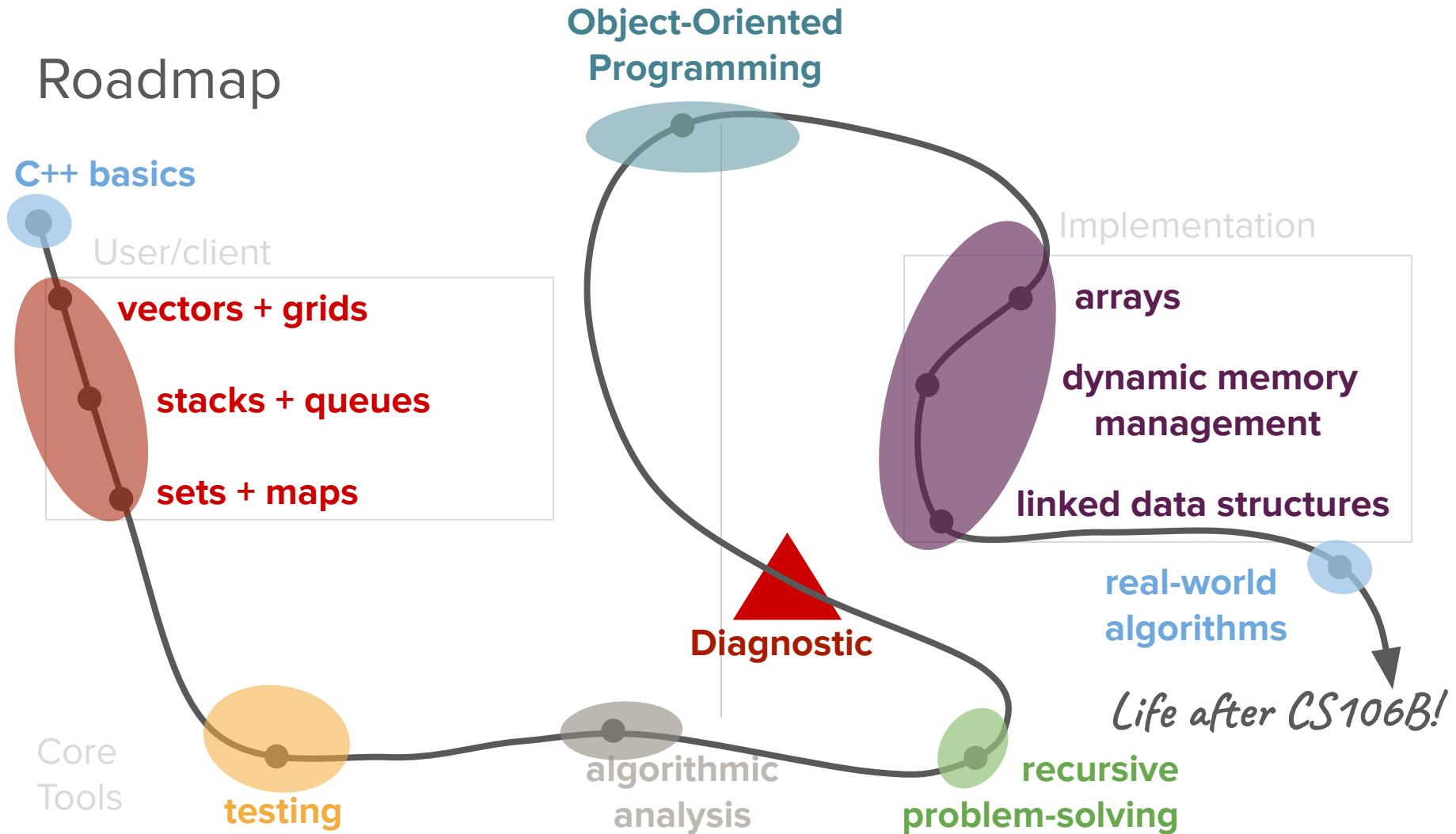
Binary Search Trees

What is your favorite type of tree?

(put your answers the chat - e.g. oak, redwood, maple, etc.)



Roadmap



Roadmap

C++ basics

User/client

vectors + grids

stacks + queues

sets + maps

Core
Tools

testing

algorithmic
analysis

**recursive
problem-solving**

Object-Oriented
Programming

Implementation

arrays

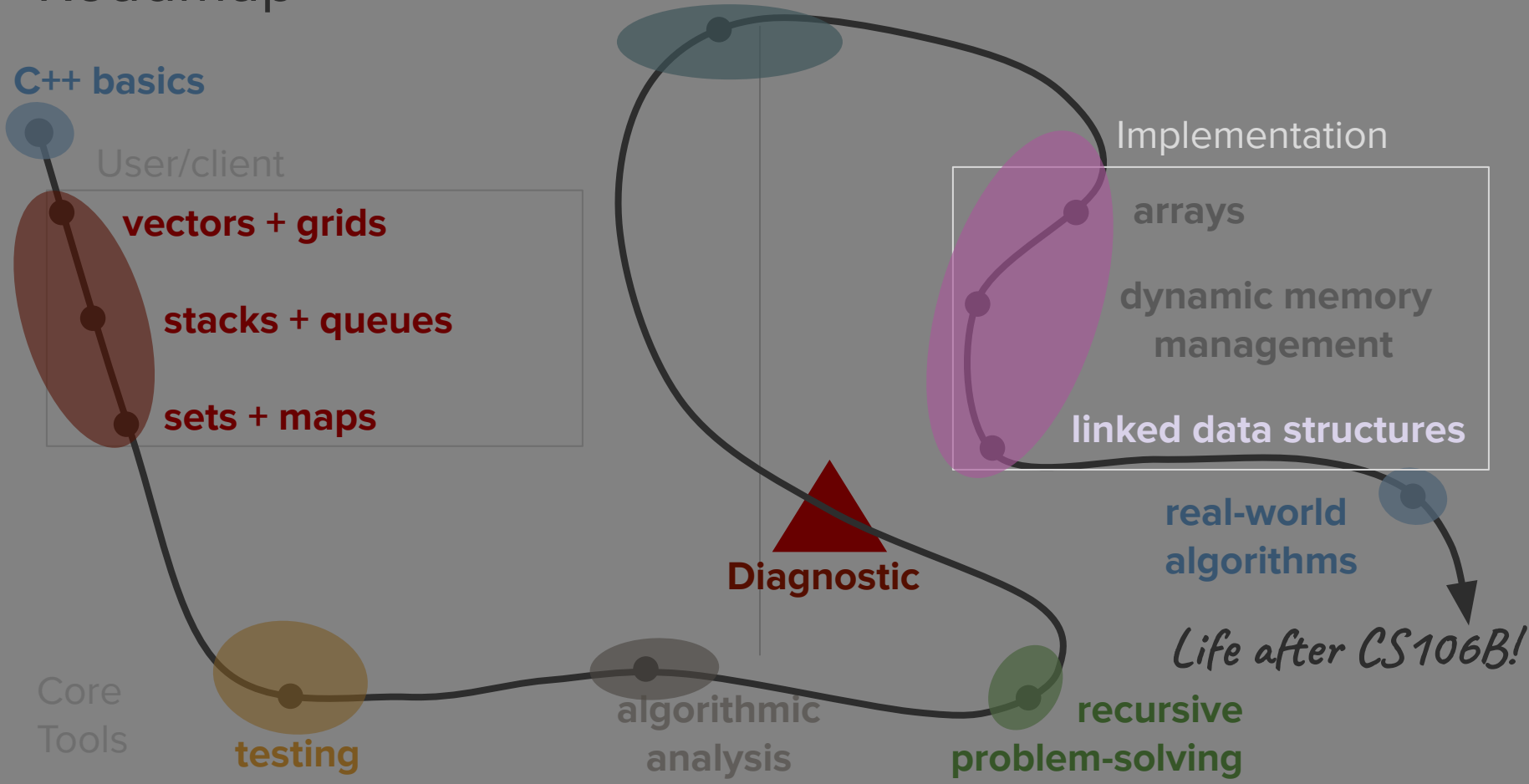
dynamic memory
management

linked data structures

**real-world
algorithms**

Life after CS106B!

Diagnostic



Today's questions

How can we take
advantage of trees to
structure and efficiently
manipulate data?

Today's topics

1. What is a binary search tree (BST)?
2. Building efficient BSTs
3. Implementing Sets with BSTs

Review

[trees]

Definition

tree

A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

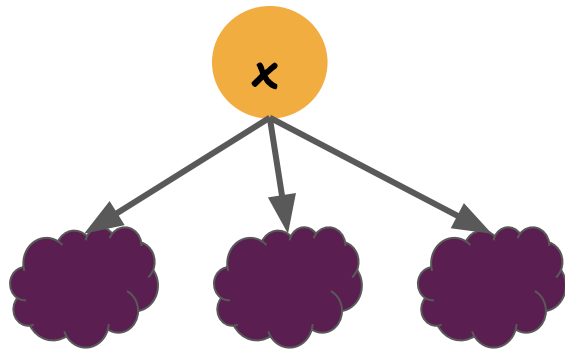
What is a tree?

A tree is either...

An empty data
structure, or...



A single node
(parent), with zero or
more non-empty
subtrees (children)

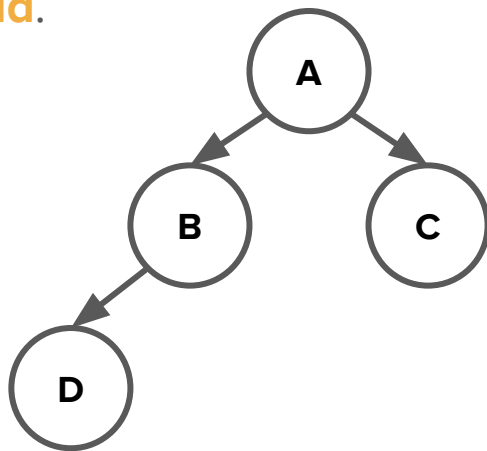


Tree terminology

- Types of nodes
 - The **root** node defines the "top" of the tree.
 - Every node has 0 or more **children** nodes descended from it.
 - Nodes with no children are called **leaf nodes**.
 - Every node in a tree has exactly one **parent** node (except for the root node).
- Terminology for quantifying trees
 - A **path** *between two nodes* traverses edges between parents and their children, and **length** *of a path* is the number of edges between the two nodes.
 - The **depth** *of a node* is the length of the path (# of edges) between the root and that node.
 - The **height** *of a tree* is the number of nodes in the longest path through the tree (i.e. the number of **levels** in the tree).

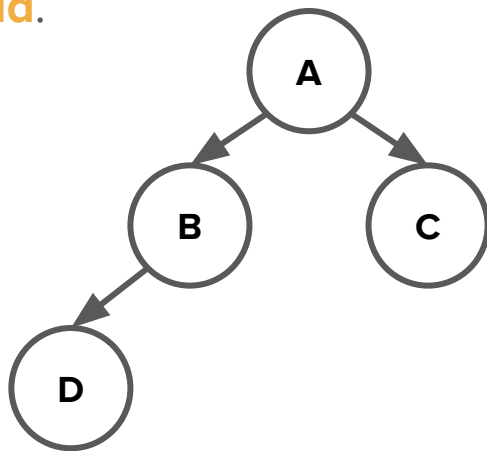
Binary trees

- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.
- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.



Binary trees

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```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

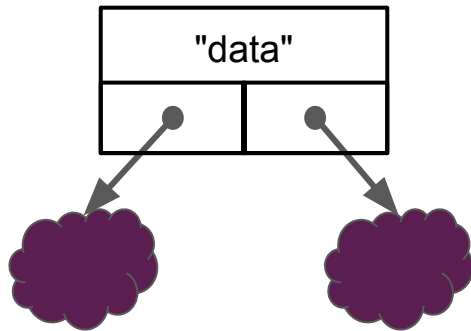
What is a tree in C++?

A tree is either...

An empty tree
represented by
nullptr, or...



A single **TreeNode**,
with 0, 1, or 2
non-null pointers to
other **TreeNode**s



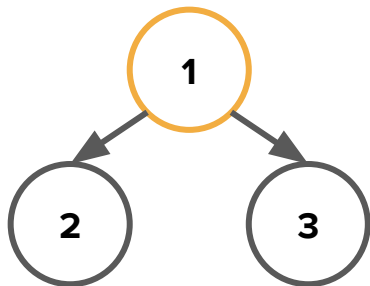
Building a tree

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We start by first creating the leaf nodes and then creating their parents.
- We integrate the parents into the tree by rewiring their **left** and **right** pointers to the already-created children.

Traversing a tree - recursively!

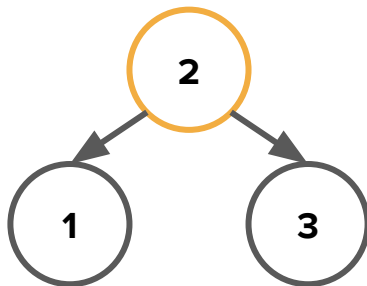
Pre-order

1. **"Do something" with the current node**
2. Traverse the left subtree
3. Traverse the right subtree



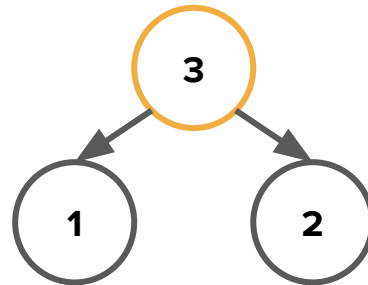
In-order

1. Traverse the left subtree
2. **"Do something" with the current node**
3. Traverse the right subtree



Post-order

1. Traverse the left subtree
2. Traverse the right subtree
3. **"Do something" with the current node**



Try it yourself:
Freeing a tree!

Key Idea: The distance from each element (node) in a tree to the top of the tree (the root) is small, even if there are many elements.

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How can we take advantage of trees to structure and efficiently manipulate data?

Revisiting our levels of
abstraction...

Levels of abstraction

What is the interface for the user?



How is our data organized?
(binary heaps, BSTs, Huffman trees)

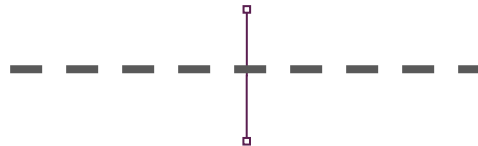


What stores our data?
(arrays, linked lists, **trees**)



How is data represented electronically?
(RAM)

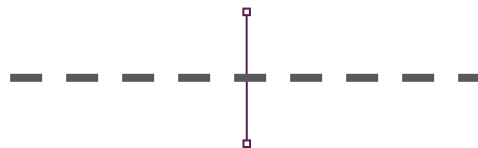
**Abstract Data
Structures**



**Data Organization
Strategies**



**Fundamental C++
Data Storage**



**Computer
Hardware**

Levels of abstraction

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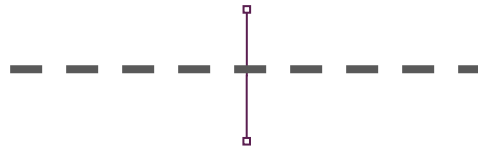


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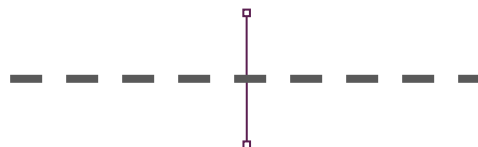
**Abstract Data
Structures**



**Data Organization
Strategies**



**Fundamental C++
Data Storage**



**Computer
Hardware**

ADT Big-O Matrix

● Vectors

- `.size()` - $O(1)$
- `.add()` - $O(1)$
- `v[i]` - $O(1)$
- `.insert()` - $O(n)$
- `.remove()` - $O(n)$
- `.clear()` - $O(n)$
- `traversal` - $O(n)$

● Grids

- `.numRows()/.numCols()`
- $O(1)$
- `g[i][j]` - $O(1)$
- `.inBounds()` - $O(1)$
- `traversal` - $O(n^2)$

● Queues

- `.size()` - $O(1)$
- `.peek()` - $O(1)$
- `.enqueue()` - $O(1)$
- `.dequeue()` - $O(1)$
- `.isEmpty()` - $O(1)$
- `traversal` - $O(n)$

● Stacks

- `.size()` - $O(1)$
- `.peek()` - $O(1)$
- `.push()` - $O(1)$
- `.pop()` - $O(1)$
- `.isEmpty()` - $O(1)$
- `traversal` - $O(n)$

● Sets

- `.size()` - $O(1)$
- `.isEmpty()` - $O(1)$
- `.add()` - $O(\log(n))$
- `.remove()` - $O(\log(n))$
- `.contains()` - $O(\log(n))$
- `traversal` - $O(n)$

● Maps

- `.size()` - $O(1)$
- `.isEmpty()` - $O(1)$
- `m[key]` - $O(\log(n))$
- `.contains()` - $O(\log(n))$
- `traversal` - $O(n)$

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Levels of abstraction

What is the interface for the user?
(**Sets**, Maps, etc.)



How is our data organized?
(binary heaps, **BSTs**, Huffman trees)

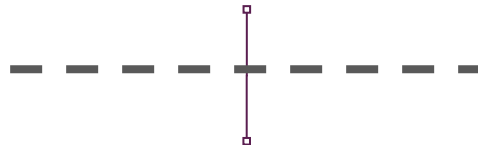


What stores our data?
(arrays, linked lists, **trees**)



How is data represented electronically?
(RAM)

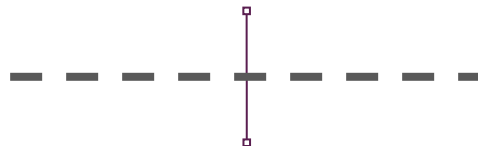
Abstract Data Structures



Data Organization Strategies



Fundamental C++ Data Storage

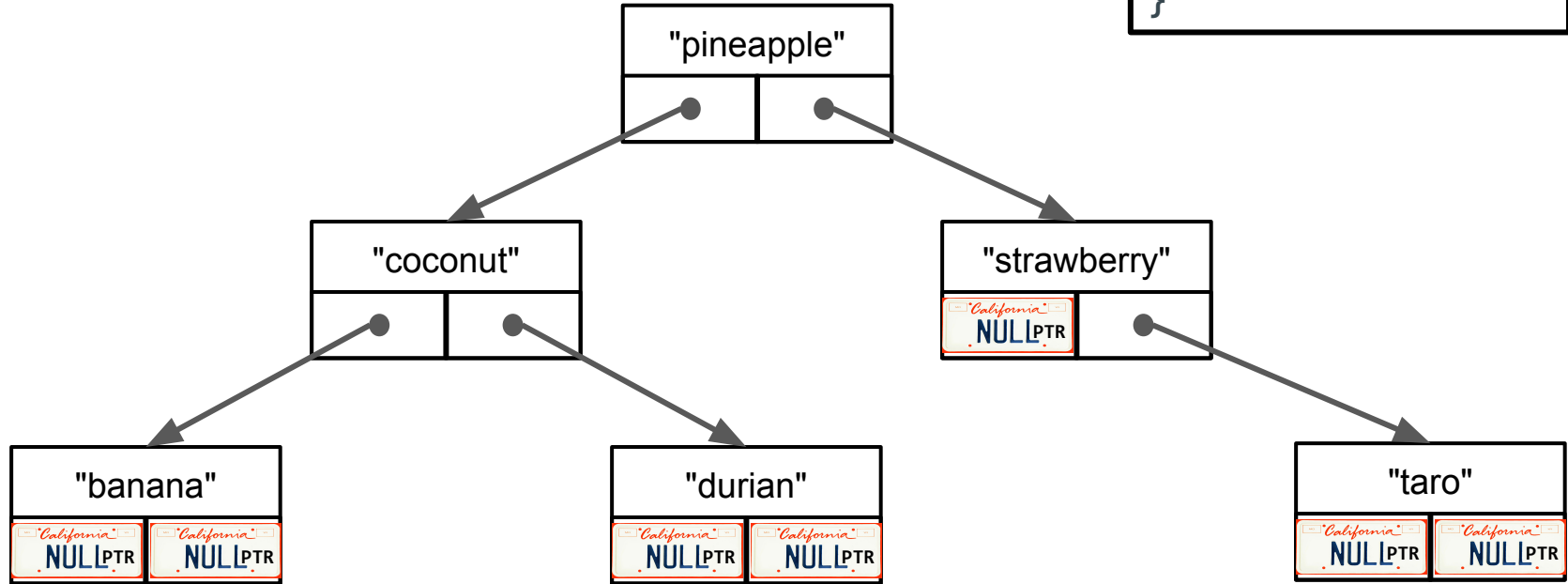


Computer Hardware

What is a binary search tree
(BST)?

Building Trees Programmatically

```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

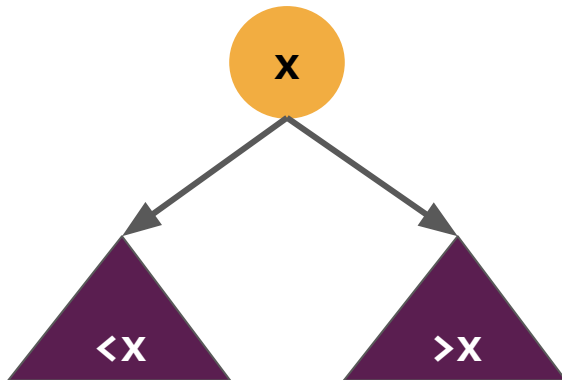


A binary search tree is either...

an empty data
structure represented
by nullptr or...

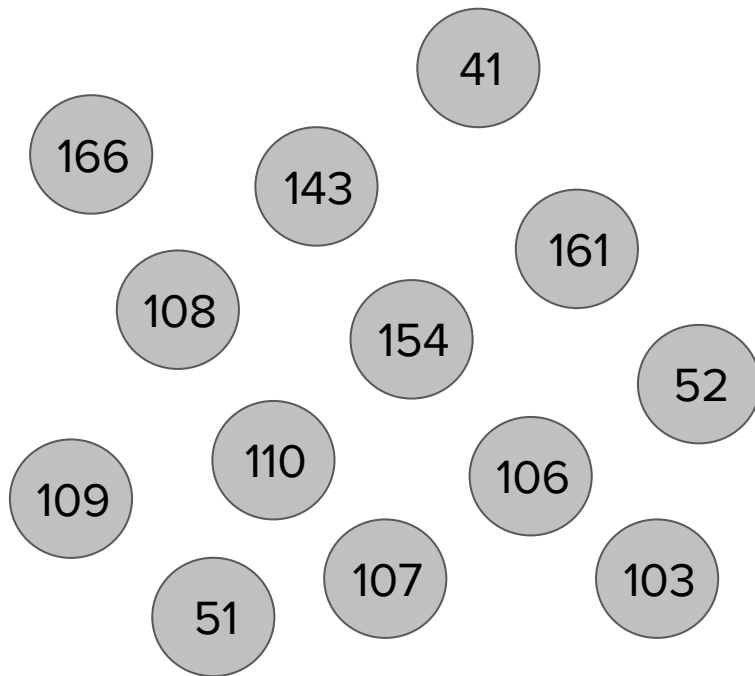


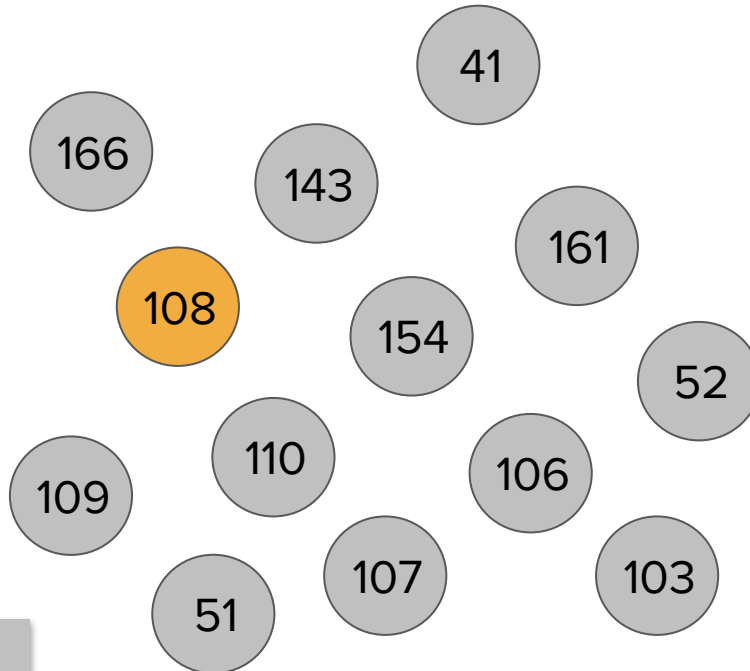
a single node,
whose left subtree is
a BST of smaller
values than **x**...



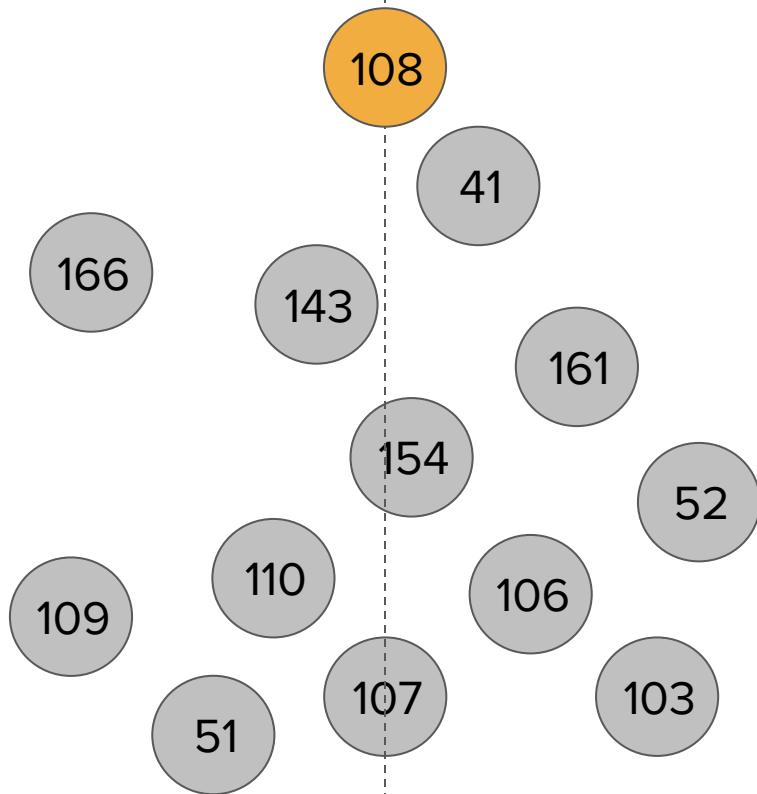
and whose right
subtree is a BST of
larger values than **x**.

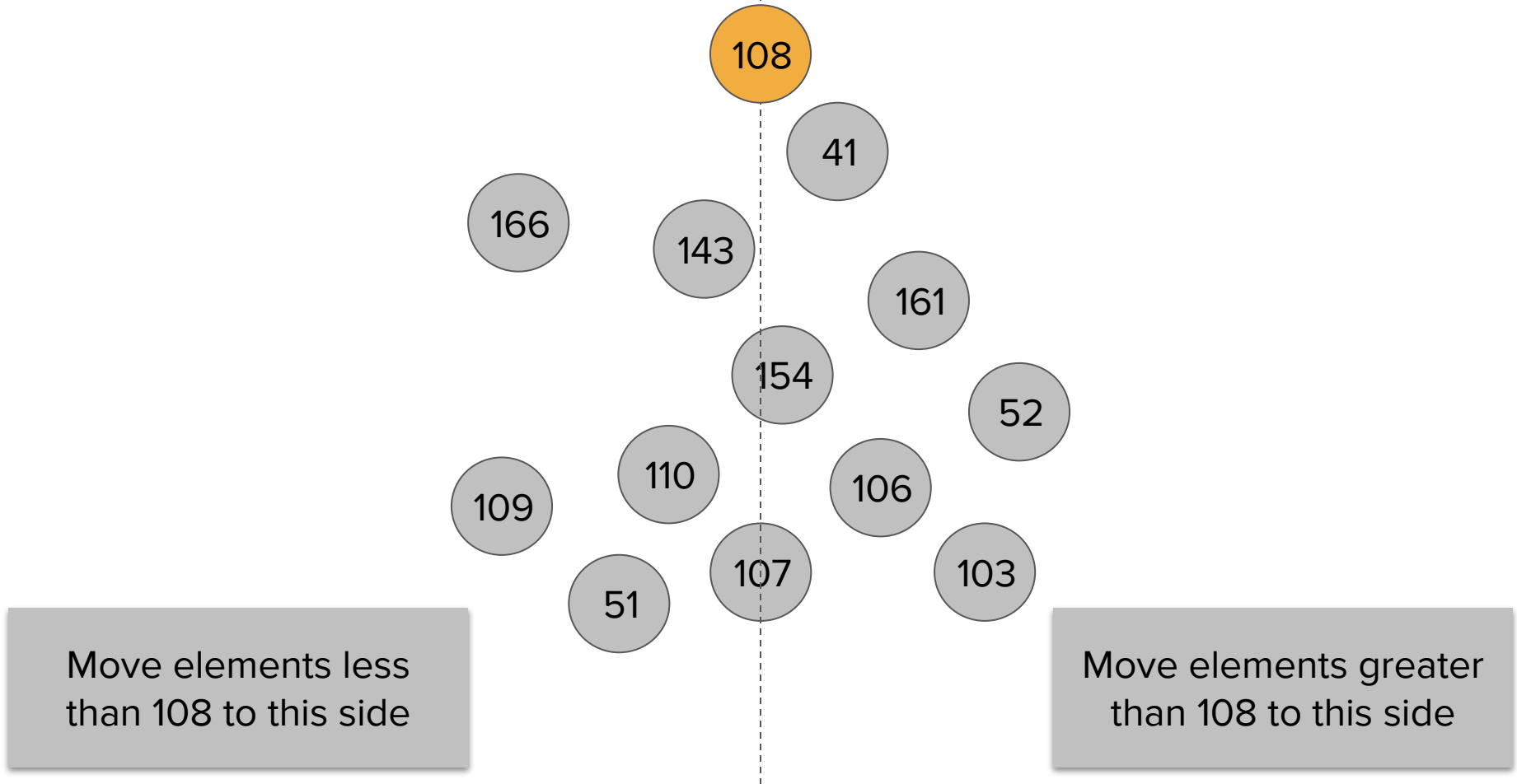
Building a BST





Pick the median element





108

51

41

107

52

106

103

Move elements less
than 108 to this side

143

166

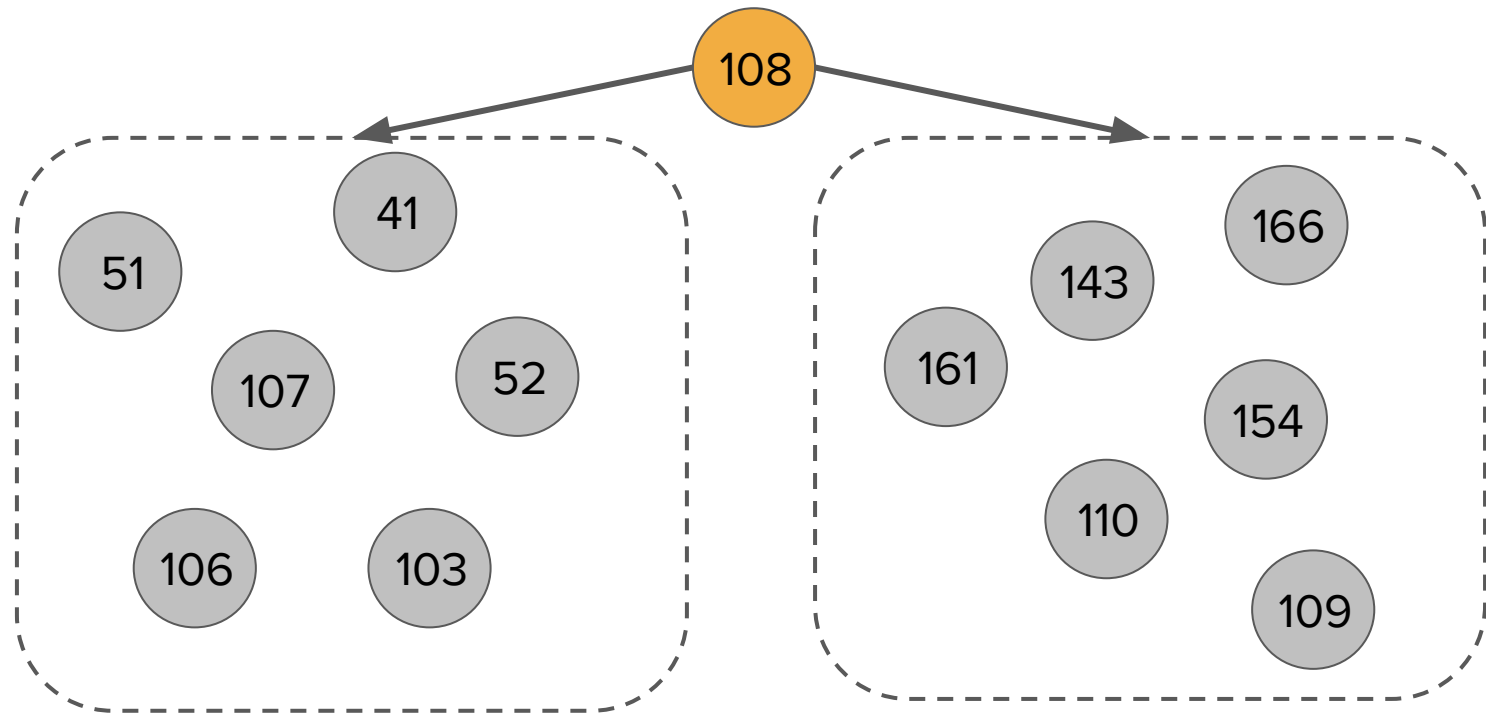
161

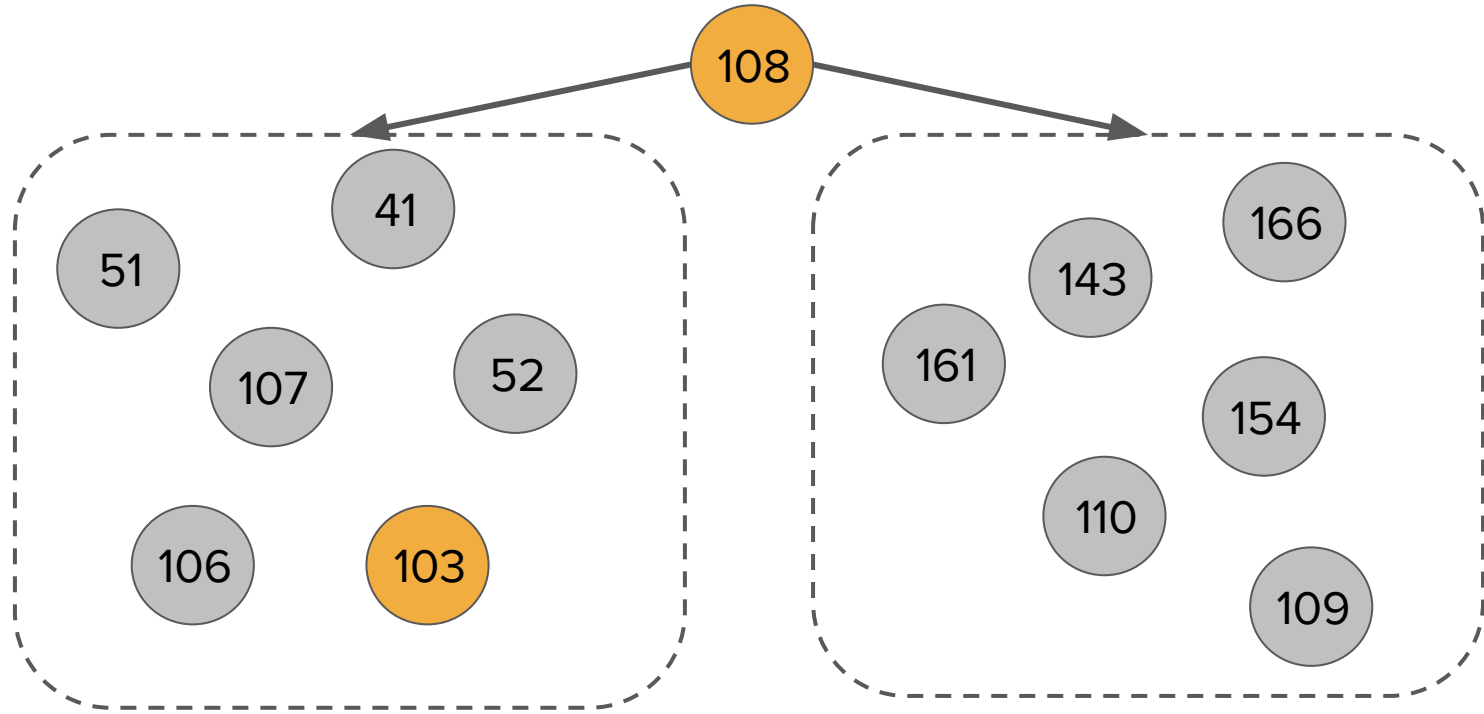
154

110

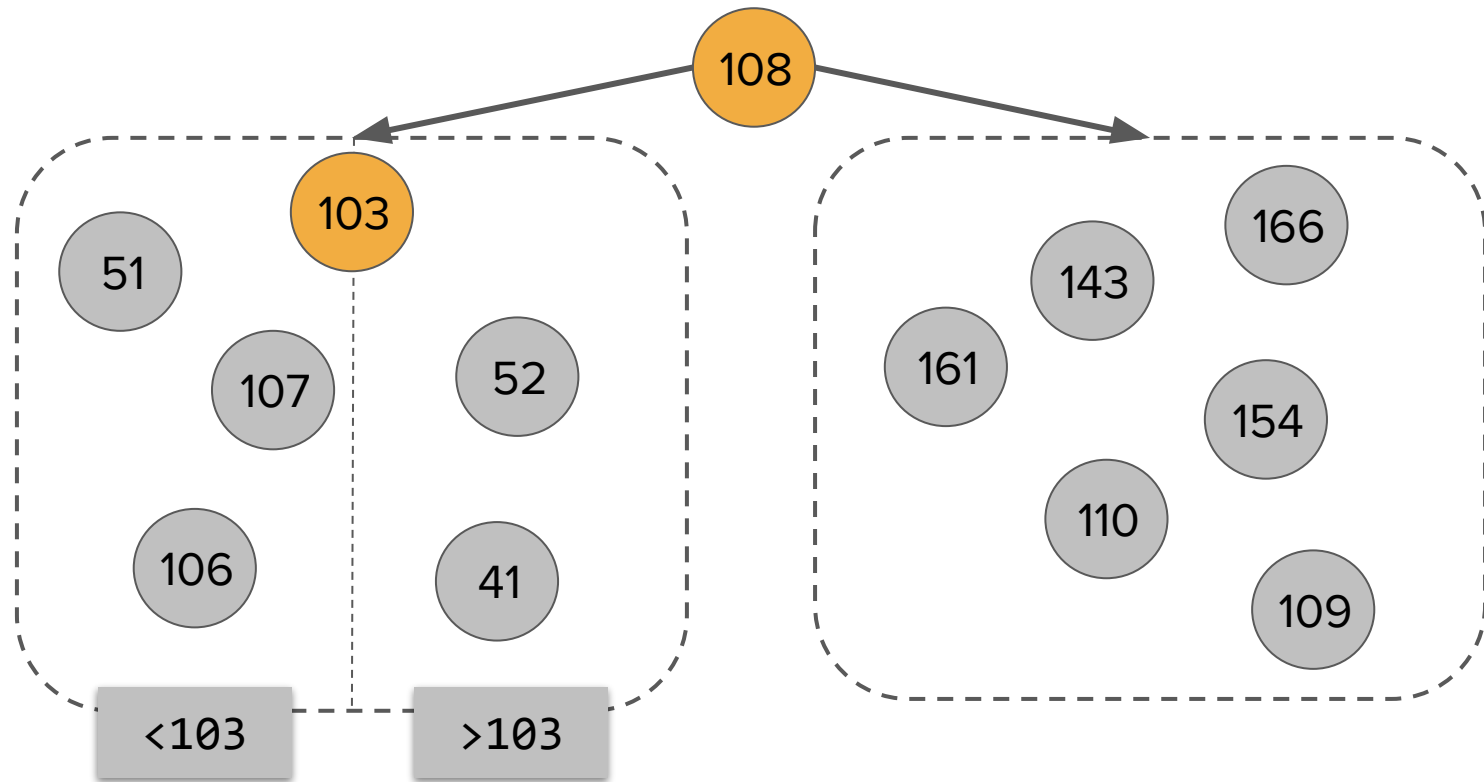
109

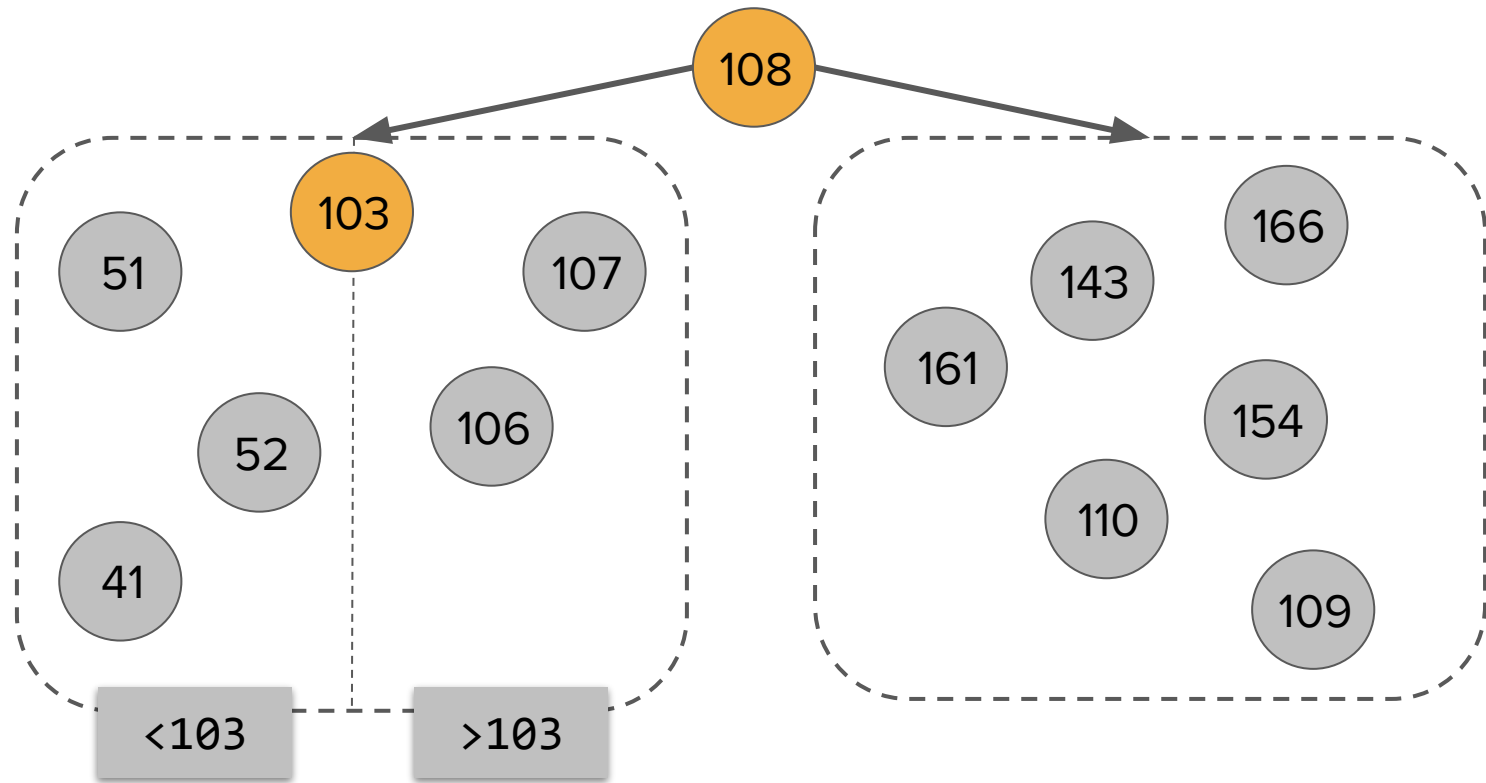
Move elements greater
than 108 to this side

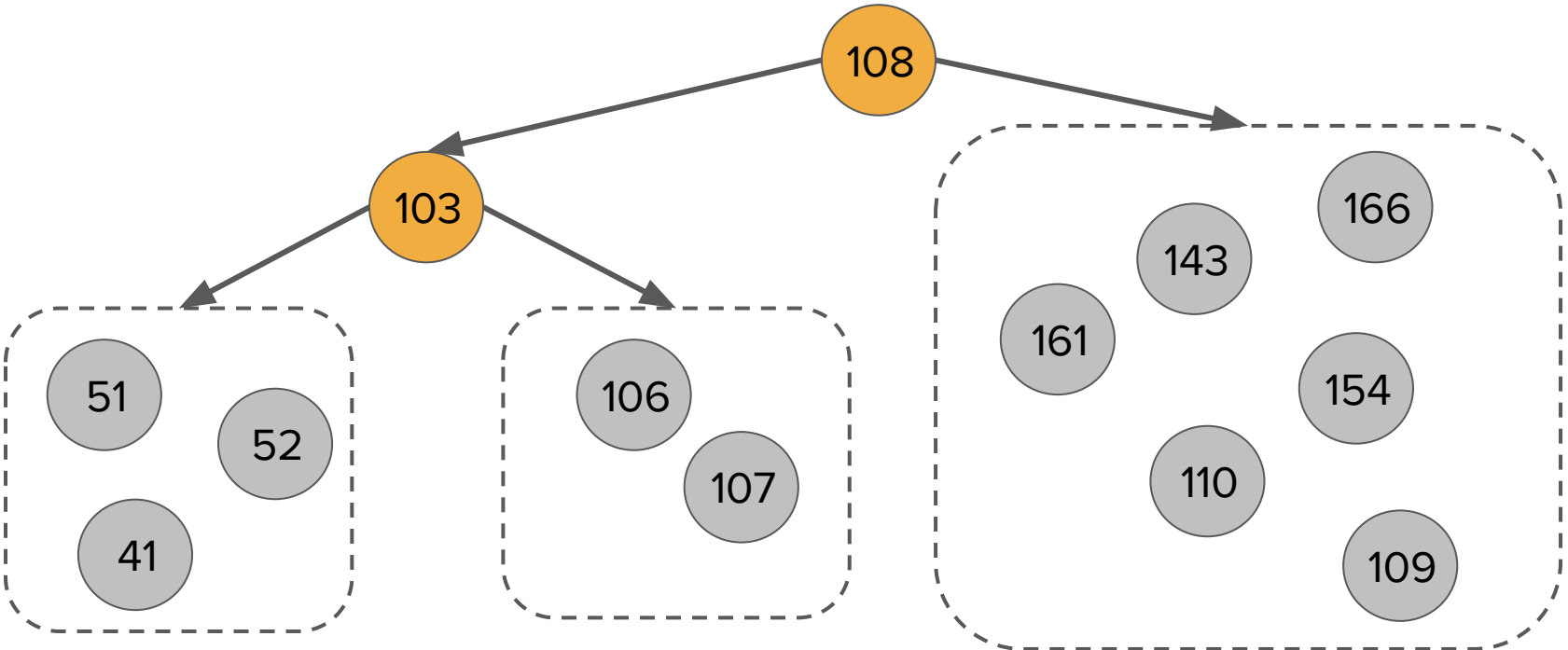


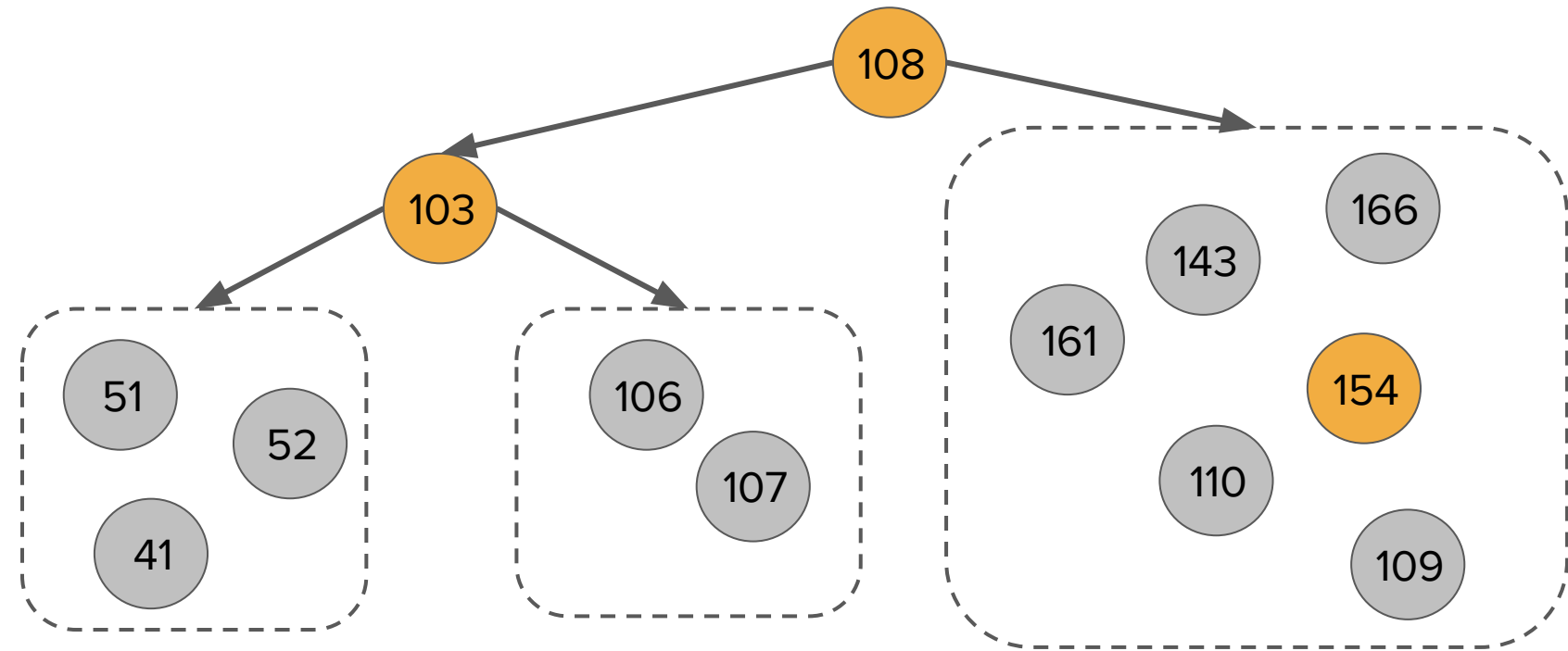


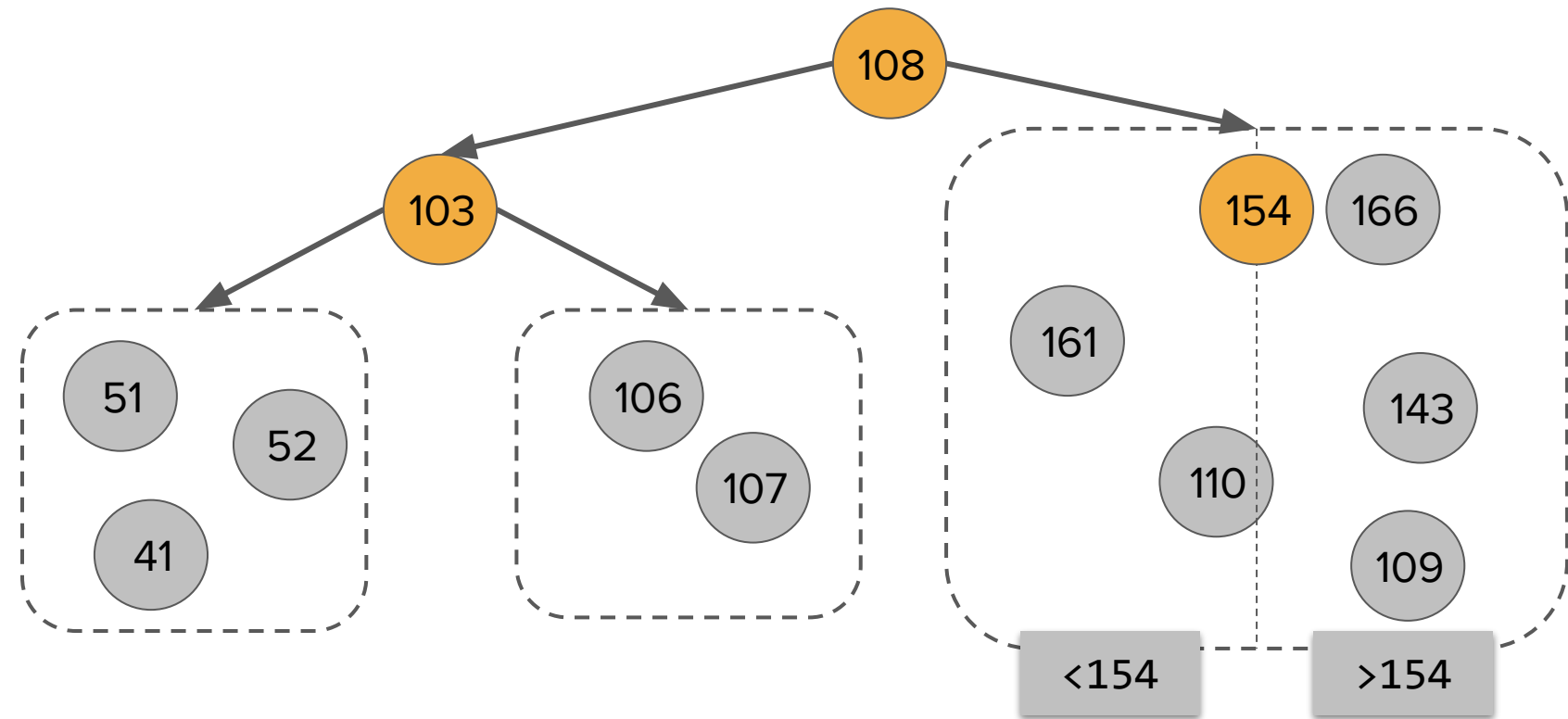
Pick the median element
of the left side

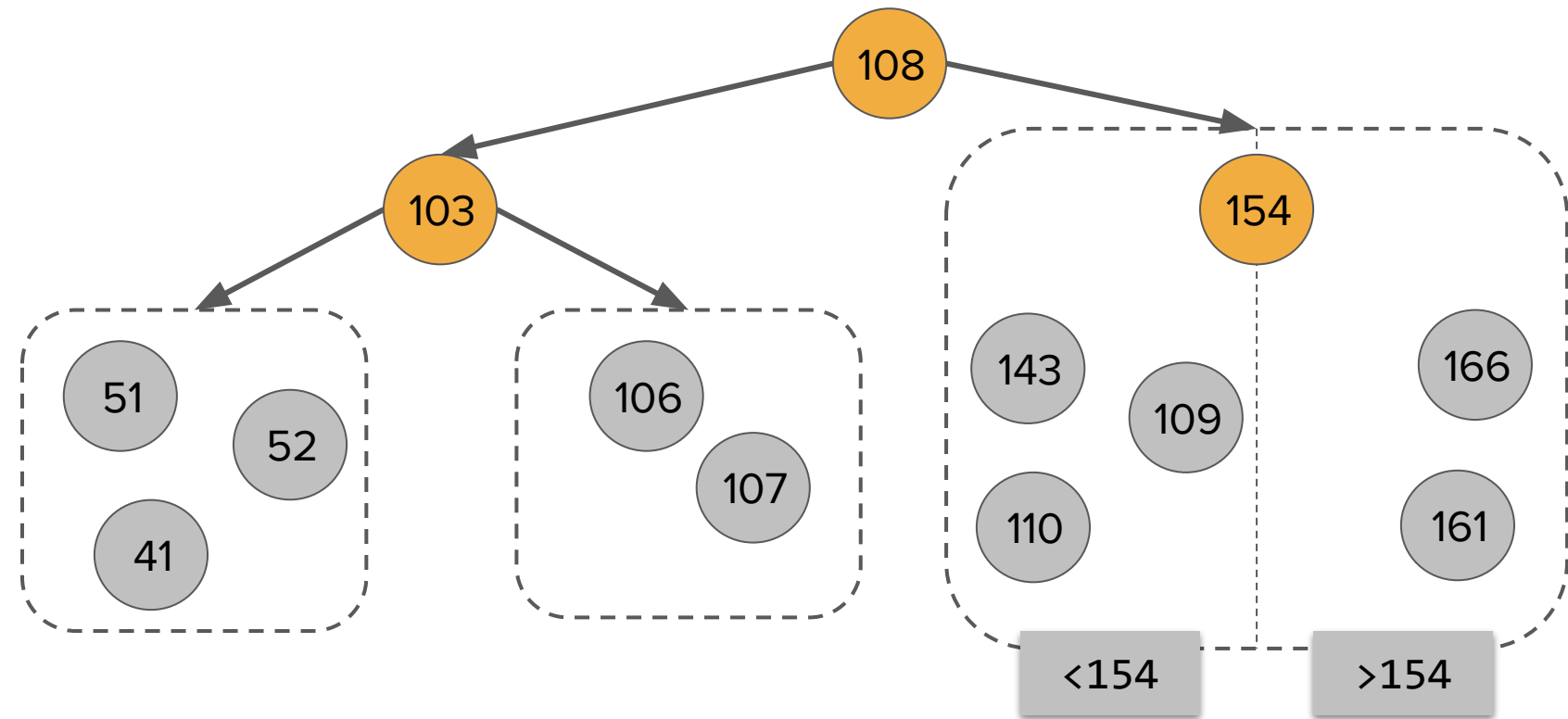


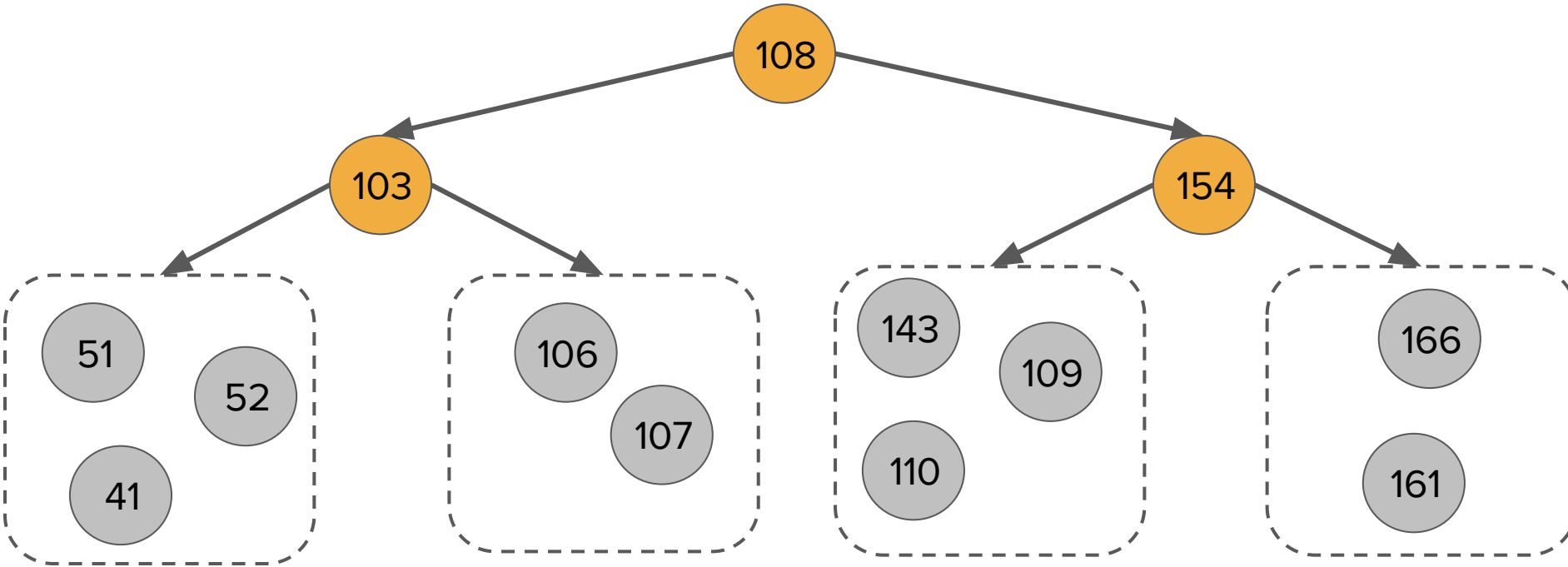


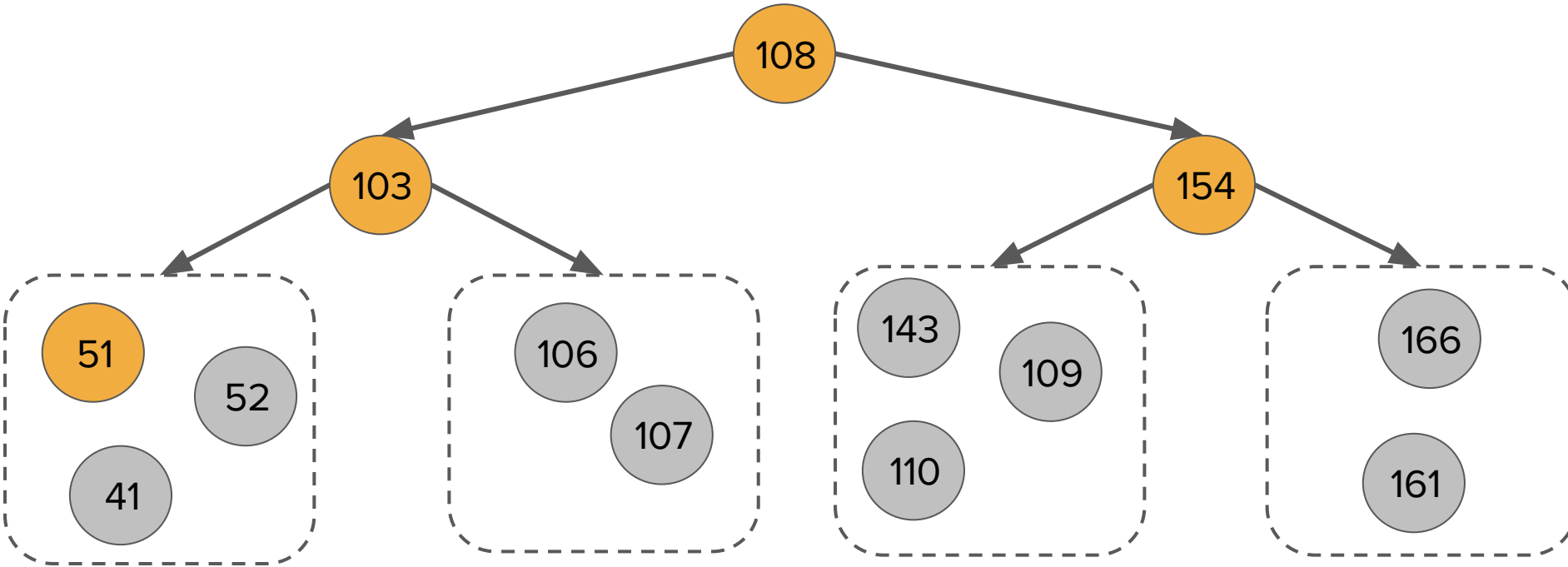




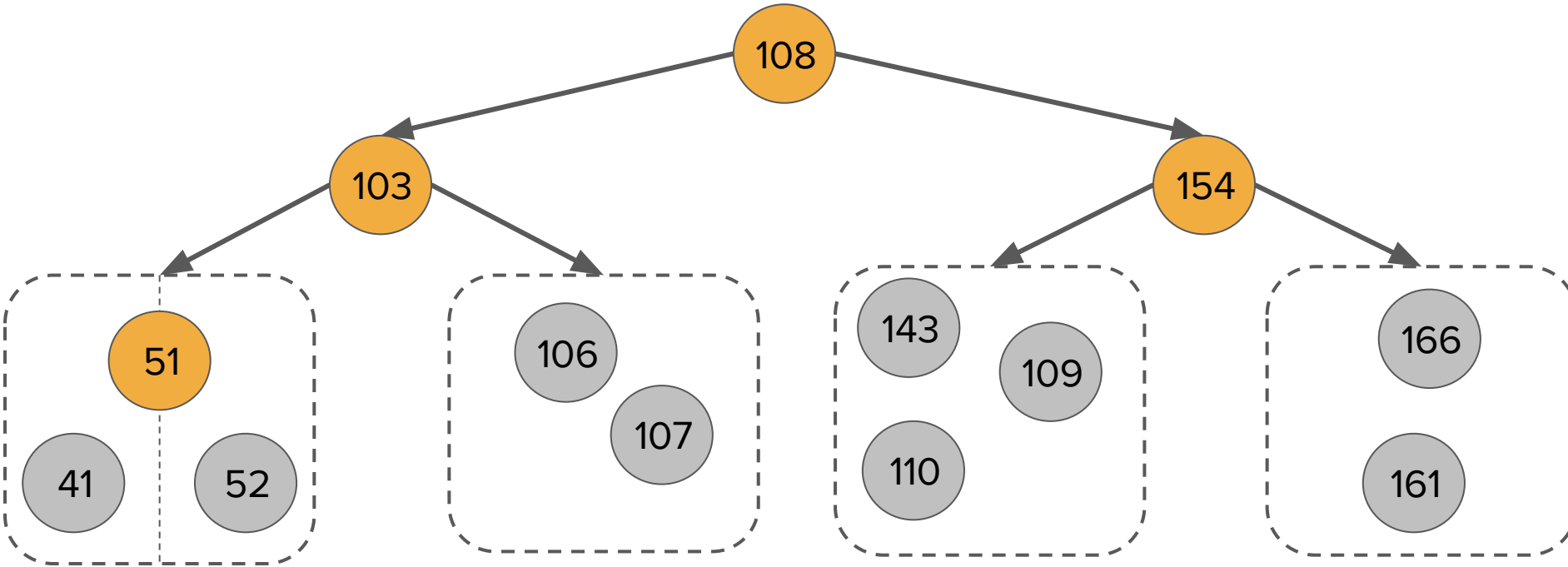


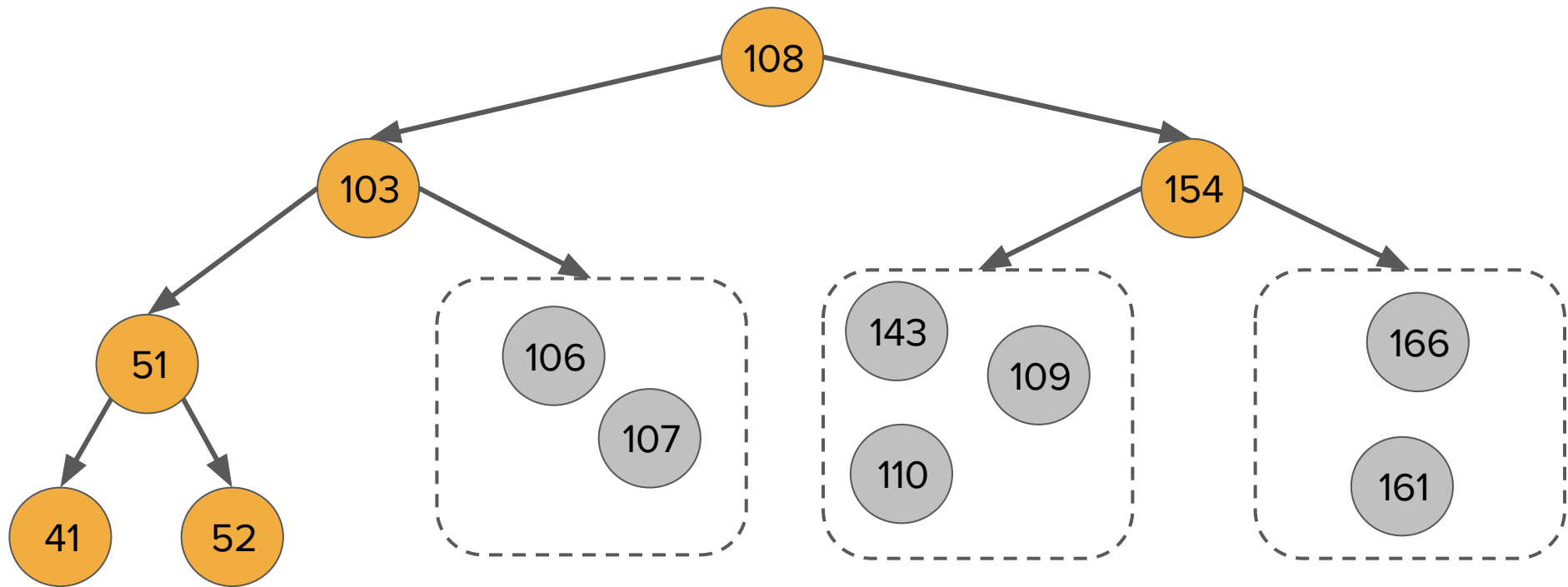


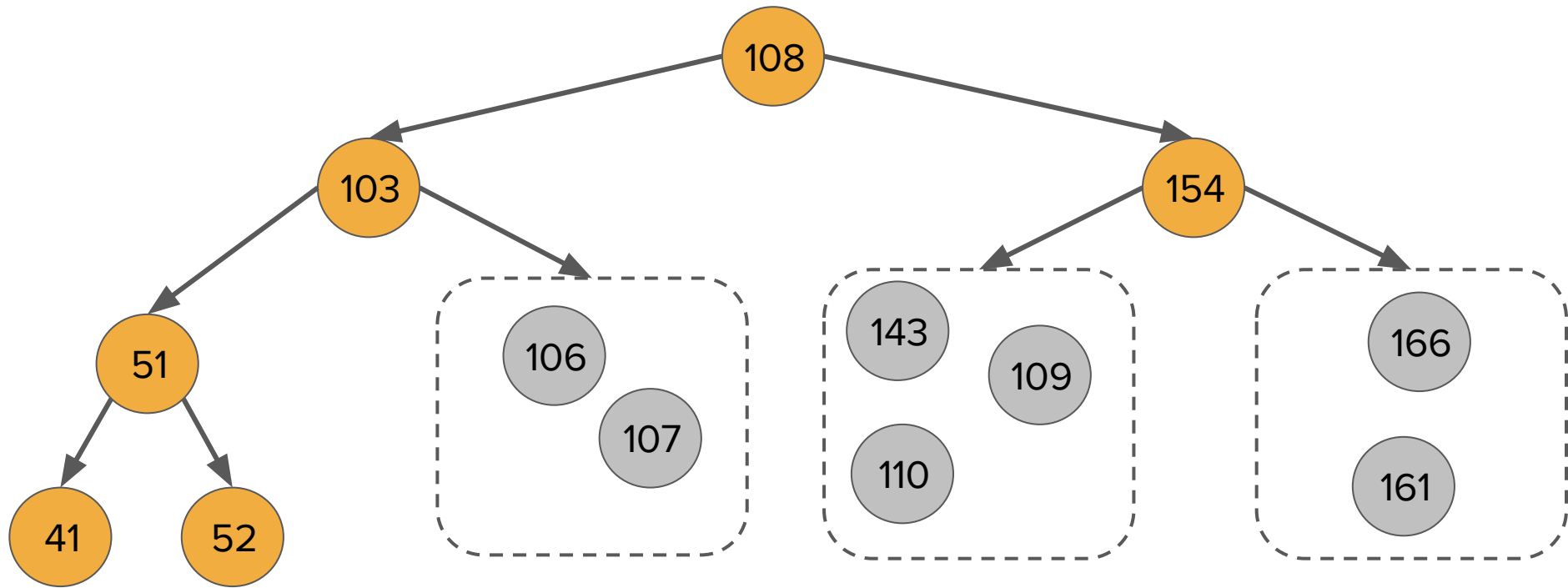




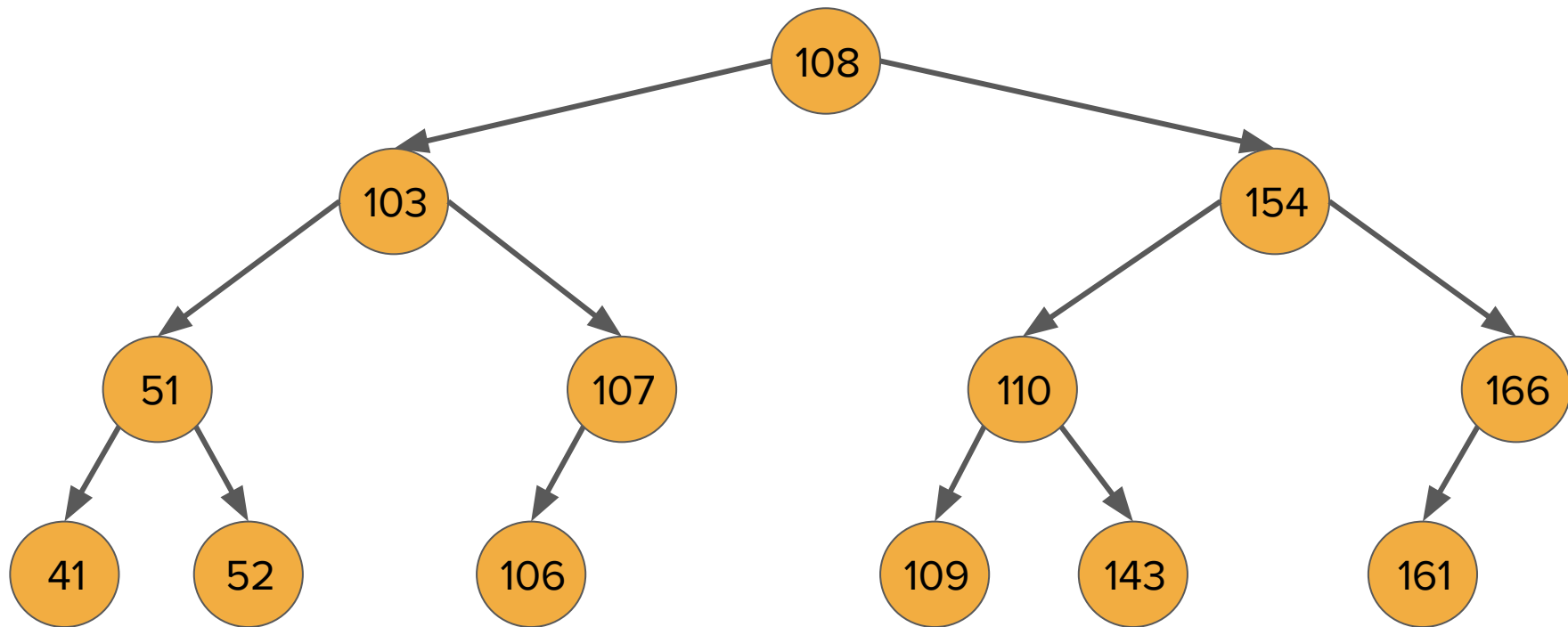
Pick the median element
of the left side

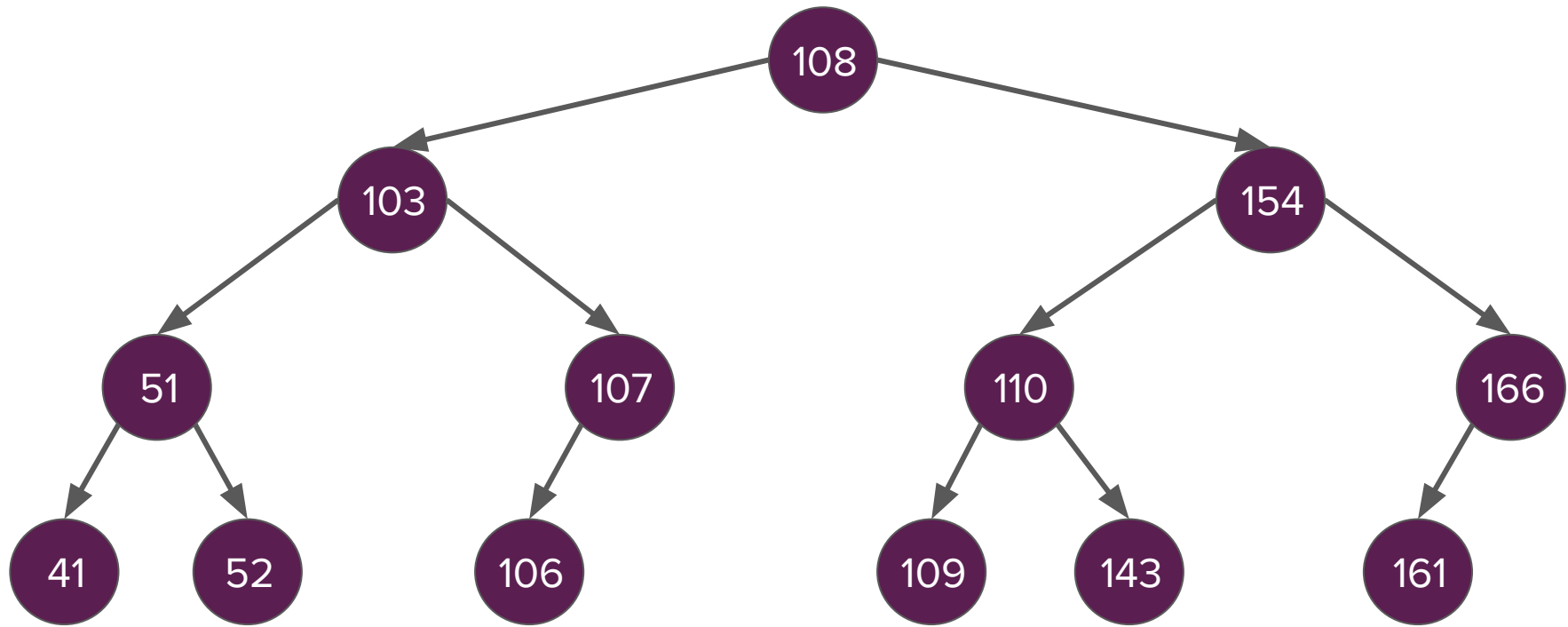




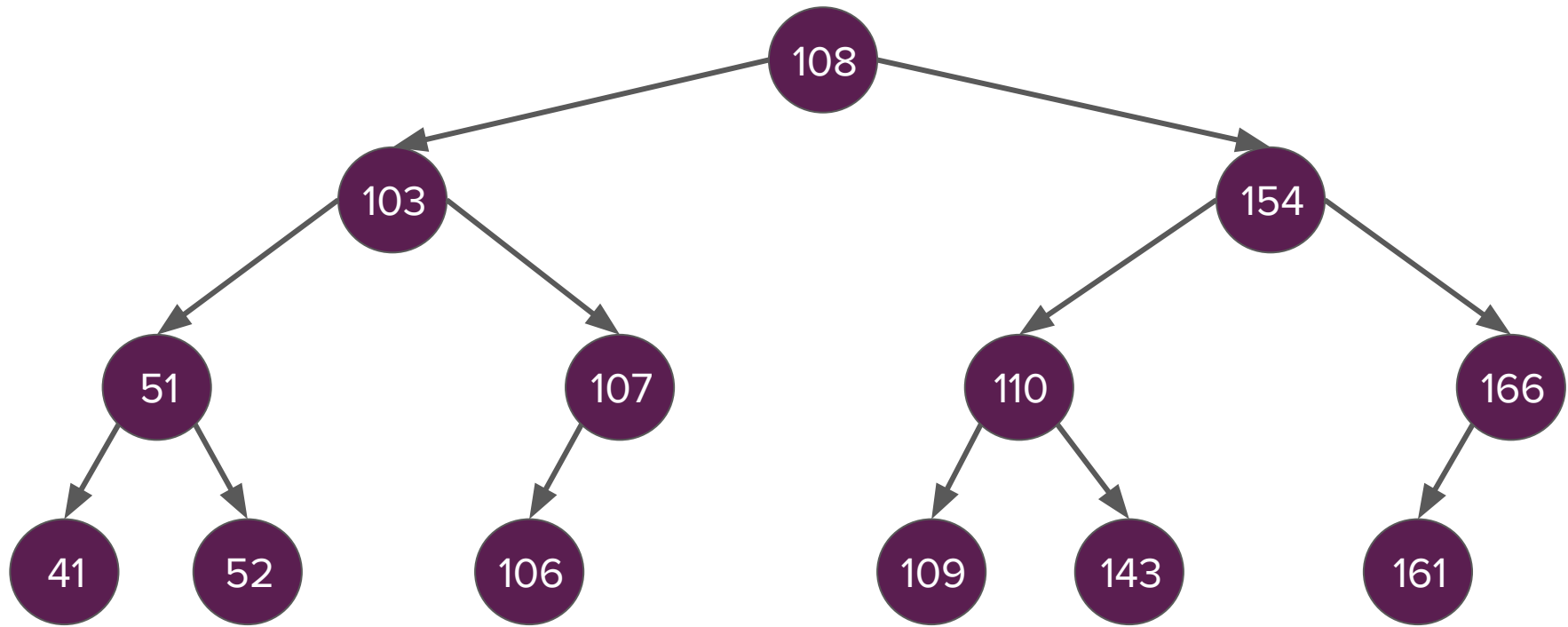


Keep repeating this process
for all the subtrees!



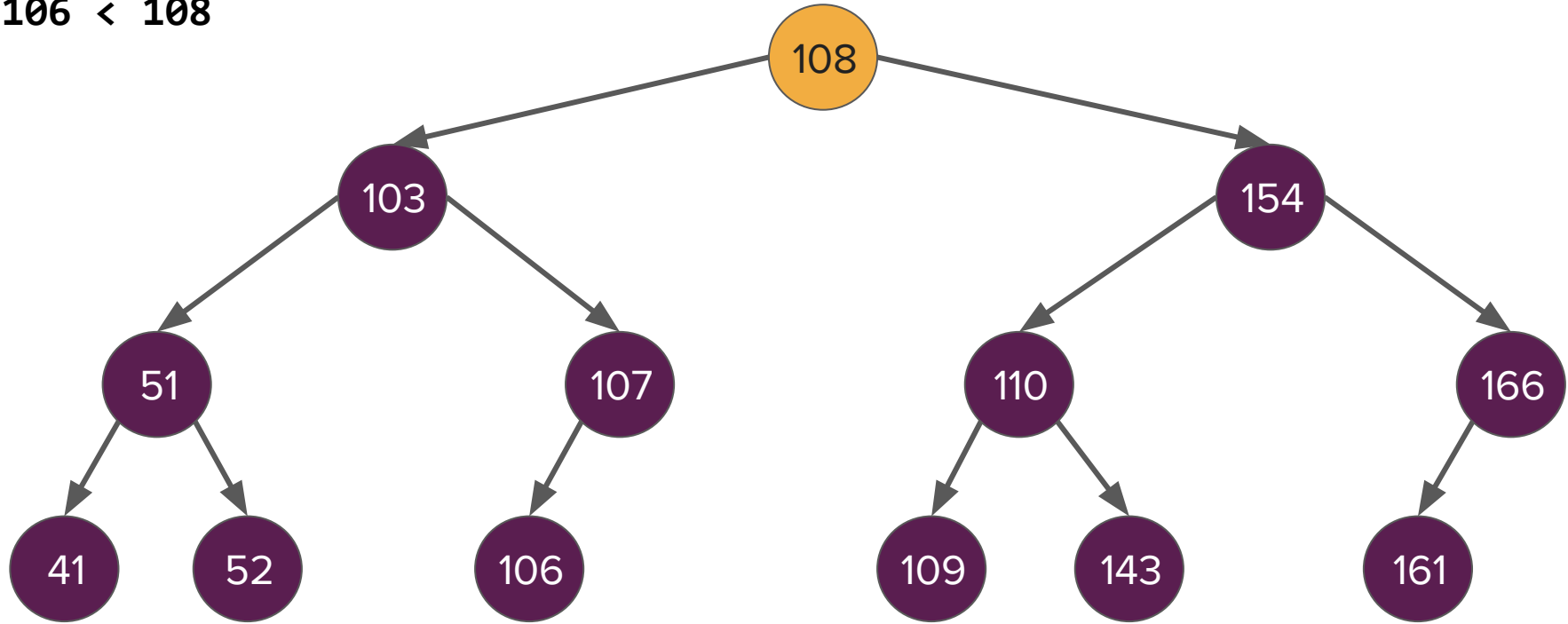


There are 13 nodes in the tree, but
the path to each node is short
 $\sim O(\log 13)$!



How could we check if **106** is in this tree?

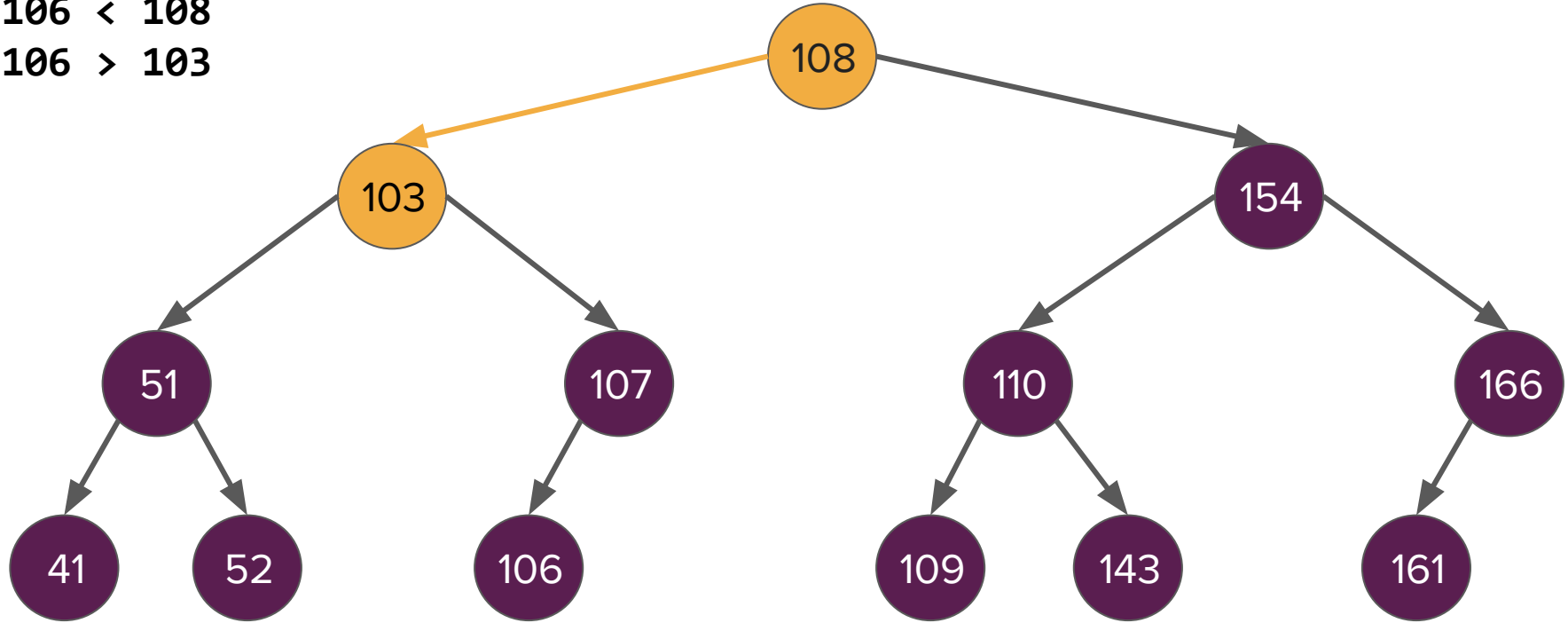
$106 < 108$



How could we check if **106** is in this tree?

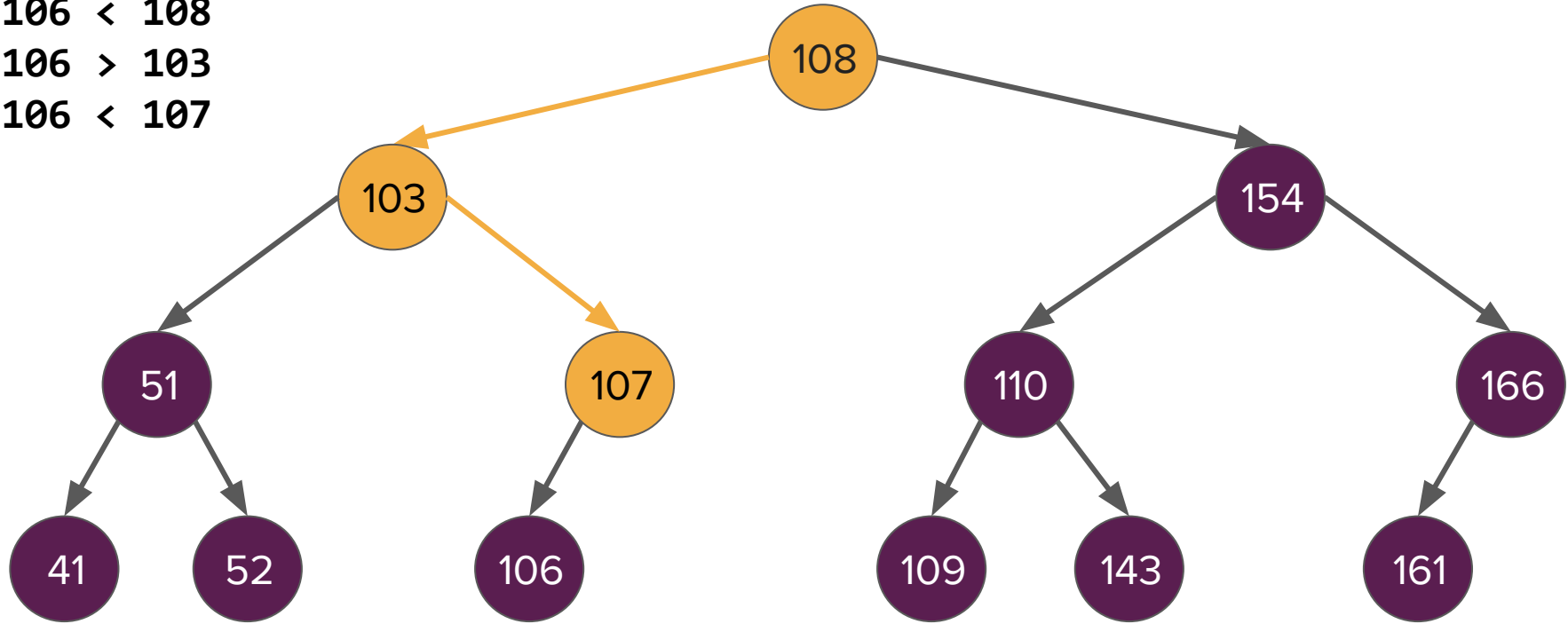
$106 < 108$

$106 > 103$



How could we check if **106** is in this tree?

$106 < 108$
 $106 > 103$
 $106 < 107$

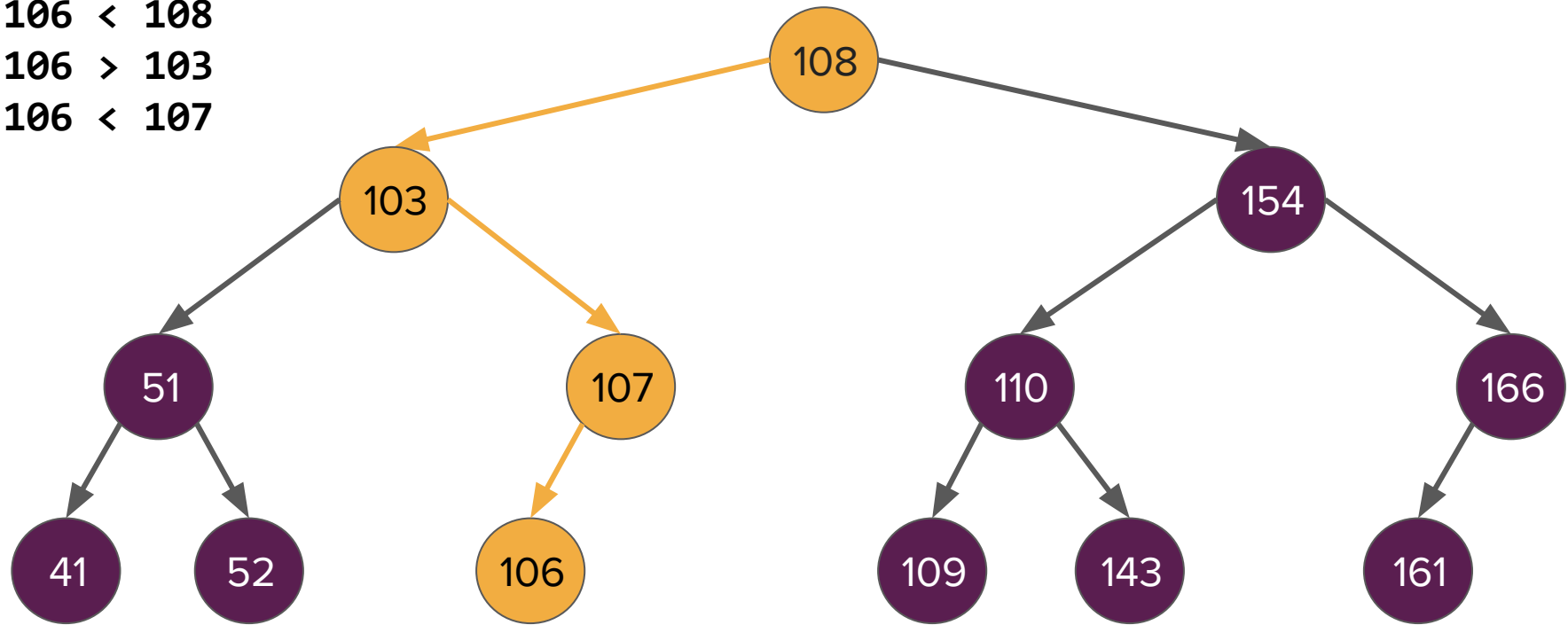


How could we check if **106** is in this tree?

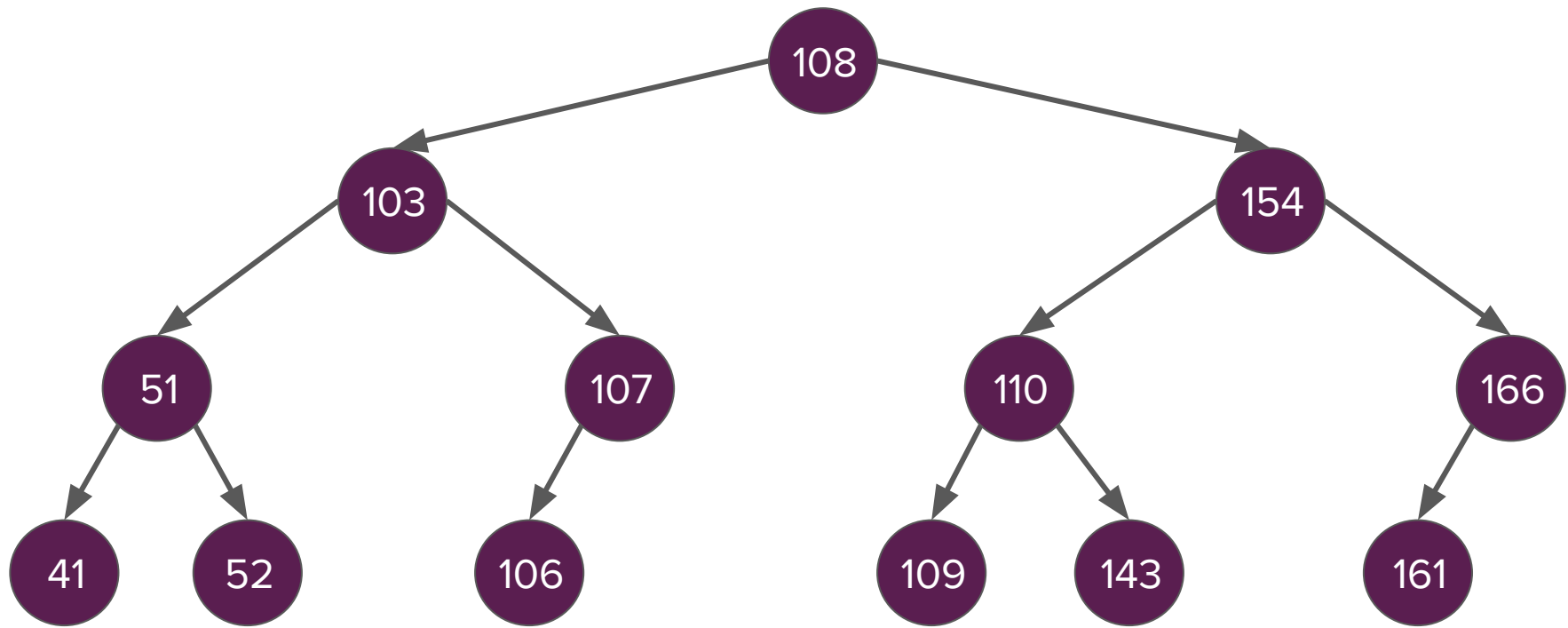
$106 < 108$

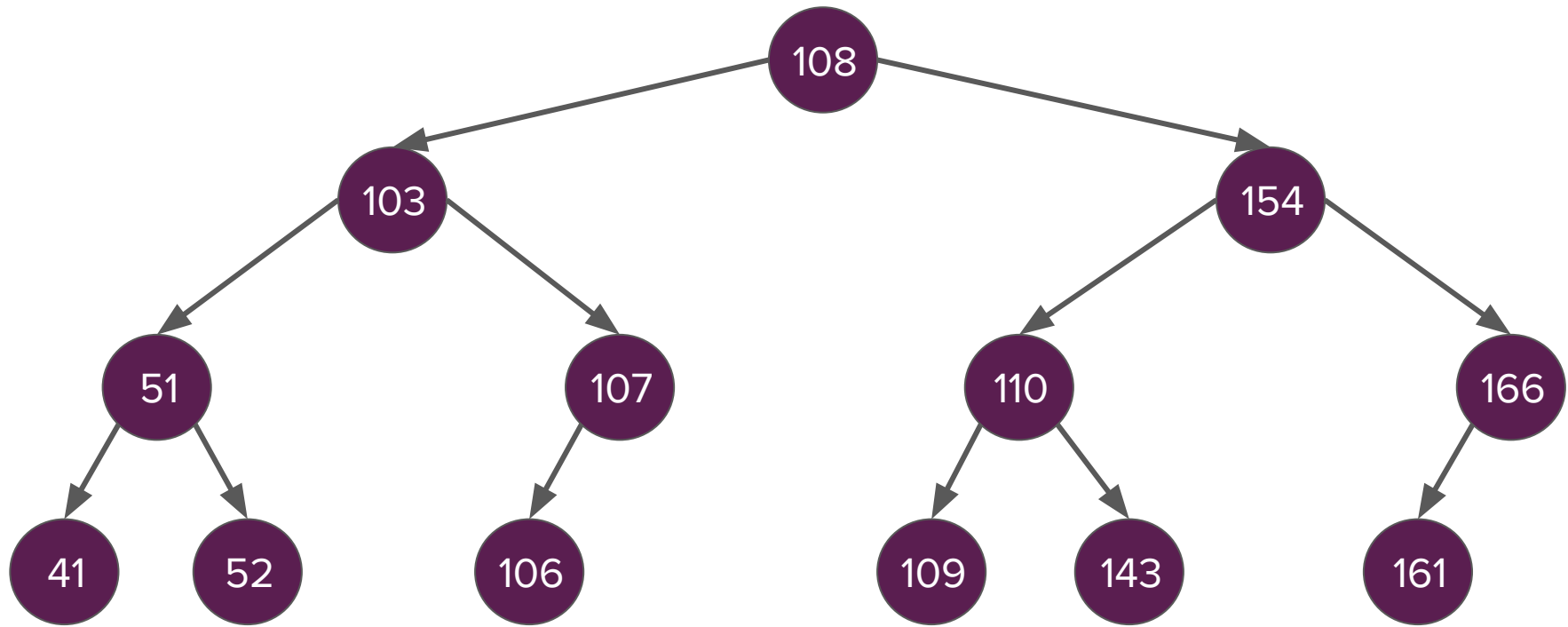
$106 > 103$

$106 < 107$



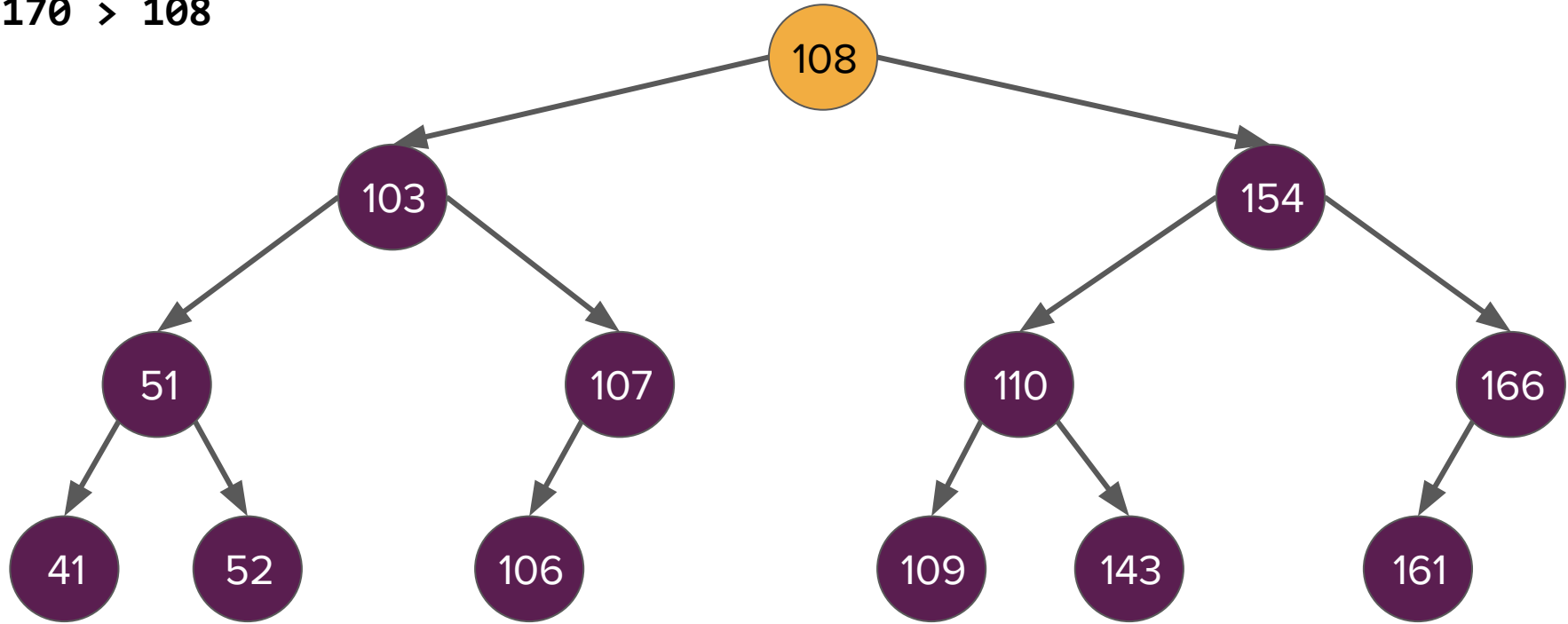
We found **106** so we're done!





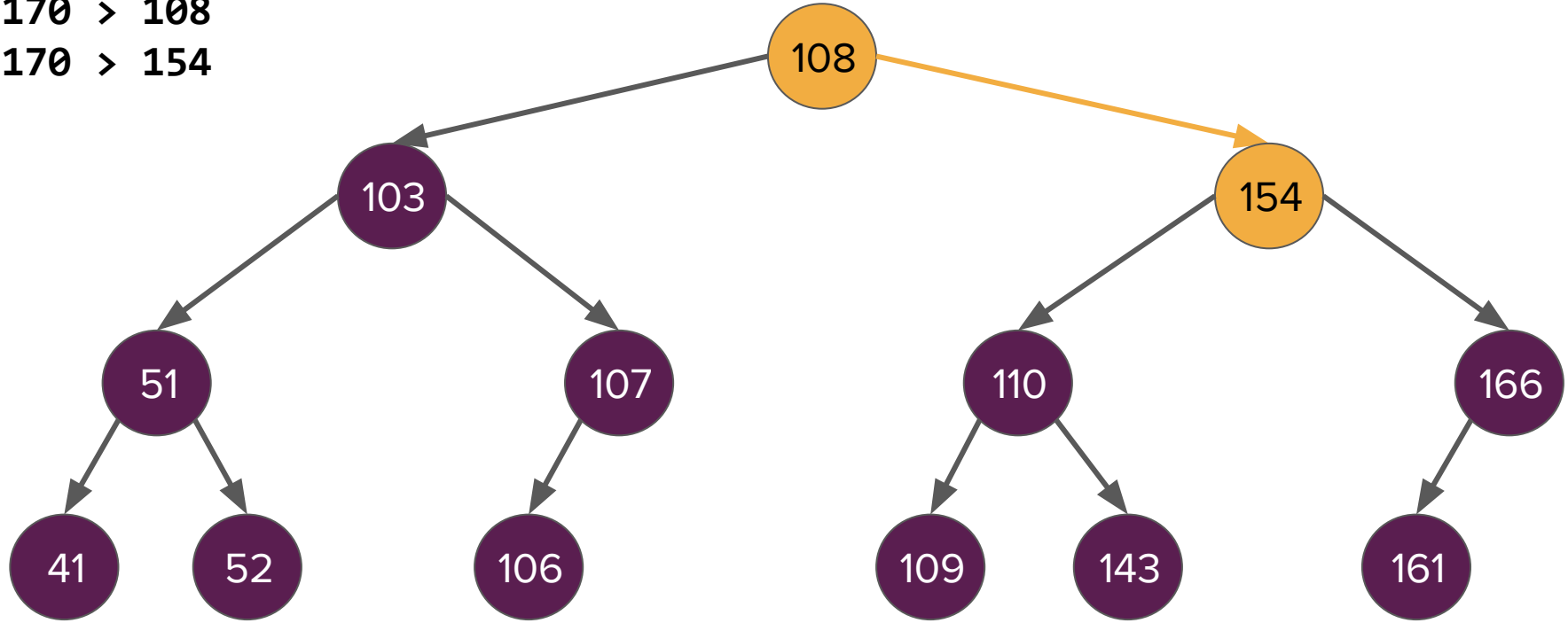
How could we check if **170** is in this tree?

$170 > 108$



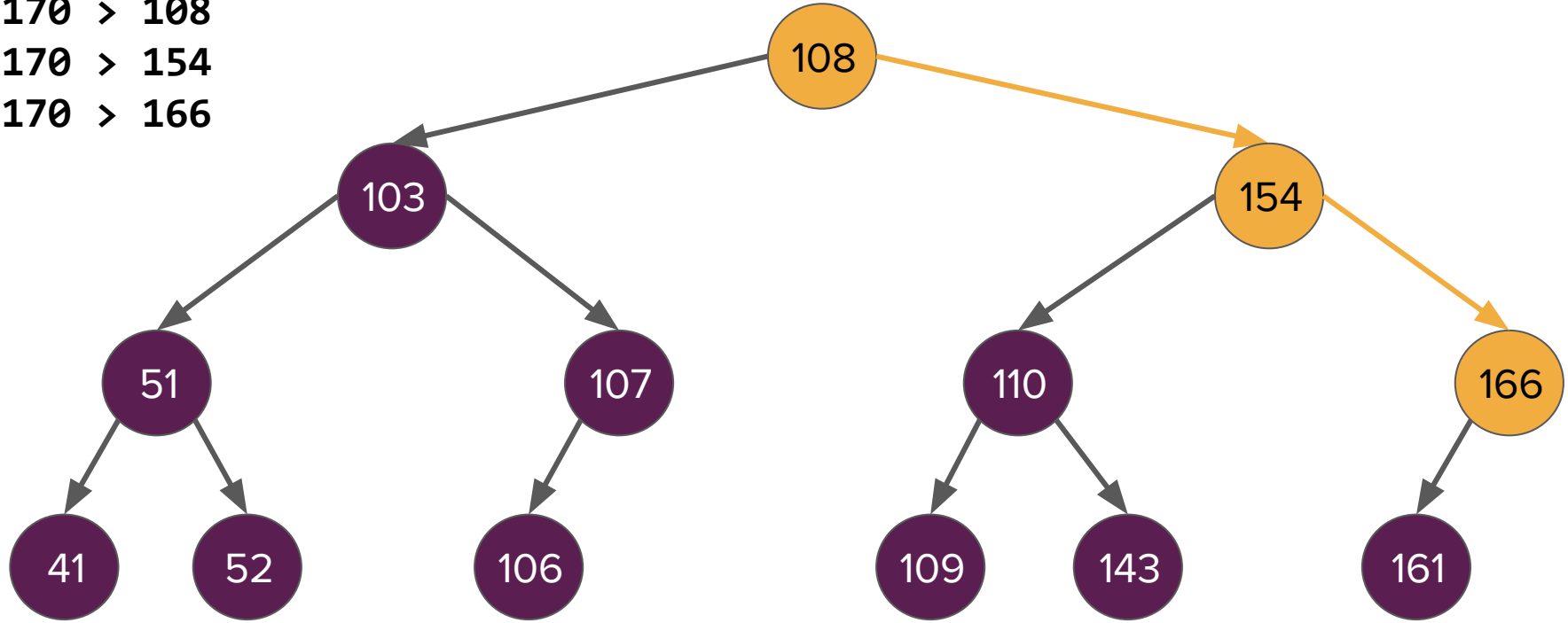
How could we check if **170** is in this tree?

$170 > 108$
 $170 > 154$



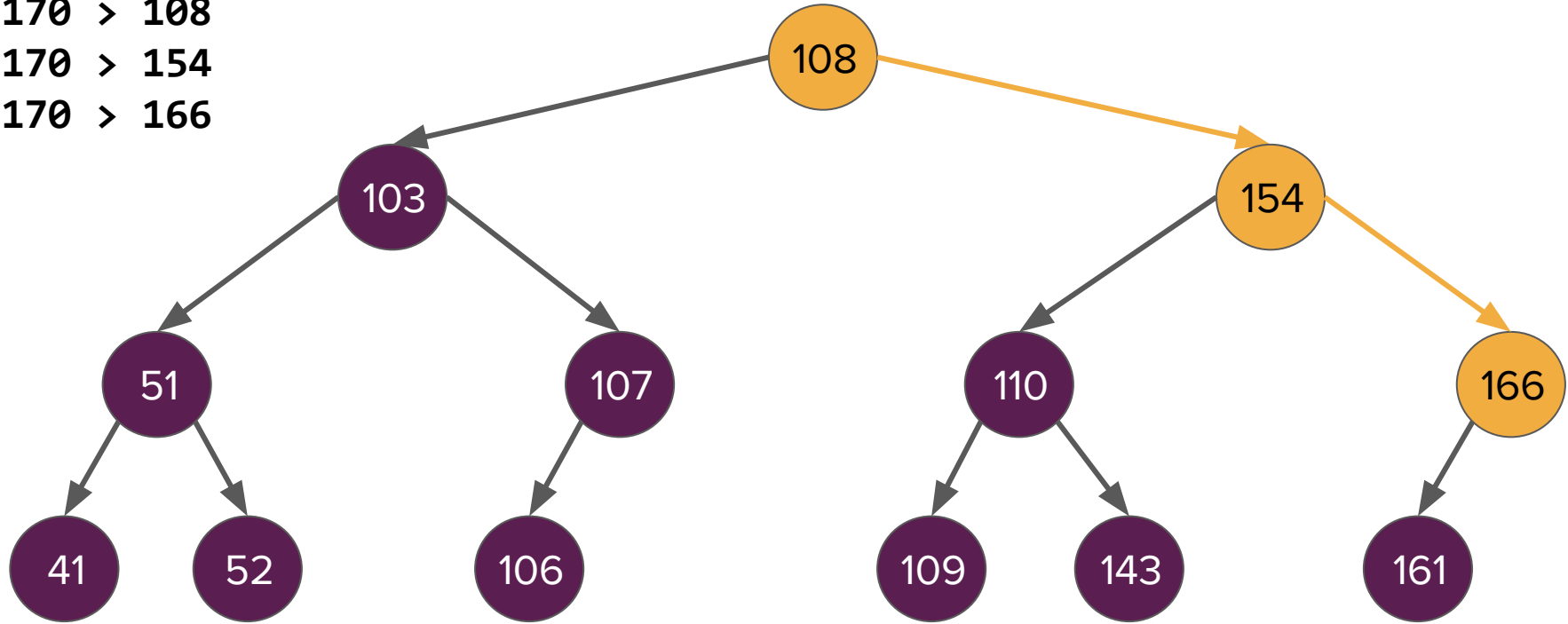
How could we check if **170** is in this tree?

$170 > 108$
 $170 > 154$
 $170 > 166$



How could we check if **170** is in this tree?

170 > 108
170 > 154
170 > 166



Right child is **nullptr** so we're done!

Building a BST

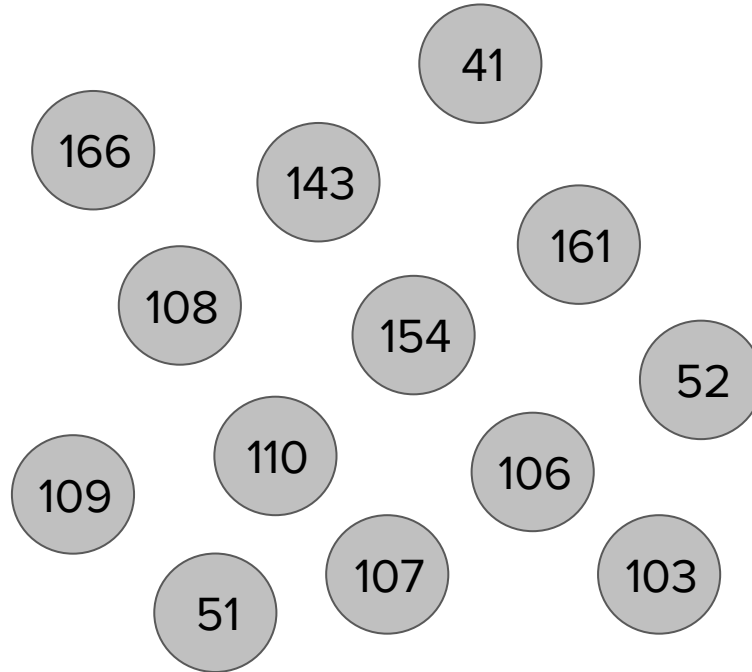
- An **optimal BST** is built by repeatedly choosing the median element as the root node of a given subtree and then separating elements into groups less than and greater than that median.

Building a BST

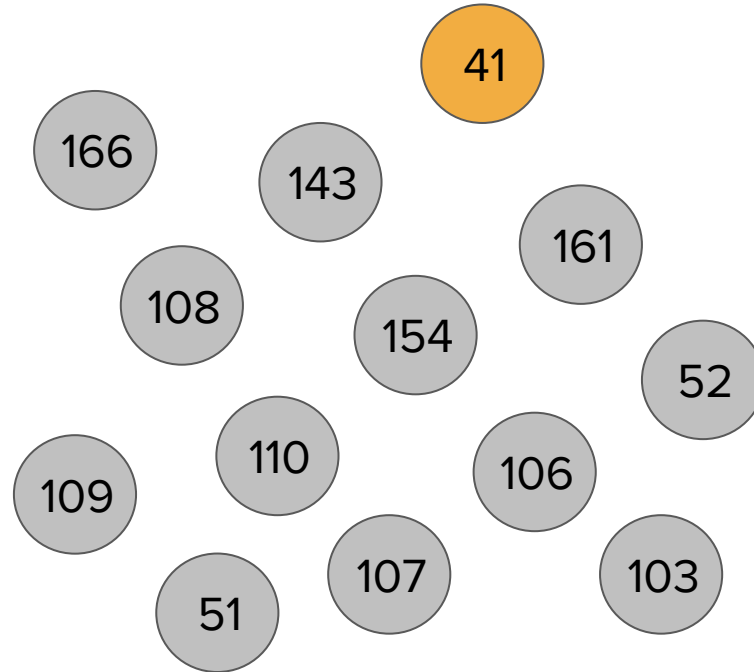
- An **optimal BST** is built by repeatedly choosing the median element as the root node of a given subtree and then separating elements into groups less than and greater than that median.

What does “optimal” mean?

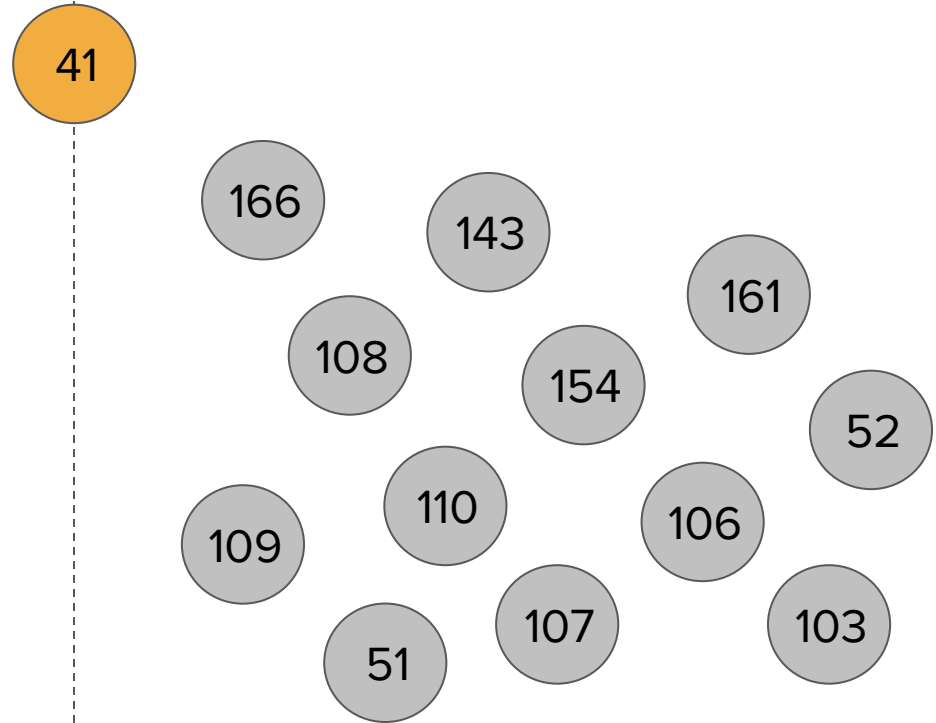
What if we didn't choose the median?



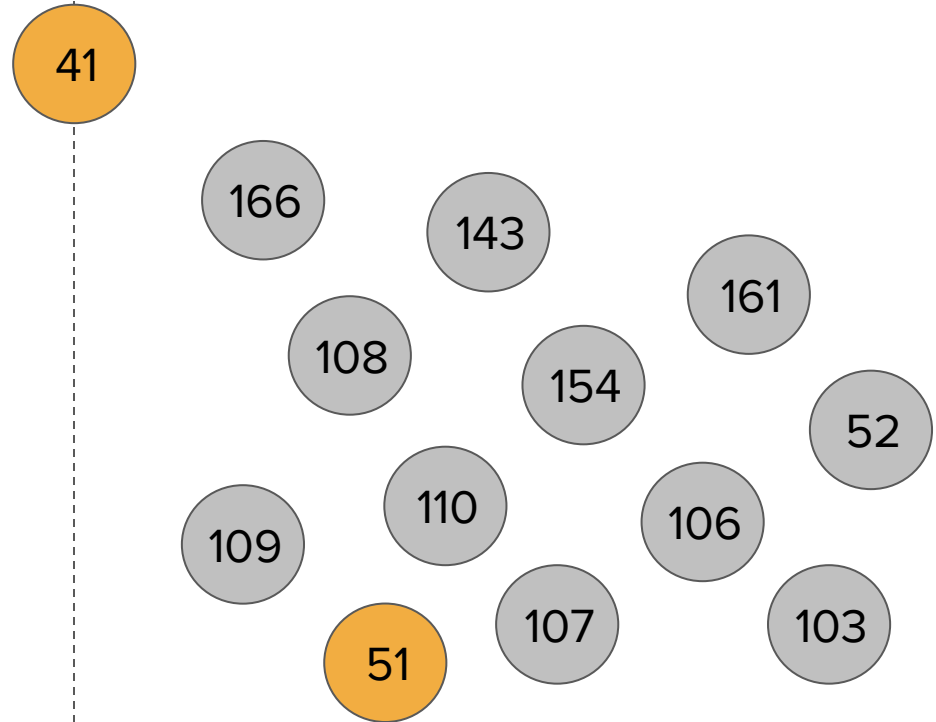
Let's choose the smallest element instead...



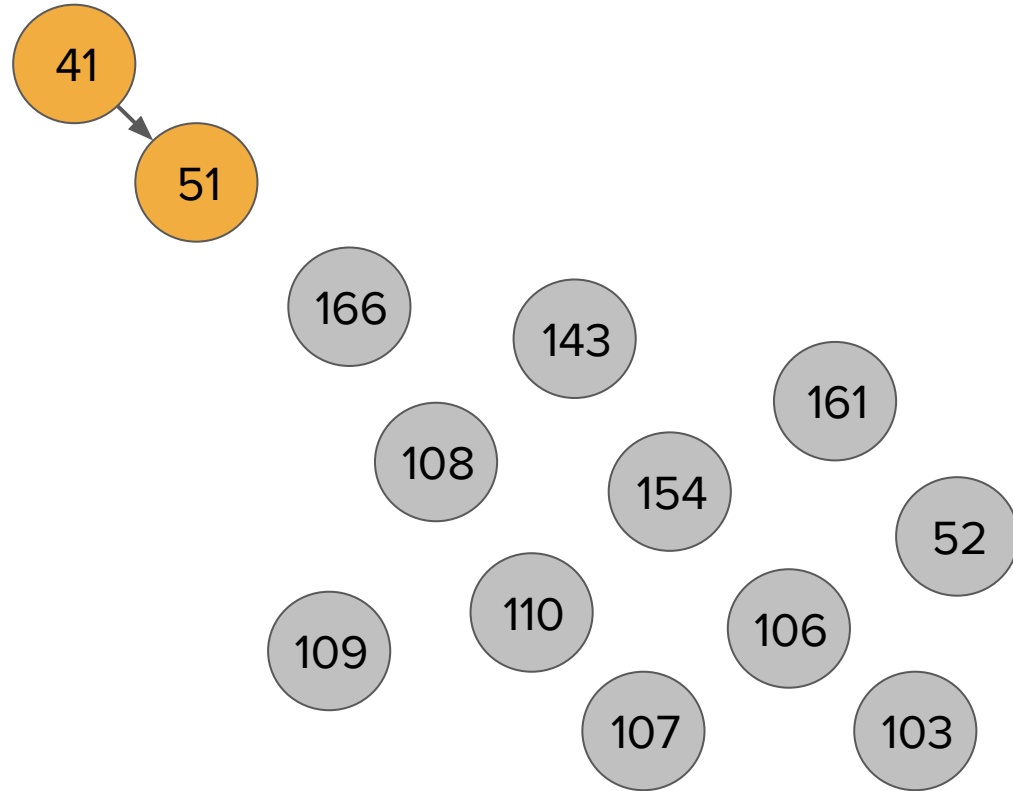
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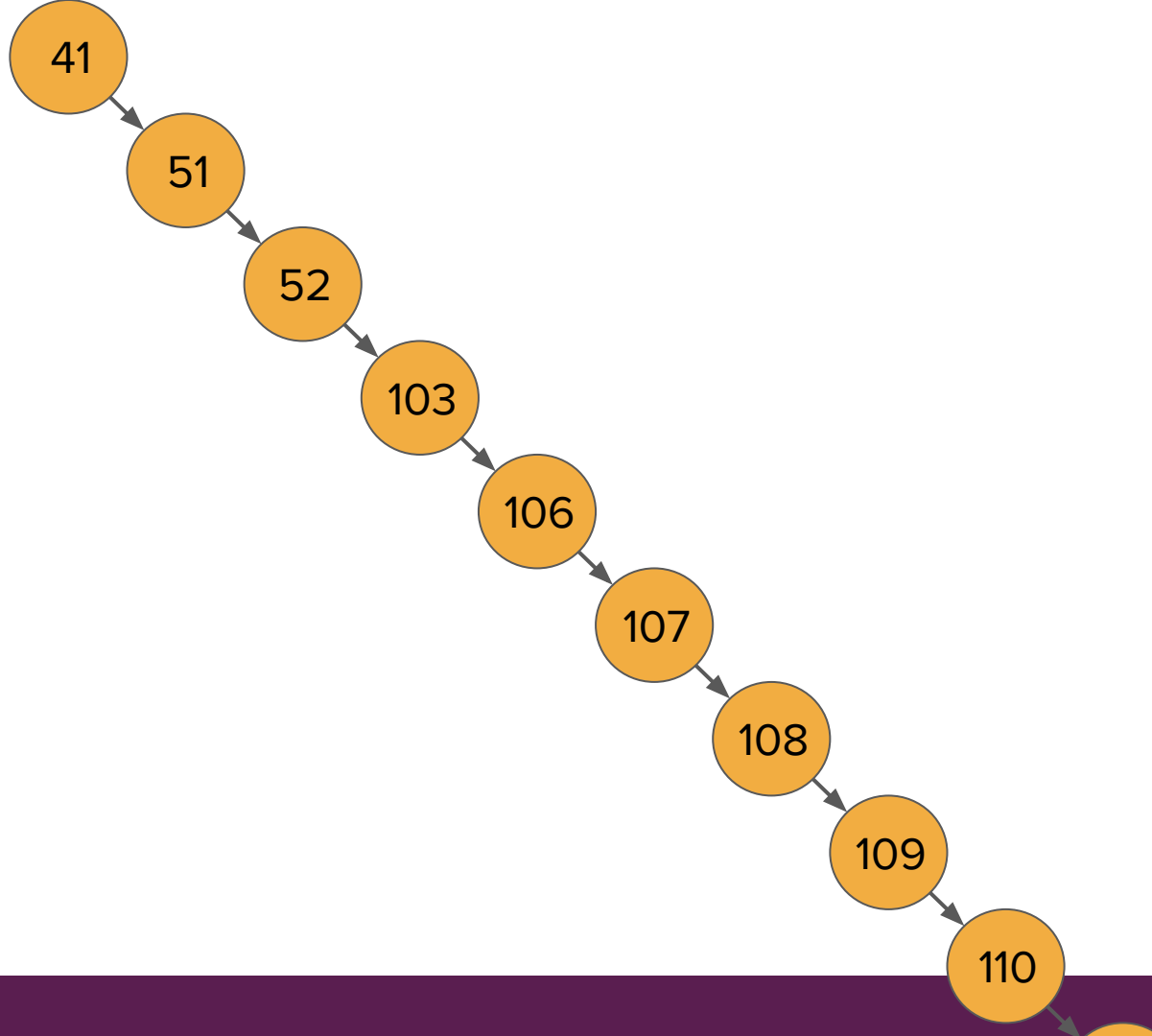


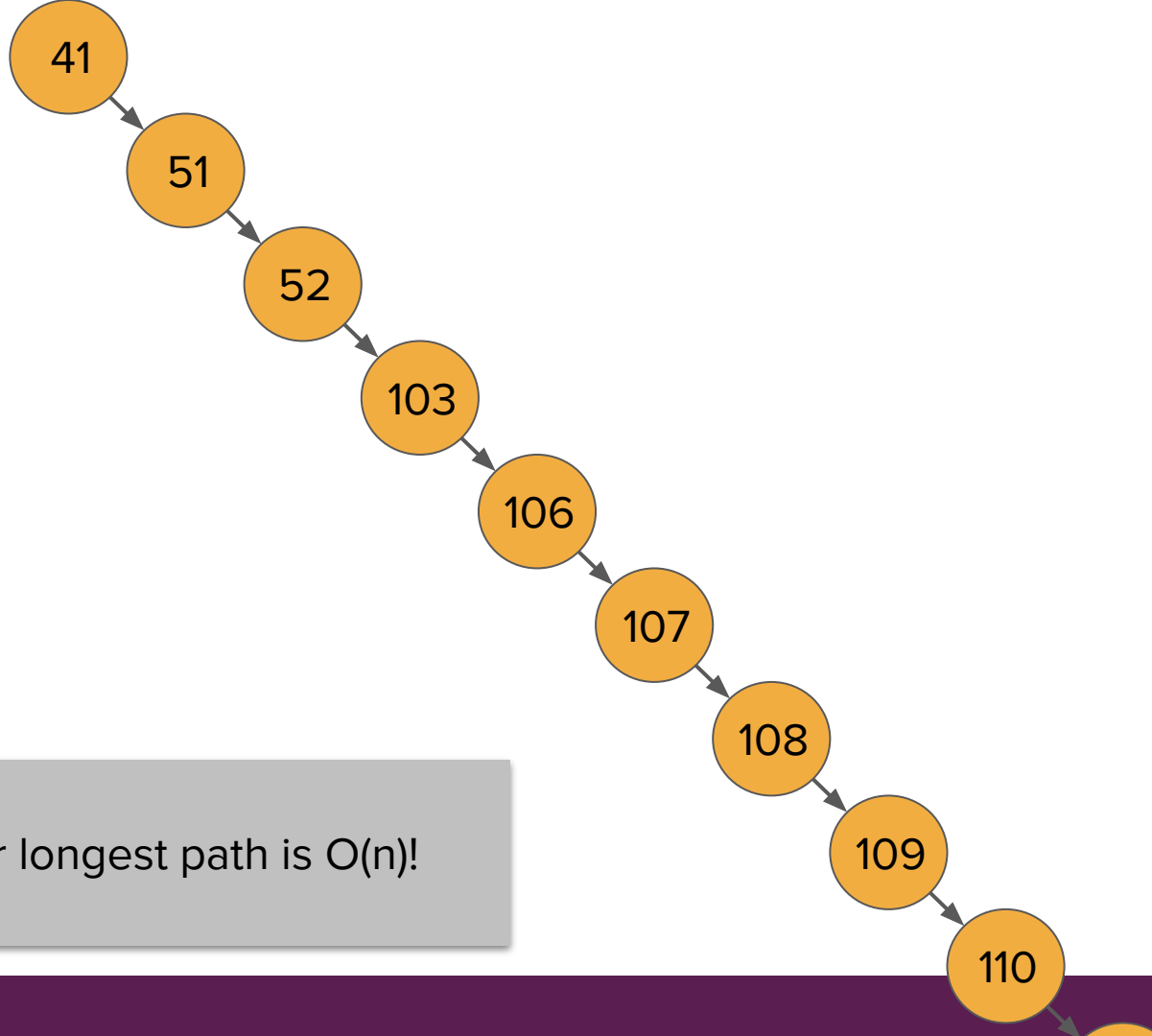
Let's choose the smallest element instead...



Let's choose the smallest element instead...







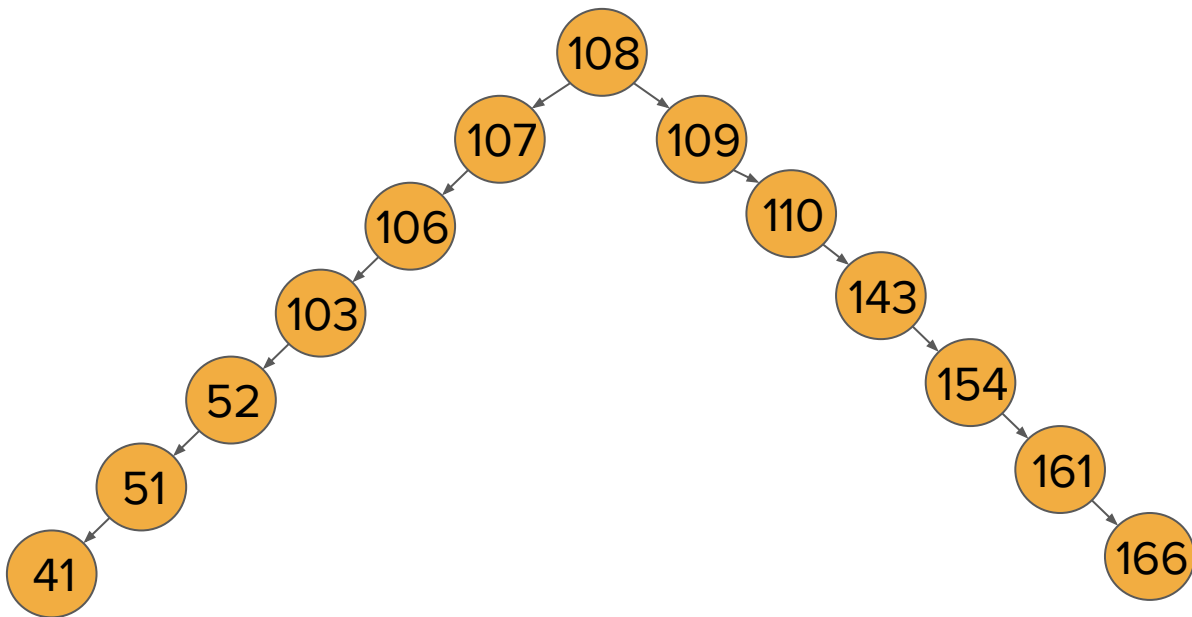
Now our longest path is $O(n)$!

Takeaways

- There are multiple valid BSTs for the same set of data.

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 - Another example with the previous dataset:



Takeaways

- There are multiple valid BSTs for the same set of data.
- How you construct the tree/the order in which you add the elements to the tree matters!

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- There are multiple valid BSTs for the same set of data.
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- A binary search tree is **balanced** if its height is $O(\log n)$, where n is the number of nodes in the tree (i.e. left/right subtrees don't differ in height by more than 1).
 - Lookup, insertion, and deletion with balanced BSTs all operate in $O(\log n)$ runtime.

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 - Lookup, insertion, and deletion with balanced BSTs all operate in $O(\log n)$ runtime.
 - **Theorem:** If you start with an empty tree and add in random values, then with high probability the tree is balanced. → take CS161 to learn why!

Takeaways

- There are multiple valid BSTs for the same set of data.
- How you construct the tree/the order in which you add the elements to the tree matters!
- A binary search tree is **balanced** if its height is $O(\log n)$, where n is the number of nodes in the tree (i.e. left/right subtrees don't differ in height by more than 1).
 - Lookup, insertion, and deletion with balanced BSTs all operate in $O(\log n)$ runtime.
 - **Theorem:** If you start with an empty tree and add in random values, then with high probability the tree is balanced. → take CS161 to learn why!
 - A **self-balancing** BST reshapes itself on insertions and deletions to stay balanced (how to do this is beyond the scope of this class).

Announcements

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- Assignment 5 is due on **tonight at 11:59pm PDT**.
- Assignment 6 will be released by the end of the day tomorrow and will be due on **Wednesday, August 12 at 11:59pm PDT**. This is a hard deadline – there is **no grace period and no submissions will be accepted after this time**.
- Due to the end of quarter timeline, there will be **no revisions on Assignments 5 and 6**.
- Final project reports are due on **Sunday, August 9 at 11:59pm PDT**. You will have the opportunity to schedule your final presentation time after submitting.

Implementing Sets with BSTs

We're going to implement a Set using a BST!

- Our Set will only store strings as its data type.

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```
struct TreeNode {  
    std::string data;  
    TreeNode* left;  
    TreeNode* right;  
  
    // default constructor does not initialize  
    TreeNode() {}  
    // 3-arg constructor sets fields from arguments  
    TreeNode(std::string d, TreeNode* l, TreeNode* r) {  
        data = d;  
        left = l;  
        right = r;  
    }  
};
```

We're going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.

OurSet Public Interface

```
class OurSet {  
public:  
    OurSet();    // constructor  
    ~OurSet();   // destructor  
  
    bool contains(string value);  
    void add(string value);  
    void remove(string value);  
    void clear();  
    int size();  
    bool isEmpty();  
    void printSetContents();  
  
private:  
    /* To be defined soon! */  
};
```


We're going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.
- As we write the Set methods, think about how their runtimes would change for a balanced vs. an unbalanced BST.
 - Note: Actual sets are self-balancing, but we won't go into the details of how to implement that!

How do we design **OurSet**?

We must answer the following three questions:

1. Member functions: *What public interface should **OurSet** support? What functions might a client want to call?*
2. Member variables: *What private information will we need to store in order to keep track of the data stored in **OurSet**?*
3. Constructor: *How are the member variables initialized when a new instance of **OurSet** is created?*

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```

Let's code it!

(constructor, destructor, **clear()**, etc.)

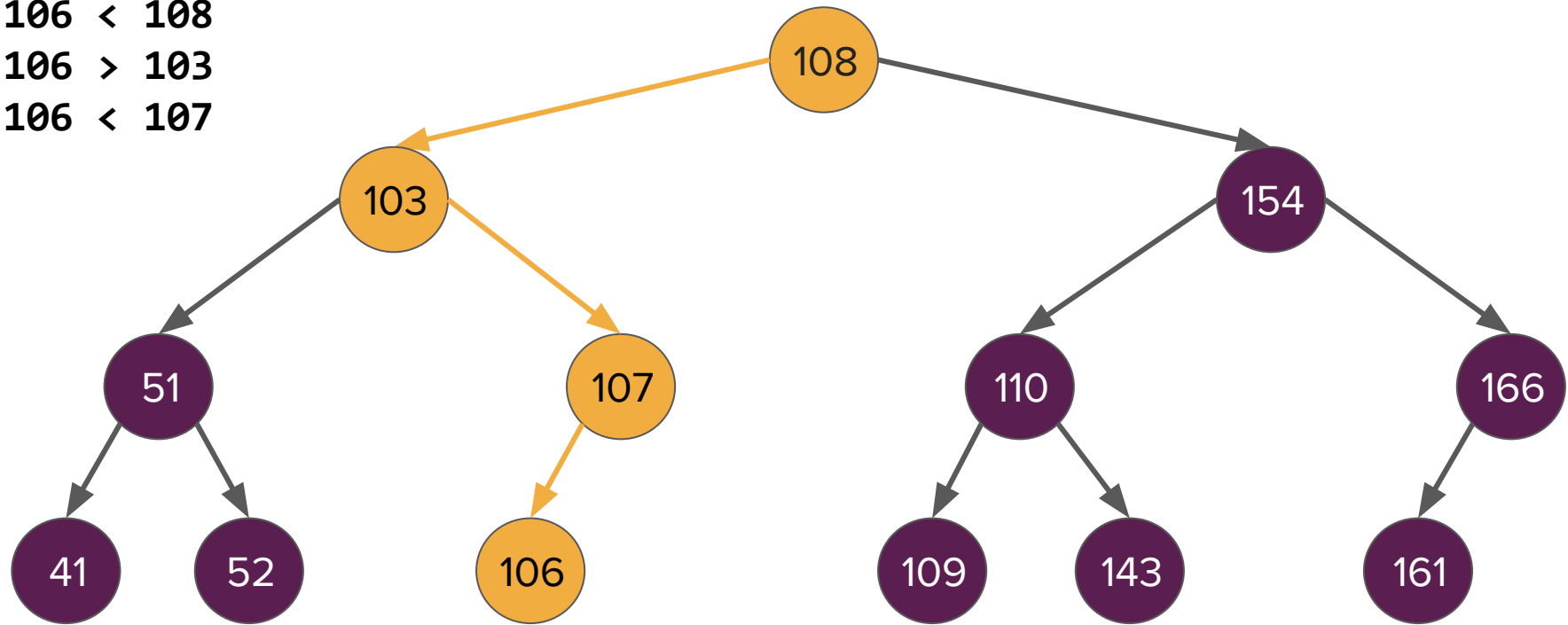
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private:  
    /* To be defined soon! */  
};
```

$106 < 108$

$106 > 103$

$106 < 107$



We found **106** so we're done!

Let's code it!

`(contains(), add())`

OurSet summary

- Our tree utility functions (**inorderPrint**, **freeTree**) showed up as private member functions/helpers!
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OurSet summary

- Our tree utility functions (**inorderPrint**, **freeTree**) showed up as private member functions/helpers!
 - In-order traversal prints our elements in the correctly sorted order!
- Using a BST allowed us to take advantage of recursion to traverse our data and get an $O(\log n)$ runtime for our methods.
- Rewiring trees can be complicated!
 - Make sure to consider when nodes need to be passed by reference.
 - Check out the remove method after class if you're interested in seeing an example of tree rewiring (you won't be required to do anything this complex with tree rewiring).

What's next?

Roadmap

C++ basics

User/client

vectors + grids

stacks + queues

sets + maps

Core
Tools

testing

algorithmic
analysis

recursive
problem-solving

Object-Oriented
Programming

Implementation

arrays

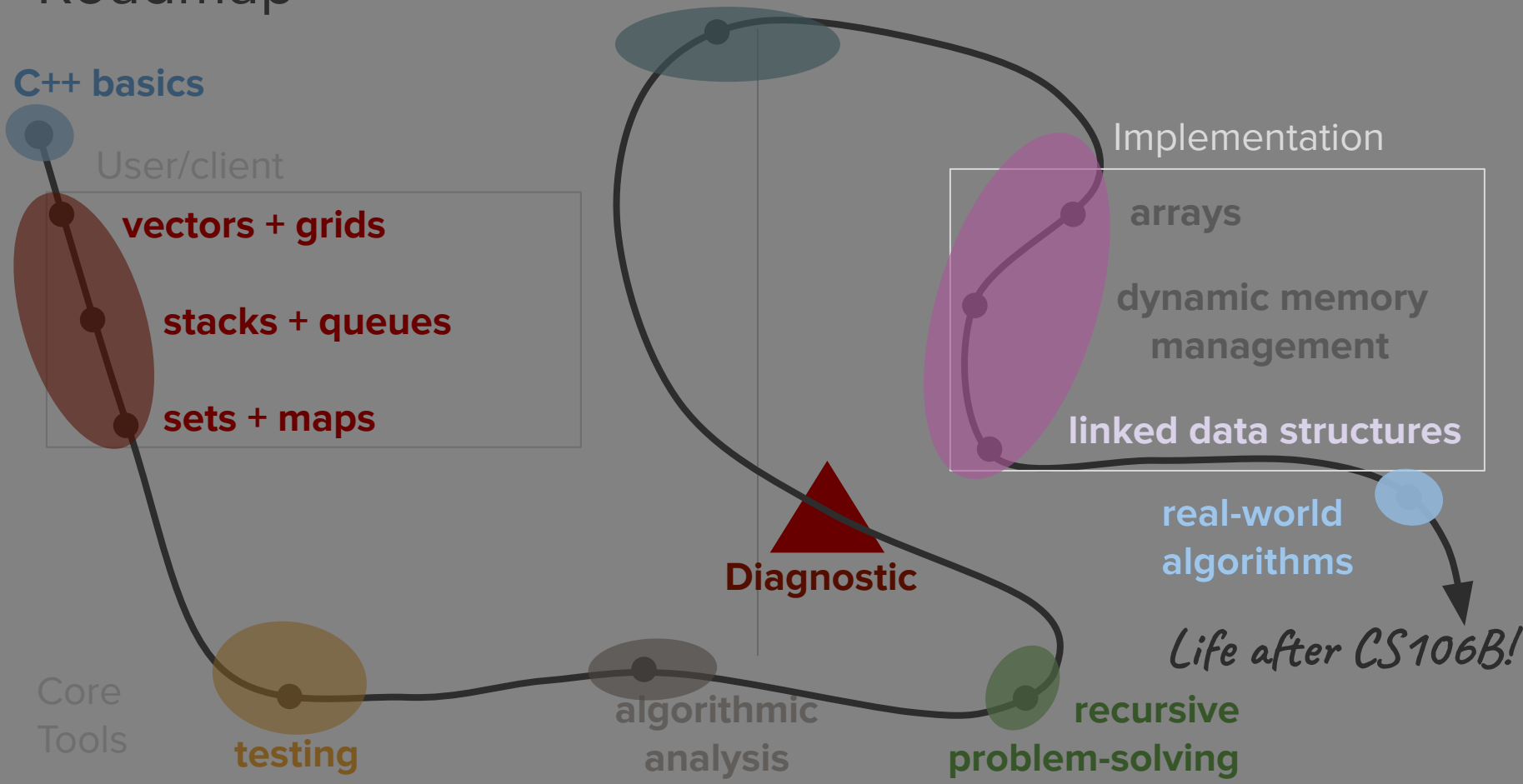
dynamic memory
management

linked data structures

real-world
algorithms

Life after CS106B!

Diagnostic



Levels of abstraction

What is the interface for the user?



How is our data organized?
(binary heaps, BSTs, **Huffman trees**)

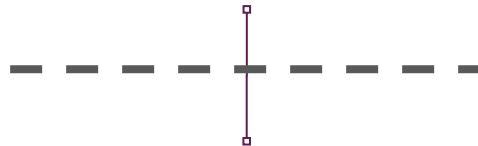


What stores our data?
(arrays, linked lists, **trees**)



How is data represented electronically?
(RAM)

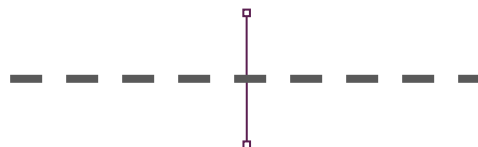
**Abstract Data
Structures**



**Data Organization
Strategies**



**Fundamental C++
Data Storage**



**Computer
Hardware**

Huffman coding

