Programming Abstractions

CS106B

Cynthia Bailey Lee
Julie Zelenski

Topics:

- Priority Queue ADT
  - Heap data structure implementation
    - What are binary trees?
    - What are heaps?
    - How do we do insert/remove operations on heaps?

Priority Queue

Emergency Department waiting room operates as a priority queue: patients are sorted according to priority (urgency), not "first come, first serve" (in computer science, "first in, first out" or FIFO).

Contents of one element of a Priority Queue

- Individual elements of our priority queue will have two pieces to them:
  - An integer indicating the **priority** of this element
    - We will use smaller number means higher priority, but could be done either way
  - A “**payload**” of whatever the actual element data is
    - Examples:
      - **a class MedicalRecord** that has many fields and is the patient’s entire medical history
      - **a string** that is the name of a student waiting in the Lair queue (in a world where Lair is based on urgency of request, rather than FIFO)
      - etc.

6	"SooMin"	13	"Diego"	15	"Muhammad"	22	"Sasha"

Two priority queue implementation options

0		1		2		3		4	
22	"Sasha"	6	"SooMin"	15	"Muhammad"	13	"Diego"		

Unsorted array

- Always insert new element *at the end of the array*
- Remove by searching entire array for highest-priority item, then removing it, and (if needed) scooting elements over to fill in the gap

0		1		2		3		4	
22	"Sasha"	15	"Muhammad"	13	"Diego"	6	"SooMin"		

Sorted array

- Always insert new elements *where they go* in priority-sorted order, with the highest-priority item at the end of the array
- Remove by taking the last element of the array

Priority queue implementations

Unsorted array

Add is FAST

- Just throw it in the array at the back
- **O(1)**

Remove/peek is SLOW

- Hard to find item the highest priority item—could be anywhere
- Might need to scoot over elements to fill gap
- **O(N)**

0		1		2		3		4	
22	"Sasha"	6	"SooMin"	15	"Muhammad"	13	"Diego"		

Priority queue implementations

Sorted array

Add is **SLOW**

- Need to step through the array to find where item goes in priority-sorted order
- If proper place is in the front/middle, need to scoot over other elements to make room
- **O(N)**

Remove/peek is **FAST**

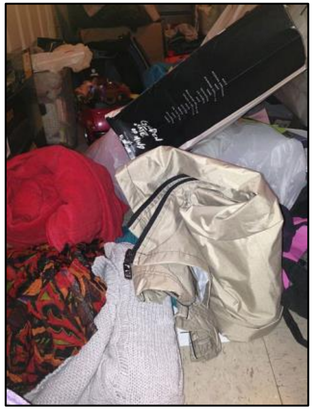
- Easy to find item you are looking for (last in array)
- No need to scoot over elements when removing last
- **O(1)**

Image is in the public domain.
http://commons.wikimedia.org/wiki/File:Wall_Closet.jpg

0		1		2		3		4	
6	"SooMin"	13	"Diego"	15	"Muhammad"	22	"Sasha"		

Would be great if we could get the best of both…

Fast add *and* fast remove/peek


Fast add

+

Fast remove/peek

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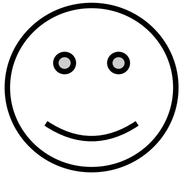
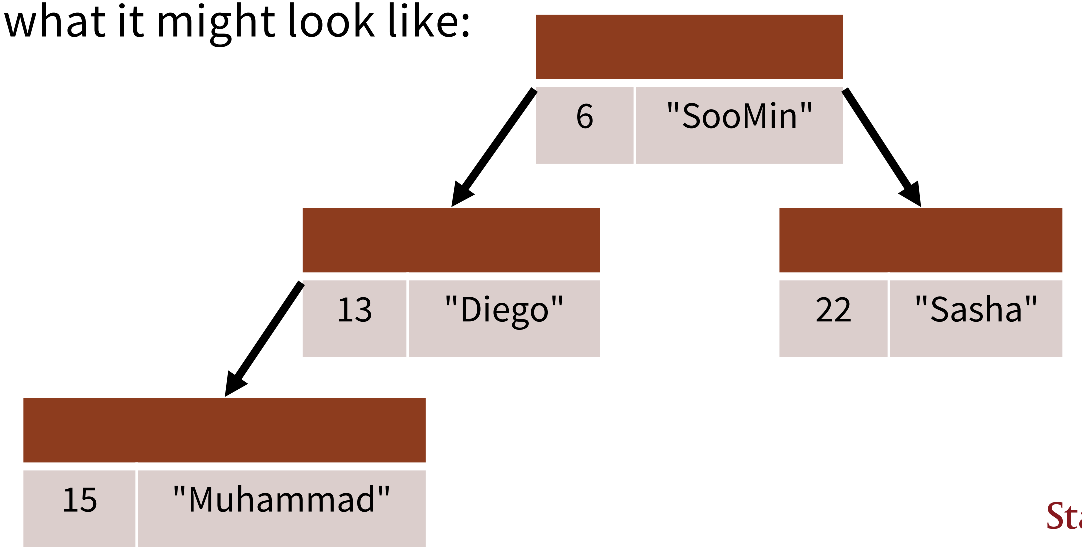


Binary heap for our priority queue

- Instead of storing our priority queue nodes entirely sorted or entirely unsorted, we will store them *partially-sorted*.
- The partial sorting will still be stored in an array, but it's best to imagine it as what we call a "tree" in computer science (computer science trees are upside-down for some reason ¯_(ツ)_/¯)
- Here's what it might look like:



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Binary trees

Before we delve into how to construct a binary heap, let’s take a step back and introduce computer science binary trees generally

A binary tree

“In computer science, a **binary tree** is a tree data structure in which each node has at most two child nodes, usually distinguished as "left" and "right."”

(Thanks, Wikipedia!)

How many of these are valid binary trees?

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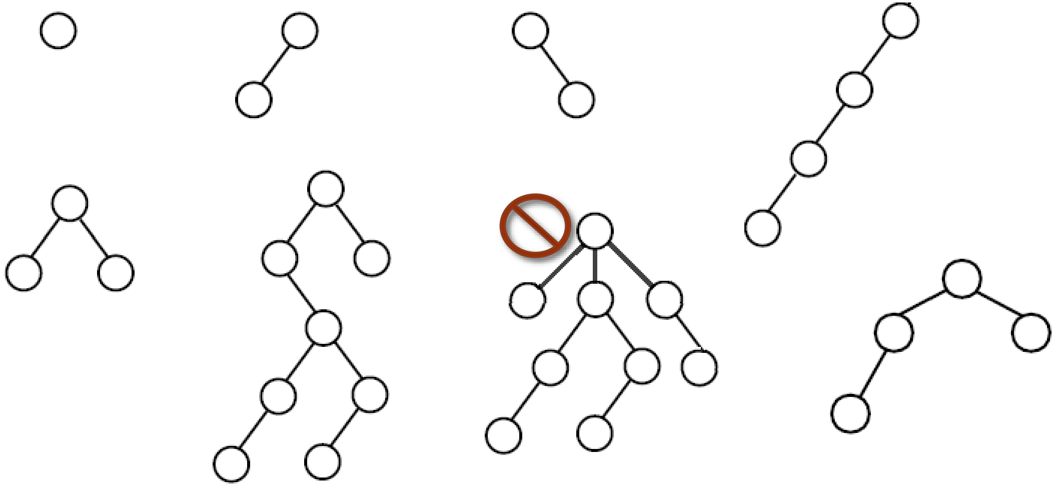

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Heaps!

Binary Heaps*

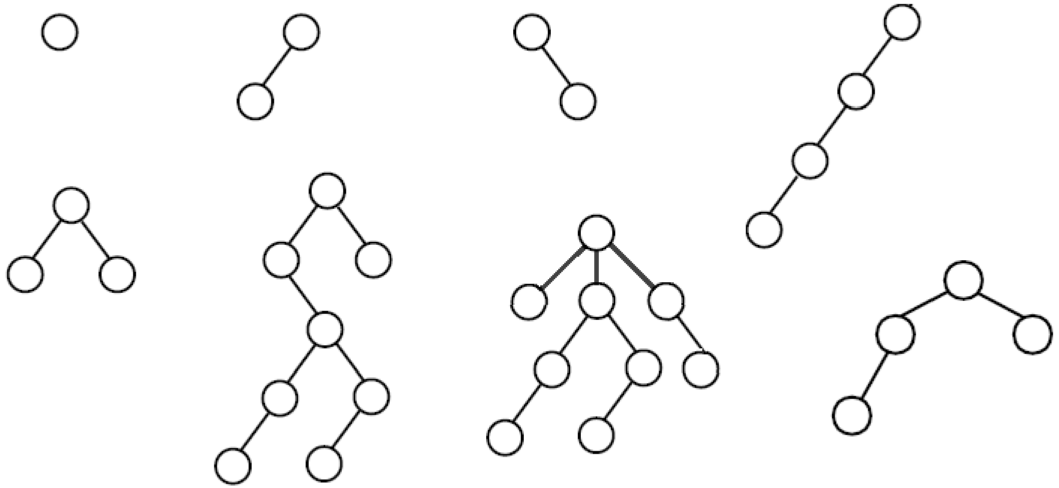
Binary heaps are **one kind** of binary tree

They have a few special restrictions, in addition to the usual binary tree:

- Must be **complete**
  - No “gaps”—nodes are filled in left-to-right on each level (row) of the tree
- Ordering of data must obey **heap property**
  - Min-heap version: a parent’s priority is always ≤ both its children’s priority
  - Max-heap version: a parent’s priority is always ≥ both its children’s priority

* There are other kinds of heaps as well. For example, binomial heap is an extra-fun one!

How many of these could be valid binary heaps?



A. 0-1

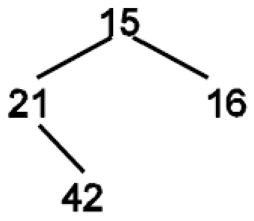
B. 2

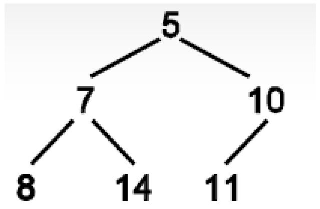
C. 3

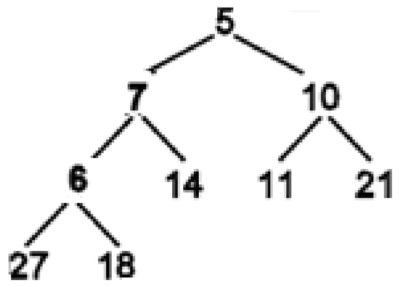
D. 4

E. 5-8

How many of these are valid min-binary-heaps?



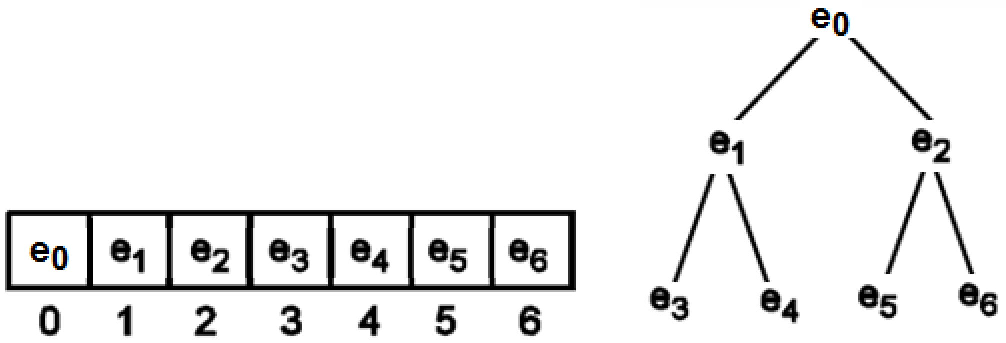




Binary heap in an array

Binary heap in an array

- Because of the special constraint that they must be **complete**, binary heaps fit nicely into an **array**
  - *As we'll see later, this is not true of some other kinds of tree data structures, and we'll use a different approach for those*

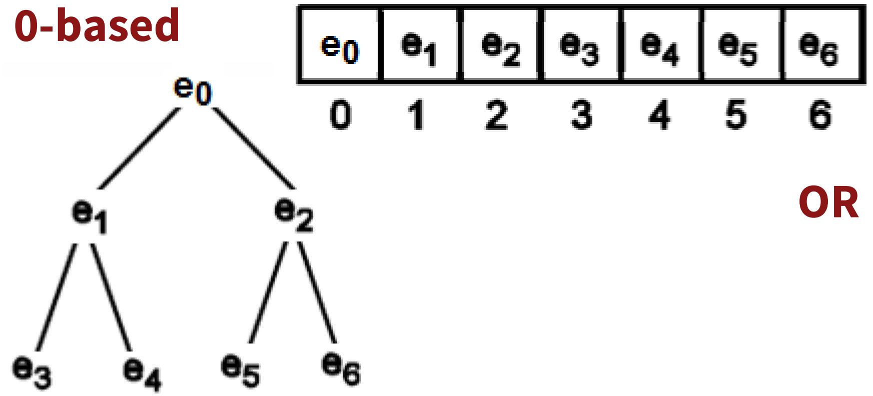


Two approaches: Binary heap in an array

Wait, but the homework handout starts storing the elements at array index 1!

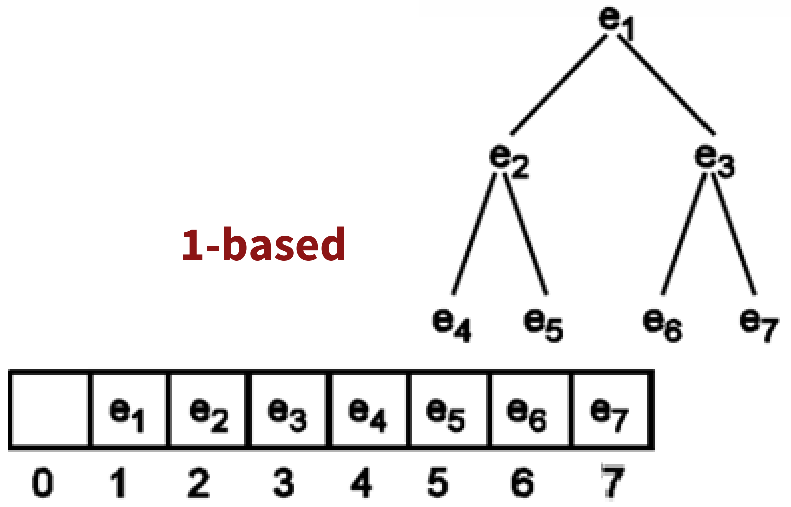
- Either way is ok for the assignment.
- You should understand both ways, so we're teaching both ways

0-based

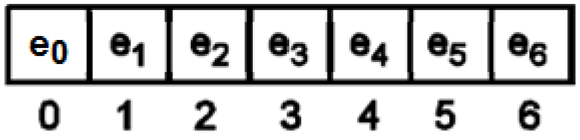


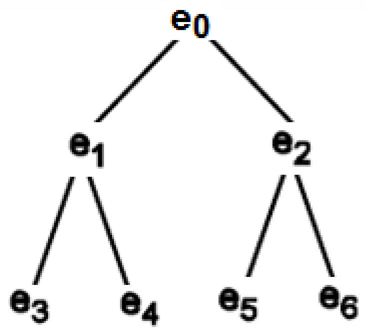
OR

1-based



Heap in an array

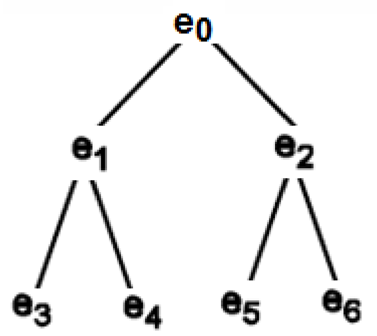


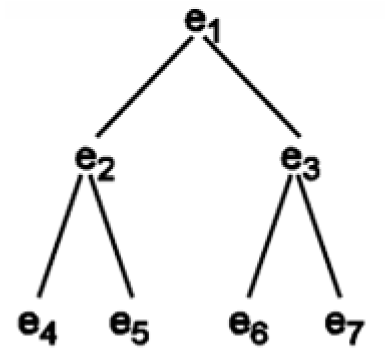


For a node in array index i:

- Q: The parent of that node is found where?
- A: at index:
  - A. i – 2
  - B. i / 2
  - C. (i – 1)/2
  - D. 2i

Fact summary: Binary heap in an array





0-based:

For tree of height h, array length is 2^h-1

For a node in array index i:

- Parent is at array index: (i – 1)/2
- Left child is at array index: 2i + 1
- Right child is at array index: 2i + 2

1-based:

For tree of height h, array length is 2^h

For a node in array index i:

- Parent is at array index: i /2
- Left child is at array index: 2i
- Right child is at array index: 2i + 1

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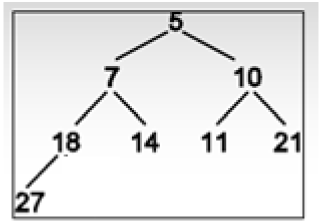
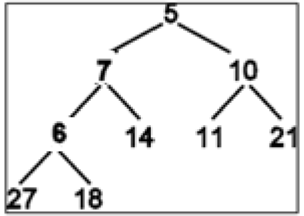
Binary heap enqueue and dequeue

Binary heap enqueue example (insert 6 + "bubble up")



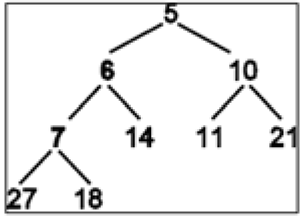
Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	18	14	11	21	27	6	?	…	?



Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	6	14	11	21	27	18	?	…	?



Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	6	10	7	14	11	21	27	18	?	…	?

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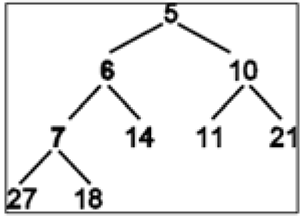
Binary heap dequeue (delete min)



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Size=9, Capacity=15
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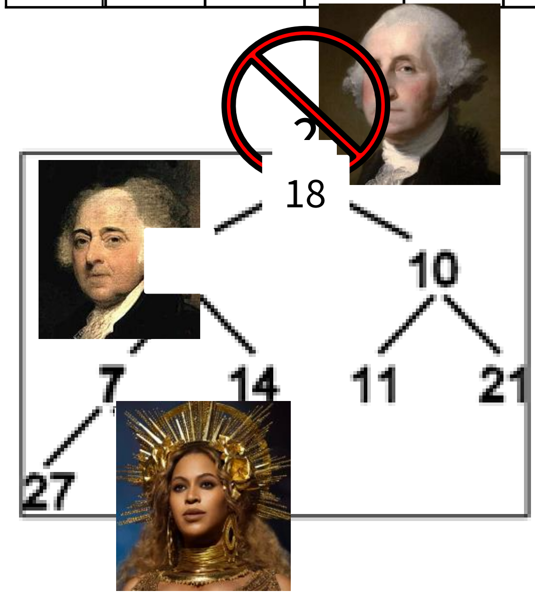
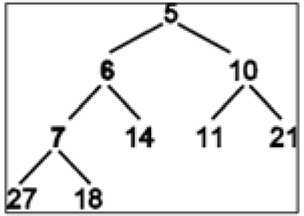
0	1	2	3	4	5	6	7	8	9	…	14
18	6	10	7	14	11	21	27		…	…	…

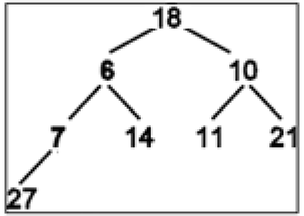


Binary heap dequeue (delete min + "trickle down")



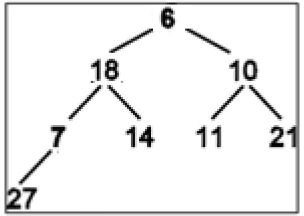
Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	6	10	7	14	11	21	27	18	?	…	?



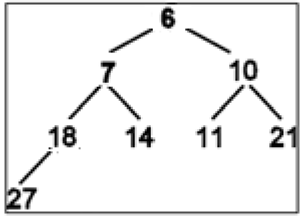
Size=**8**, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
18	6	10	7	14	11	21	27	18	?	…	?



Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
6	**18**	10	7	14	11	21	27	18	?	…	?



Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
6	**7**	10	**18**	14	11	21	27	18	?	…	?

Summary analysis

Comparing our priority queue options

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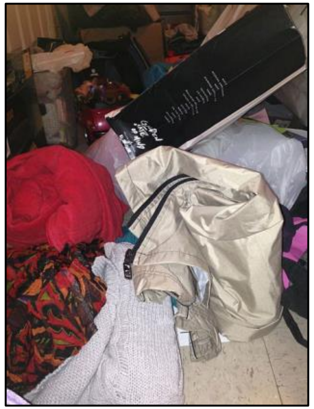
Would be great if we could get the best of both…

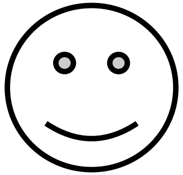
Fast add *and* fast remove/peek


Fast add

+

Fast remove/peek

=



Review: priority queue implementation options performance

Unsorted array

- Insert new element in back: O(1)
- Remove by searching list and scooting over: O(N)

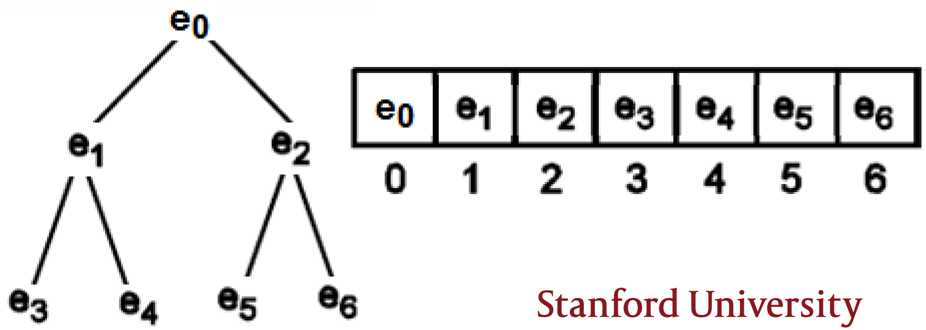
Sorted array

- Always insert in sorted order: O(N)
- Remove from back: O(1)

Binary heap

- Insert + "bubble up": O(logN)
- Delete + "trickle down": O(logN)

☺



Final aside on terminology

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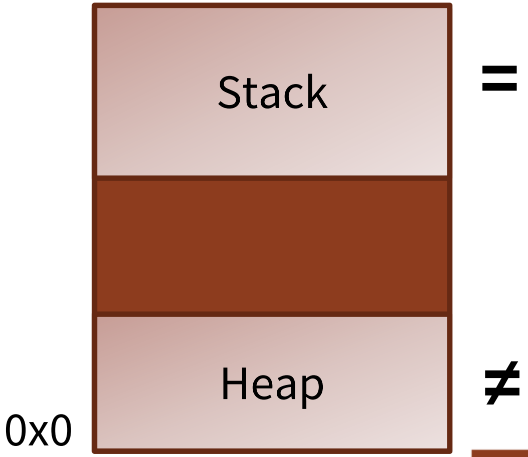
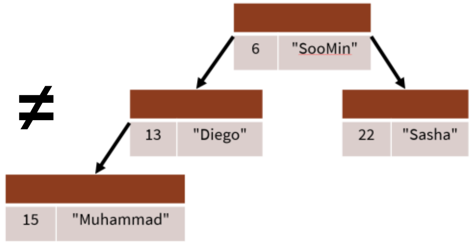
Aside: Binary Heap, not to be confused with Heap memory!

- The Stack section of memory **is** a Stack like the ADT
- The Heap section of memory **has nothing to do** with the Heap structure.



Stack ADT



Heap data structure

- Probably just happened to reuse the same word ☹

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