

# Trees

**What are real-life examples of where you've seen sorting algorithms in action?**  
(put your answers the chat)



# Roadmap

C++ basics

User/client

vectors + grids

stacks + queues

sets + maps

Core  
Tools

testing

algorithmic  
analysis

recursive  
problem-solving

Object-Oriented  
Programming

Implementation

arrays

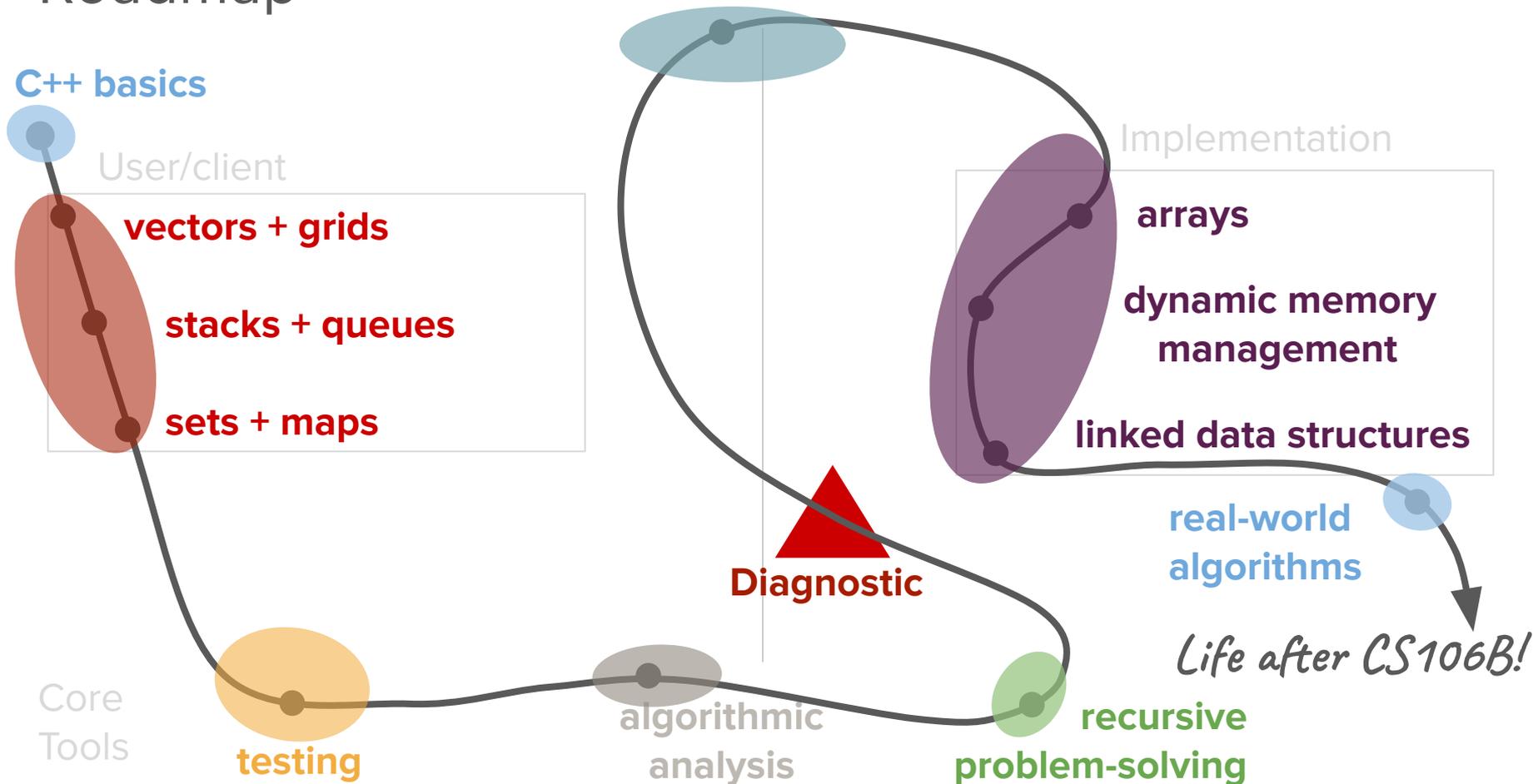
dynamic memory  
management

linked data structures

real-world  
algorithms

*Life after CS106B!*

**Diagnostic**



# Roadmap

## Object-Oriented Programming

### C++ basics

User/client

**vectors + grids**

**stacks + queues**

**sets + maps**

Implementation

arrays

dynamic memory management

linked data structures

real-world algorithms

*Life after CS106B!*

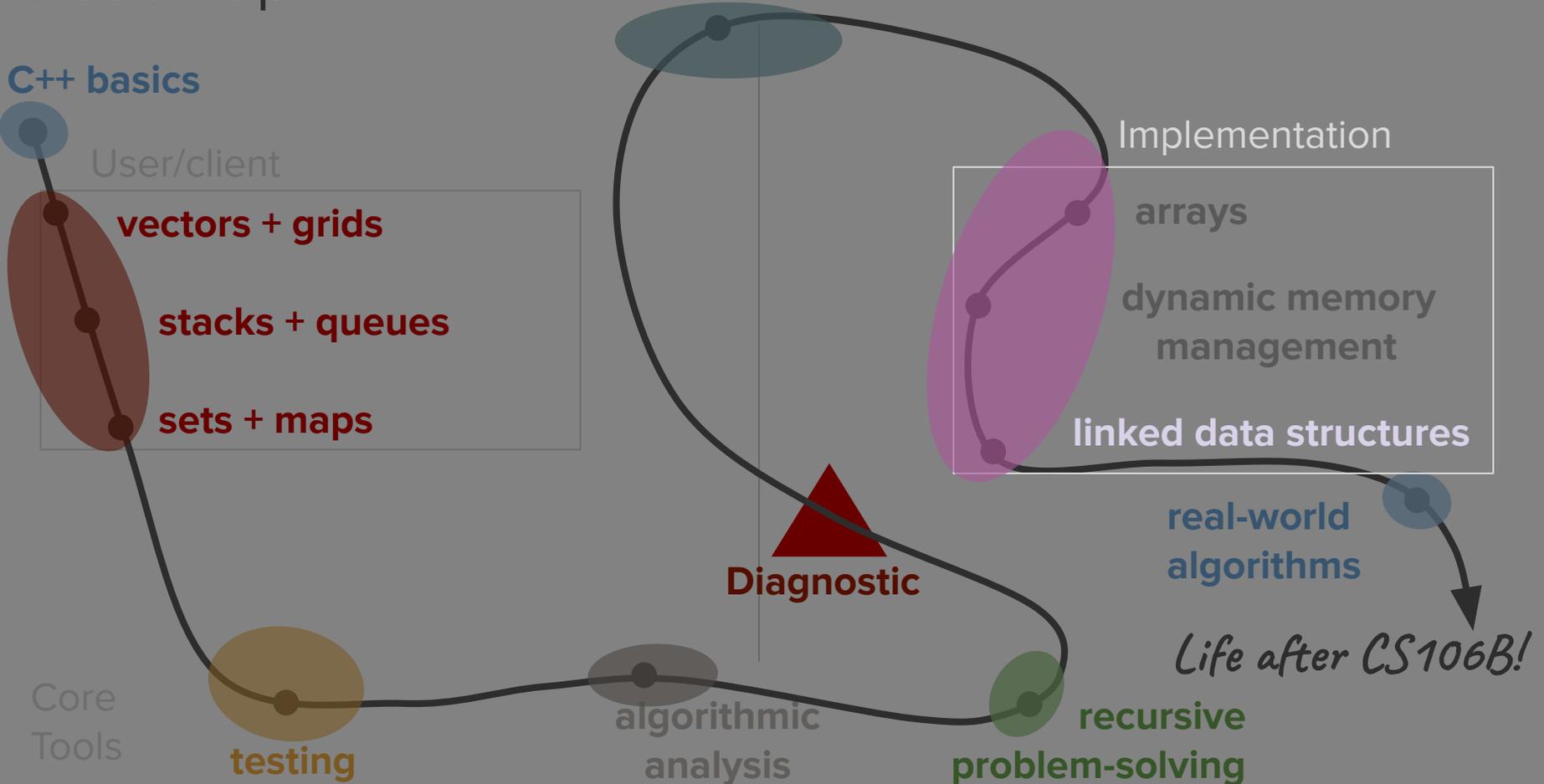
Core Tools

**testing**

algorithmic analysis

**recursive problem-solving**

**Diagnostic**



# Today's questions

How can we better  
organize data stored in a  
linked data structure?

# Today's topics

1. Sorting + Linked Data Structure Overview
2. Introduction to Trees
3. Trees in C++

# Review

[sorting + linked data structures]

# Sorting

- Sorting is a powerful tool for organizing data in a meaningful format!
- There are many different methods for sorting data:
  - Selection Sort
  - Insertion Sort
  - Mergesort
  - Quicksort
  - And many more...
- Understanding the different runtimes and tradeoffs of the different algorithms is important when choosing the right tool for the job!

## Sorting Big-O Cheat Sheet

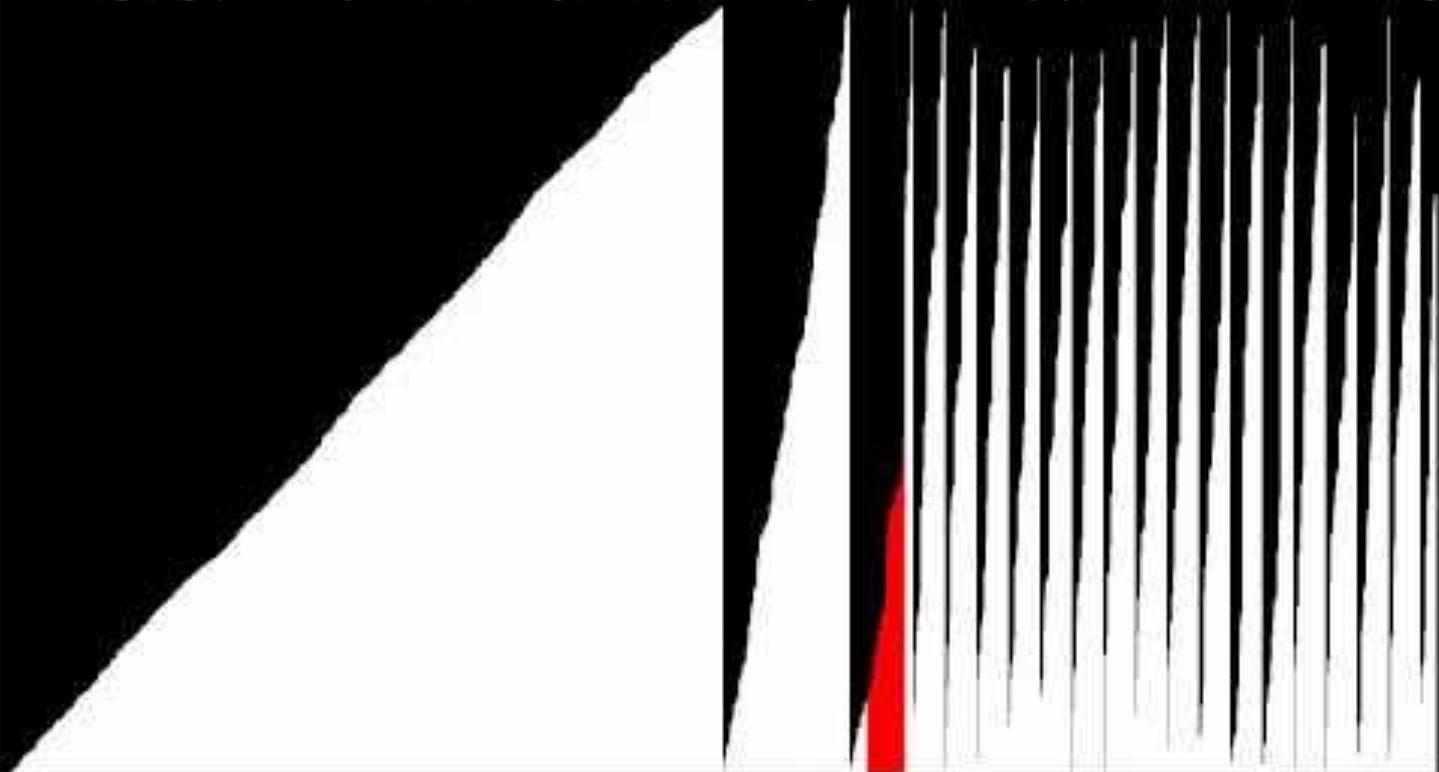
Sort	Worst Case	Best Case	Average Case
Insertion	$O(n^2)$	$O(n)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quicksort	$O(n^2)$	$O(n \log n)$	$O(n \log n)$

<https://www.toptal.com/developers/sorting-algorithms>

 Play All	 Insertion	 Selection	 Bubble	 Shell	 Merge	 Heap	 Quick	 Quick3
 Random								
 Nearly Sorted								
 Reversed								
 Few Unique								

std::stable\_sort (gcc) - 8950 comparisons, 20268 array accesses, 1.00 ms delay

<http://panthema.net/2013/sound-of-sorting>



# Assignment 6 Tips

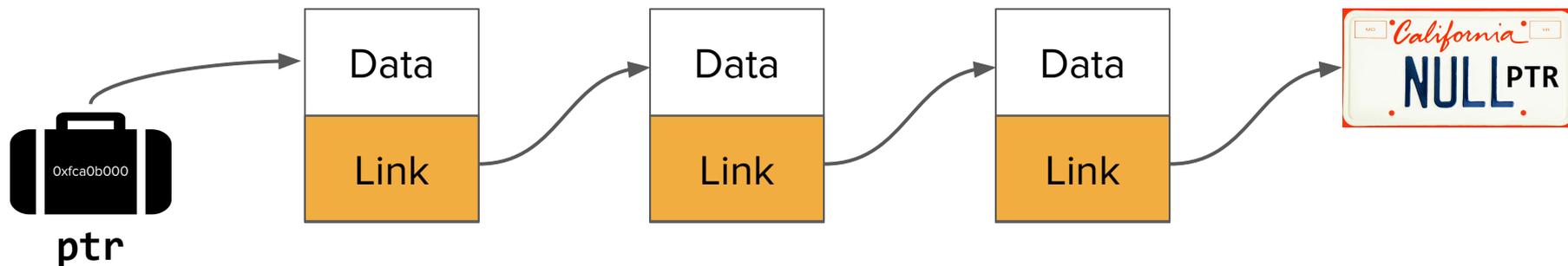
- When implementing the sorting algorithm on linked lists, it is strongly recommended to implement helper functions for the divide/join components of the algorithm.
  - For quicksort this means having helper functions for the partition and concatenate operations
- These helper functions should be implemented iteratively, but the overall sorting algorithm itself operates recursively. Mind the distinction!
- Write tests for your helper functions first! Then, write end-to-end tests for your sorting function.

# Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.

# Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.



# Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.
- Linked data structures are distinguished by the fact that they stored data in a **distributed** manner. This means that the data is stored across many different locations in computer memory.

# Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.
- Linked data structures are distinguished by the fact that they stored data in a **distributed** manner. This means that the data is stored across many different locations in computer memory.
- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."

# Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.
- Linked data structures are distinguished by the fact that they stored data in a **distributed** manner. This means that the data is stored across many different locations in computer memory.
- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."
- Using pointers allows us to **create links** to other nodes to impose structure.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
  - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
  - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room.
  - Because all the data was stored in dynamic memory, expanding the size of the linked list was very easy and never required an expensive "re-sizing" operation that had to copy all the data.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists:

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists:
  - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists:
  - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.
  - Finding elements in a linked list is an  **$O(n)$**  operation, which can get slow when we want to store many elements.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists:
  - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.
  - Finding elements in a linked list is an  **$O(n)$**  operation, which can get slow when we want to store many elements.
  - We couldn't feasibly write recursive algorithms that traversed linked lists, due to stack frame limits that came into play since traversal algorithms required one stack frame per node.

# Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists.
- **Question:** Can we organize data in a linked data structure in such a way that the path between the "front" and any element in the structure is short (better than  $O(n)$ ) even if there are many elements?

How can we better organize  
data stored in a linked data  
structure?

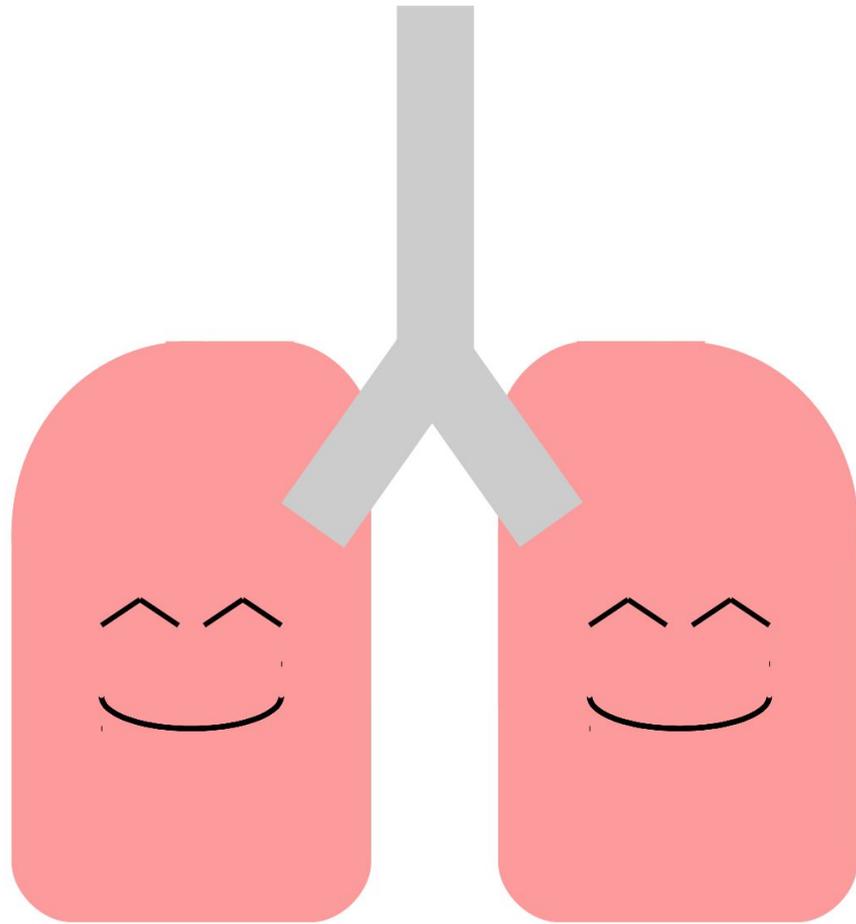
# Interactive Exercise

[borrowed from Keith Schwarz]

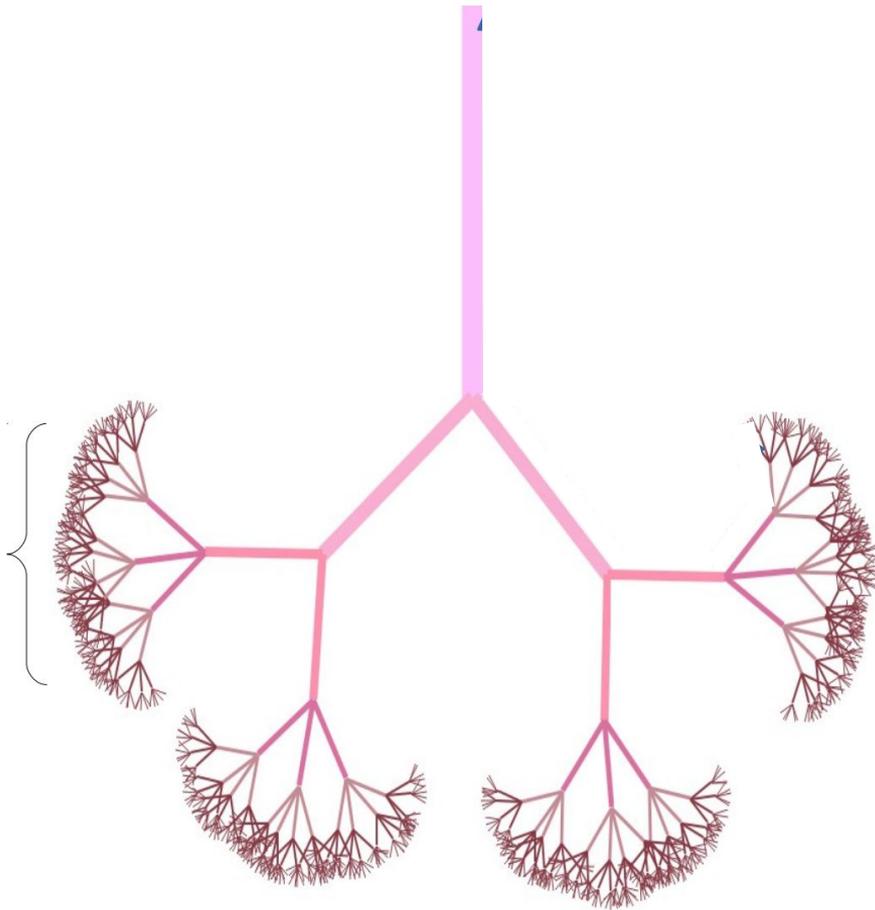
**Take a deep breath.**

**And exhale...**

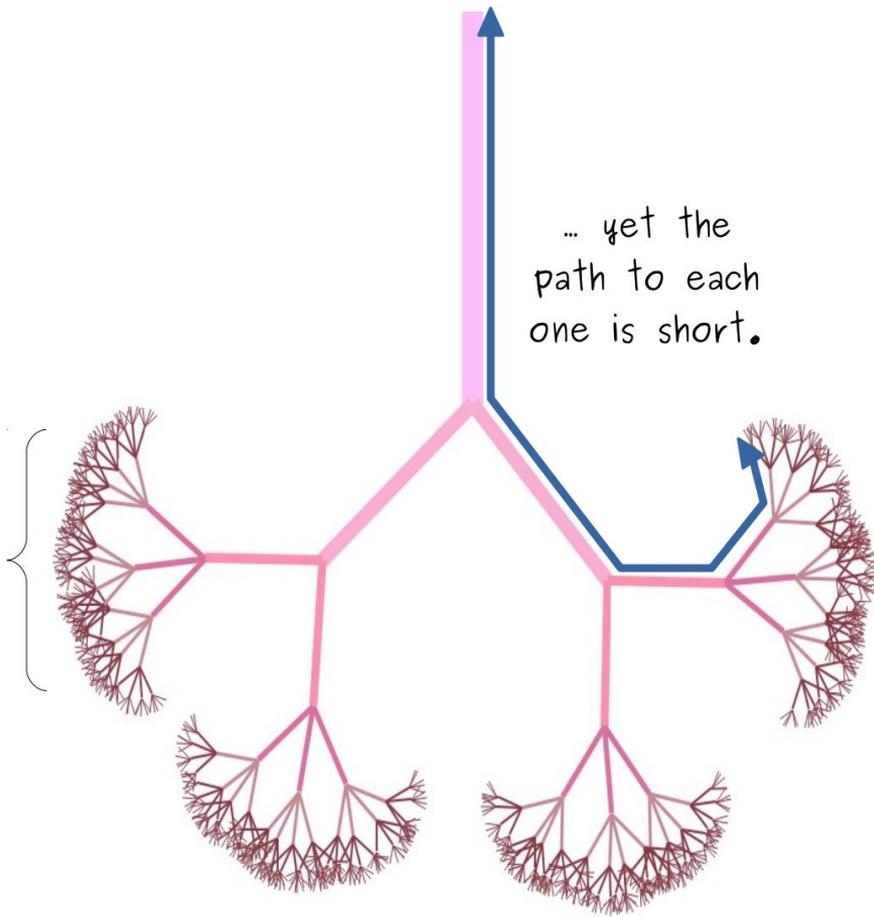
**Feel nicely oxygenated?**



Your lungs  
have about  
500 million  
alveoli...



Your lungs  
have about  
500 million  
alveoli...



... yet the  
path to each  
one is short.

**Key Idea:** The distance from each element in this structure to the top of the structure is small, even if there are many elements.

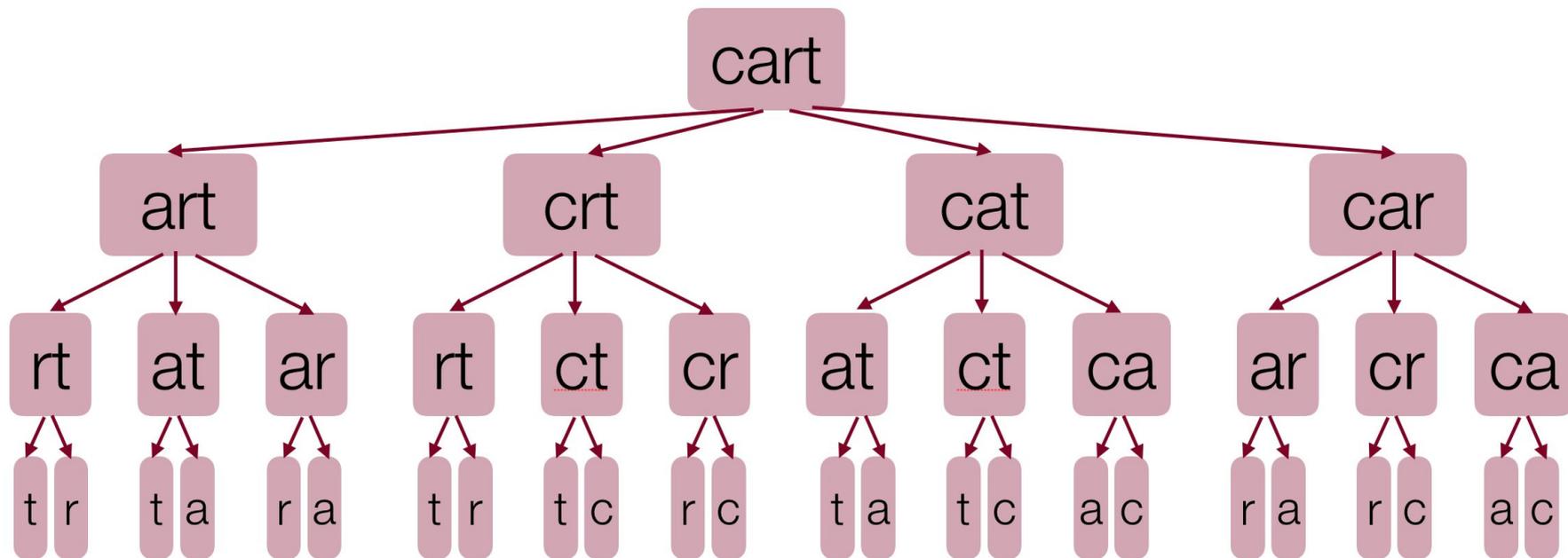
Trees

# Throwback Thursday (on Wednesday)

- We've already seen trees before in this class... decision trees!

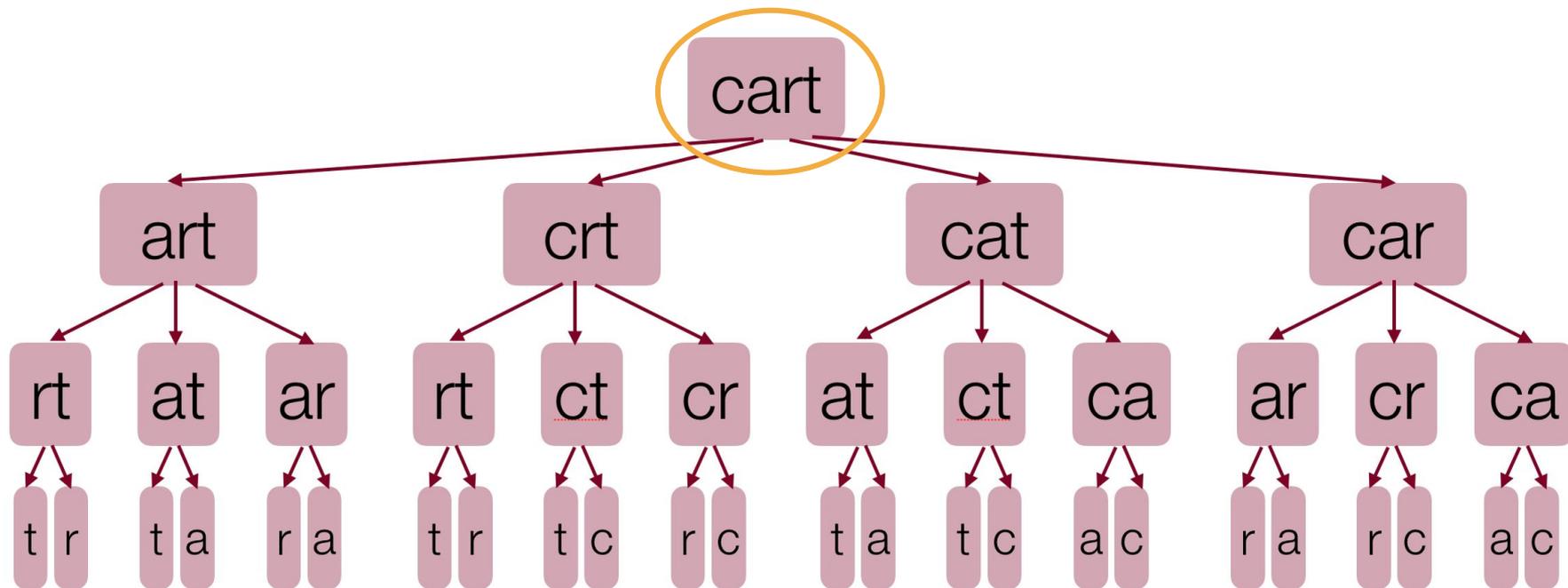
# Throwback Thursday (on Wednesday)

- We've already seen trees before in this class... decision trees!



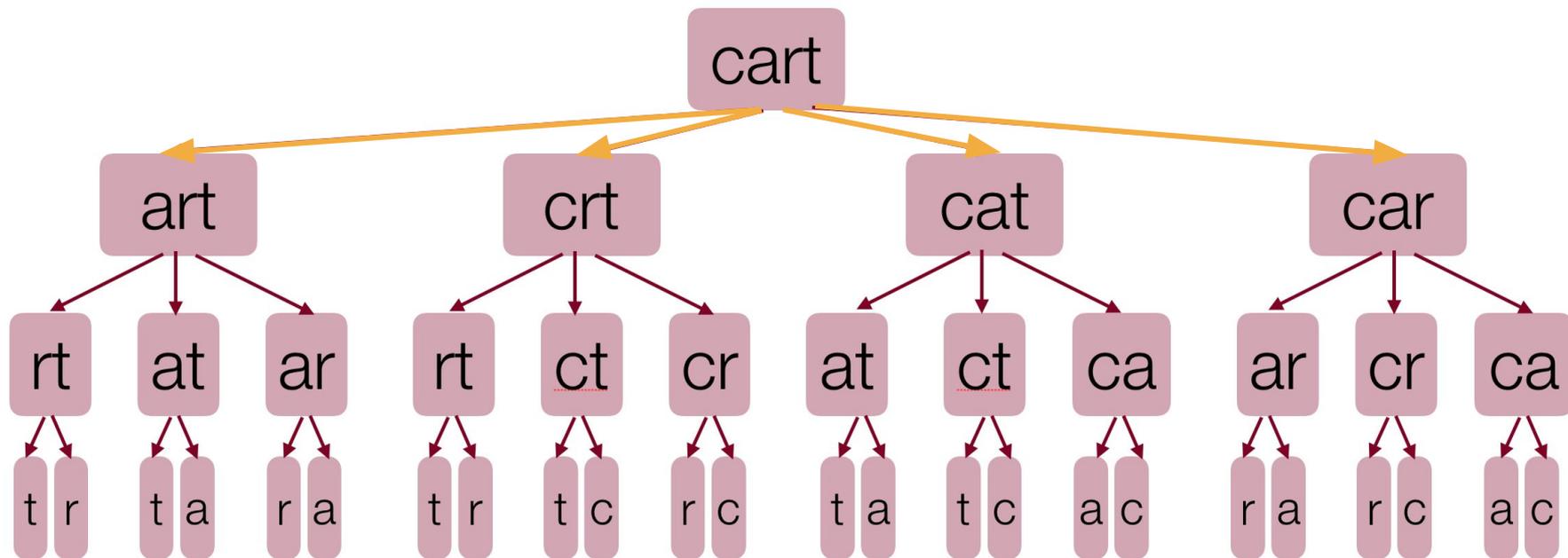
# Throwback Thursday (on Wednesday)

- We've already seen trees before in this class... decision trees!



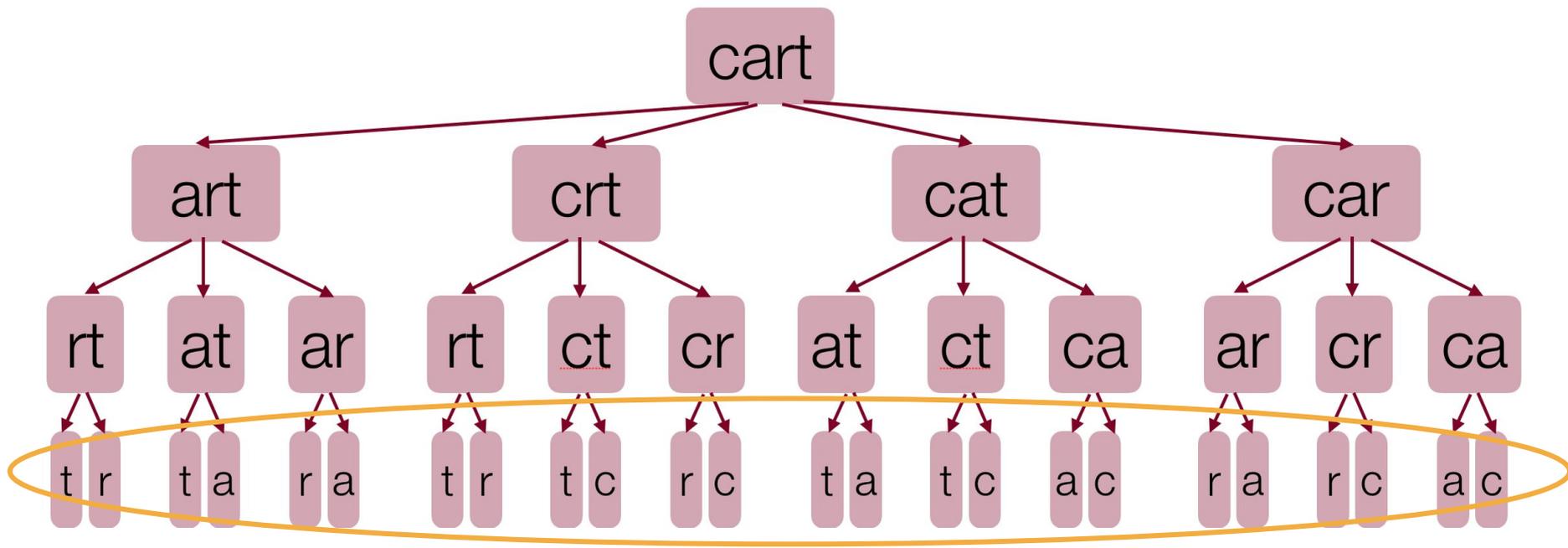
# Throwback Thursday (on Wednesday)

- We've already seen trees before in this class... decision trees!



# Throwback Thursday (on Wednesday)

- We've already seen trees before in this class... decision trees!

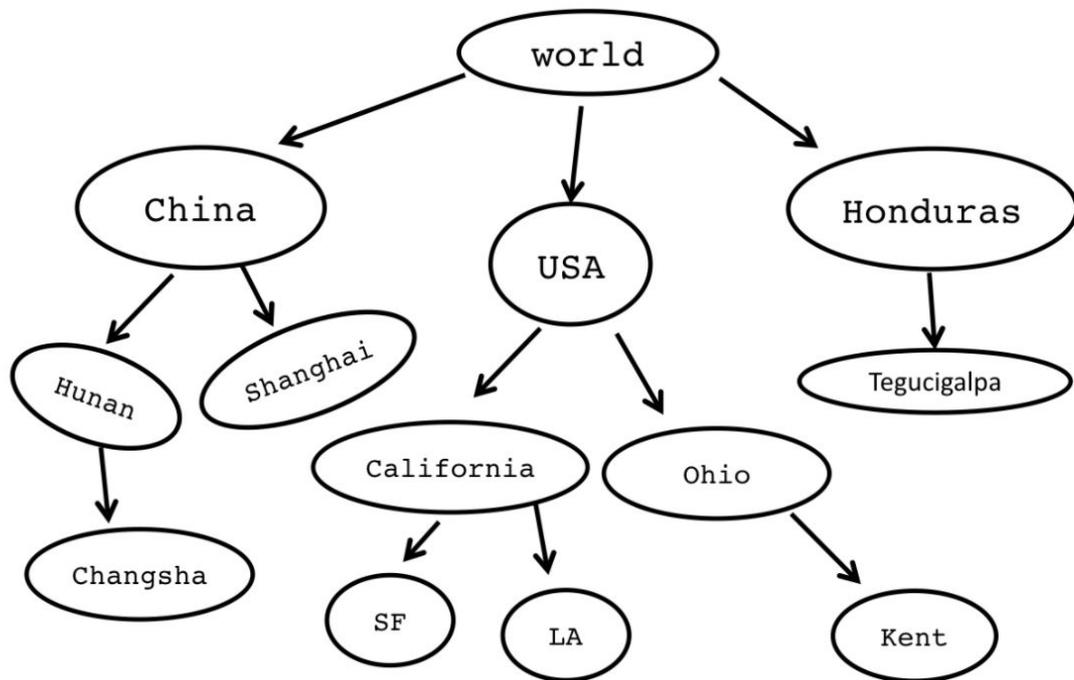


# Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

# Trees in the Wild

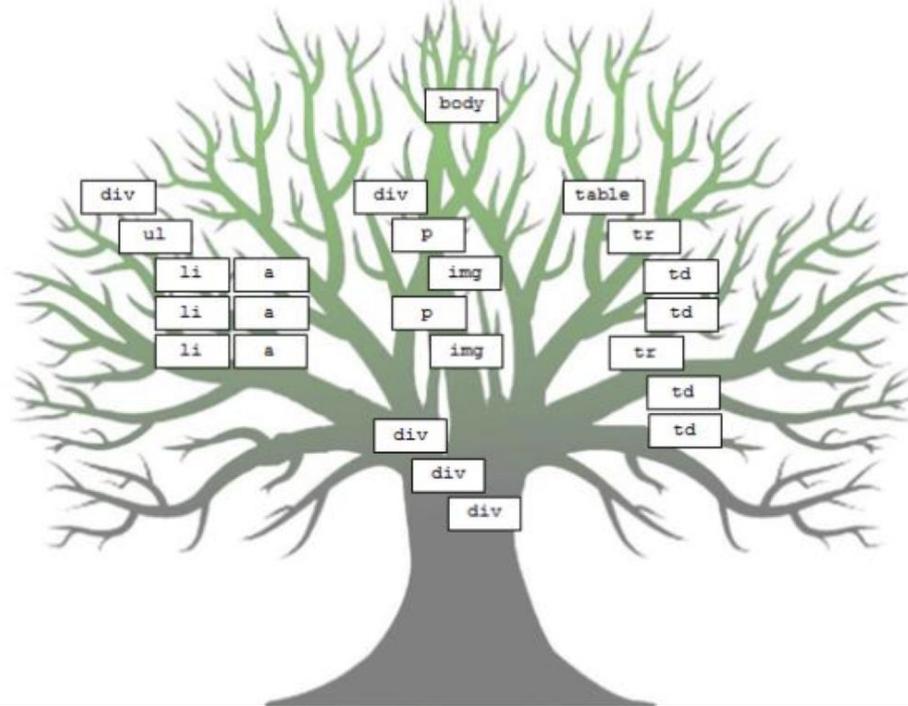
- Trees are useful in other ways besides just visualizing recursive backtracking.



*Trees can be used to describe hierarchies.*

# Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

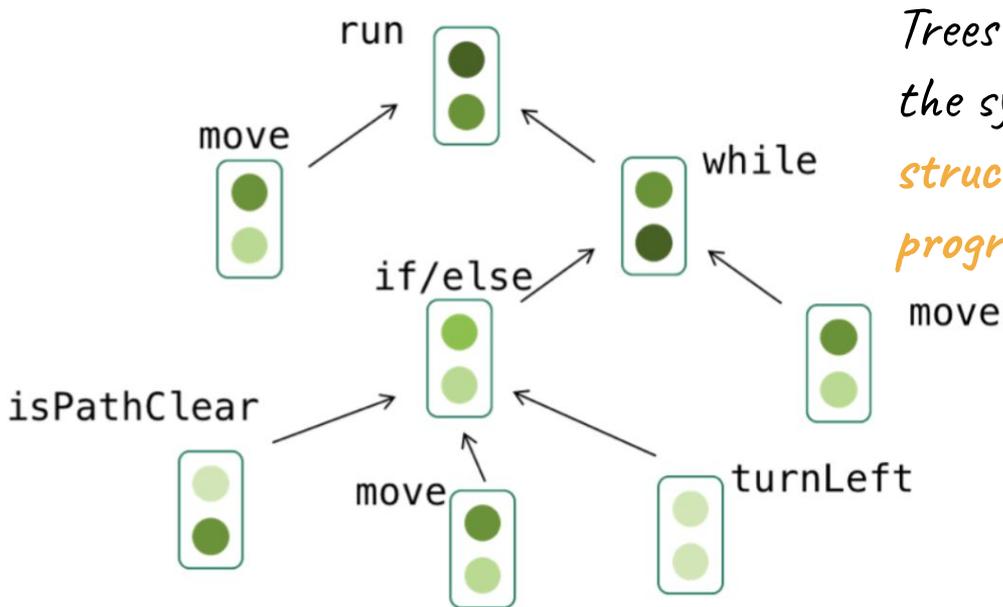


*Trees are used to model the structure of websites.*

# Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

```
def run() {  
  move();  
  while (notFinished()) {  
    if (isPathClear()) {  
      move();  
    } else {  
      turnLeft();  
    }  
    move();  
  }  
}
```



*Trees describe  
the syntax  
structure of  
programs.*

# Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.
- But, it is not a coincidence that we first saw them appear in conjunction with recursion.

# Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.
- But, it is not a coincidence that we first saw them appear in conjunction with recursion.
- Trees are inherently defined recursively!

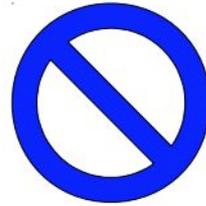
What is a tree?

**A tree is either...**

# What is a tree?

**A tree is either...**

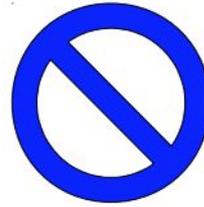
An empty data  
structure, or...



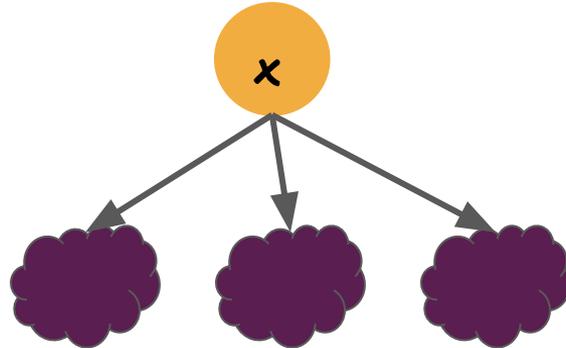
# What is a tree?

## A tree is either...

An empty data structure, or...



A single node (parent), with zero or more non-empty subtrees (children)



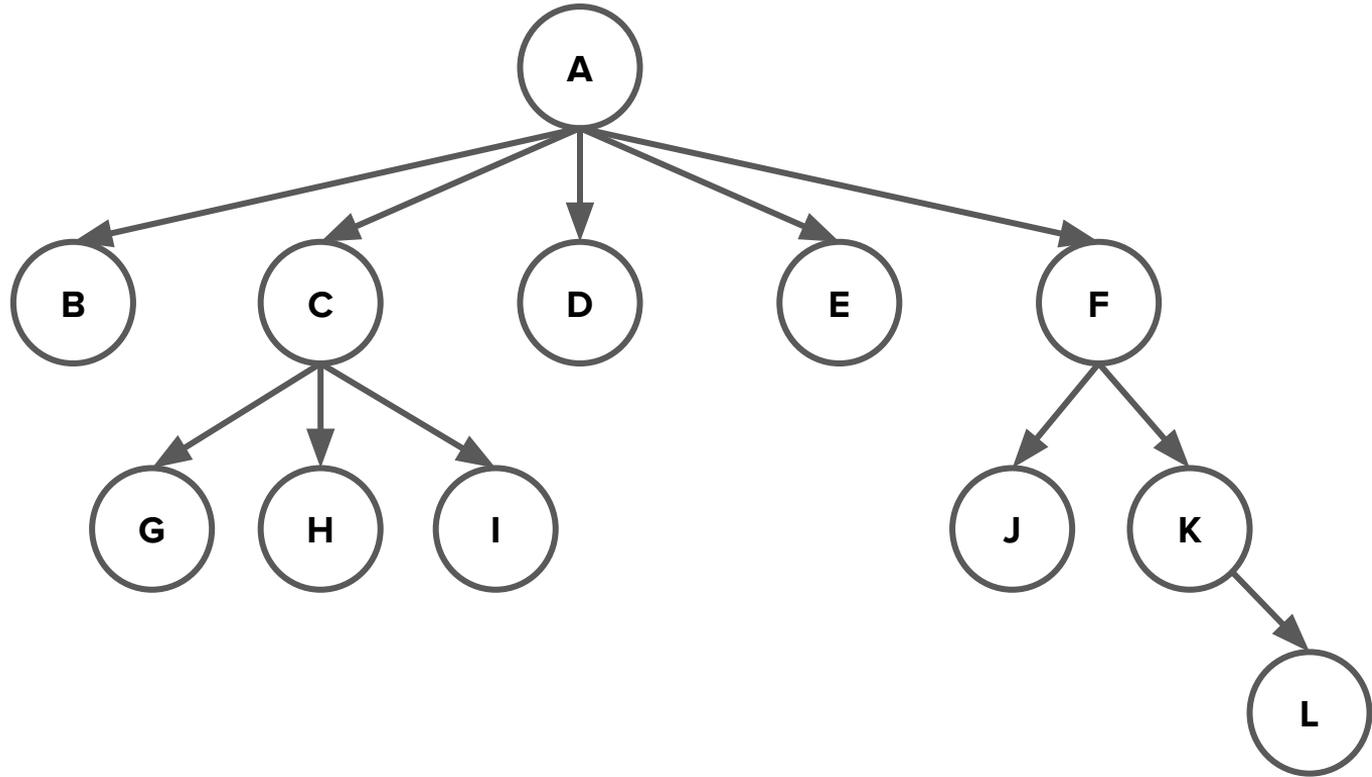
## *Definition*

### **tree**

A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

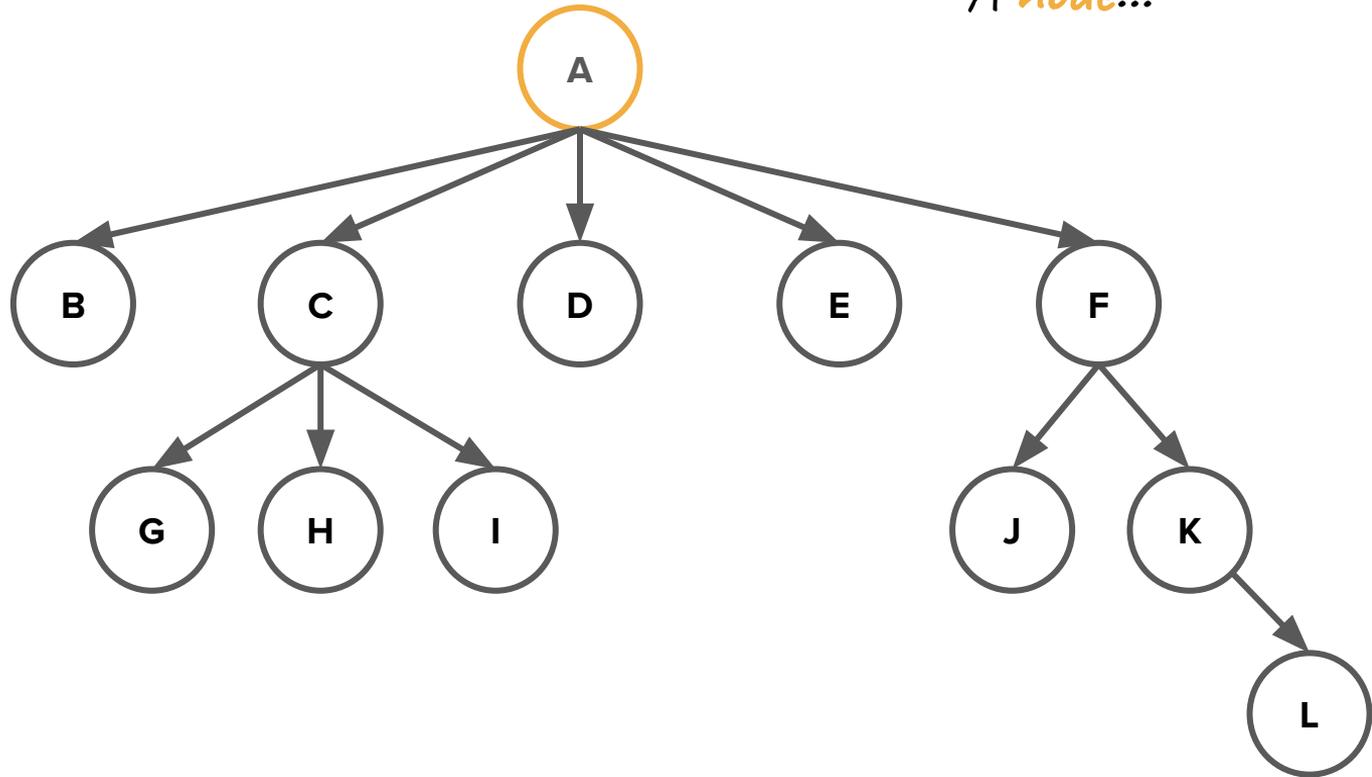
# Tree Terminology

# Tree Terminology

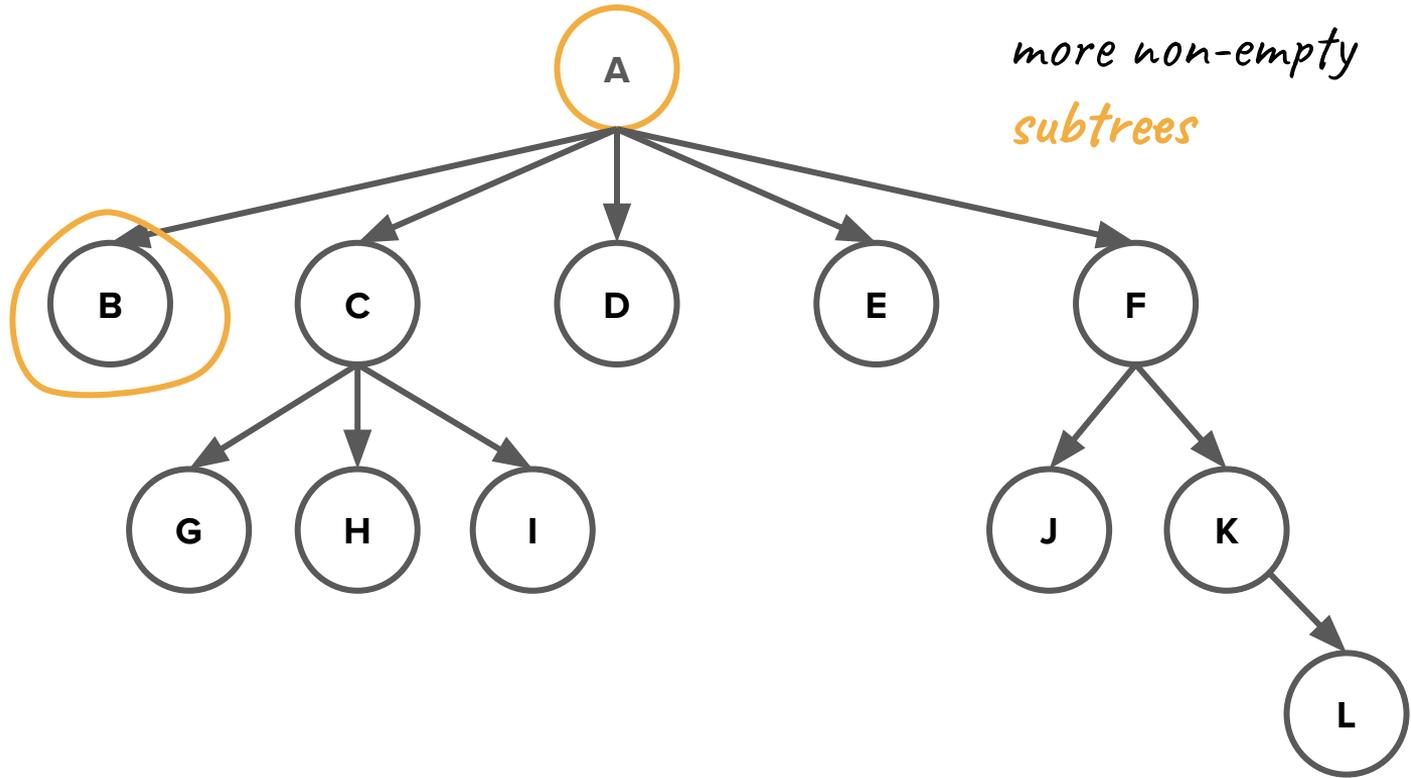


# Tree Terminology

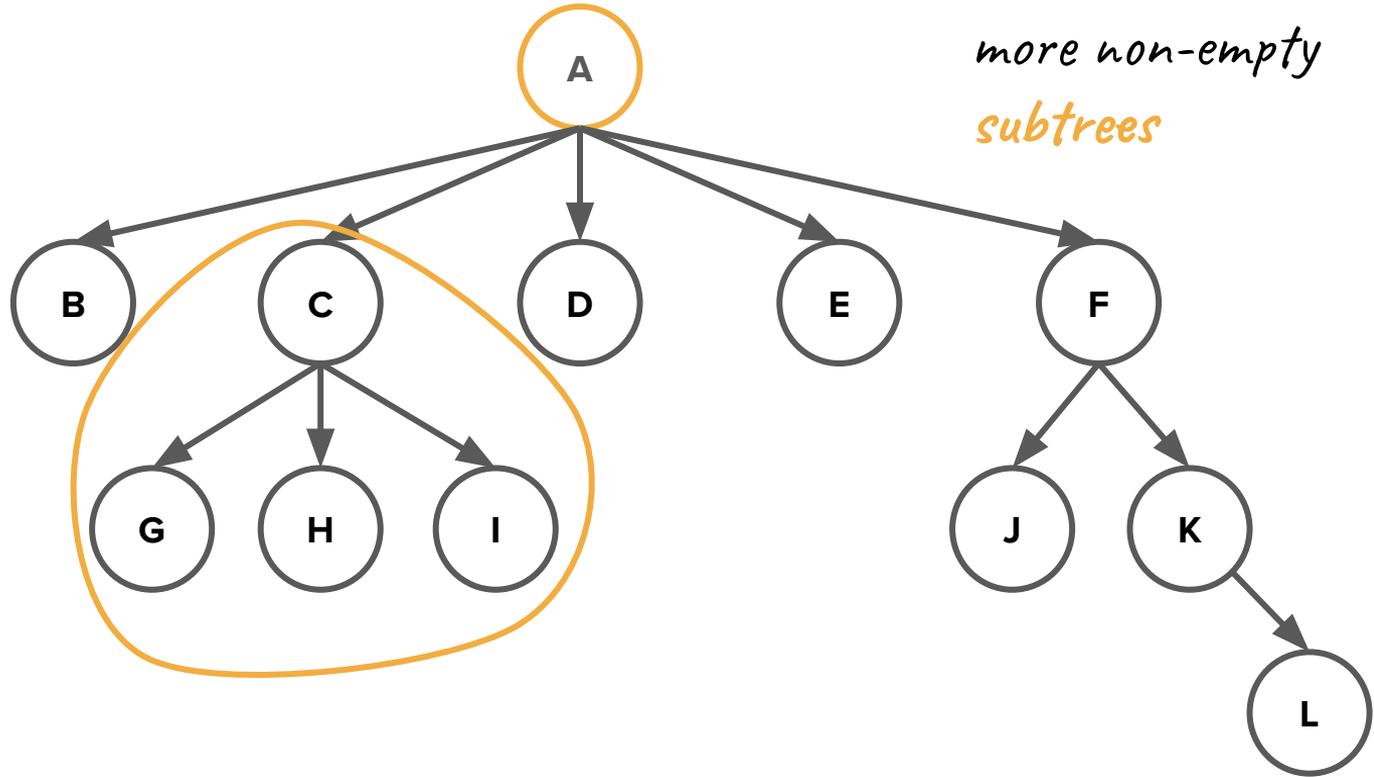
*A node...*



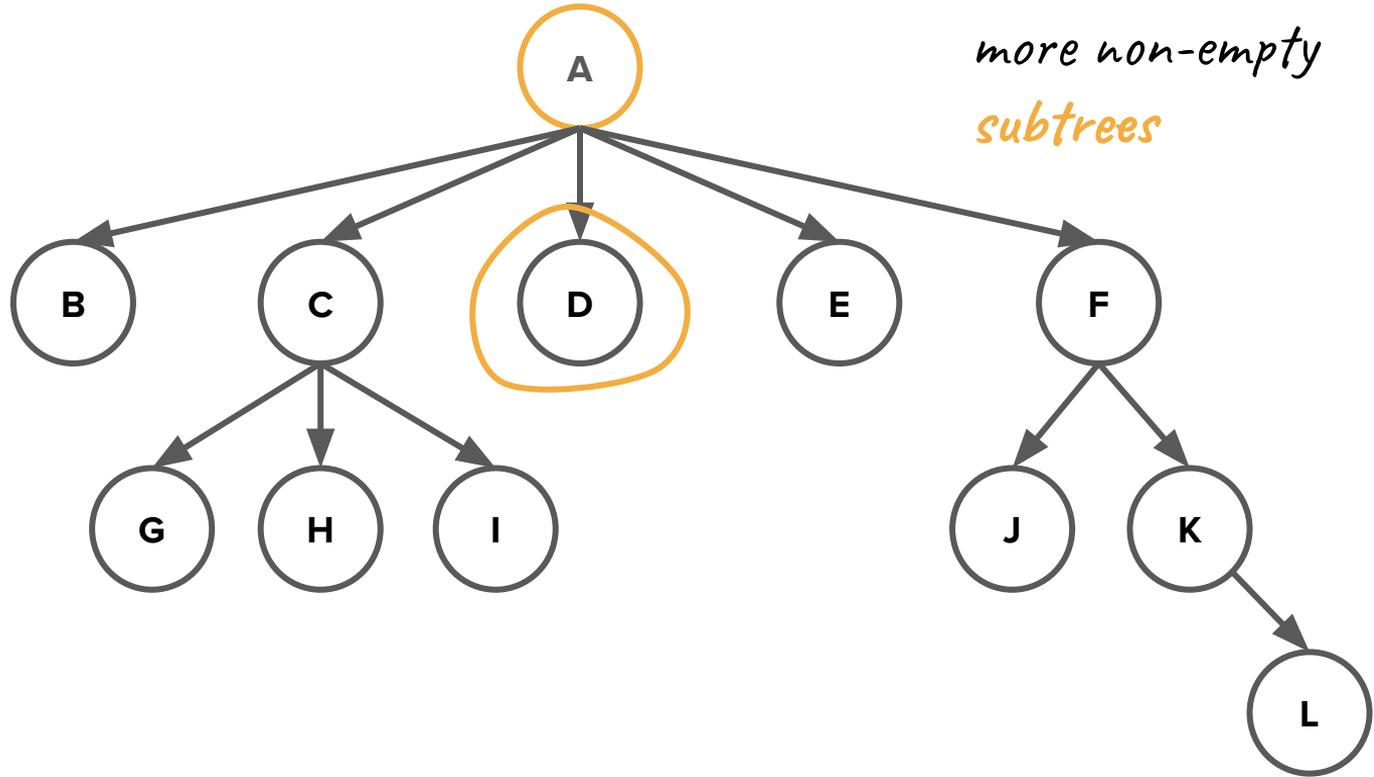
# Tree Terminology



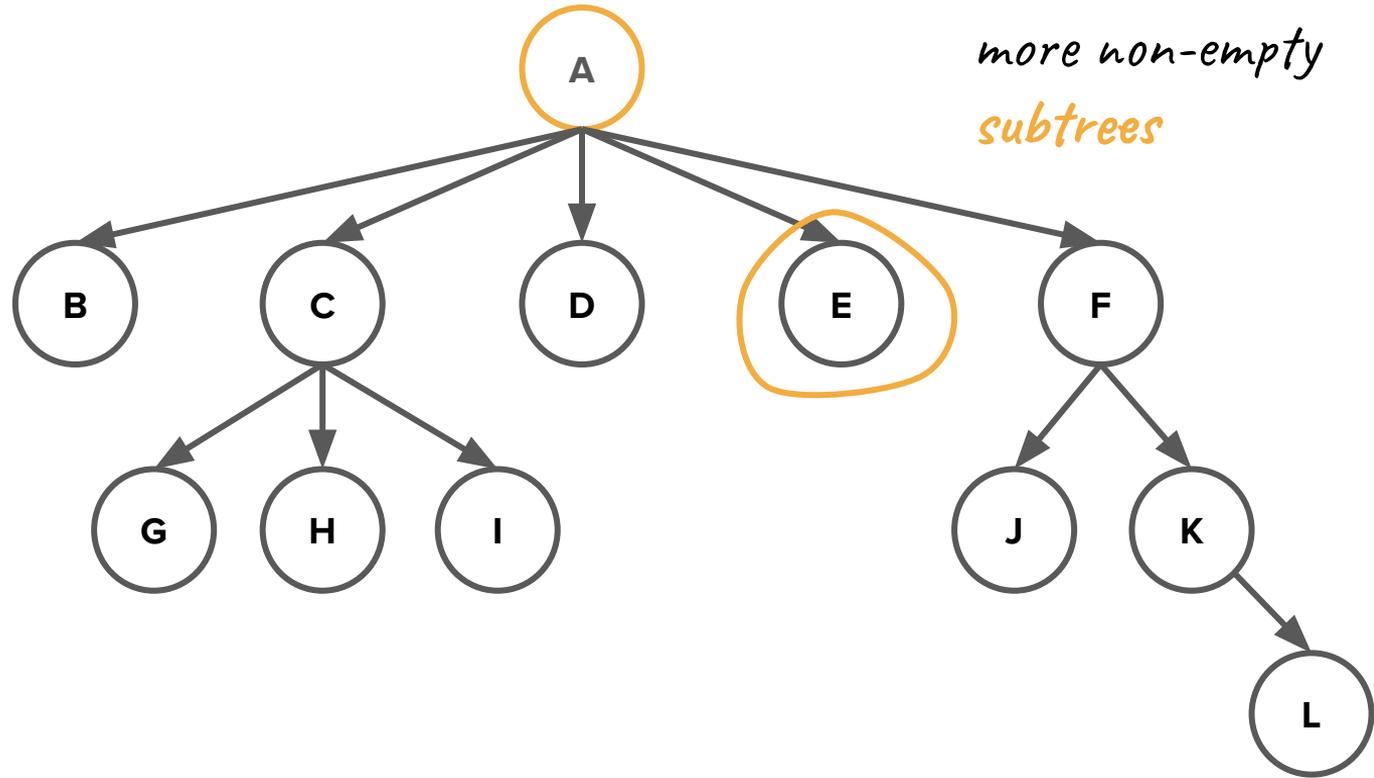
# Tree Terminology



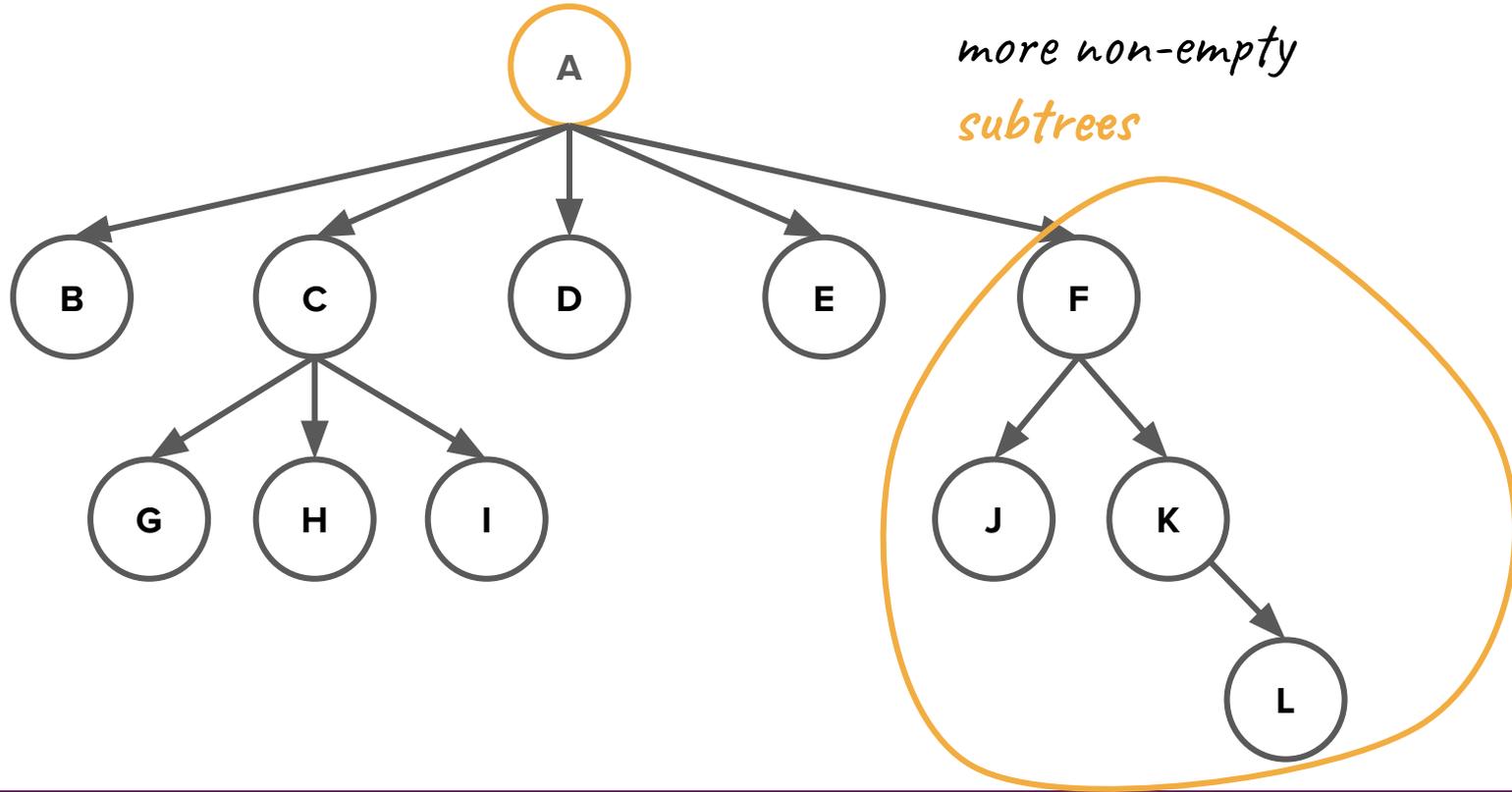
# Tree Terminology



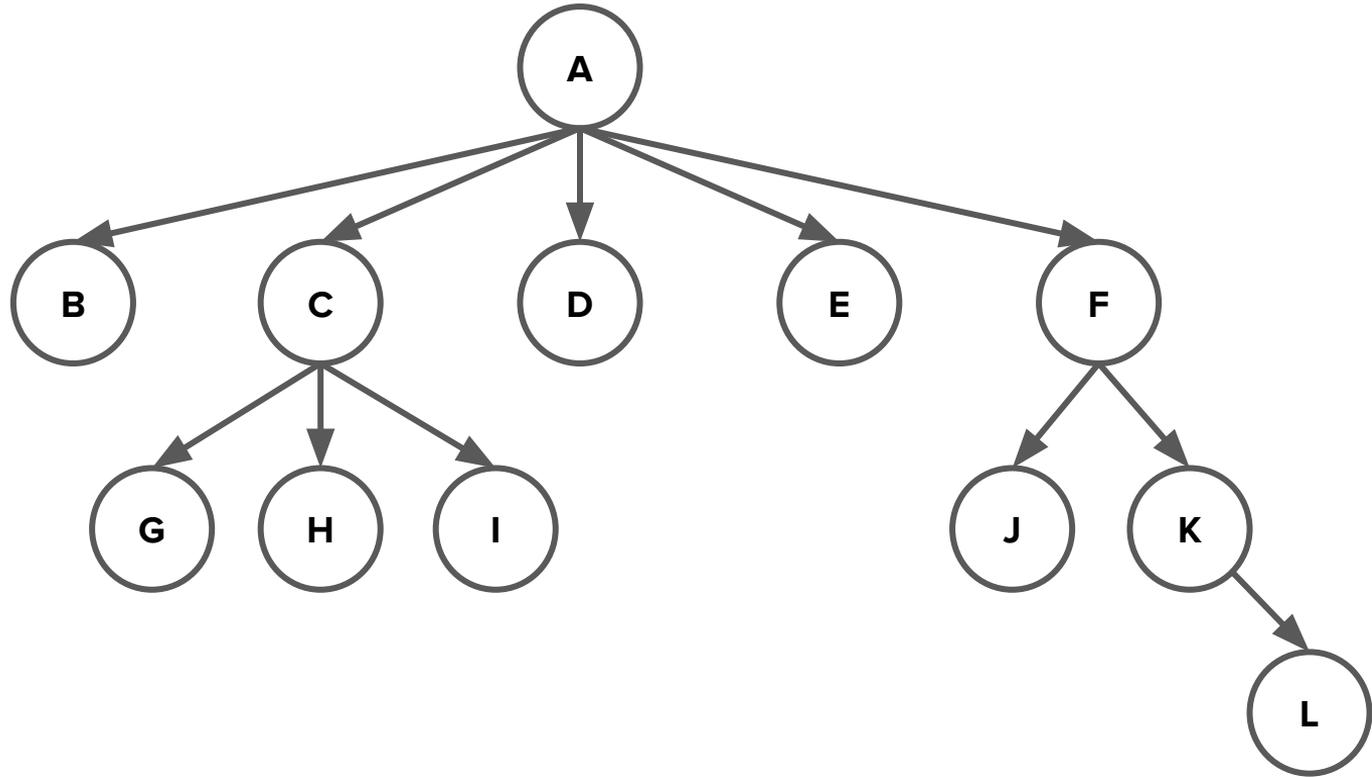
# Tree Terminology



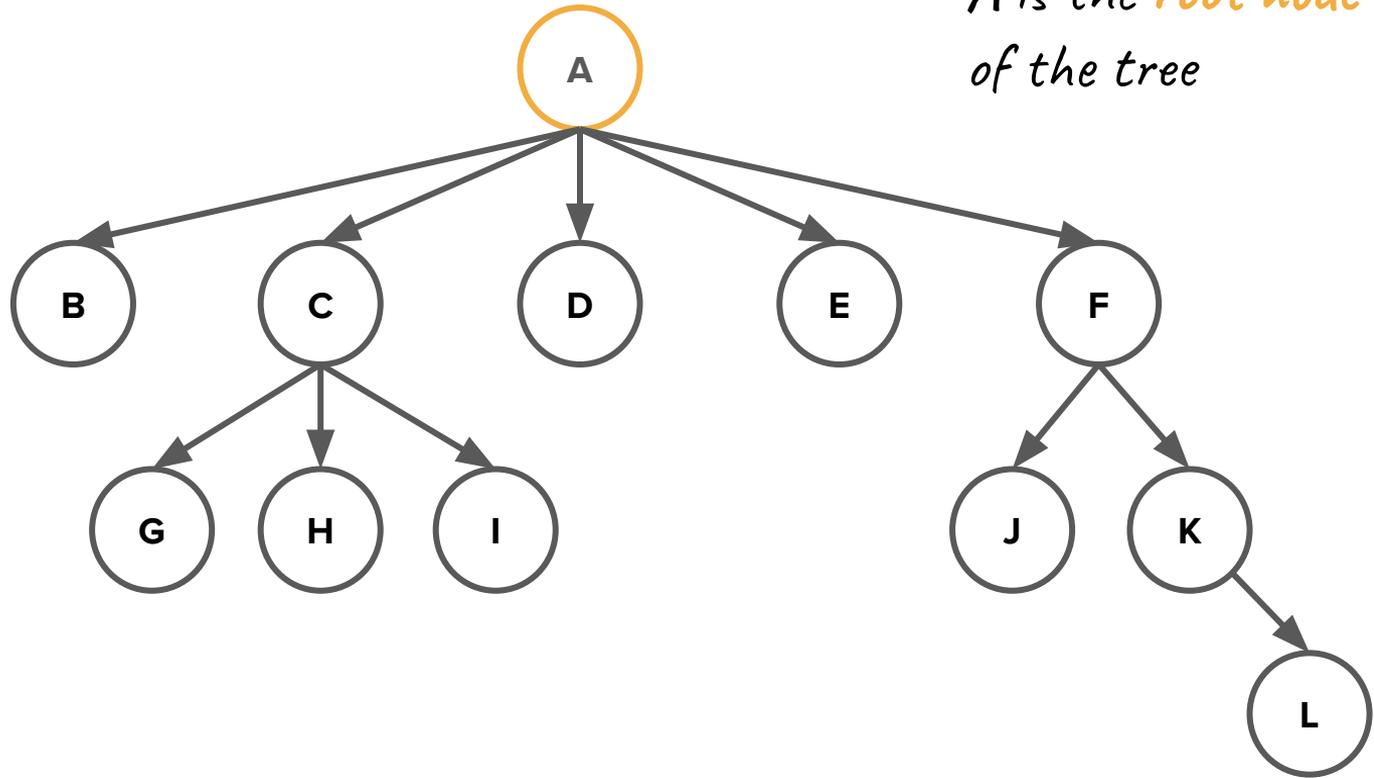
# Tree Terminology



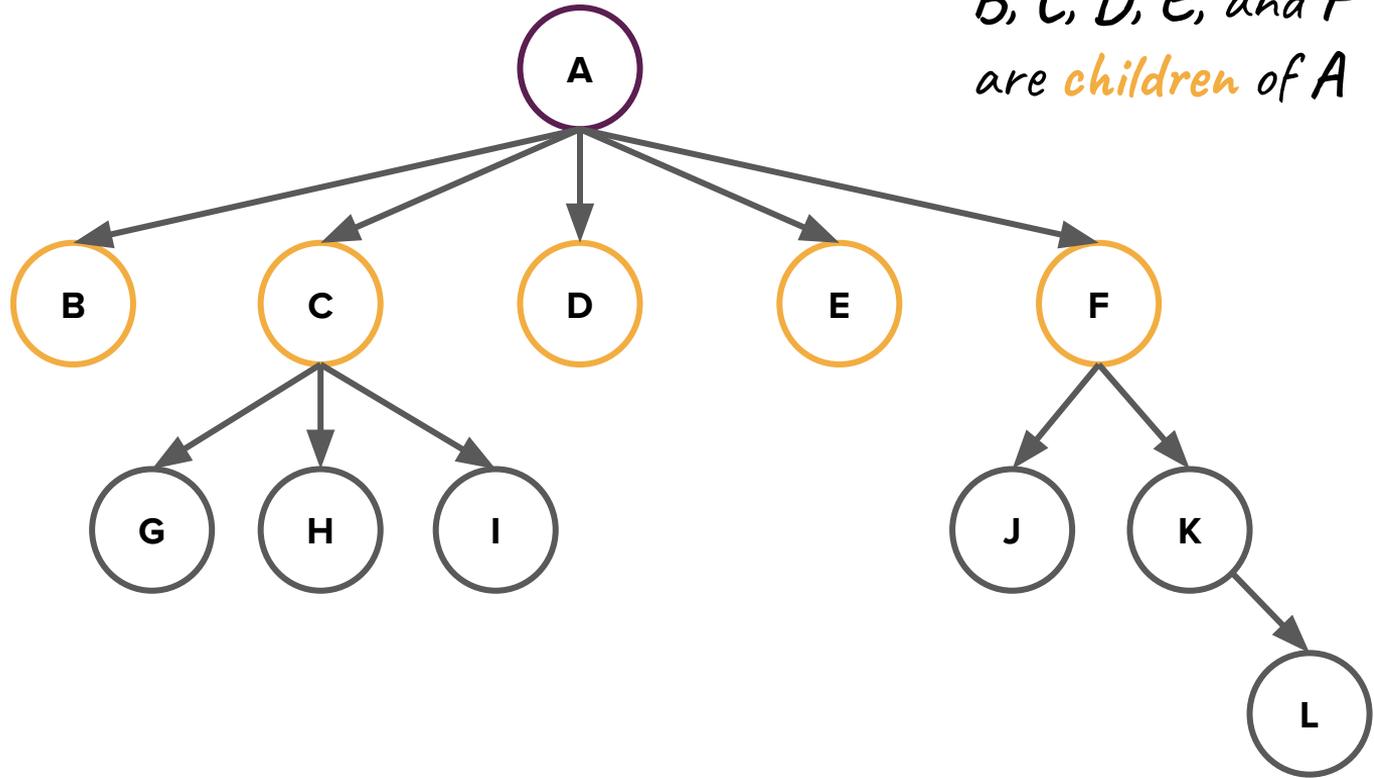
# Tree Terminology



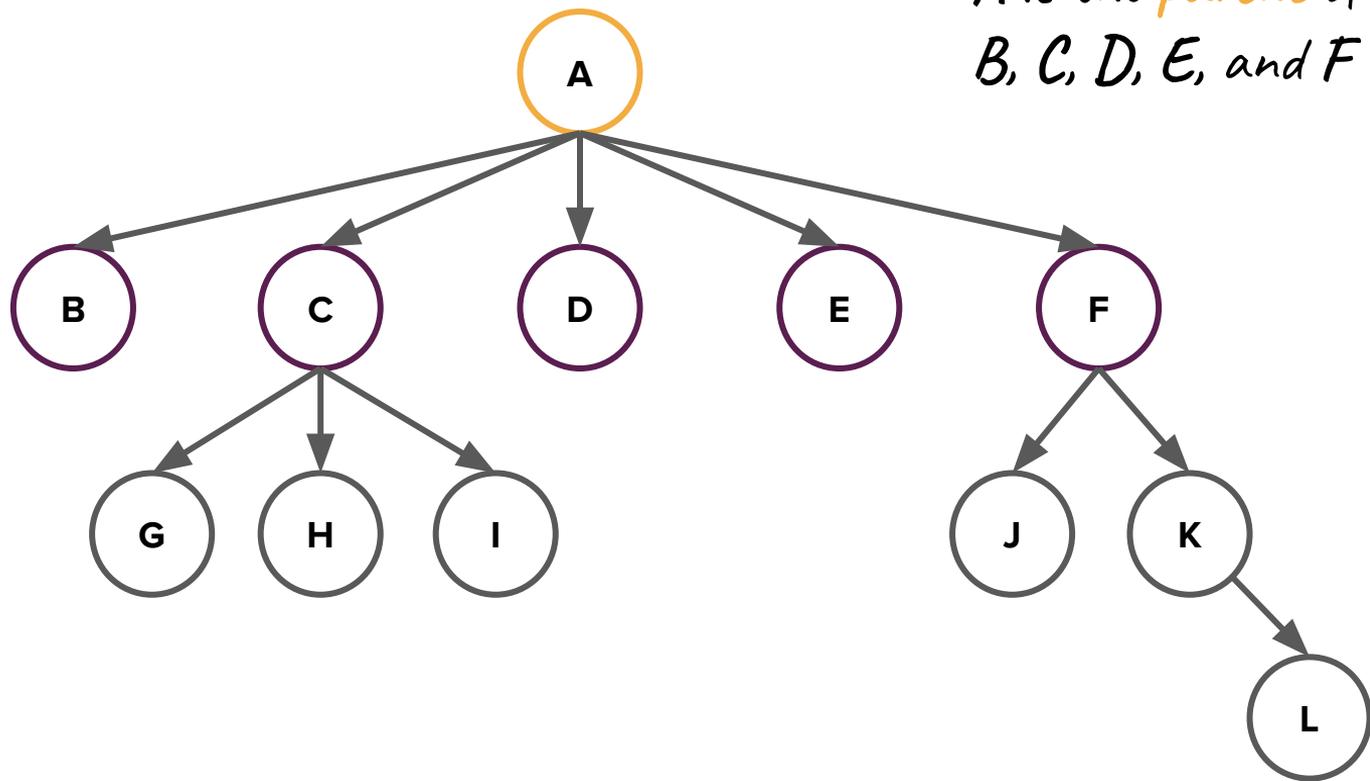
# Tree Terminology



# Tree Terminology

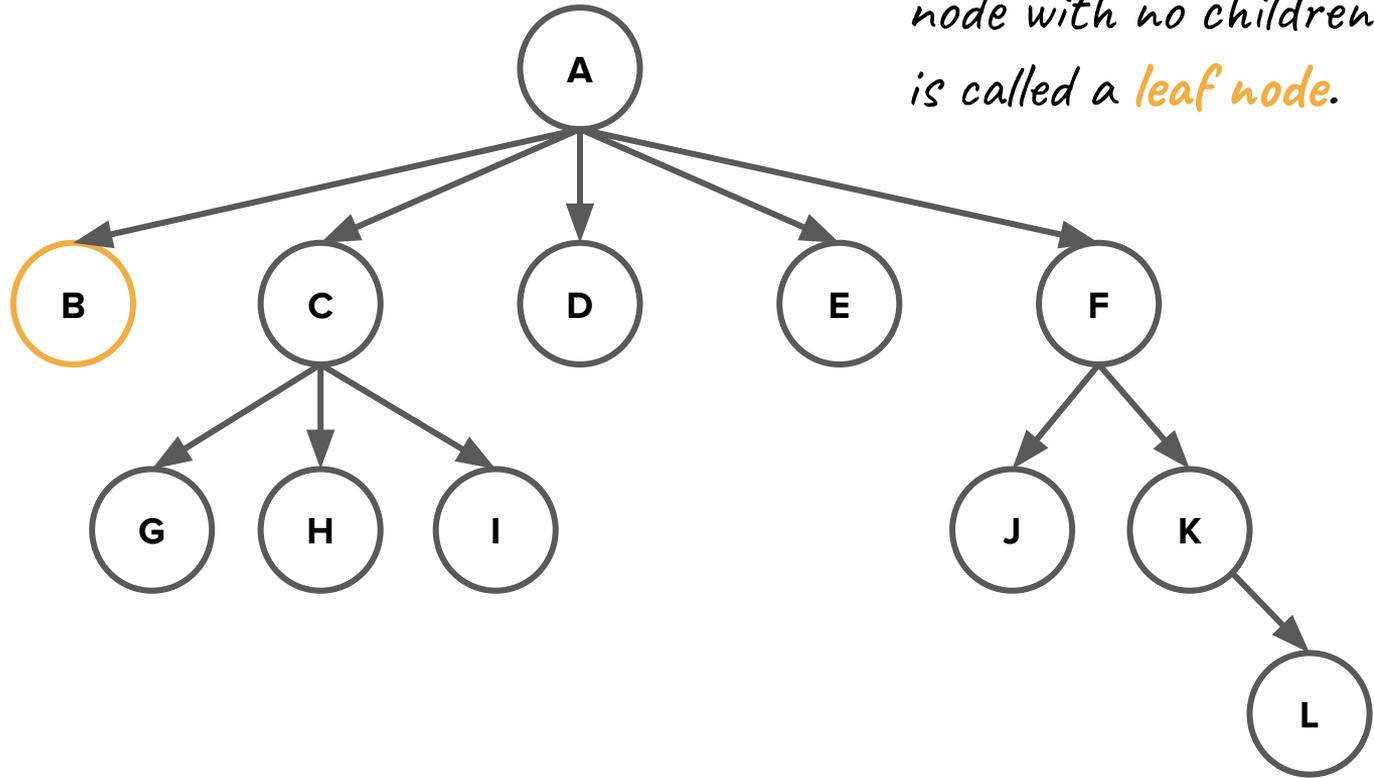


# Tree Terminology



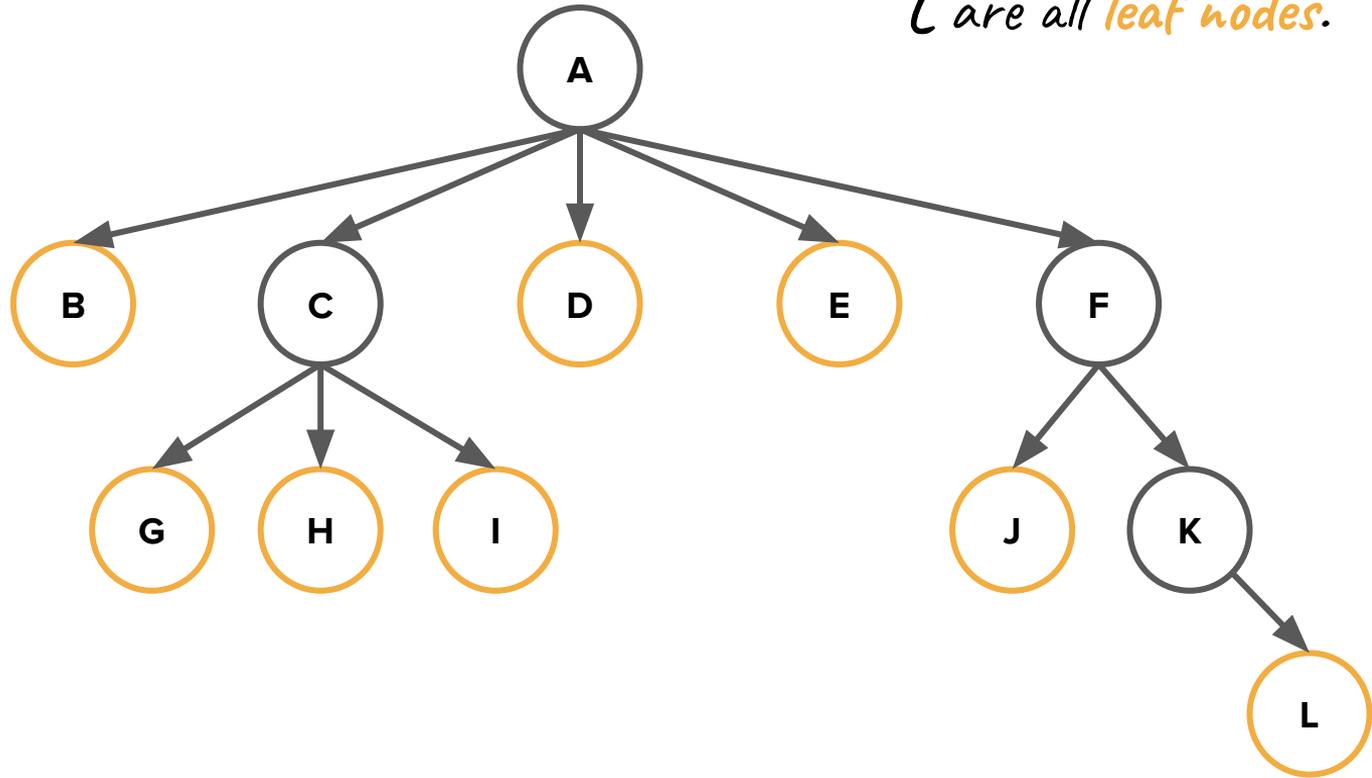
# Tree Terminology

*B* has no children. A node with no children is called a **leaf node**.



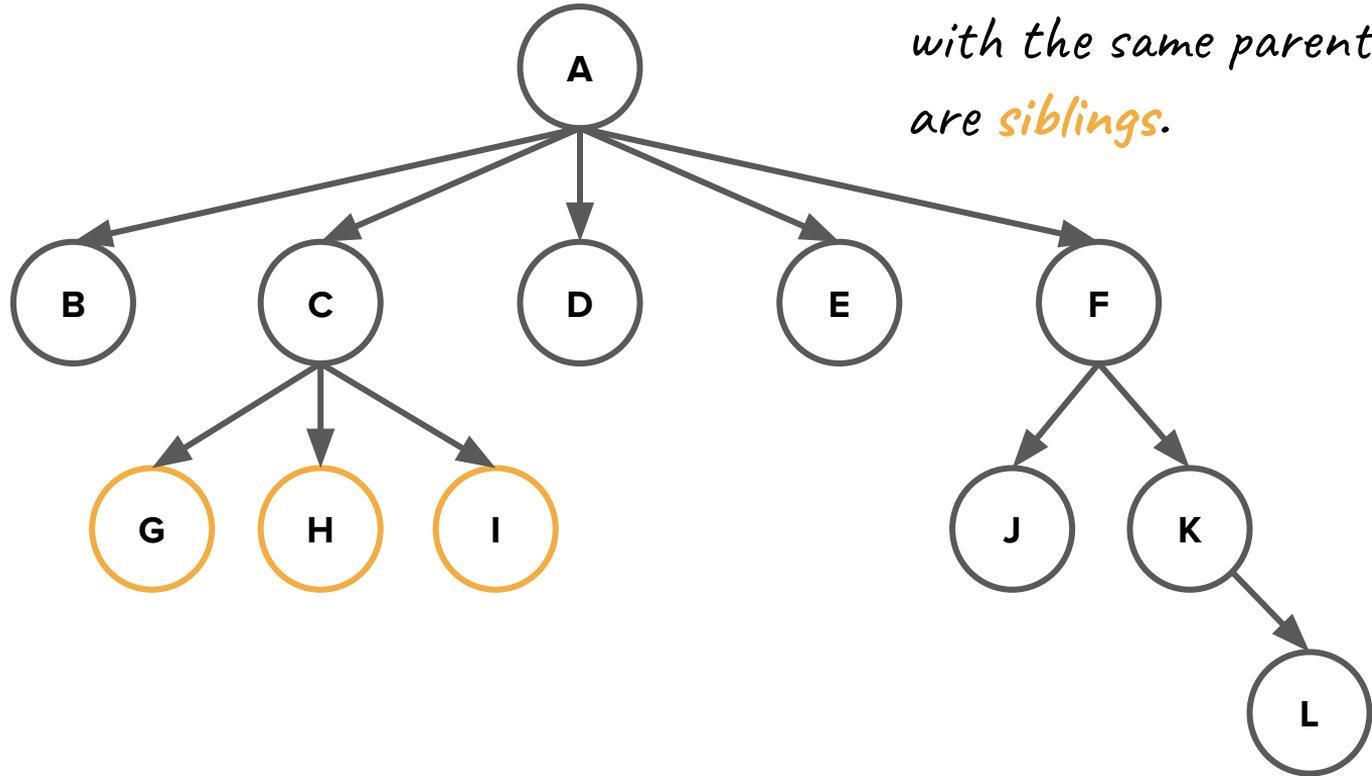
# Tree Terminology

*B, G, H, I, D, E, J, and L are all leaf nodes.*



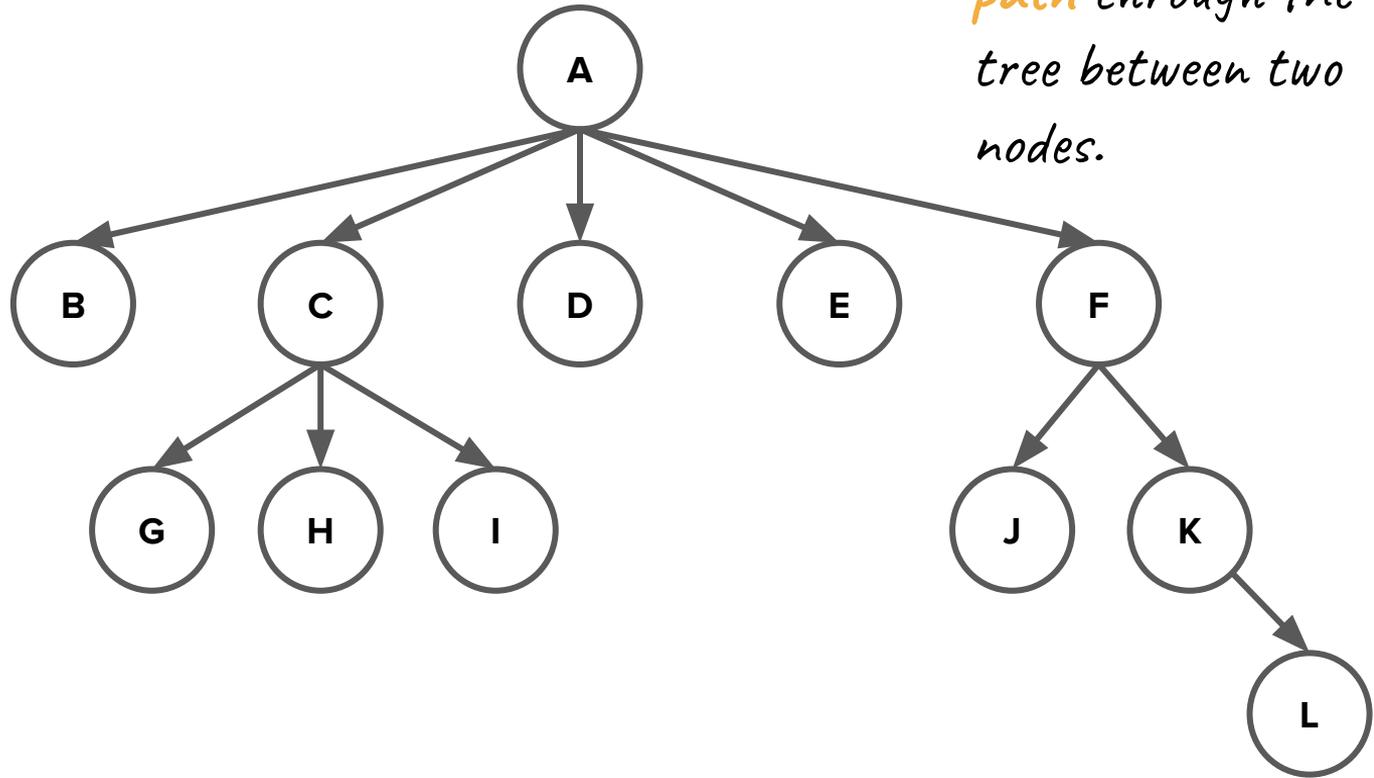
# Tree Terminology

*G, H and I all have the same parent. Nodes with the same parent are **siblings**.*



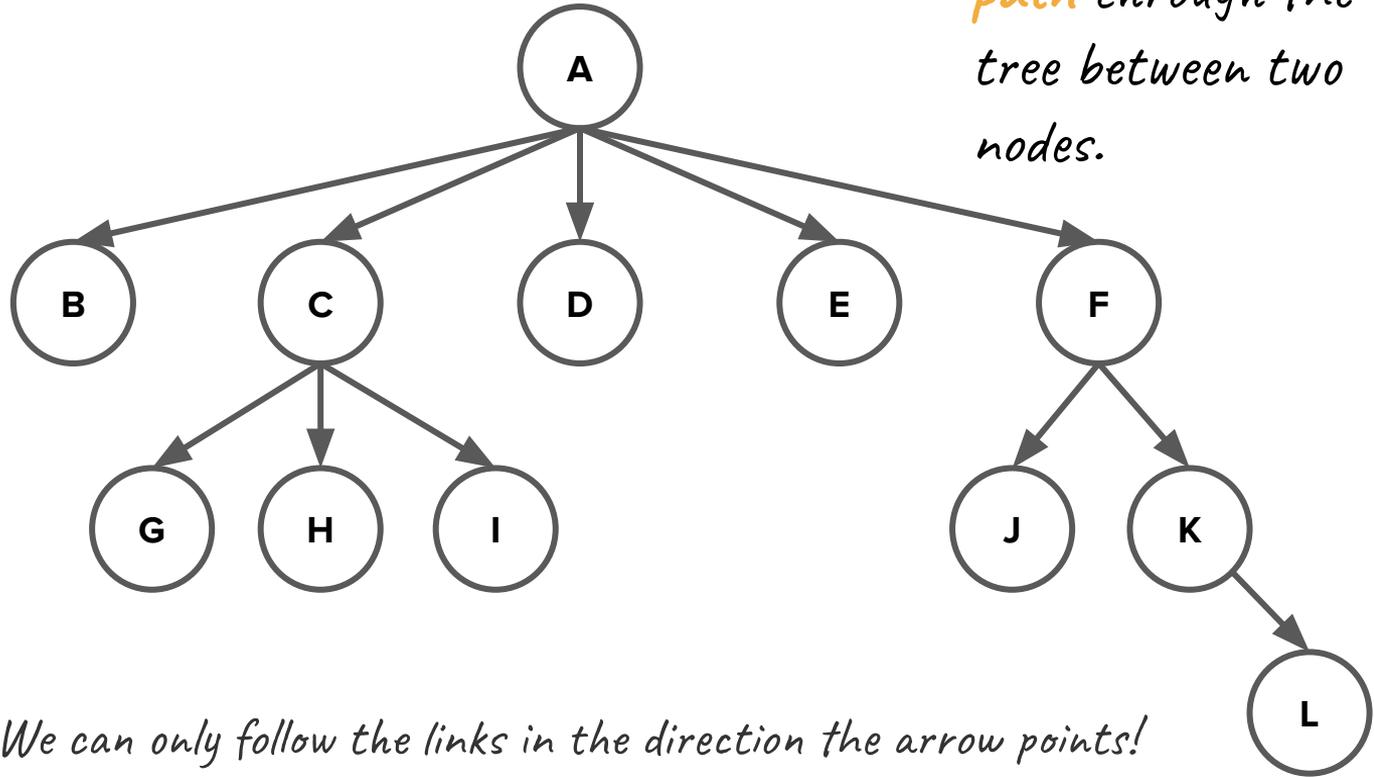
# Tree Terminology

We can define a **path** through the tree between two nodes.



# Tree Terminology

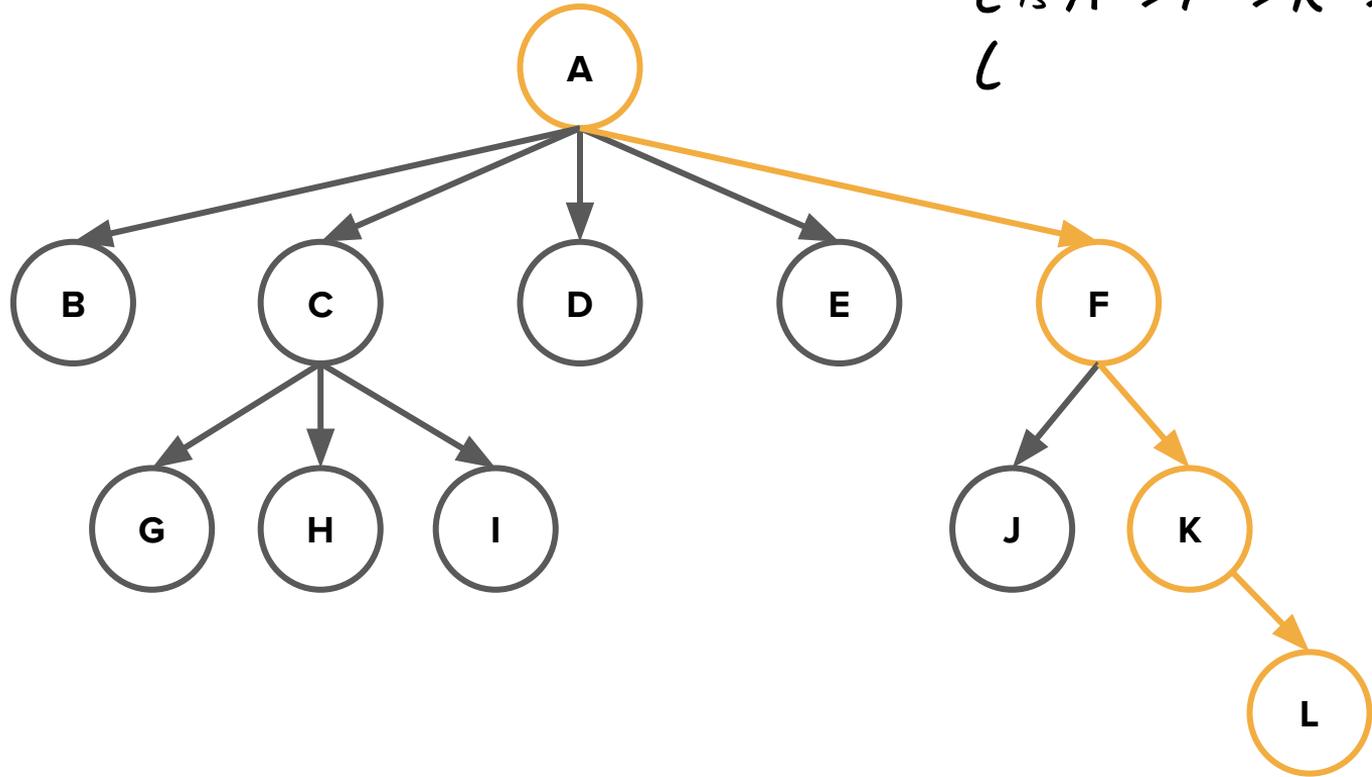
We can define a *path* through the tree between two nodes.



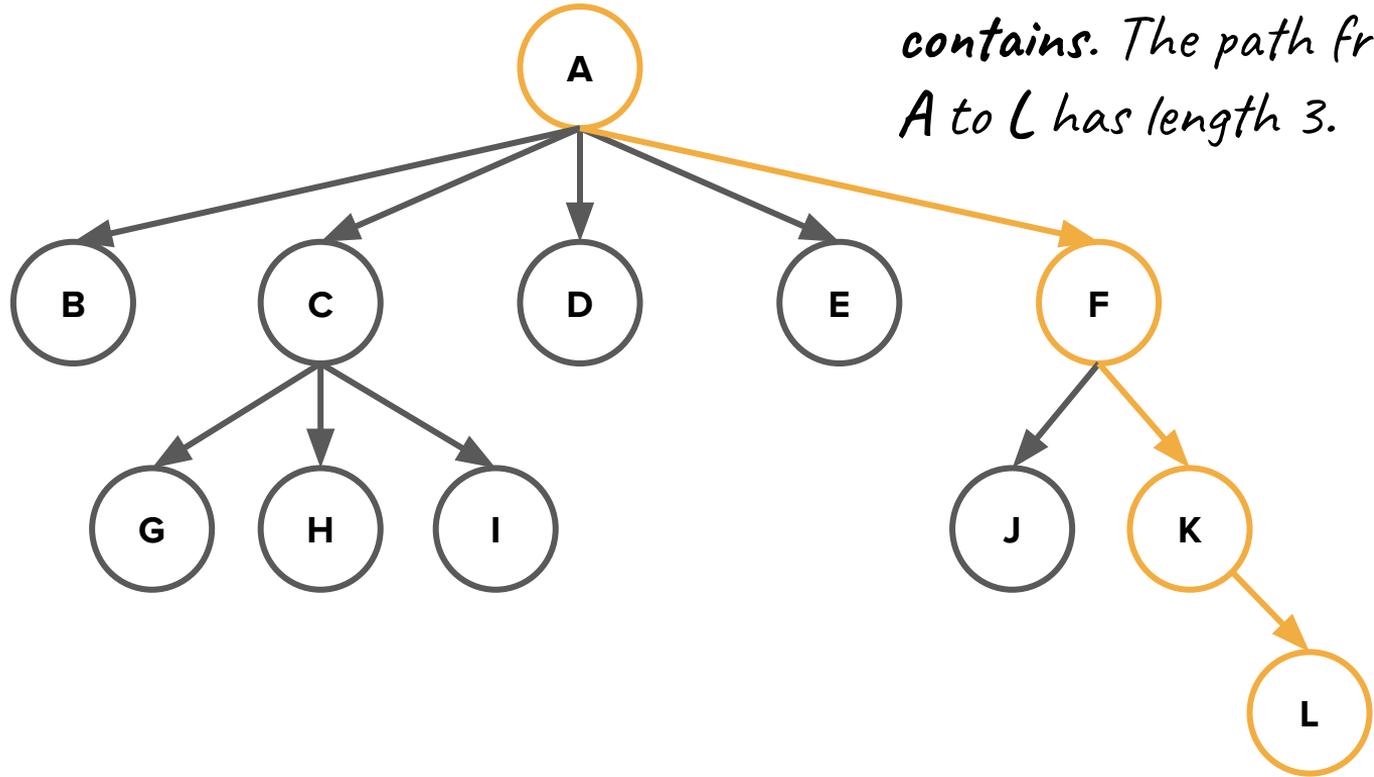
*Note:* We can only follow the links in the direction the arrow points!

# Tree Terminology

The *path* from A to L is A → F → K → L



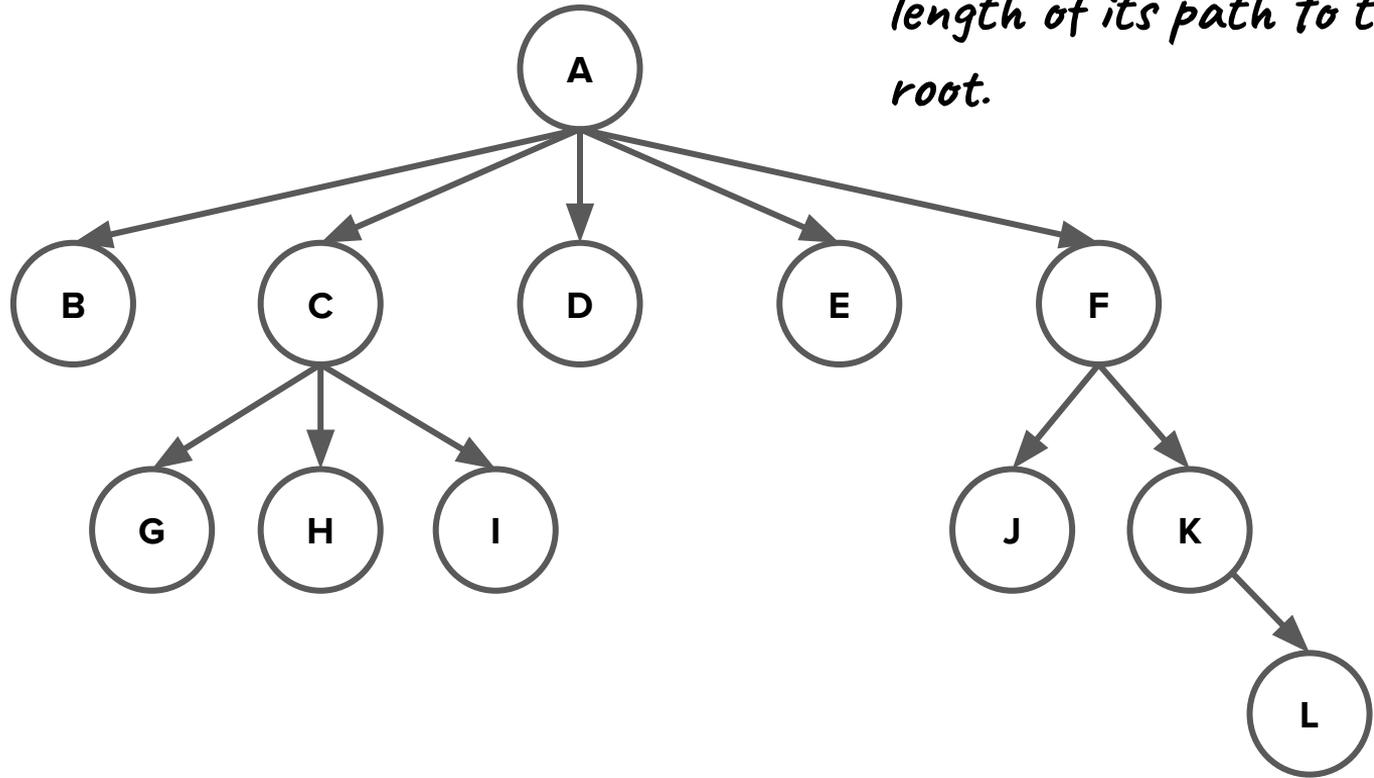
# Tree Terminology



The *length* of the path is number of edges it contains. The path from A to L has length 3.

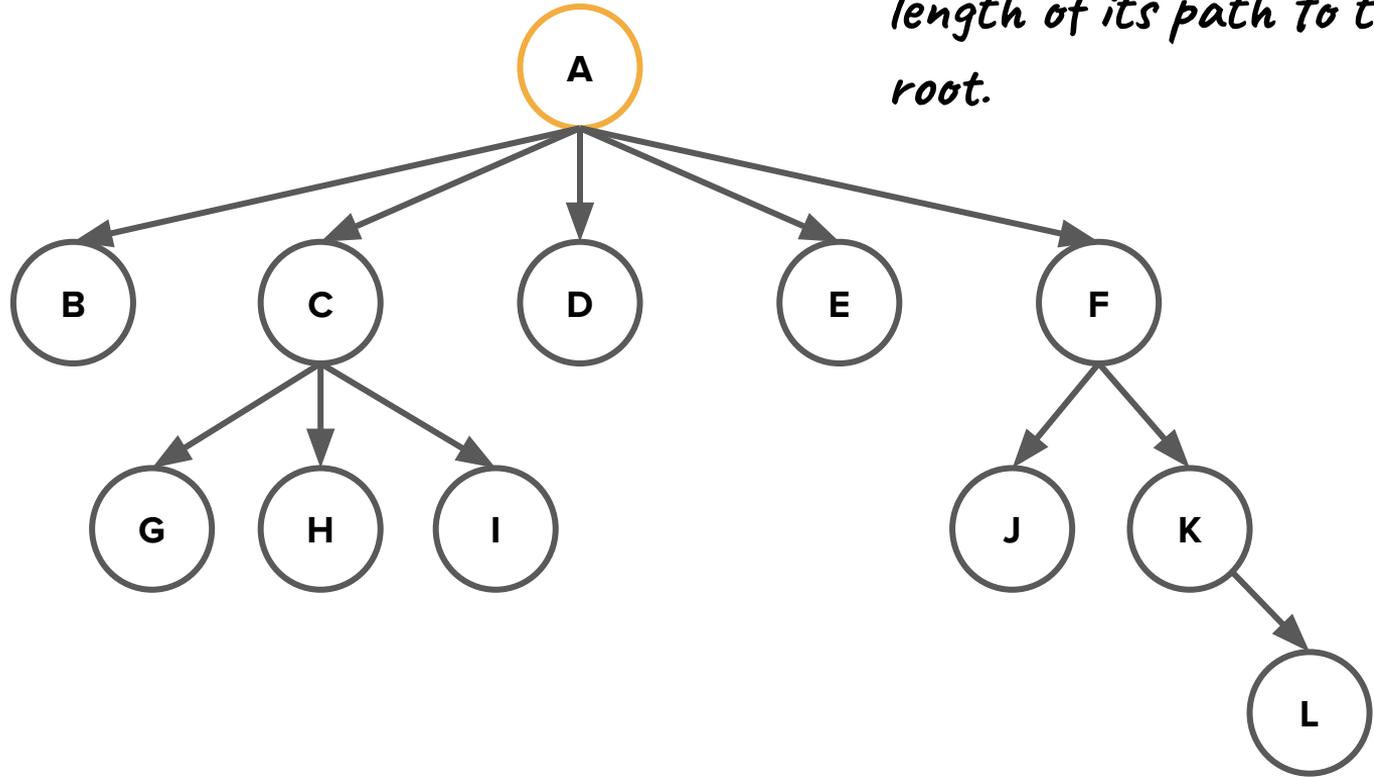
# Tree Terminology

The **depth** of a node is the length of its path to the root.



# Tree Terminology

depth: 0



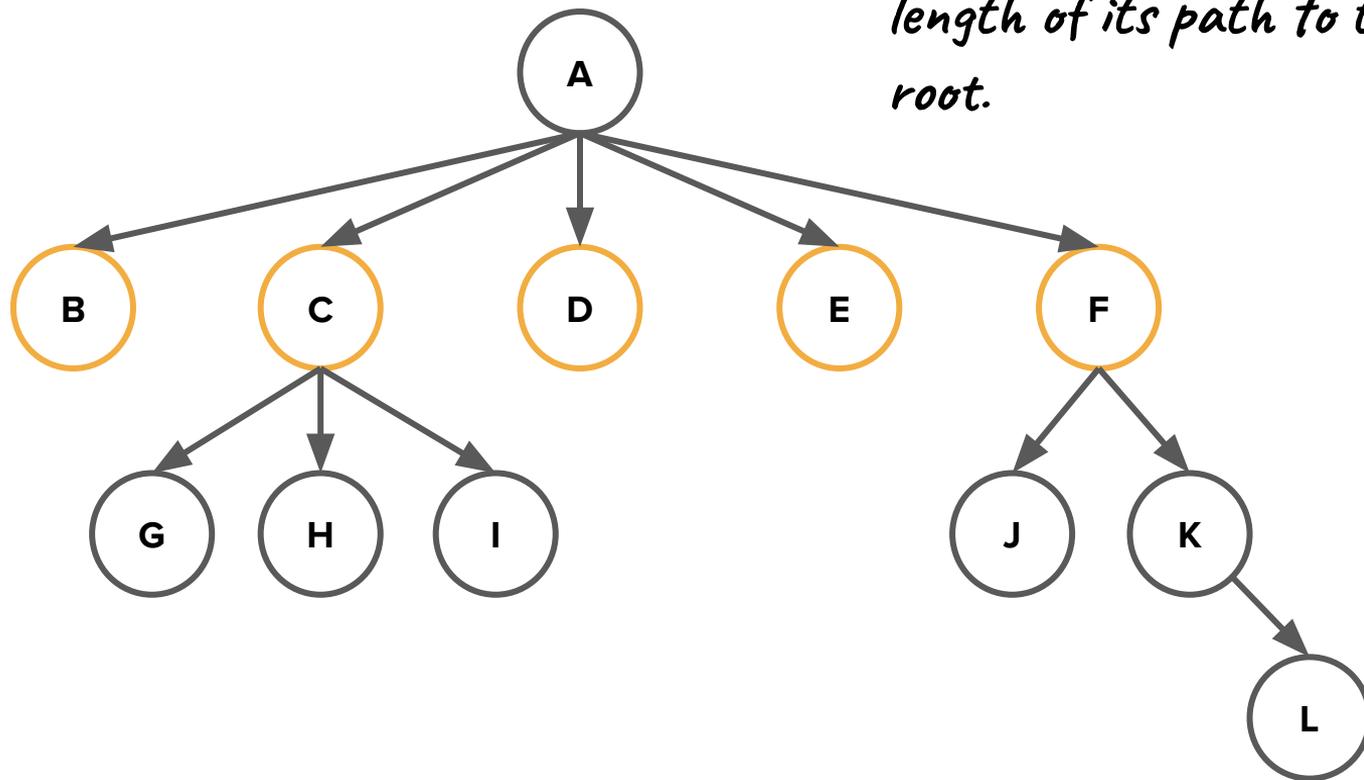
The **depth** of a node is the length of its path to the root.

# Tree Terminology

The *depth* of a node is the length of its path to the root.

depth: 0

depth: 1



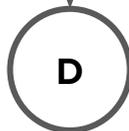
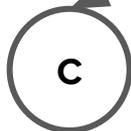
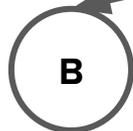
# Tree Terminology

The *depth* of a node is the length of its path to the root.

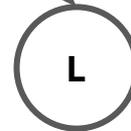
depth: 0



depth: 1



depth: 2



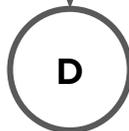
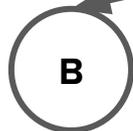
# Tree Terminology

The *depth* of a node is the length of its path to the root.

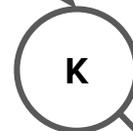
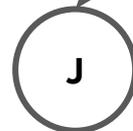
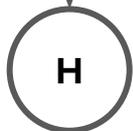
depth: 0



depth: 1



depth: 2

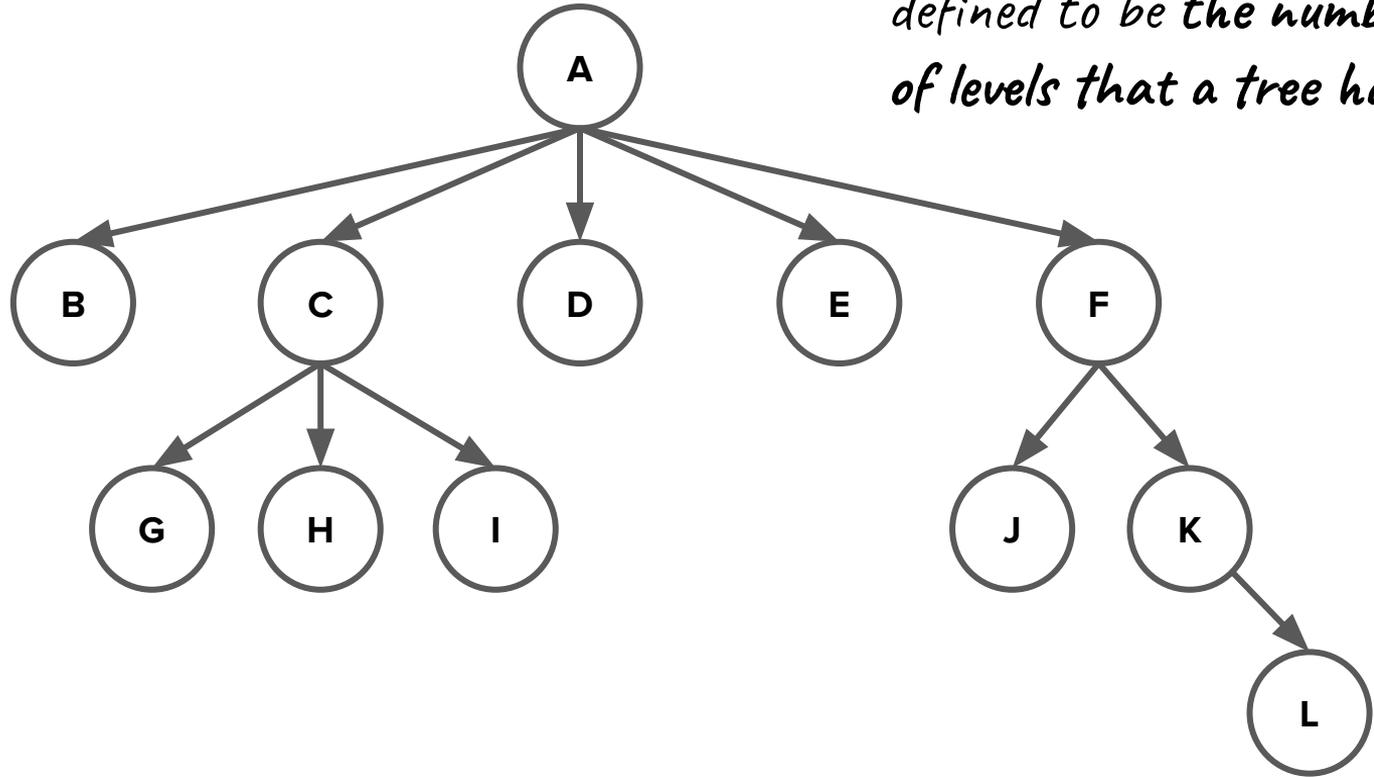


depth: 3



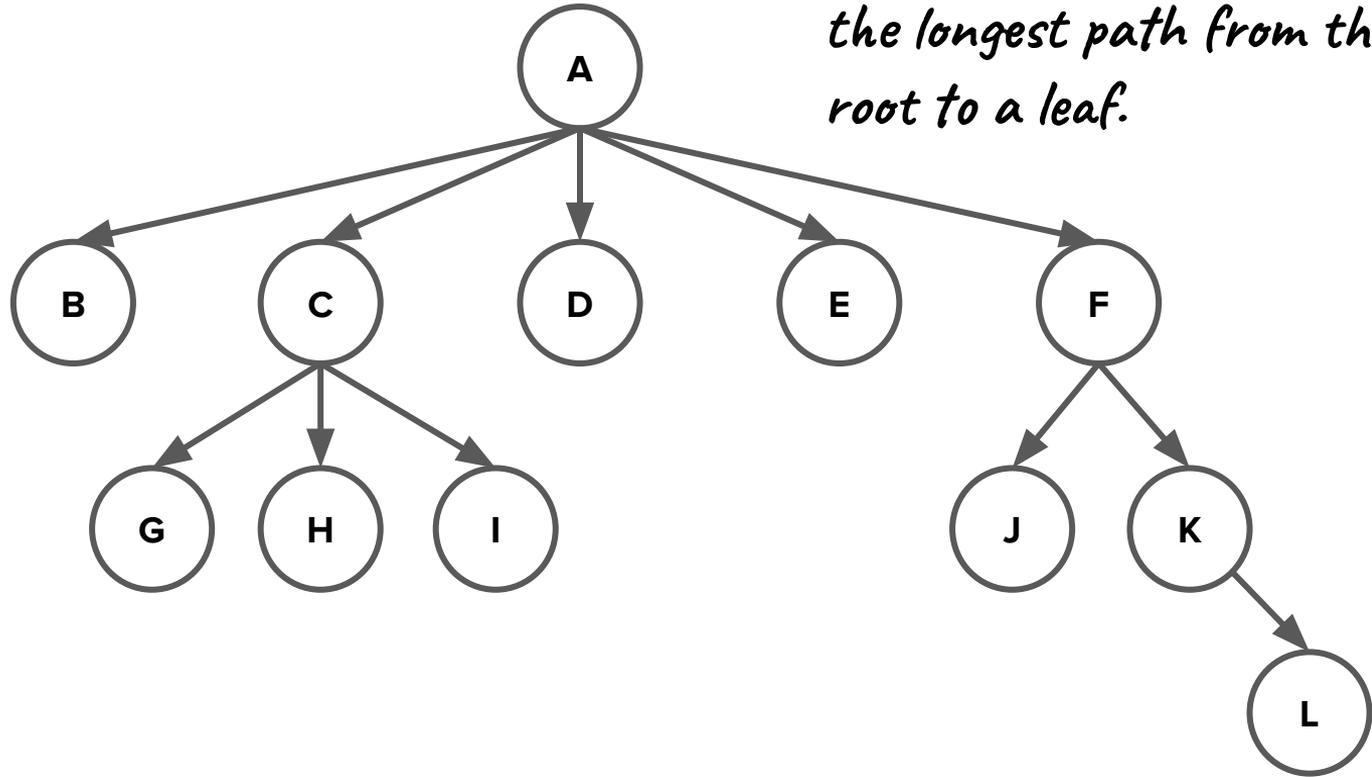
# Tree Terminology

The **height** of a tree is defined to be the number of levels that a tree has.



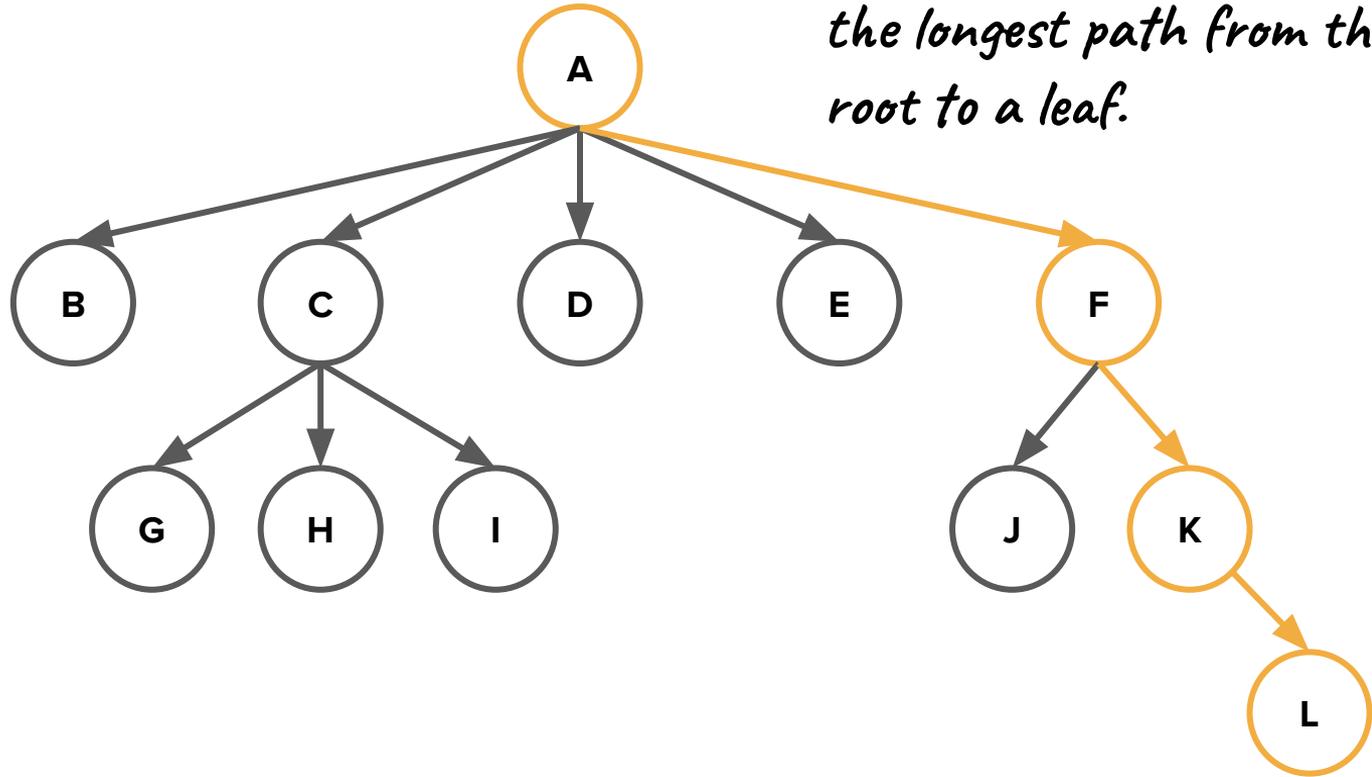
# Tree Terminology

The **height** can also be defined as the number of nodes along the longest path from the root to a leaf.



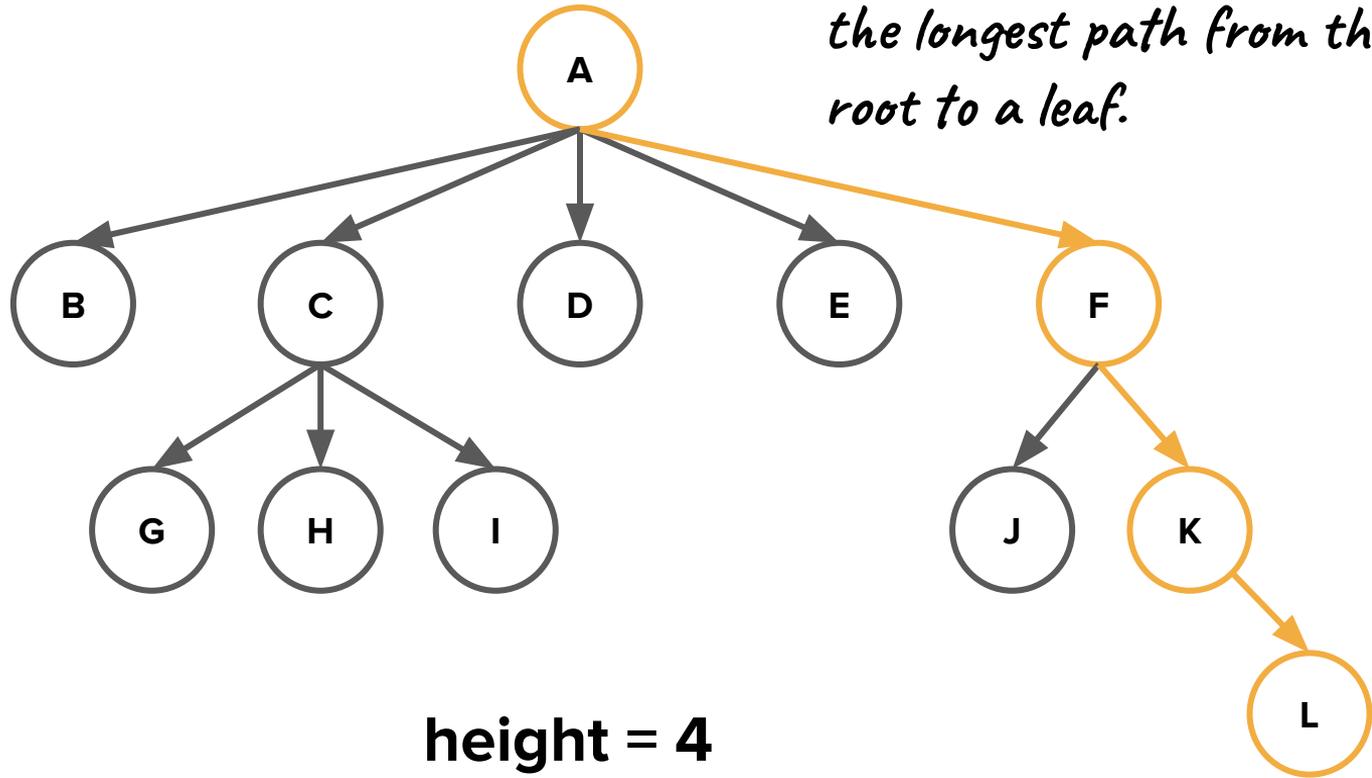
# Tree Terminology

The **height** can also be defined as the number of nodes along the longest path from the root to a leaf.



# Tree Terminology

The **height** can also be defined as the number of nodes along the longest path from the root to a leaf.



# Tree Terminology Summary

- Every non-empty tree has a **root node** that defines the "top" of the tree.
- Every node has 0 or more **children** nodes descended from it. Nodes with no children are called **leaf nodes**.
- Every node in a tree has exactly one **parent** node (except for the root node).
- A **path** through the tree traverses edges between parents and their children.
- The **depth** of a node is the length of the path between the root and that node. A tree's **height** is the number of nodes in the longest path through the tree.

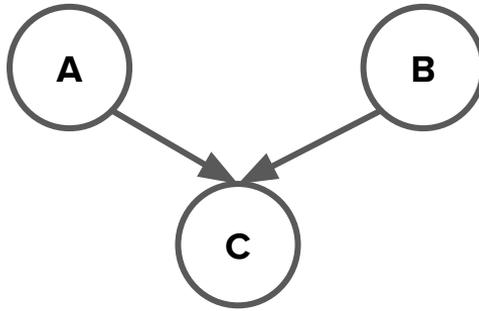
# Tree Properties

# Tree Properties

- Any node in a tree can only have one parent.

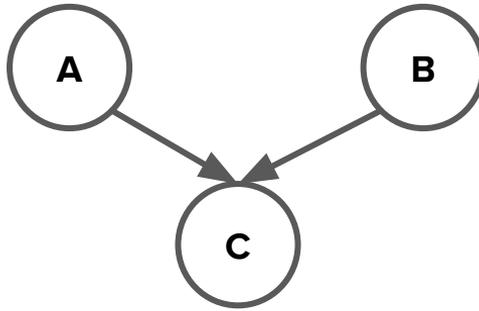
# Tree Properties

- Any node in a tree can only have one parent.



# Tree Properties

- Any node in a tree can only have one parent.



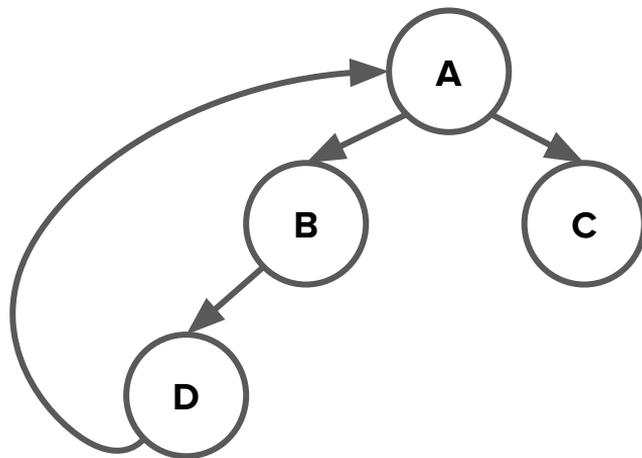
*Not a tree!*

# Tree Properties

- Any node in a tree can only have one parent.
- The tree cannot have any cycles. That is, there should be no way to make a complete loop through the tree.

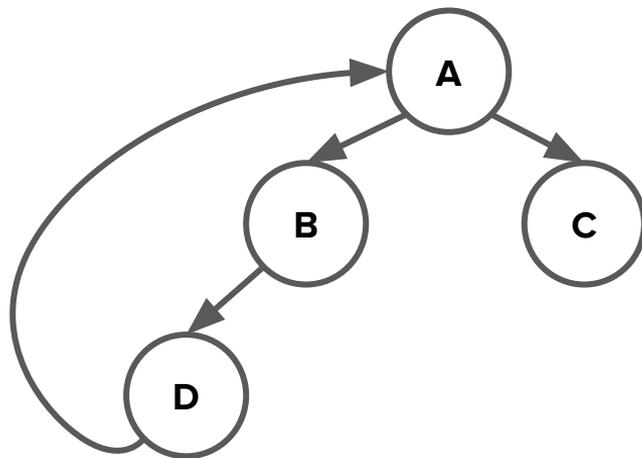
# Tree Properties

- Any node in a tree can only have one parent.
- The tree cannot have any cycles. That is, there should be no way to make a complete loop through the tree.



# Tree Properties

- Any node in a tree can only have one parent.
- The tree cannot have any cycles. That is, there should be no way to make a complete loop through the tree.



*Not a tree!*

# Announcements

# Announcements

- Final project feedback was released this morning!
  - Some of you may have received feedback requesting that you meet with one of us in order to receive full credit.
  - Next week's OH will be all group OH to accommodate more meetings. We'll also have final project work time next Friday during class when you can chat with us. Links for OH + lecture will stay the same, but there will be no individual OH appointments.
  - Please stay on track with the suggested schedule!
- Kylie's OH times will be a little bit shifted the next two weeks because of the start of the (middle school) school year. **Next Tuesday's OH (August 17) will be from 2-4pm PDT.** Updates are reflected in the calendar.

# Announcements

- Assignment 6 is due on **Friday, August 13 at 11:59pm PDT** with a 24-hour grace period.
- Assignment 5 revisions are due **Tuesday, August 17**. Due to the end of quarter timeline, there will be **no revisions on Assignments 6 and 7**.
- The change in grading basis deadline is this **Friday, August 13 at 5pm PDT**.

# Trees in C++

# Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.

# Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.

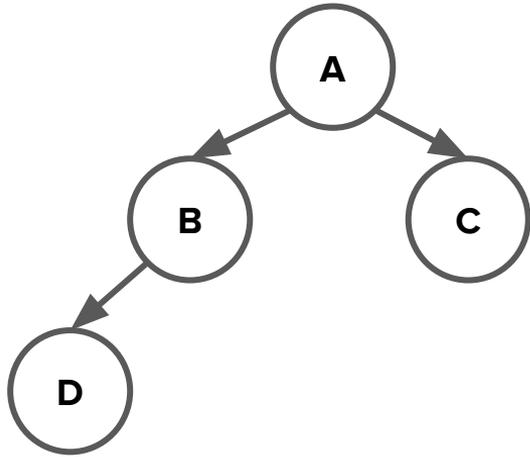
# Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.
- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.

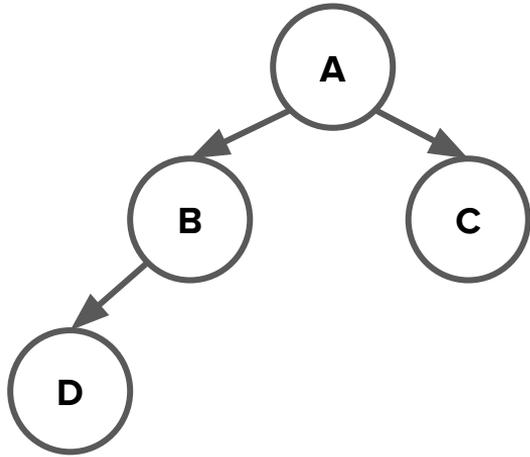
# Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.
- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.
- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.

# Binary Trees

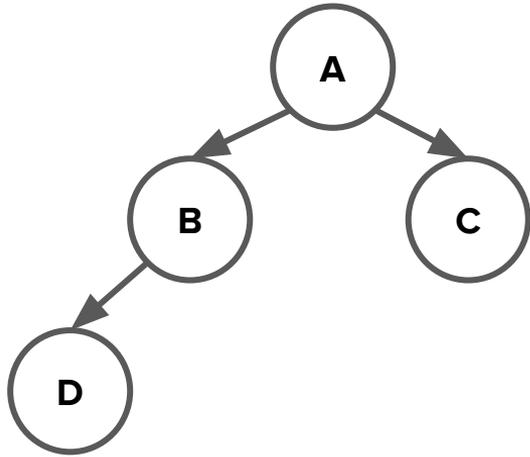


# Binary Trees

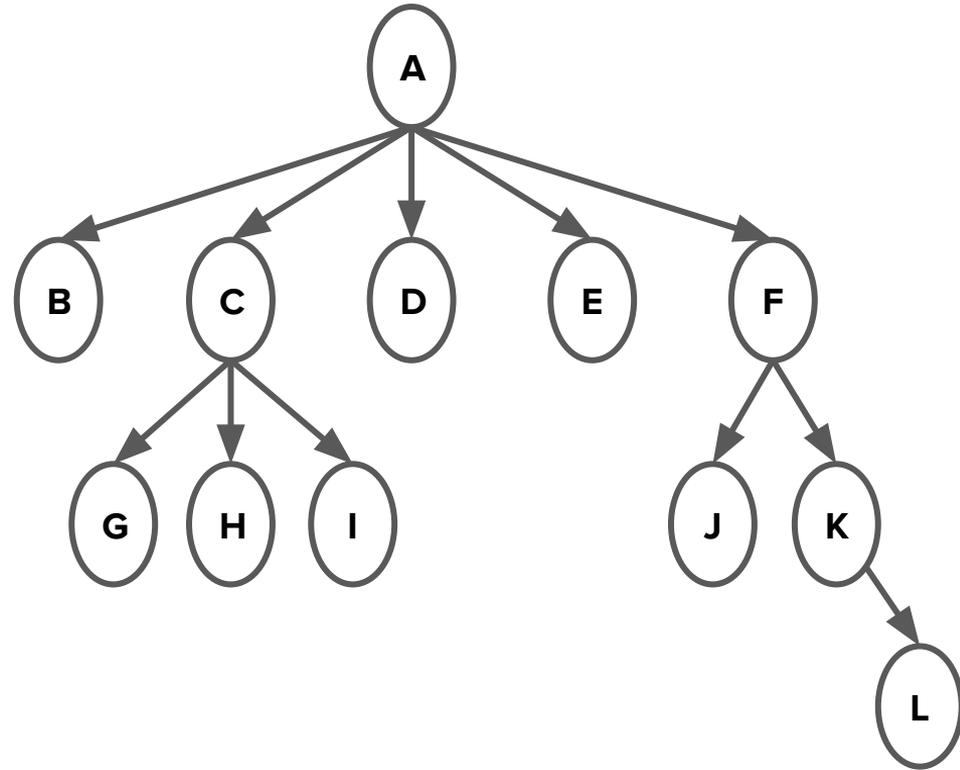


*Binary Tree!*

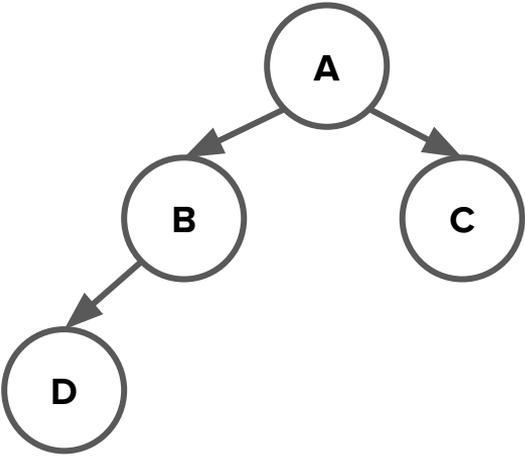
# Binary Trees



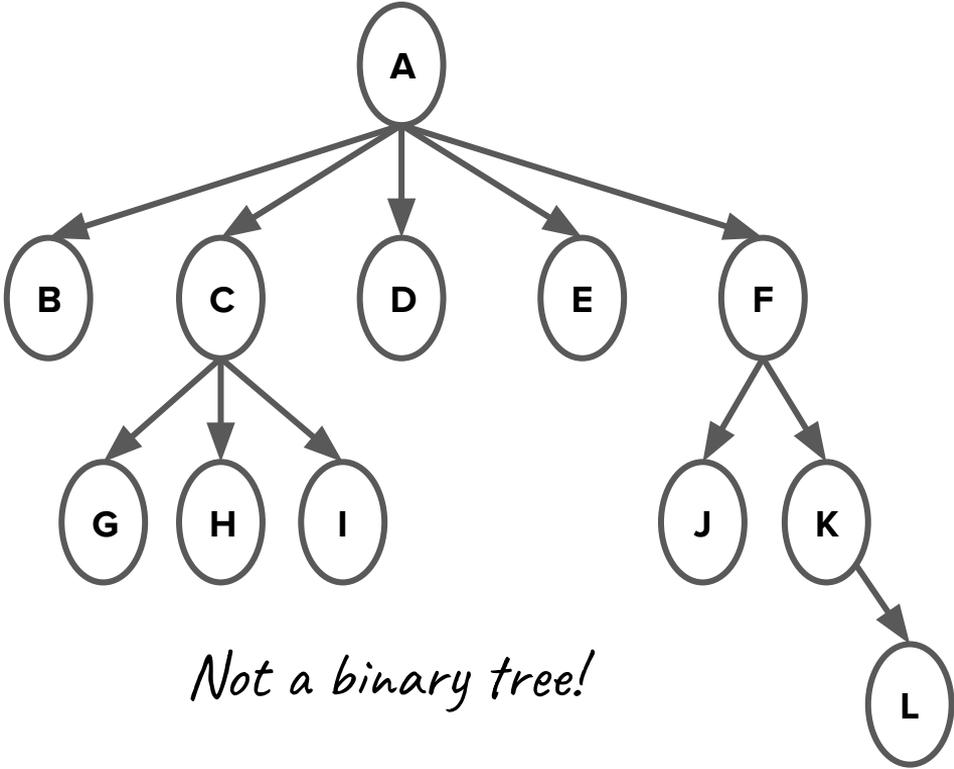
*Binary Tree!*



# Binary Trees



*Binary Tree!*



*Not a binary tree!*

# Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

# Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.
- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.

# Building Trees Programmatically

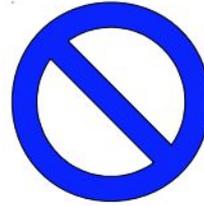
- To build a tree in C++, we need a new version of the Node struct we've seen before.
- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.

```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

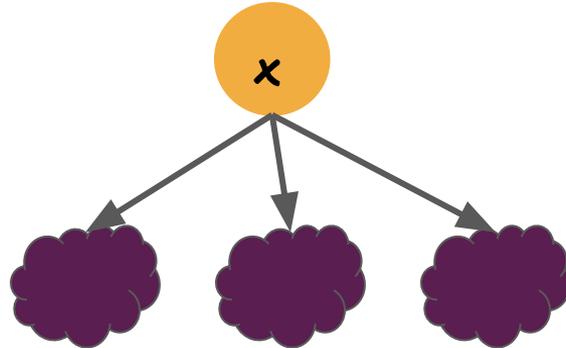
# What is a tree?

## A tree is either...

An empty data structure, or...



A single node (parent), with zero or more non-empty subtrees (children)



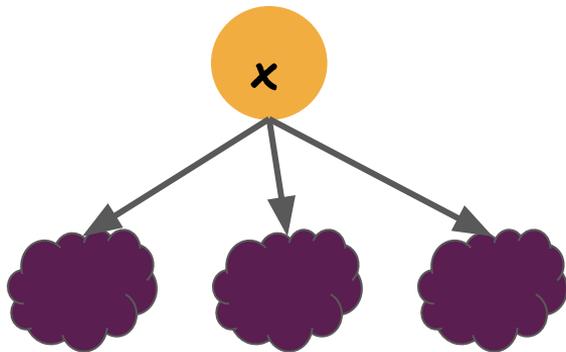
# What is a tree in C++?

## A tree is either...

An empty data structure, or...



A single node (parent), with zero or more non-empty subtrees (children)



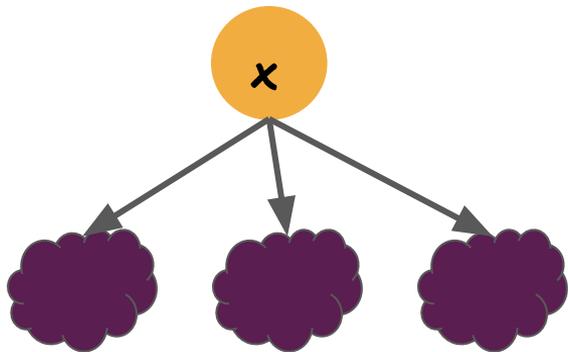
# What is a tree in C++?

## A tree is either...

An empty tree  
represented by  
`nullptr`, or...



A single node  
(parent), with zero or  
more non-empty  
subtrees (children)



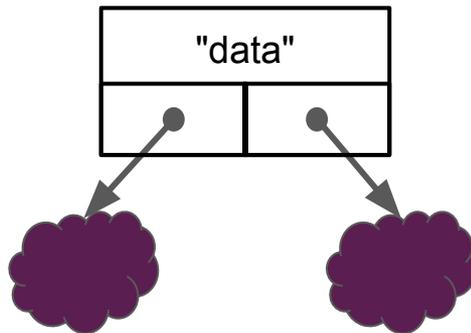
# What is a tree in C++?

## A tree is either...

An empty tree represented by `nullptr`, or...



A single `TreeNode`, with 0, 1, or 2 non-null pointers to other `TreeNode`s



# Building Trees Programmatically

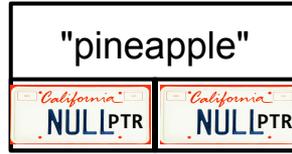
```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

# Building Trees Programmatically



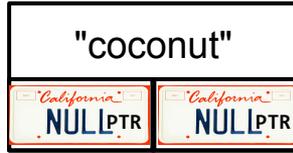
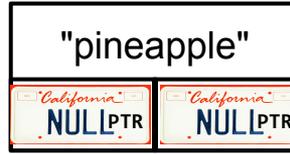
```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

# Building Trees Programmatically



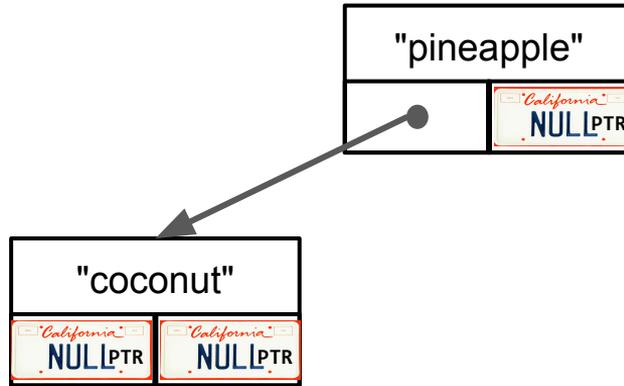
```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

# Building Trees Programmatically



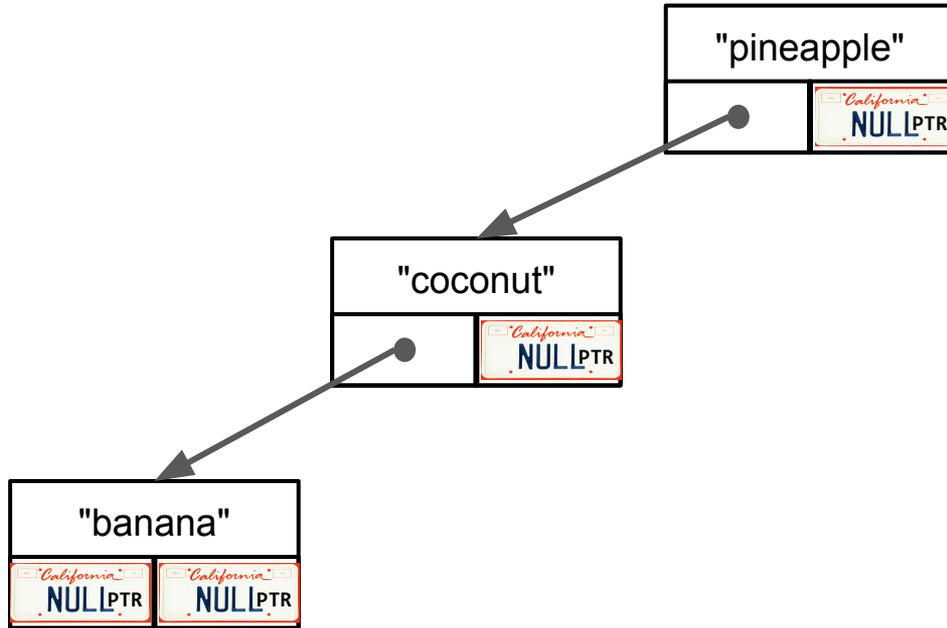
```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

# Building Trees Programmatically



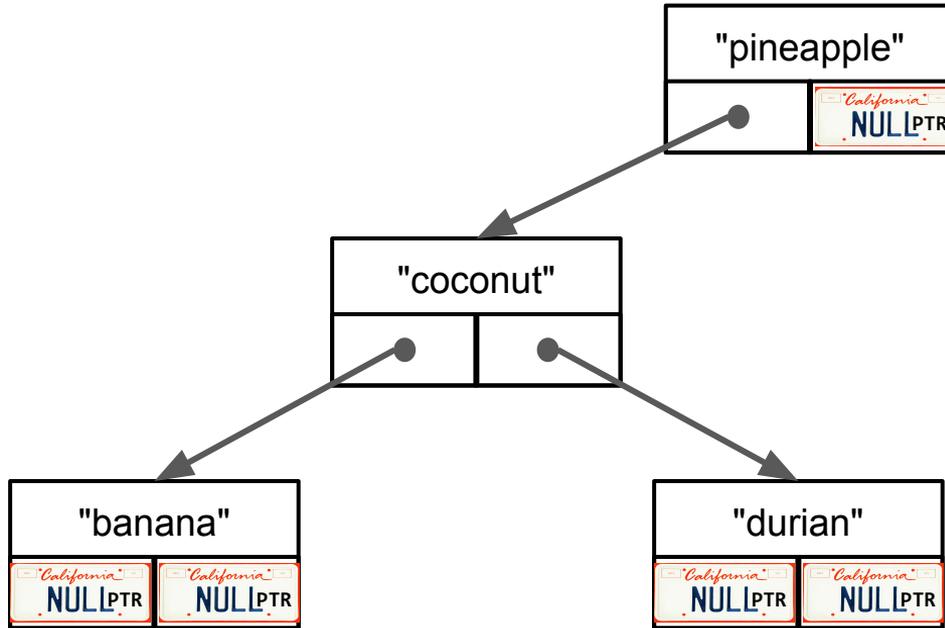
```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

# Building Trees Programmatically



```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

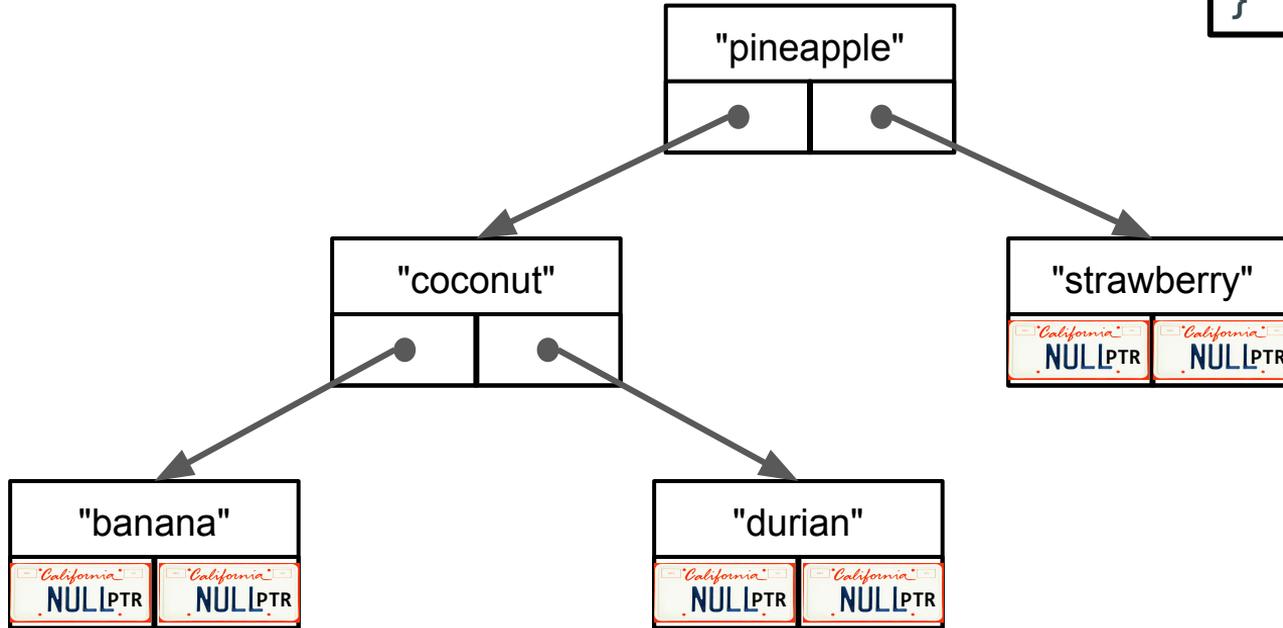
# Building Trees Programmatically



```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```

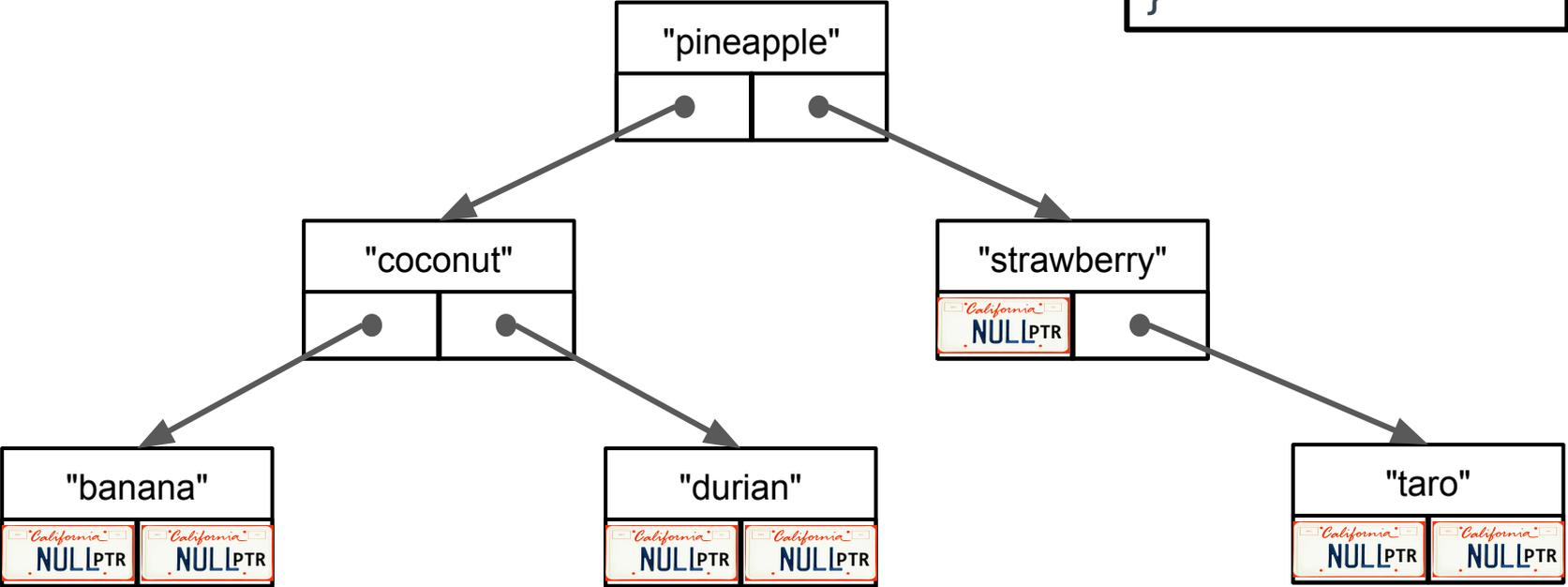
# Building Trees Programmatically

```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```



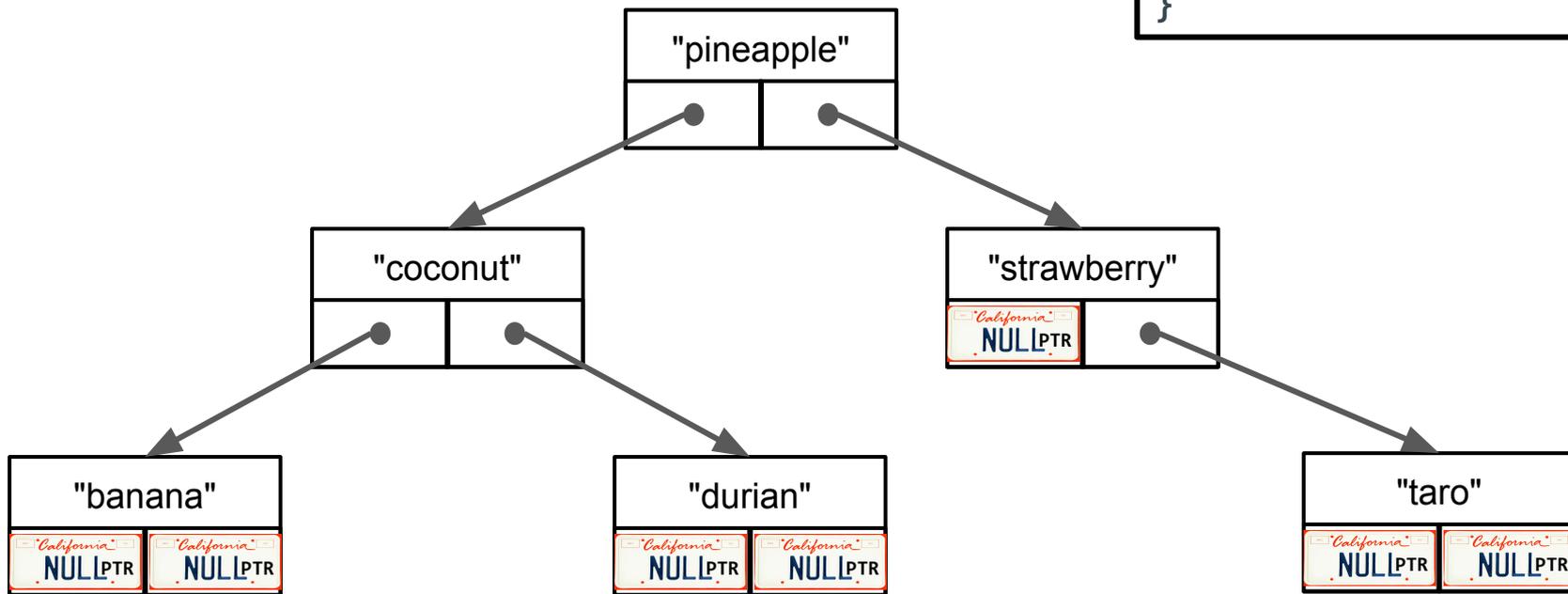
# Building Trees Programmatically

```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```



# Building Trees Programmatically

```
struct TreeNode {  
    string data;  
    TreeNode* left;  
    TreeNode* right;  
}
```



*Note: Trees do not have to be complete, like heaps. Any node can have 0, 1, or 2 children.*

Let's code it!

```
buildExampleTree()
```

# Building a Tree Takeaways

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We integrate these new nodes into the tree by rewiring the **left** and **right** pointers of existing nodes in the tree.

# Tree Traversals

# Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!

# Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!
- There are three main ways to traverse a binary tree:
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal

# Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!
- There are three main ways to traverse a binary tree:
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal
- Due to the recursive nature of trees, all of these algorithms are most easily defined **recursively**.

# Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree
- For example purposes, let's have our "do something" to be printing the contents of the current node, which will allow us to print the overall tree.

Let's code it!

`preorderPrintTree()`

# Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree
- For example purposes, let's have our "do something" be printing the contents of the current node, which will allow us to print the overall tree.
- Output: **pineapple coconut banana durian strawberry taro**

# In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

Let's code it!

**`inorderPrintTree()`**

# In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree
- Output: **banana coconut durian pineapple strawberry taro**
- Observation: The output of this traversal gives us all the values in alphabetical order. Is this a coincidence?
  - No! We'll see why Friday!

# Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

Try it yourself!

**postorderPrintTree()**

# Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node
- Output: **banana durian coconut taro strawberry pineapple**
- Application: Freeing trees! (we'll see this in lecture tomorrow)

# Summary

# Trees Summary

- Trees allow us to organize information in a linked data structure such that the distance to any element is short, even if there are many elements.
- Trees organize nodes in a hierarchical manner, where each element contains connections to children nodes that exist "lower" in the tree.
- There are three main ways to traverse the nodes in a tree, and each type of traversal visits the nodes of the tree in a distinctly different order.

What's next?

# Roadmap

## Object-Oriented Programming

### C++ basics

User/client

**vectors + grids**

**stacks + queues**

**sets + maps**

Implementation

arrays

dynamic memory management

linked data structures

real-world algorithms

*Life after CS106B!*

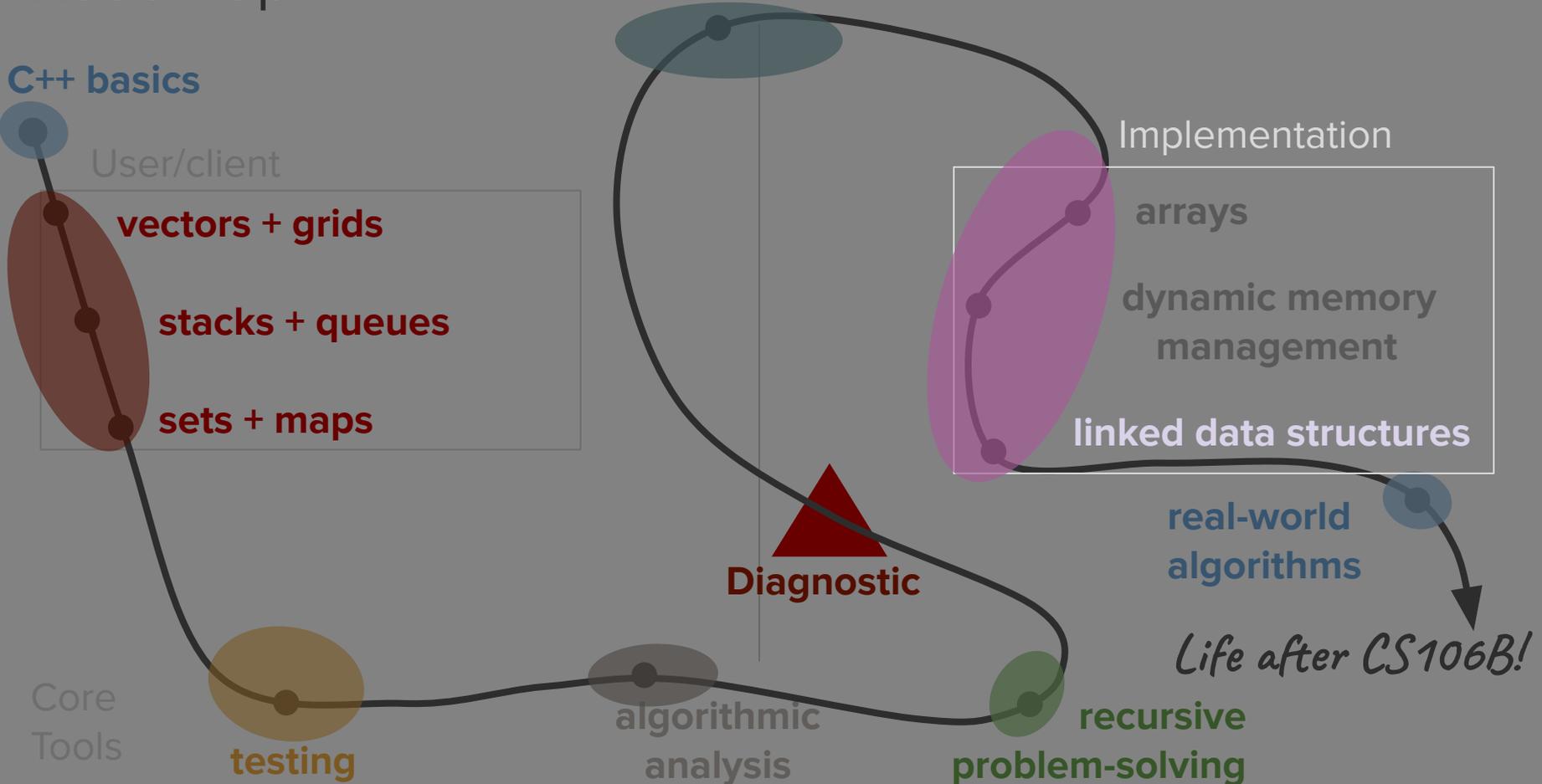
Core Tools

**testing**

algorithmic analysis

**recursive problem-solving**

**Diagnostic**



# Binary Search Trees

