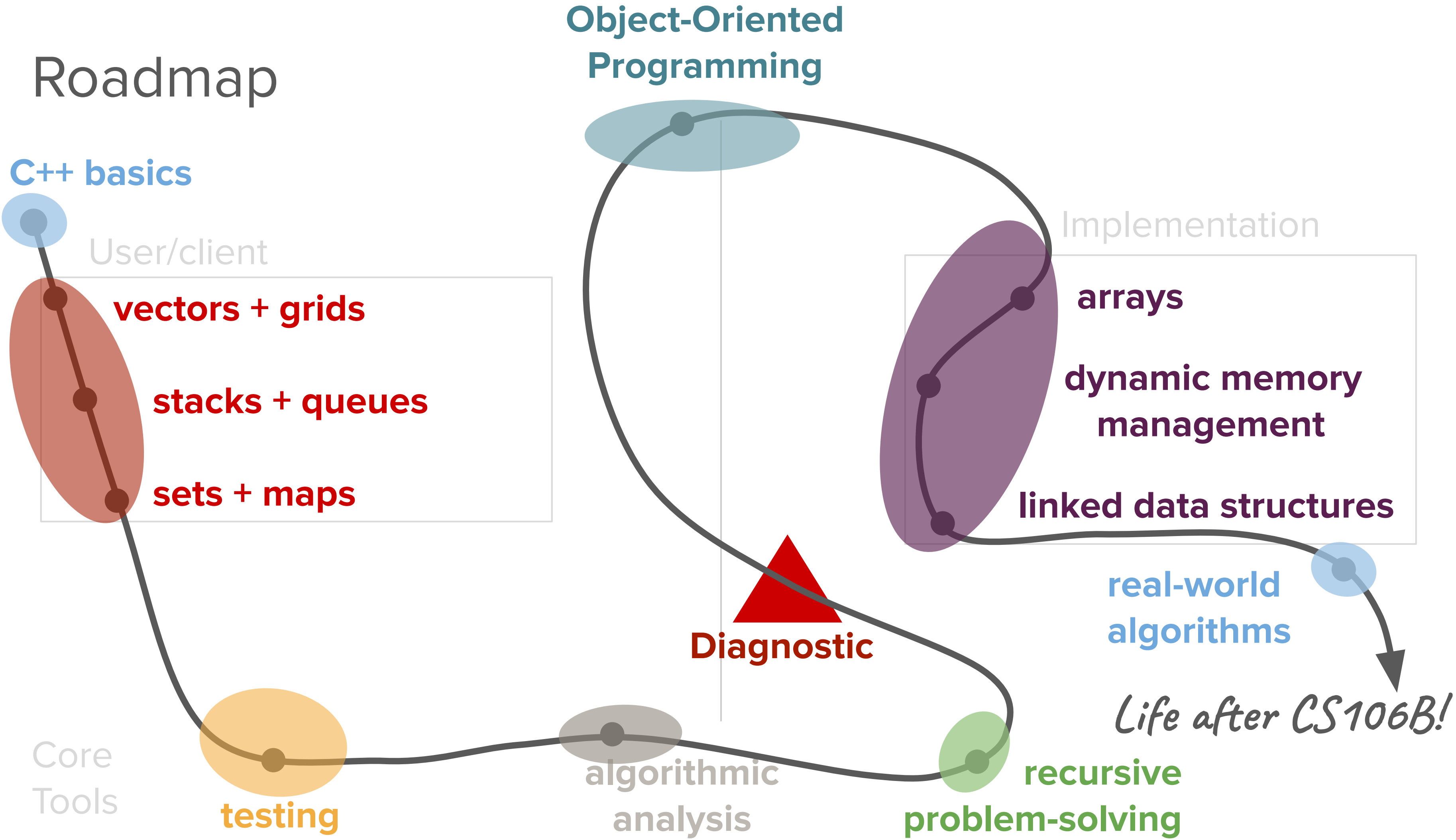
Trees

Is there any component of "Life after CS106B" that you would like us to focus on in our final lecture next week?

(put your answers the chat)

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
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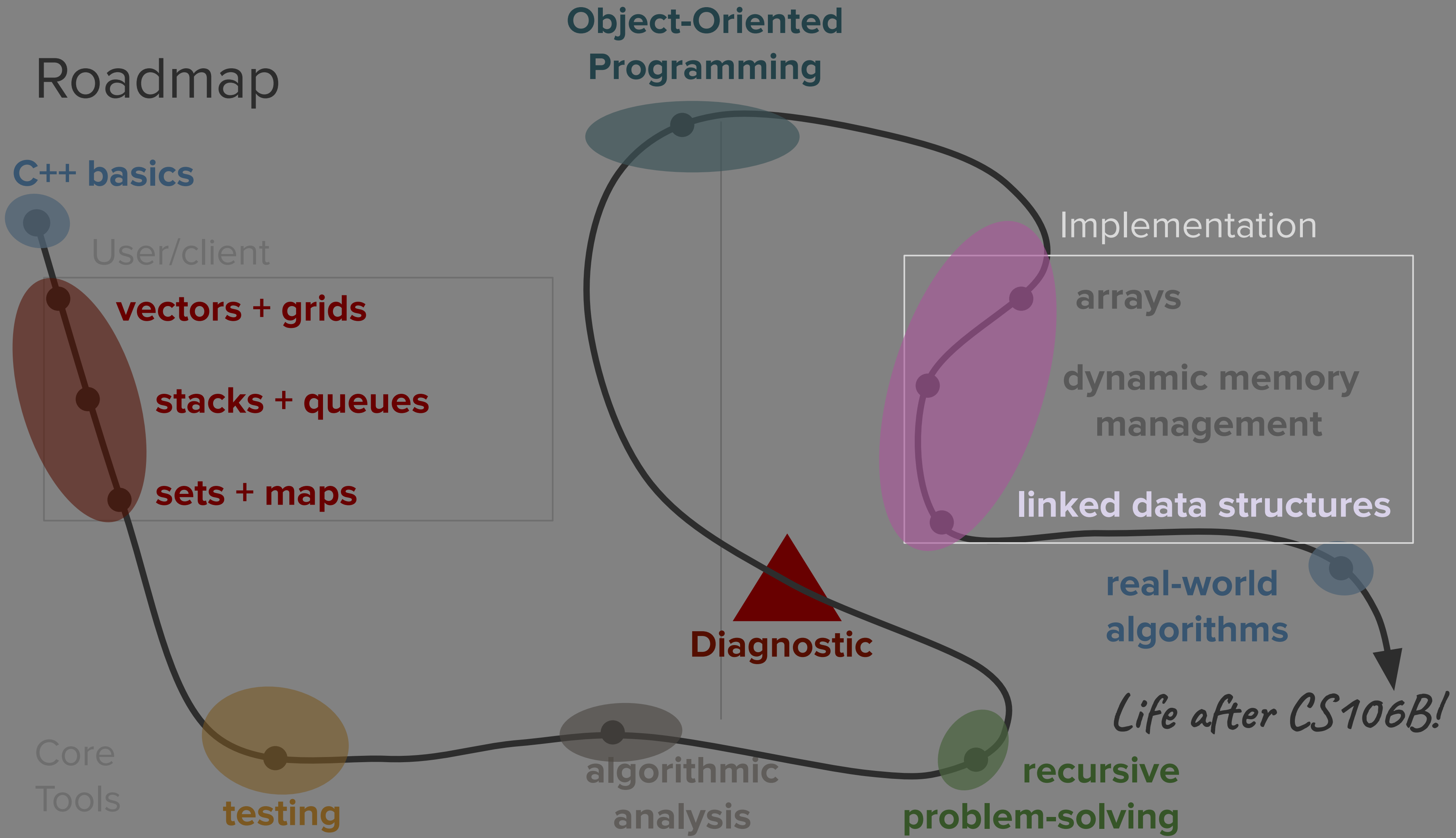

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Today's questions

How can we better organize data stored in a linked data structure?

Today's topics

1. Linked Data Structure Overview
2. Introduction to Trees
3. Trees in C++

Review

[linked data structures]

Linked Data Structures

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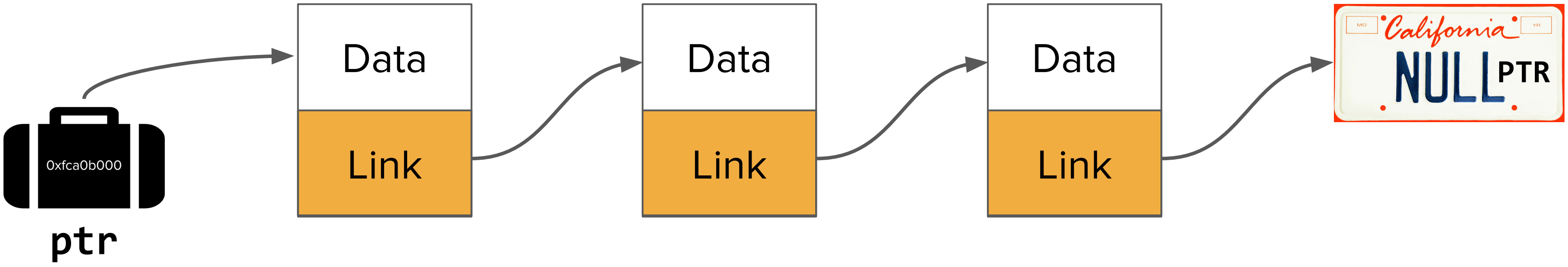

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- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."

- Using pointers allows us to **create links** to other nodes to impose structure.

Linked List Tradeoffs

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  - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room.
  - Because all the data was stored in dynamic memory, expanding the size of the linked list was very easy and never required an expensive "re-sizing" operation that had to copy all the data.

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  - Finding elements in a linked list is an **`O(n)`** operation, which can get slow when we want to store many elements.
  - We couldn't feasibly write recursive algorithms that traversed linked lists, due to stack frame limits that came into play since traversal algorithms required one stack frame per node.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

- However, we also ran into some limitations when it came to working with lists.

- **Question:** Can we organize data in a linked data structure in such a way that the path between the "front" and any element in the structure is short (better than `O(n)`) even if there are many elements?

How can we better organize data stored in a linked data structure?

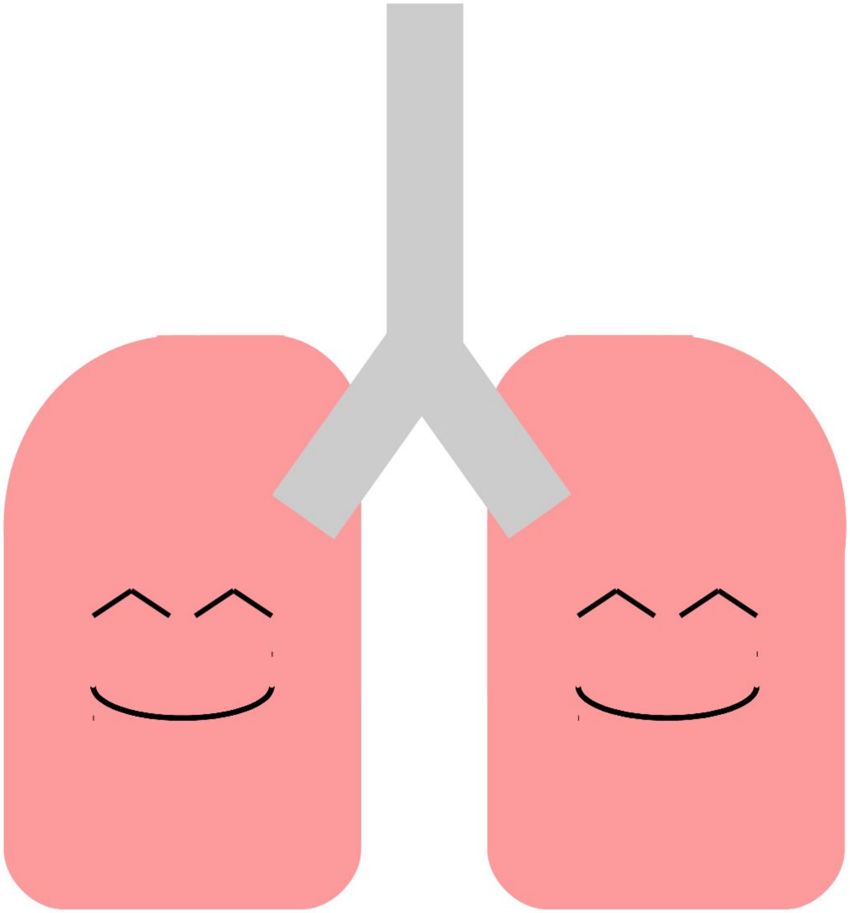
Interactive Exercise

[borrowed from Keith Schwarz]

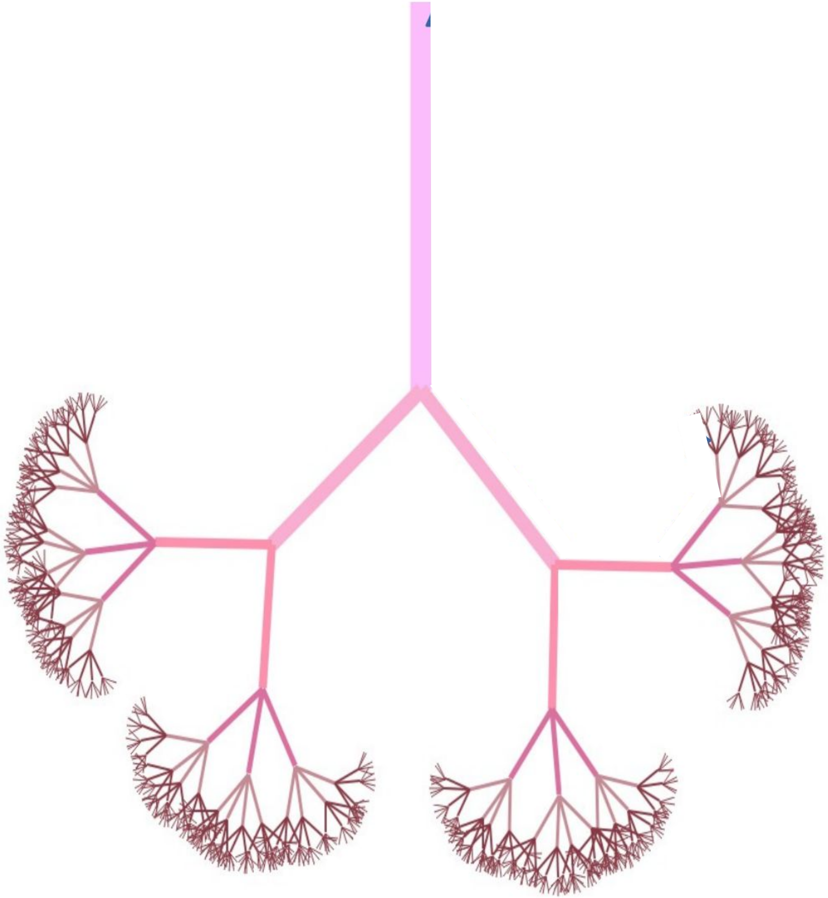
Take a deep breath.

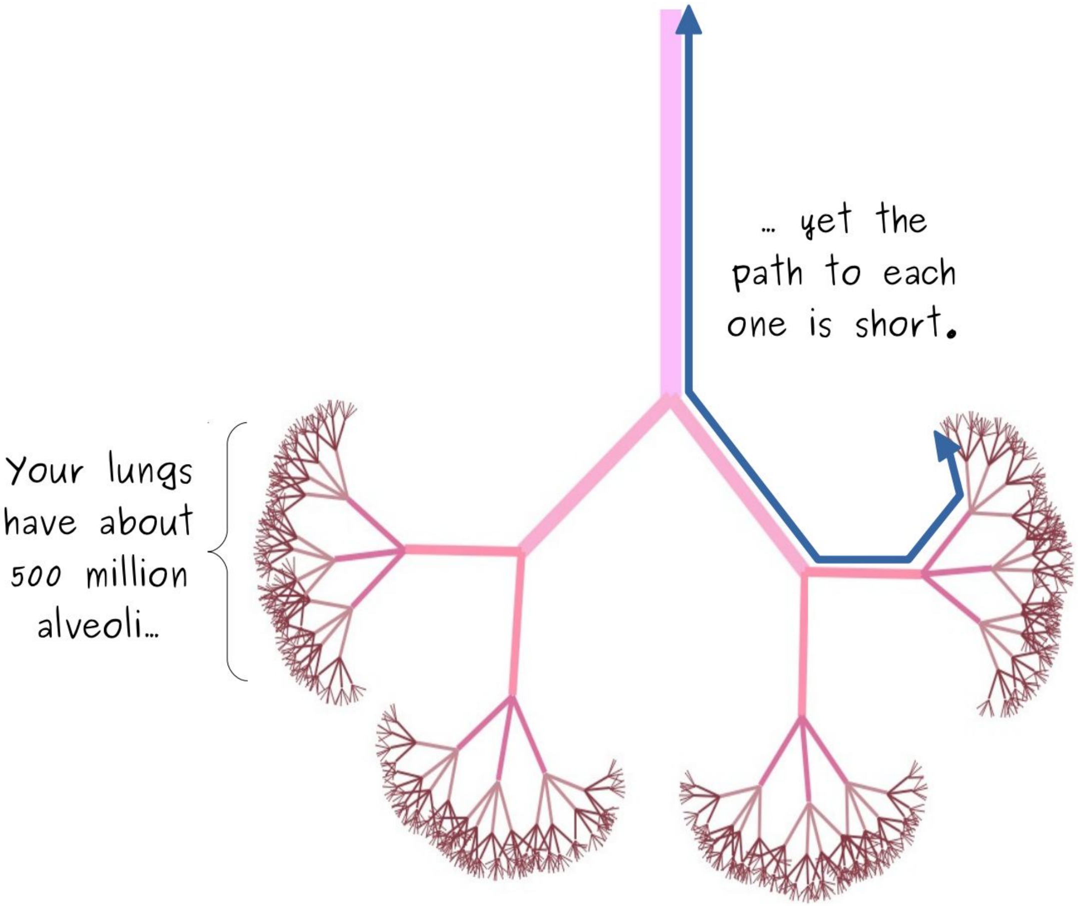
And exhale...

Feel nicely oxygenated?



Your lungs have about 500 million alveoli…





... yet the path to each one is short.
Your lungs have about 500 million alveoli...

Key Idea: The distance from each element in this structure to the top of the structure is small, even if there are many elements.

Trees

Throwback Thursday (on Monday)

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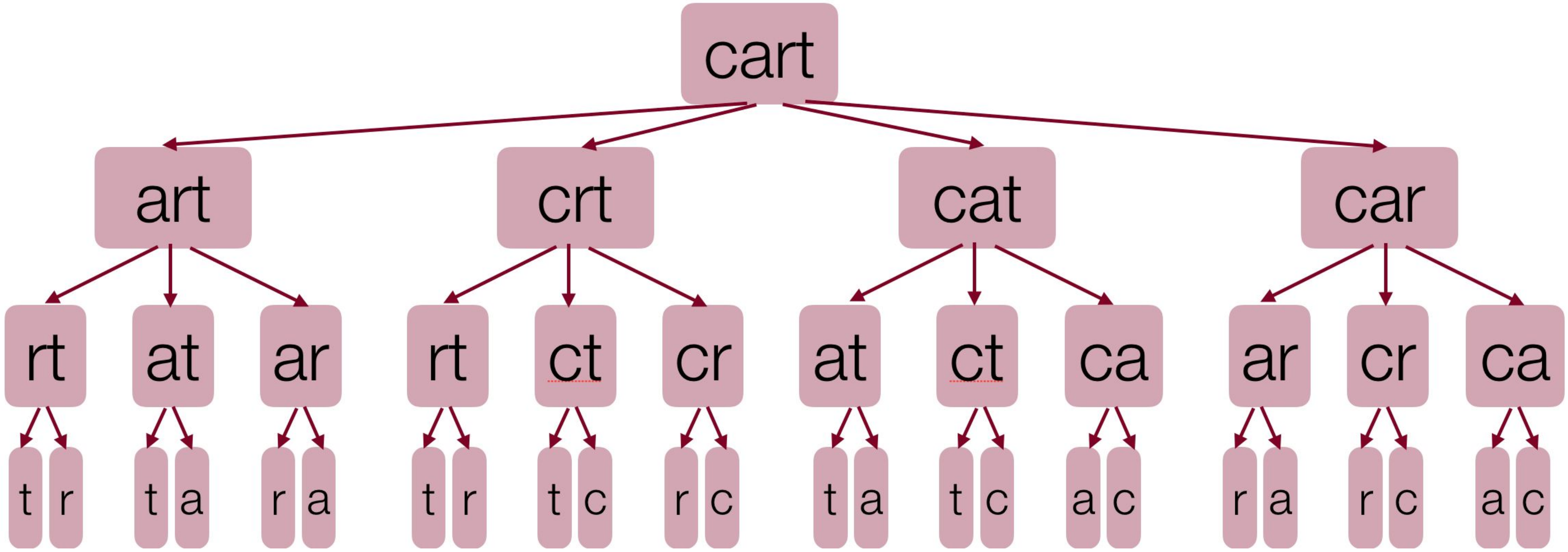

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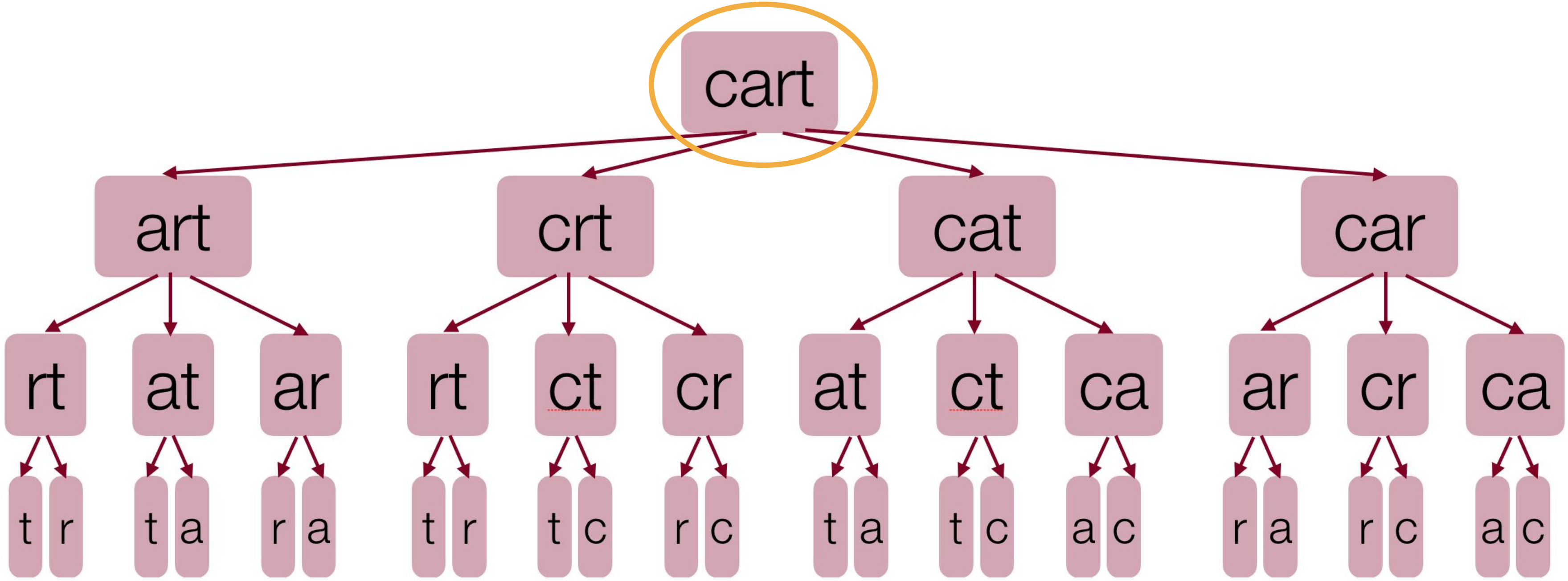

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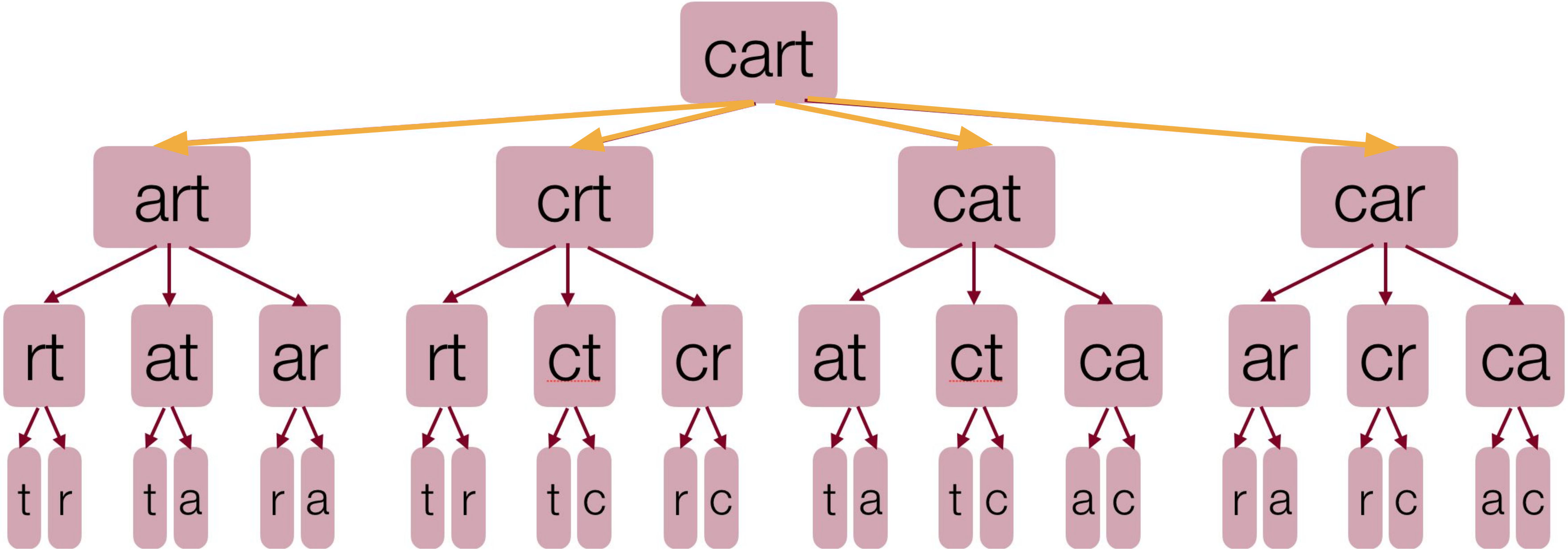

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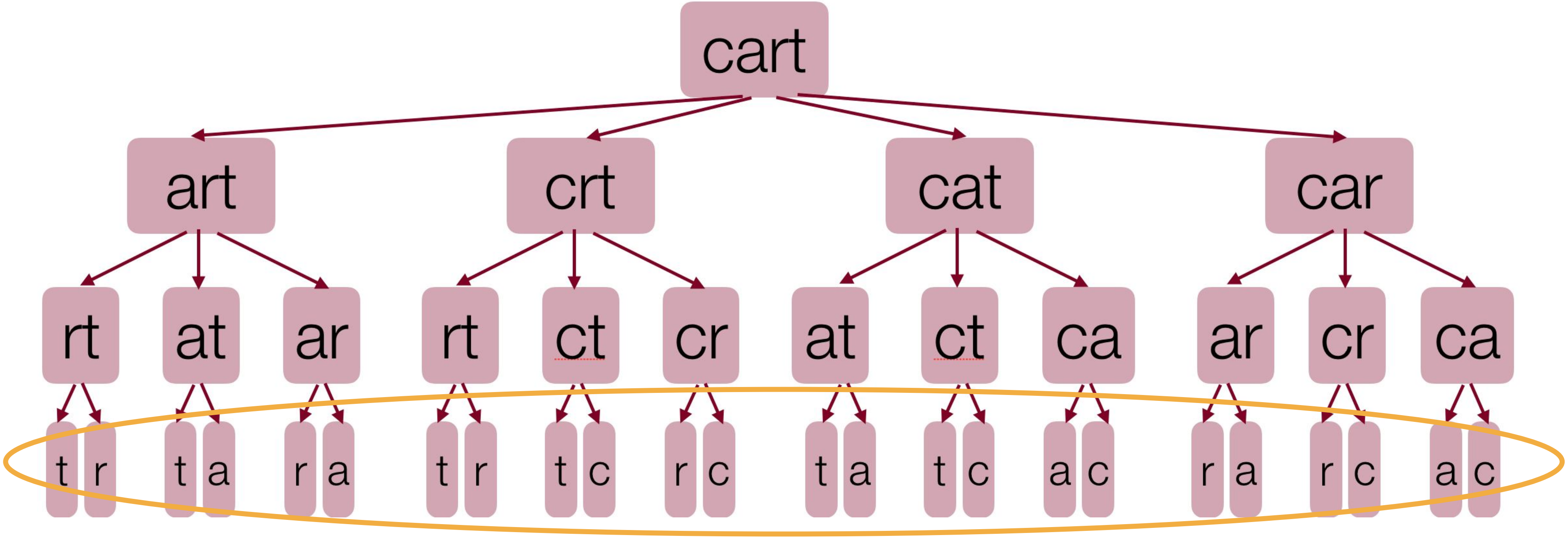

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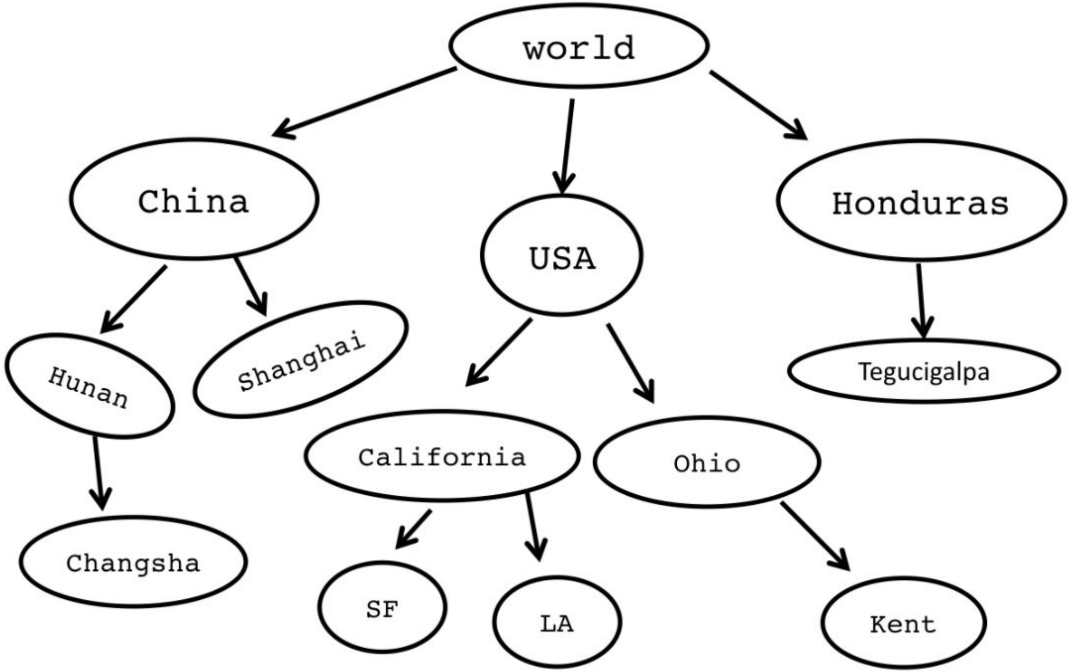

Trees can be used to describe *hierarchies*.

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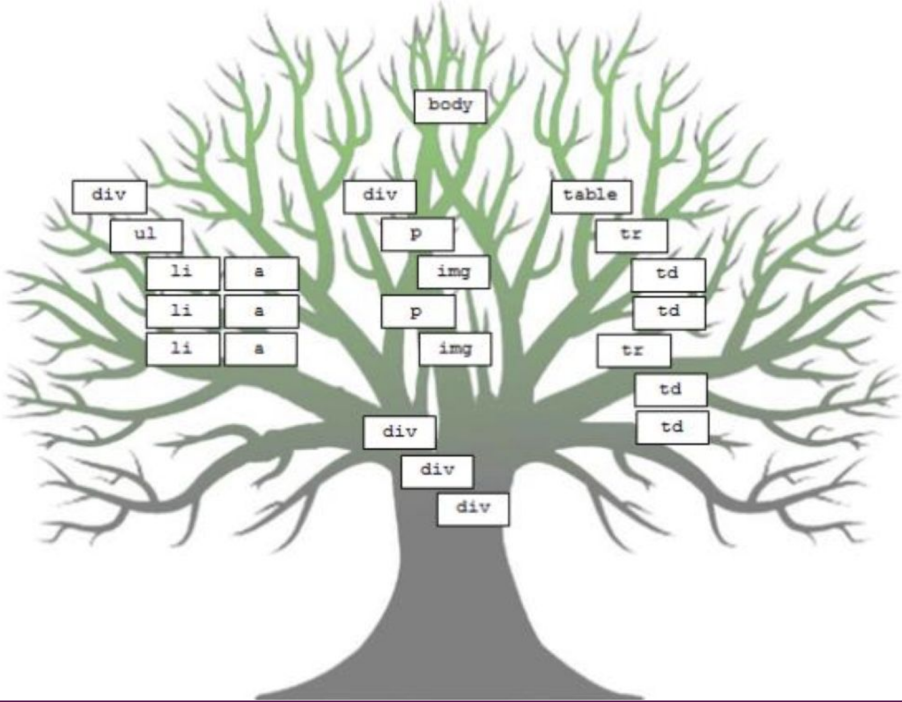
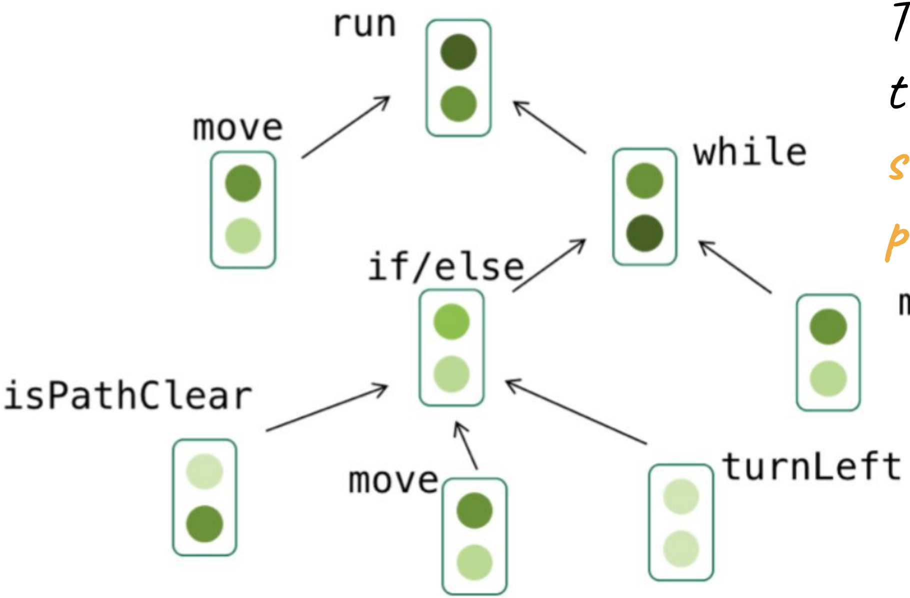


Trees are used to model the structure of websites.

Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

```
def run() {
  move();
  while (notFinished()) {
    if (isPathClear()) {
      move();
    } else {
      turnLeft();
    }
    move();
  }
}
```



Trees describe the syntax structure of programs.

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- But, it is not a coincidence that we first saw them appear in conjunction with recursion.

- Trees are inherently defined recursively!

What is a tree?

A tree is either...

What is a tree?

A tree is either...

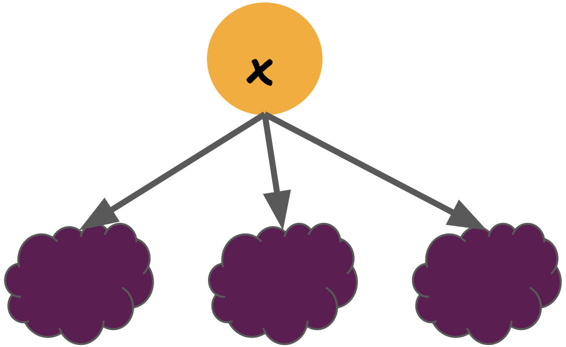
An empty data structure, or...

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



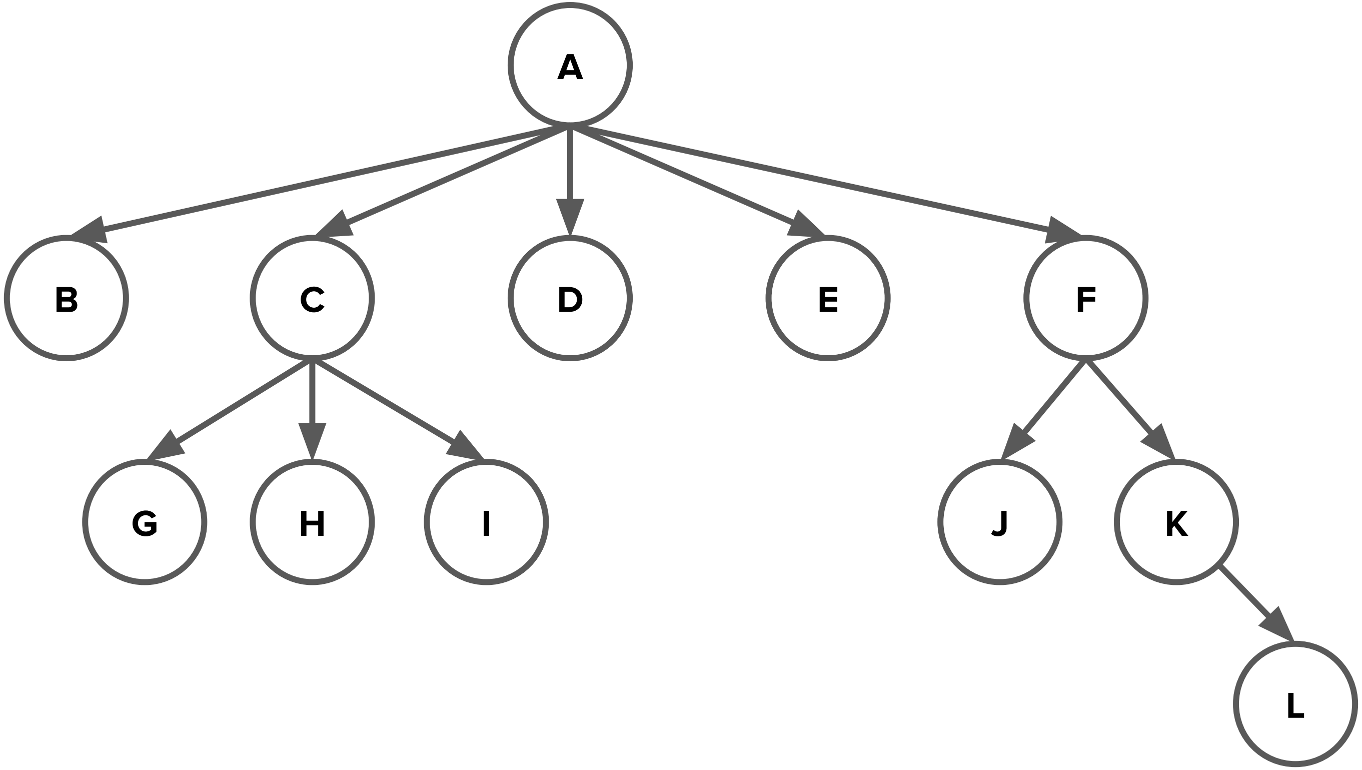
Definition

tree

A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

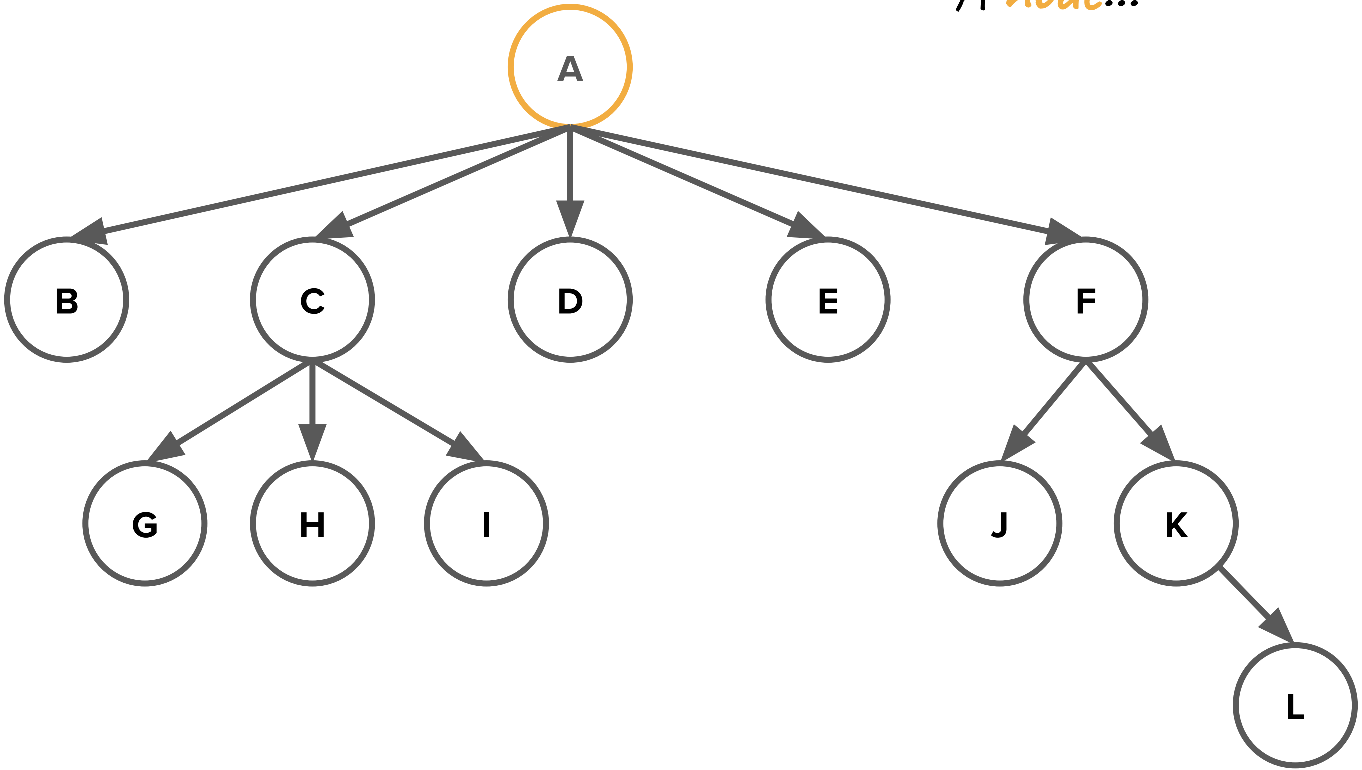
Tree Terminology

Tree Terminology



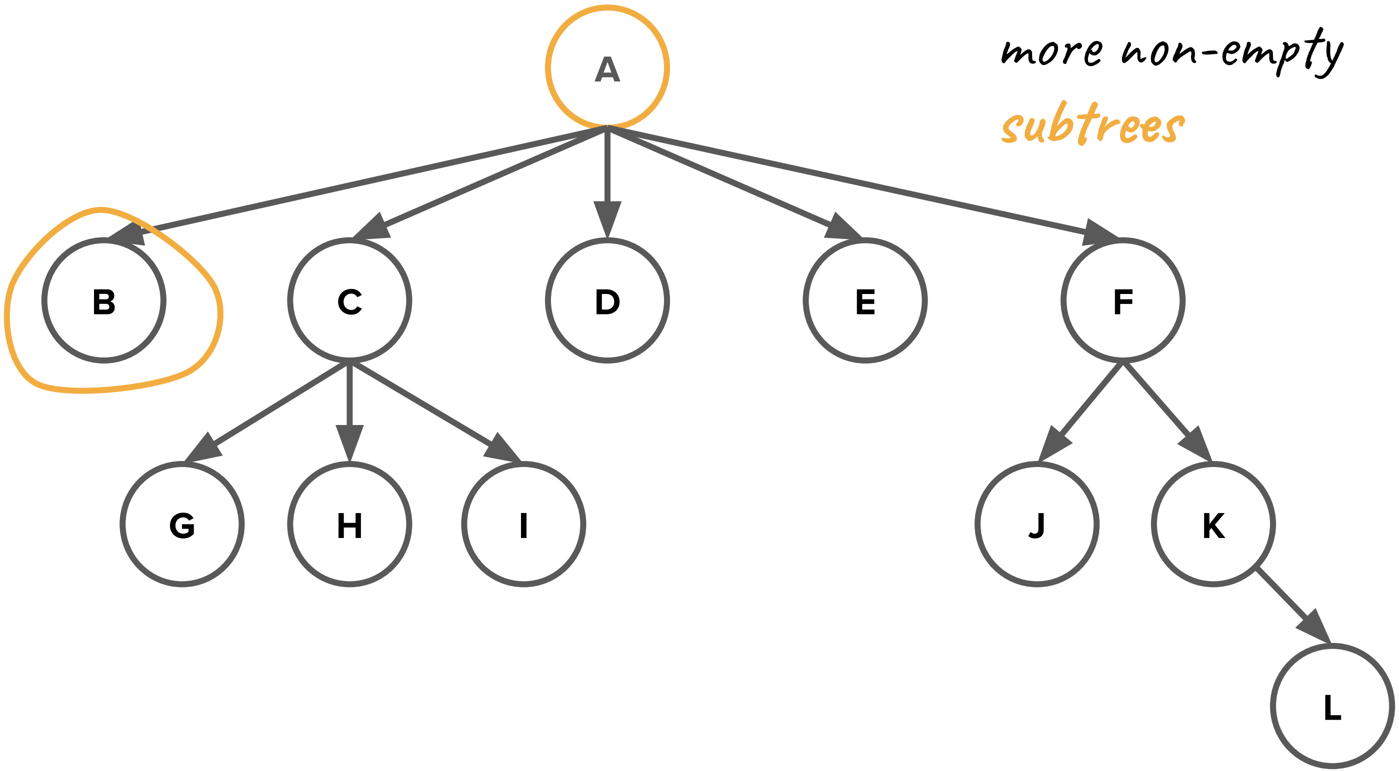
Tree Terminology

A node...



Tree Terminology

A node with 0 or more non-empty subtrees



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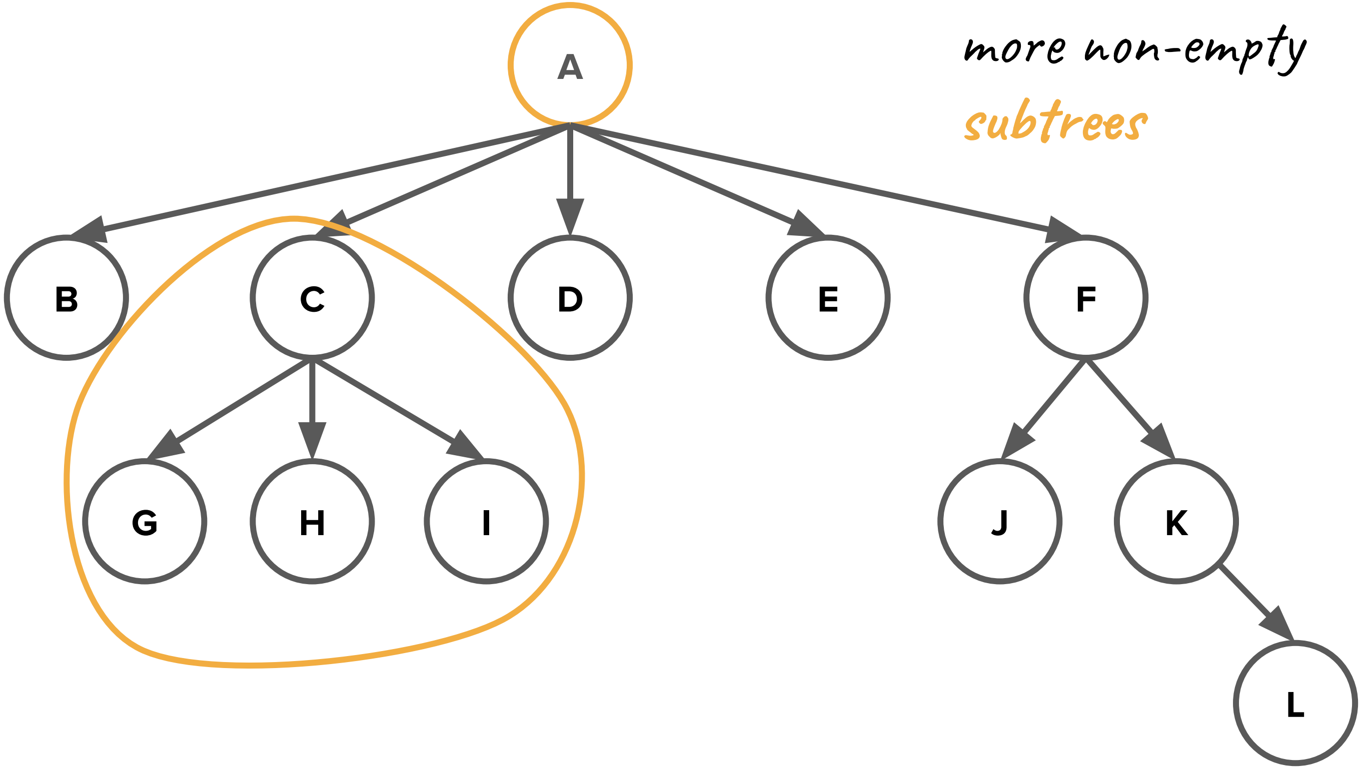

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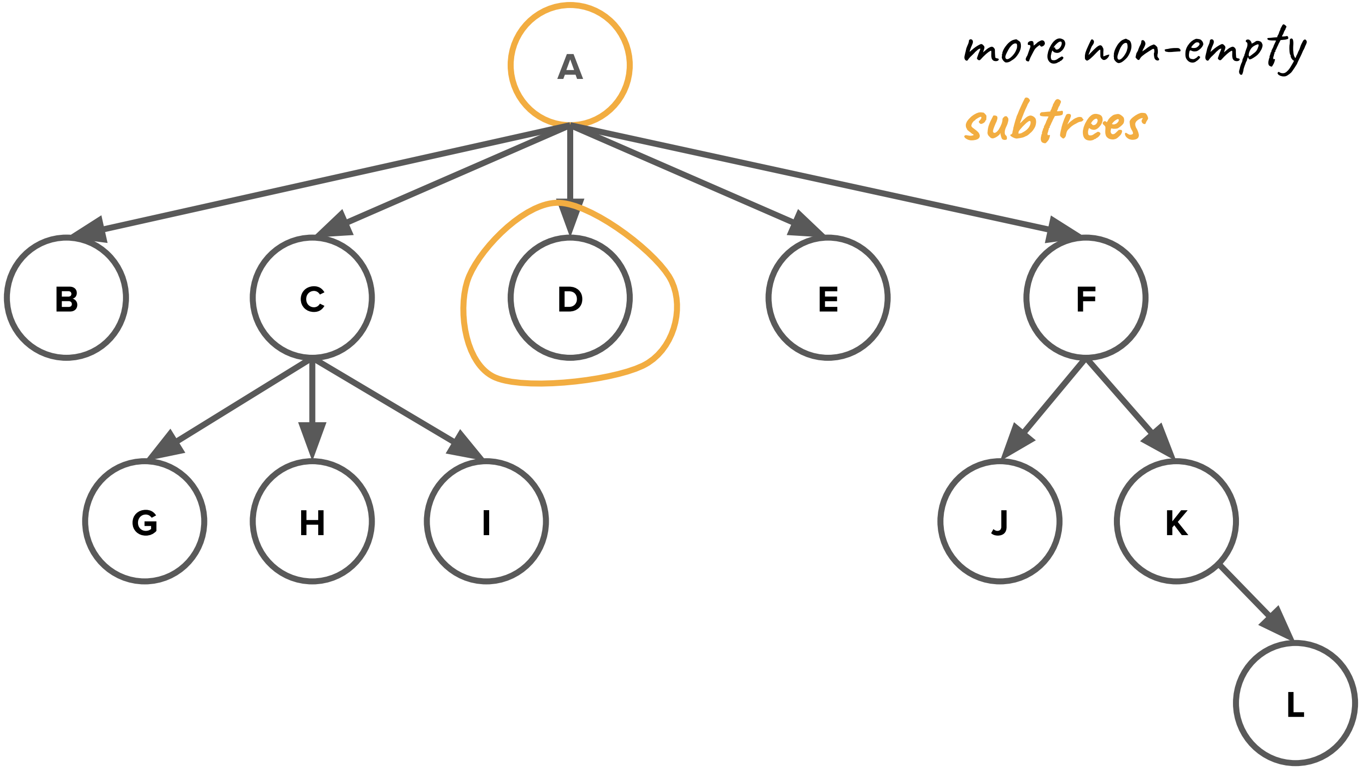
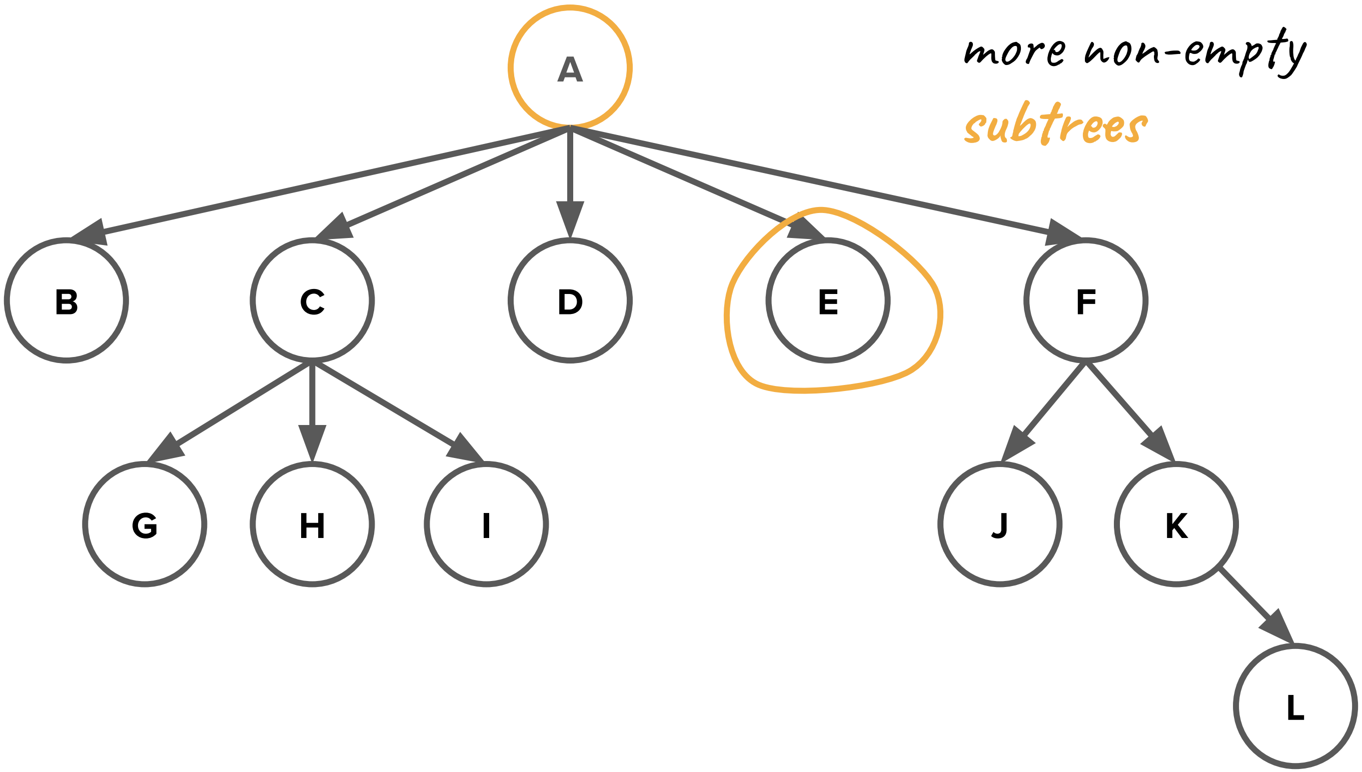


A *node* with 0 or more non-empty *subtrees*

Tree Terminology



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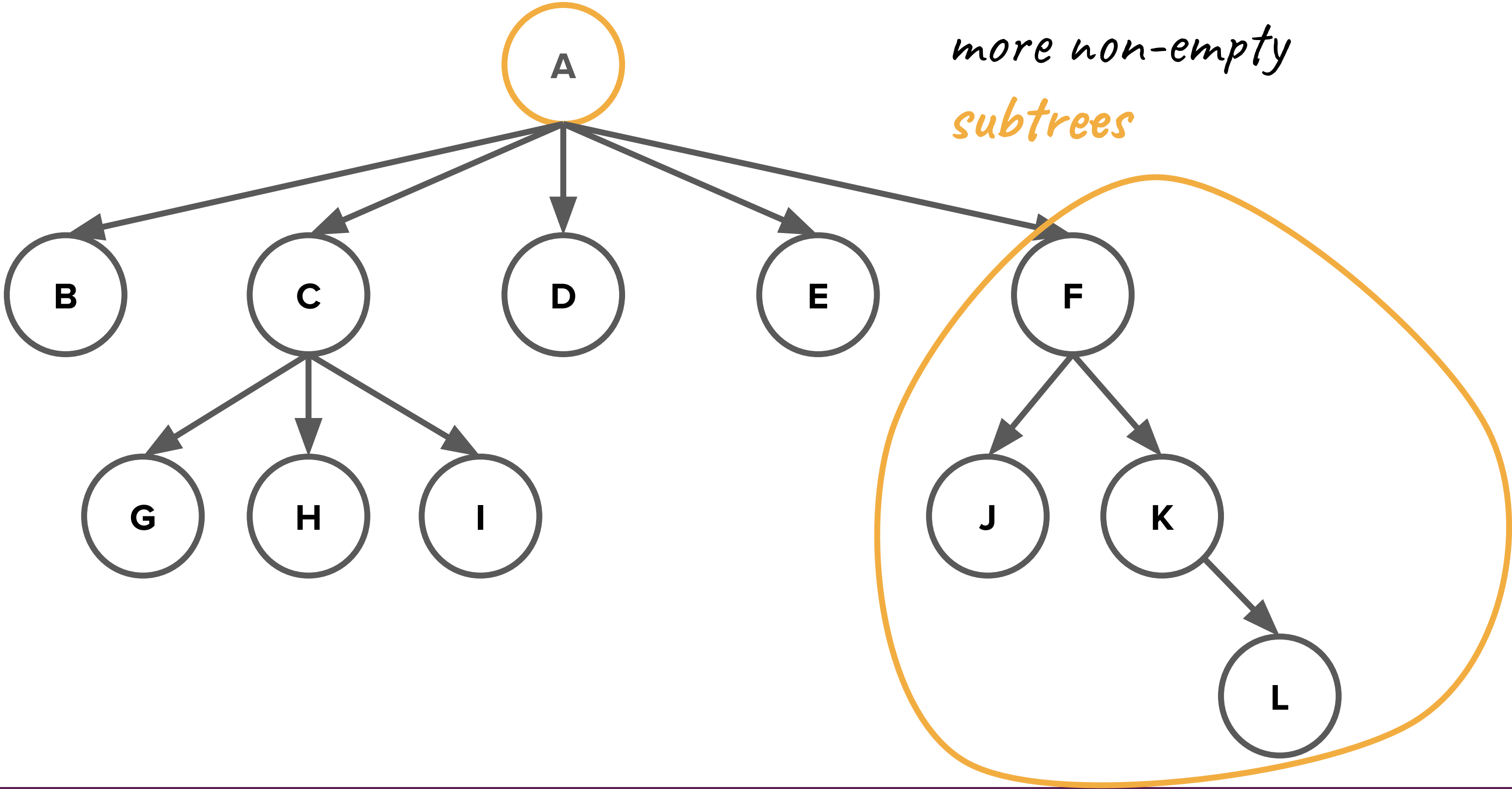

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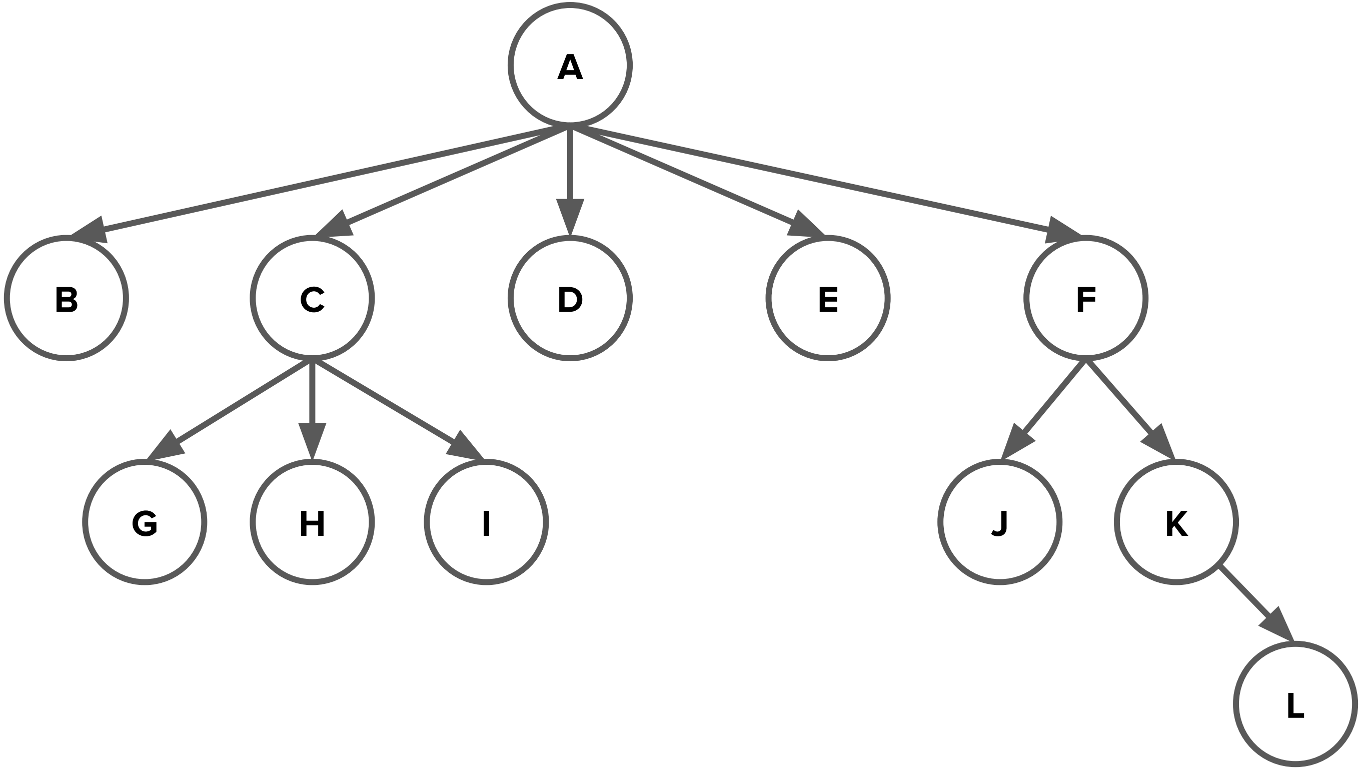
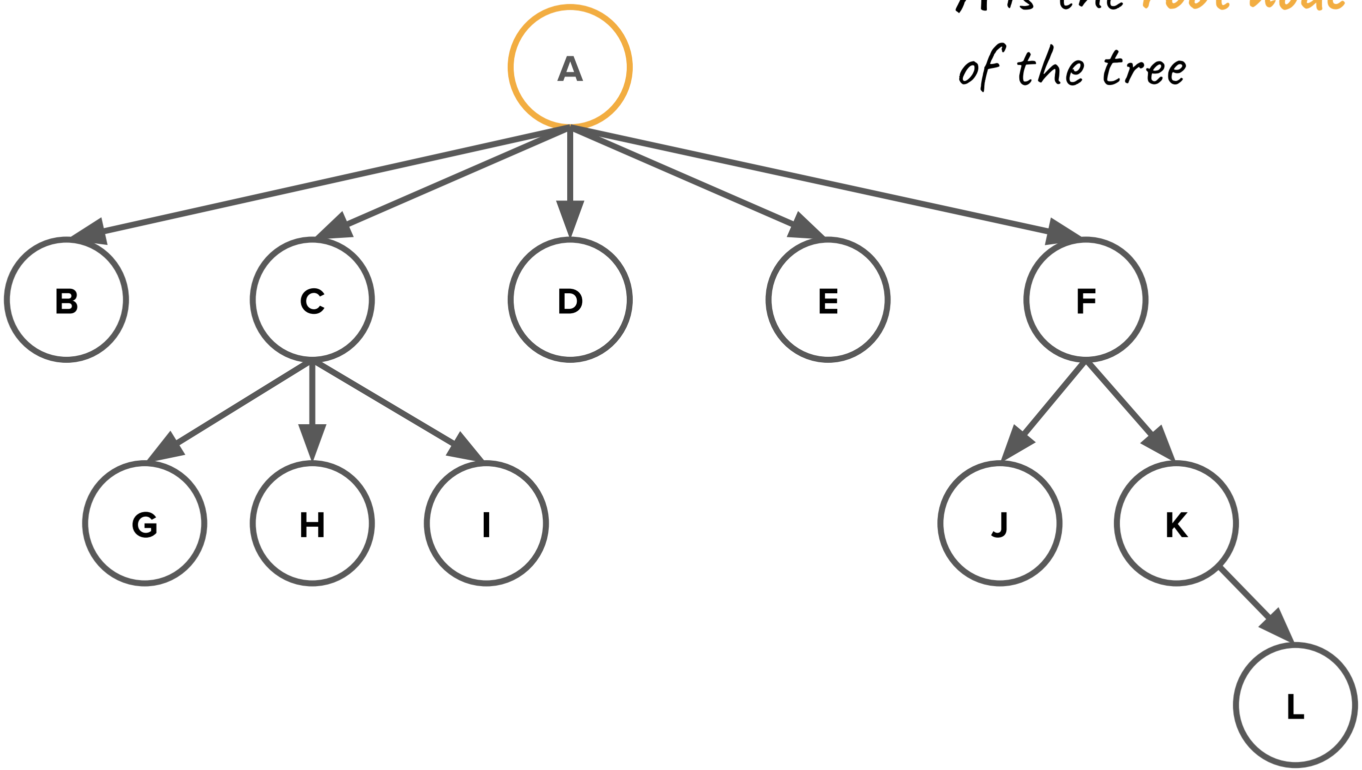
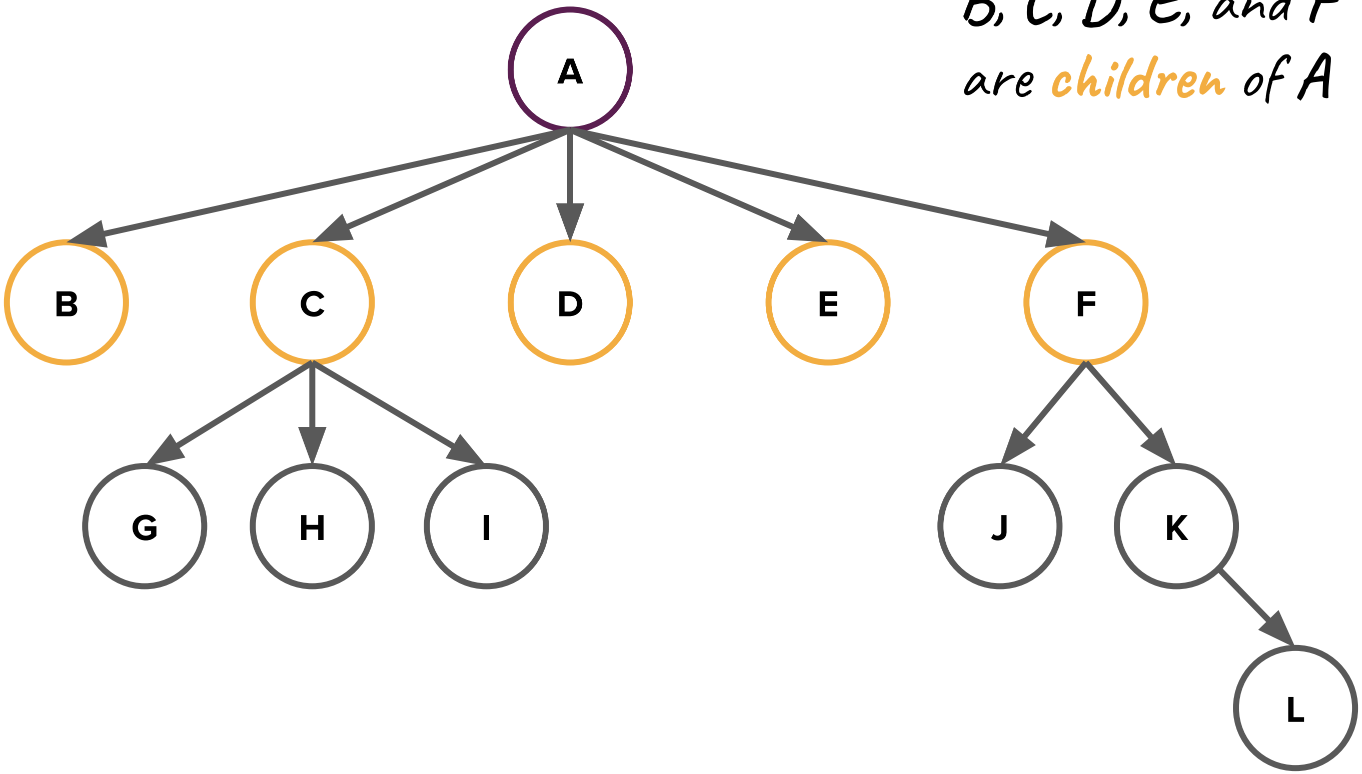


Tree Terminology

A is the root node of the tree

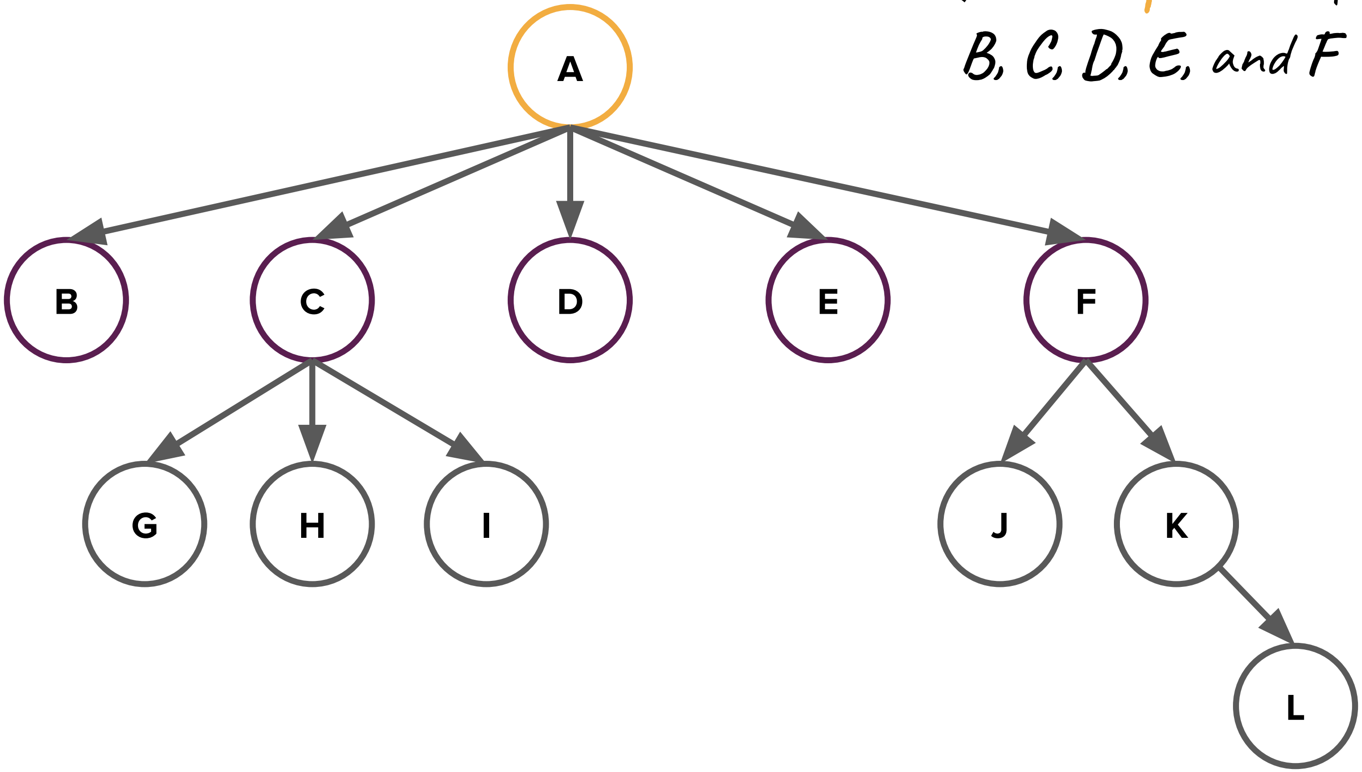


Tree Terminology



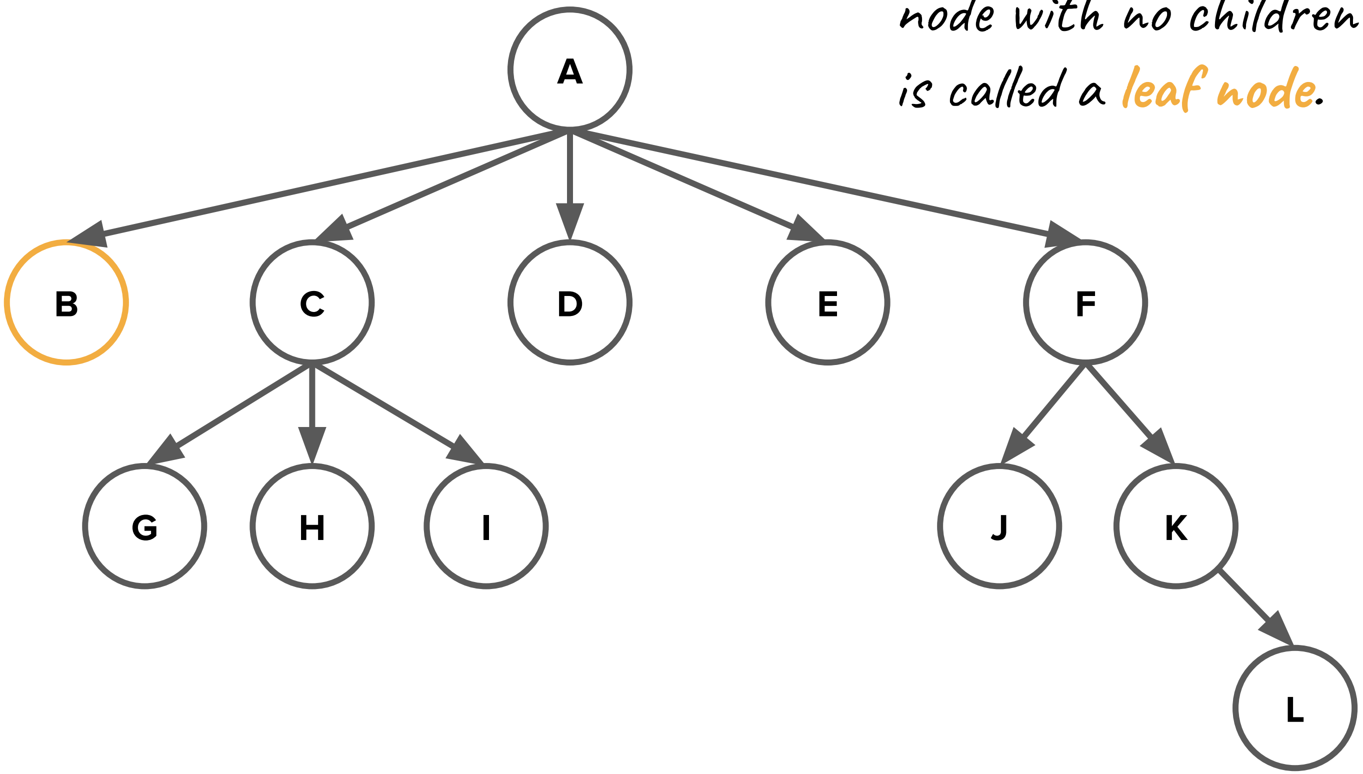
B, C, D, E, and F are *children* of A

Tree Terminology



A is the parent of B, C, D, E, and F

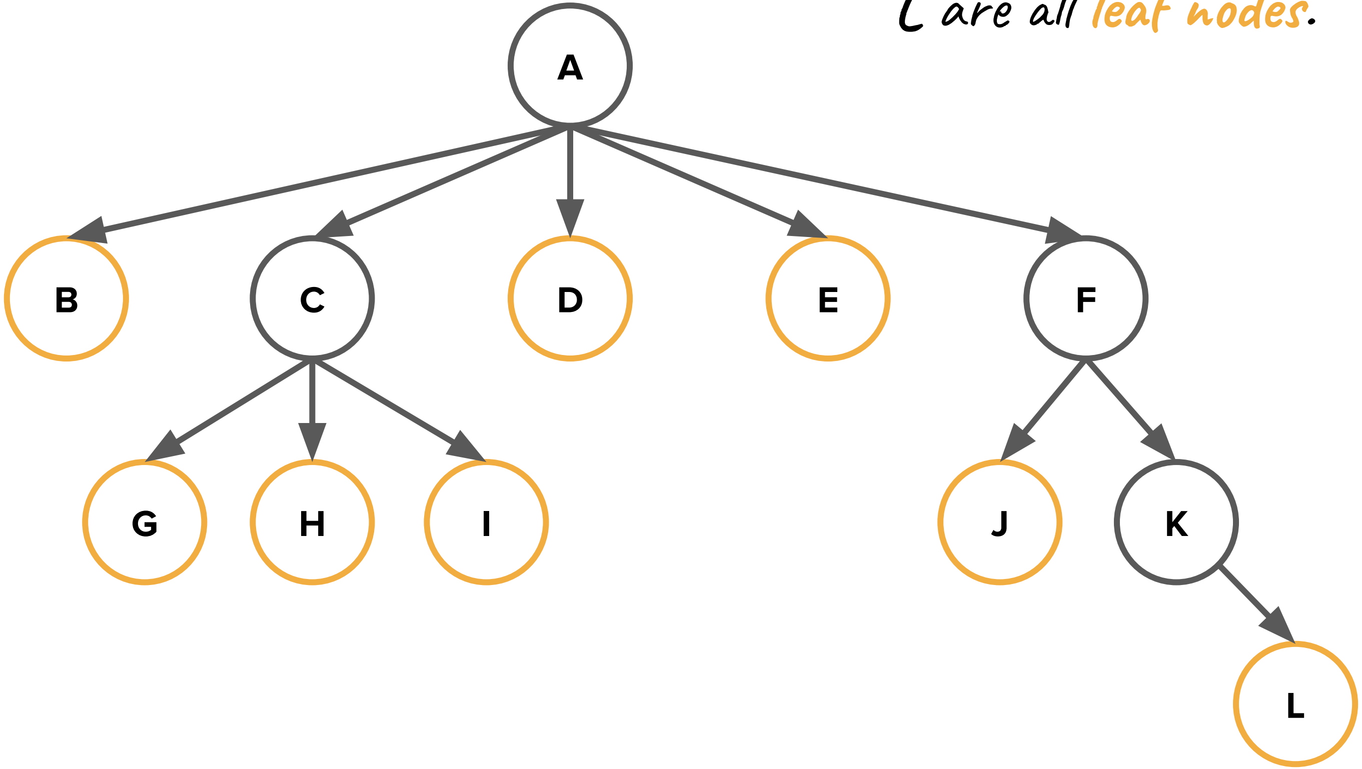
Tree Terminology



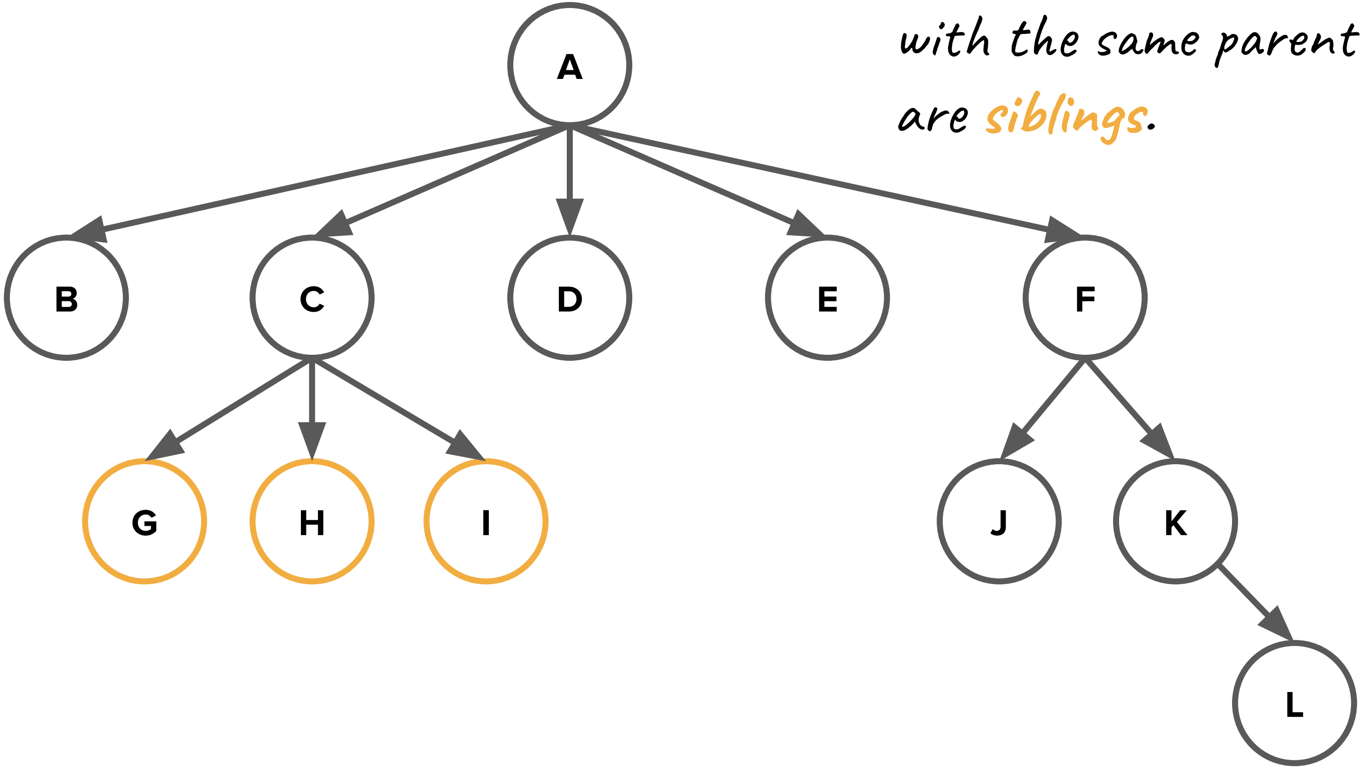
B has no children. A node with no children is called a leaf node.

Tree Terminology

B, G, H, I, D, E, J, and L are all **leaf nodes**.



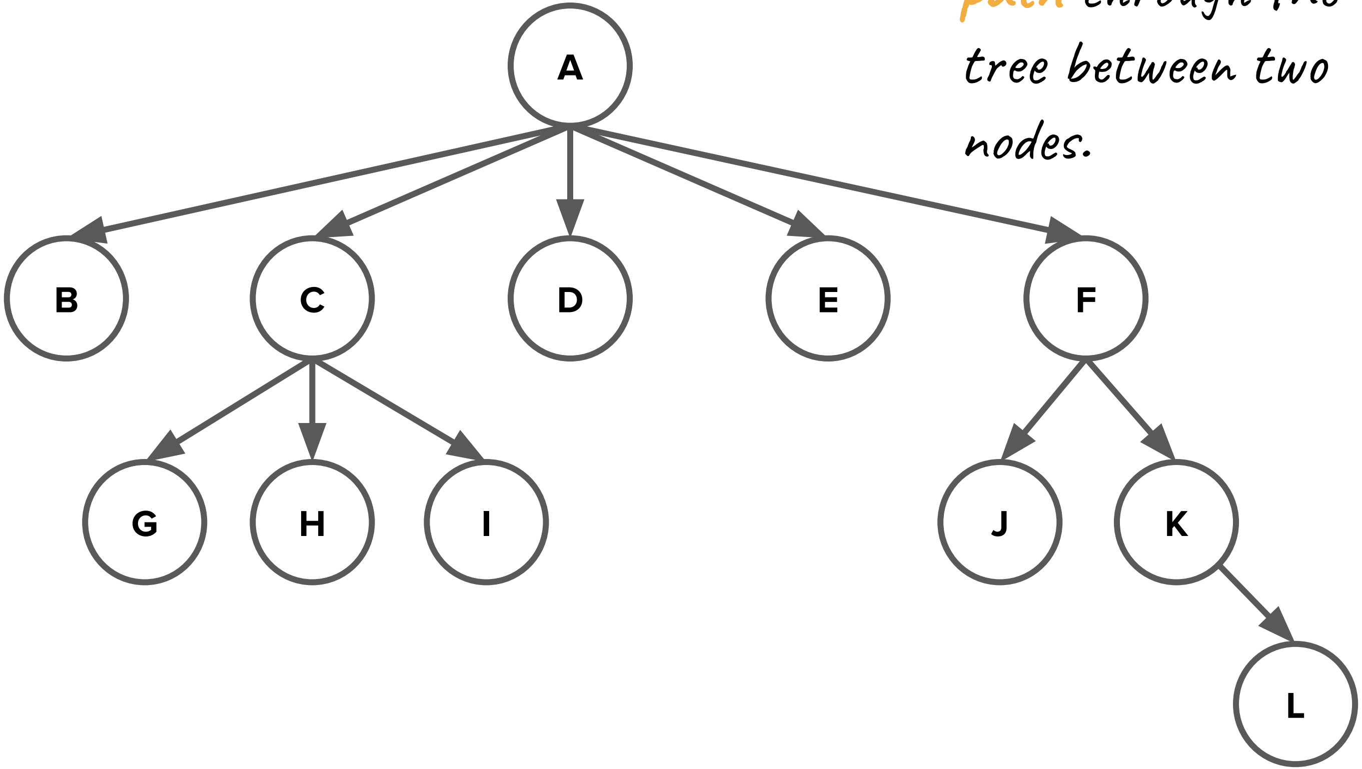
Tree Terminology



G, H and I all have the same parent. Nodes with the same parent are siblings.

Tree Terminology

We can define a path through the tree between two nodes.



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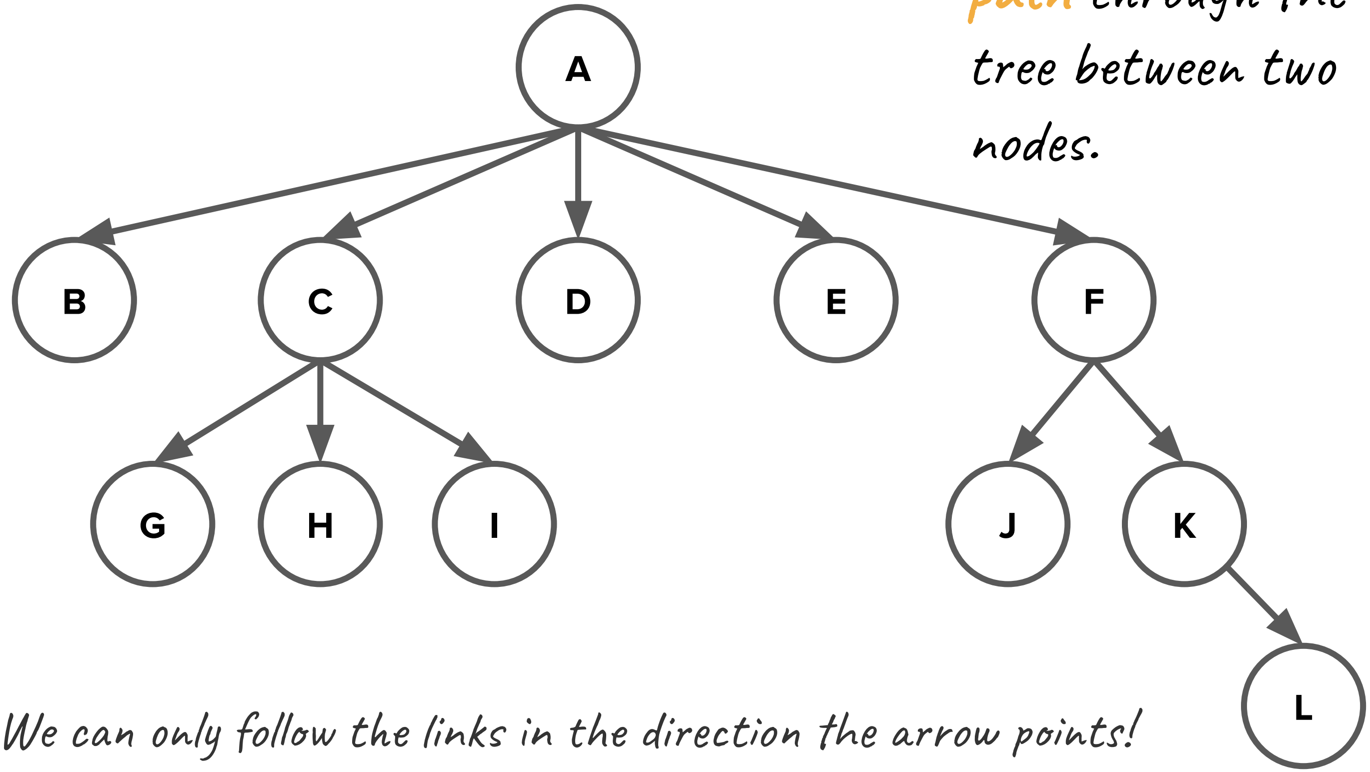
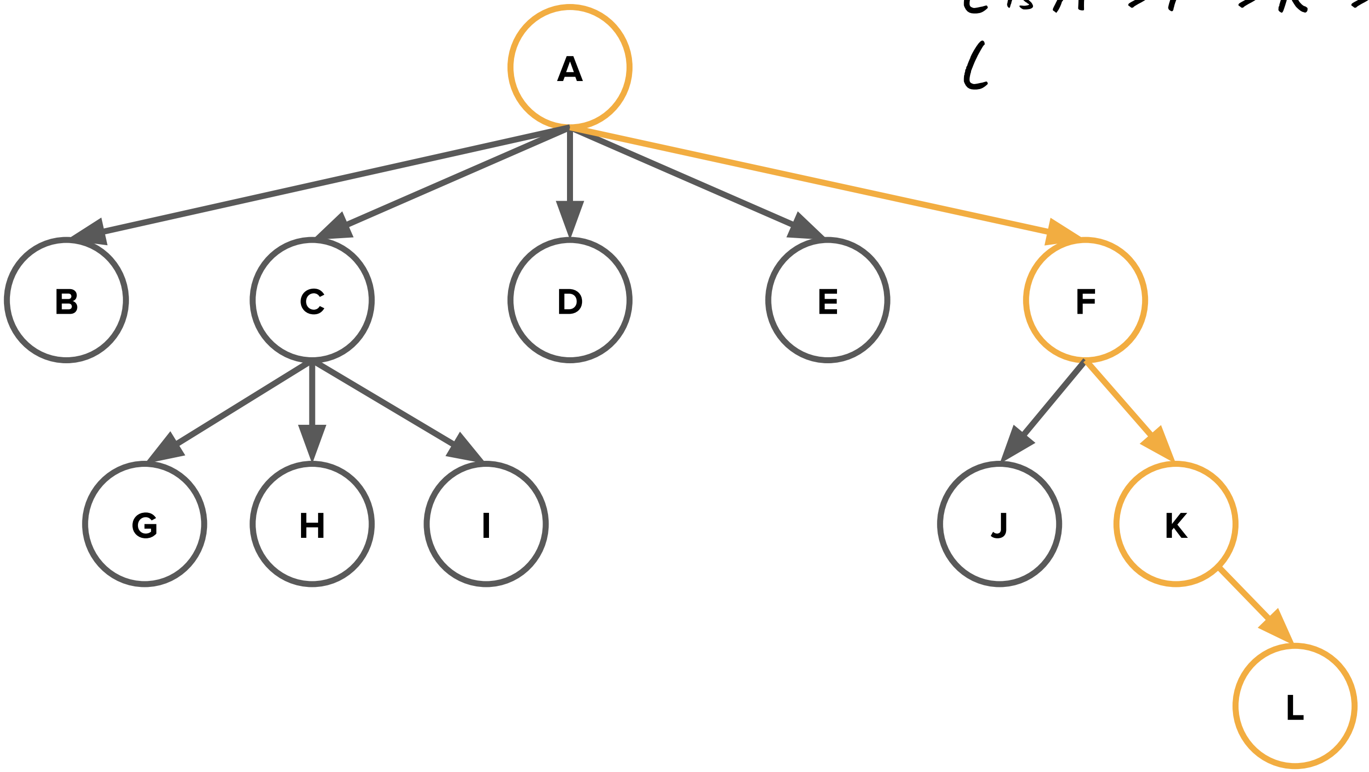


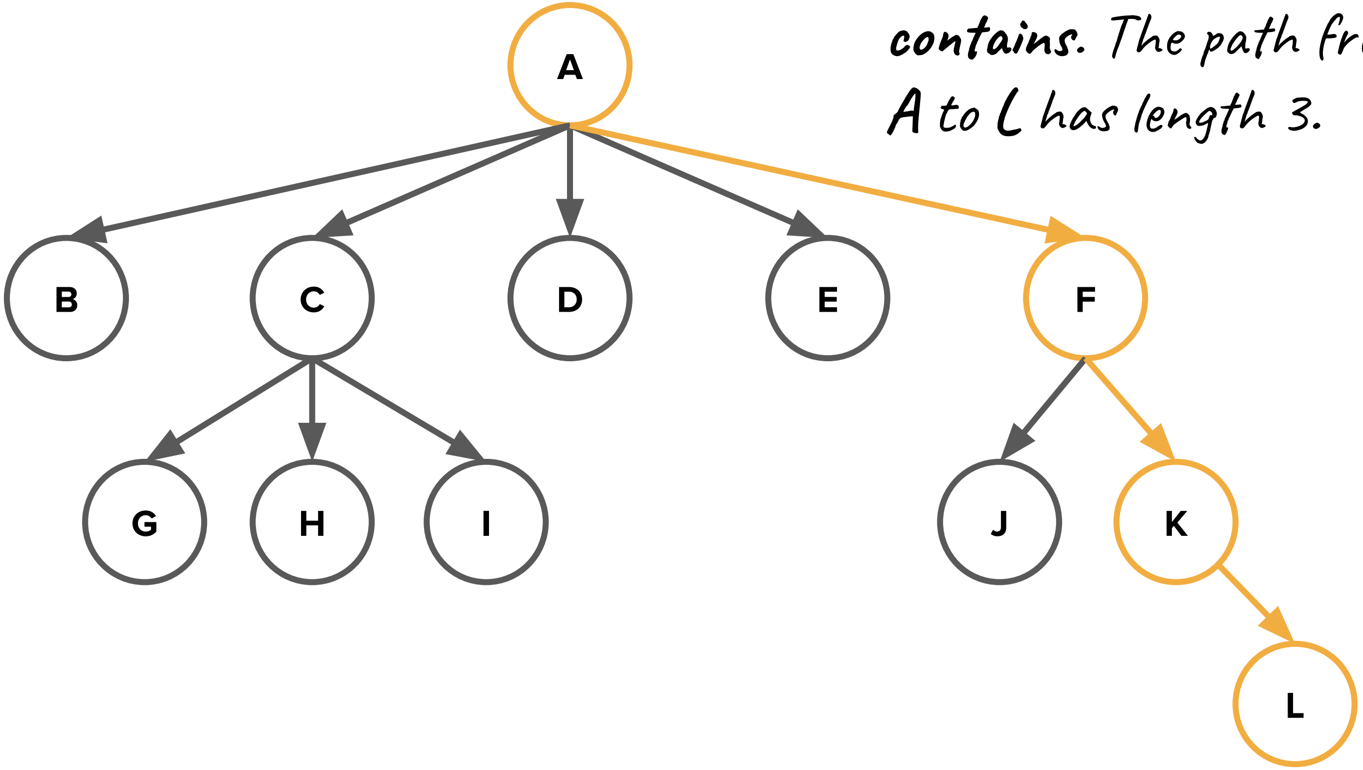
Note: We can only follow the links in the direction the arrow points!

Tree Terminology

The path from A to L is A -> F -> K -> L



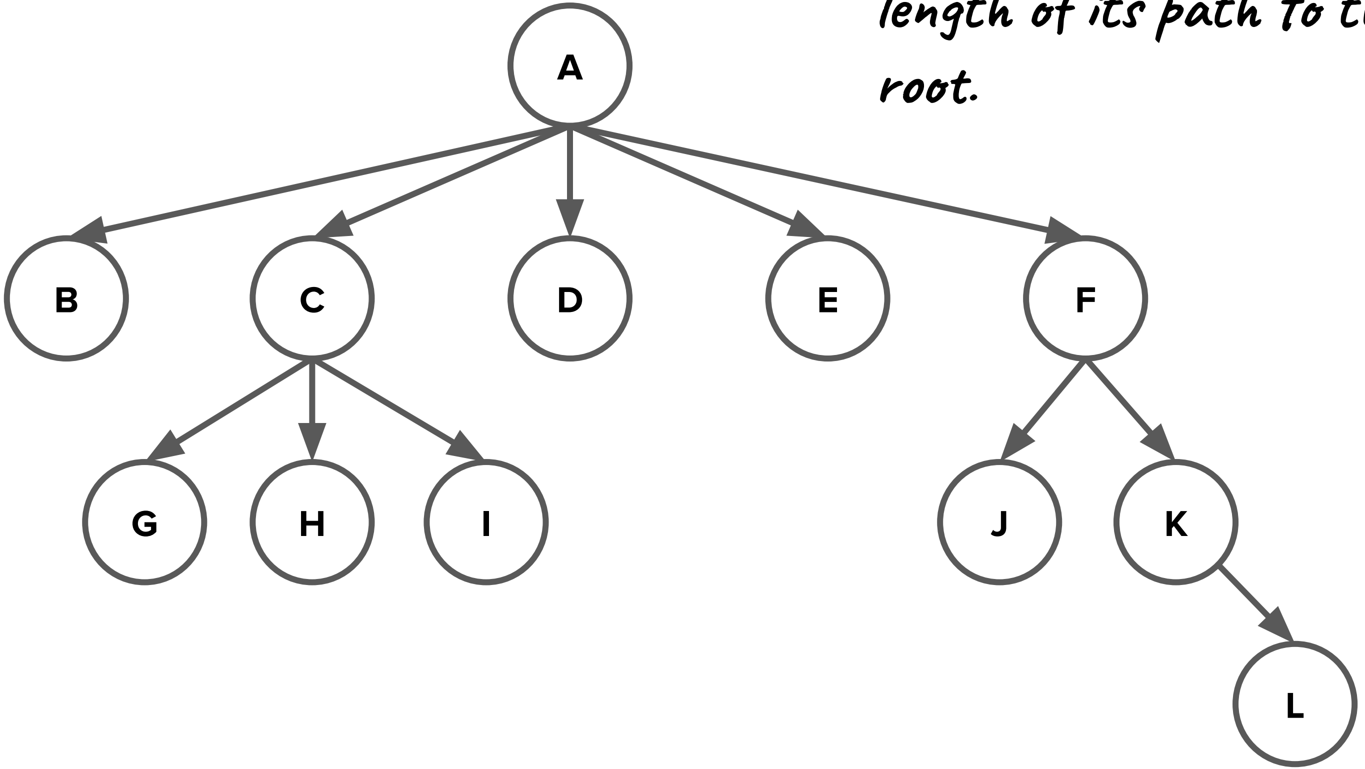
Tree Terminology



The length of the path is **number of edges it contains.** The path from A to L has length 3.

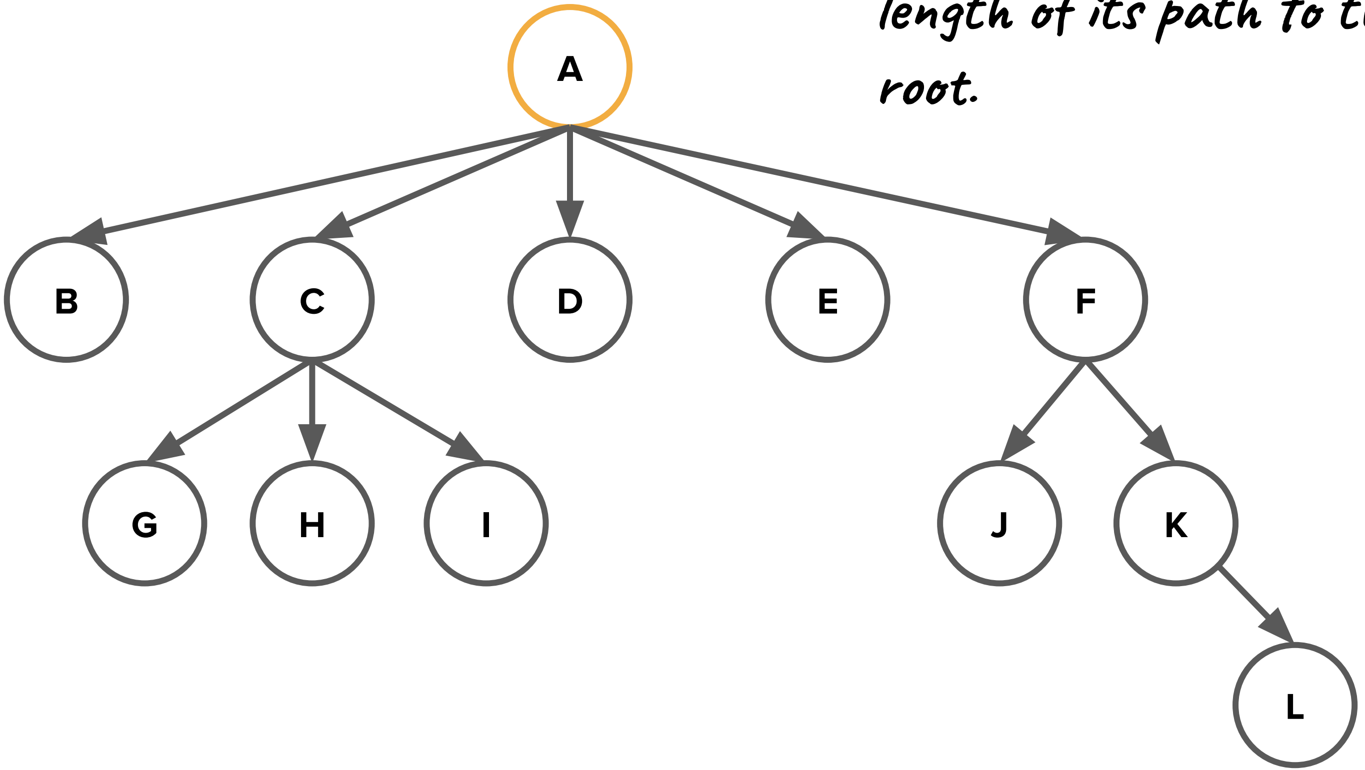
Tree Terminology

The *depth* of a node is the length of its path to the root.



Tree Terminology

depth: 0



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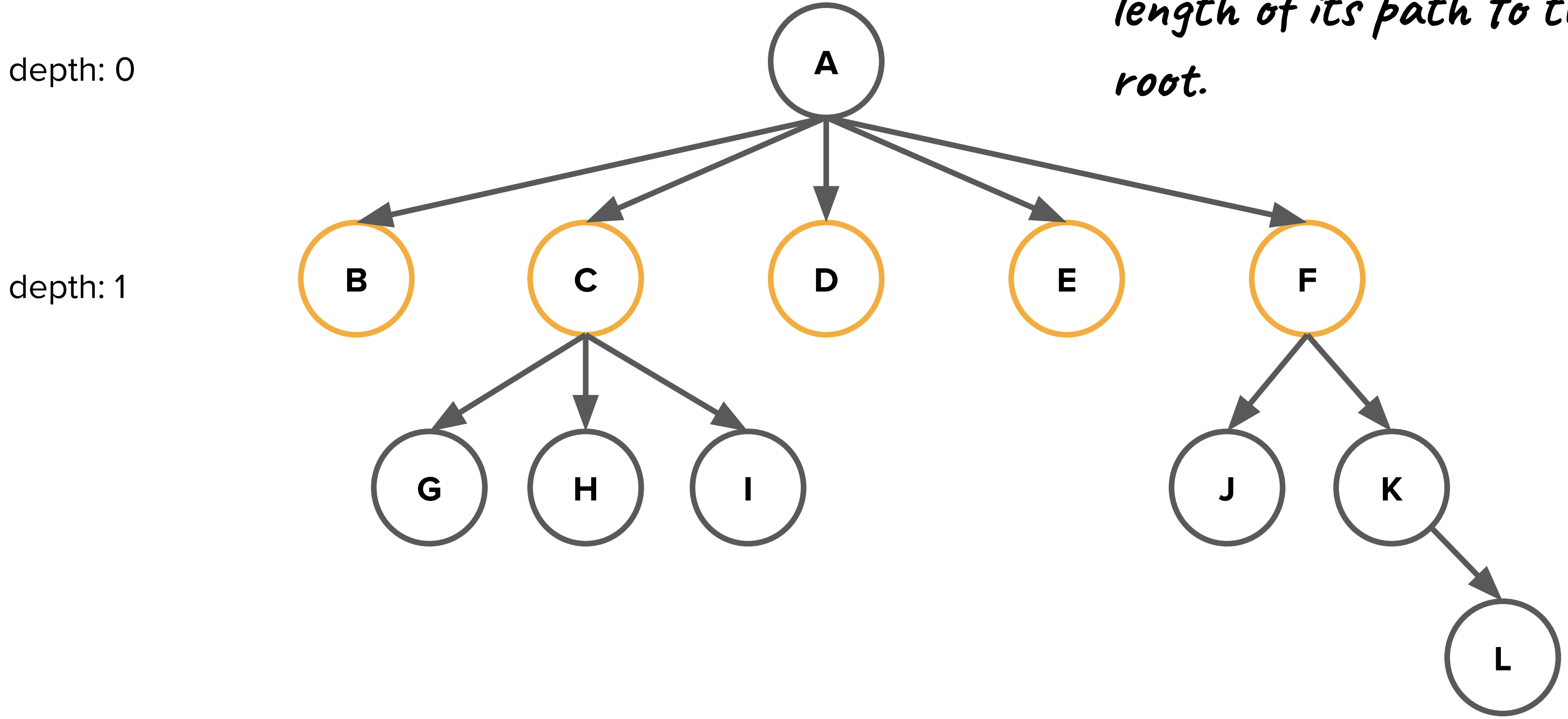

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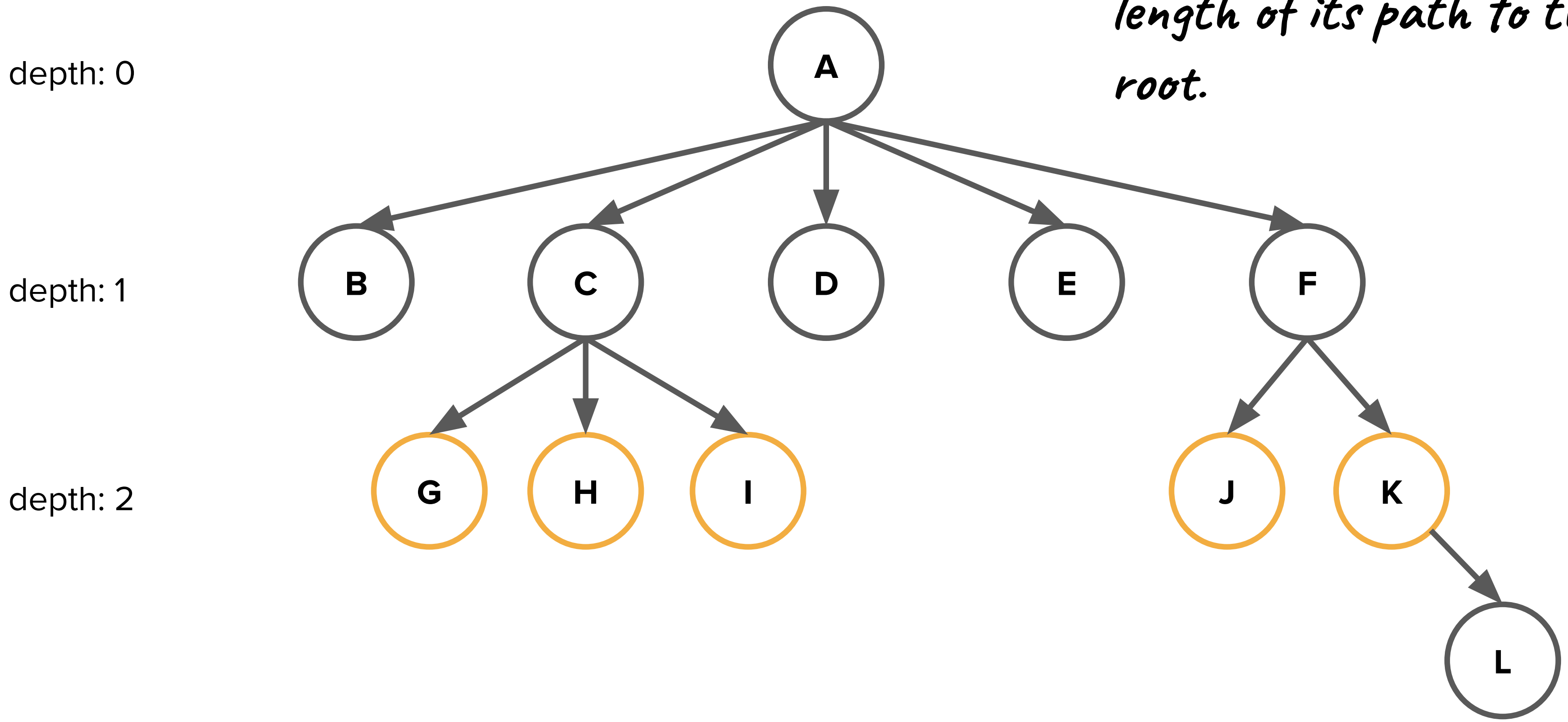

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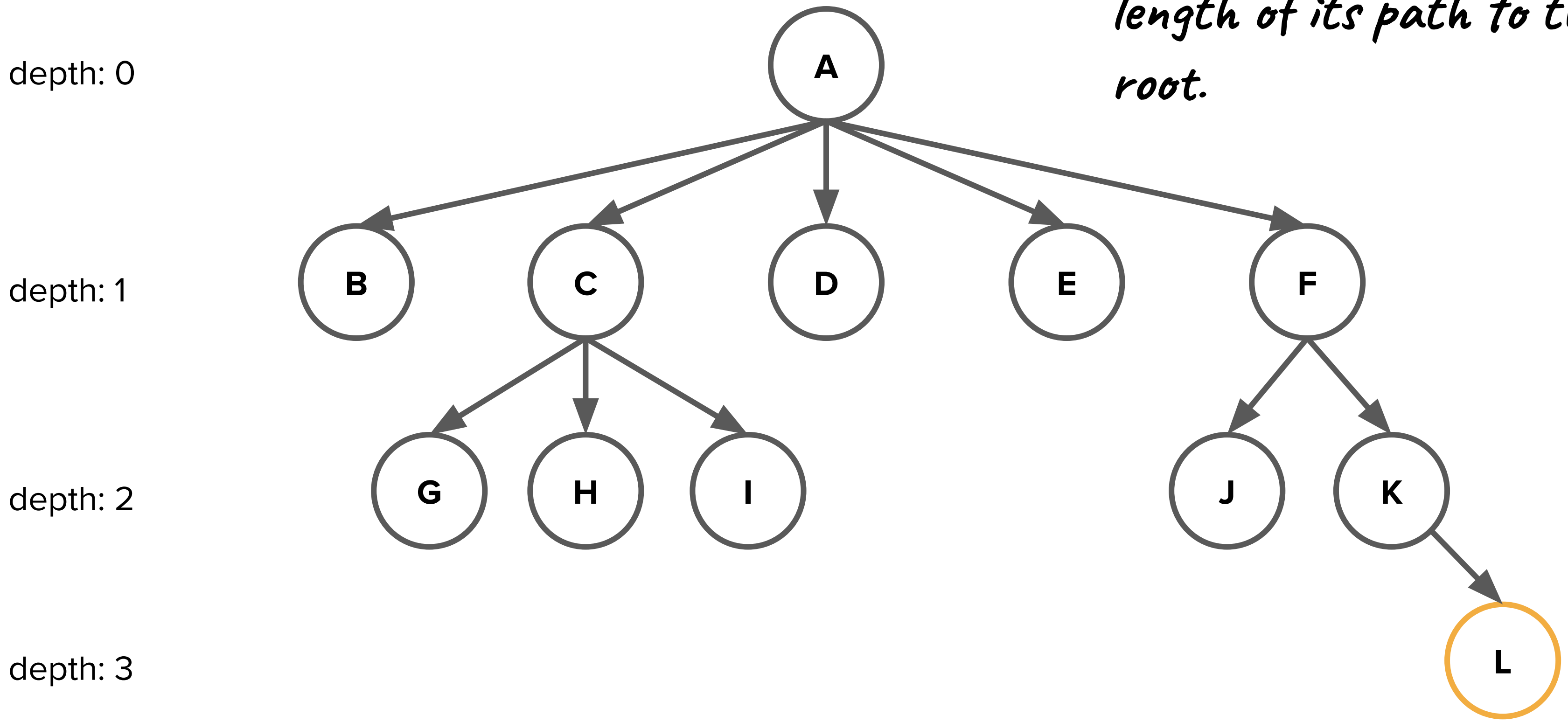
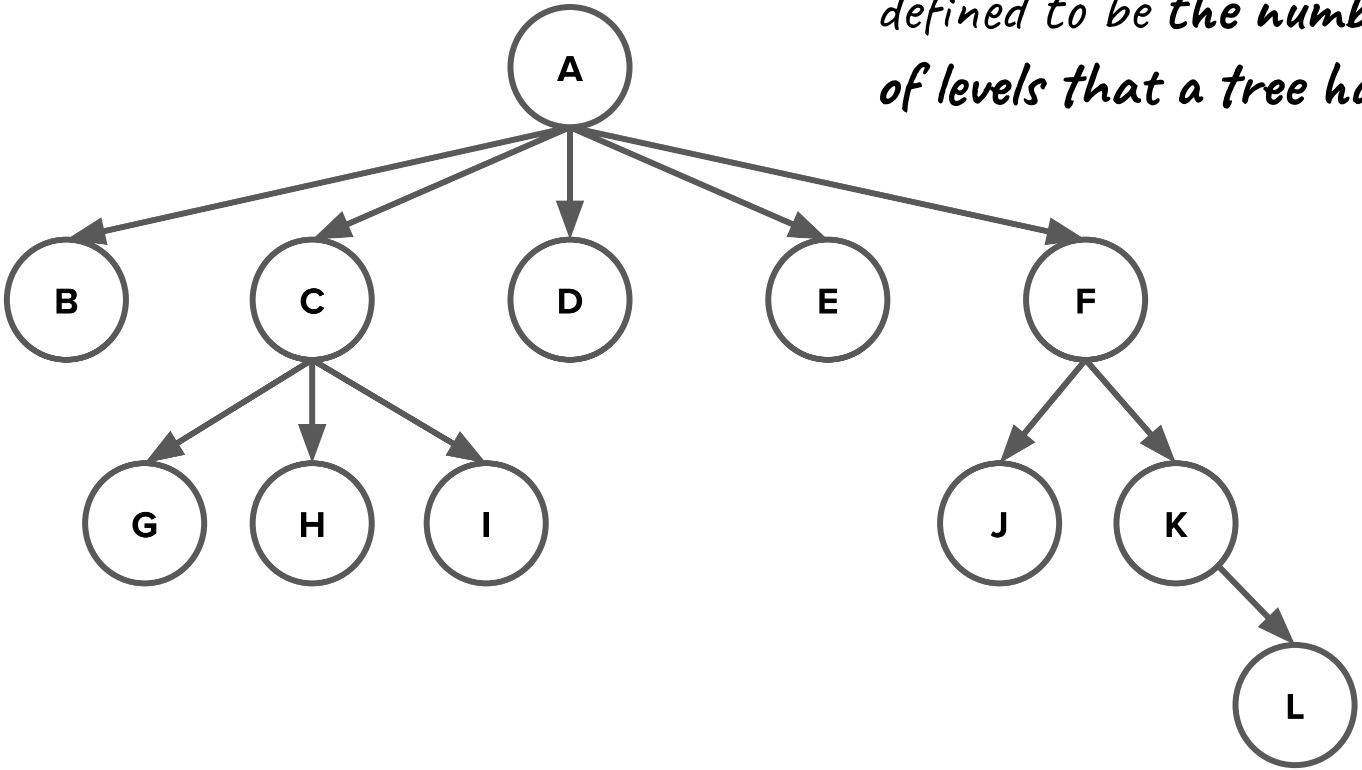


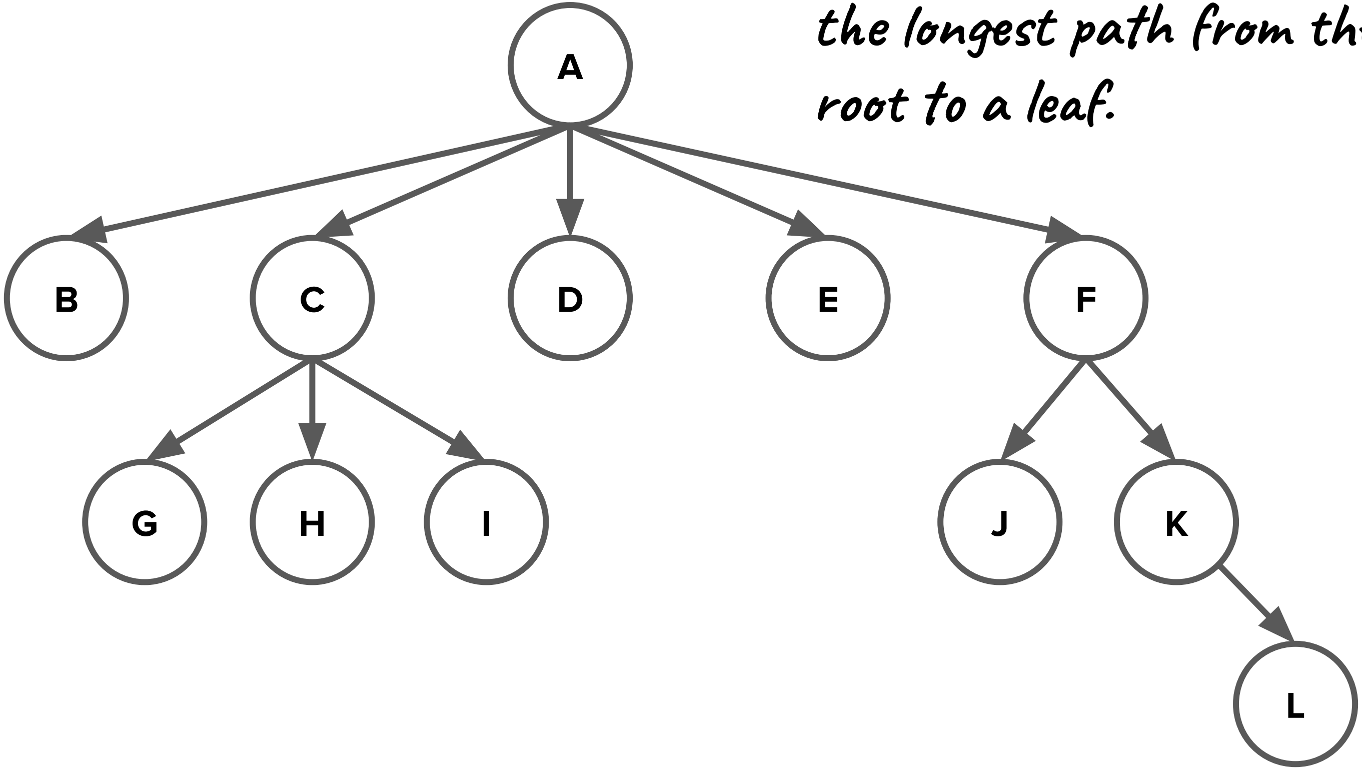
Tree Terminology

*The **height** of a tree is defined to be **the number of levels that a tree has.***



Tree Terminology

The height can also be defined as the number of nodes along the longest path from the root to a leaf.



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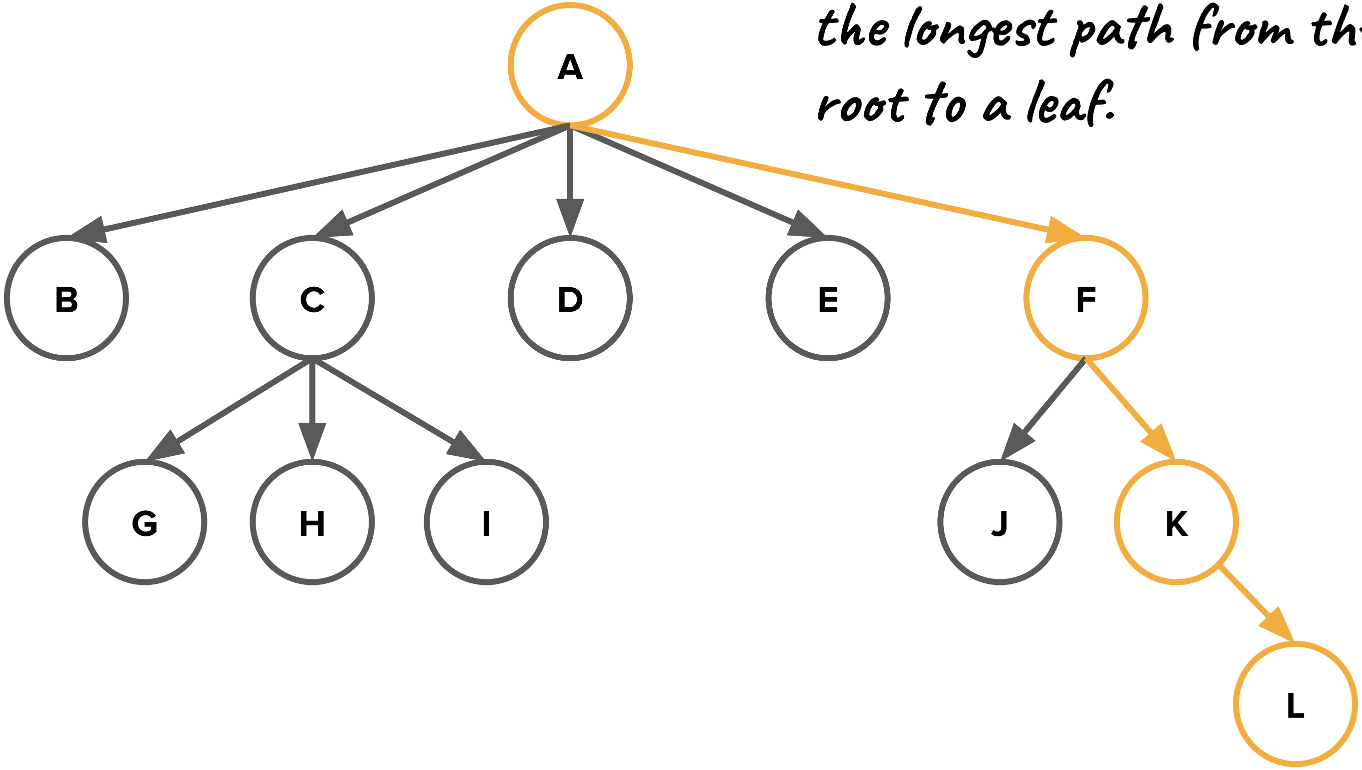

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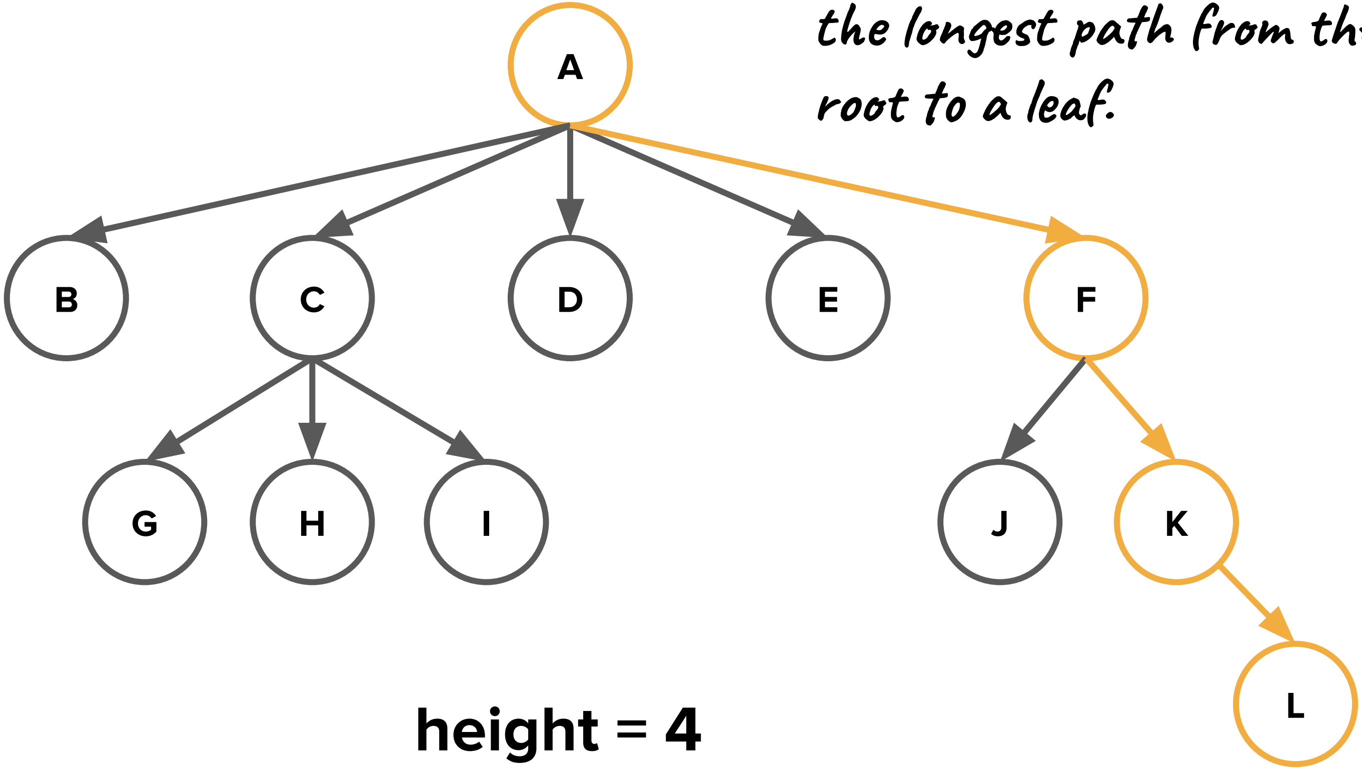

height = 4

Tree Terminology Summary

- Every non-empty tree has a **root node** that defines the "top" of the tree.
- Every node has 0 or more **children** nodes descended from it. Nodes with no children are called **leaf nodes**.
- Every node in a tree has exactly one **parent** node (except for the root node).
- A **path** through the tree traverses edges between parents and their children.
- The **depth** of a node is the length of the path between the root and that node. A tree's **height** is the number of nodes in the longest path through the tree.

Tree Properties

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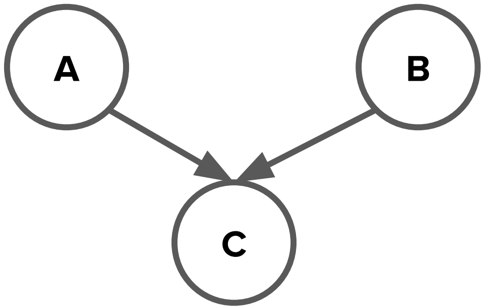

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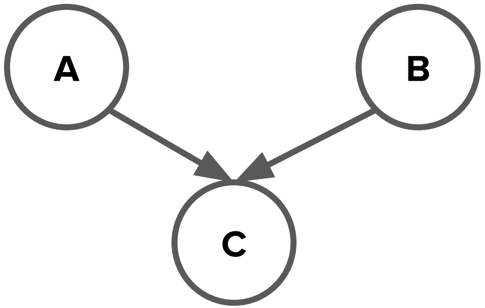

Not a tree!

Tree Properties

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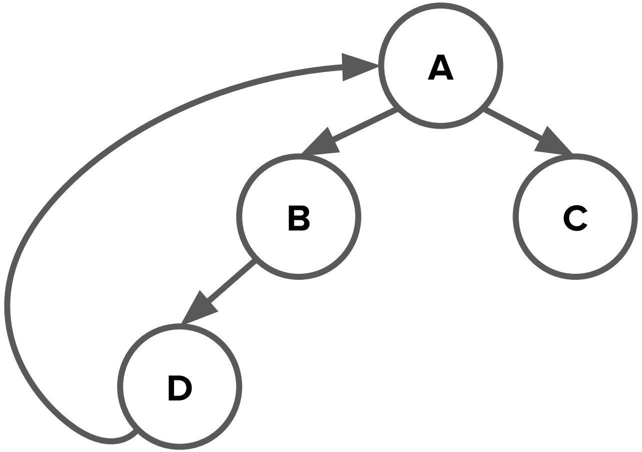

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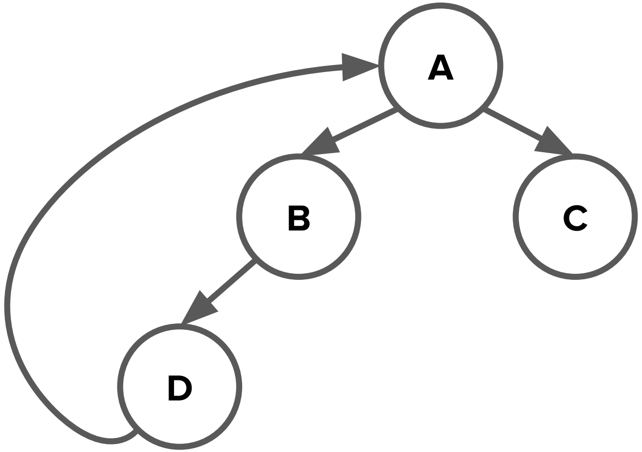

Not a tree!

Announcements

Announcements

- Assignment 5 is due on **Tuesday, August 4 at 11:59pm PDT**.
- Assignment 6 will be released by the end of the day on Wednesday and will be be due on **Wednesday, August 12 at 11:59pm PDT**. This is a hard deadline – there is **no grace period and no submissions will be accepted after this time**.
- Due to the end of quarter timeline, there will be **no revisions on Assignments 5 and 6.**
- Final project reports are due on Sunday, August 9 at 11:59pm PDT. You will have the opportunity to schedule your final presentation time after submitting.

Trees in C++

Binary Trees

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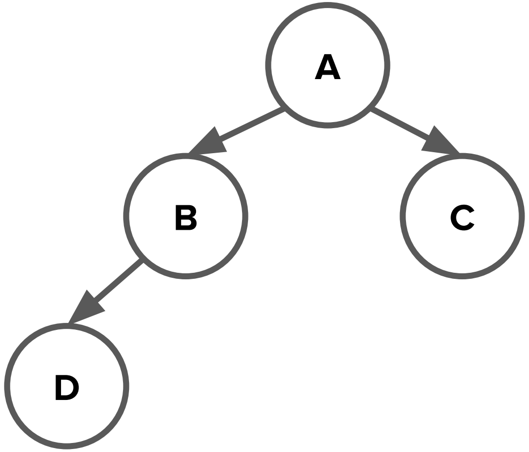
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.

- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.

- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.

Binary Trees



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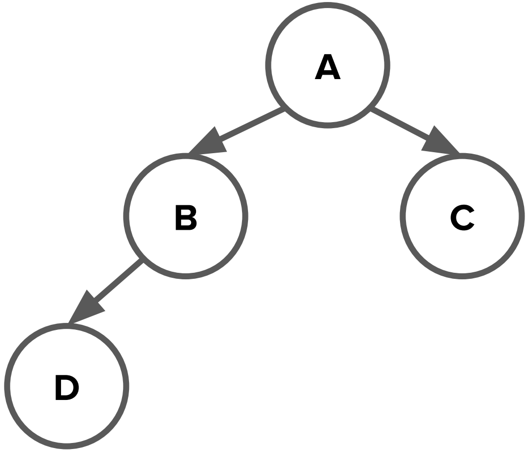
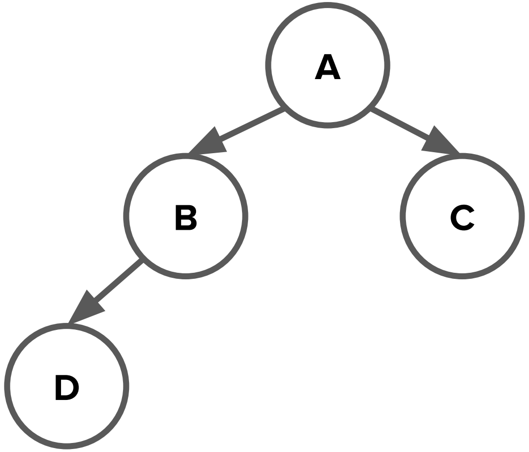


Binary Tree!

Binary Trees



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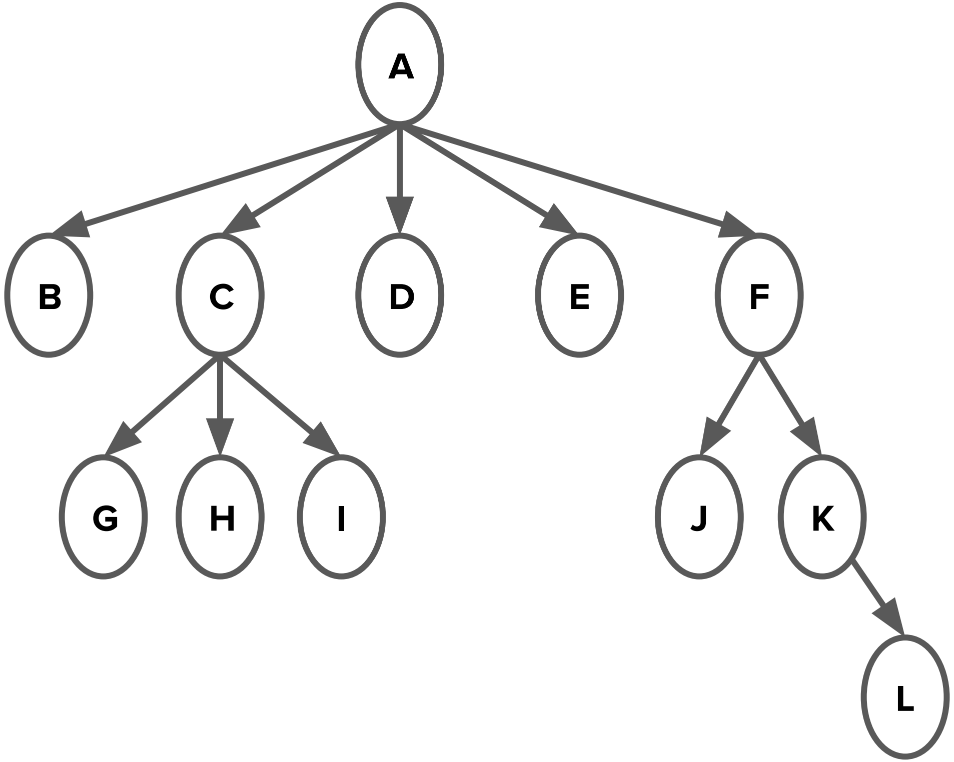

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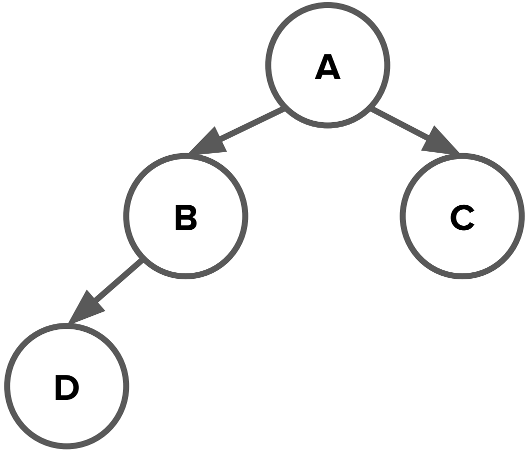

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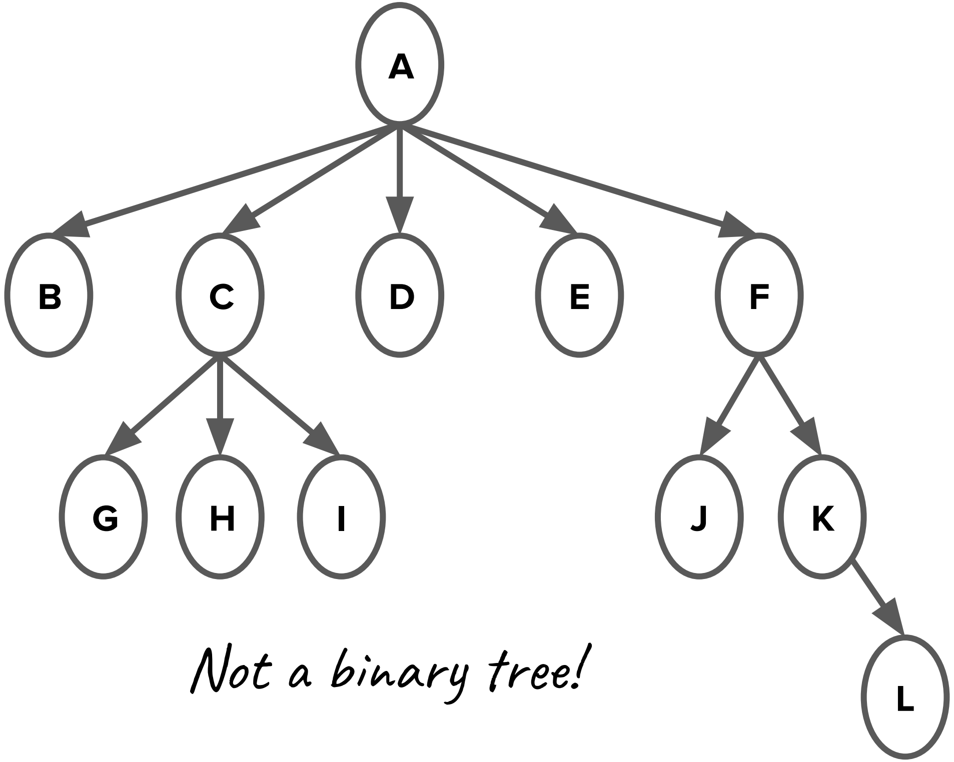

Not a binary tree!

Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

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struct TreeNode {
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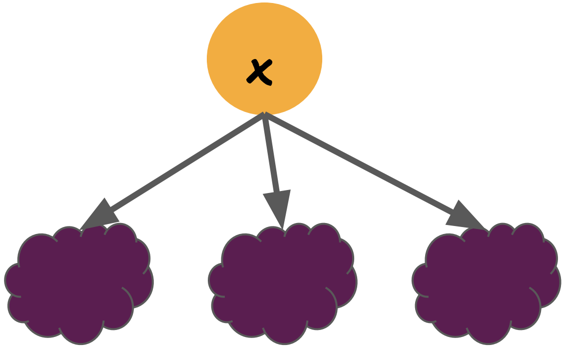
What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



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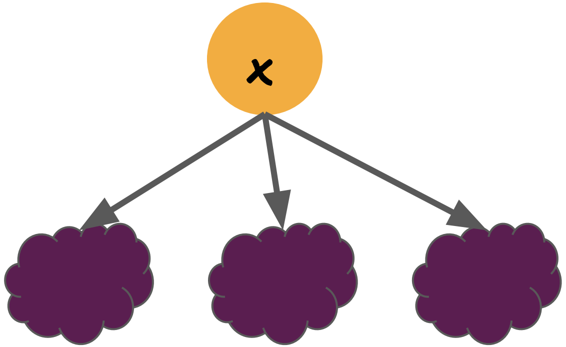

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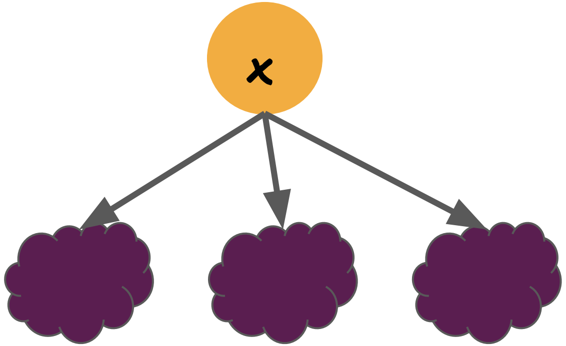

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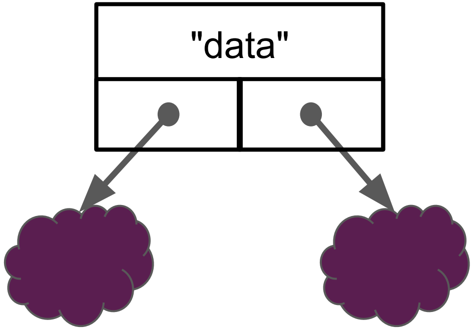

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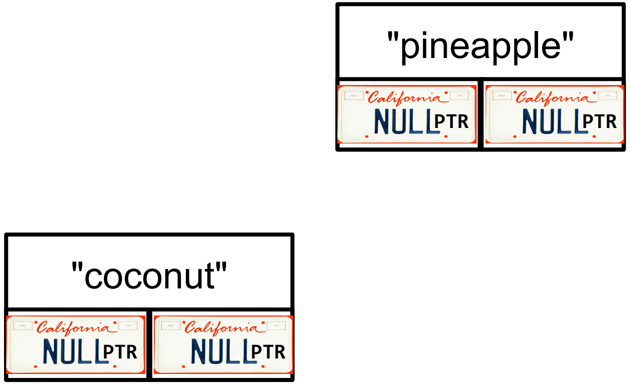

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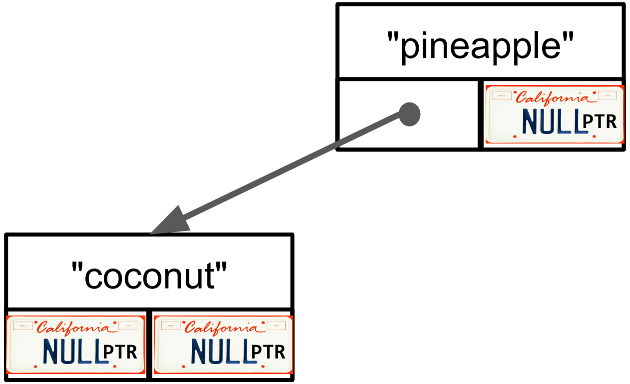

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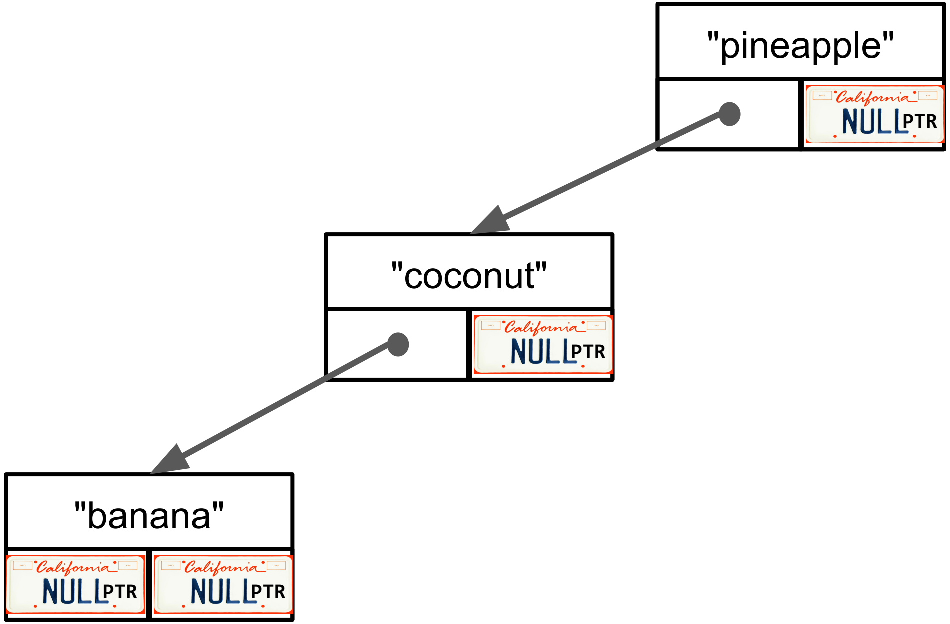

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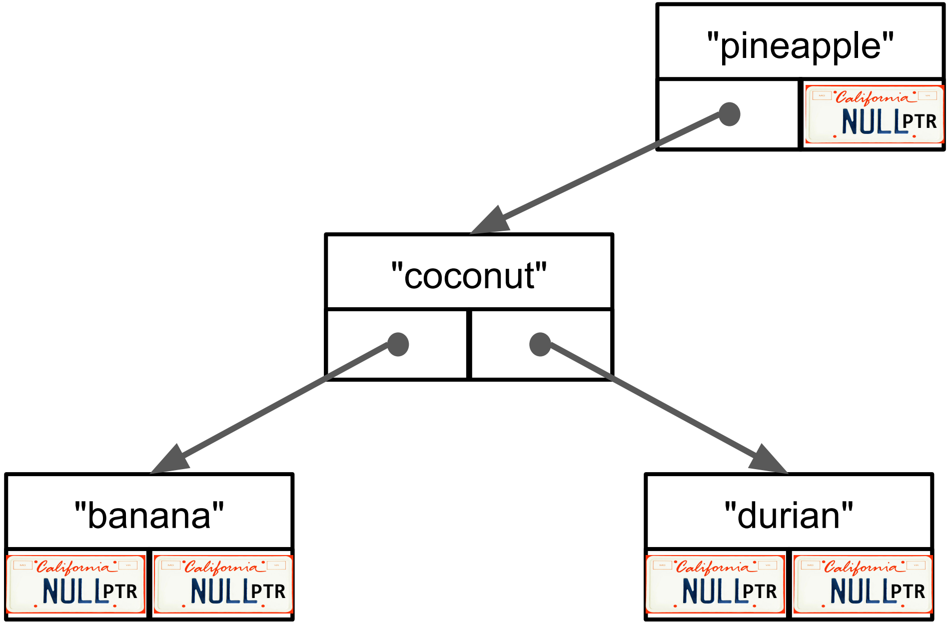

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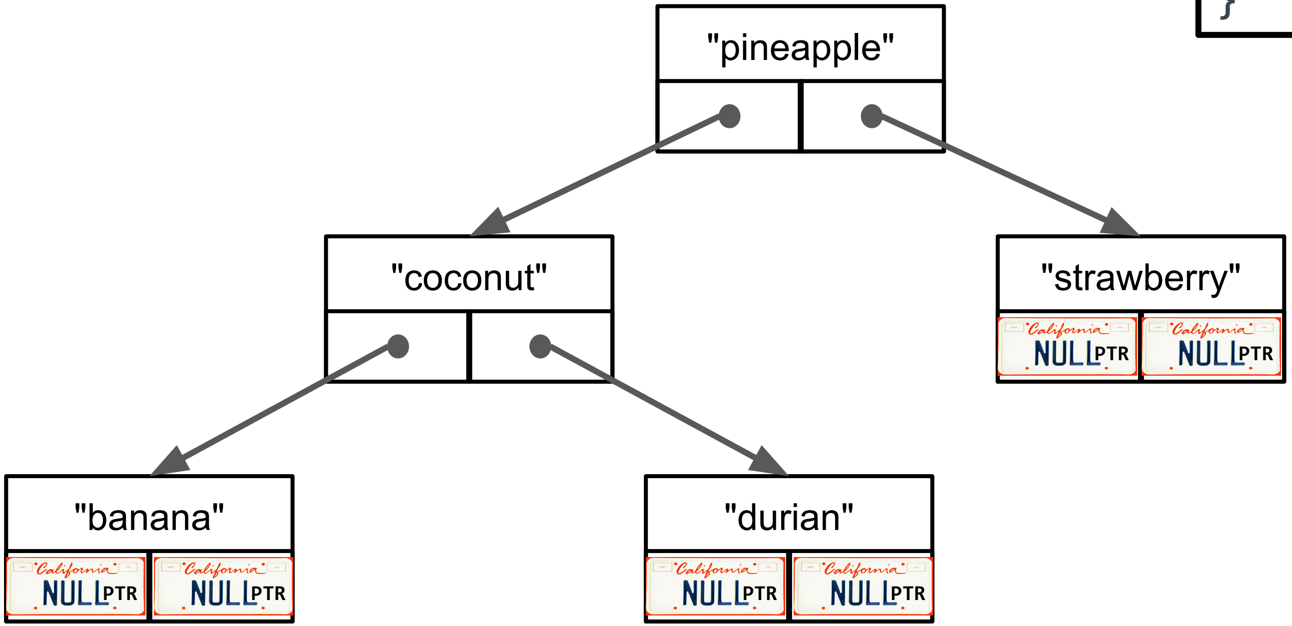

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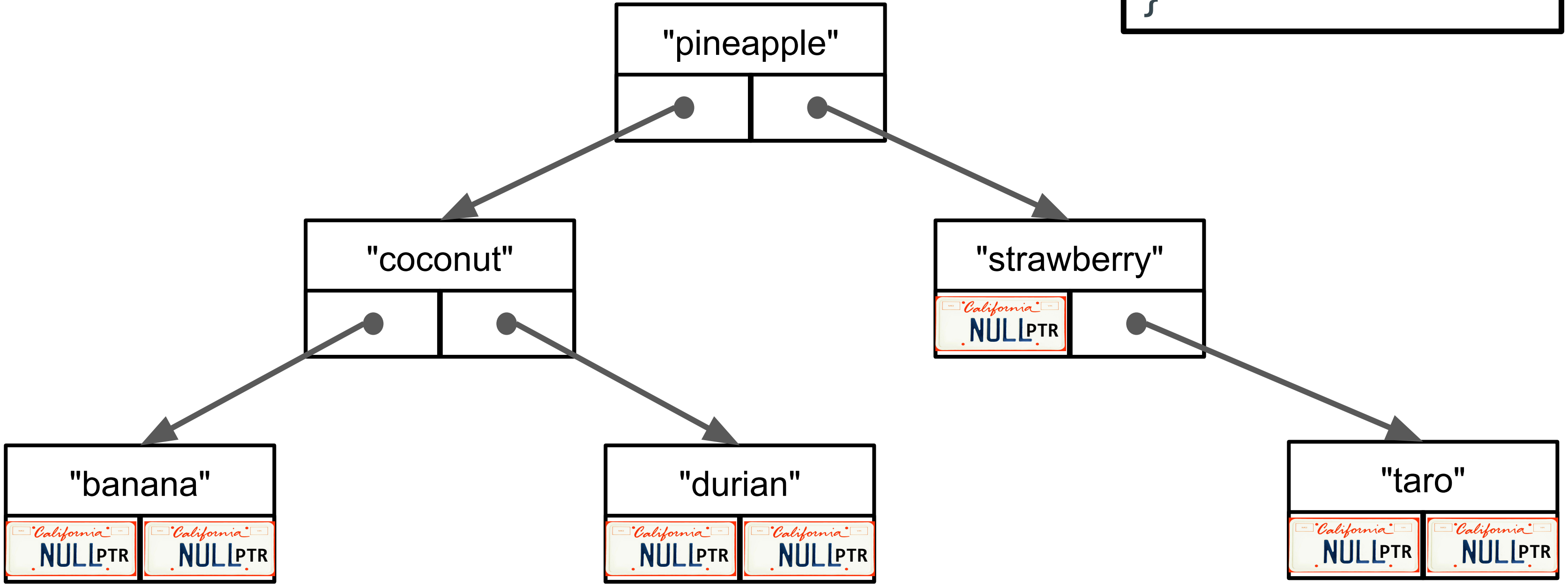

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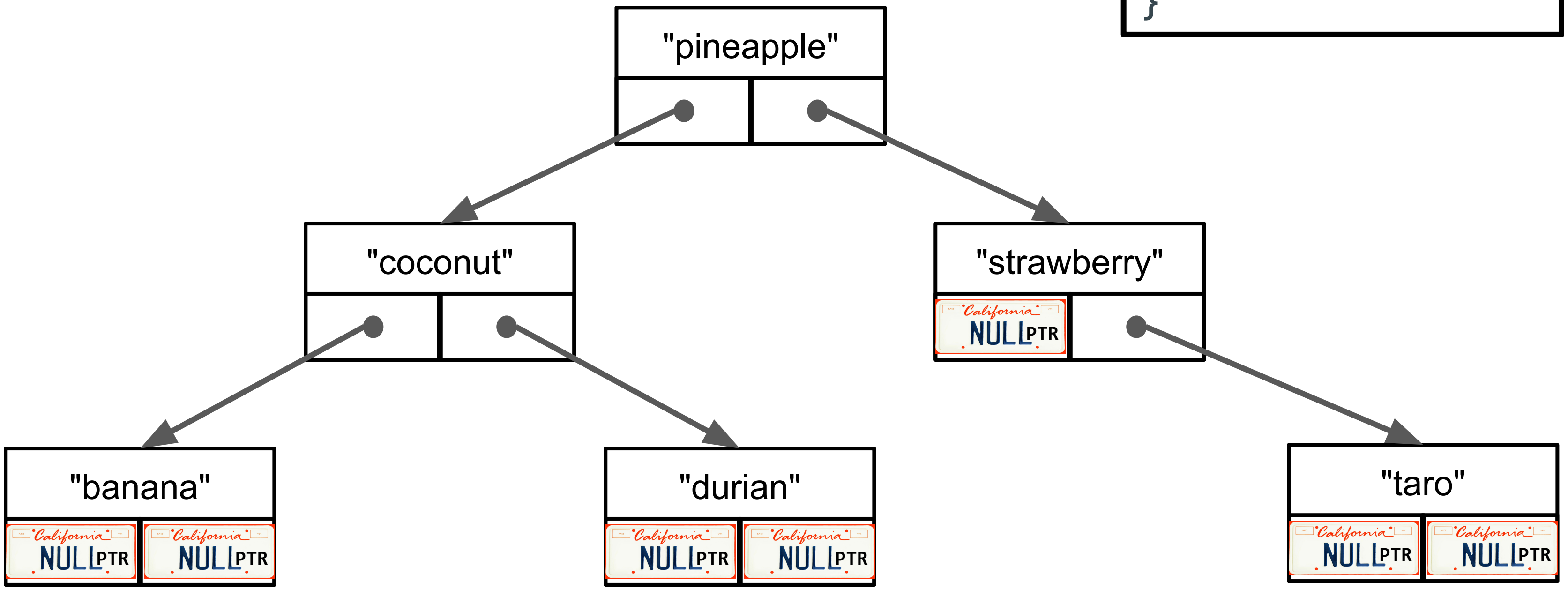

Note: Trees do not have to be complete, like heaps. Any node can have 0, 1, or 2 children.

Let's code it!

`buildExampleTree()`

Building a Tree Takeaways

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We integrate these new nodes into the tree by rewiring the **`left`** and **`right`** pointers of existing nodes in the tree.

Tree Traversals

Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!

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  - Pre-order traversal
  - In-order traversal
  - Post-order traversal

Tree Traversals

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- There are three main ways to traverse a binary tree:
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal

- Due to the recursive nature of trees, all of these algorithms are most easily defined **recursively**.

Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" to be printing the contents of the current node, which will allow us to print the overall tree.

Let's code it!

`preorderPrintTree()`

Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
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- Output: **`pineapple coconut banana durian strawberry taro`**

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

Let's code it!

`inorderPrintTree()`

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

- Output: **`banana coconut durian pineapple strawberry taro`**

- Observation: The output of this traversal gives as all the values in alphabetical order. Is this a coincidence?
  - No! We'll see why tomorrow!

Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
    - Traverse the left subtree
    - Traverse the right subtree
    - "Do something" with the current node

Try it yourself!

`postorderPrintTree()`

Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

- Output: **`banana durian coconut taro strawberry pineapple`**

- Application: Freeing trees! (we'll see this in lecture tomorrow)

Summary

Trees Summary

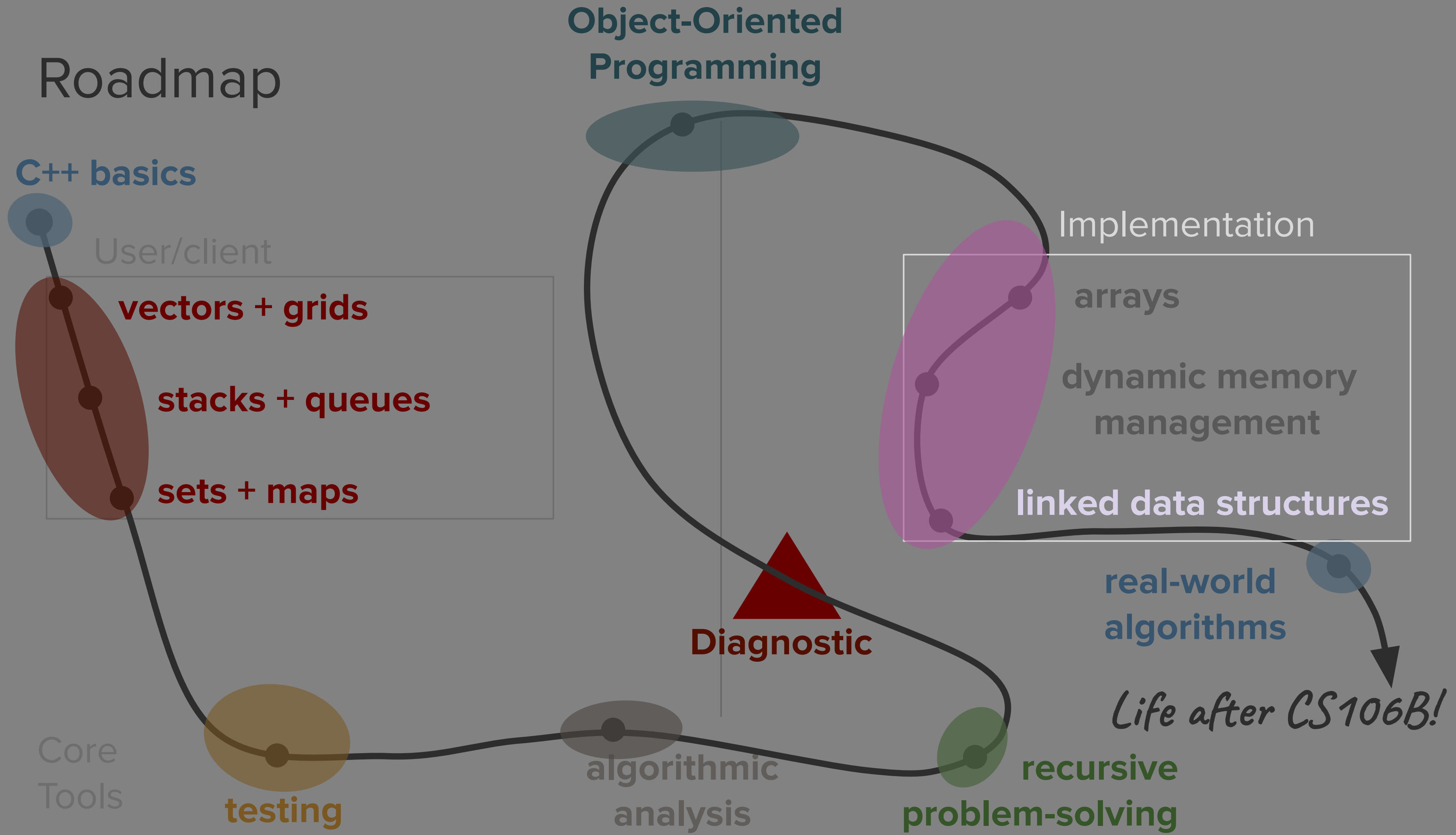
- Trees allow us to organize information in a linked data structure such that the distance to any element is short, even if there are many elements.

- Trees organize nodes in a hierarchical manner, where each element contains connections to children nodes that exist "lower" in the tree.

- There are three main ways to traverse the nodes in a tree, and each type of traversal visits the nodes of the tree in a distinctly different order.

What's next?

Roadmap



Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Binary Search Trees