Recursive Backtracking
Logistics

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- Once again, you are allowed to work in pairs on this assignment. We think this one’s pretty tricky, so be sure to start early!
Part 1: Warmups

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- In the first one, you’ll be examining The Towers of Hanoi, a famous recursive problem. You’ll be responsible for stepping through the recursive function and reporting back various info to us. Here are the helpful steps in the debugger:
  - Stepping over a recursive call can be helpful when thinking holistically. A recursive call is simply a “magic” black box that completely handles the smaller subproblem.
  - Stepping into a recursive call allows you to trace the nitty-gritty details of moving from an outer recursive call to the inner call.
  - Stepping out of a recursive call allows you to follow along with the action when backtracking from an inner recursive call to the outer one.
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- In the second warmup, you’ll be examining a buggy implementation of code that finds permutations. It’ll be your job to figure out the little error that causes the bug, and after, you’ll need to reflect as to why the that bug was so catastrophic.
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- In the second warmup, you’ll be examining a buggy implementation of code that finds permutations. It’ll be your job to figure out the little error that causes the bug, and after, you’ll need to reflect as to why the bug was so catastrophic.
  - We designed these warmups specifically because we’ve seen these errors come up many times in the past! If you have a good understanding of why the permutations code doesn’t work, you’ll have a much better understanding of recursion / backtracking!
Let’s hop into the code!

Poster for the 2015 film “Backtrack.” Critics gave it a paltry 30% on Rotten Tomatoes, citing “not enough recursion.”
Part 2: Doctors Without Orders

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- A growing application of CS is in the field of Healthcare.

Stanford AI in Healthcare
Part 2: Doctors Without Orders

- The more programming knowledge we know, the more we can use CS to solve real life problem!
- A growing application of CS is in the field of Healthcare.
- Let’s utilize the recursion skill we have practiced so far to solve a cool problem!
Part 2: Doctors Without Orders

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- Each doctor has a maximum number of hours they can work for.
  - `Map<string, int> doctors;`
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- Each doctor has a maximum number of hours they can work for.
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- If a valid matching exist, we also want to keep track of such a matching.
  - `Map<string, Set<string>> schedule;`
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Part 2: Doctors Without Orders

- Let’s walk through a quick example (low animation budget ahead!)
Strategy 1: Grab-n-Go!

D

10

8

8

8

P

2

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6
Strategy 1: Grab-n-Go!

[D]

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Strategy 1: Grab-n-Go!
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*Surprise! That’s didn’t work!*
Strategy 2: Greedy

```
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Strategy 2: Greedy
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Surprise (this time less sarcastically)!
Strategy 2: Greedy

*Surprise (this time less sarcastically)!*

Note: This is provable! Take CS 161 to find out :)
“Strategy” 3: Oracle

There is indeed a solution here.
Part 2: Doctors Without Orders

- Some notes:
  - Each patient can only be assigned to ONE doctor.

```cpp
bool canAllPatientsBeSeen(const Map<string, int>& doctors,
const Map<string, int>& patients,
Map<string, Set<string>>& schedule);
```
Part 2: Doctors Without Orders

- Some notes:
  - Each patient can only be assigned to ONE doctor.
  - Doctors don’t have to use up all of their hours.
    - In fact, it is totally fine to have a doctor not matched with any patient.

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  - Doctors don’t have to use up all of their hours.
    - In fact, it is totally fine to have a doctor not matched with any patient.
  - We don’t care what “schedule” contains if not matching is possible.
    - Although we imagine that you don’t have to explicitly address this case.

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    - 1. Go one doctor at a time, deciding which subset of patients that doctor should see.

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  - `map.firstKey()` can give you a “random” key from the map.

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bool canAllPatientsBeSeen(const Map<string, int>& doctors,
                          const Map<string, int>& patients,
                          Map<string, Set<string>>>&& schedule);
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  - Not the first priority, but make sure your code is efficient (there is a stress test).

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  - Not the first priority, but make sure your code is efficient (there is a stress test).
    - Don’t make repeated schedules.
    - Don’t go down impossible paths (like intentionally ignoring a patient).

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Questions?
Part 3: Disaster Planning

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- A few limitations:
  - Limited supplies: We can’t afford to stockpile all cities, so we can only pick a strict subset of the vulnerable cities to cover.
  - Need for proximity: A city cannot react to a disaster fast enough if the closest emergency supply is too far away.
Part 3: Disaster Planning

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- A region is represented by a set of cities, and each pair of cities can be optionally connected by a road.
  - `Map<string, Set<string>> roadNetwork;`
Part 3: Disaster Planning

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- A region is represented by a set of cities, and each pair of cities can be optionally connected by a road.
  - `Map<string, Set<string>>& roadNetwork;`
- There are only a limited number of cities we can directly supply.
  - `int numCities;`
Part 3: Disaster Planning

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• A region is represented by a set of cities, and each pair of cities can be optionally connected by a road.
  ○ Map<string, Set<string>>& roadNetwork;
• There are only a limited number of cities we can directly supply.
  ○ int numCities;
• We would like to know, if possible, what cities should be supplied so, for every city in the region, it is either directly supplied, or adjacent to a city that is directly supplied.
  ○ Set<string> supplyLocations;
Part 3: Disaster Planning

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Say we have 5 Cities, A, B, X, D and C!
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Suppose we cover city A...
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Question, what’s the minimum number of cities we need to supply to “cover” all cities in this region?
Part 3: Disaster Planning

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One! City X!
Part 3: Disaster Planning

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Remember that cities can be covered by their neighbors! This will come in handy in this assignment!
Part 3: Disaster Planning

```cpp
bool canBeMadeDisasterReady(const Map<string, Set<string>>& roadNetwork,
                              int numCities,
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```

Let’s go over the algorithm to solve this problem:
Part 3: Disaster Planning

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2. For each way that it can be covered (i.e. covering the city or ANY of its neighbors), try covering that option, and then see whether the result of covering it returns true. If so, then covering that option was the correct choice, and you can return true. If all choices return false, then there’s no way to cover that city, meaning that you cannot cover all cities -- you should return false.
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How many ways can we cover H1?
Part 3: Disaster Planning

Let’s see some brief examples of this:

Only 2 ways: either by covering H1 or by covering C1
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Let’s say that we first try covering C1:
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In technical terms, by covering some city C, you can remove roadNetwork[C] (all of C’s neighbors) from the set of uncovered cities, including C.
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Be sure to account for these changes in every recursive call!
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What’s the next optimal choice to cover all cities?
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C2! Even though it’s already “covered,” it is worth stockpiling the city because it will cover the last 3 uncovered cities!
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Your computer won’t be able to analyze the diagrams, so it needs to try all options :p
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  - In a similar vein, **don’t modify roadNetwork**. We strongly encourage that you add parameters via a wrapper function, but if you change `roadNetwork`, all bets are off WRT your functionality.
  - `numCities` can be zero, but you should raise an `error()` if it’s negative.
  - Keith recommends getting the return statement correct before filling the outparameter `supplyLocations`. Once you’ve gotten the functionality correct that determines whether a region can be supplied, *then* you can start working on the outparameter.
bool canBeMadeDisasterReady(const Map<string, Set<string>>& roadNetwork,
int numCities,
Set<string>& supplyLocations);