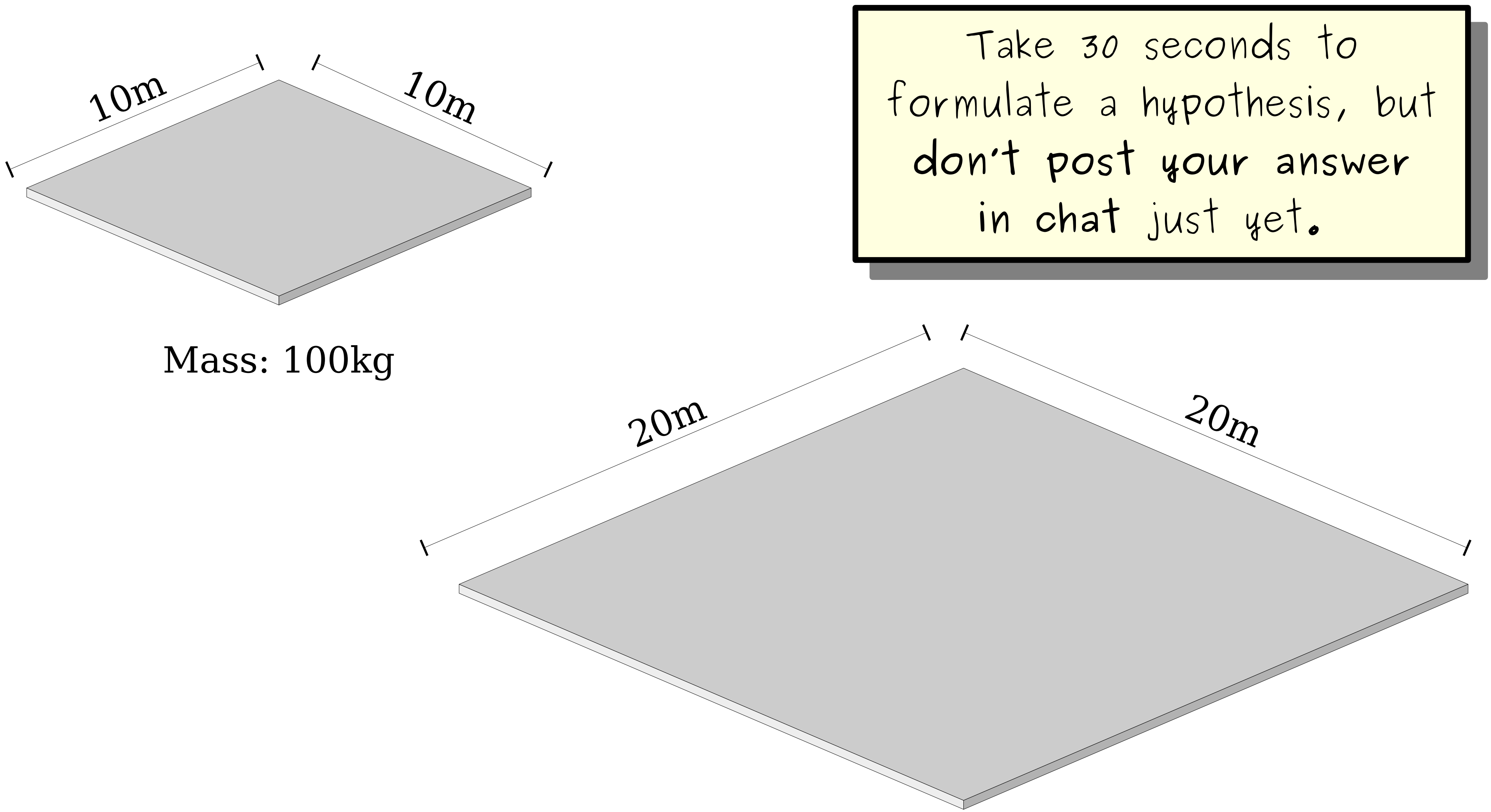
Big-O Notation

Apply to Section Lead!

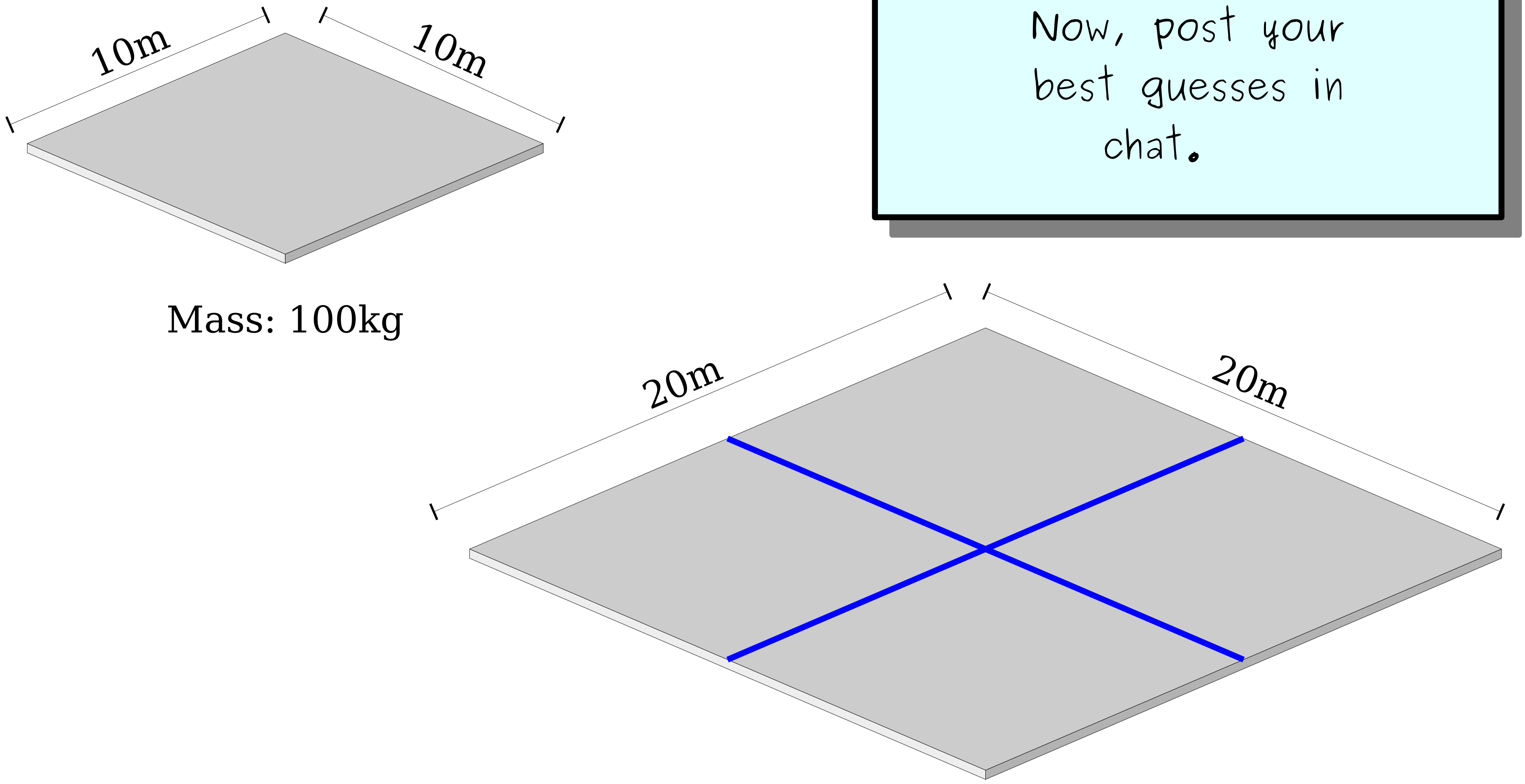
Don't just take my word for it…

Estimating Quantities



These two square plates are made of the same material.
They have the same thickness.

What's your best guess for the mass of the second square?



Now, post your best guesses in chat.

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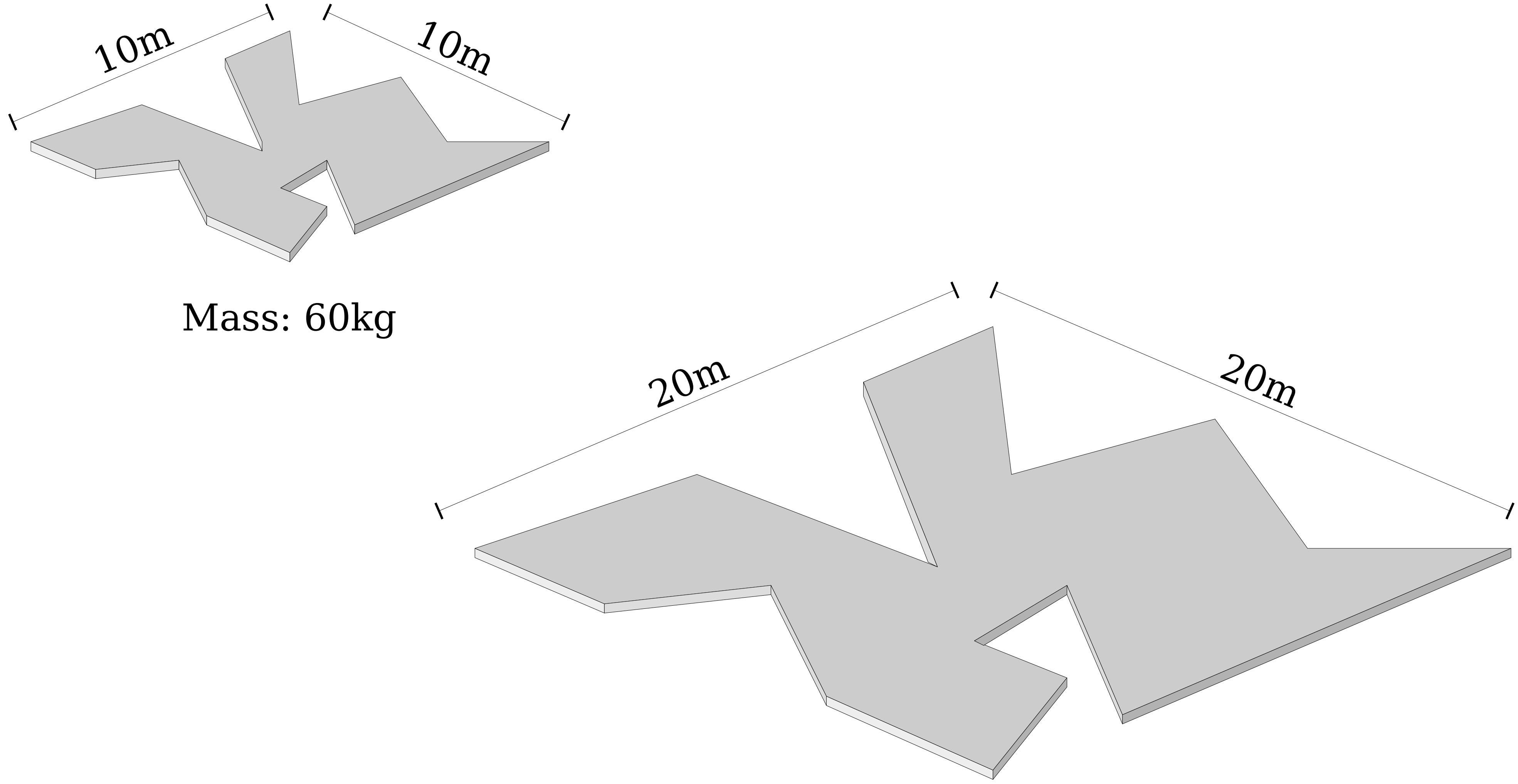
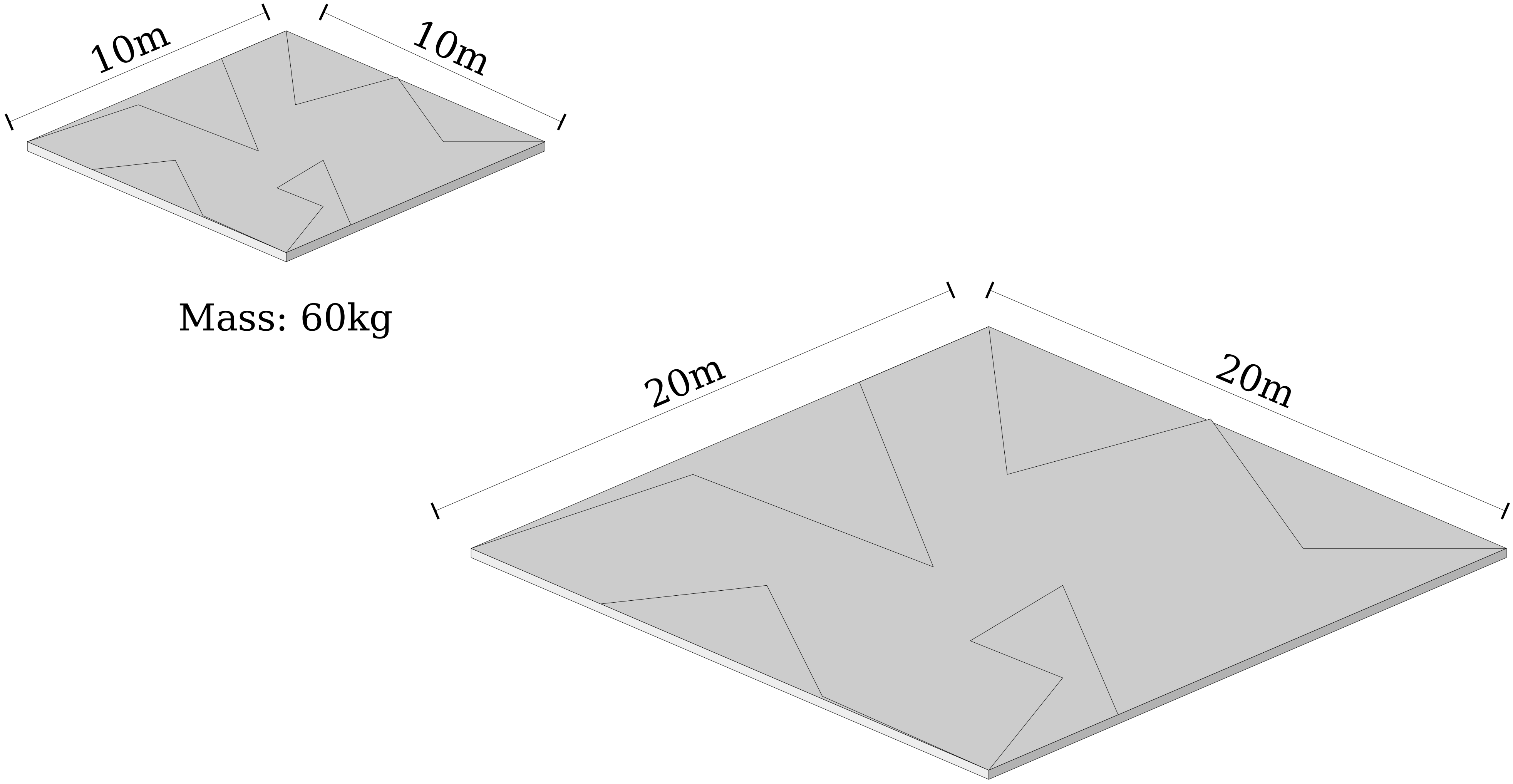


These two figures are made of the same material.
They have the same thickness.

What's your best guess for the mass of the second figure?



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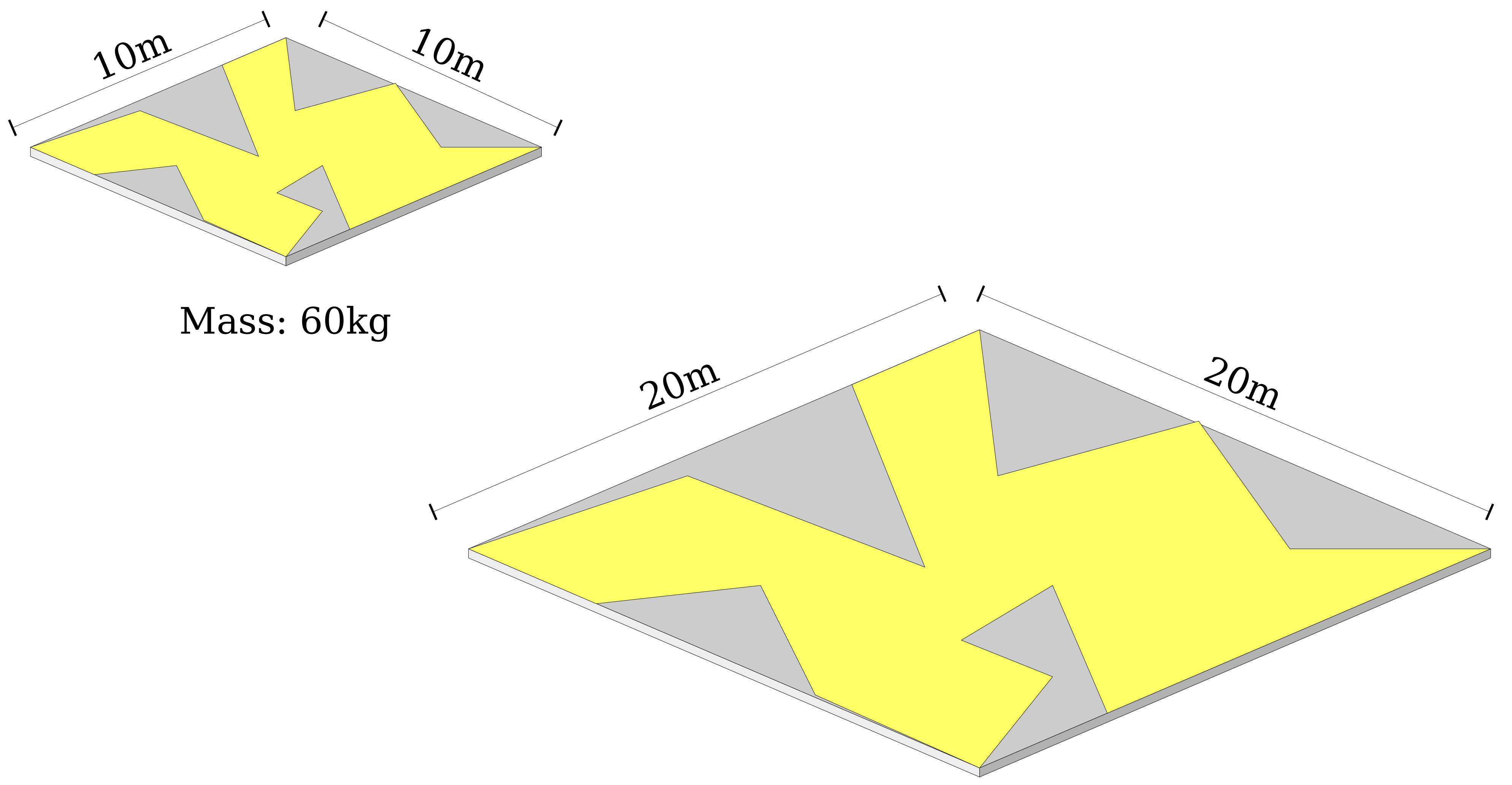

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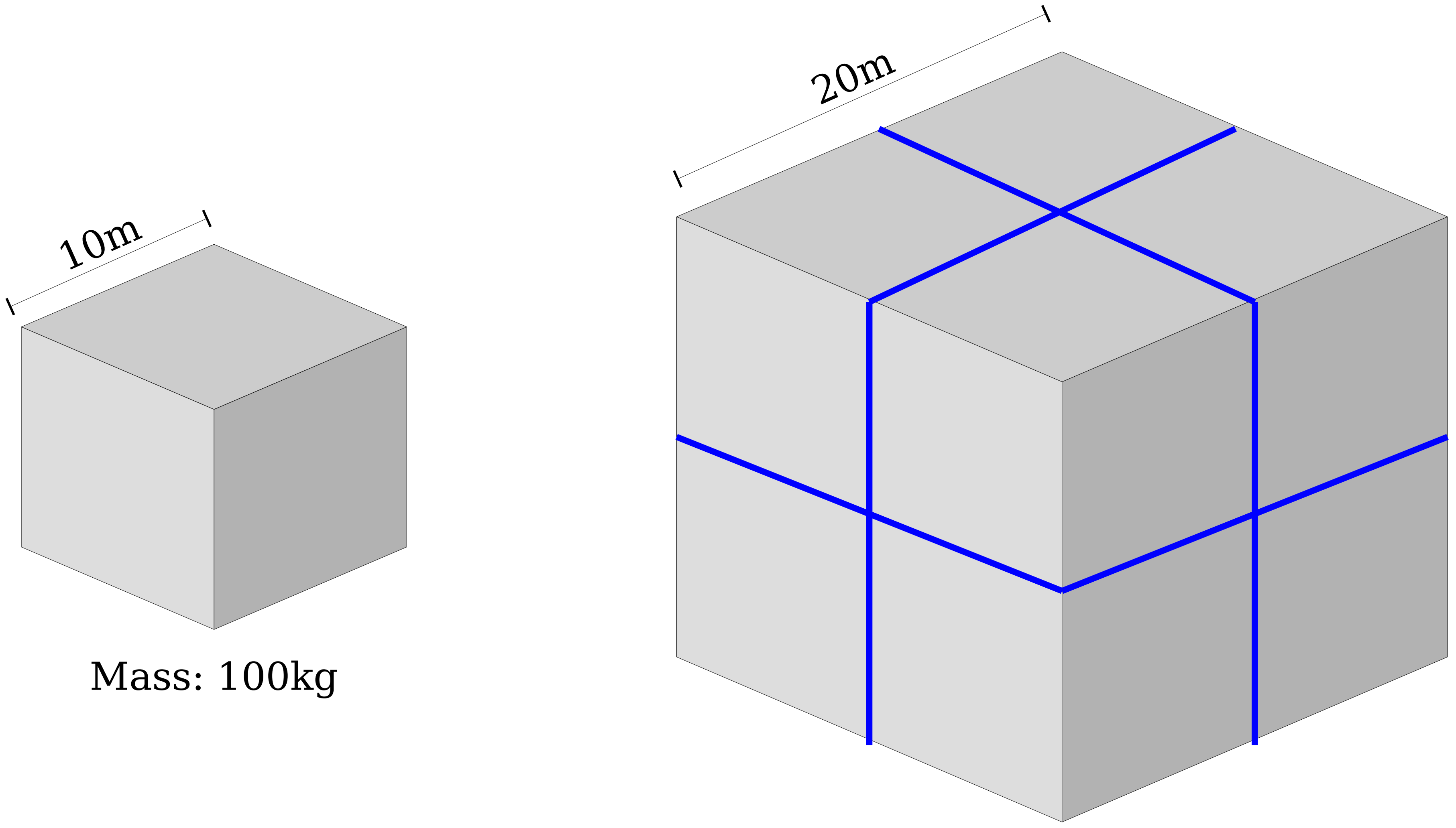
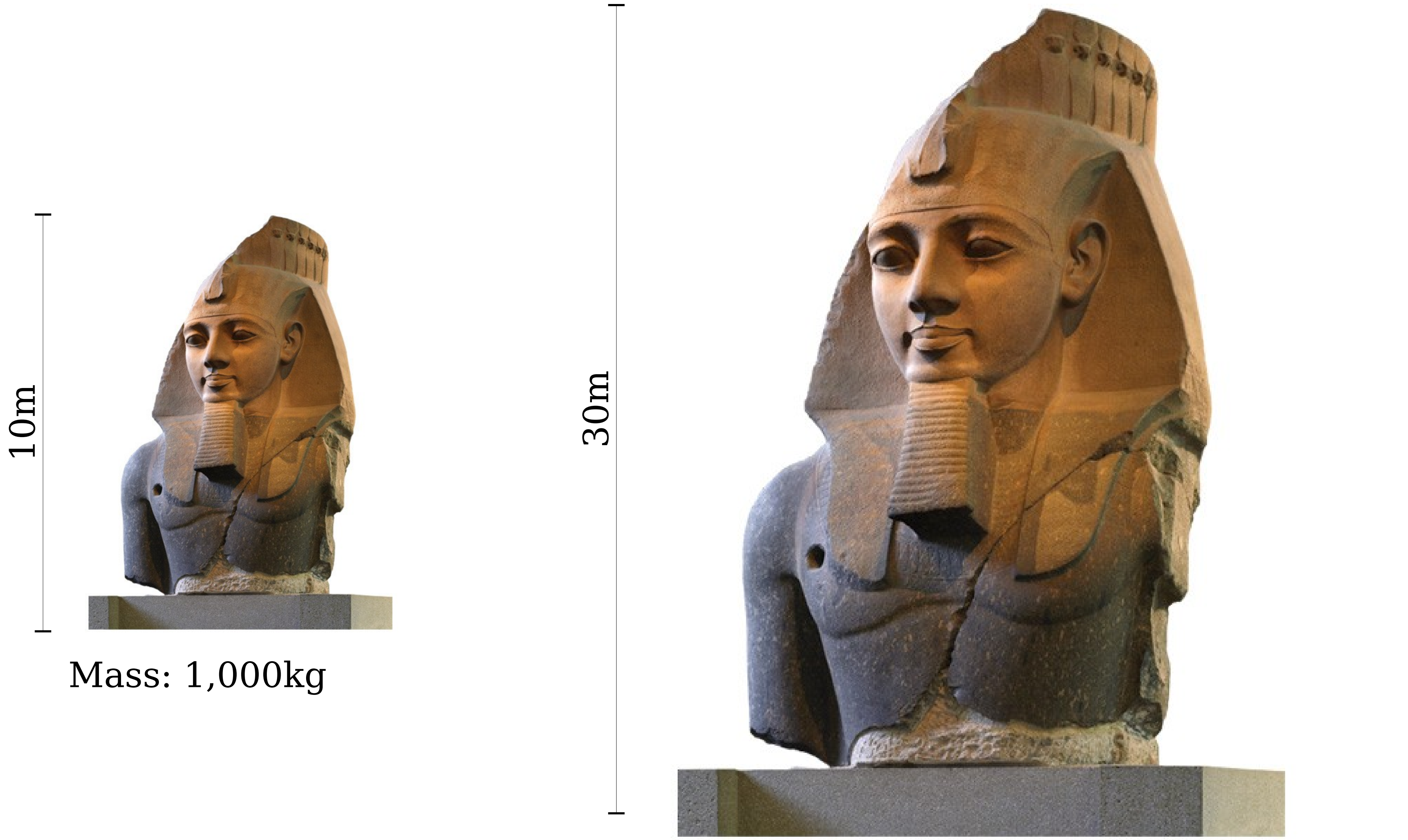


These two cubes are made of the same material.

What’s your best guess for the mass of the second cube?



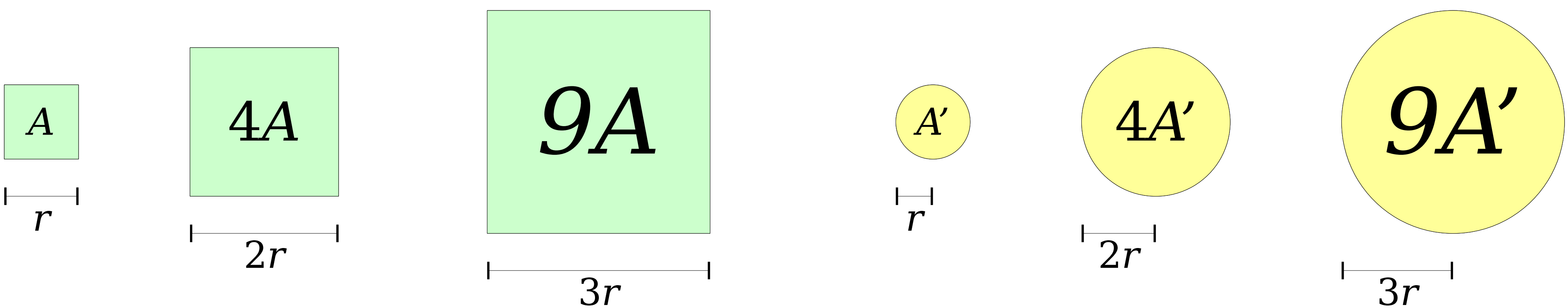
These two statues are made of the same material.

What's your best guess for the mass of the second statue?

Knowing the rate at which some quantity scales allows you to predict its value in the future, even if you don’t have an exact formula.

Big-O Notation

- ***Big-O notation*** is a way of quantifying the rate at which some quantity grows.
- For example:
  - A square of side length r has area $O(r^2)$.
  - A circle of radius r has area $O(r^2)$.



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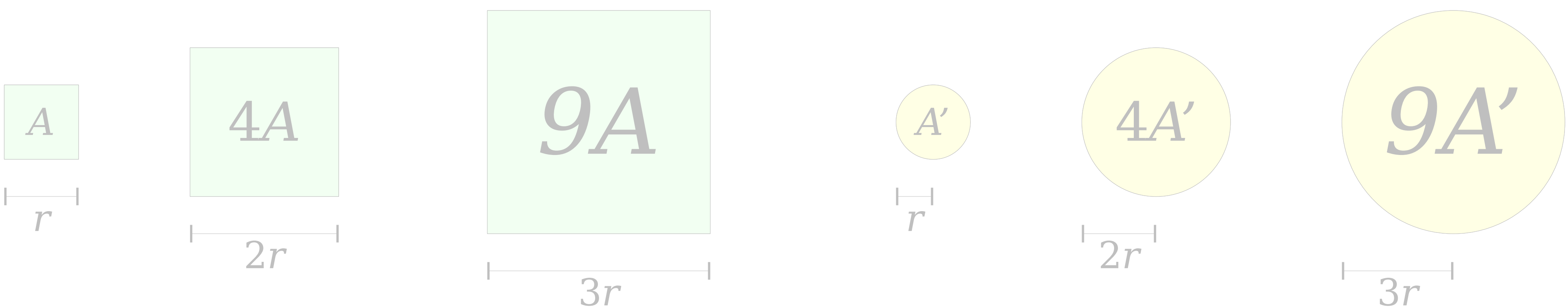

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Big-O Notation

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This just says that these quantities grow at the same relative rates. It does not say that they're equal!

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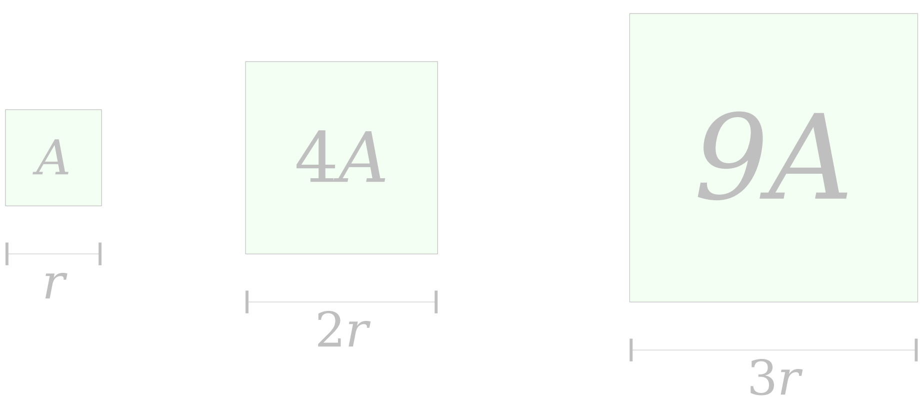

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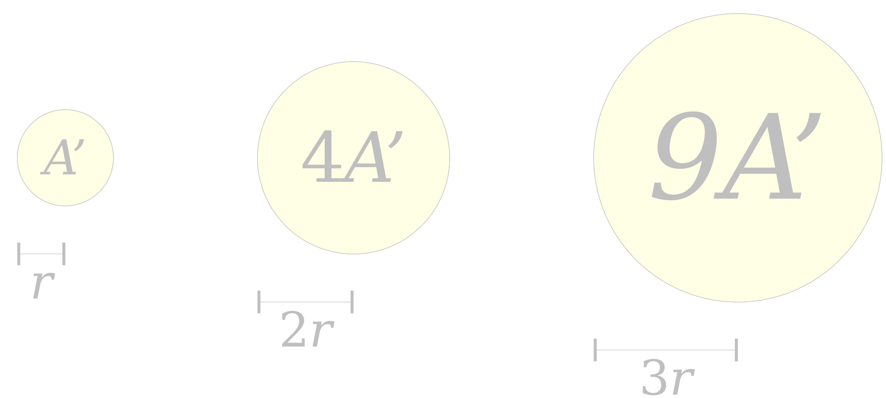

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  - A circle of radius r has area $O(r^2)$.
  - A cube of side length r has volume $O(r^3)$.
  - A sphere of radius r has volume $O(r^3)$.
  - A sphere of radius r has surface area $O(r^2)$.
  - A cube of side length r has surface area $O(r^2)$.

Example: Network Value

- ***Metcalfe's Law*** says that

  The value of a communications network with n users is $O(n^2)$.

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> Take 45 seconds to formulate a hypothesis, but **don't post your answer in chat** just yet.

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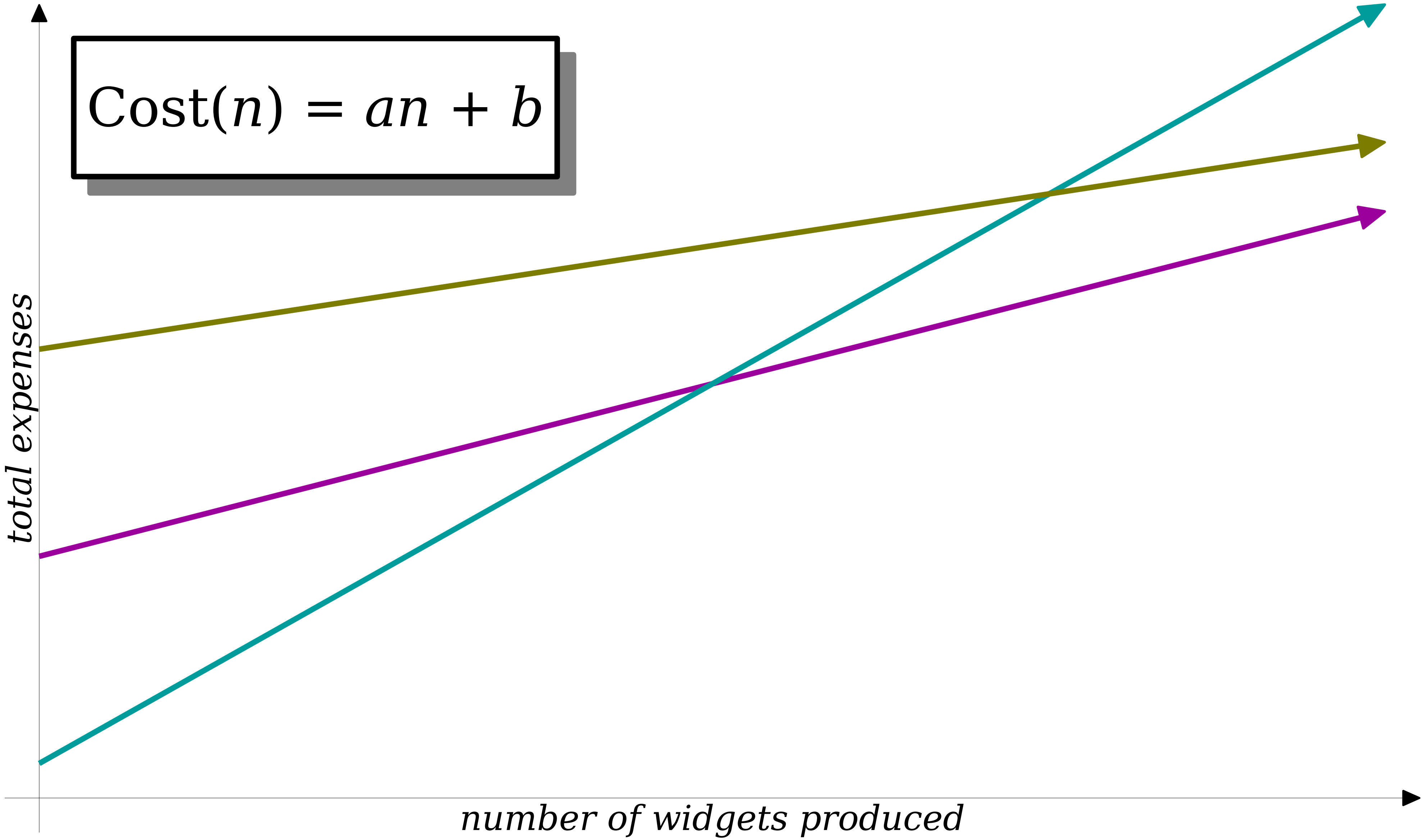
  The value of a communications network with n users is $O(n^2)$.

- Imagine a social network has 10,000,000 users and is worth $10,000,000. Estimate how many users it needs to have to be worth $1,000,000,000.
- ***Reasonable guess:*** The network needs to grow its value 100×. Since value grows quadratically with size, it needs to grow its user base 10×, requiring 100,000,000 users.

A Messier Example: Manufacturing

Making Widgets



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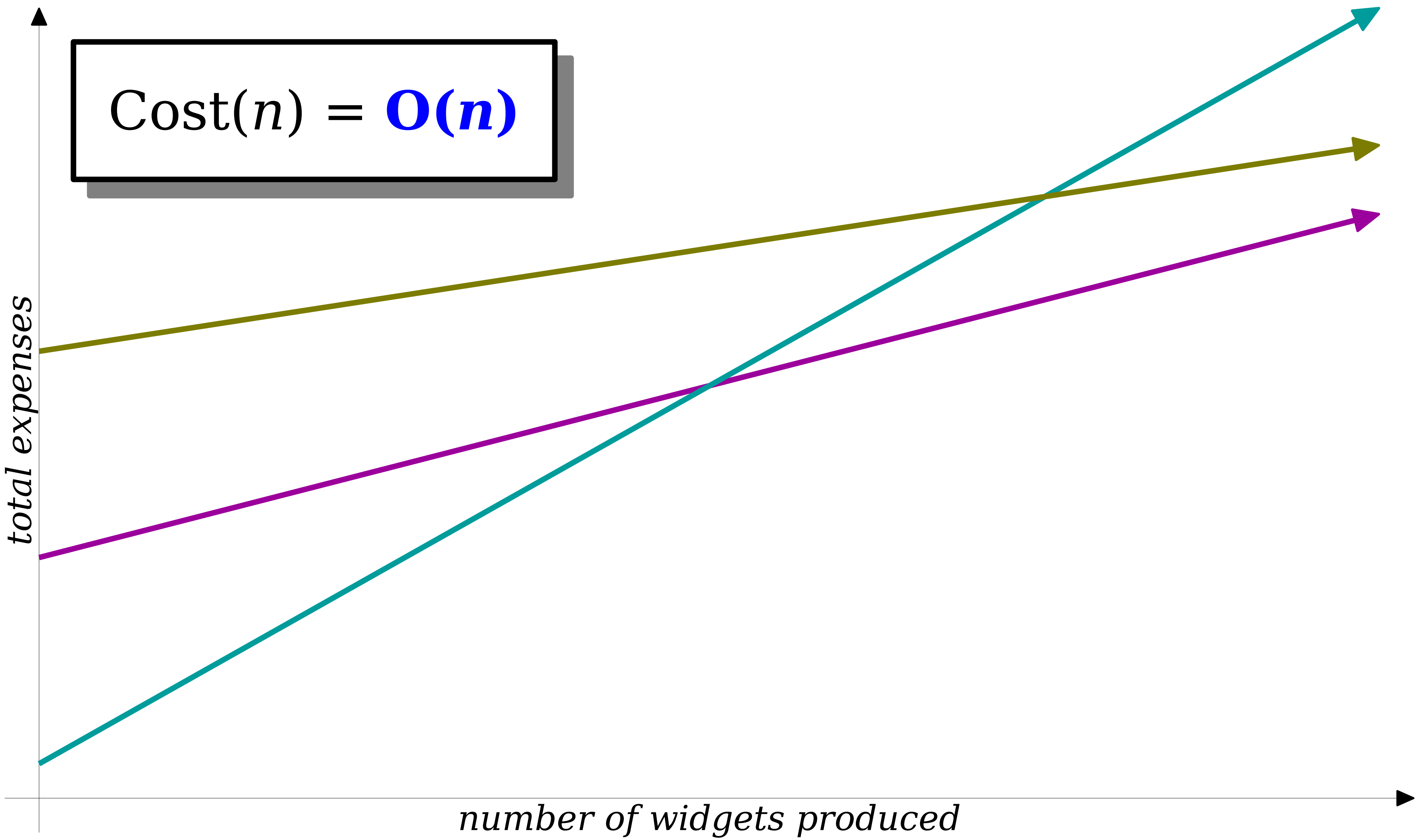

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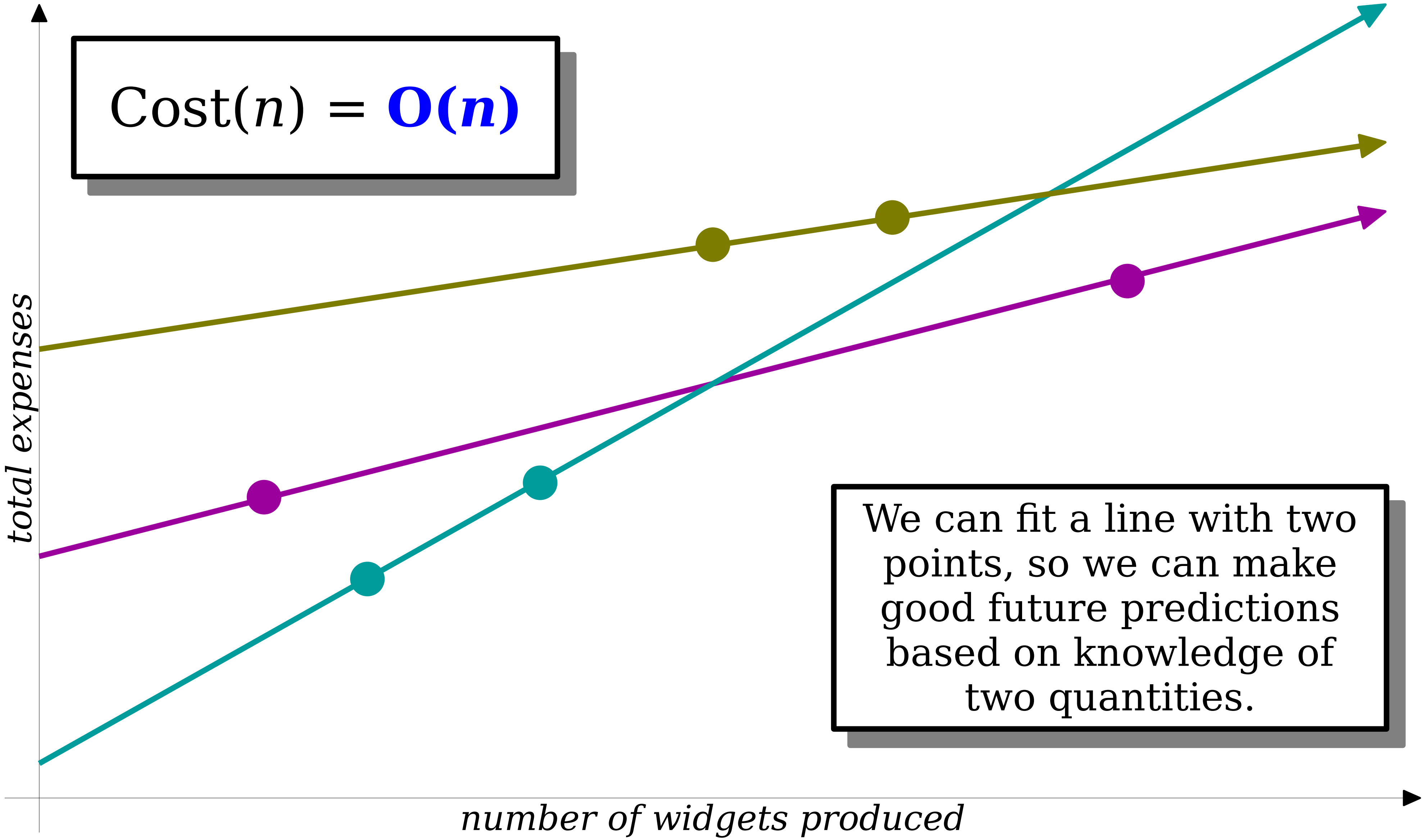

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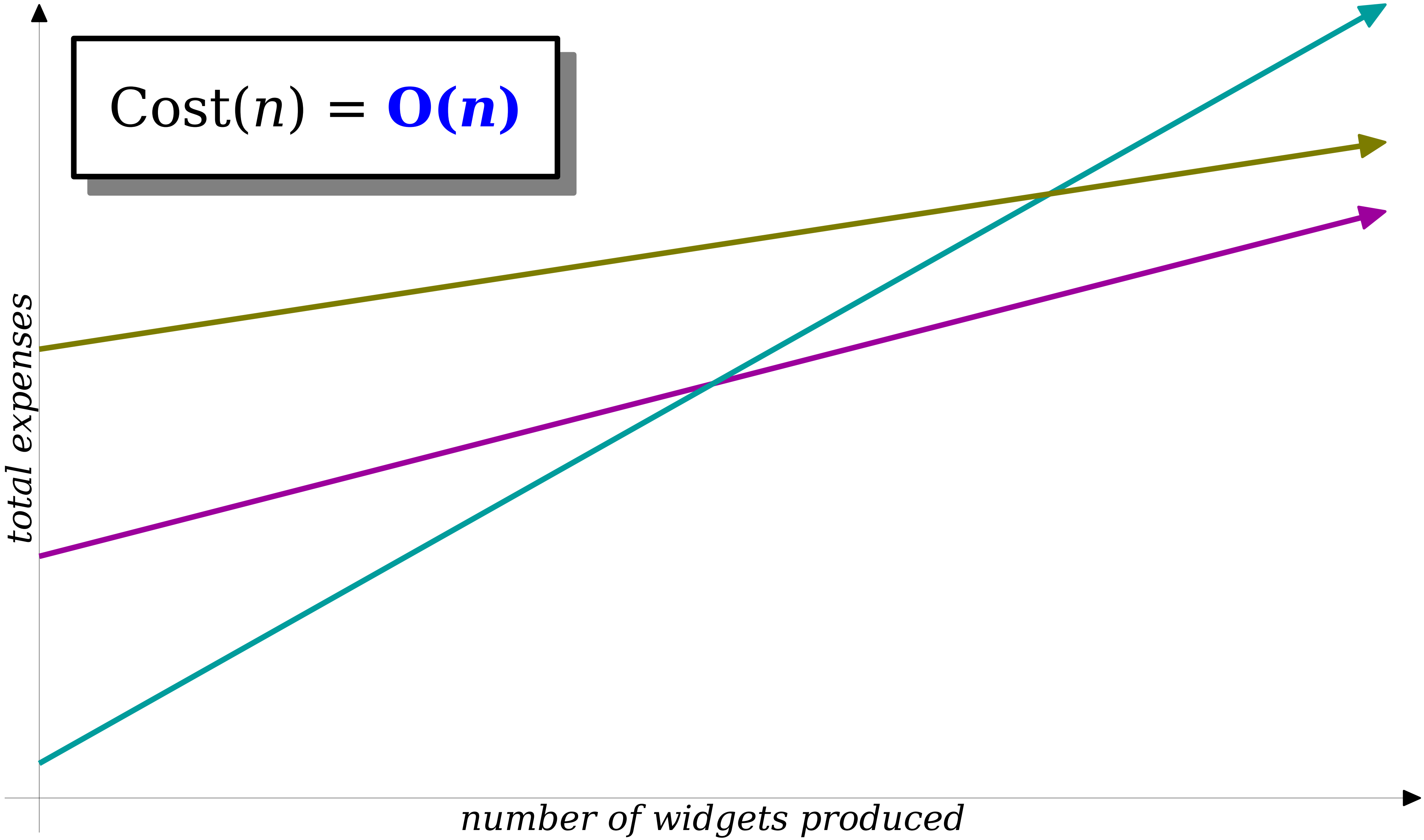

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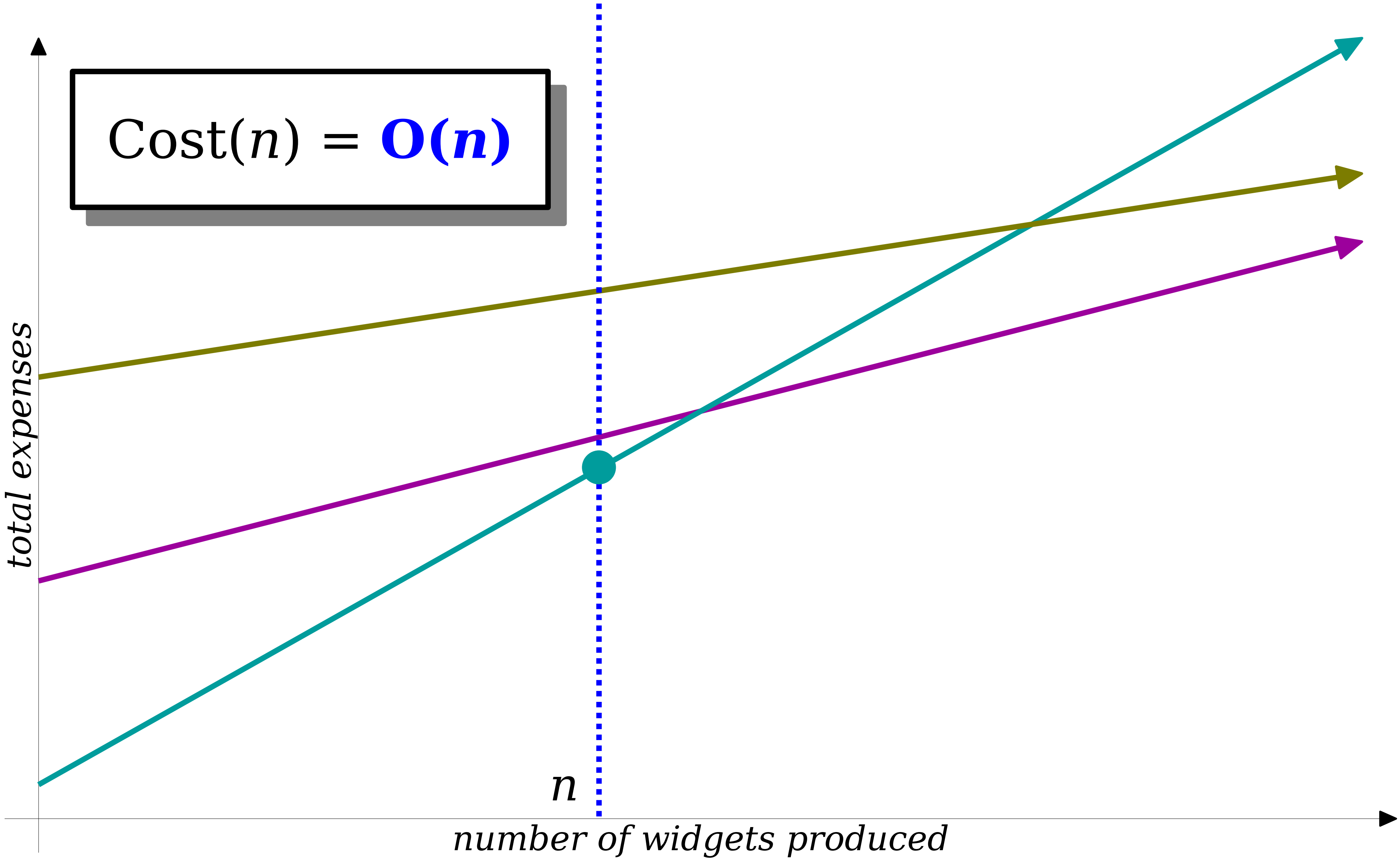

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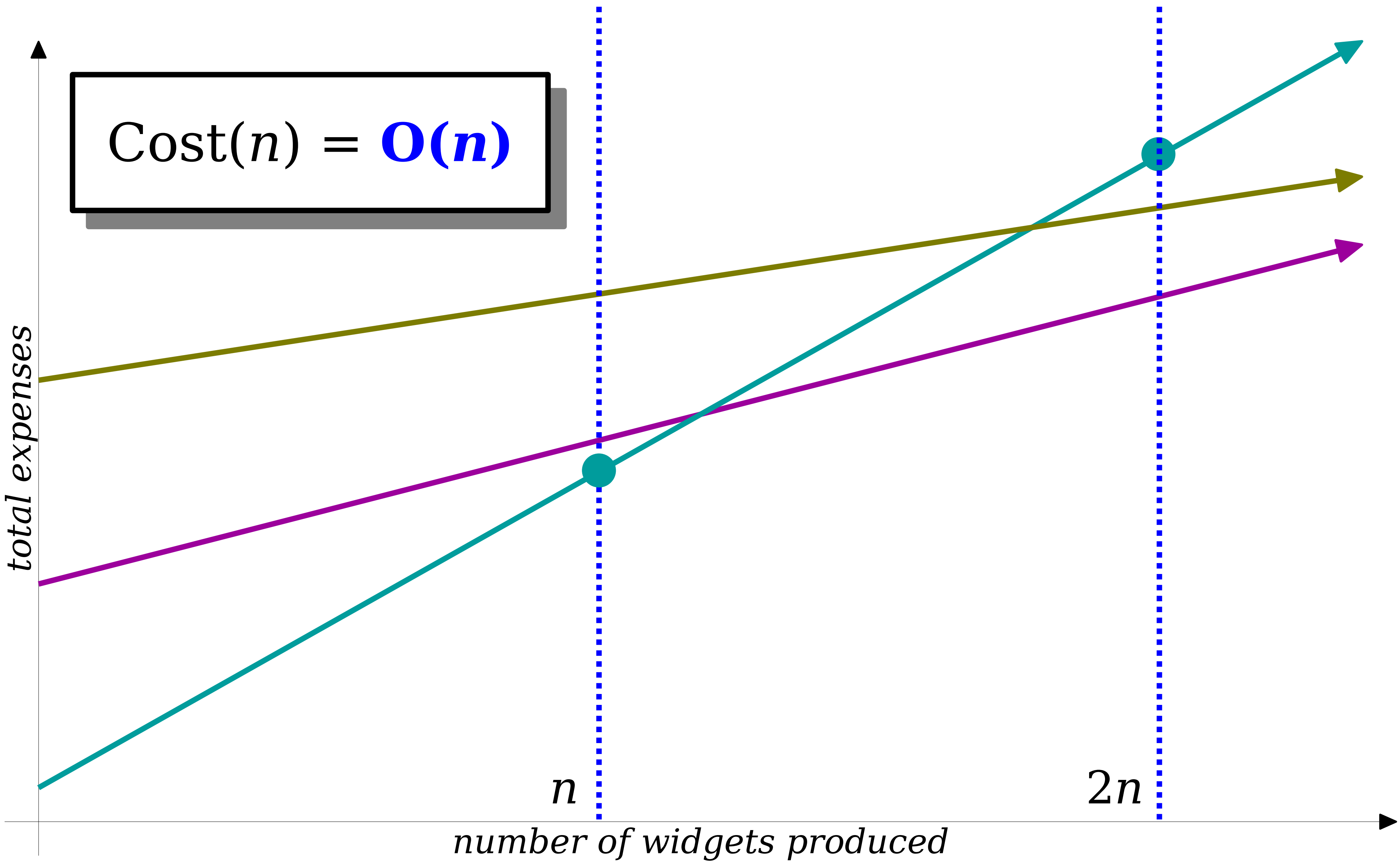

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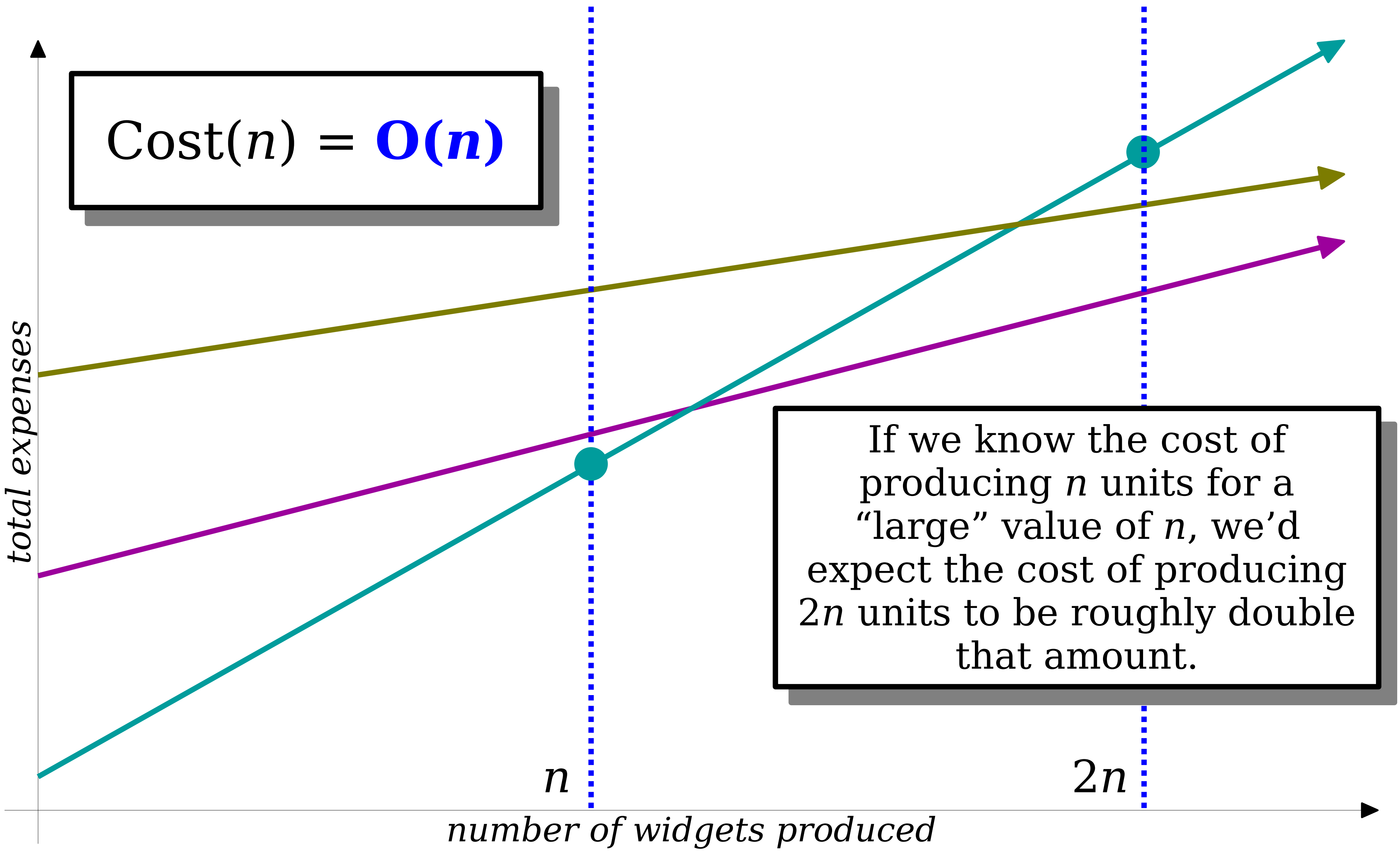

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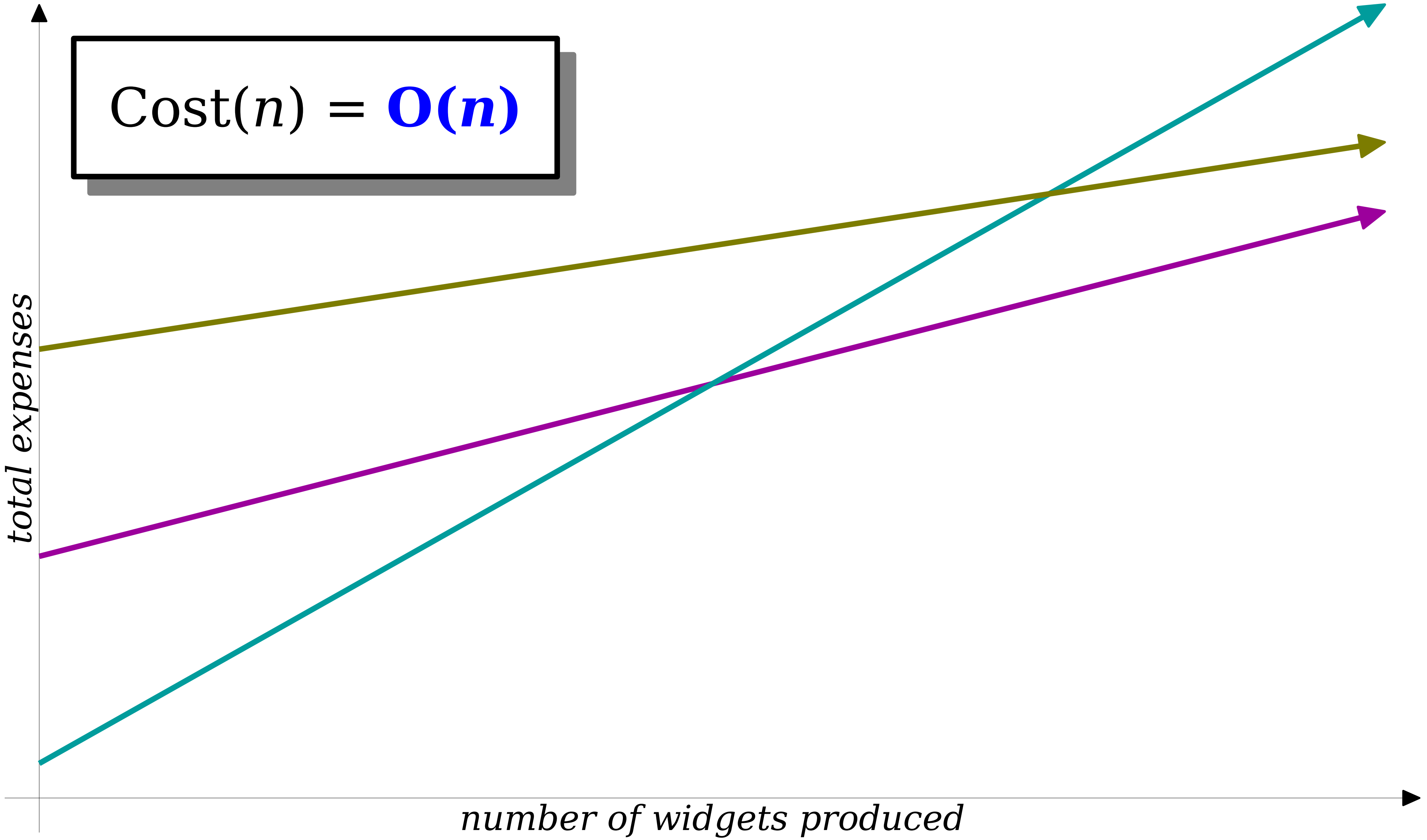

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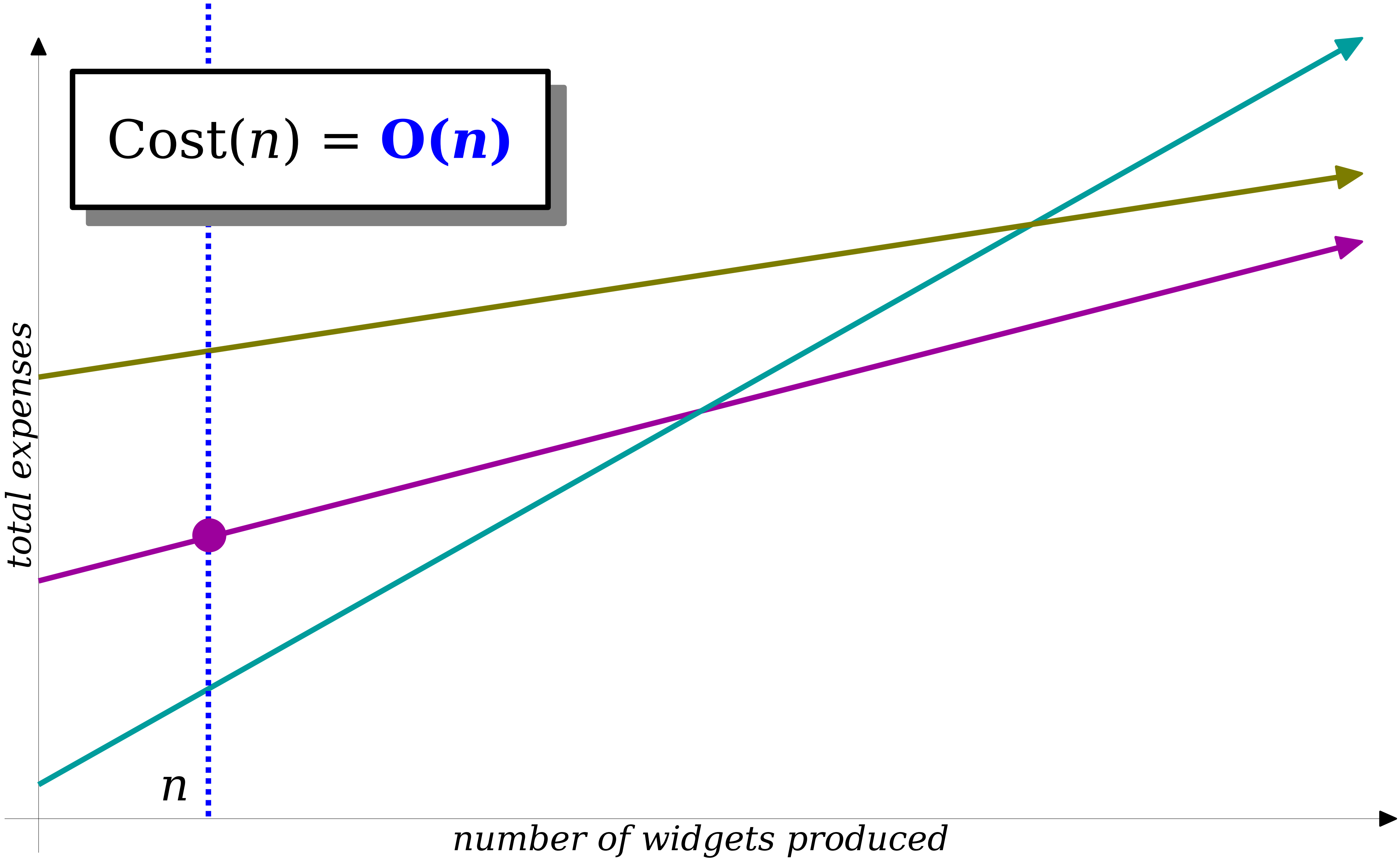
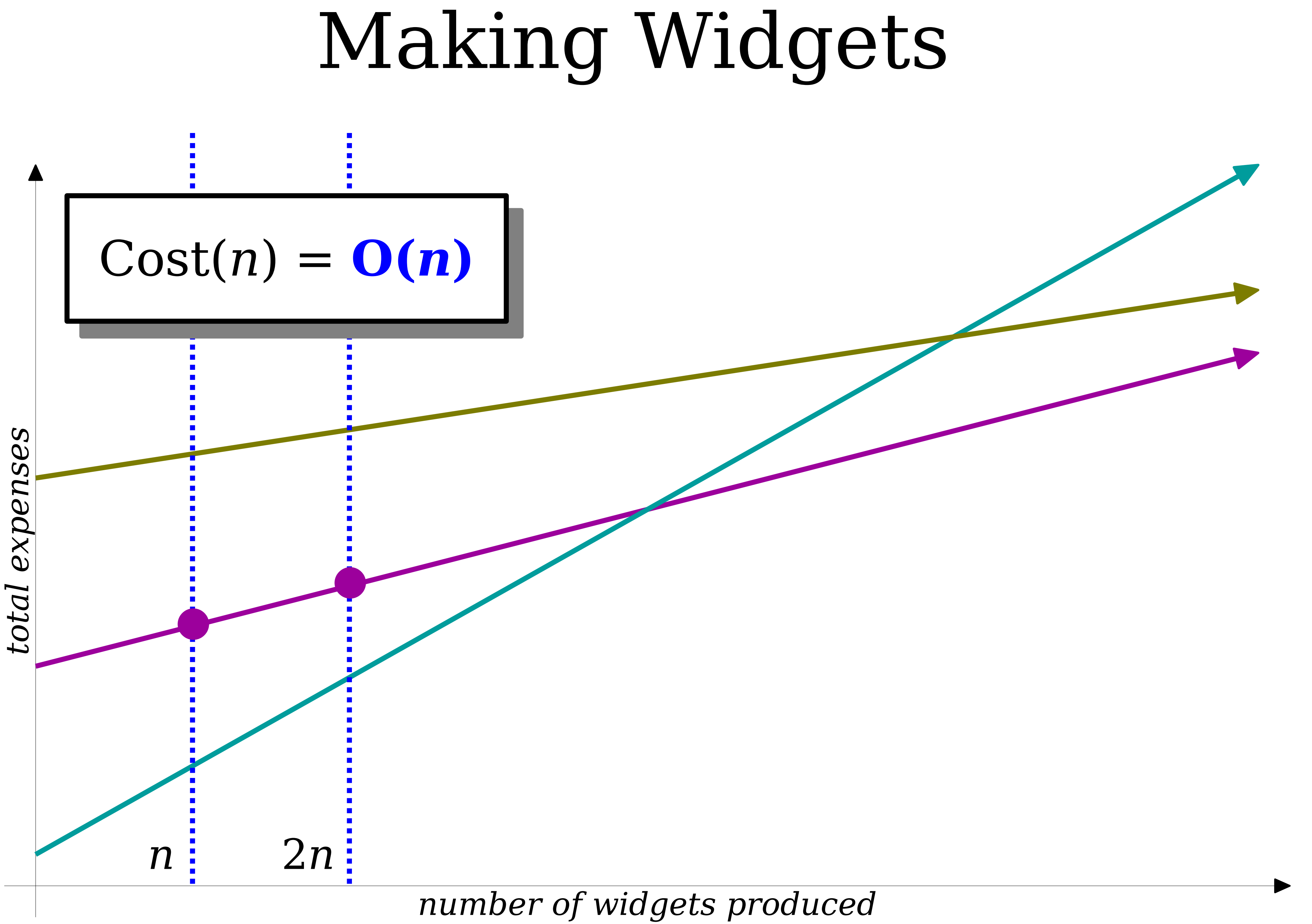
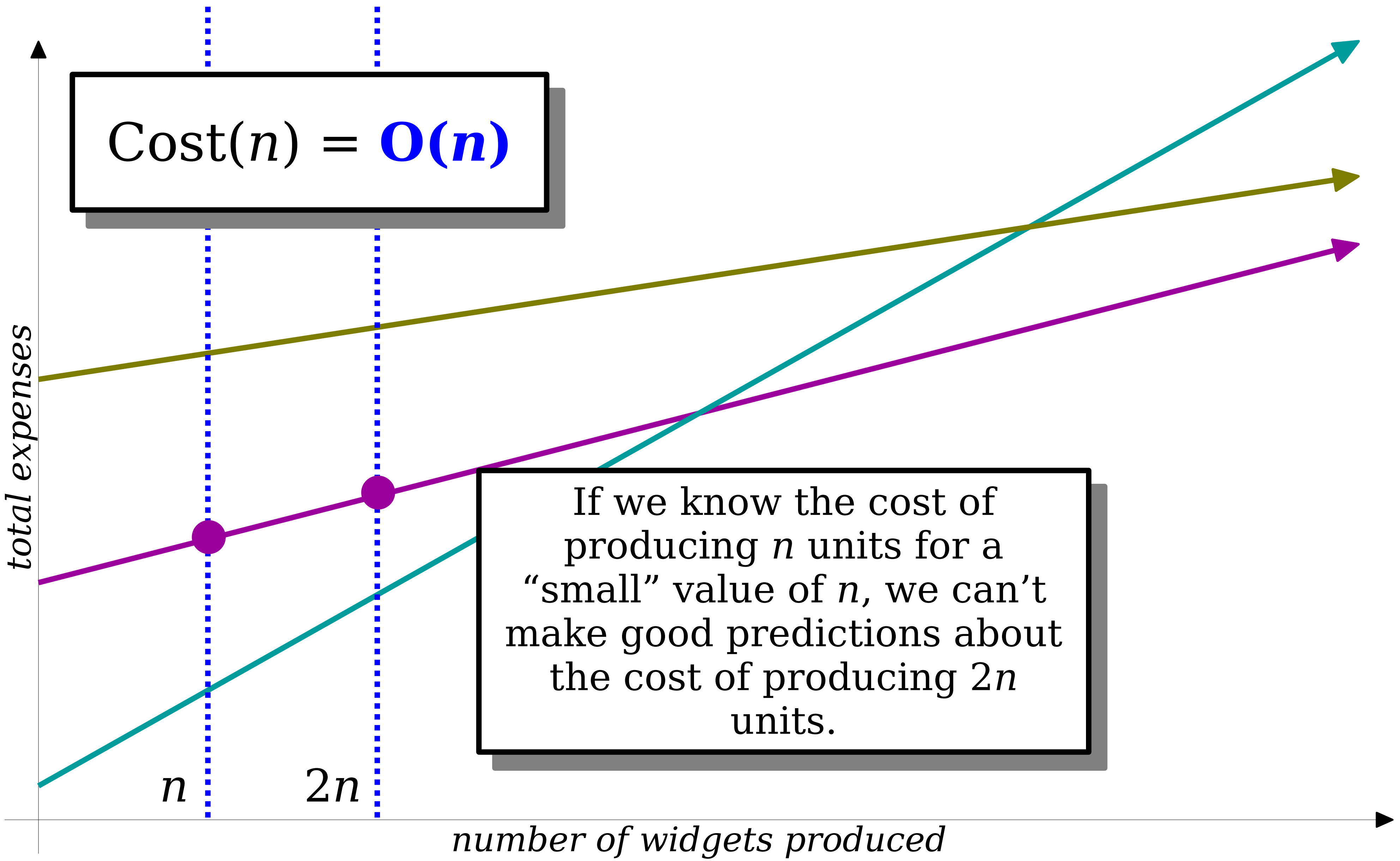



Making Widgets
Cost(n) = O(n)
total expenses
n
2n
number of widgets produced

Making Widgets



Nuances of Big-O Notation

- Big-O notation is designed to capture the ***rate at which a quantity grows***.
- It does not capture information about
  - leading coefficients: the area of a square of side length r and a circle of radius r are each $O(r^2)$.
  - lower-order terms: the functions n, $5n$, and $137n + 42$ are all $O(n)$.
- However, it's still a powerful tool for predicting behavior.

What does big-O notation have to do with computer science?

Time-Out for Announcements!

Assignment 4

- Assignment 3 was due today at 11:30AM Pacific.
  - Grace period ends Sunday at 11:30AM Pacific.
- Assignment 4 (***Recursion to the Rescue!***) goes out today. It's due next Friday.
  - Get a better feel for how to debug recursive code!
  - See how to put recursion to good use in a pair of powerful, practical programs.
- As always, feel free to ask for help when you need it! Ping us on EdStem, stop by the LaIR, call into our office hours, or email your section leader!

Midterm Exam Logistics

- Our first midterm exam runs from 12:30PM Friday, February 12th to 12:30PM Sunday, February 14th, Pacific time.
- It's a take-home coding exam. It will be designed to take three hours to complete, though you'll have the full 48-hour window to complete it.
- We'll post starter files and instructions along the lines of what we've done for the programming assignments.
- Topic coverage is as follows:
  - Primary focus is on material and concepts from Lectures 00 – 09 and Assignments 0 – 3.
  - Secondary focus is on material and concepts from Lectures 10 – 13 and Assignment 4.
- More information can be found online in the "Midterm Information" handout. We'll talk more about the exam over the next week in lecture, too!

fg

(The Unix command to resume a program that was paused)

What does big-O notation have to do with computer science?

Fundamental Question:

How do we measure efficiency?

One Idea: ***Runtime***

Why Runtime Isn't Enough

- Measuring wall-clock runtime is less than ideal, since
  - it depends on what computer you're using,
  - what else is running on that computer,
  - etc.
- Worse, ***individual runtimes can't predict future runtimes***.

```cpp
double averageOf(const Vector<int>& vec) {
  double total = 0.0;

  for (int i = 0; i < vec.size(); i++) {
      total += vec[i];
  }

  return total / vec.size();
}
```

Assume any individual statement takes one unit of time to execute. If the input `Vector` has n elements, how many time units will this code take to run?

```
double averageOf(const Vector<int>& vec) {
1 double total = 0.0;

               1              n+1          n
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}
```

Is this useful?

What does that tell us?

One possible answer: $3n + 4$.

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    return total / vec.size();
}
```

1 (`double total = 0.0;`)

1 (`int i = 0`), $n+1$ (`i < vec.size()`), n (`i++`)

n (`total += vec[i];`)

Doubling the size of the input roughly doubles the runtime.

If we get some data points, we can extrapolate runtimes to good precision.

~~One possible answer: $3n + 4$.~~

More useful answer: **$O(n)$**.

```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cout << '*' << endl;
        }
    }
}
```

How much time will it take for this code to run, as a function of *n*? Answer using big-O notation.

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void printStars(int n) {
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        do O(n) units of work;

    }
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```
void printStars(int n) {

    do O(n²) units of work;

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How much time will it take for this code to run, as a function of *n*? Answer using big-O notation.

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void printStars(int n) {
    for (int i = 0; i < n; i++) {
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        }
    }
}
```

If we time this code on input n, how much longer will it take to run on the input $2n$?

Answer: **$O(n^2)$**.

Take 45 seconds to formulate hypotheses, but **don't post your answer in chat** just yet.

How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

```cpp
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
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    }
}

void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
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void beni(int n) {
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How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {

        do 5n units of work;

    }
}
void pando(int n) {
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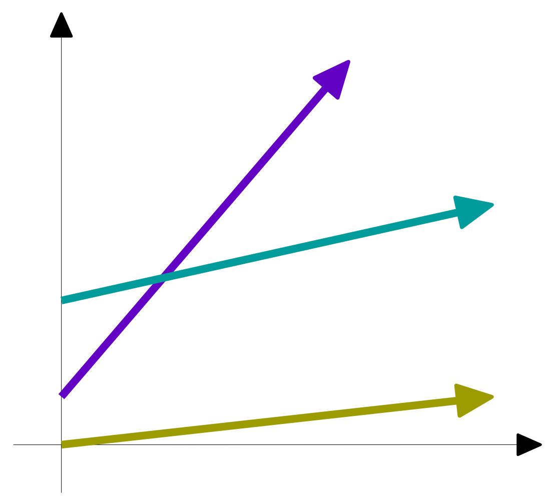

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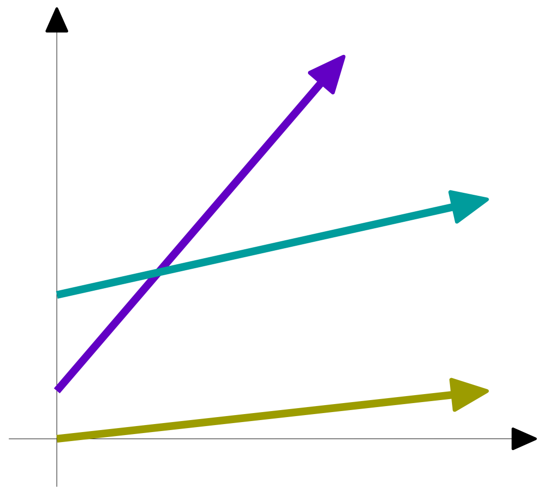

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$O(n^2)$

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    for (int i = 0; i < 3 * n; i++) {
        cout << "*" << endl;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

How much time will it take for these functions to run, as a function of n? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
        for (int j = 0; j < 5 * n; j++) {
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        }
    }
}
```

$O(n^2)$

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}
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$O(n^2)$

```
void pando(int n) {
    for (int i = 0; i < 3 * n; i++) {
        do one unit of work;
    }
    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
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}
```

$O(n^2)$

```
void pando(int n) {

    do 3n units of work;

    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
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            cout << '*' << endl;
        }
    }
}
```

$O(n^2)$

```
void pando(int n) {

    do O(n) units of work;

    for (int i = 0; i < 8; i++) {
        cout << "*" << endl;
    }
}
```

How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

```
void beni(int n) {
    for (int i = 0; i < 2 * n; i++) {
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            cout << '*' << endl;
        }
    }
}
```

$O(n^2)$

```cpp
void pando(int n) {

    do O(n) units of work;


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How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

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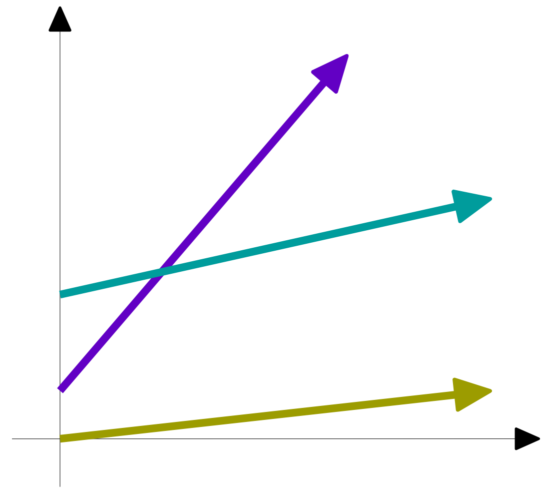

Any linear function is O(n). The y-intercept doesn't matter.

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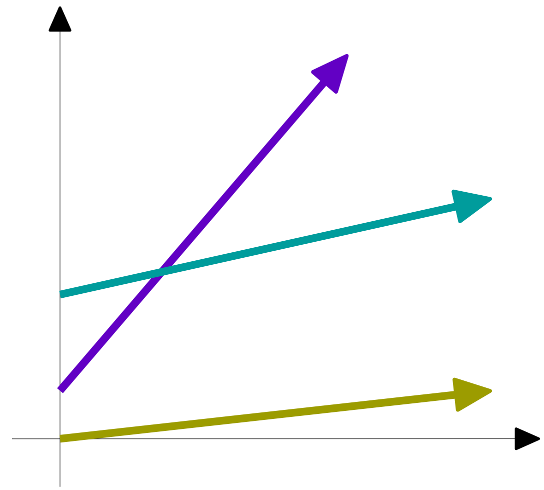

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How much time will it take for these functions to run, as a function of *n*? Answer using big-O notation.

Recap from Today

- Big-O notation captures the rate at which a quantity grows or scales as the input size increases.
- Big-O notation ignores low-order terms and constant factors.
- “When in doubt, work inside out!” When you see loops, work from the inside out to determine the big-O complexity.

Your Action Items

- ***Read Chapter 10.1 – 10.2.***
  - It’s all about big-O and efficiency, and it’s a great complement to what we covered today.
- ***Start Assignment 4*.**
  - If you want to follow our suggested timetable, aim to complete the debugging exercise and Doctors Without Orders by Monday.

Next Time

- ***Sorting Algorithms***
  - How do we get things in order?
- ***Designing Better Algorithms***
  - Using predictions from big-O notation.