Thinking Recursively

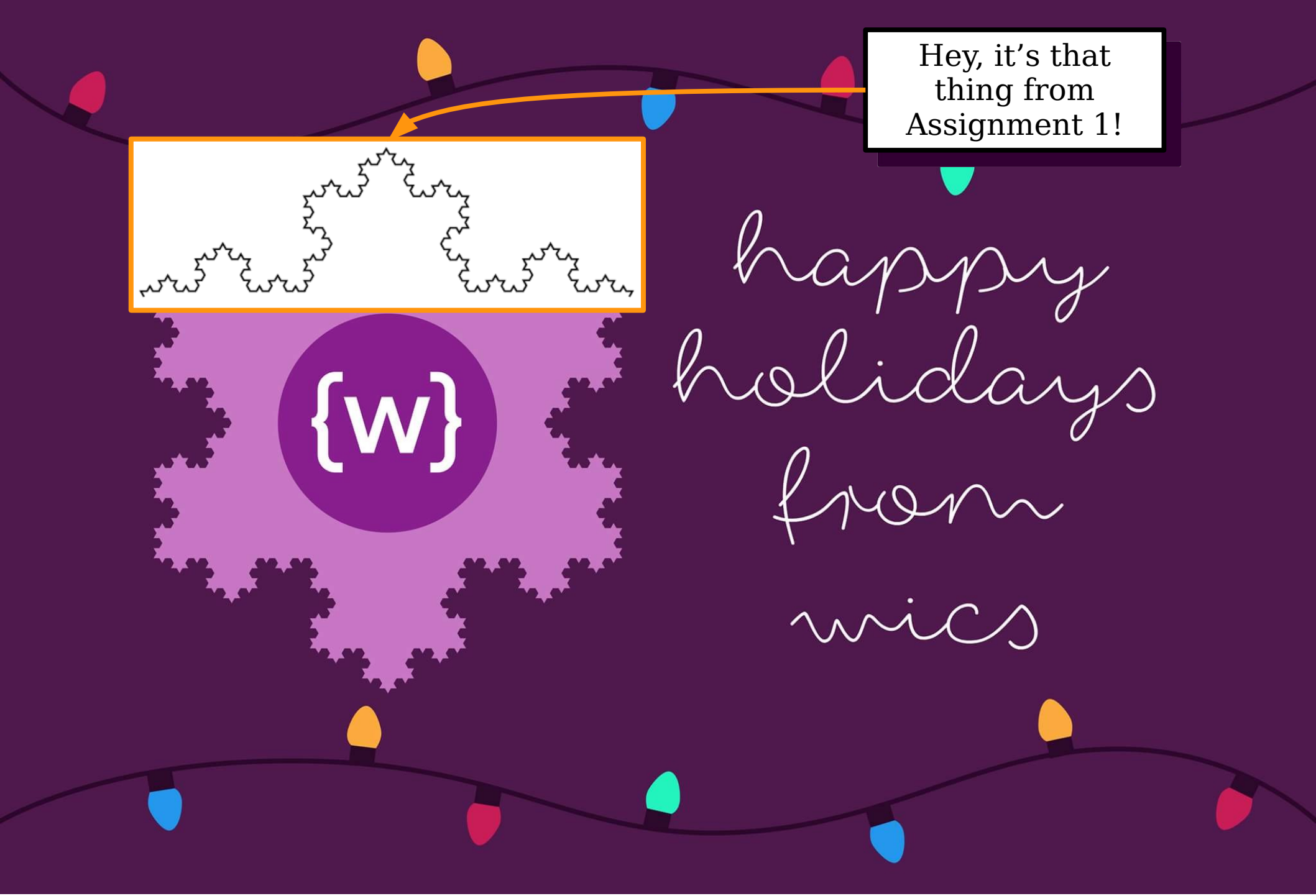
Part I

Outline for Today

- ***Self-Similarity***
  - Recursive patterns are everywhere!
- ***Recursive Trees***
  - Elegant structures from simple code.
- ***Information Flow***
  - How to send information around in recursion.

Self-Similarity

An object is ***self-similar*** if it contains a smaller copy of itself.



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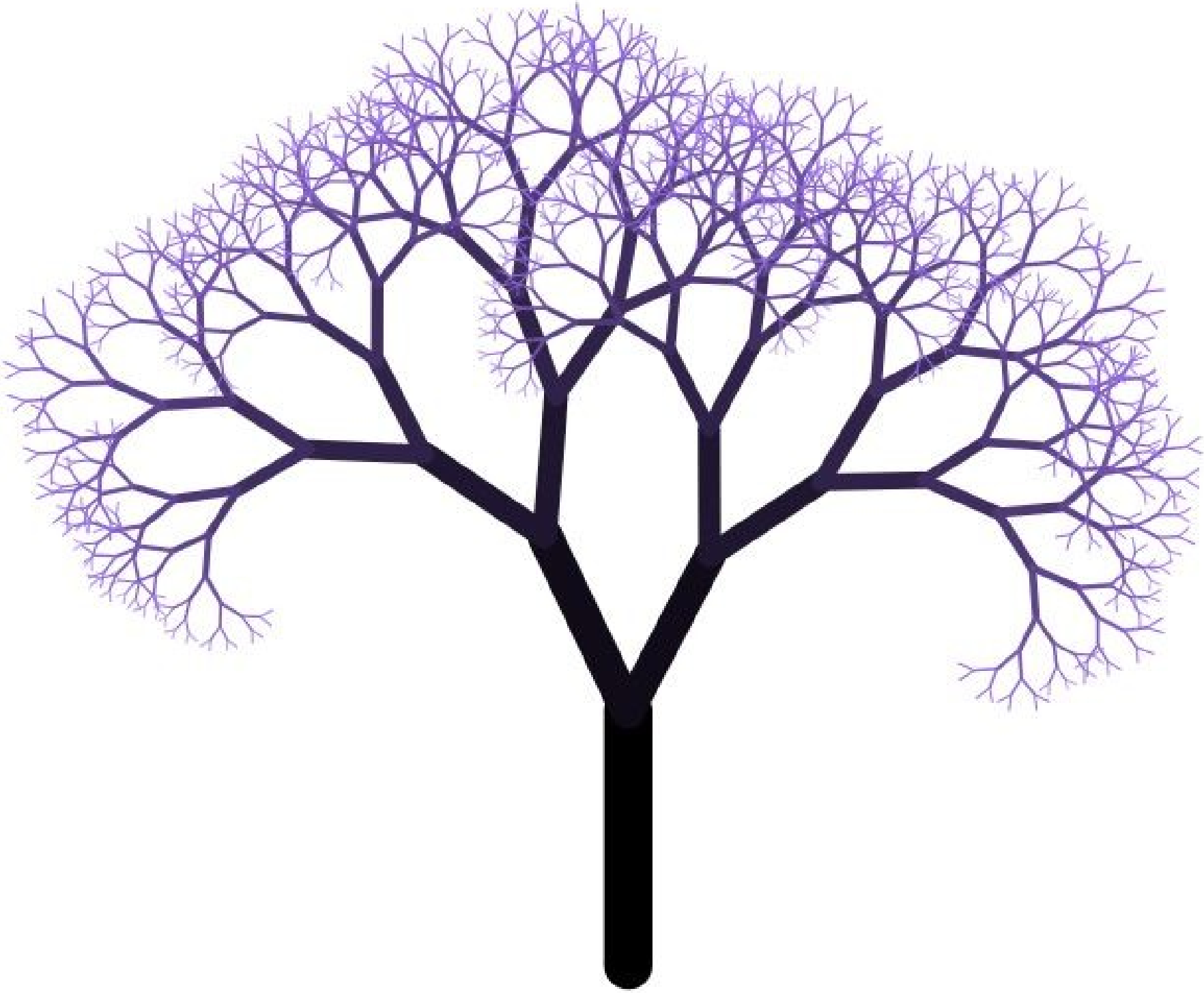
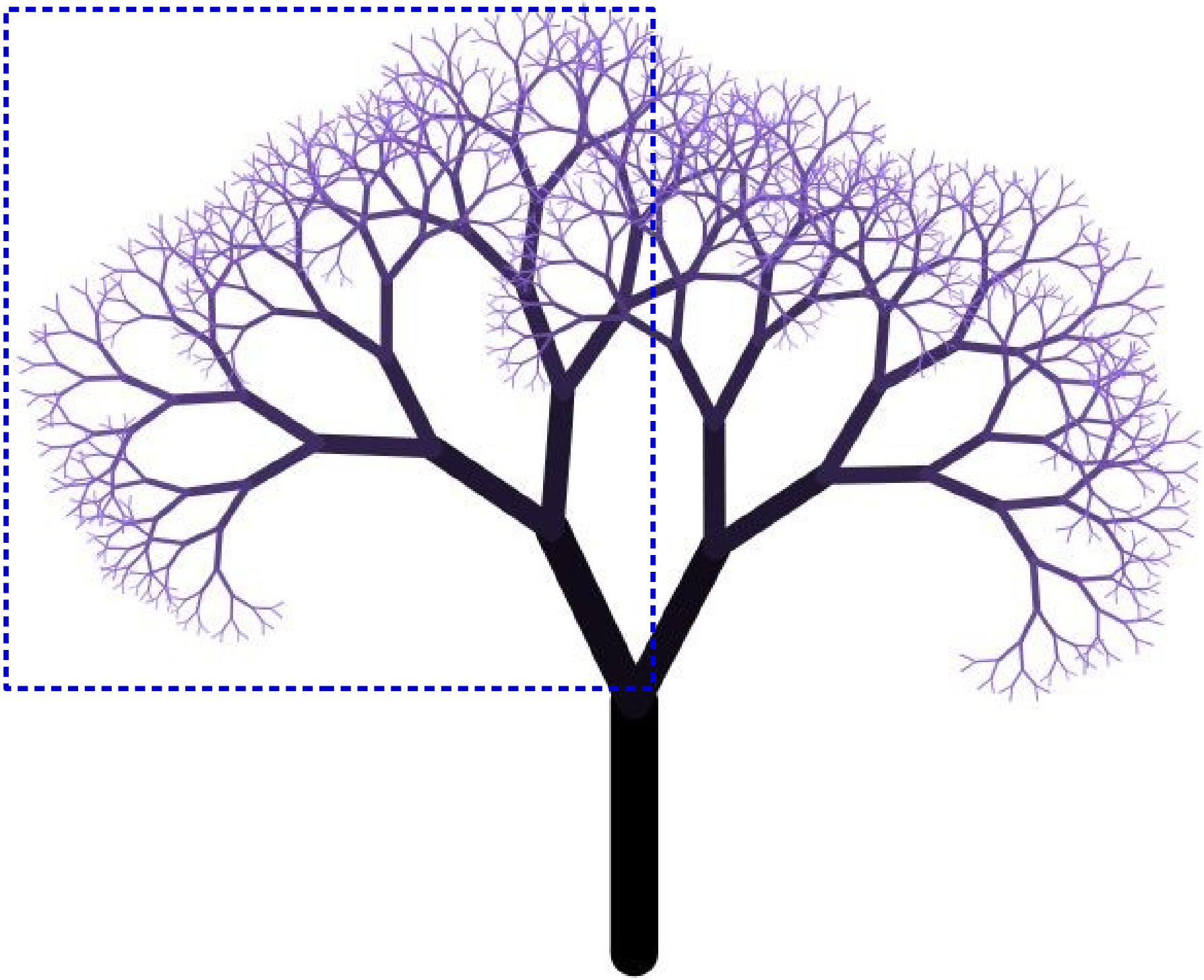
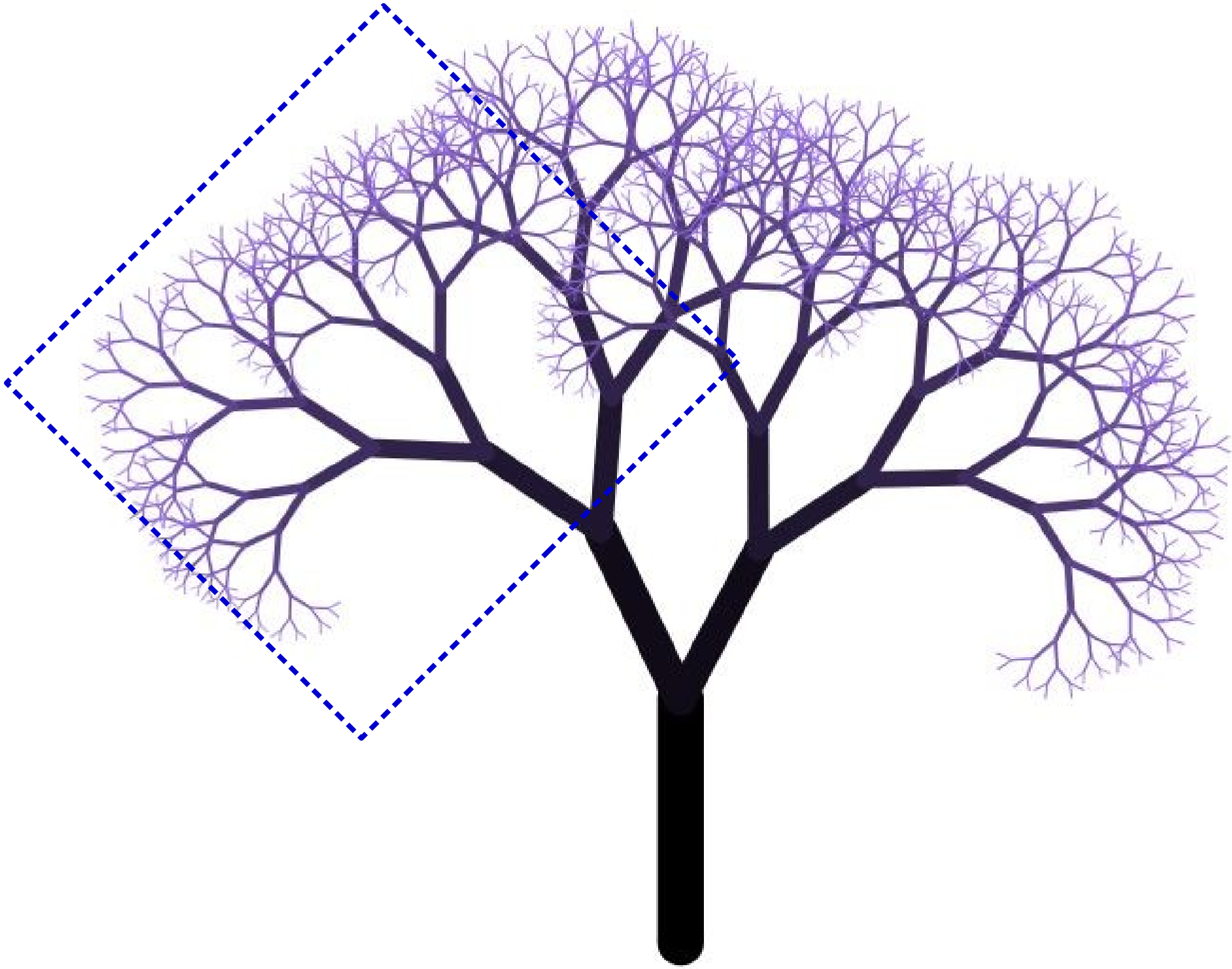
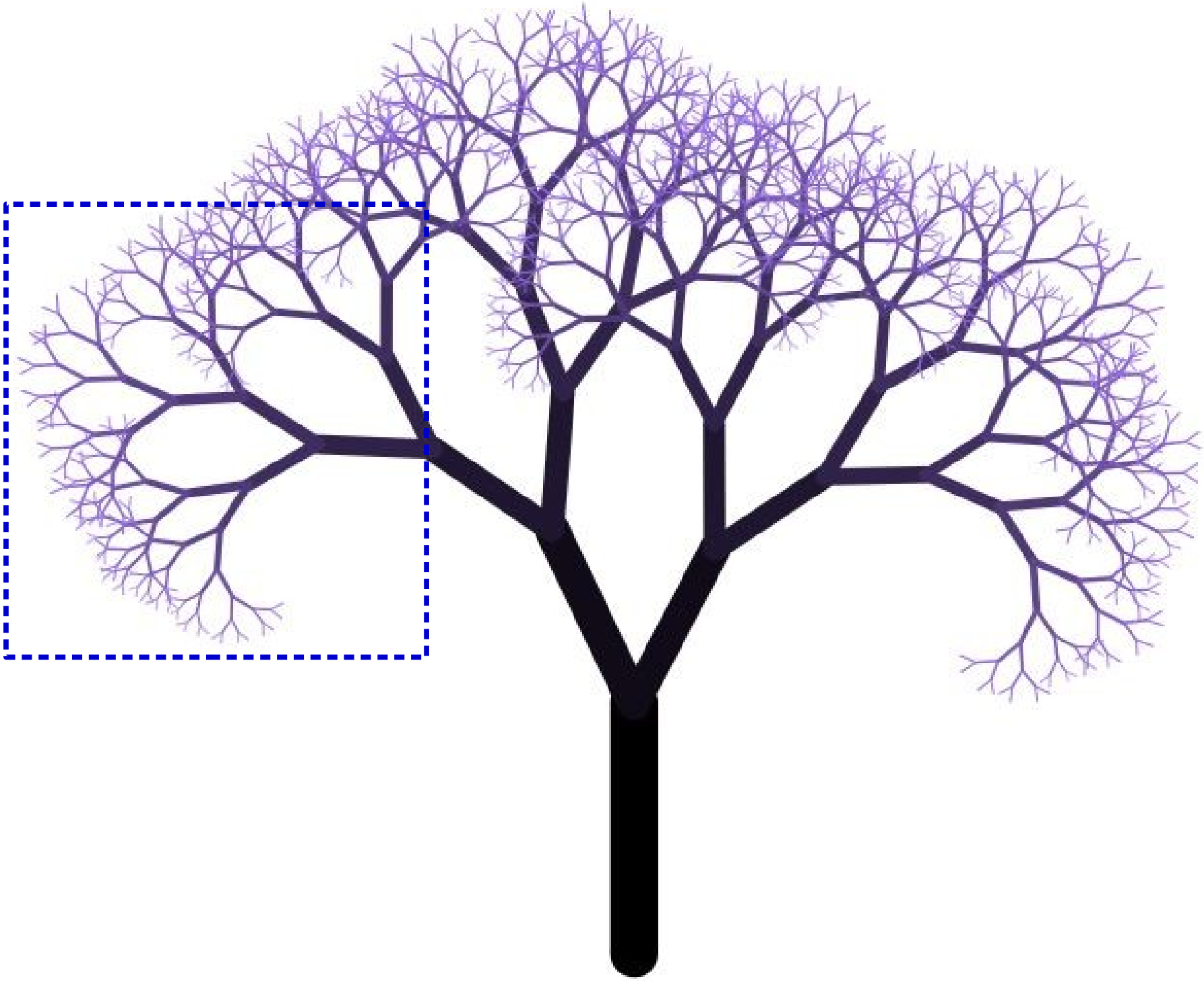
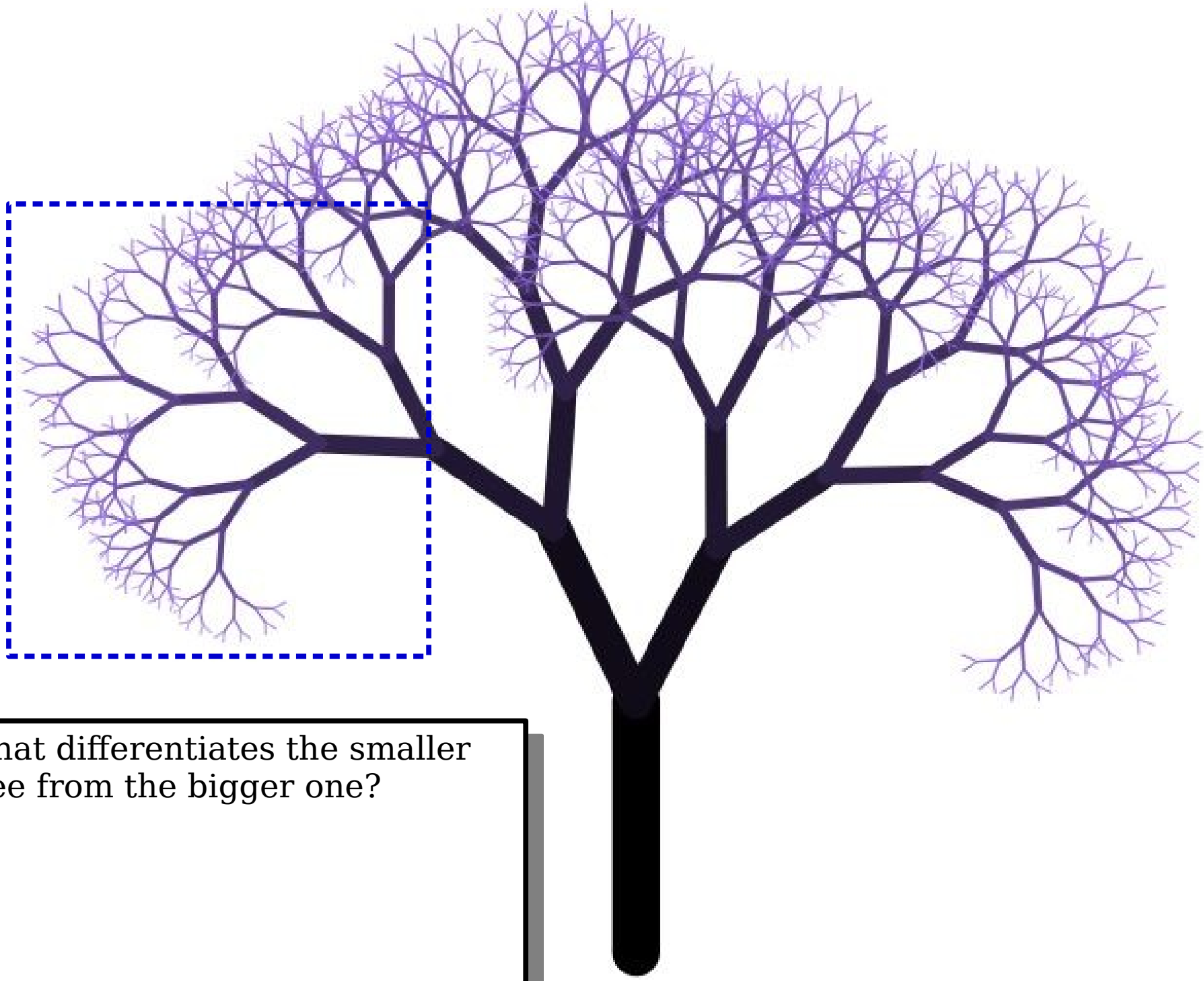
Drawing Self-Similar Shapes



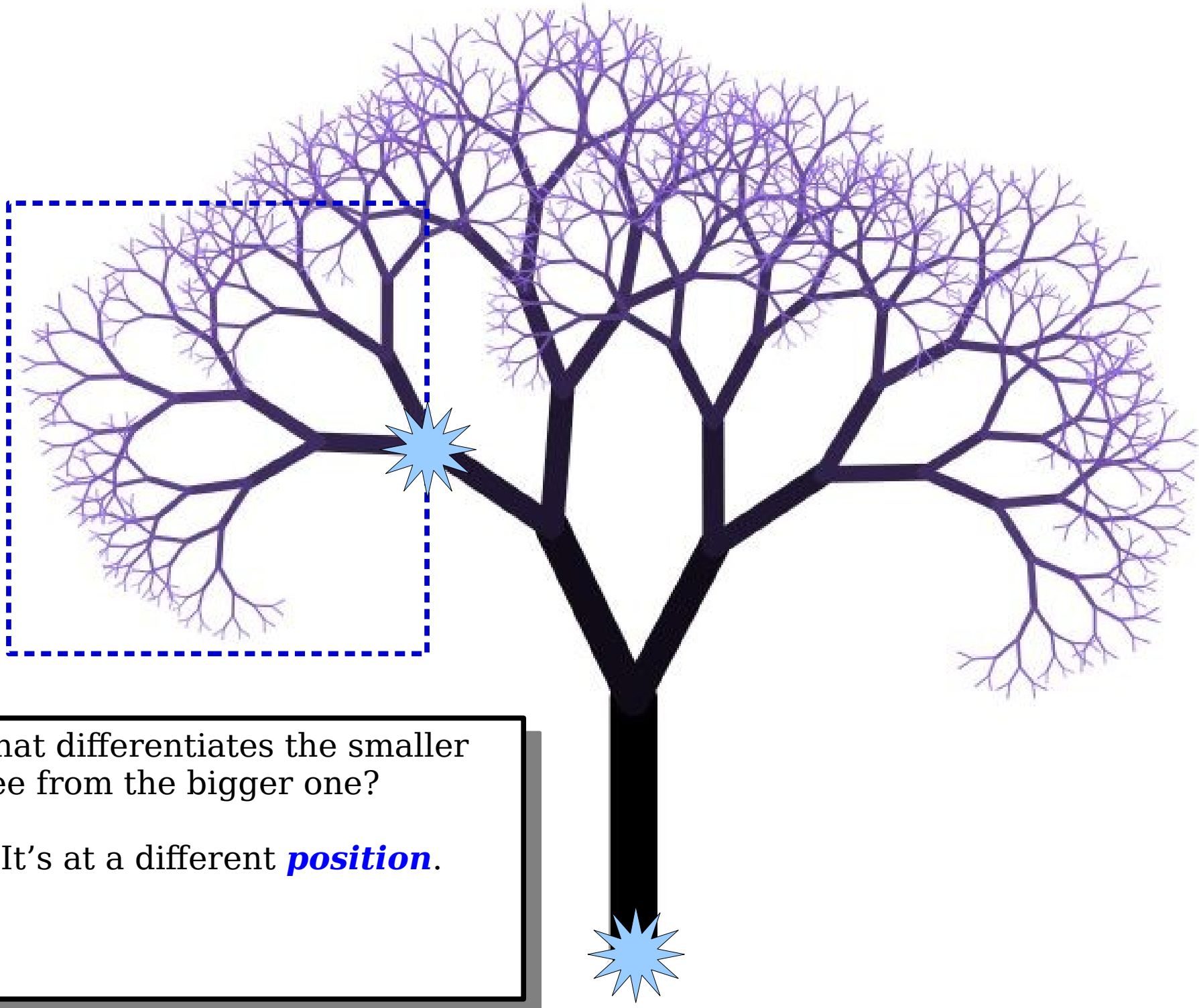








What differentiates the smaller tree from the bigger one?



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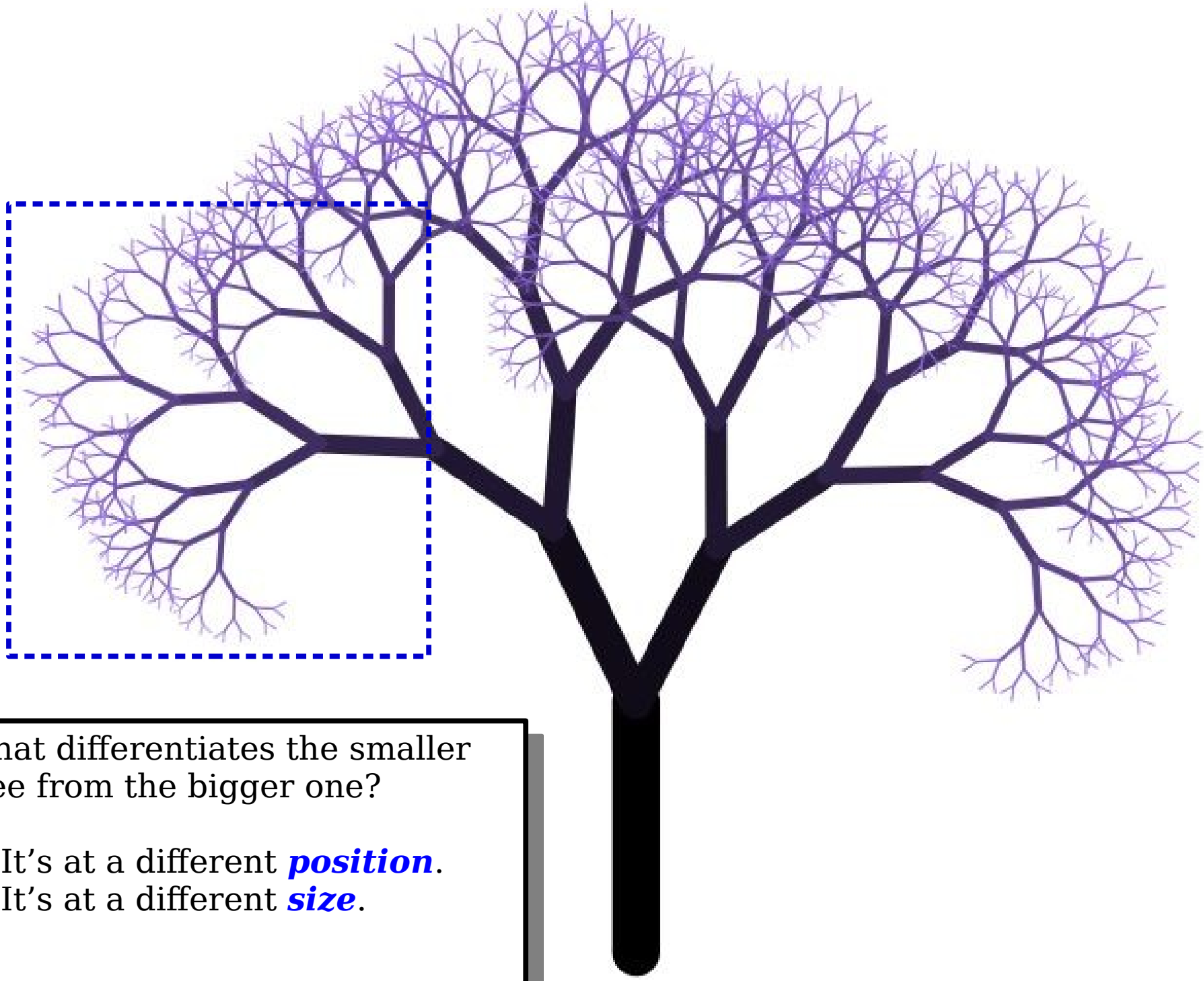

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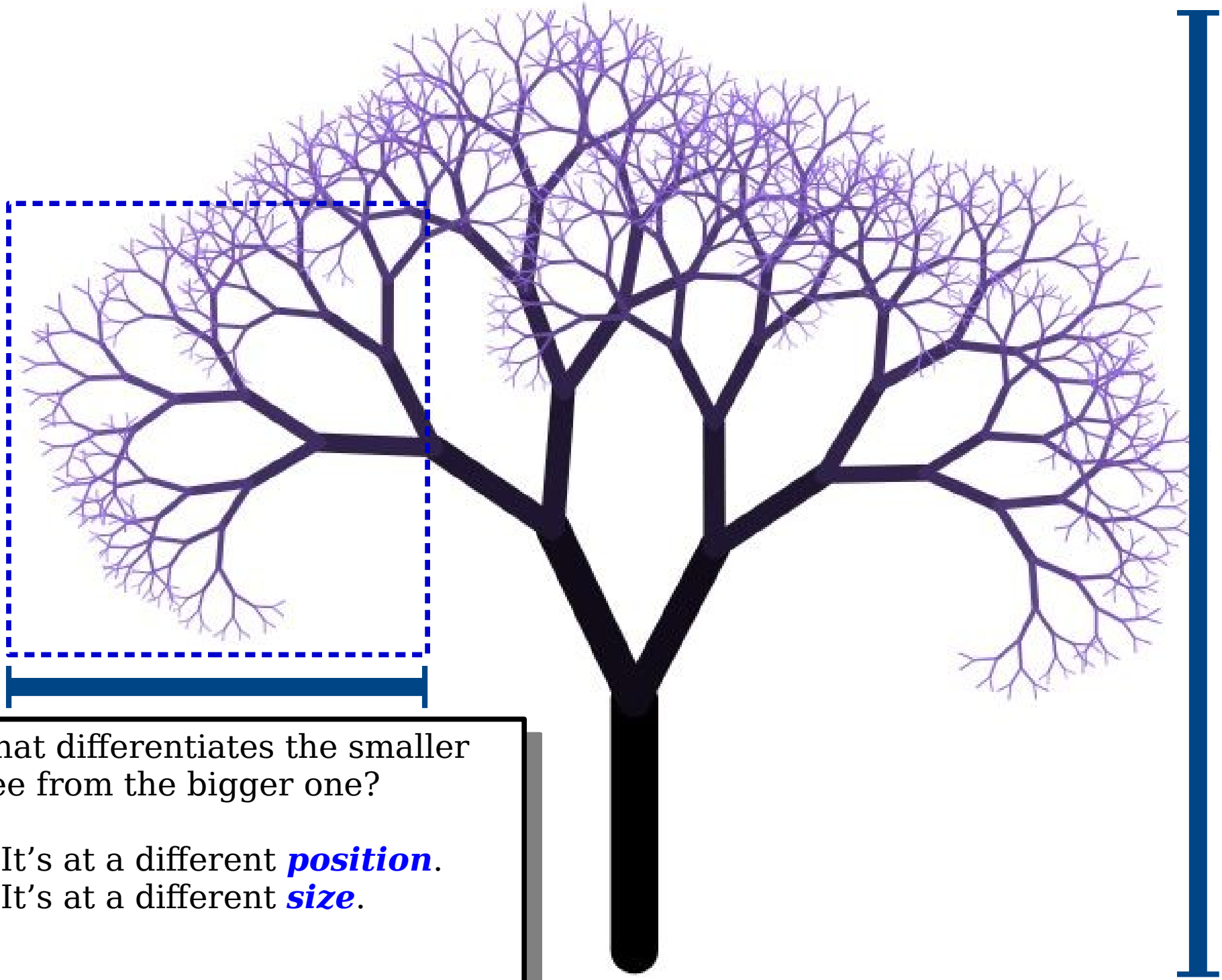

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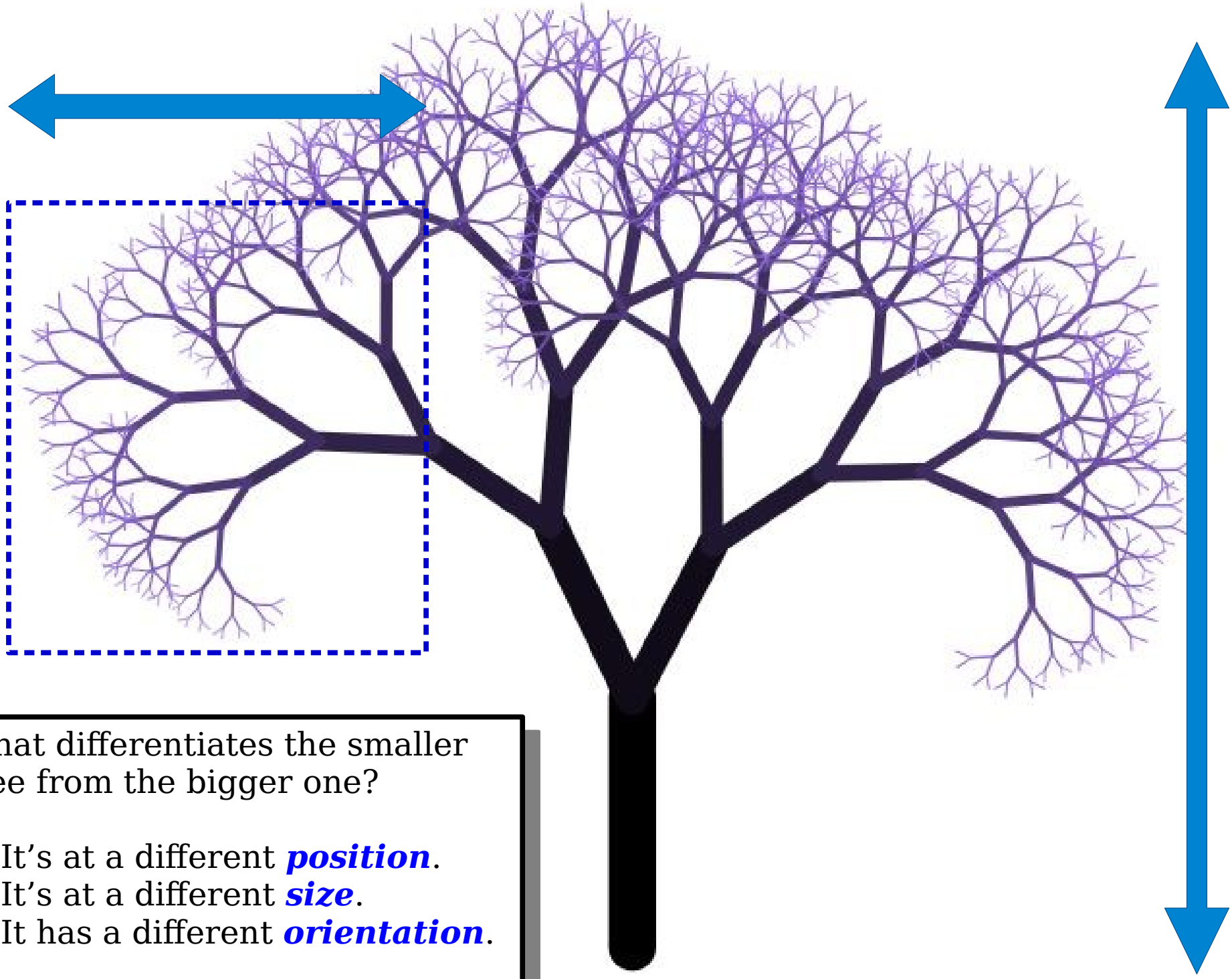

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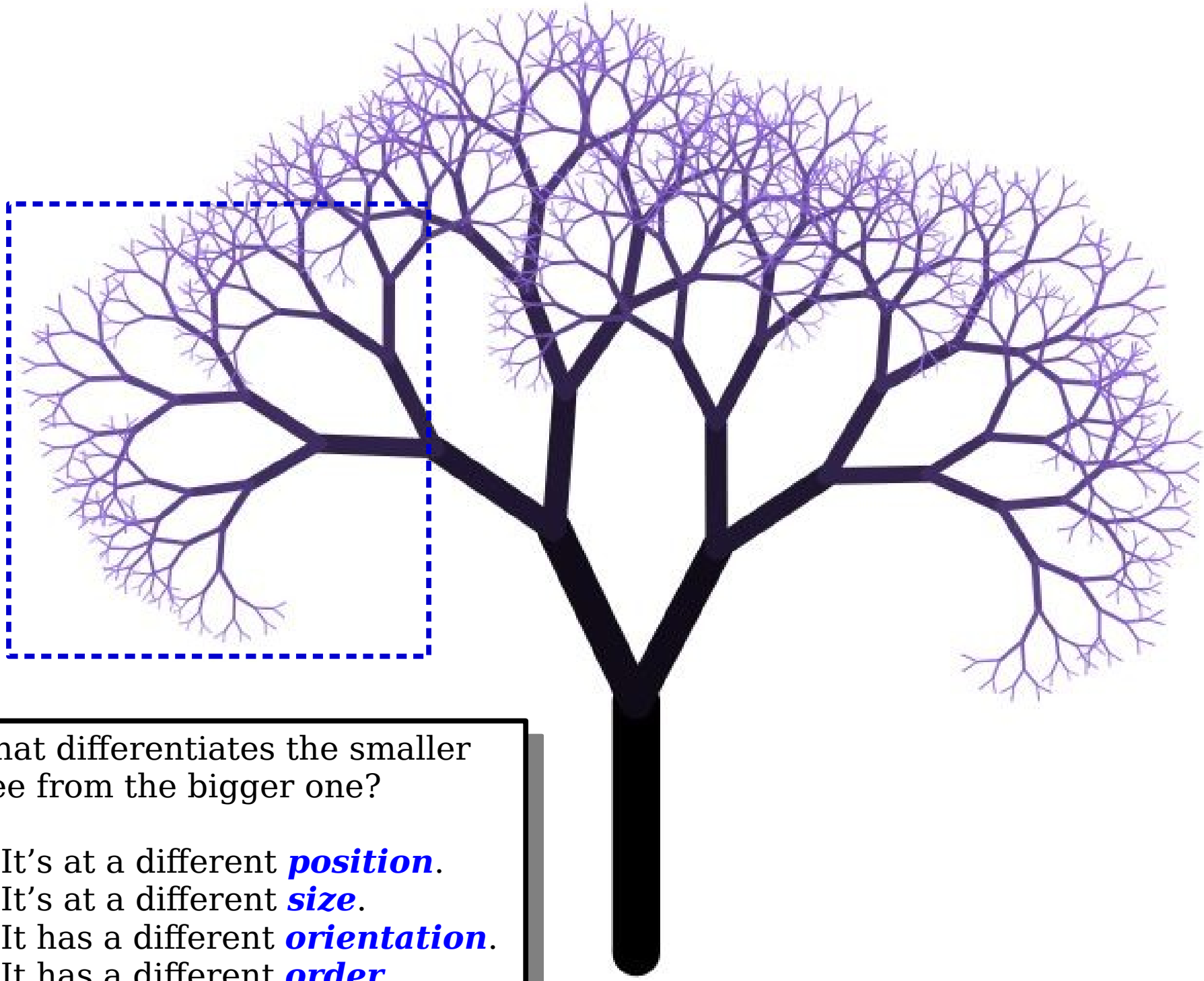

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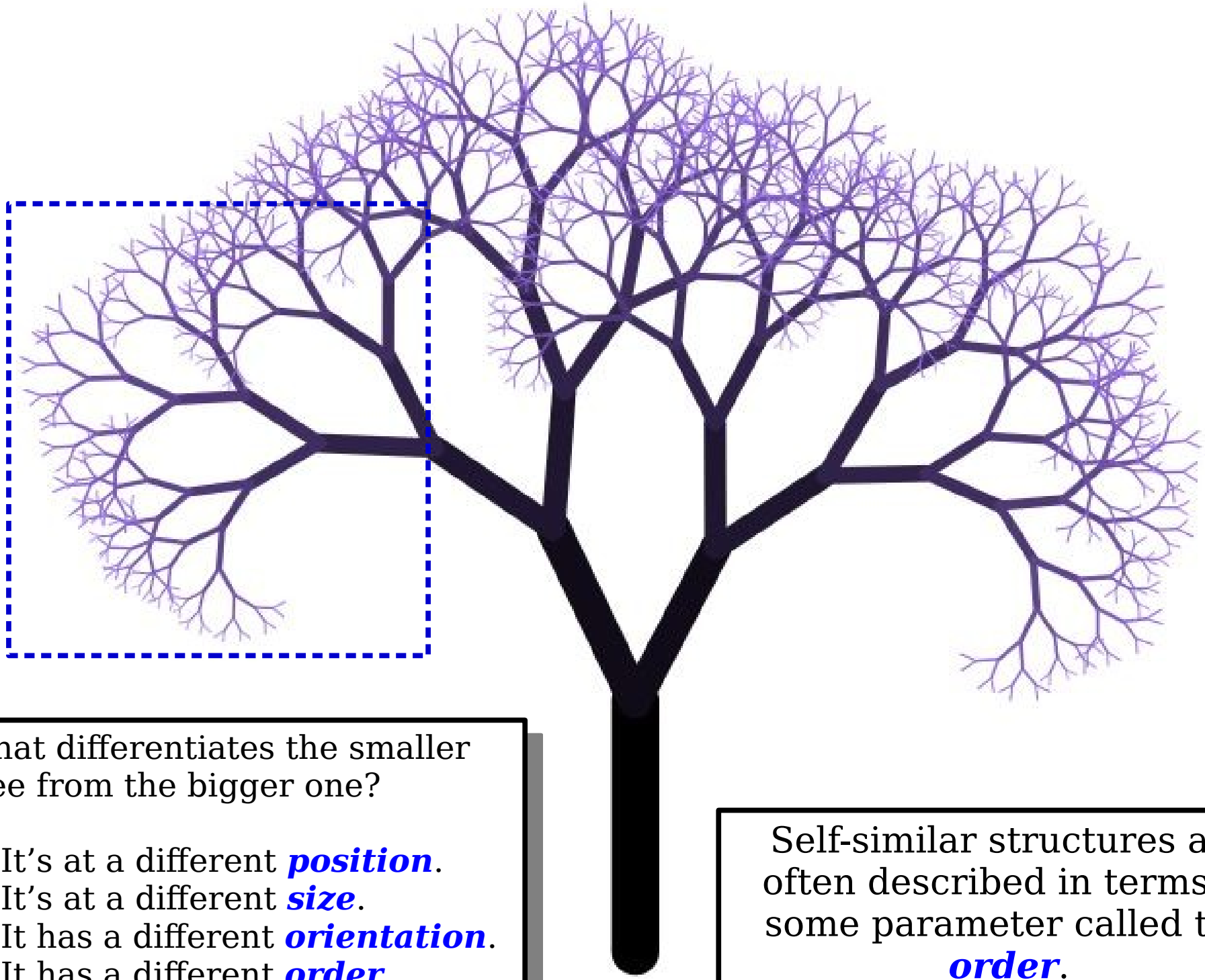

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Self-similar structures are often described in terms of some parameter called the ***order***.

An order-0 tree.

What differentiates the smaller tree from the bigger one?

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An order-1 tree.

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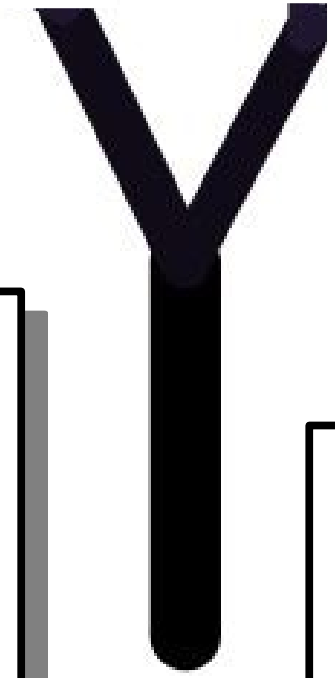
An order-2 tree.



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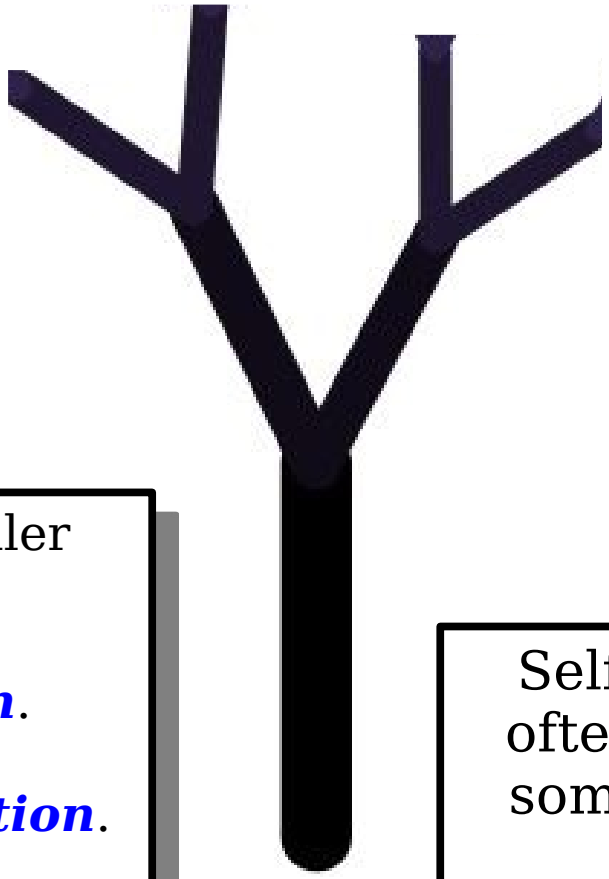

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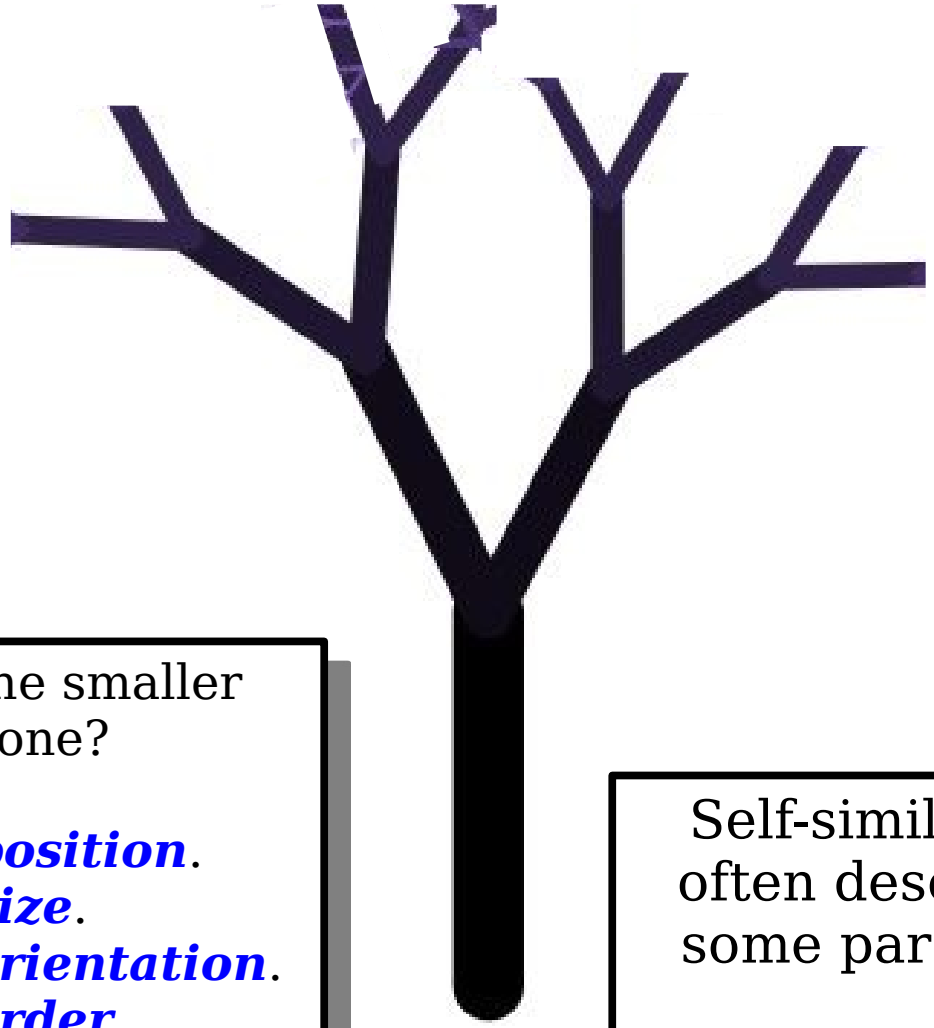
An order-4 tree.



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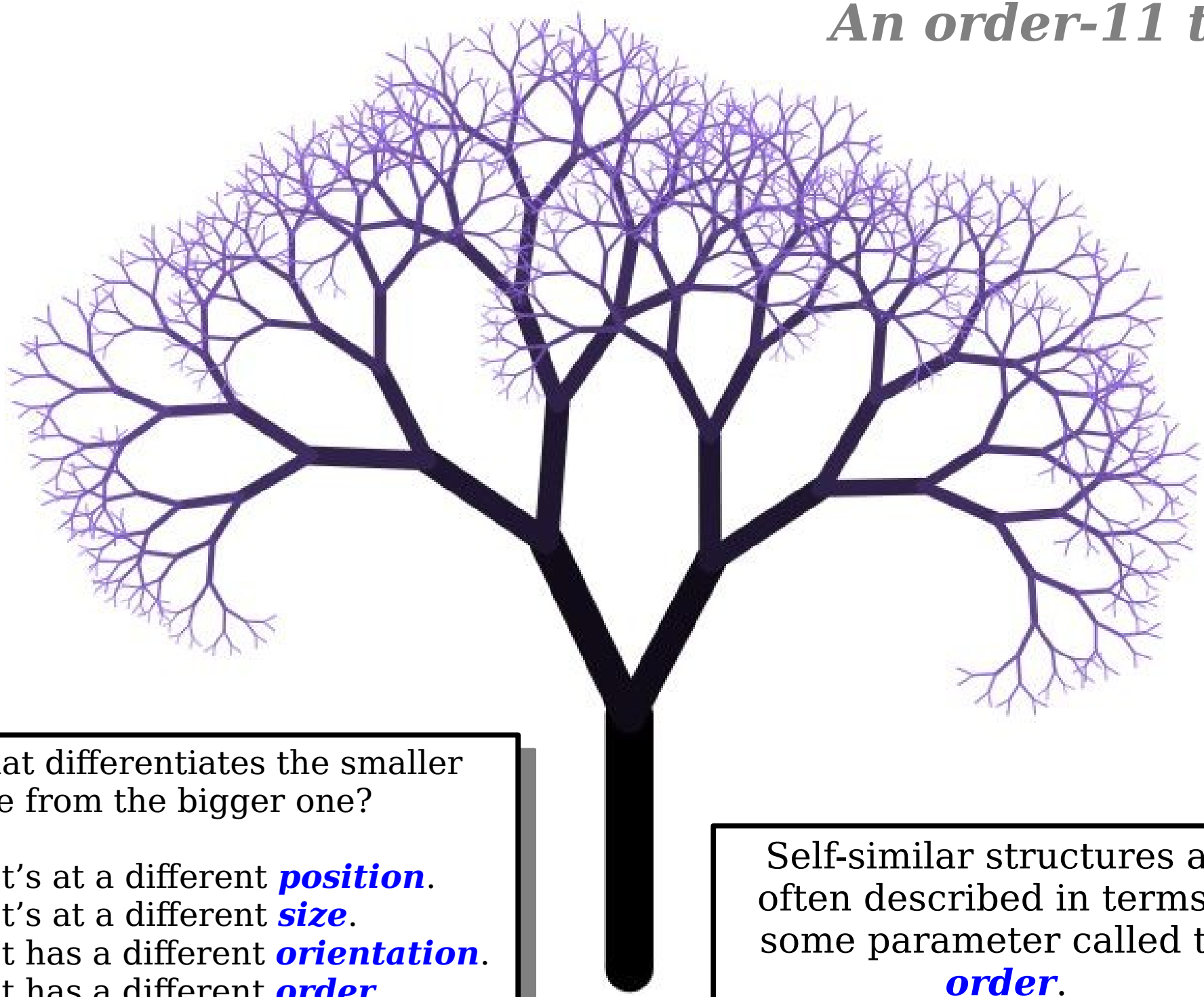
An order-11 tree.



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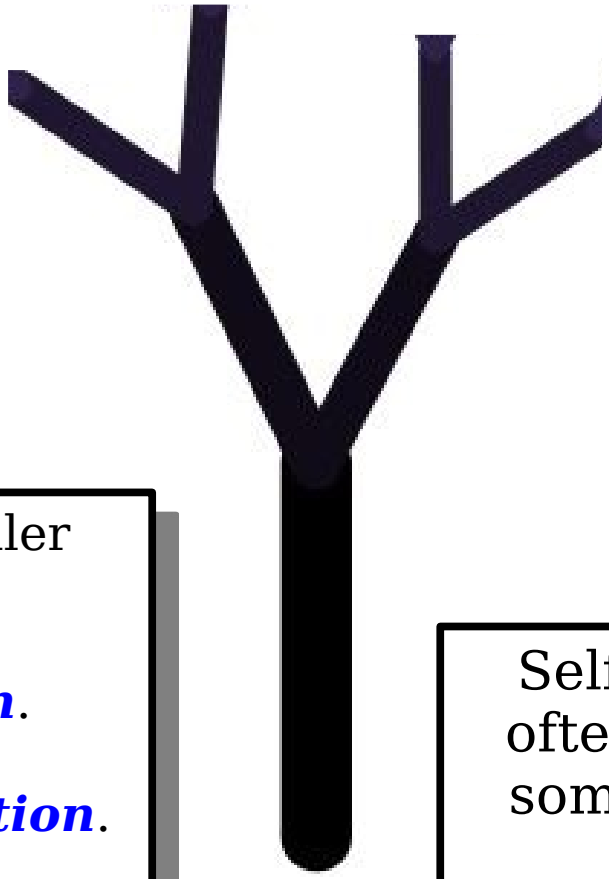

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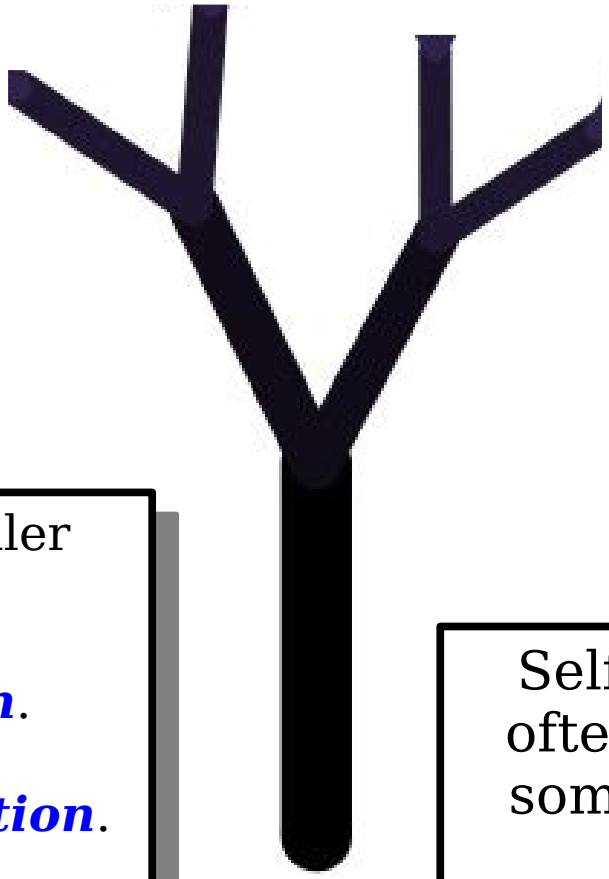
Self-similar structures are often described in terms of some parameter called the ***order***.

An order-3 tree.

An order-0 tree is nothing at all.

An order-n tree is a line with two smaller order-(n-1) trees starting at the end of that line.



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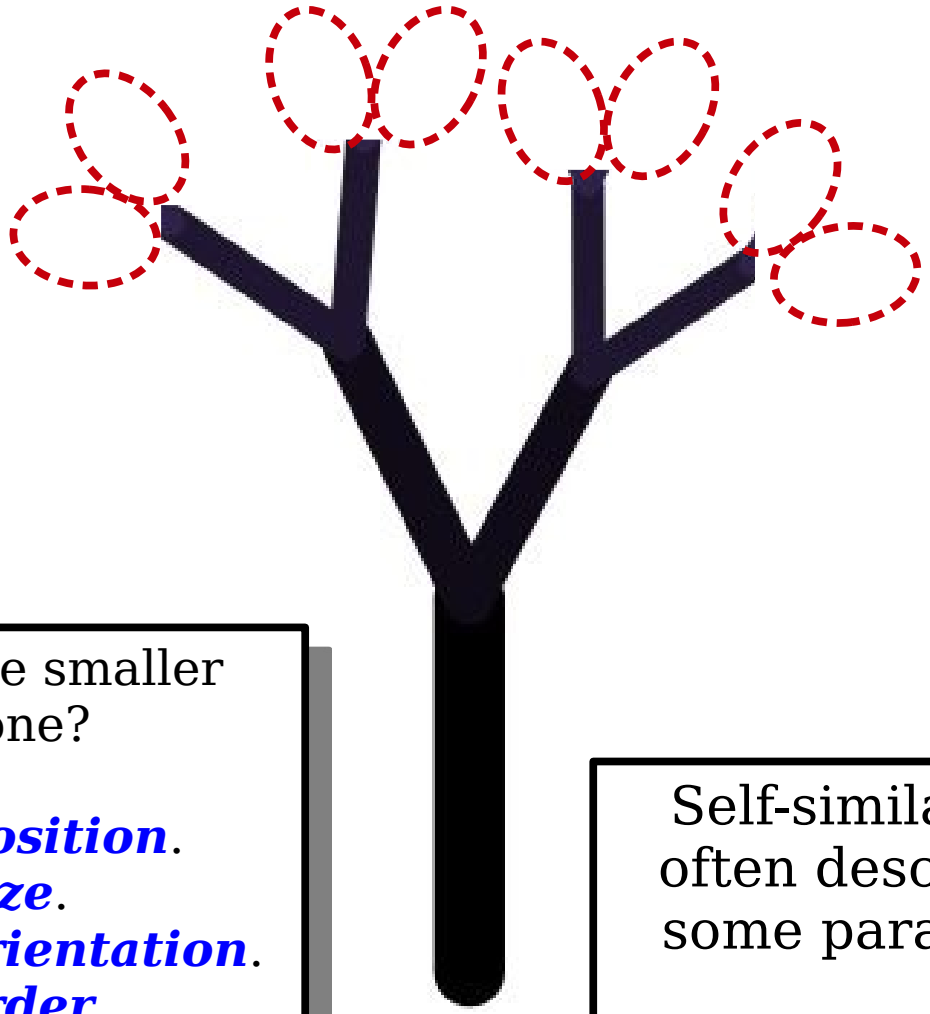

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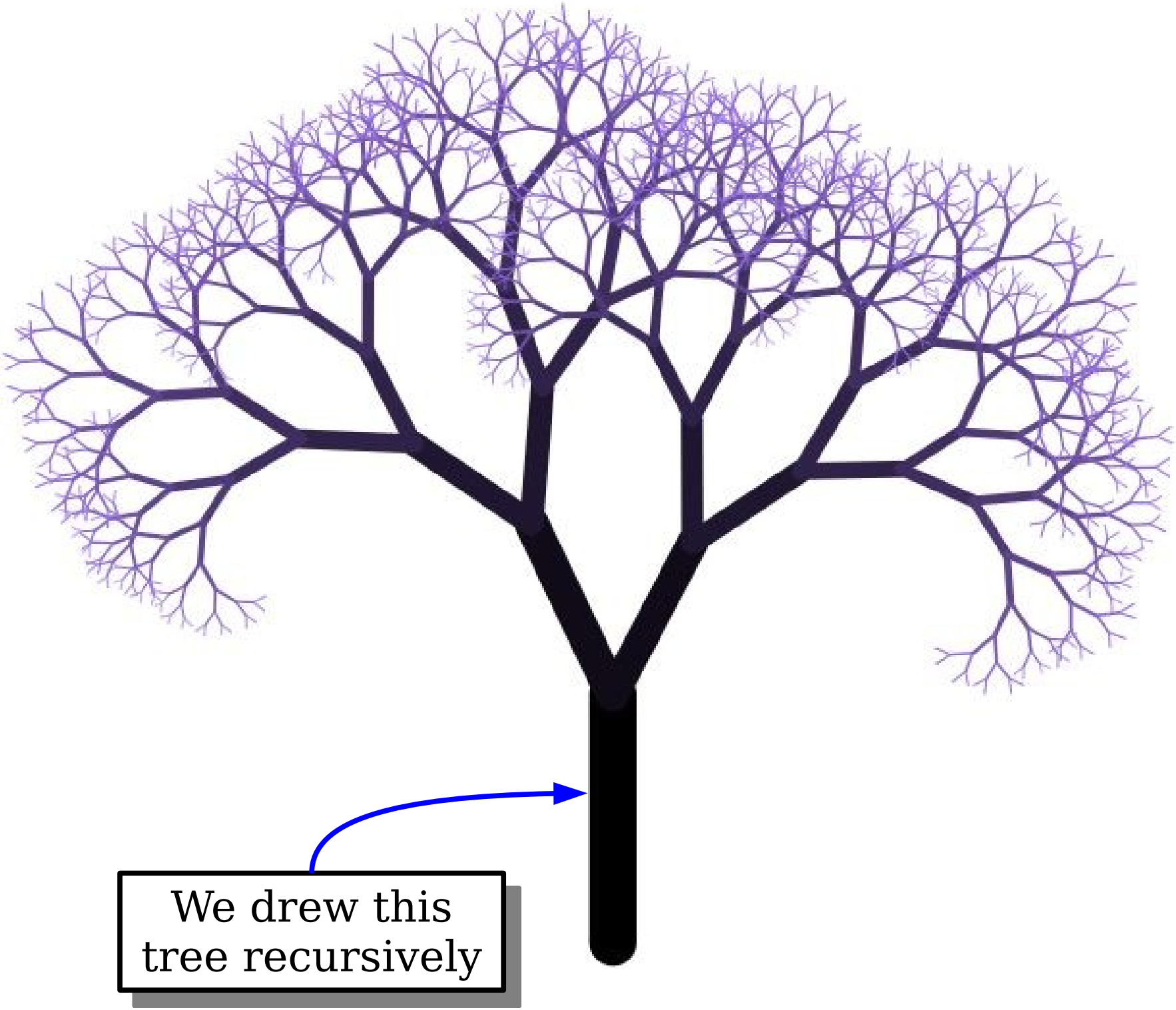
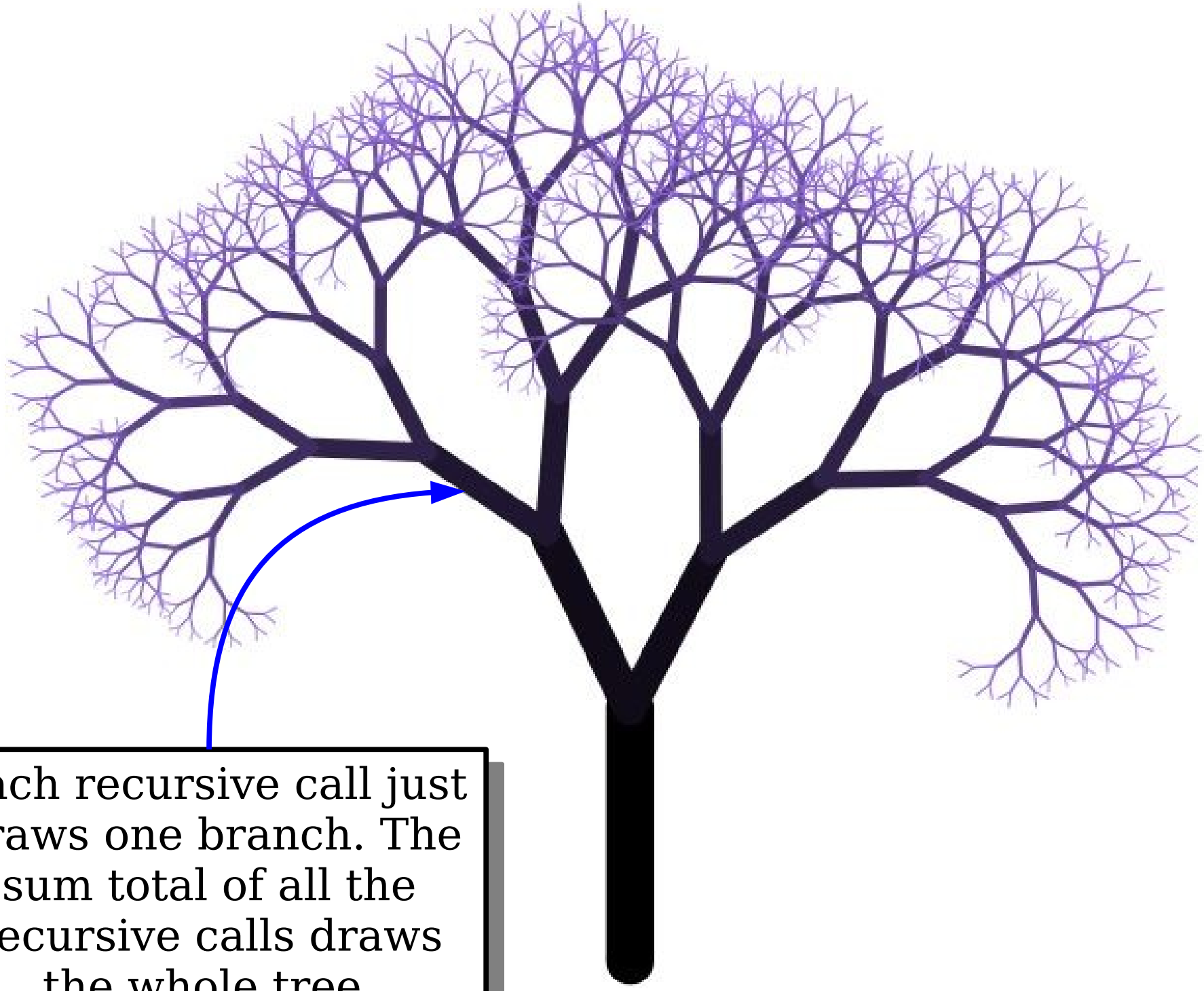
To Summarize


We drew this
tree recursively



Each recursive call just draws one branch. The sum total of all the recursive calls draws the whole tree.

An Amazing Website

http://recursivedrawing.com/

Time-Out for Announcements!

Assignment 2

- Assignment 2 is due on Friday.
  - If you're following our suggested timetable, you should be finished with Rosetta Stone at this point and should be working on Rising Tides.
- Have questions?
  - Call into the LaIR!
  - Email your section leader!
  - Ask on EdStem!
  - Visit Keith's or Neel's office hours!

Submitting Your Work

- Each assignment has a “Submission Instructions” section at the end with information about what files to submit.
- ***Please submit all the files listed there***. Otherwise, we can’t grade all the work you’ve done.
- Thanks!

Looking Forward to Partners

- Starting with Assignment 3, which goes out Friday, you'll be allowed to work on the assignment in pairs.
- To work in a pair, you must be in the same section as your partner.
- Need to switch sections? Fill out ***this form*** to request a section swap.
  - The link is also on EdStem.
- Please fill this out by 5:00PM this Friday, Pacific time.

Onward and Forward!

A Quick, Relevant Tangent

Reasoning By Analogy

- What's wrong with this code?

```
double areaOfCircle(double radius) {
    return M_PI * radius * radius;
}

int main() {
    double radius = 1.61;
    areaOfCircle(radius);
    return 0;
}
```

Formulate a hypothesis, but ***don't post anything in chat just yet***.

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Now, ***post your best guess in chat***. Not sure? Just answer "??"

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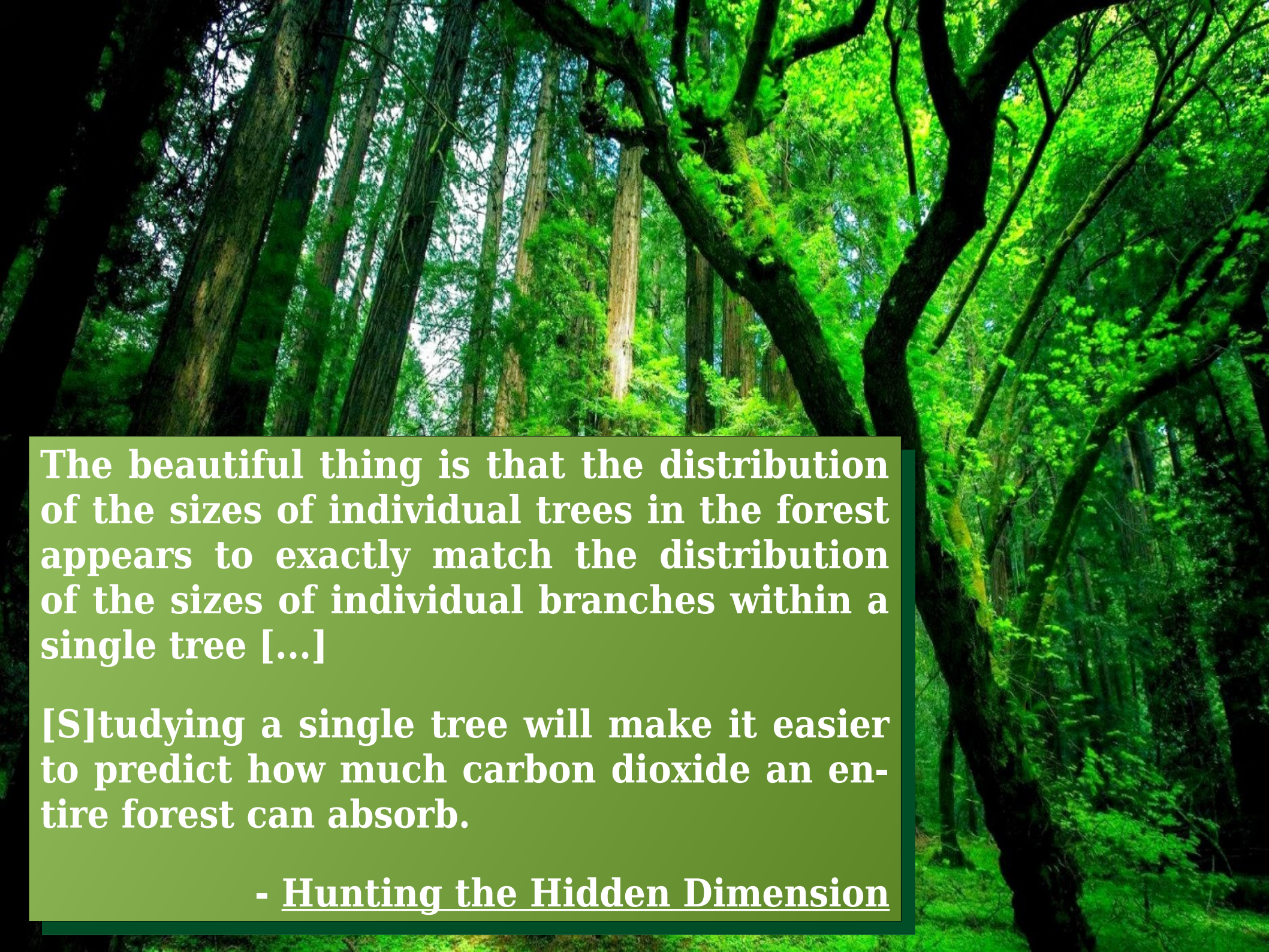
}
```

It's odd to call a function that returns a value and then not use that value.

If you don't use a function's return value, it's likely a bug!

Back to Recursion...

A Practical Application

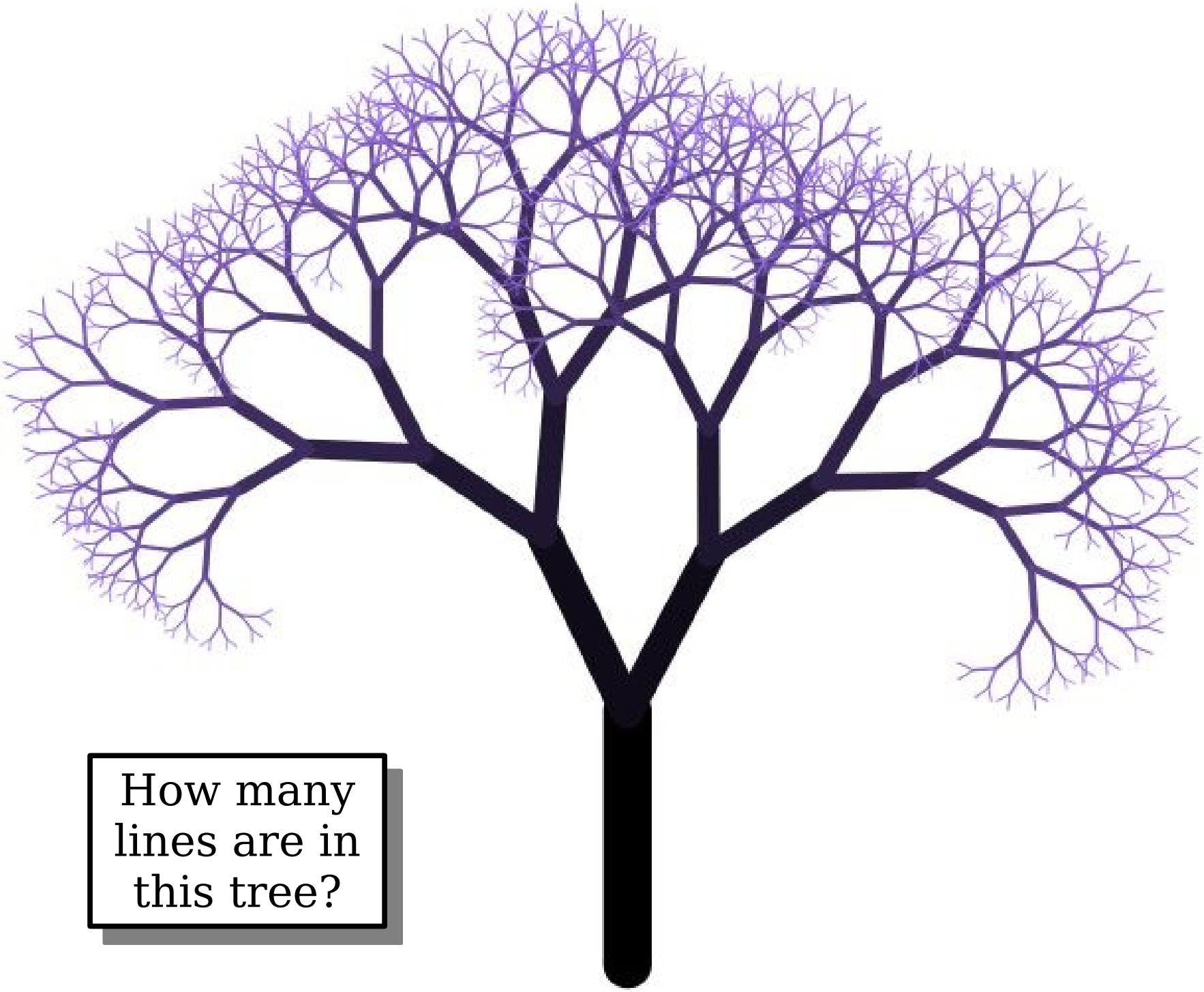


The beautiful thing is that the distribution of the sizes of individual trees in the forest appears to exactly match the distribution of the sizes of individual branches within a single tree [...]

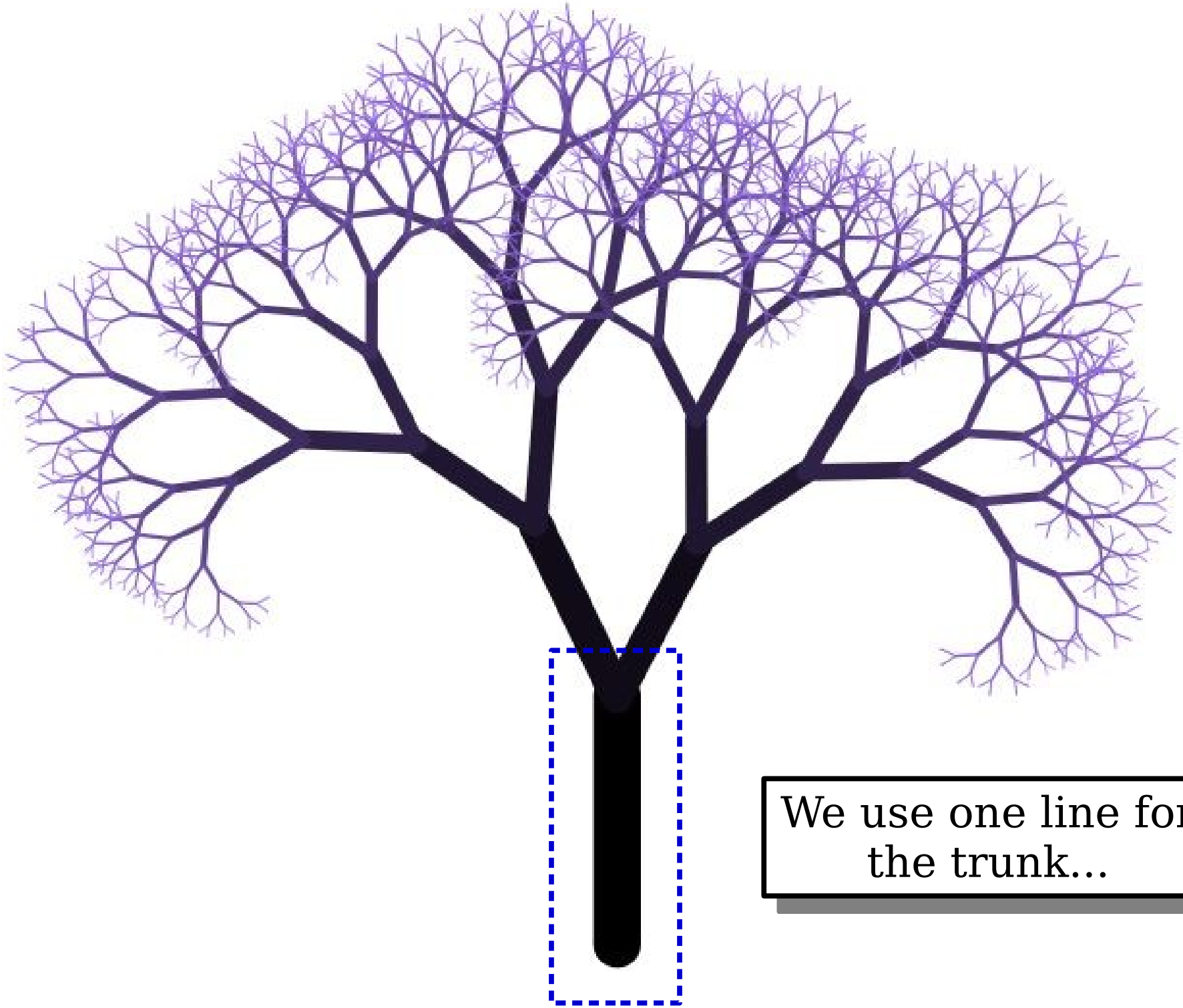
[S]tudying a single tree will make it easier to predict how much carbon dioxide an entire forest can absorb.

- Hunting the Hidden Dimension

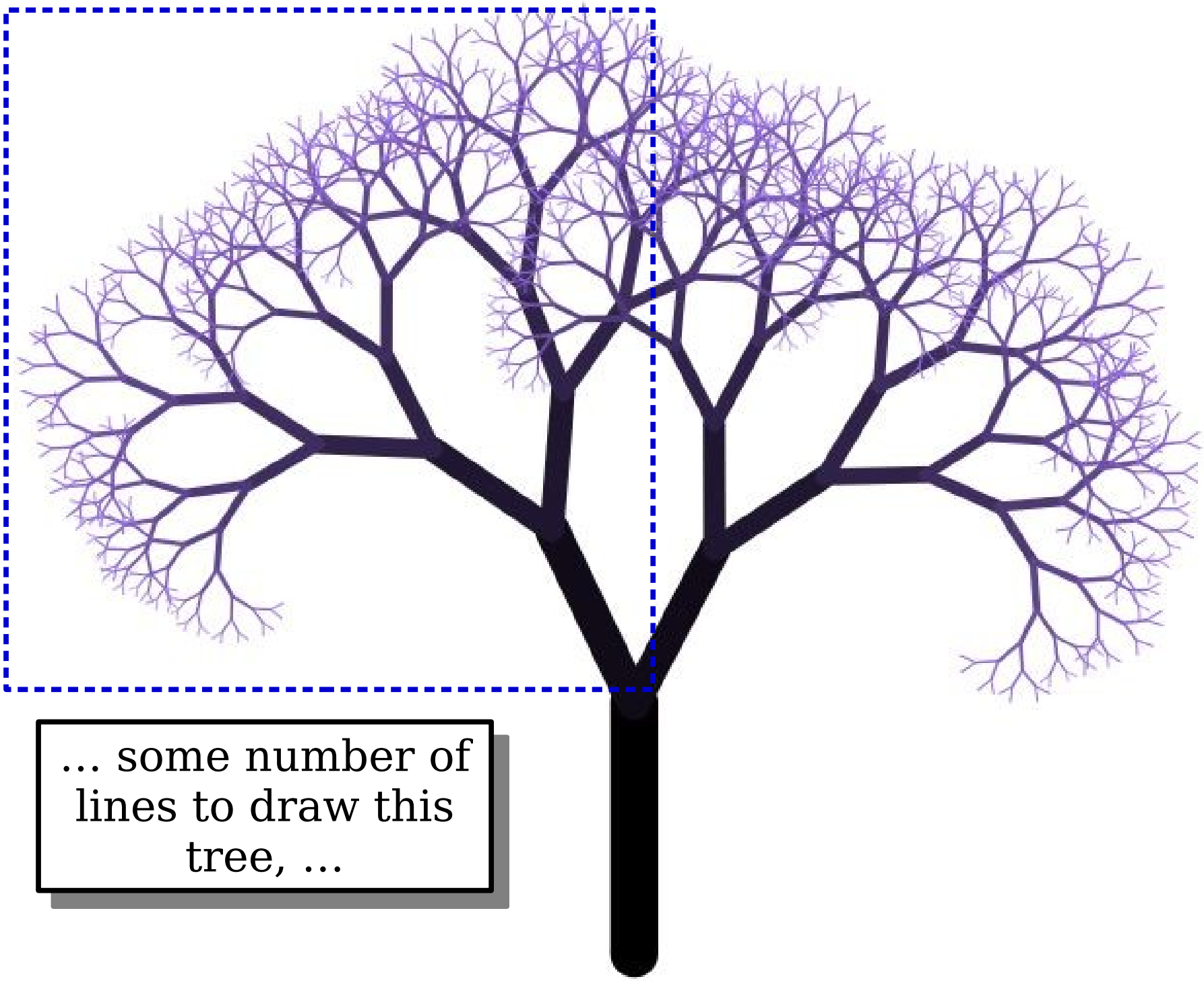
How many lines make up each tree?



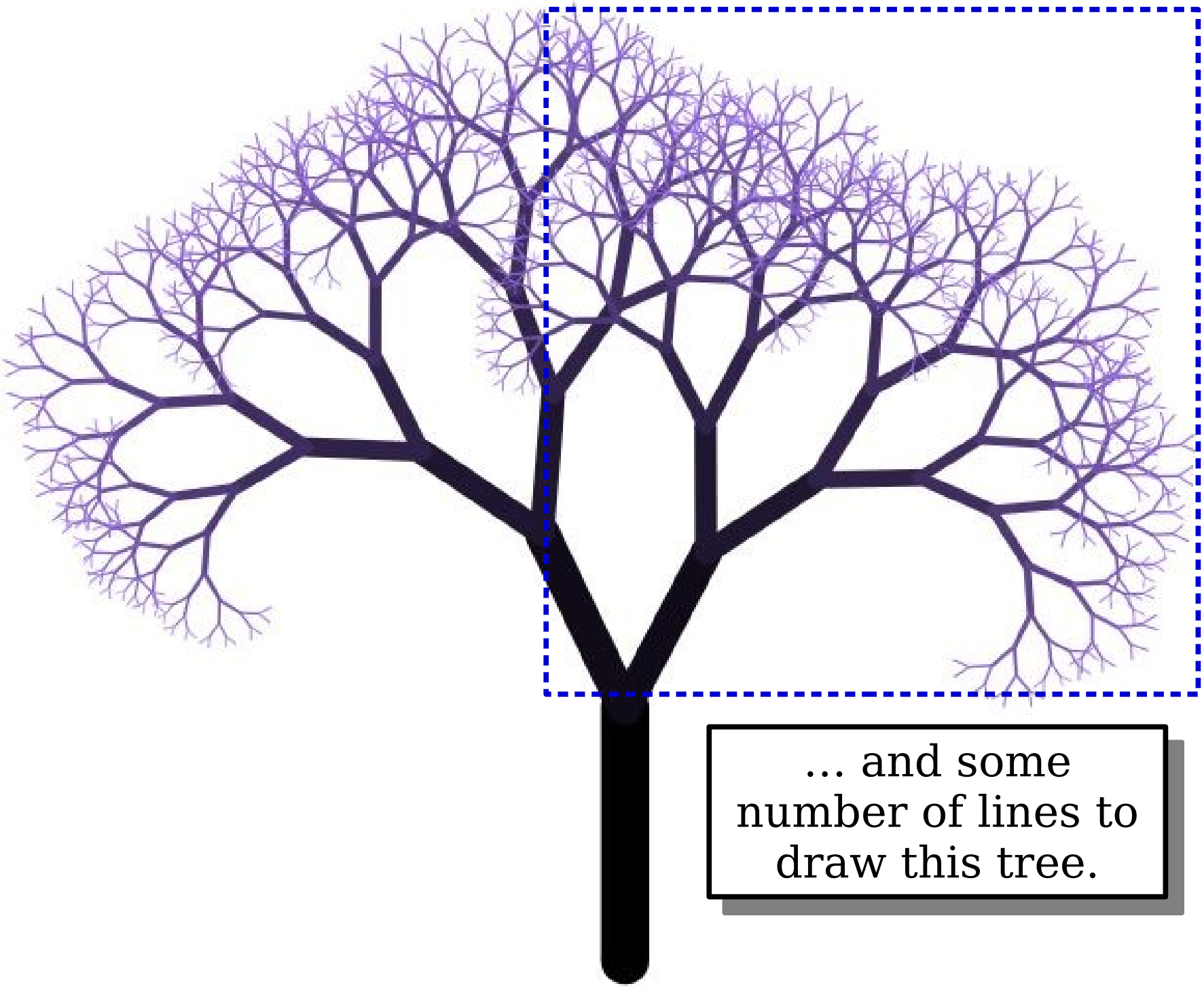
How many lines are in this tree?



We use one line for the trunk...



... some number of lines to draw this tree, ...



... and some number of lines to draw this tree.

Why didn’t this code print the correct number of lines drawn?

Formulate a hypothesis, but ***don’t post anything in chat just yet.***

Why didn’t this code print the correct number of lines drawn?

Now, ***post your best hypothesis in chat***. Not sure? Just answer “??”

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    if (order == 0) {
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}
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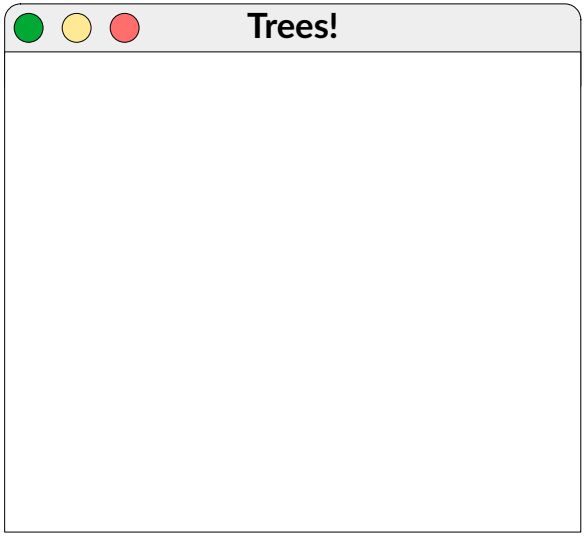

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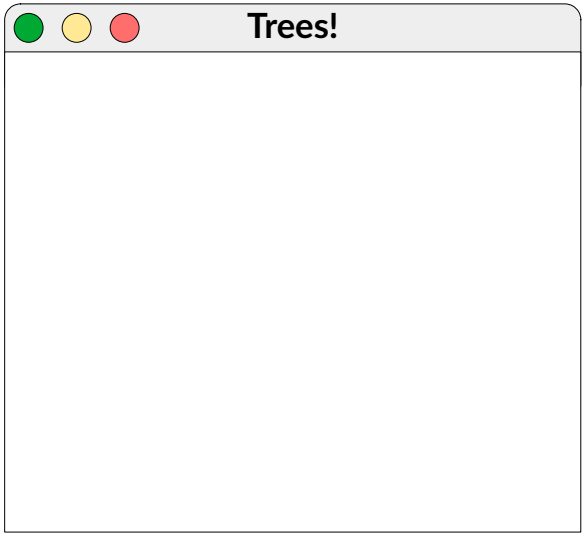

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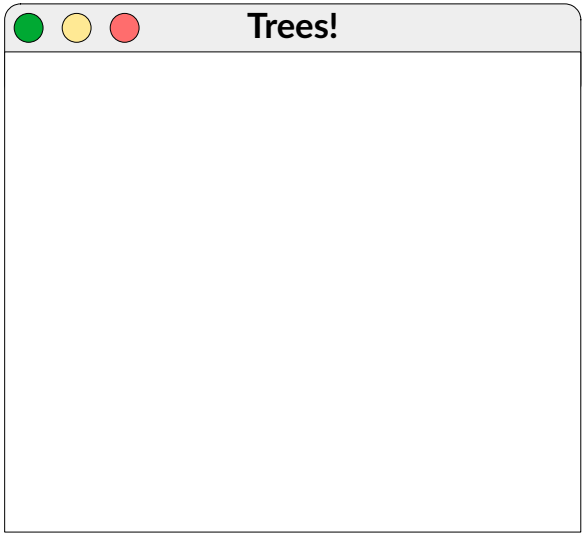

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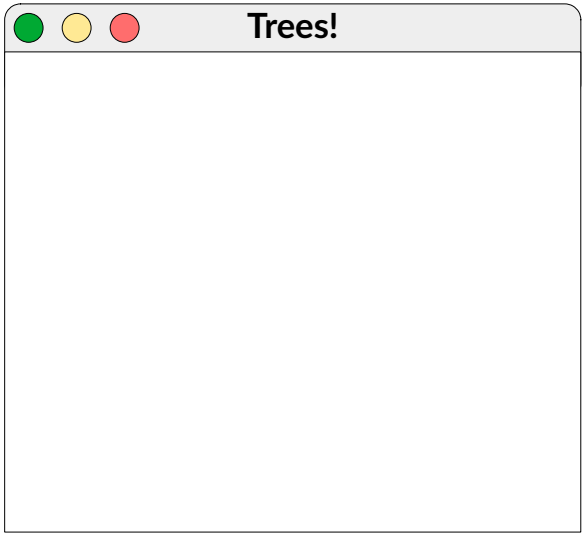

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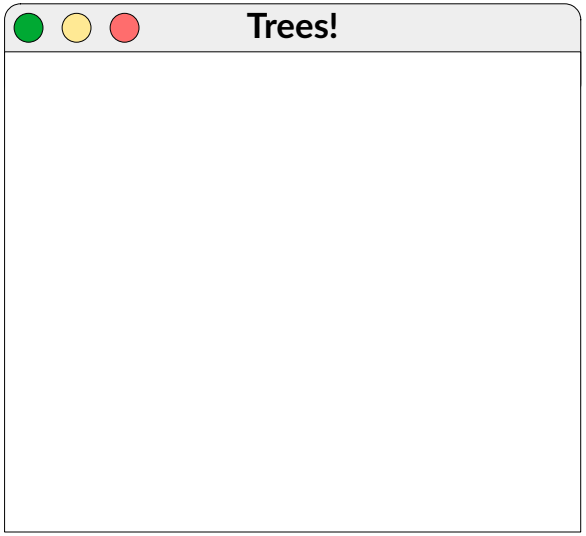

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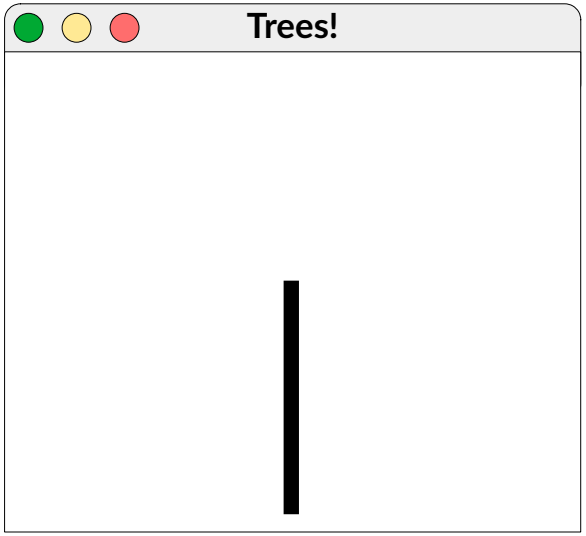

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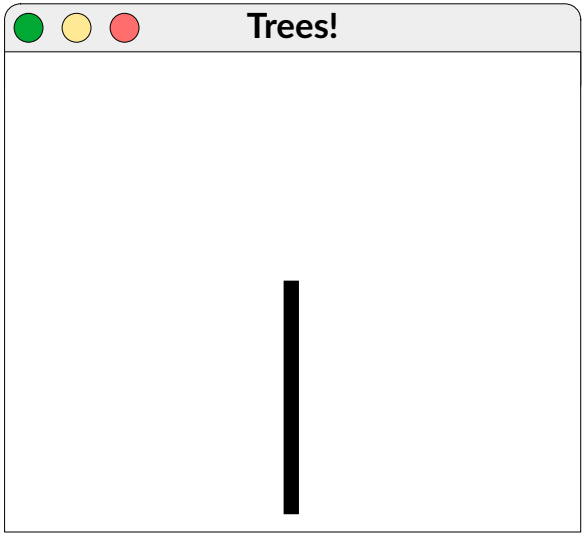

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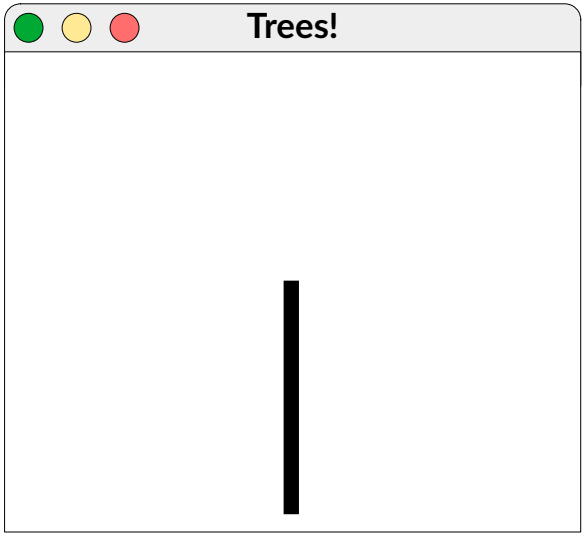

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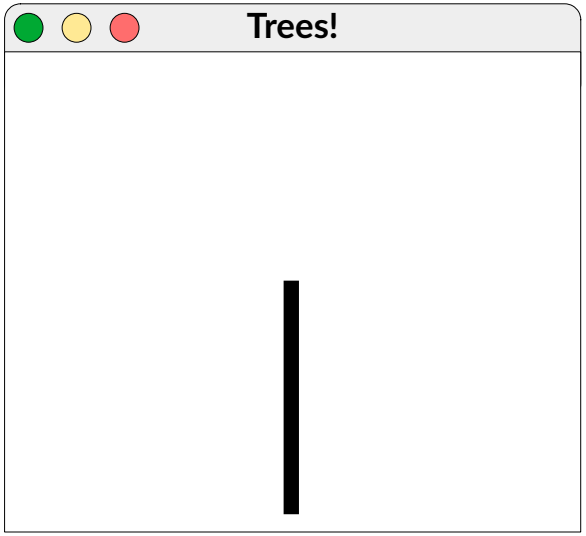

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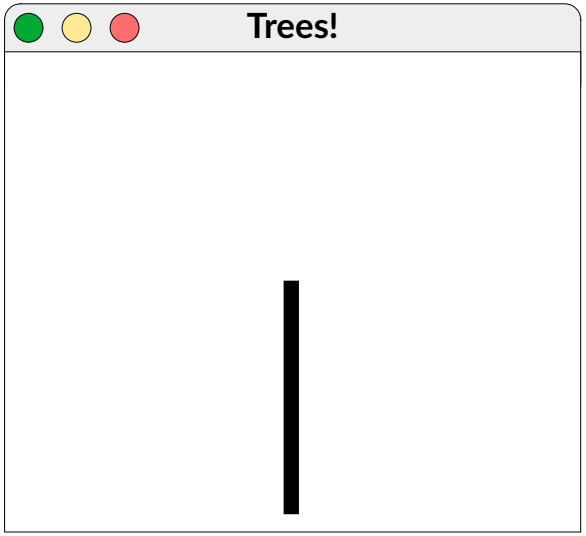

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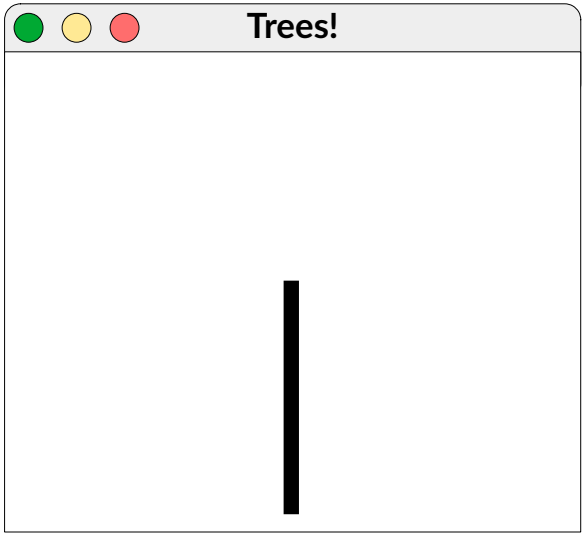

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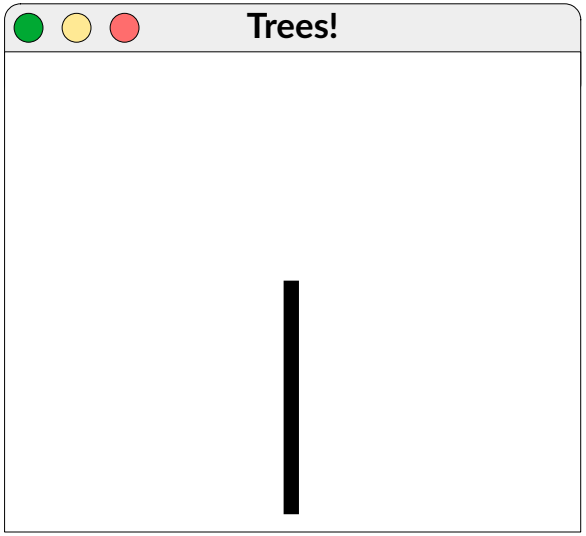
    int numLines = 0;
    CPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

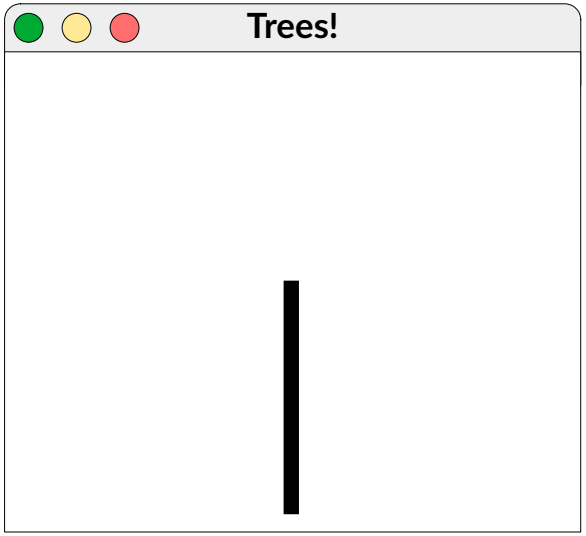
    int numLines = 0;
    CPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

0

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    CPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

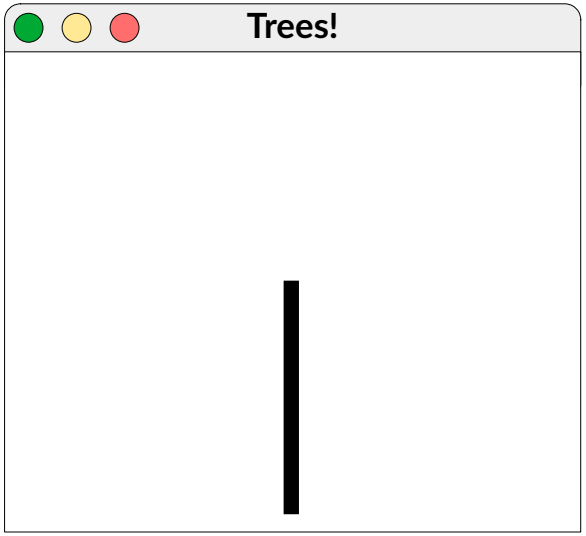
    return numLines;
}
```

0

numLines

It's reasonable to guess that this line is the problem because it looks like it resets `numLines` to zero at each call.

But that's not actually the issue. Remember – every recursive call gets its own copies of all local variables.



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

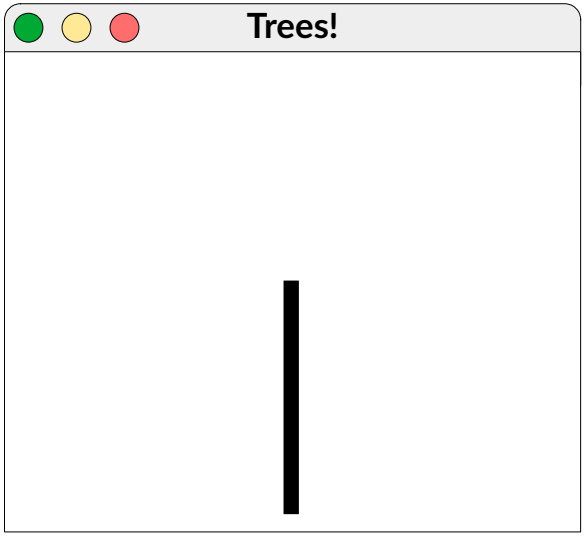
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

0

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

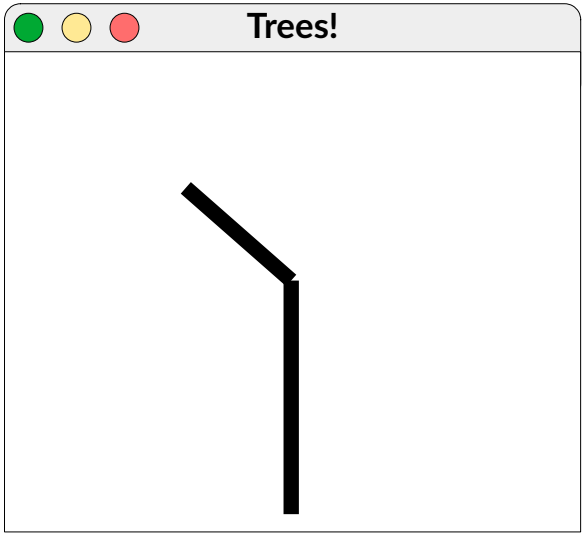
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

0

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

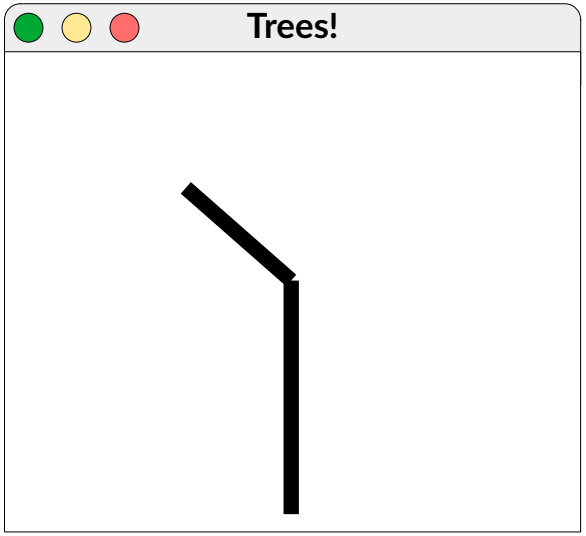
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

0

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

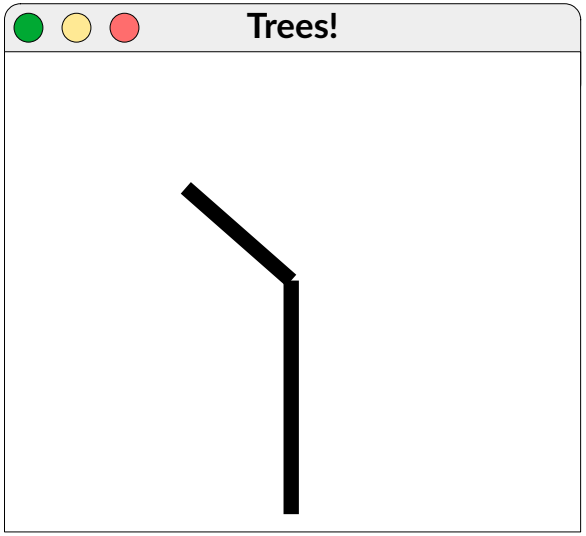
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

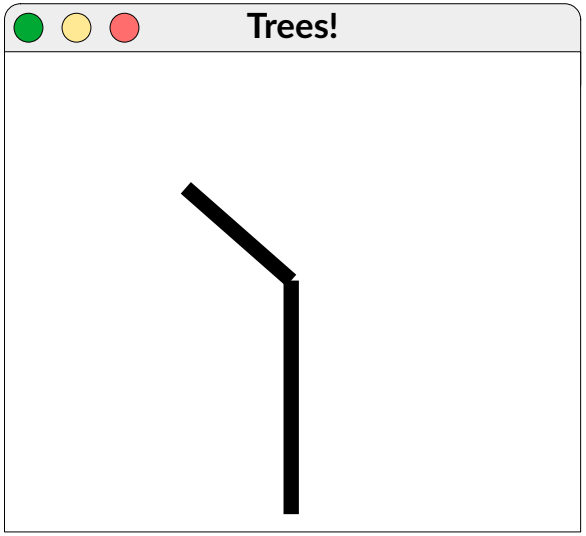
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



It's also reasonable to guess that the error is that this isn't incrementing the copy of `numLines` inside of the top-level call.

While it's true that this doesn't increment the top-level copy of `numLines`, that isn't an error *per se*. This function says it will return the number of lines drawn, not update a global total somewhere.

```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

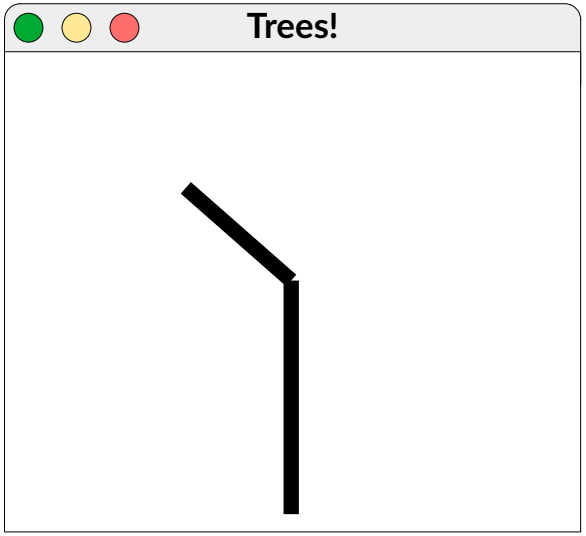
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

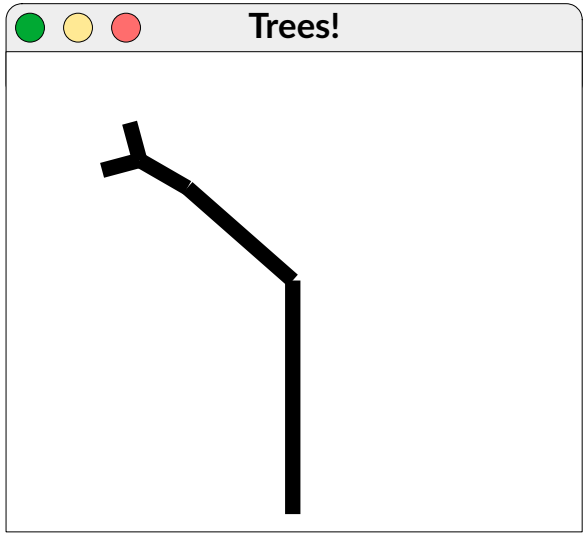
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

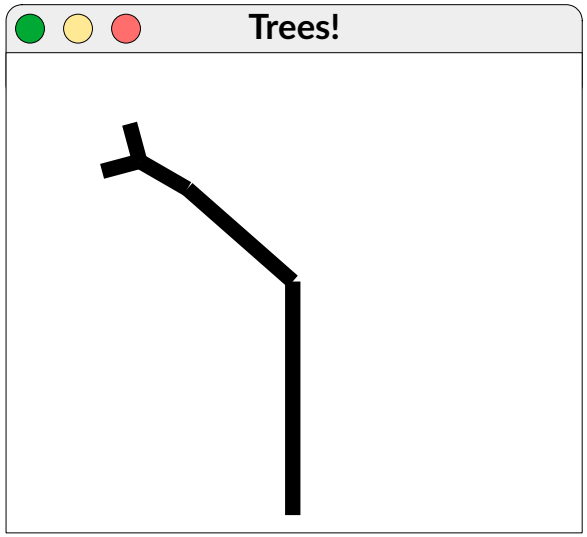
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



This function returns an integer, but we didn't do anything with that integer! It would be like writing this line of code:

```
sqrt(137);
```

This computes a square root, but doesn't store it anywhere. Oops! Our total is now wrong.

```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

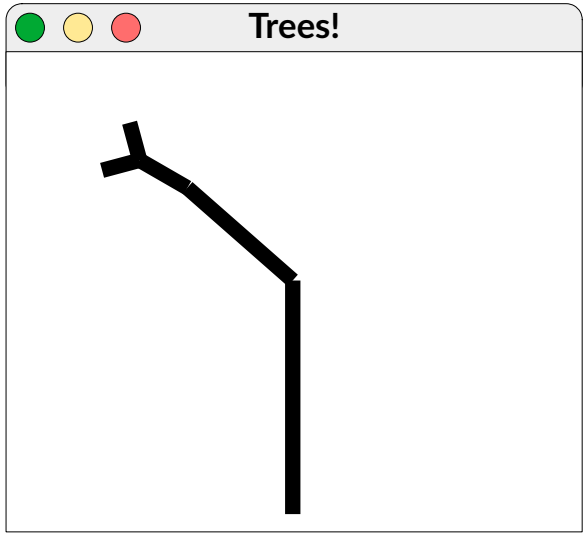
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

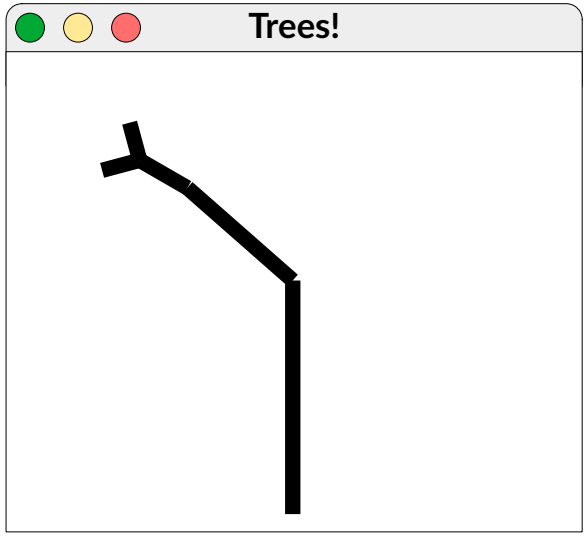
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

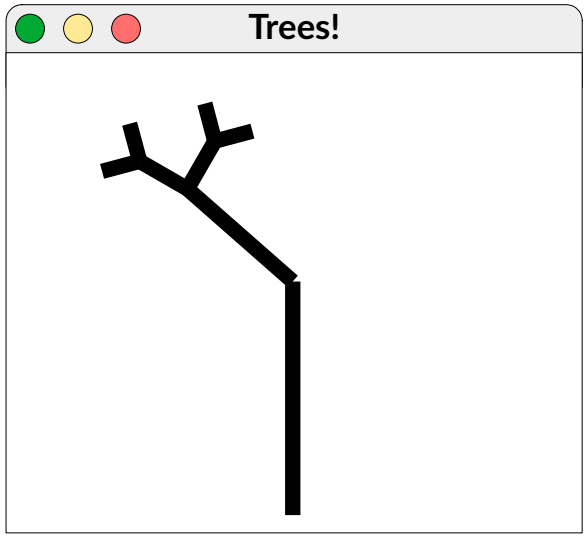
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

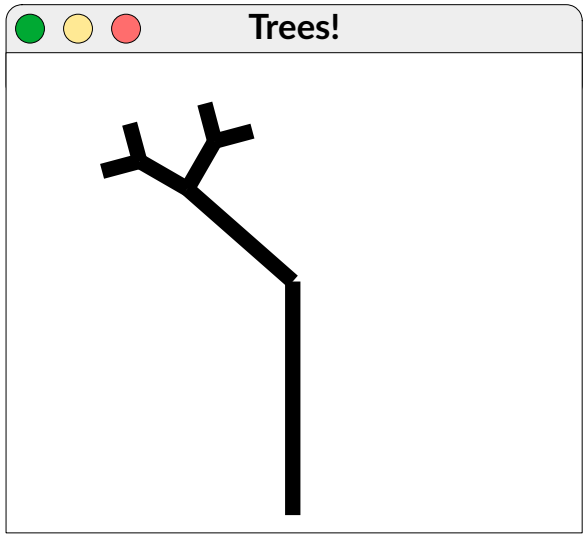
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



Oops – we didn't do anything with the return value.

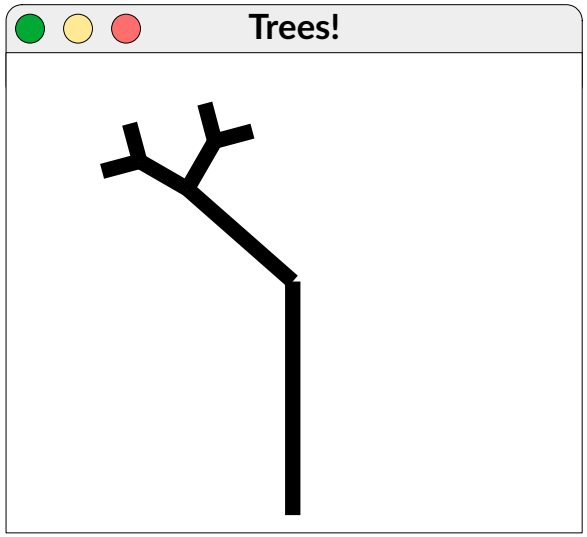
```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);
    return numLines;
}
```

1

numLines



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    int numLines = 0;
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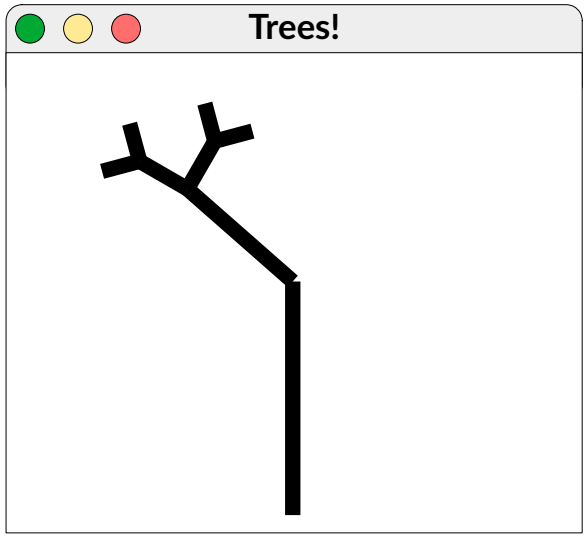
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

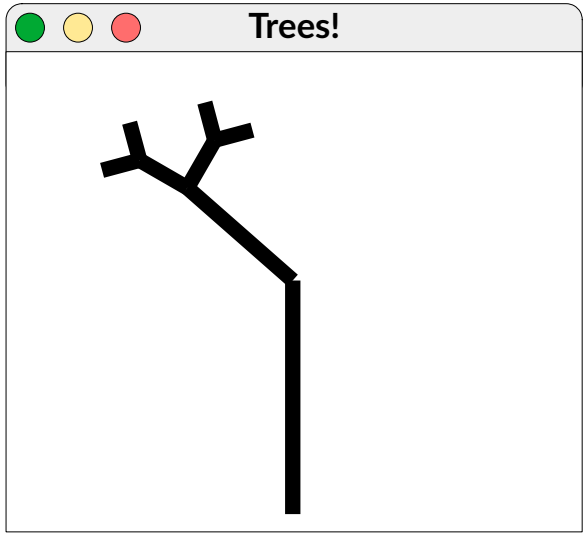
    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines

Oops – we didn't do anything with the return value.



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int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

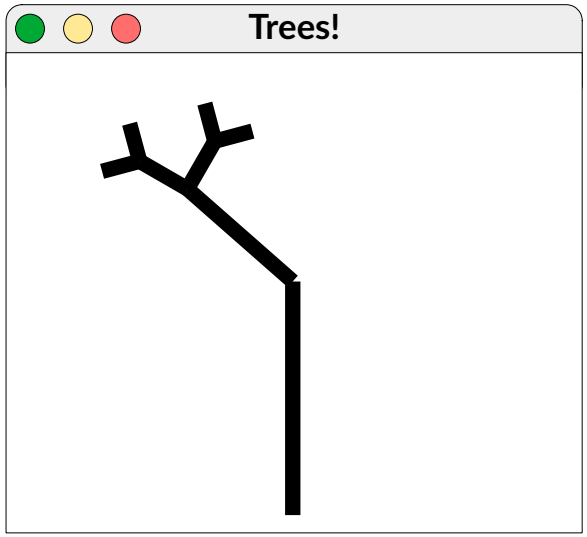
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

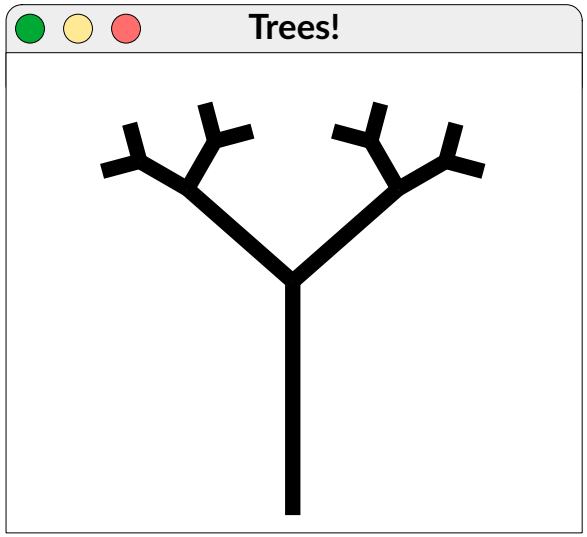
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



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int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

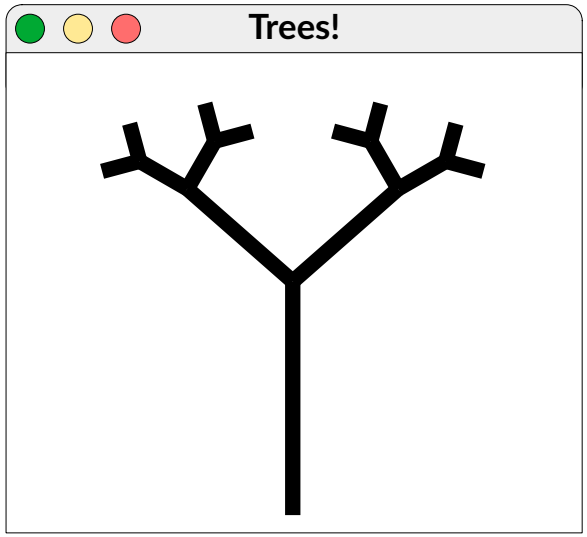
    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines

Oops – we didn't do anything with the return value.



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int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

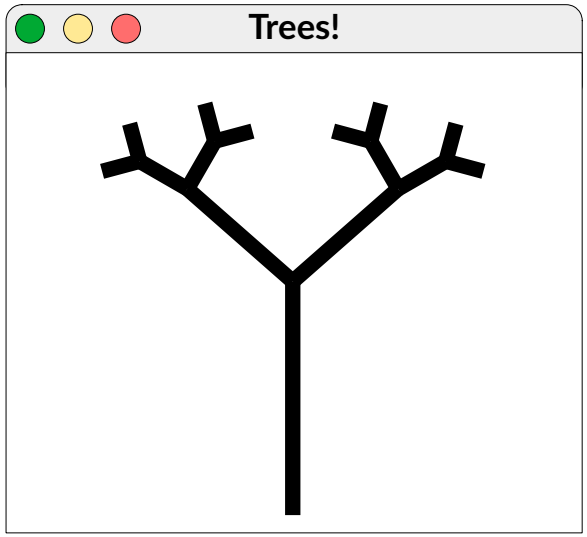
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    drawTree(/* … */);
    drawTree(/* … */);

    return numLines;
}
```

1

numLines



General Advice

- If a function returns a value, you should, in general, do something with that value.
  - Otherwise, the function did all this hard work for you, and you just dropped it on the floor!
- If you're writing a recursive function that returns a value, you should explicitly do something with the value returned by each recursive call.
  - Otherwise, your recursive call is trying to tell you something, and you're ignoring it!

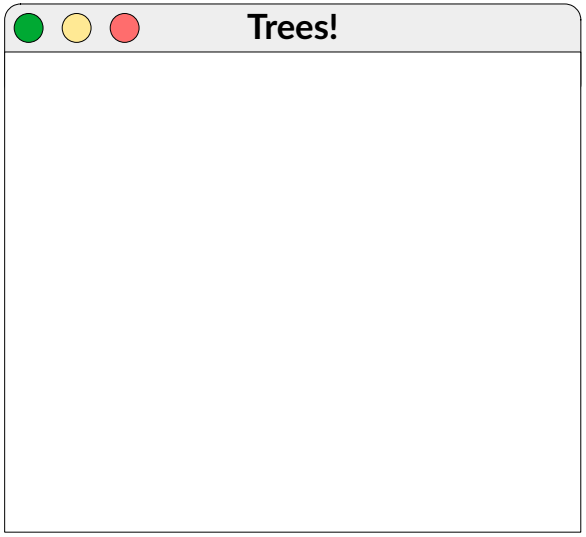
The Correction

```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

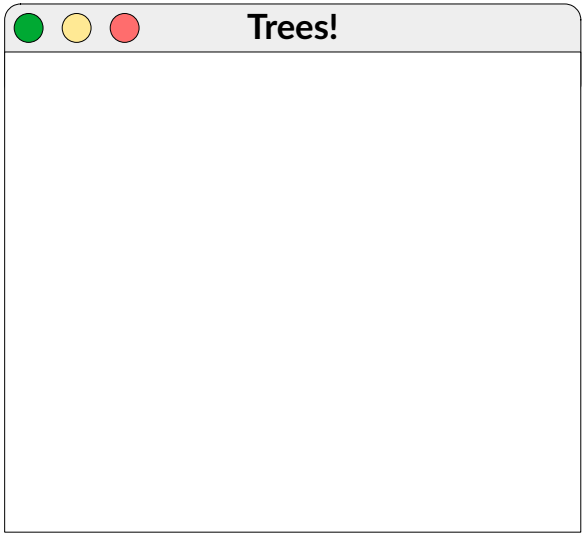


```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```



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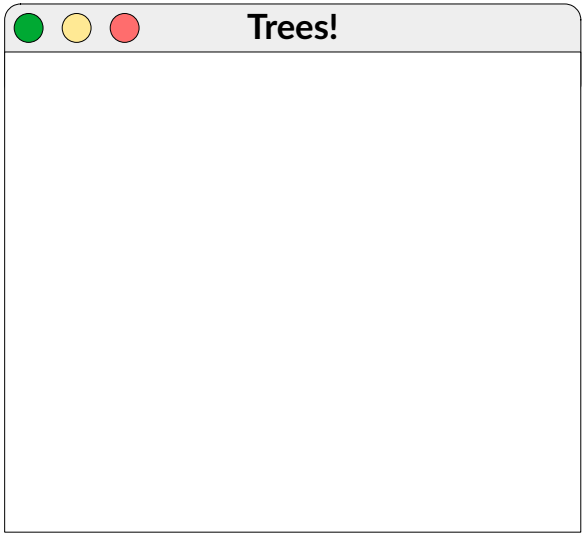
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```



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    if (order == 0) {
        return 0;
    }

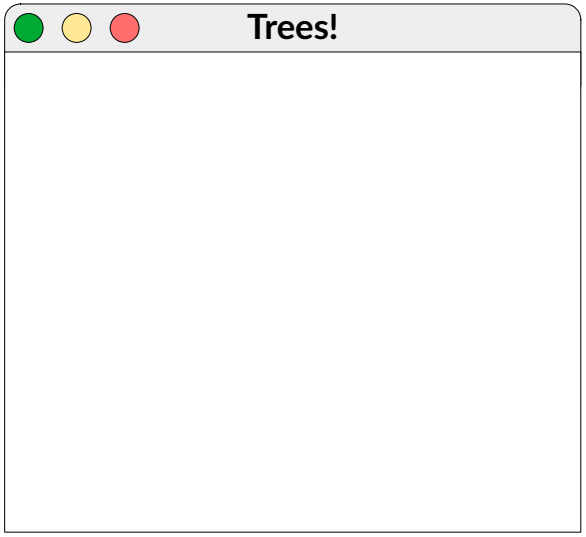
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

0

numLines



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    if (order == 0) {
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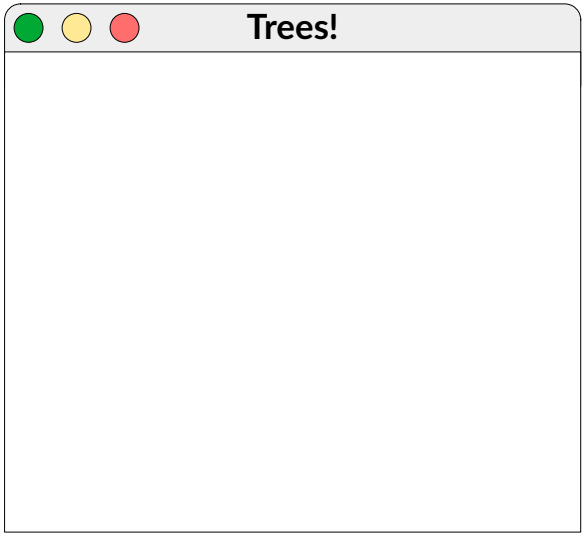
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

0

numLines



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    if (order == 0) {
        return 0;
    }

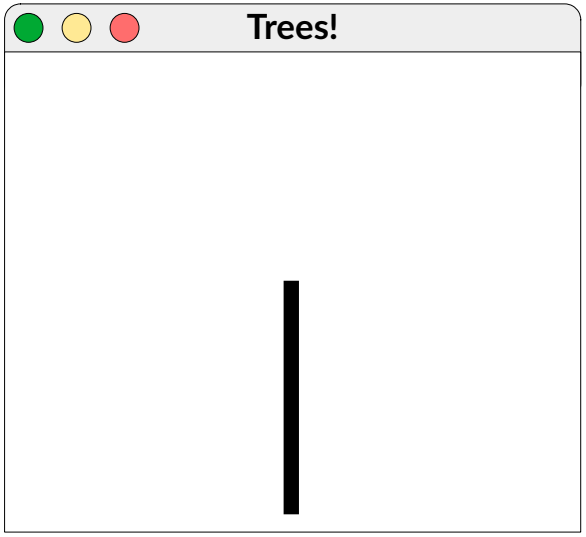
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

0

numLines



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    numLines += drawTree(/* … */);
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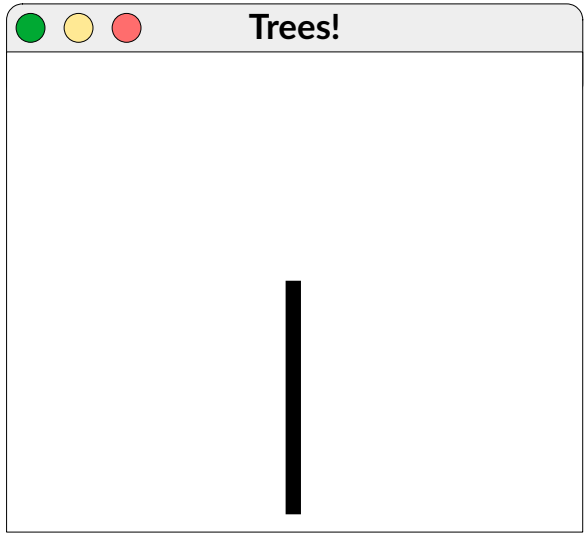
0

numLines



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    if (order == 0) {
        return 0;
    }

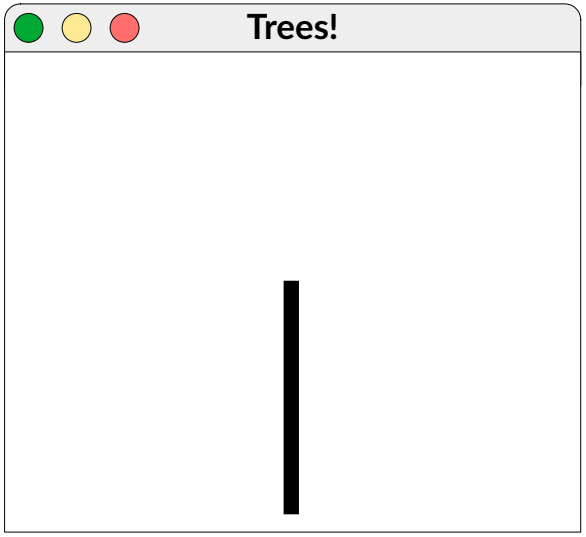
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

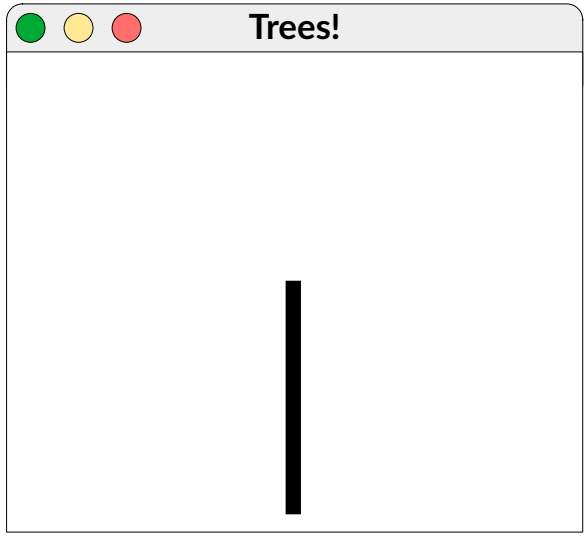
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    numLines++;

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    return numLines;
}
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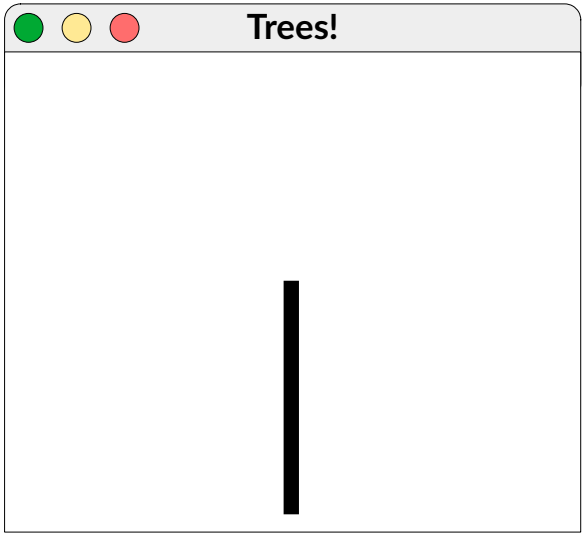
1

numLines



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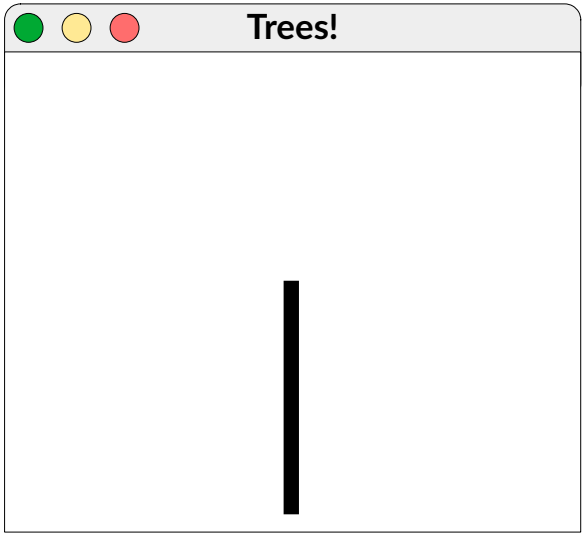
    if (order == 0) {
        return 0;
    }

    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```



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    if (order == 0) {
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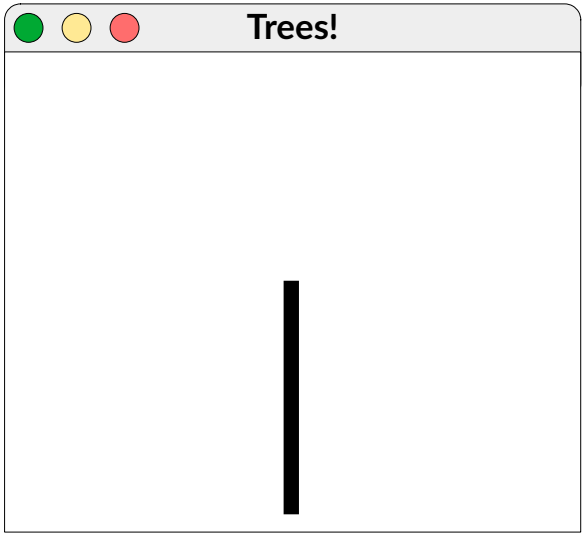
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```



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        return 0;
    }

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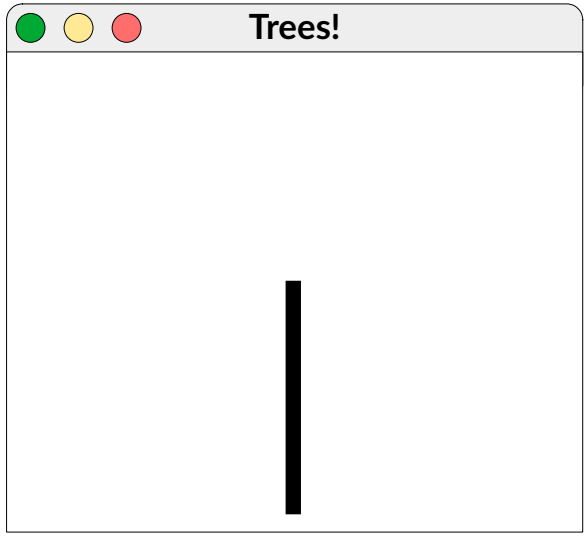
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```



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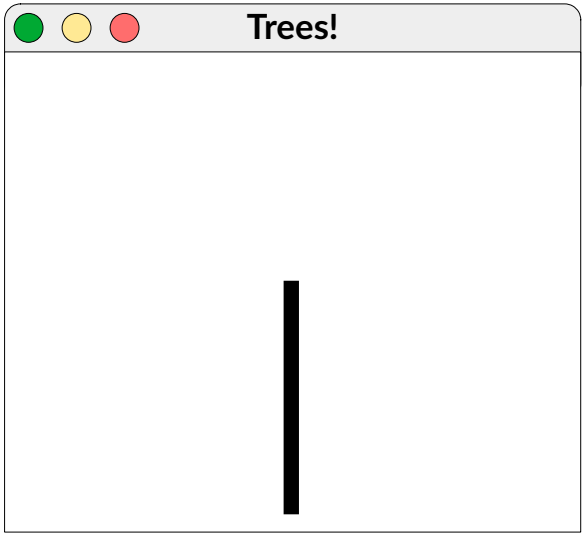
    return numLines;
}
```

0

numLines



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    numLines += drawTree(/* … */);

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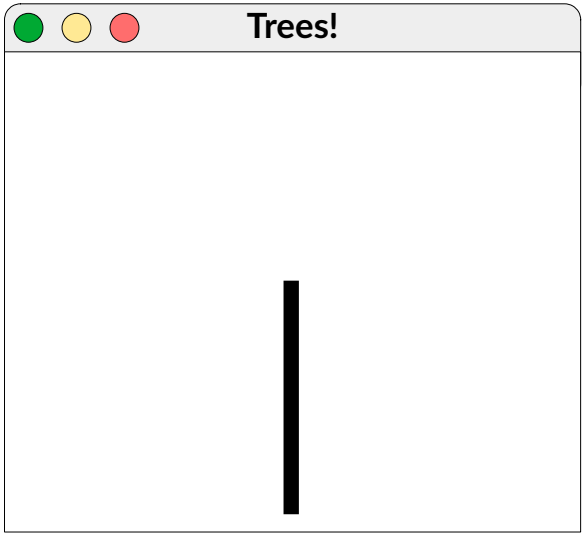
}
```

0

numLines



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    numLines += drawTree(/* … */);

    return numLines;
}
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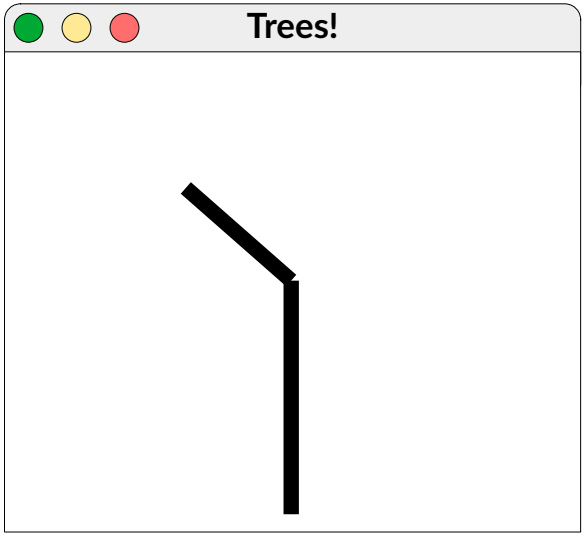
0

numLines



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    if (order == 0) {
        return 0;
    }

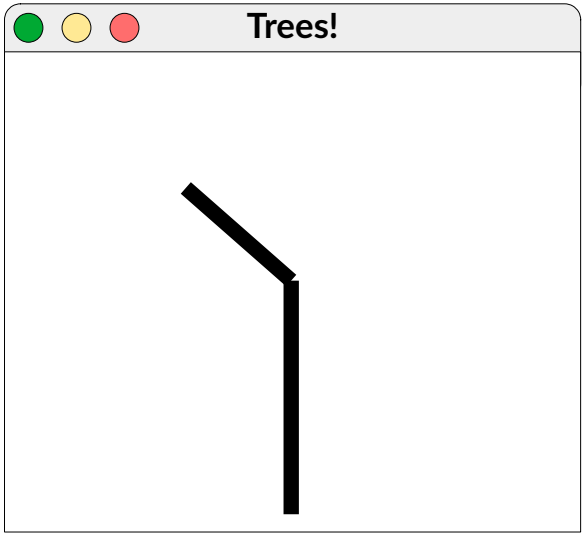
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

0

numLines



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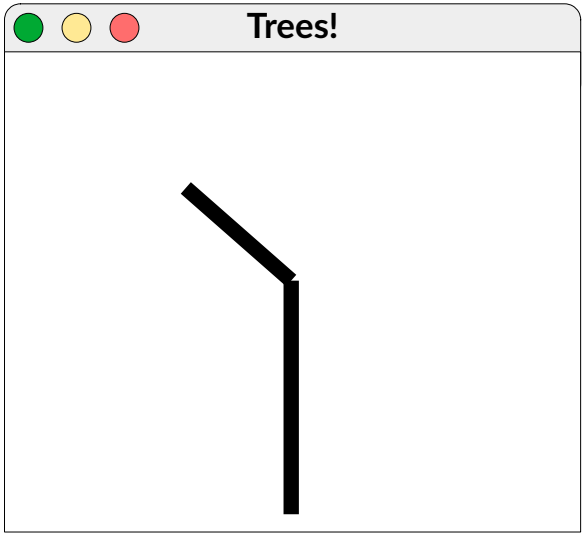
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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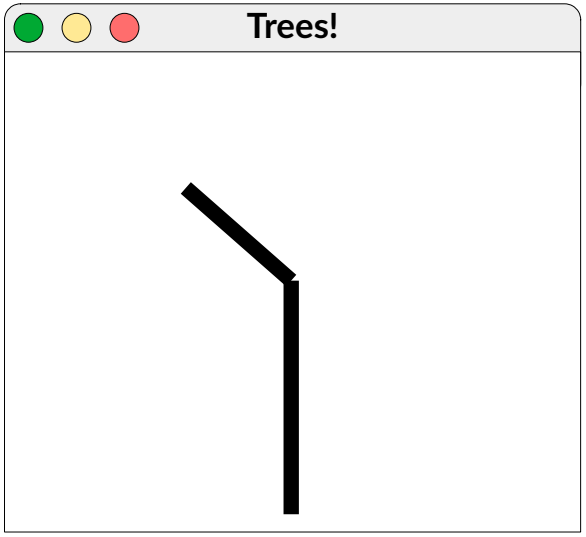
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    numLines++;

    numLines += drawTree(/* … */);
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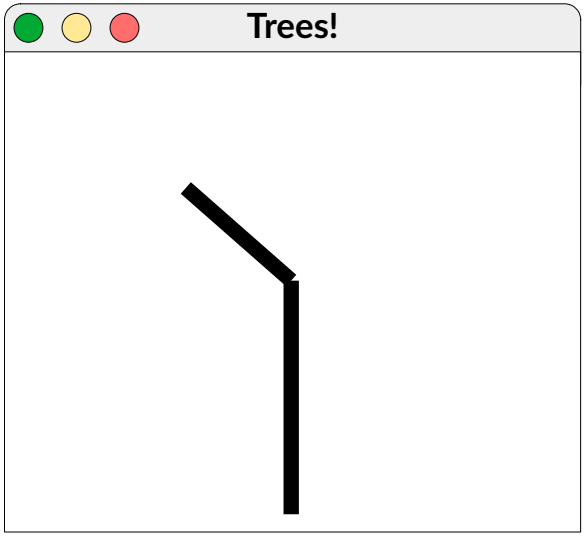
}
```

1

numLines



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int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

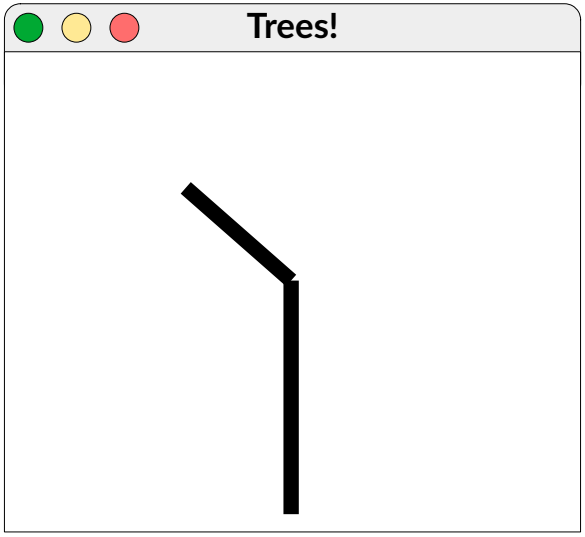
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



This call draws a recursive tree.

```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

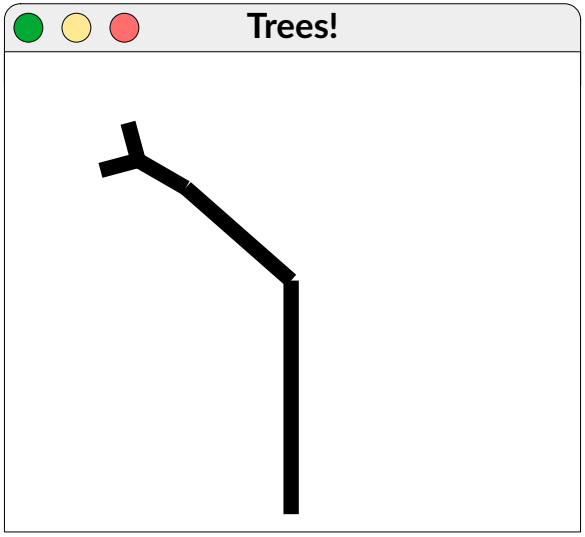
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

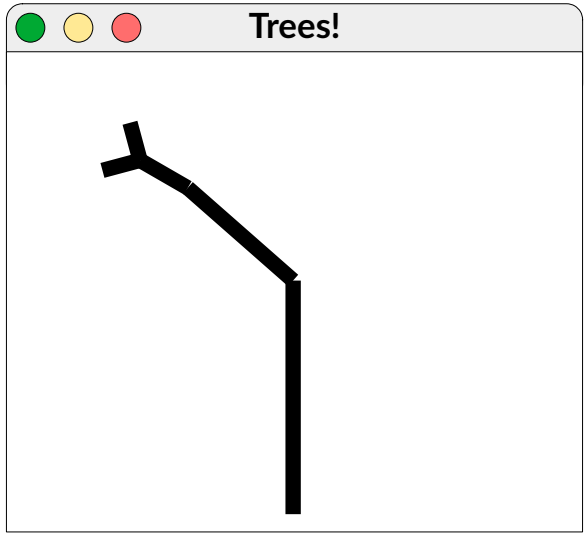
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



This call draws a recursive tree.

It then returns the number of lines drawn.

```
int drawTree(/* … */) {
    if (order == 0) {
        return 0;
    }

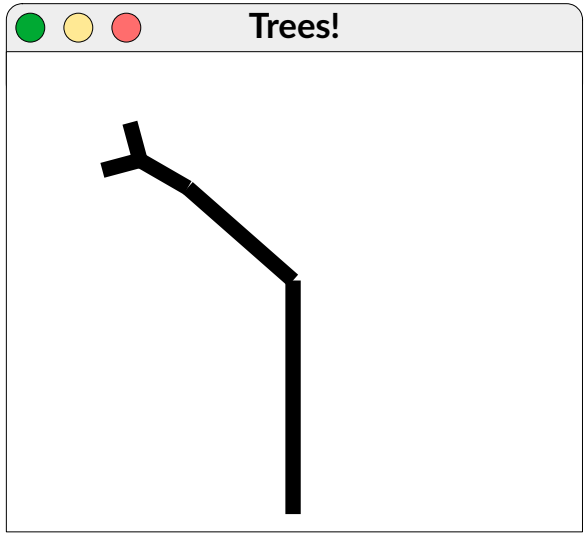
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

1

numLines



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    if (order == 0) {
        return 0;
    }

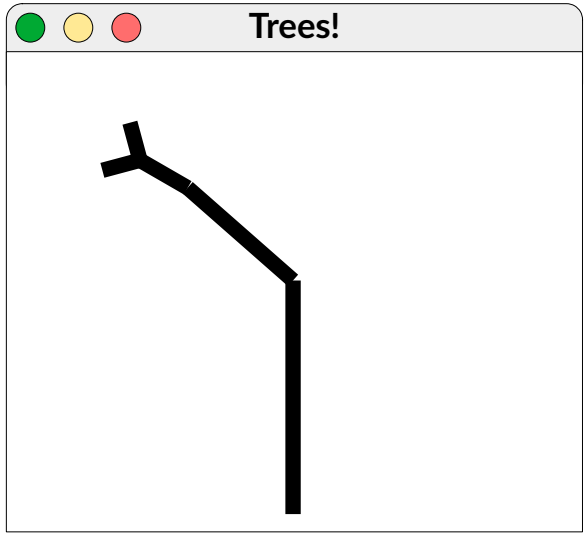
    int numLines = 0;
    GPoint endpoint = drawPolarLine(/* … */);
    numLines++;

    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

    return numLines;
}
```

4

numLines



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    if (order == 0) {
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    numLines += drawTree(/* … */);
    numLines += drawTree(/* … */);

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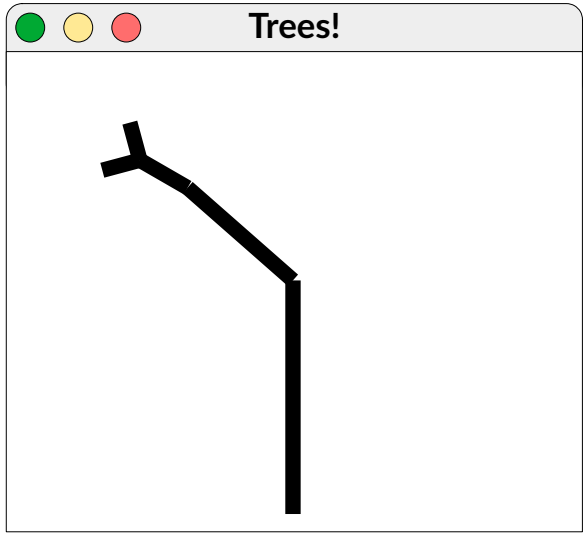
```

4

numLines



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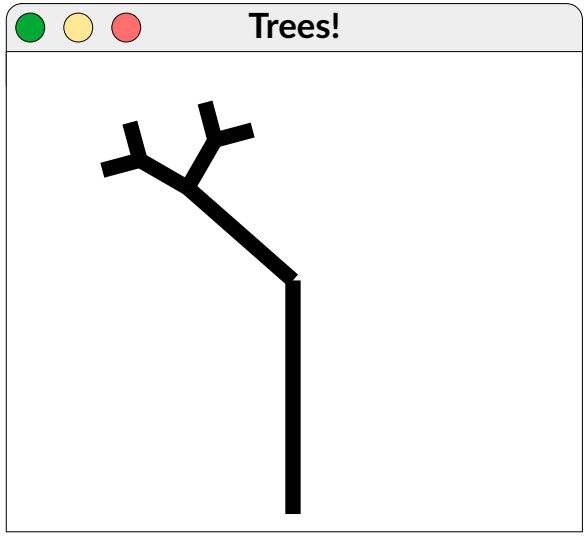
4

numLines



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    numLines += drawTree(/* … */);
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    return numLines;
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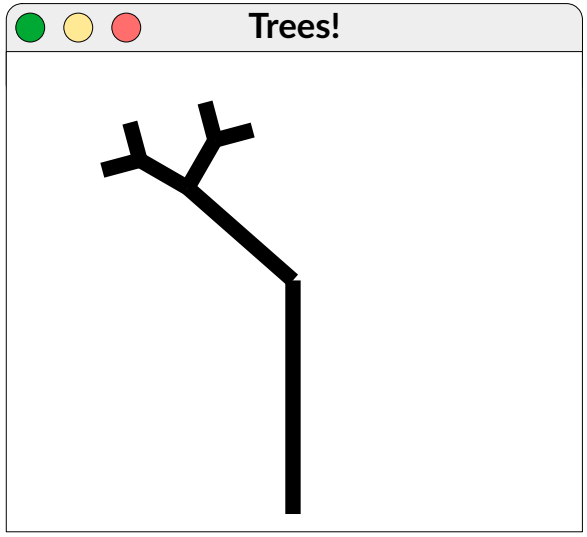
```

7

numLines



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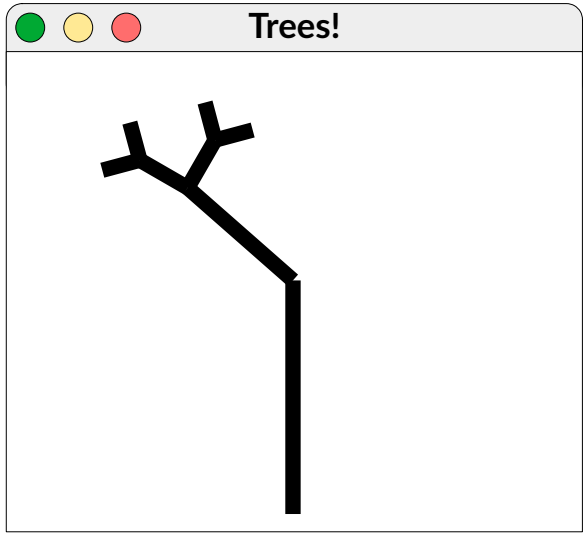
7

numLines



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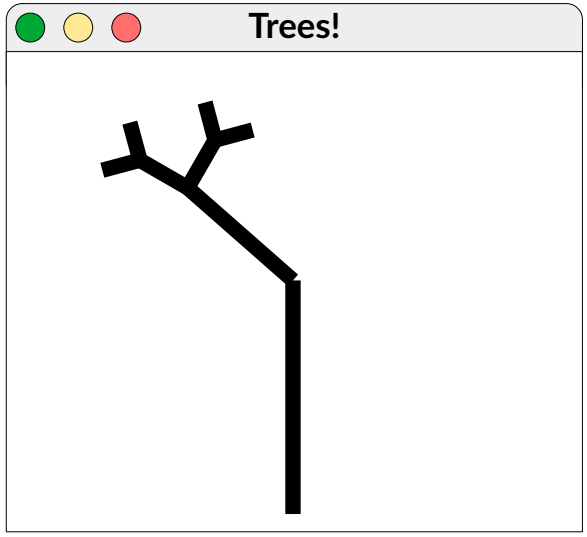
1

numLines



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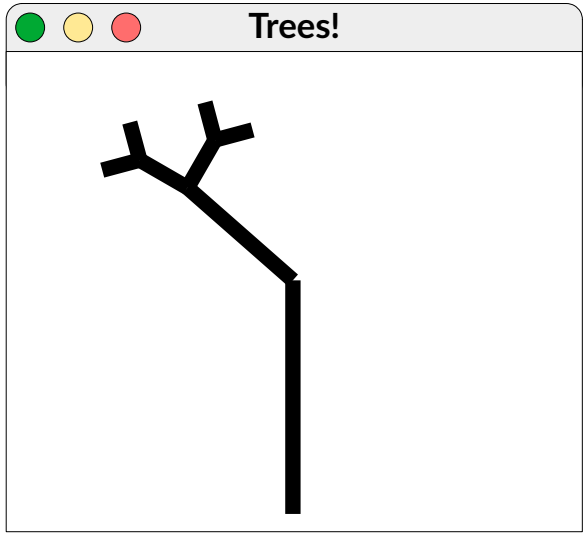
1

numLines



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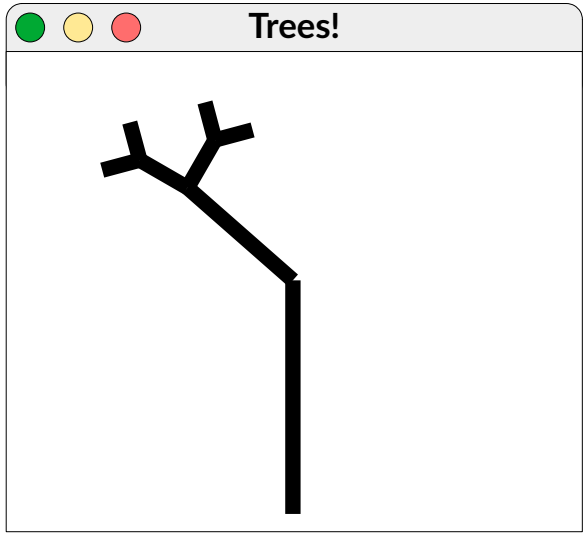
```

7

8

numLines



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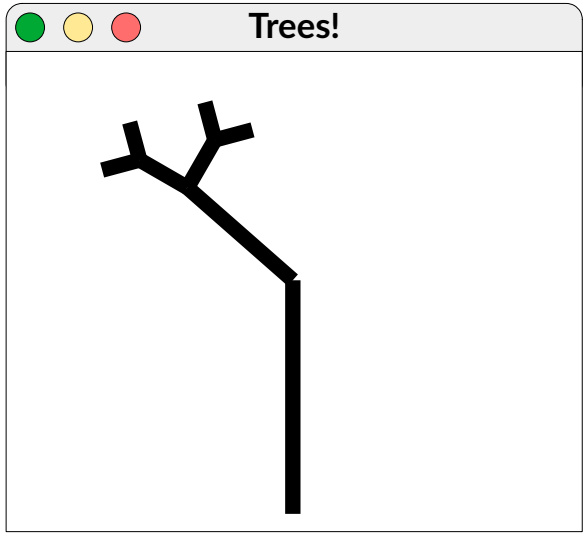
8

numLines



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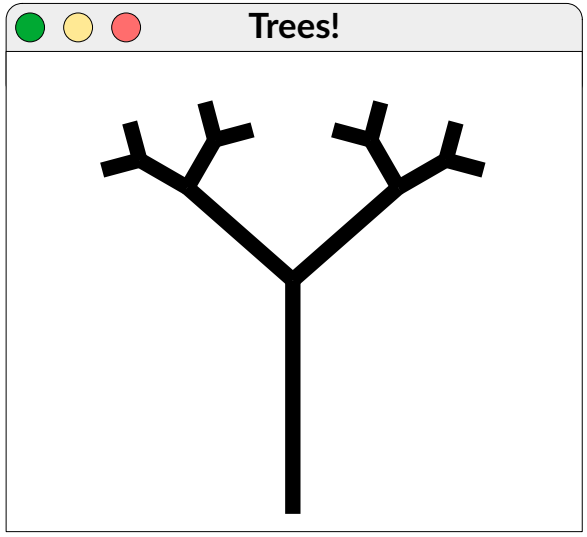
8

numLines



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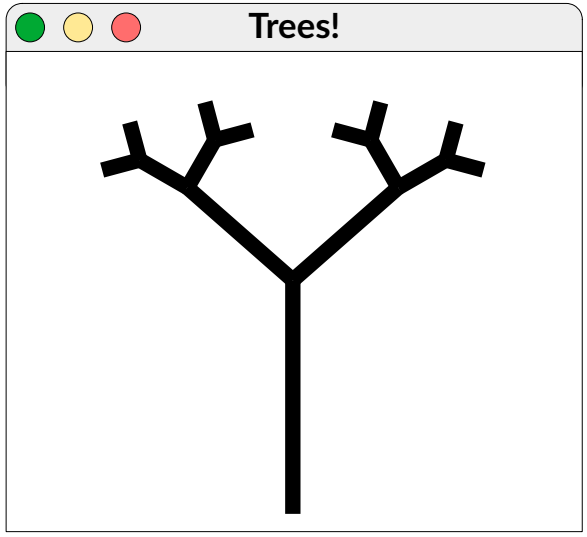
7

8

numLines



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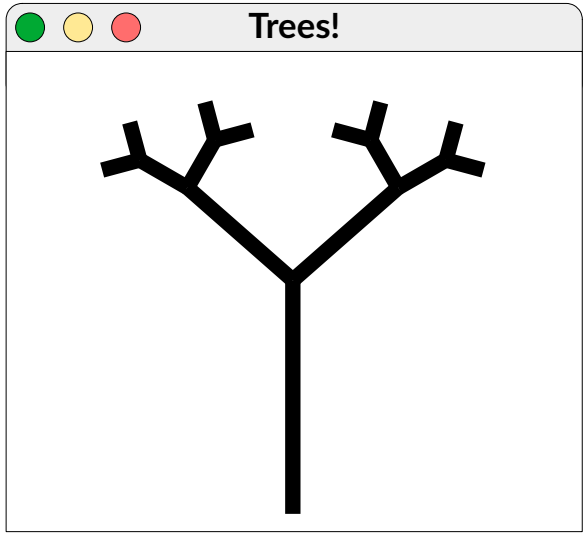
7

15

numLines



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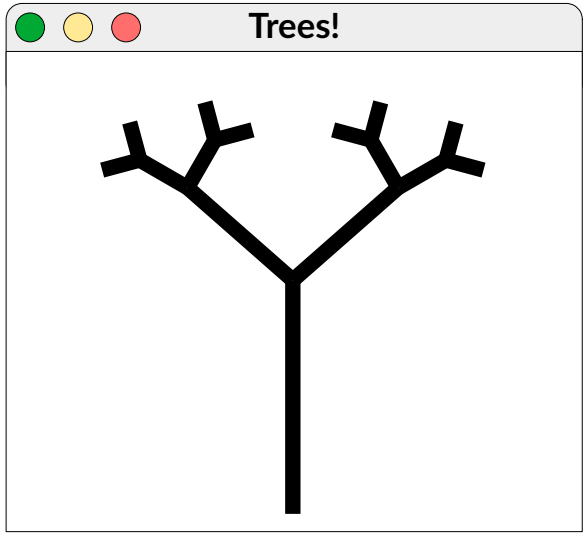
15

numLines



Summary From Today

- Self-similar figures exist in many places, and recursion is a great way to draw them.
- When drawing a self-similar figure, identify what aspects of the figure are different at different scales.
- Assigning an order to a self-similar figure is a great way to make a base case.
- When writing a recursive function that returns a value, make sure you use the result of each recursive call. Otherwise, important data can get lost.

Your Action Items

- ***Read Chapter 8.***
  - There's a ton of goodies in there! It'll help you solidify your understanding of recursion and recursive techniques.
- ***Keep Working On Assignment 2.***
  - Need help? Stop by the LaIR or post on EdStem! That's what we're here for.

Next Time

- ***Recursive Enumeration***
  - Finding all objects of a given type.
- ***Enumerating Subsets***
  - A classic combinatorial problem!