# Programming Abstractions

CS106B

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# Topics:

- **Map** implemented as a Binary Search Tree (BST)
  - Starting with a dream: binary search in a linked list?
  - How our dream provided the inspiration for the BST
  - BST insert
  - Big-O analysis of BST
  - BST balance issues
- Traversals
  - Pre-order
  - In-order
  - Post-order
  - Breadth-first
- Applications of Traversals

# BST Balance Strategies

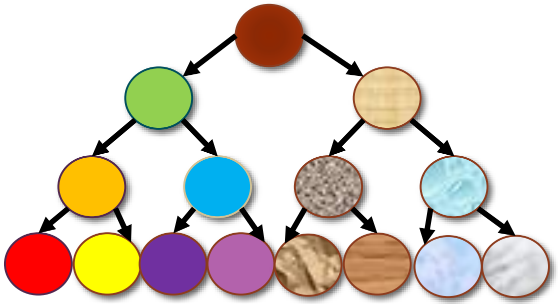
WE NEED TO BALANCE THE TREE TO KEEP PERFORMANCE O(LOGN) INSTEAD OF O(N)

# Step 1: understanding validity and equivalence in BSTs

AVL ROTATIONS: A KEY TO OUR REBALANCING ALGORITHMS

# AVL rotations: BST-order-preserving movement of nodes

- Here is a Binary Search Tree whose keys I'm not going to show you
  - (but the nodes have colors/textures so you can tell them apart)
- Let's pause and think about what we know must be true



# AVL rotations

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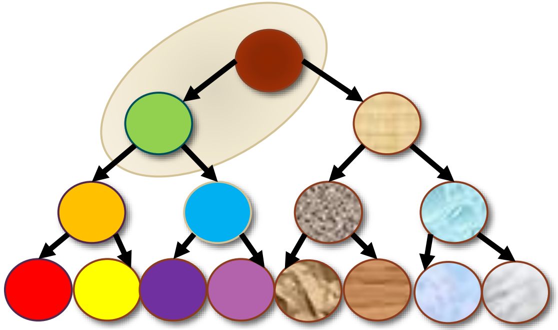
  1. Cardinal's key > green's key



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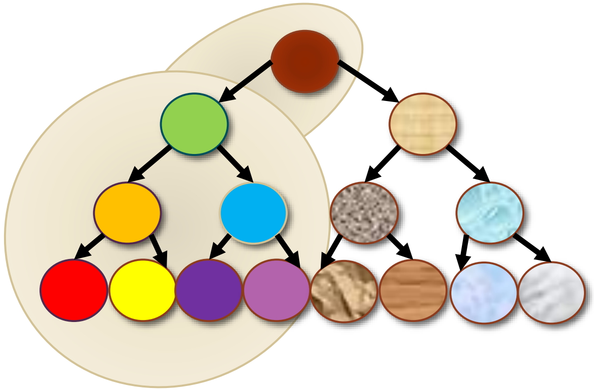
  2. Cardinal's key > all 7 keys to its left!



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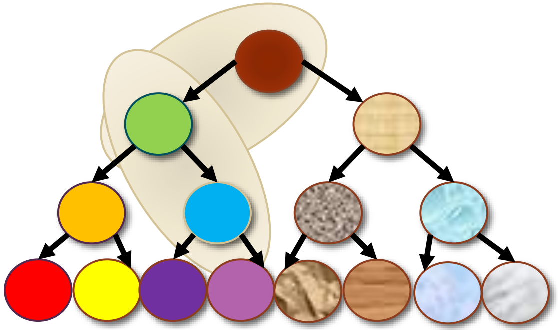
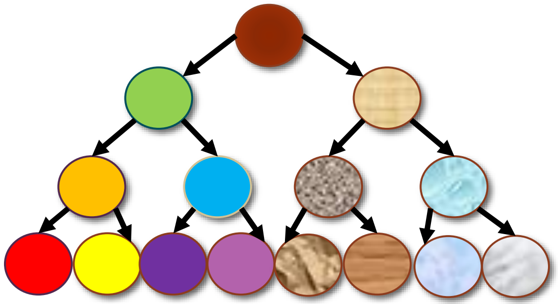
  3. Green's key < blue's key < cardinal's key



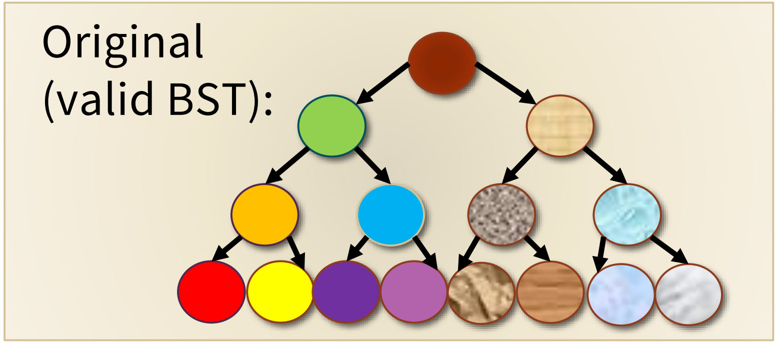
# AVL rotations

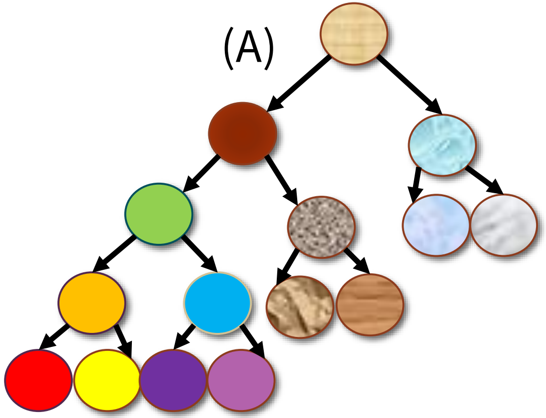
- Here is a Binary Search Tree whose keys I’m not going to show you
  - (but the nodes have colors/textures so you can tell them apart)
- Let’s pause and think about what we know must be true
  1. Cardinal’s key > green’s key
  2. Cardinal’s key > all 7 keys to its left!
  3. Green’s key < blue’s key < cardinal’s key
- Those are just a few examples of the kind of reasoning you’ll want to use for this exercise…

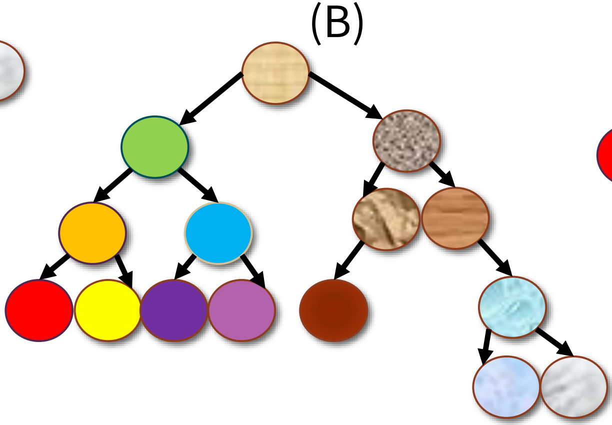


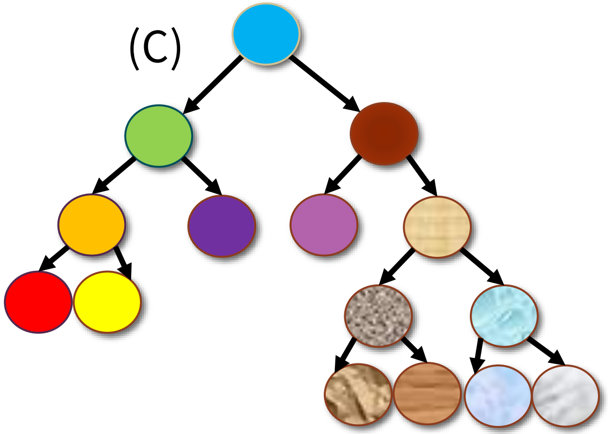
# AVL rotations

- **Your turn:** Which of the trees below are still in BST order? *(list all that apply)*



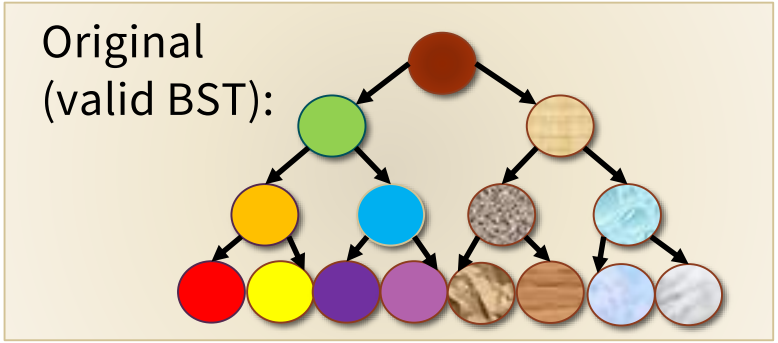



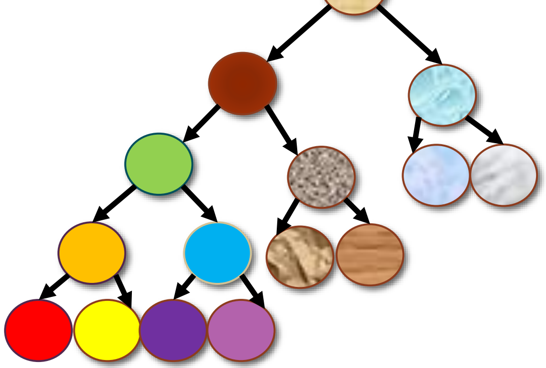



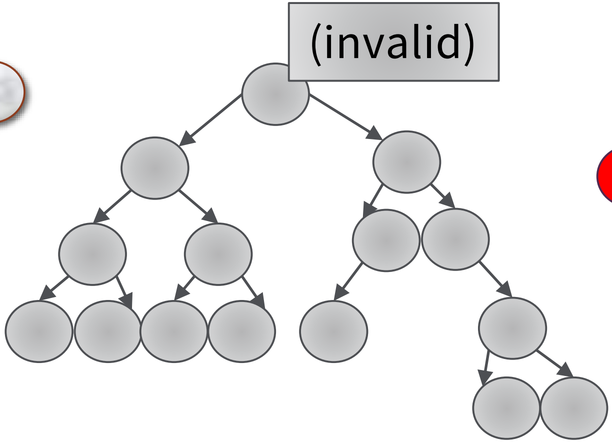



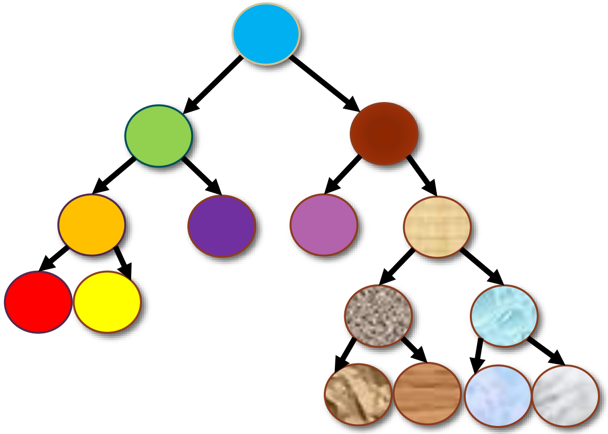
# AVL rotations

- 2/3 are actual AVL rotations!
- In this case, our BST started balanced, so the rotations made the *less* balanced. But also useful for balancing.

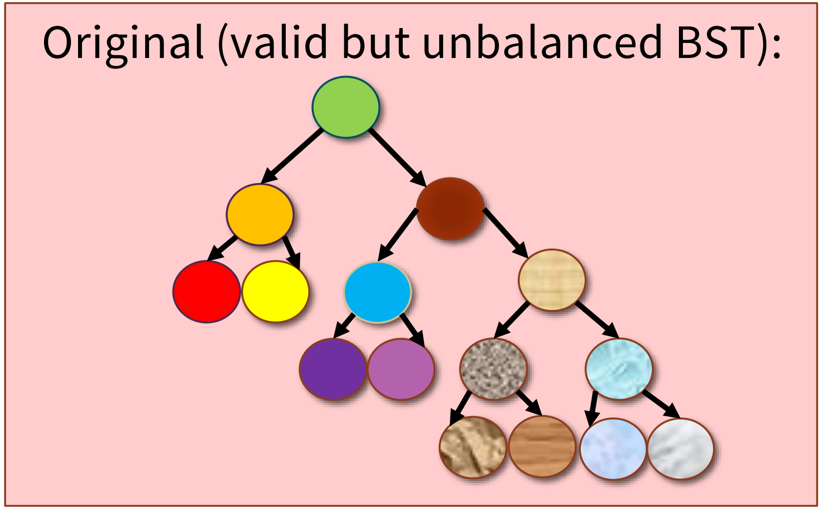


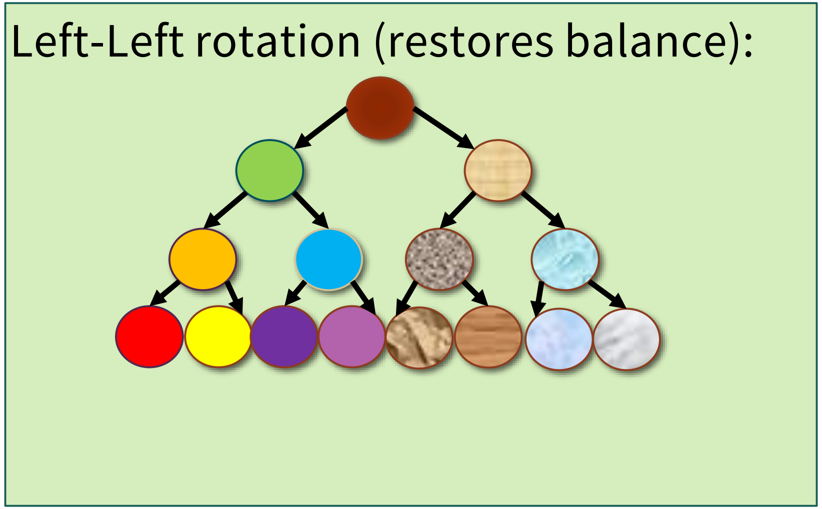





# Left-Left AVL Rotation

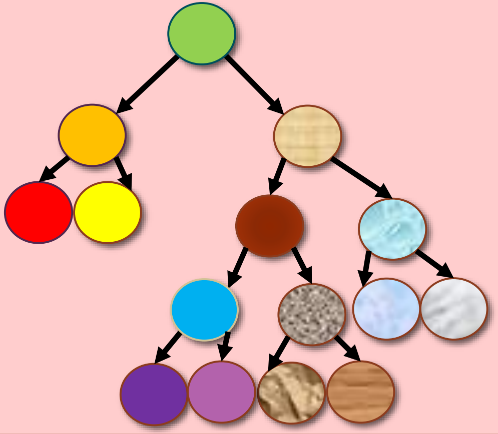


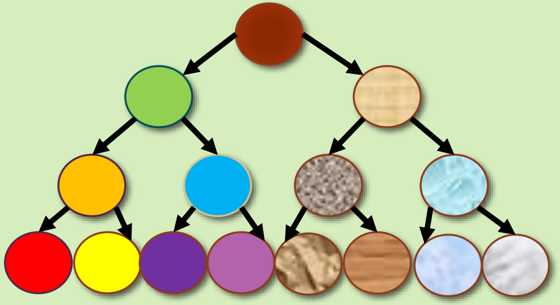

- Right-Right is just the mirror image

# Right-Left AVL Rotation

Original (valid but unbalanced BST):



Right-Left rotation (restores balance):



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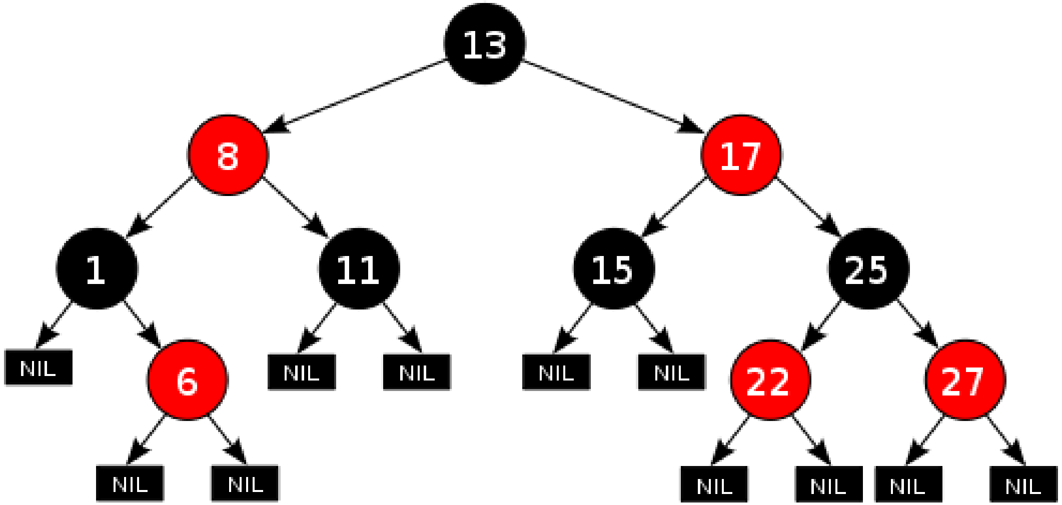
# A few BST balance strategies

- AVL tree
  - Uses AVL rotations to guarantee balance
- Red-Black tree
  - Uses AVL rotations to guarantee balance is off by no more than a constant factor (longest path from root to leaf can be at most 2x the shortest path)
- Treap
  - Each node has *two* keys and a value, one is BST key, one is a min-heap key, *both kinds of trees' order properties are maintained (!!!)*
  - Insert nodes according to BST keys and BST order
  - Then use AVL rotations to "bubble up" the newly inserted node as needed to restore the min-heap order property on the min-heap keys
  - What could be cooler than that, amirite? ♥ 😍 ♥ 😍

Video: http://www.youtube.com/watch?v=vDHFF4wjWYU

# Red-Black trees




**Every simple path from a given node to any of its descendant leaves contains the same number of black nodes.**

- (This is what guarantees "close" to balance)

# Other fun types of BST

Splay tree

- Rather than only worrying about balance, Splay Tree dynamically readjusts based on **how often users search for an item**. Most commonly-searched items move towards the root, saving time
  - Example: if Google did this, "Bieber" would be near the root, and "splay tree" would be further down by the leaves

B-Tree

- Like BST, but a node can have many children, not just two
- More branching means an even "flatter" (smaller height) tree
- Used for huge databases

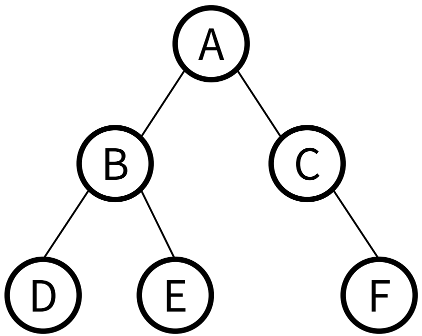
# Tree Traversals!

THESE ARE FOR ANY BINARY TREES, BUT WE OFTEN DO THEM ON BSTS

# Your Turn: What does this print?
## (assume we call traverse on the root node to start)

```
void traverse(Node* node) {
  if (node != nullptr) {
     cout << node->key << " ";
     traverse(node->left);
     traverse(node->right);
  }
}
```

A. A B C D E F
B. A B D E C F
C. D B E F C A
D. D E B F C A
E. Other/none/more



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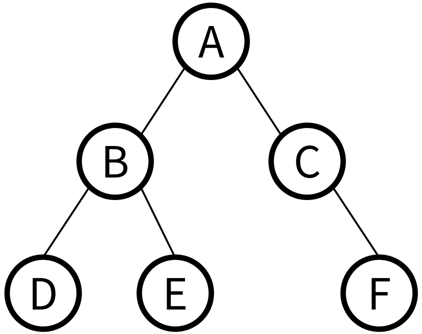


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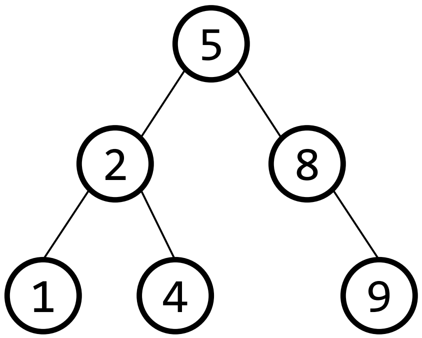



A. 1 2 4 5 8 9
B. 1 4 2 9 8 5
C. 5 2 1 4 8 9
D. 5 2 8 1 4 9
E. Other/none/more

# Applications of Tree Traversals

BEAUTIFUL LITTLE THINGS FROM AN ALGORITHMS/THEORY STANDPOINT, BUT THEY HAVE A PRACTICAL SIDE TOO!

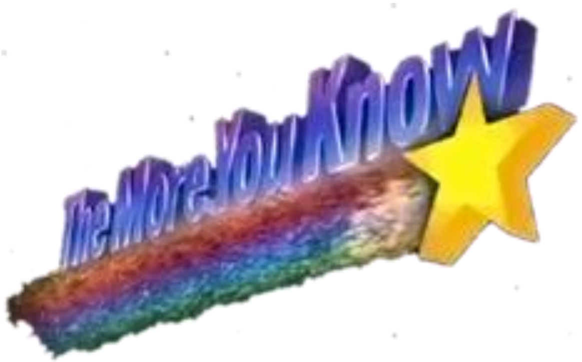
# Traversals a very commonly-used tool in your CS toolkit

```
void traverse(Node* node) {
  if (node != NULL) {
     traverse(node->left);
     // "do something”
     traverse(node->right);
  }
}
```

- Customize and move around the “do something,” and that’s the basis for dozens of algorithms and applications

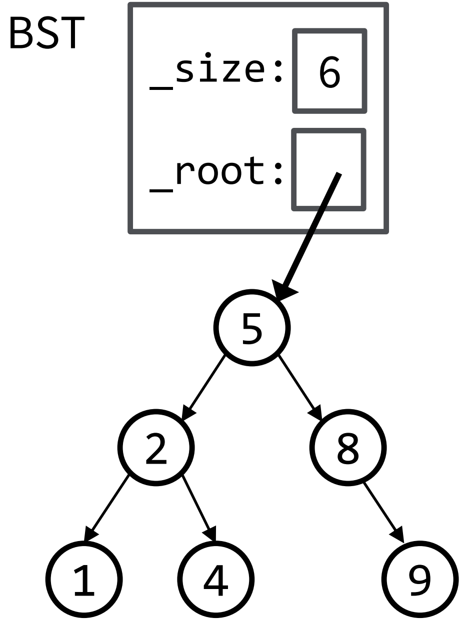
# Stanford Library Map

- Remember how when you iterate over the Stanford library Map you get the keys in sorted order?
  - (we used this for the word occurrence counting code example in class)
- Now you know why it can do that in O(N) time!
  - **Stanford library Map is a BST**
  - **In-order traversal on BST!**



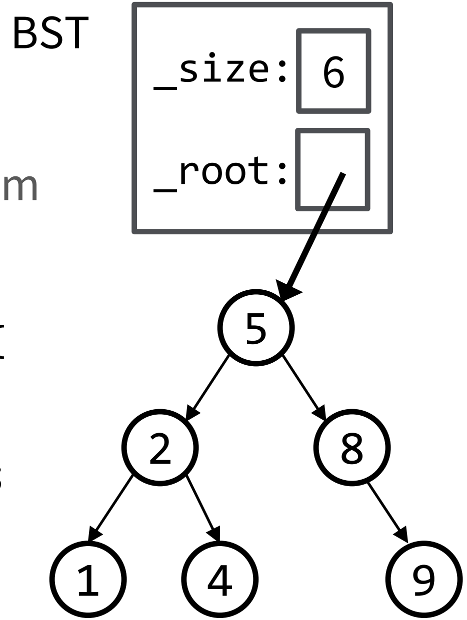
# Your Turn: Applications of the traversals

- You are writing the **destructor** for a BST class. Given a pointer to the root, it needs to free each node. Which traversal would form the foundation of your destructor algorithm?
  A. Pre-order
  B. In-order
  C. Post-order
  D. Breadth-first



# Applications of the traversals

- You are writing the **destructor** for a BST class. Given a pointer to the root, it needs to free each node. Which traversal would form the foundation of your destructor algorithm?
  - Post-order is a good choice, because we need to use the node's fields to recurse
  - Don't want to delete fields before we use them



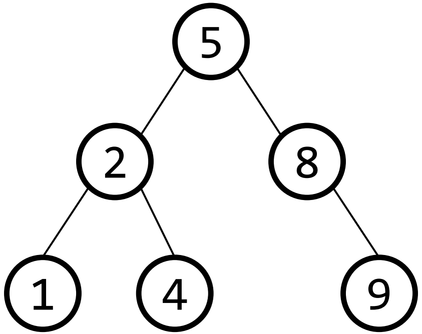

```
void bstDestructorRecursiveHelper(Node *node) {
  if (node != nullptr) {
     bstDestructorRecursiveHelper(node->left);
     bstDestructorRecursiveHelper(node->right);
     delete node; // post-order
  }
}
```

# Breadth-First Tree Traversal

A somewhat different kind of traversal

# How can we get code to print top-to-bottom, left-to-right order?

```
void traverse(Node* node) {
  if (node != nullptr) {
 ??  cout << node->key << " ";
     traverse(node->left);
     traverse(node->right);
  }
}
```




You can't do it by using this code and moving around the cout—we already tried moving the cout to all 3 possible places and it didn't print in order

- You can but you use a **queue** instead of recursion
- **"Breadth-first"** search
- *Again we see this key theme of BFS (queue) vs DFS (stack/recursion)!*