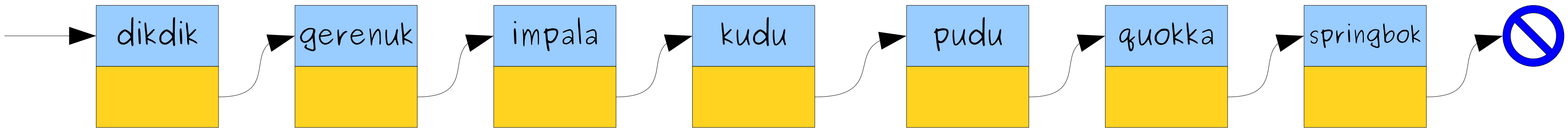
Binary Search Trees

Part One

Outline for Today

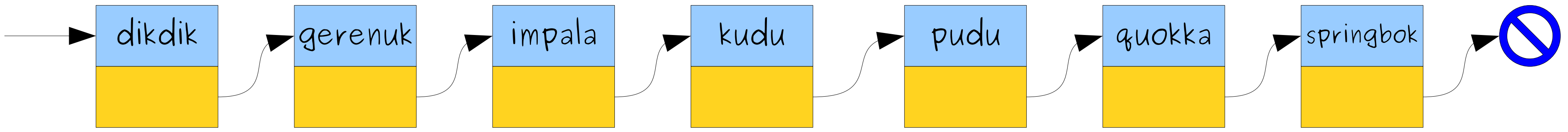
- ***Why Trees?***
  - What's so special about a tree shape?
- ***Binary Search Trees***
  - A simple and elegant way to store data.
- ***Tree Searches***
  - On knowing where to look.
- ***Printing Trees***
  - A delightful recursive algorithm.
- ***Adding to Trees***
  - Expanding things outward.

On Being Near the Front



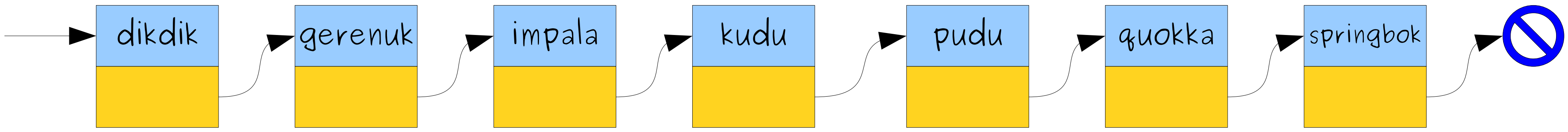
What is the average cost of searching for an element in an n-item linked list? Answer using big-O notation.

Formulate a hypothesis!



What is the average cost of searching for an element in an *n*-item linked list? Answer using big-O notation.

Chat with your neighbors!



Answer: **O(*n*)**.

Intuition: Most elements are far from the front.

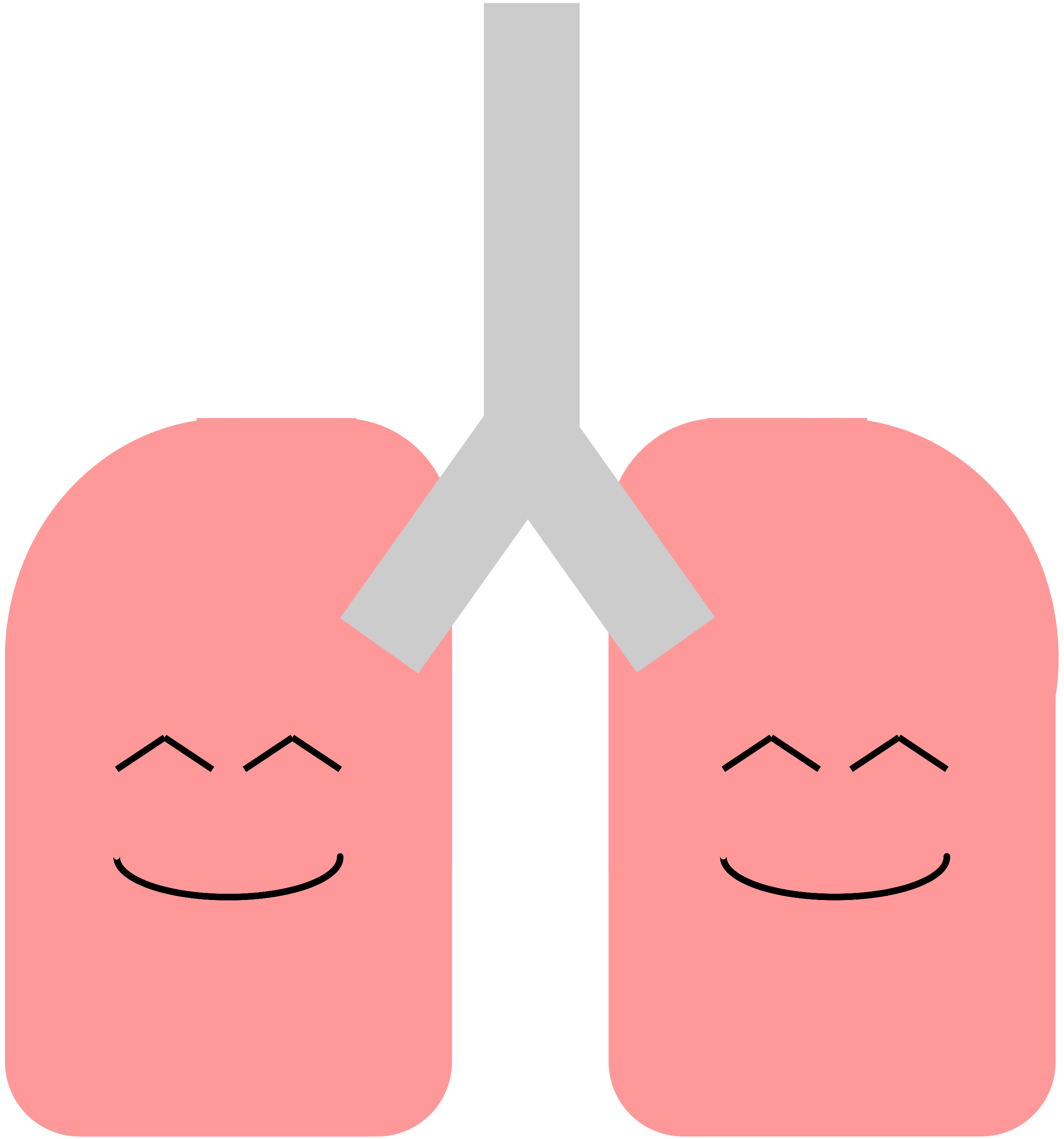
Can you chain a bunch of objects together so that most of them are near the front?

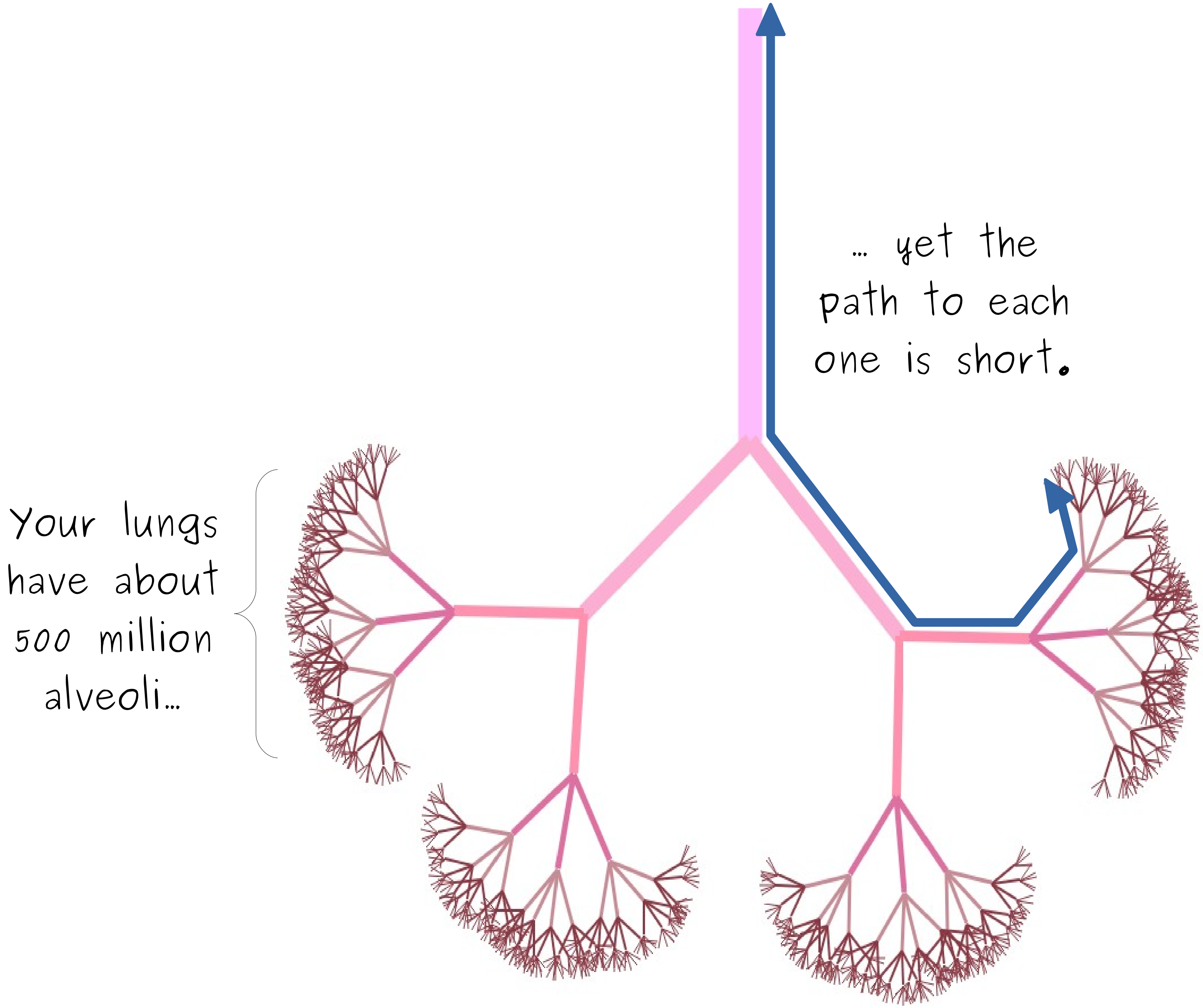
An Interactive Analogy

Take a deep breath.

And exhale.

Feel nicely oxygenated?

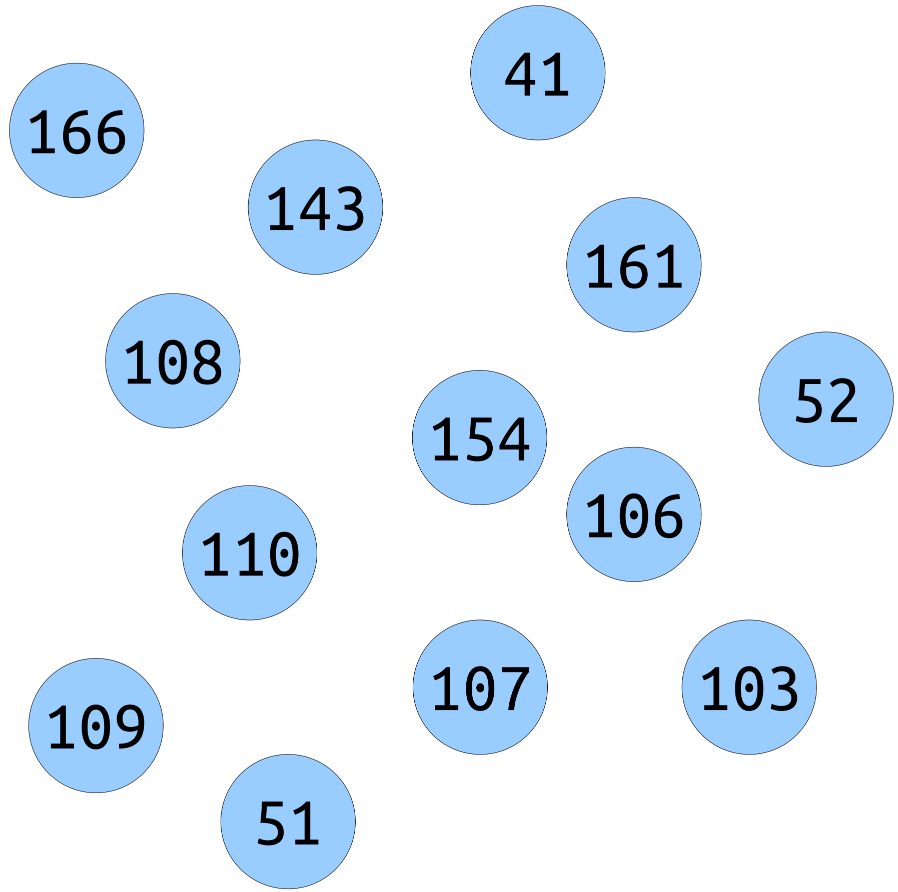




Your lungs have about 500 million alveoli...
... yet the path to each one is short.

Key Idea: The distance from the top of a tree to each node in the tree is small.

Harnessing this Insight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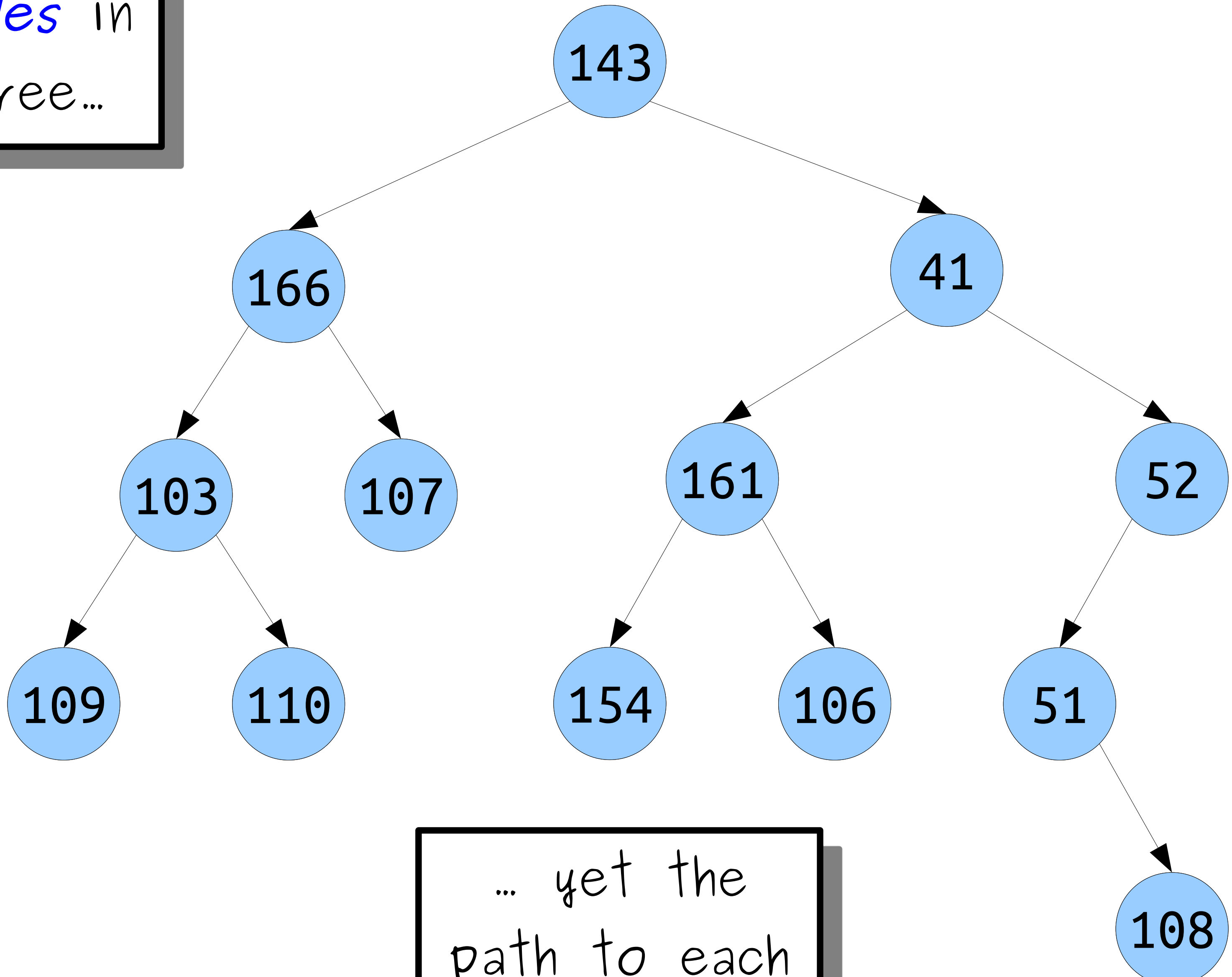
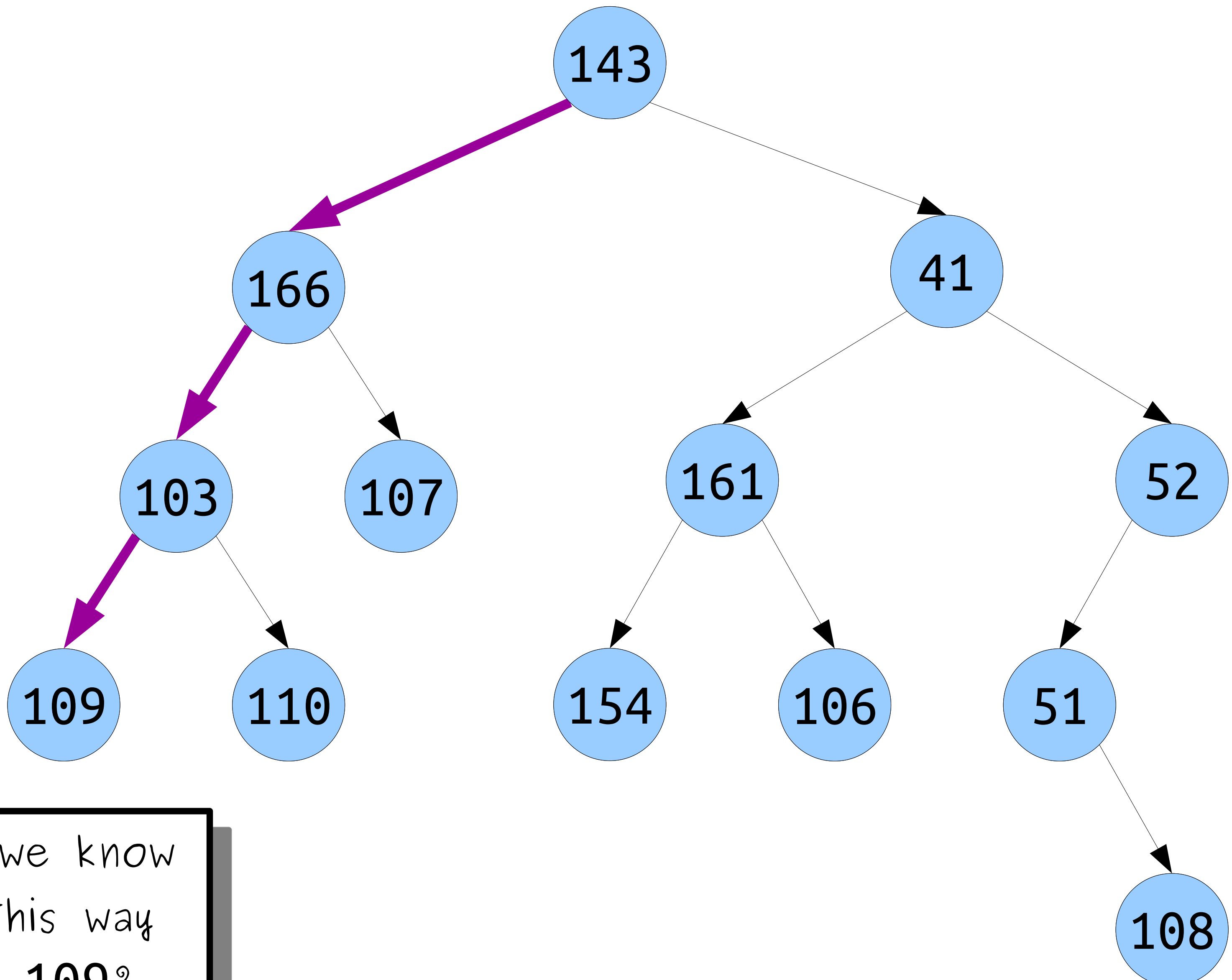
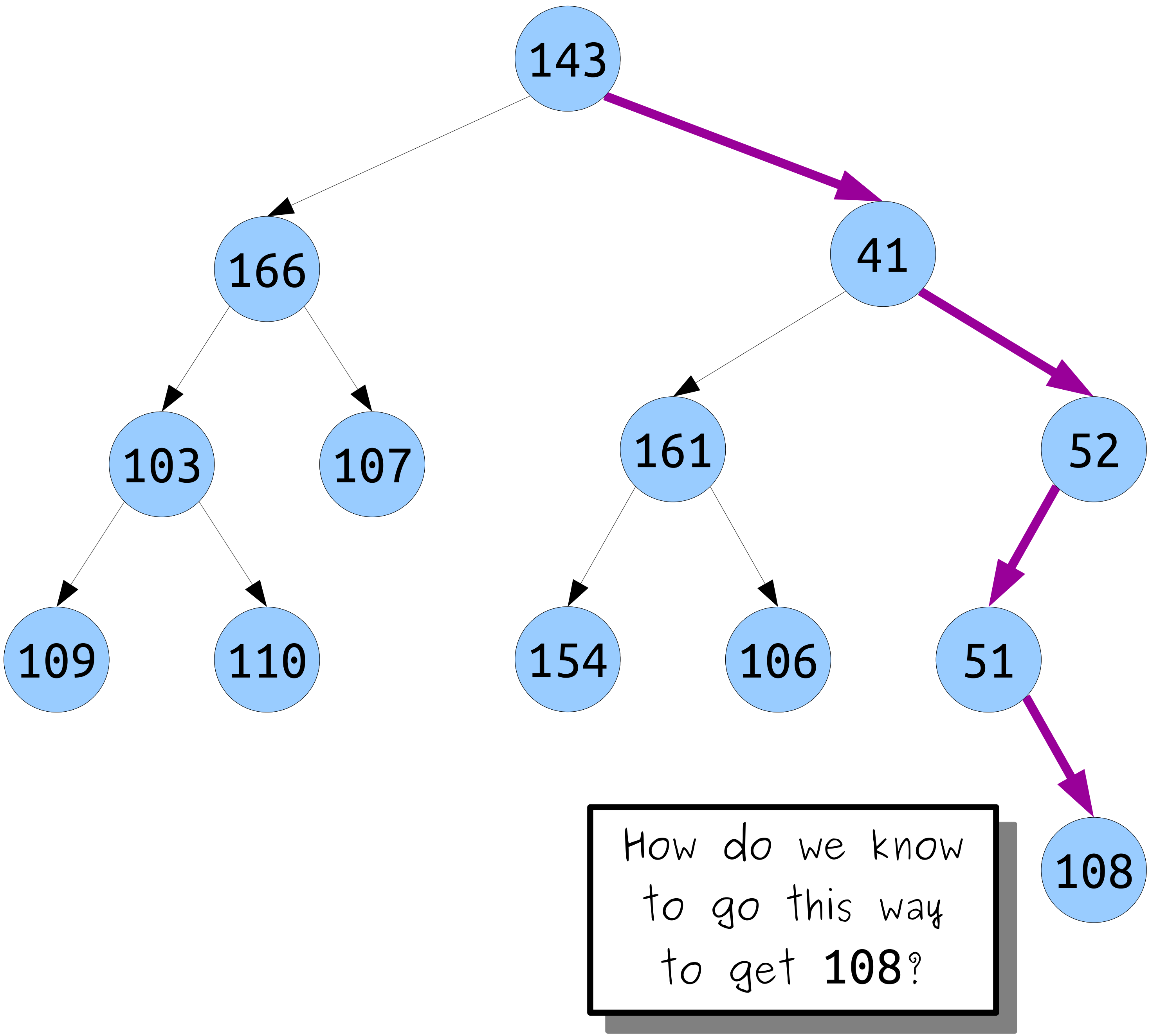
There are 13 *nodes* in this tree…



… yet the path to each one is short.



How do we know to go this way to get **109**?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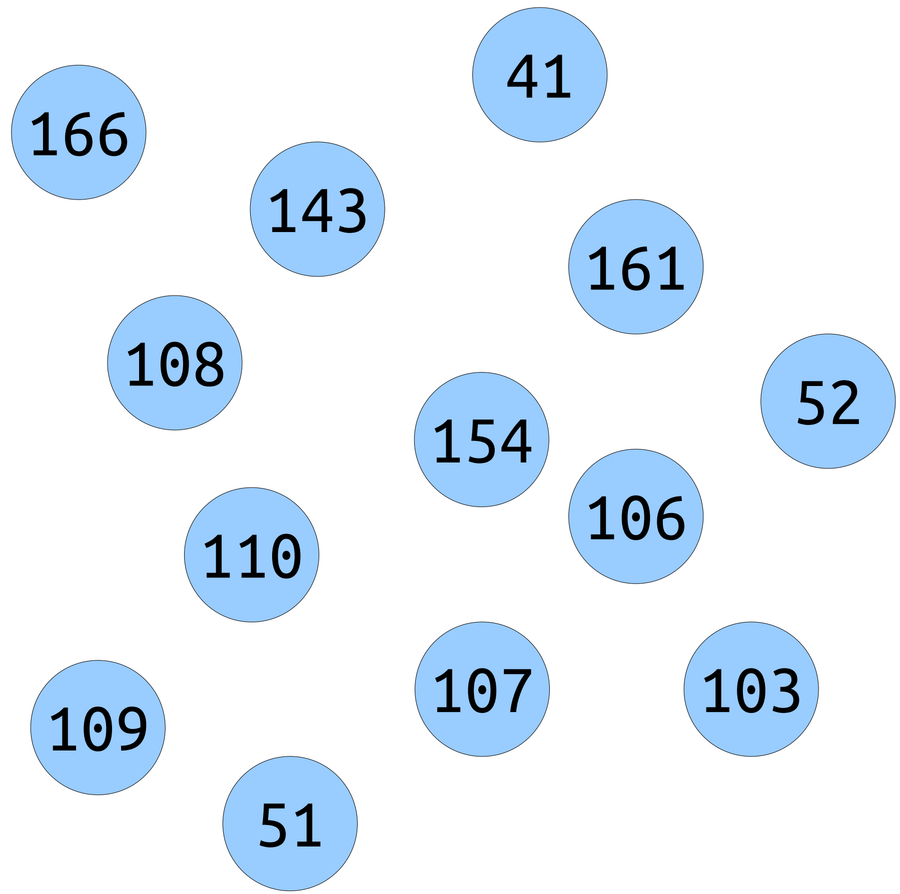
How do we know to go this way to get **108**?

Goal: Store elements in a tree structure where there's an easy way to find them.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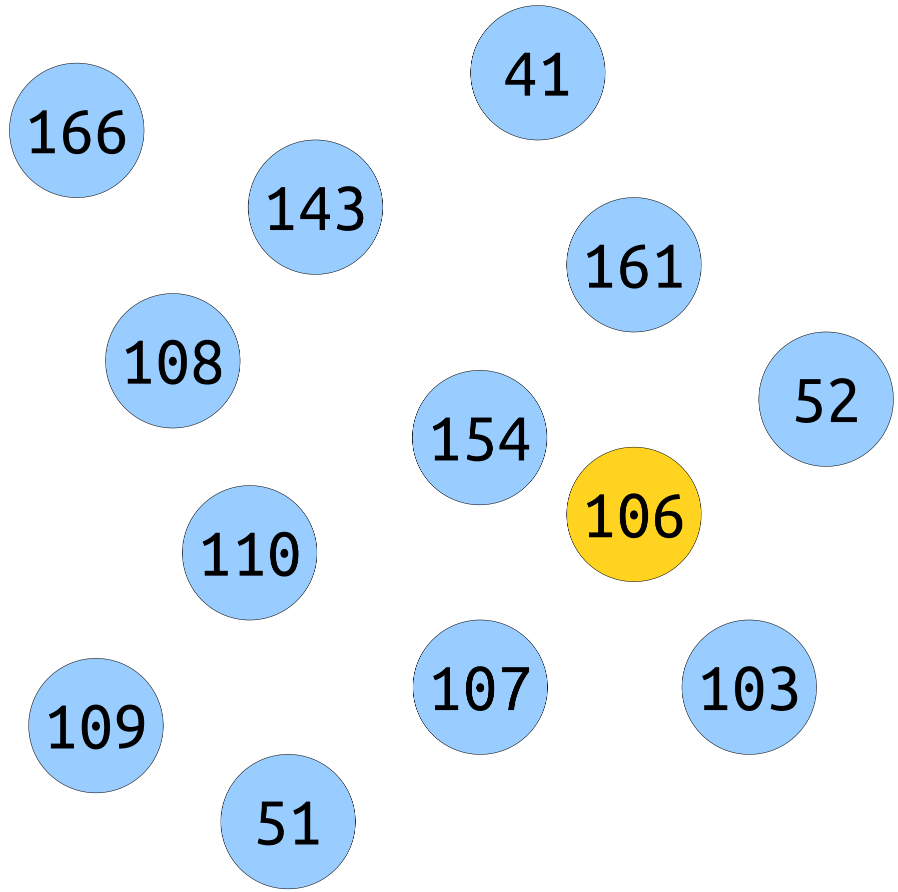
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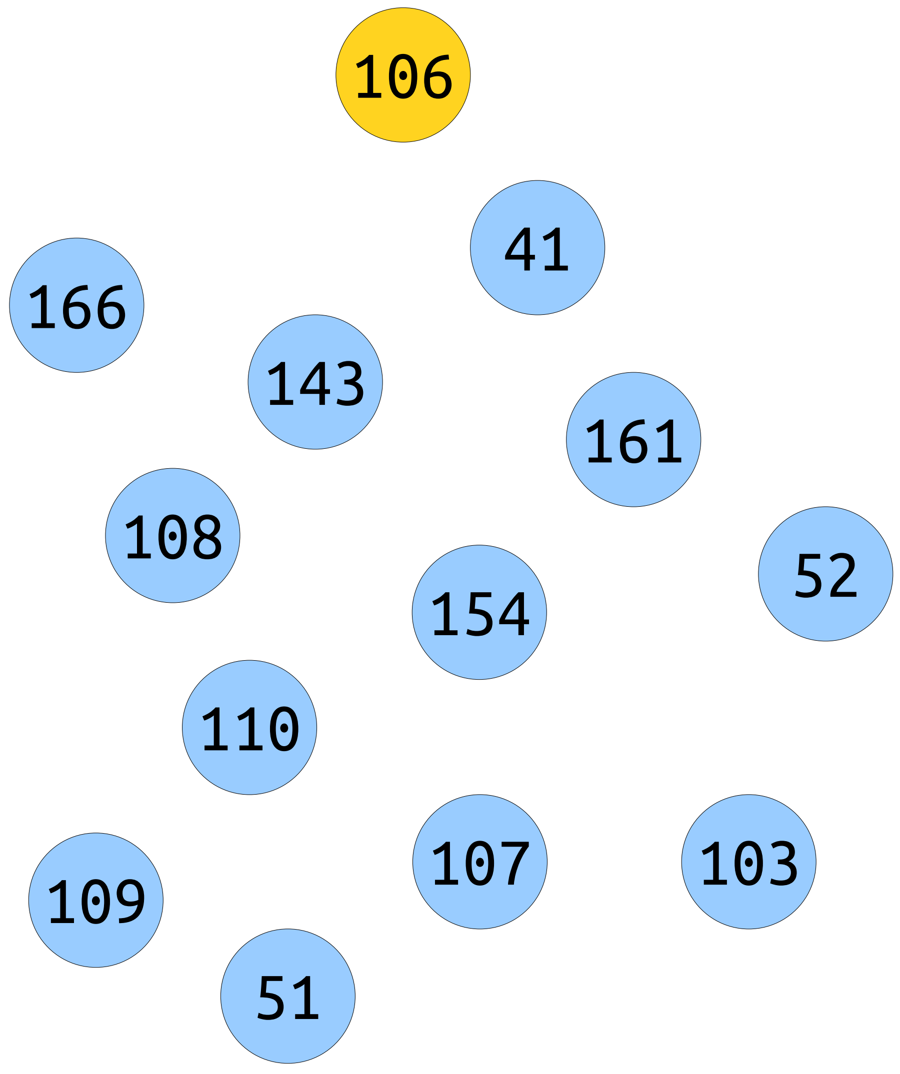
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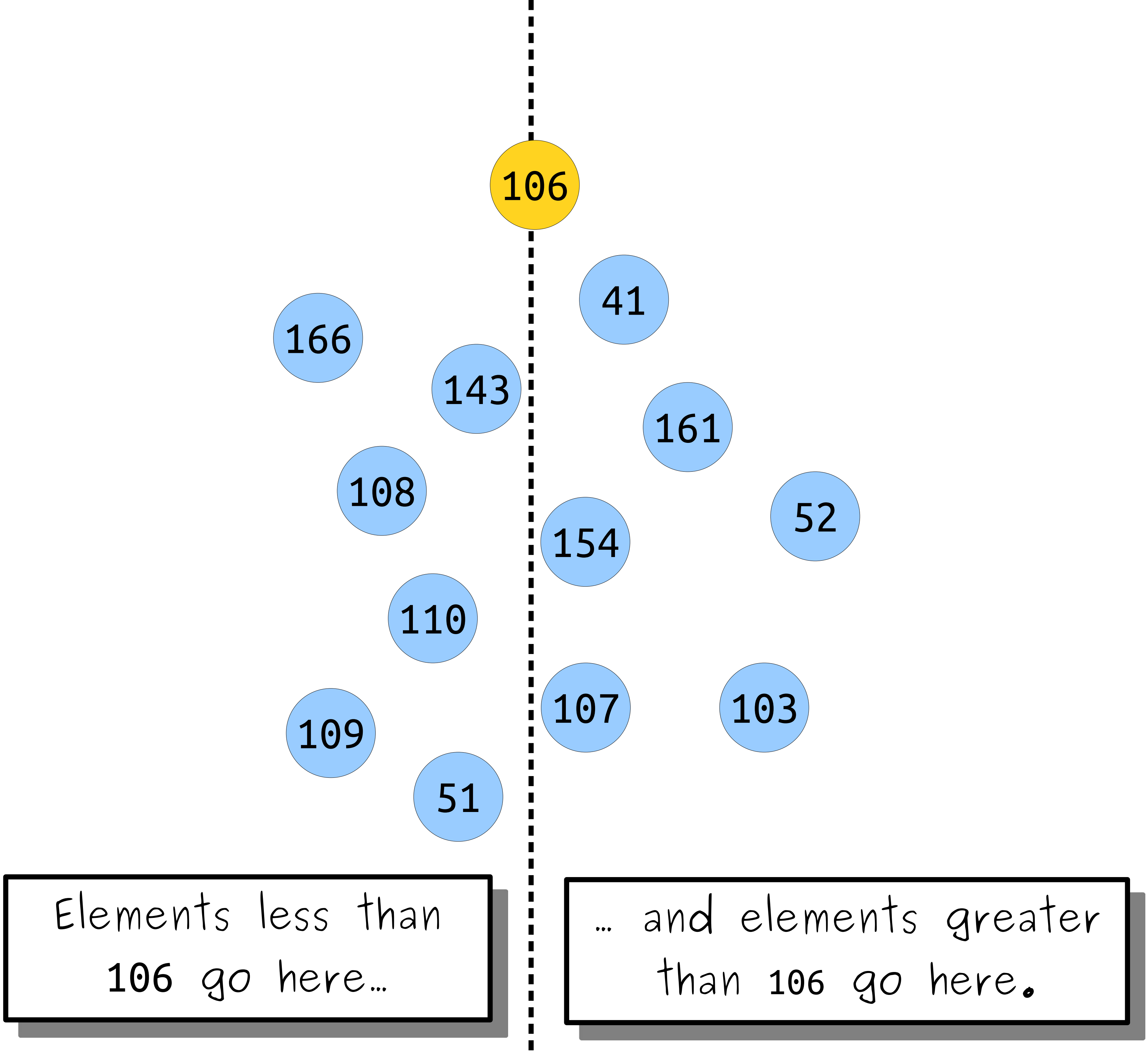
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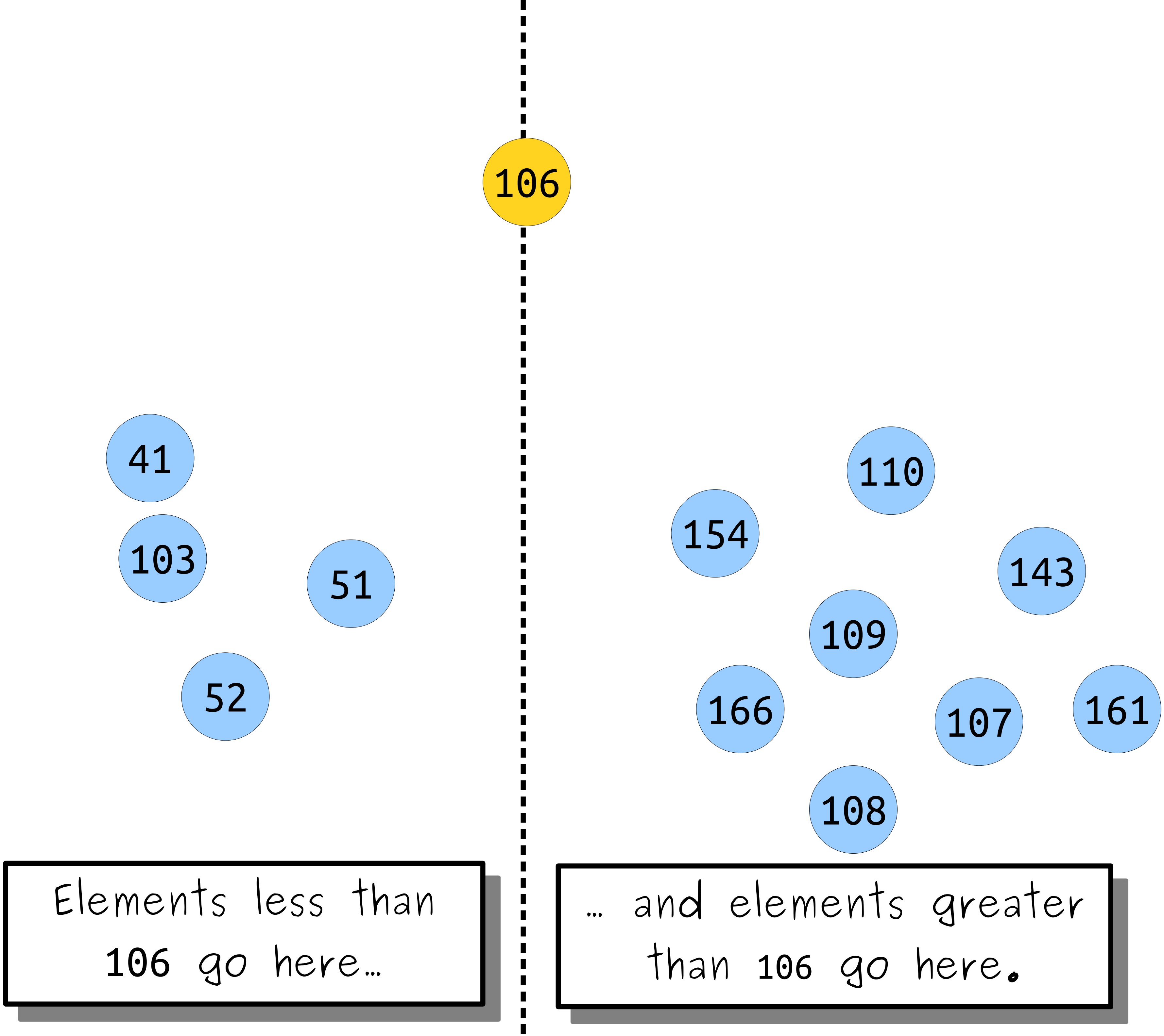
Elements less than 106 go here...
... and elements greater than 106 go here.



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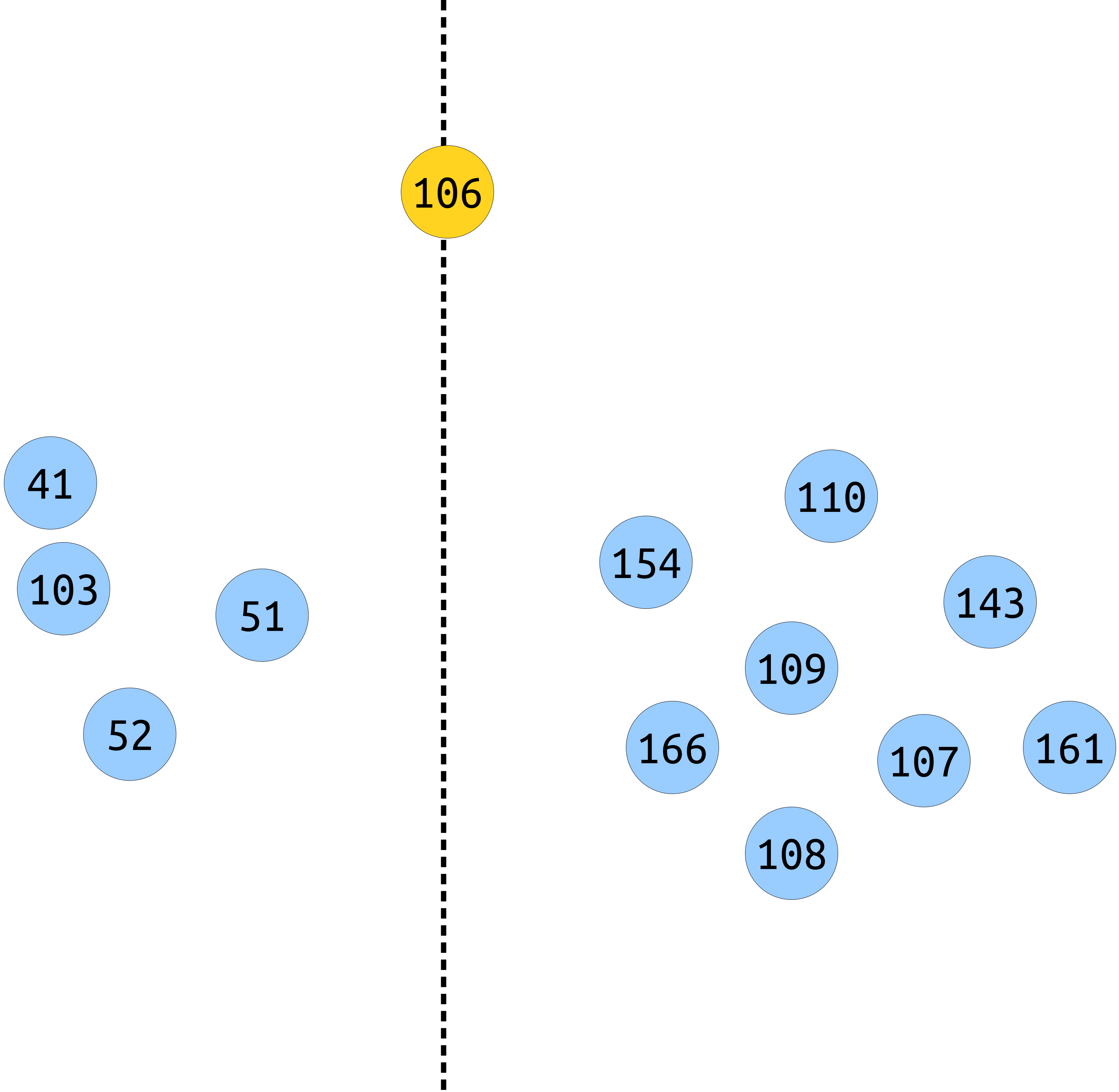
Elements less than 106 go here…
… and elements greater than 106 go here.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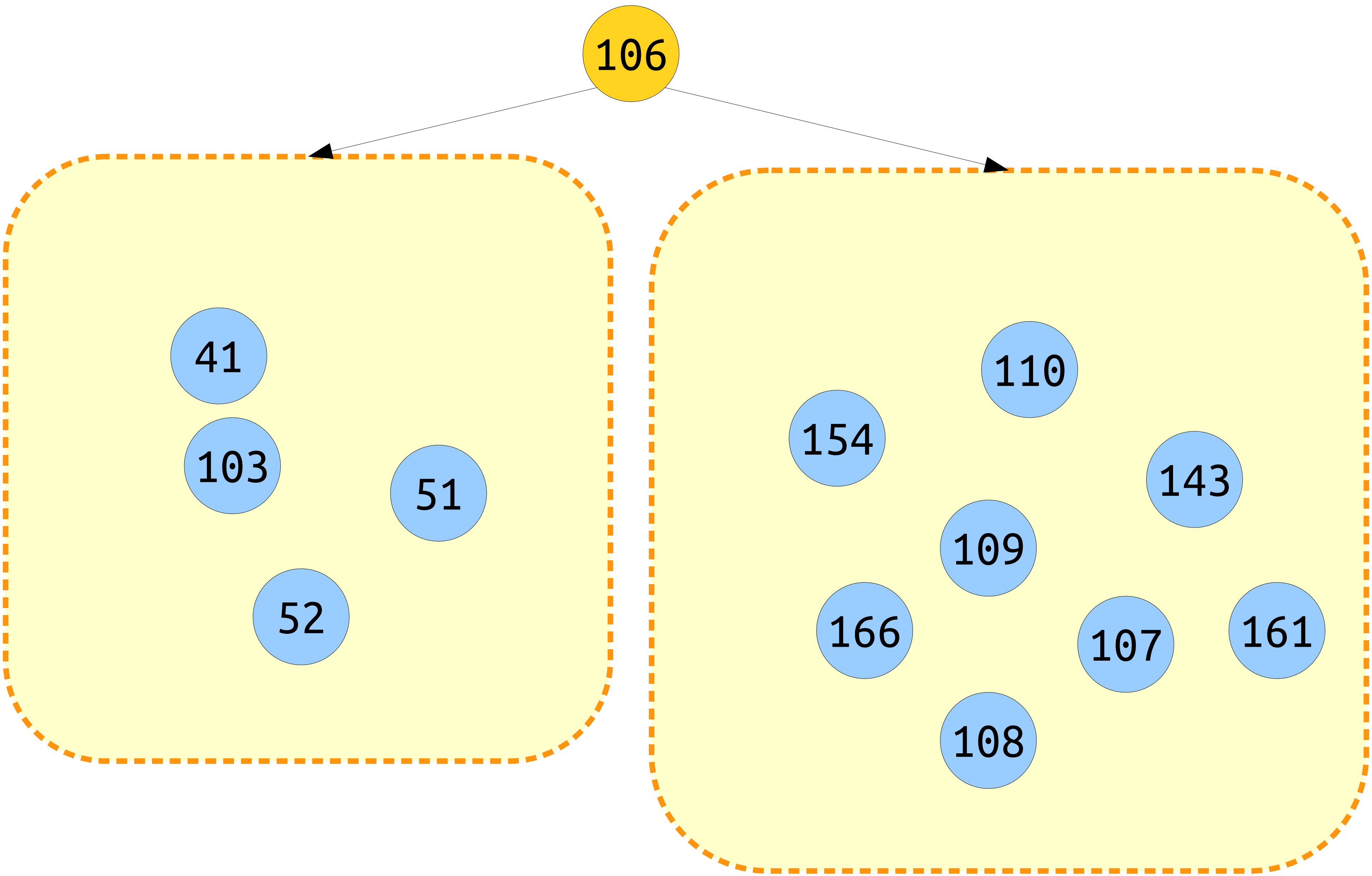
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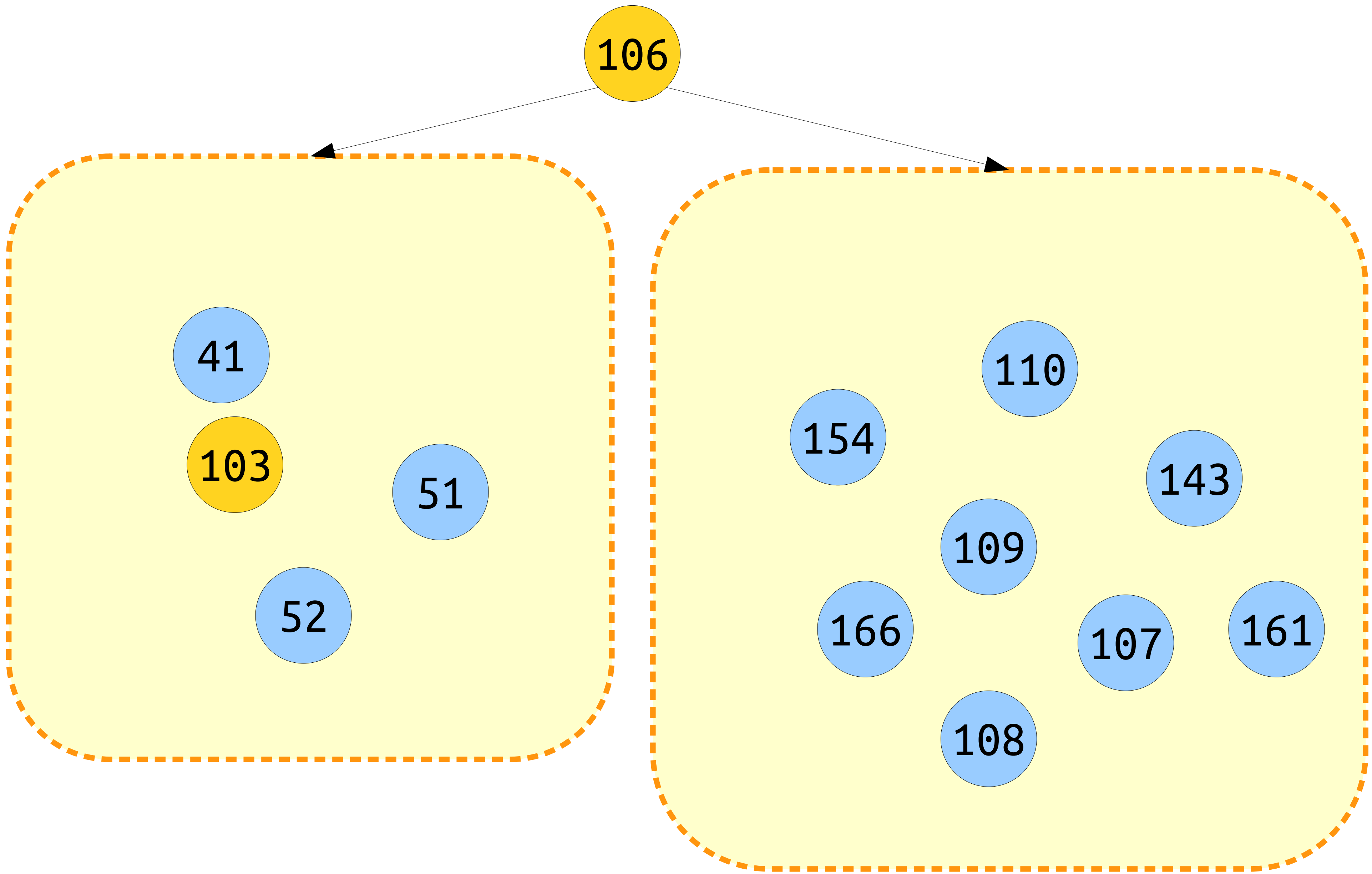
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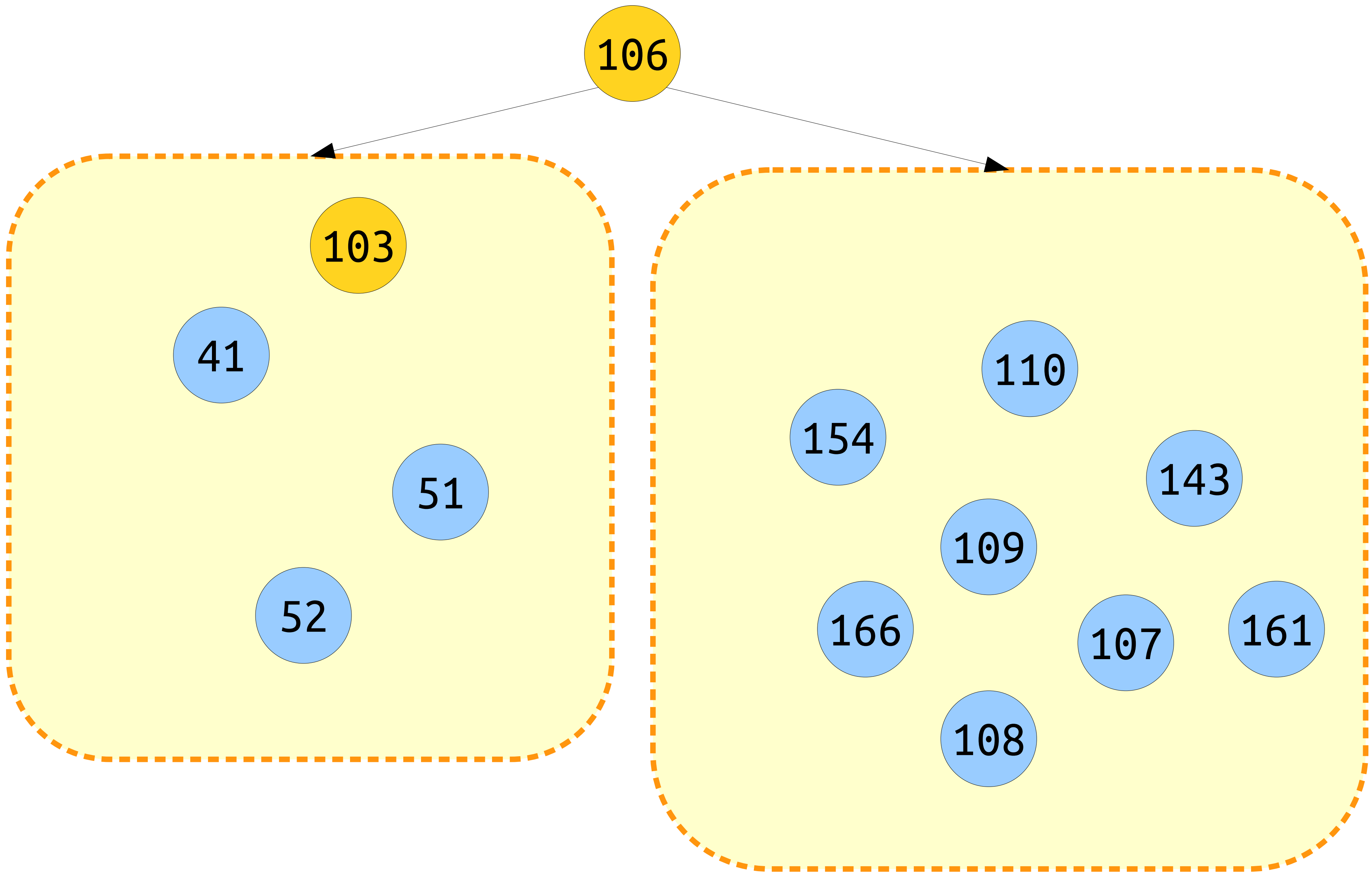
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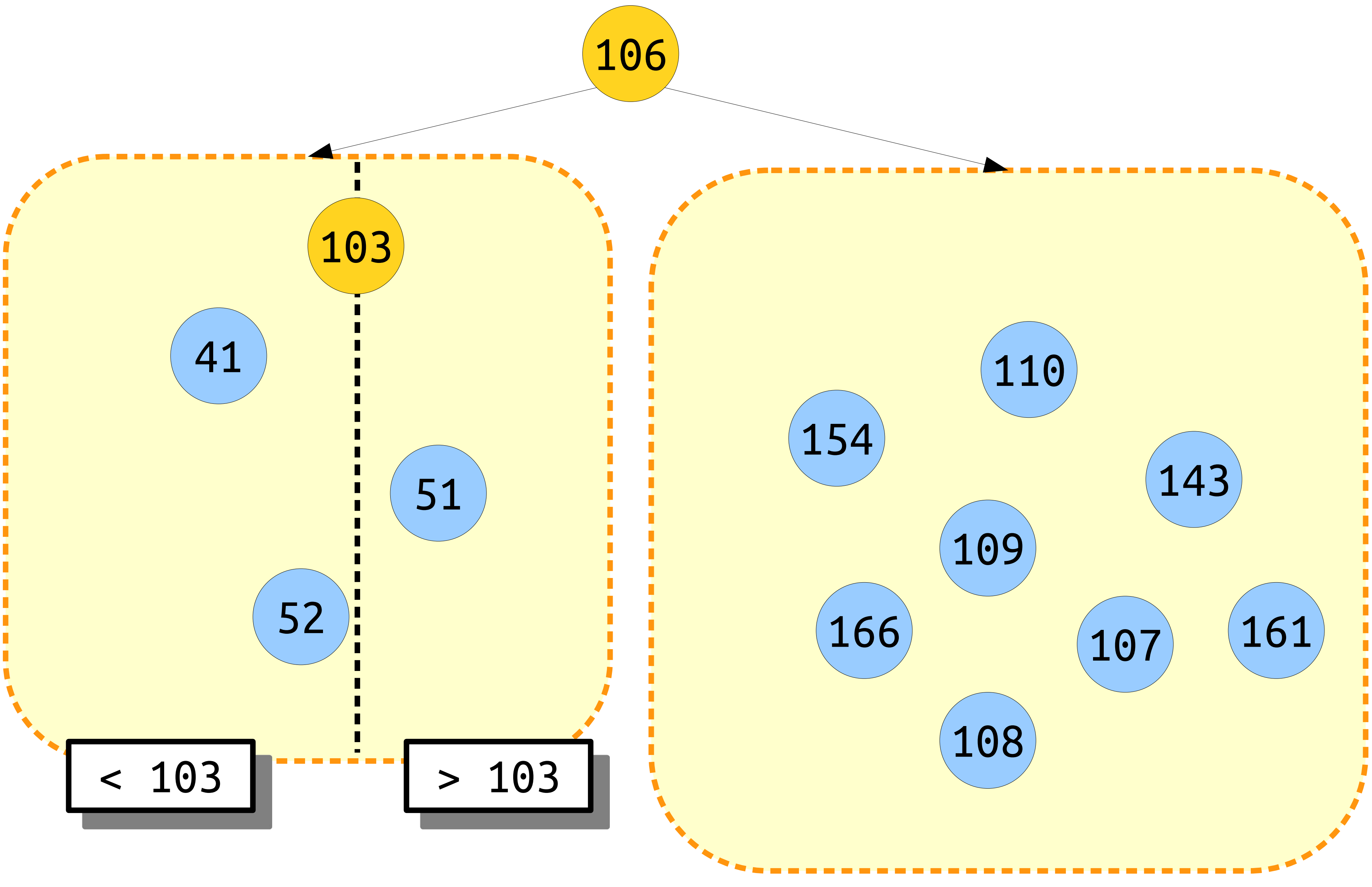
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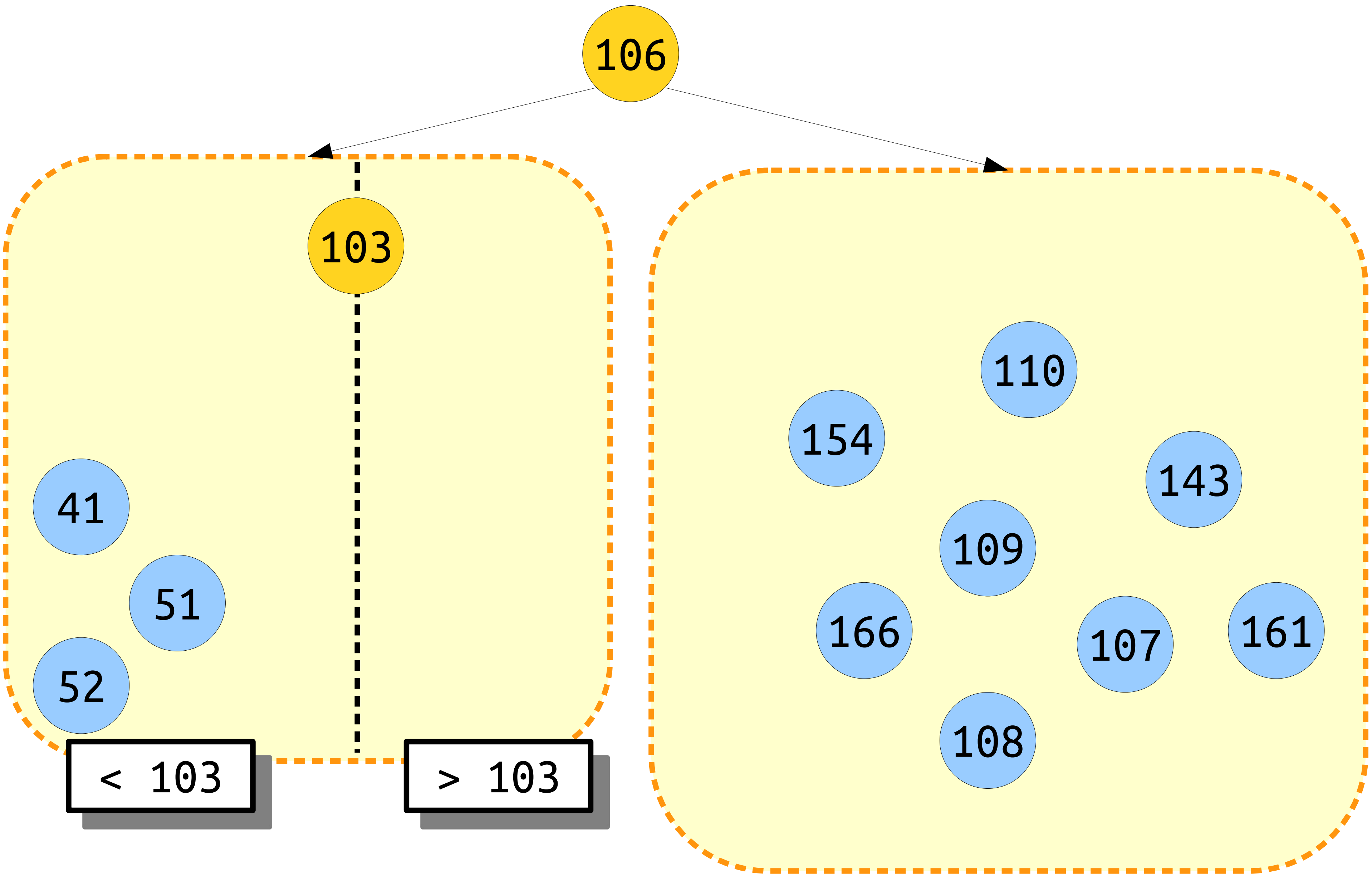
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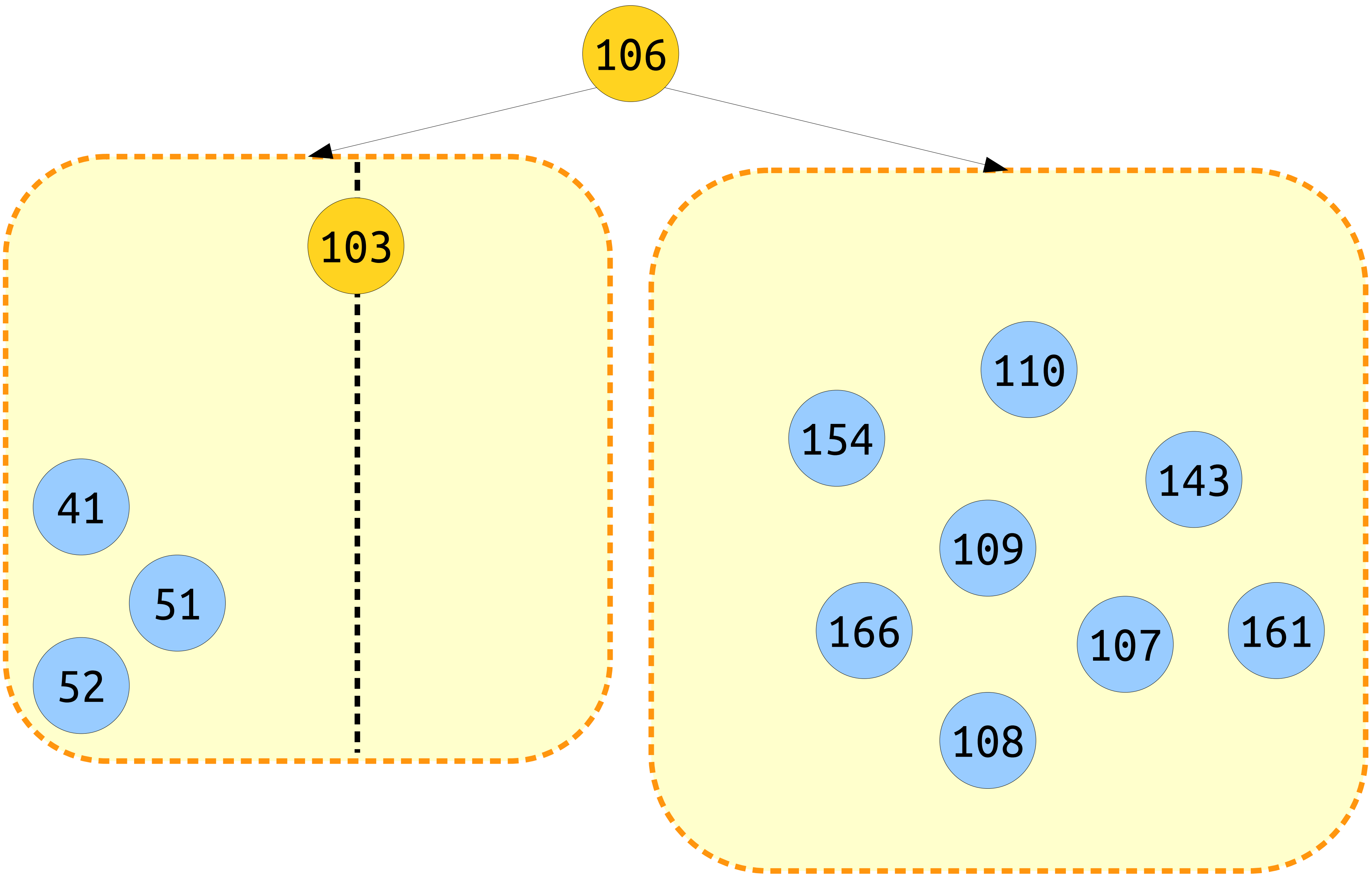
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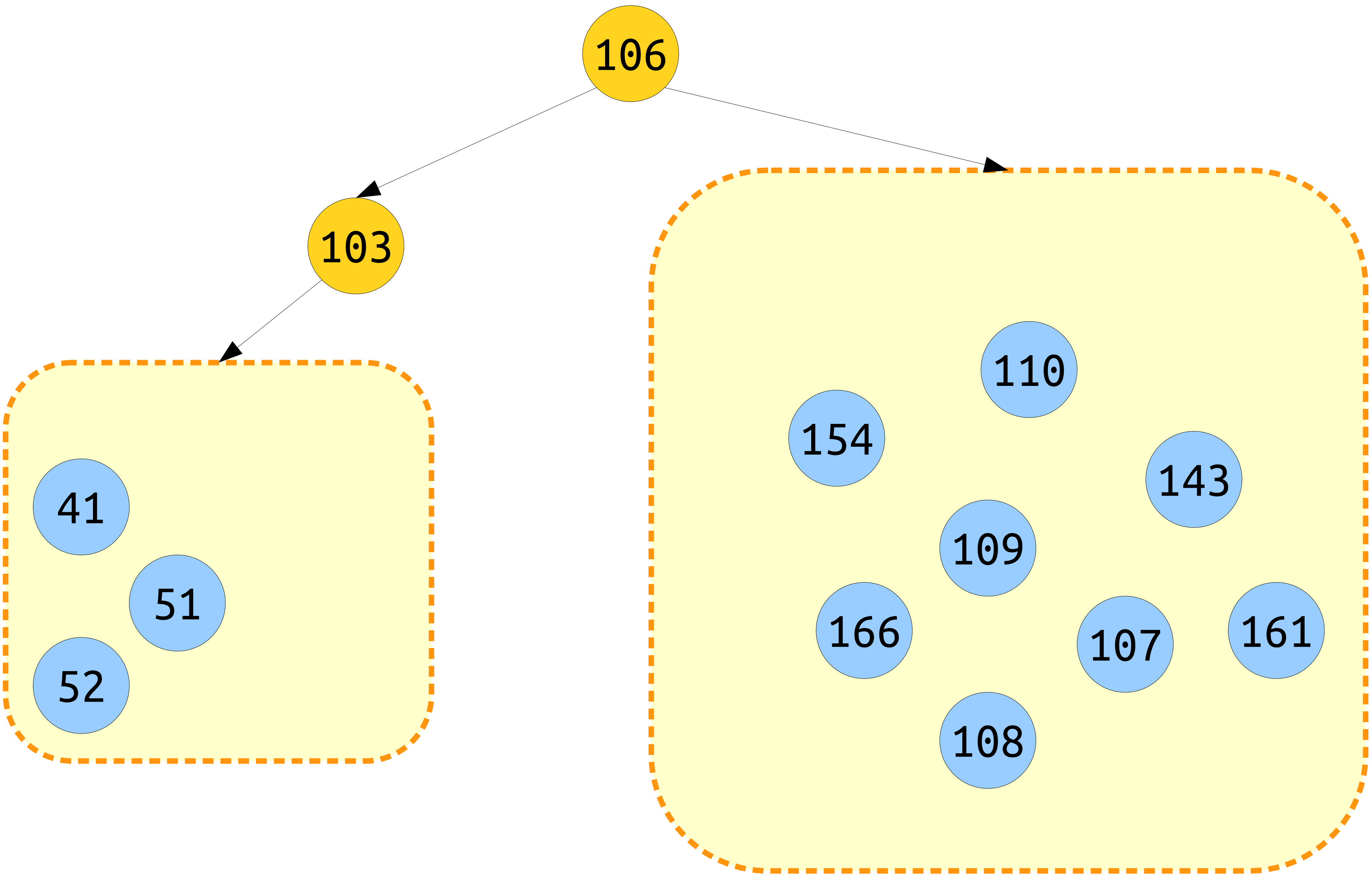

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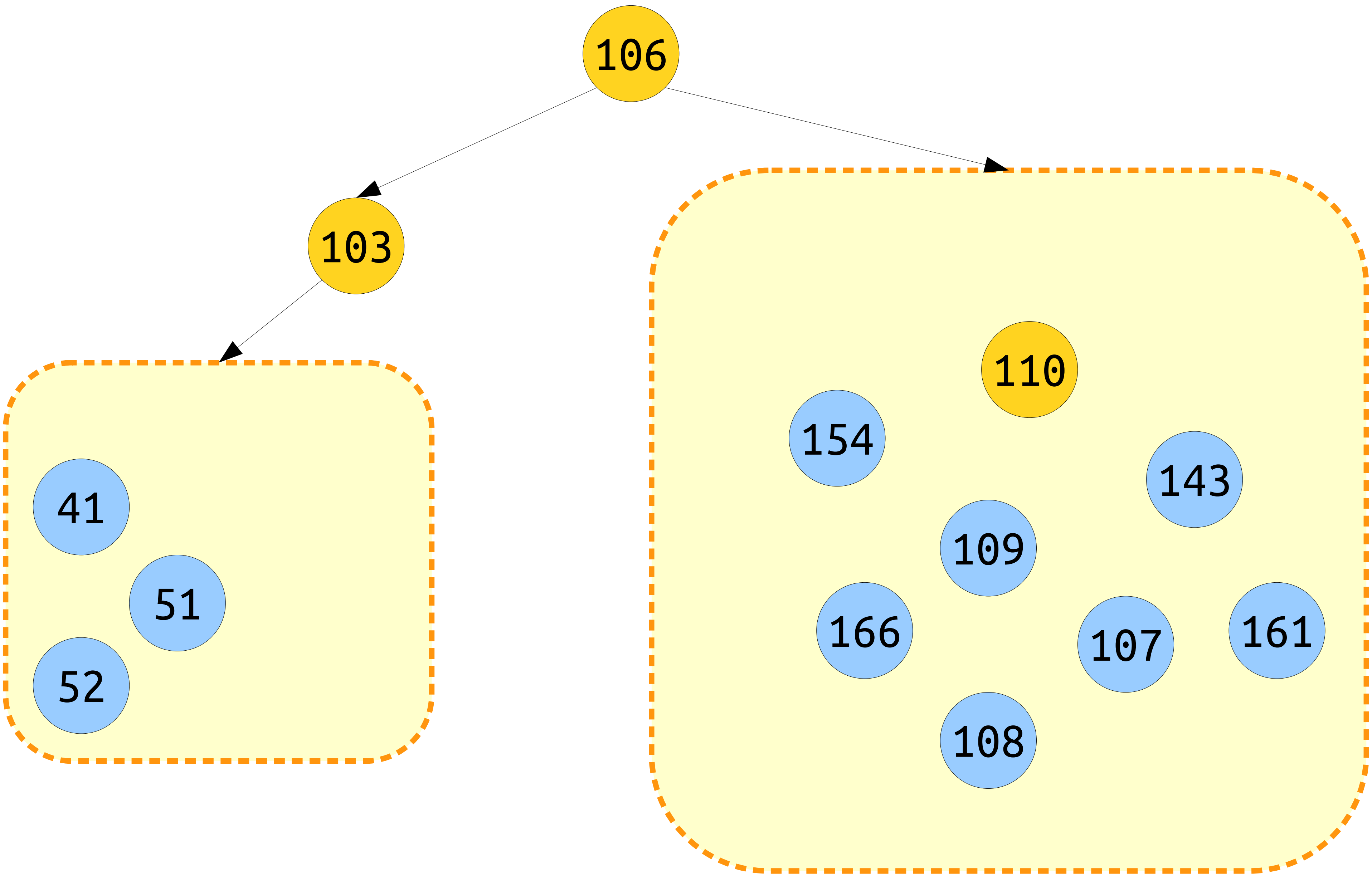
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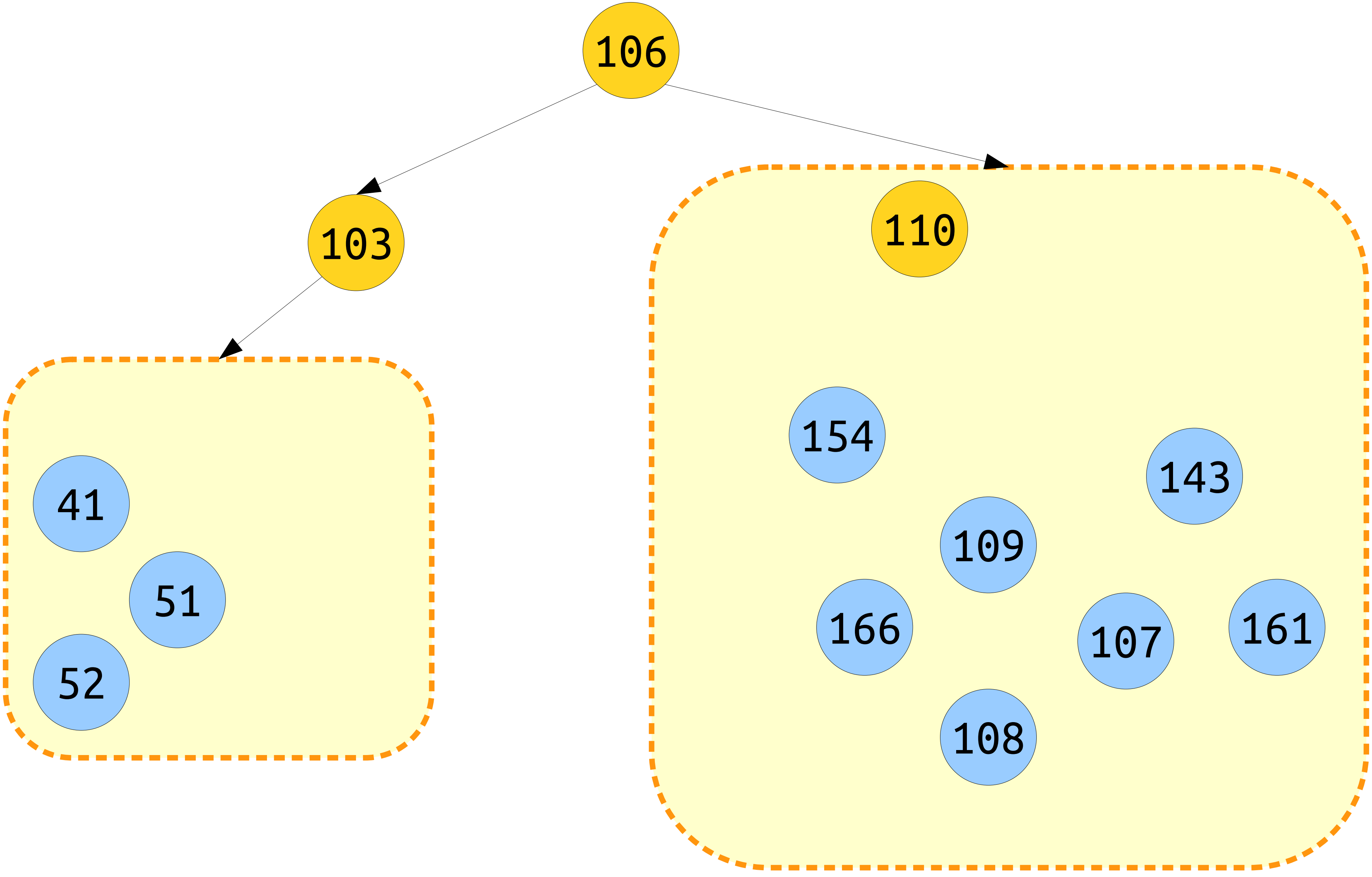
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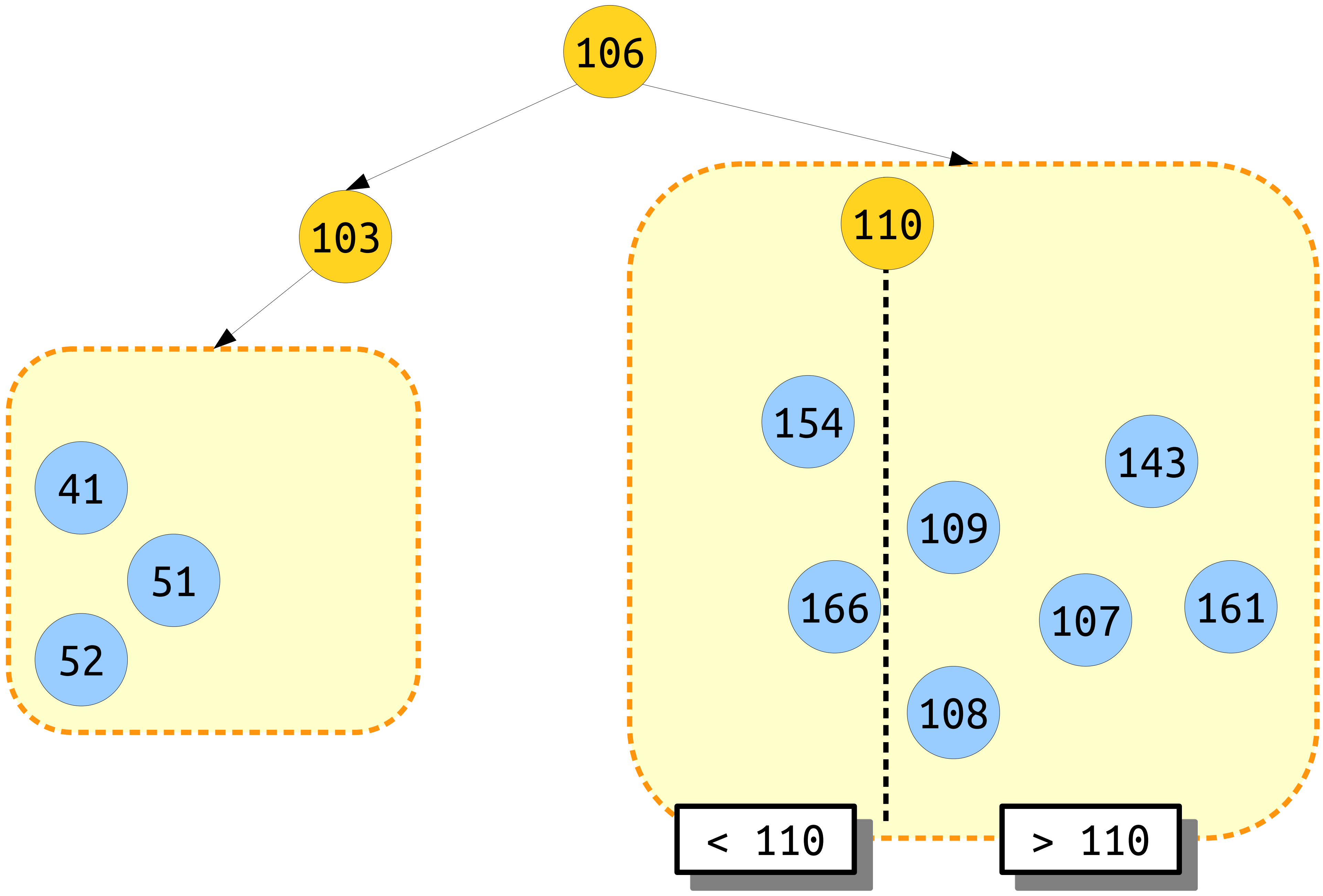
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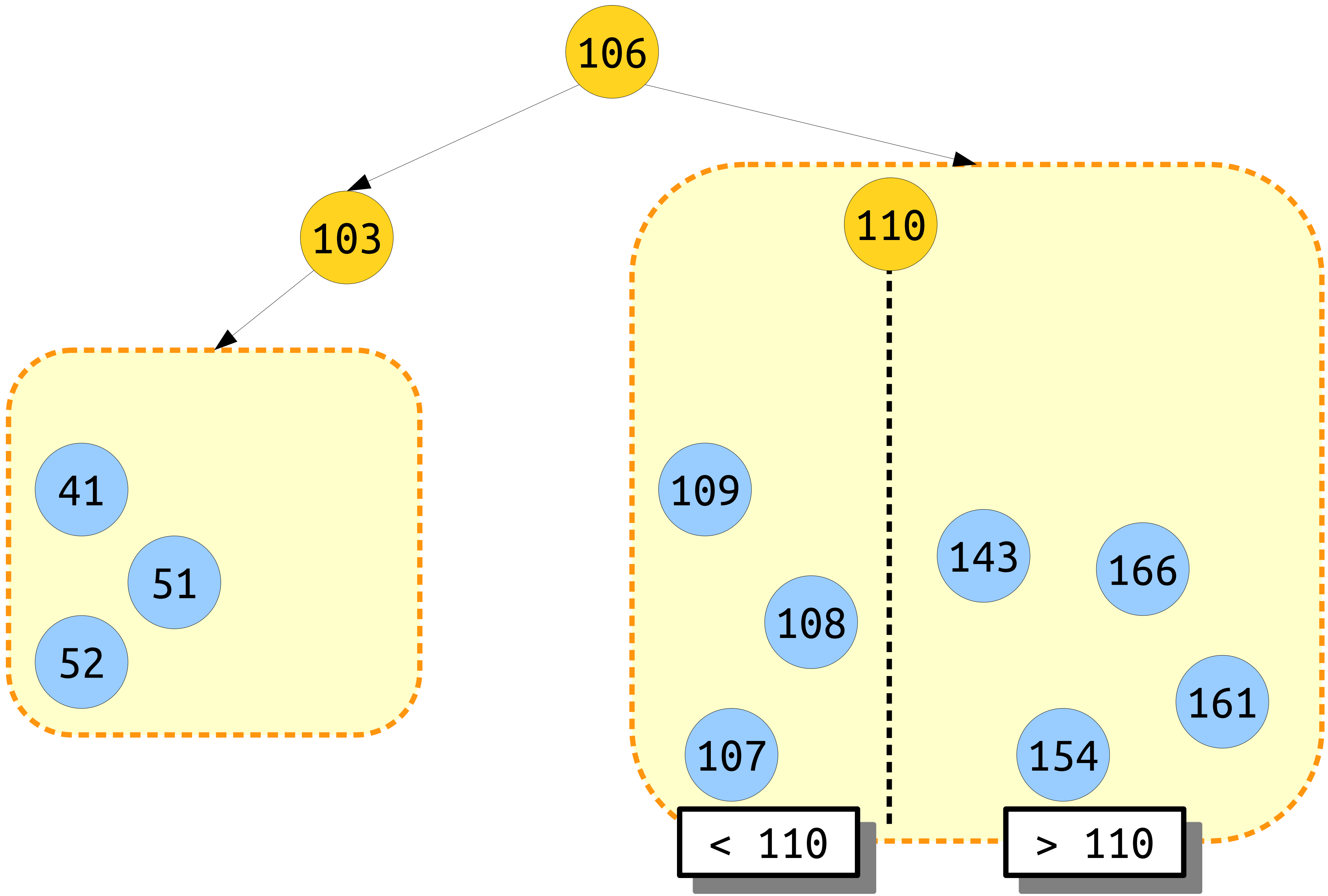
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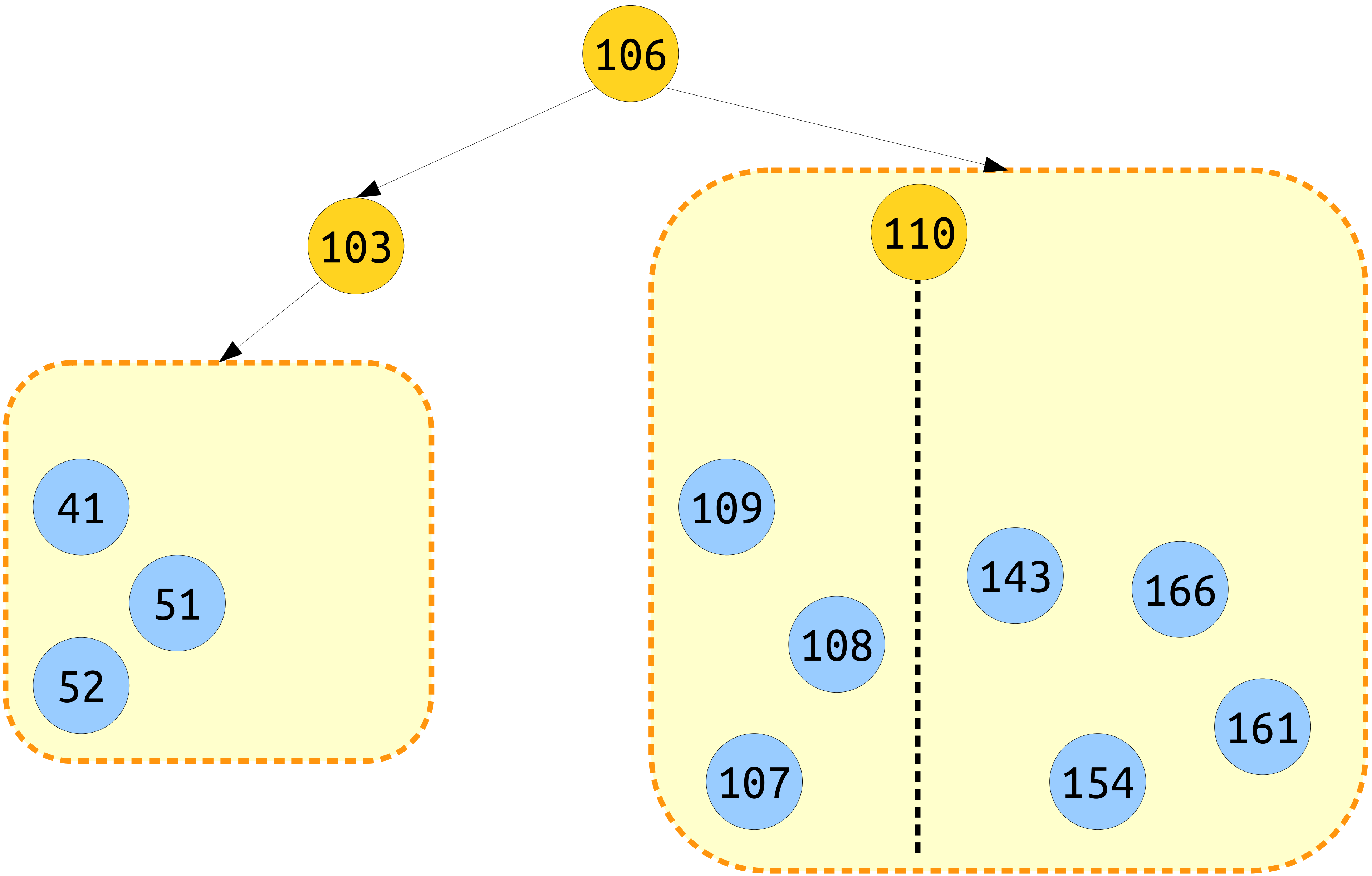
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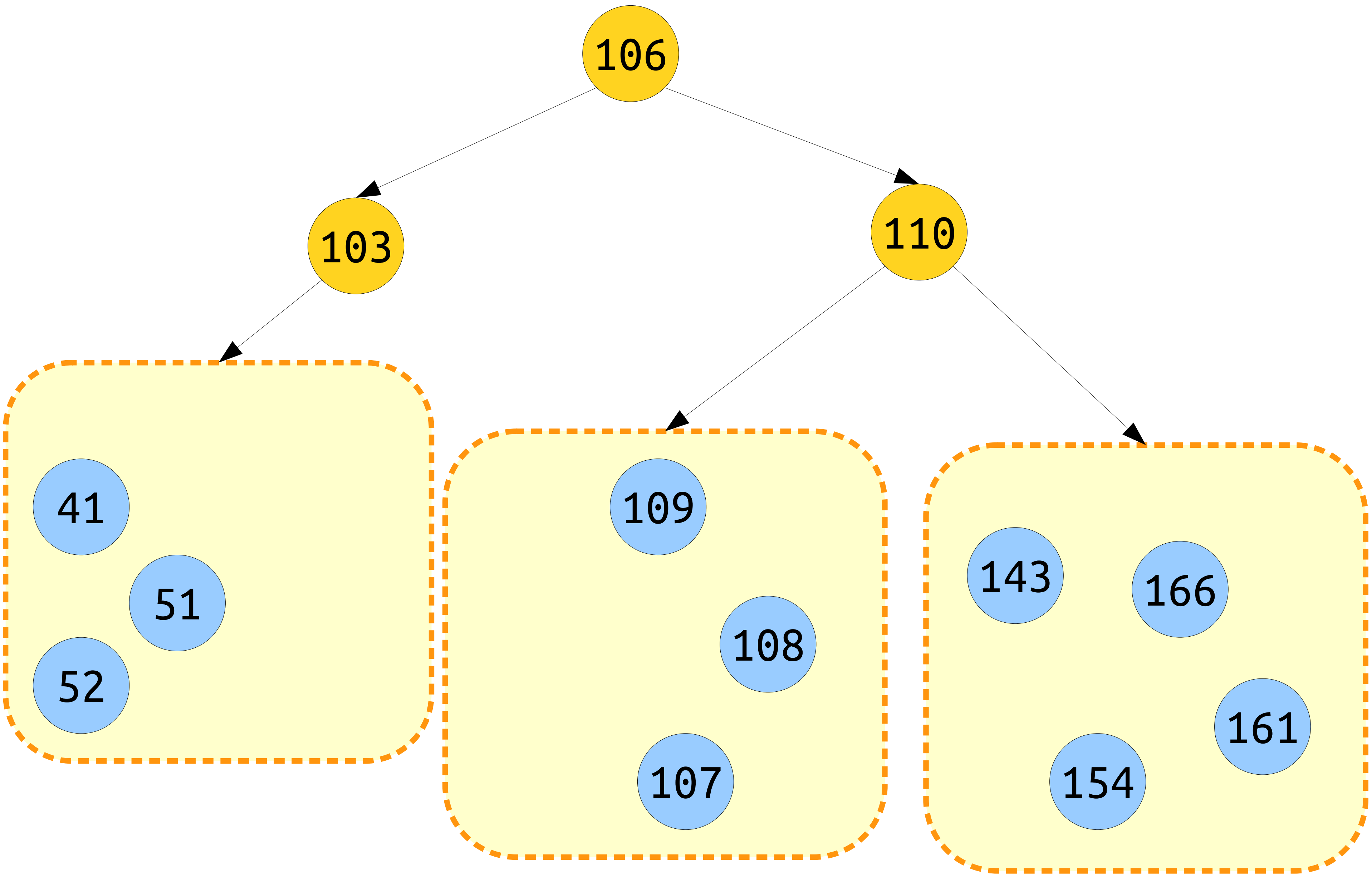
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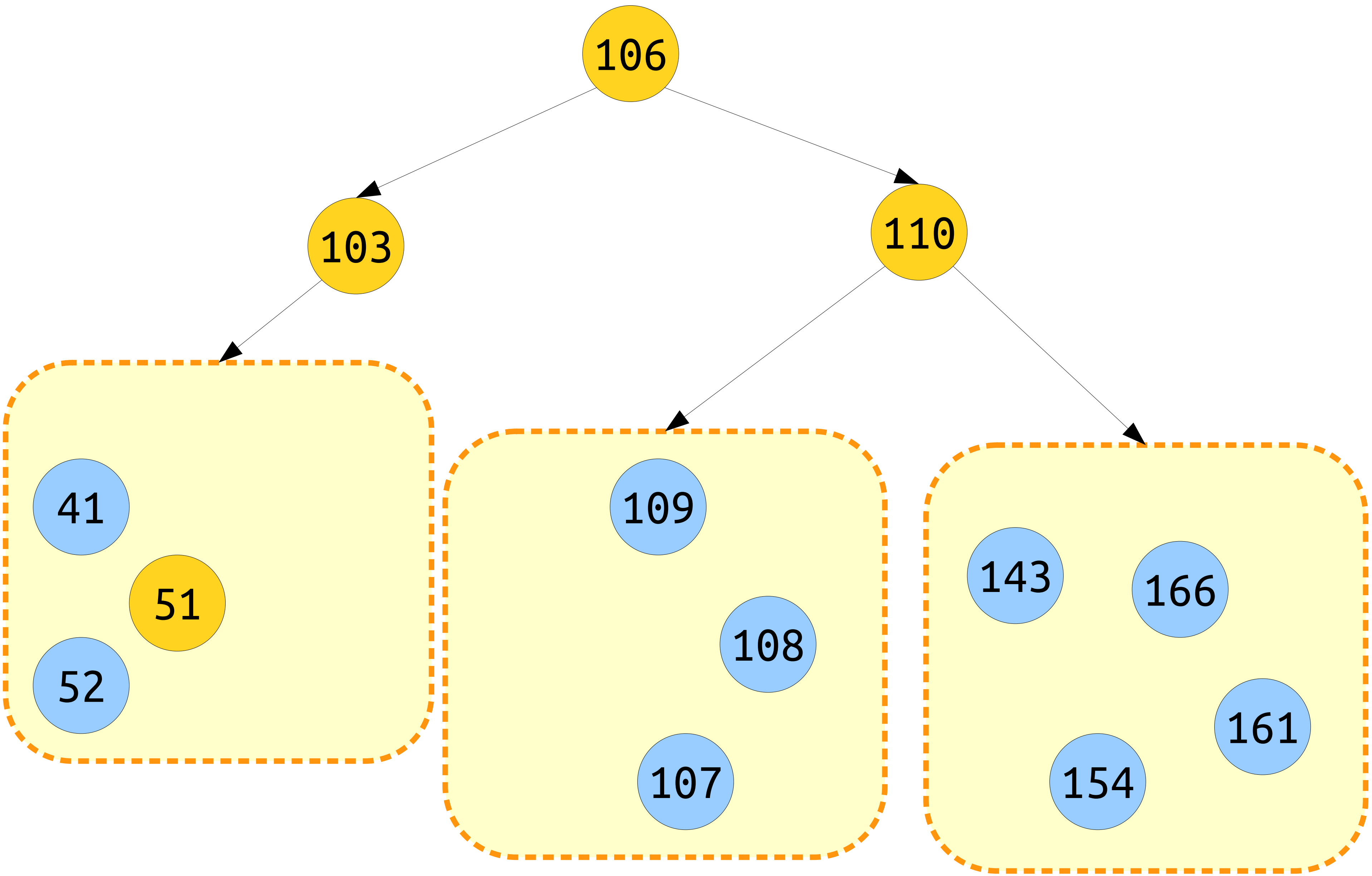
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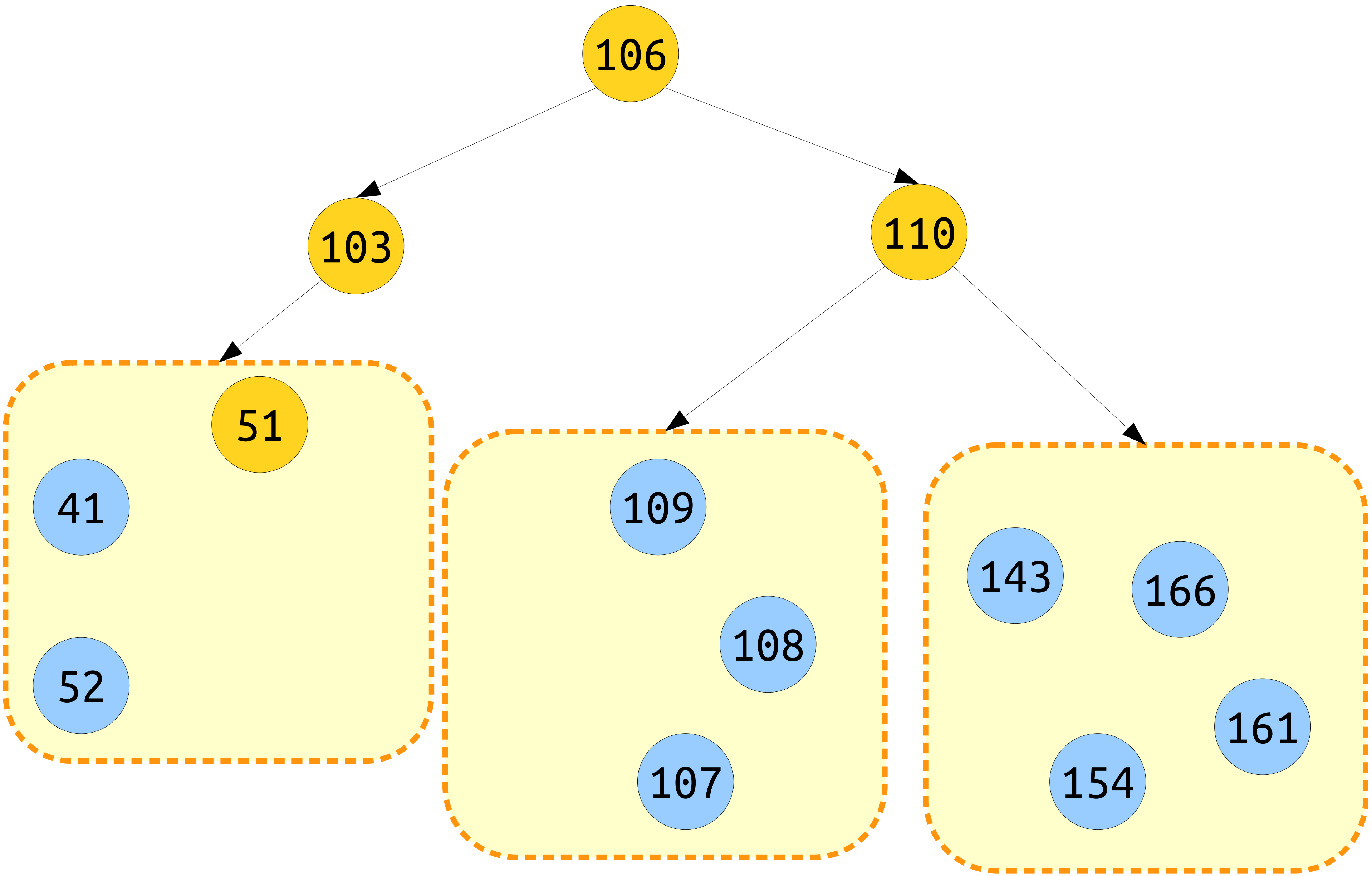
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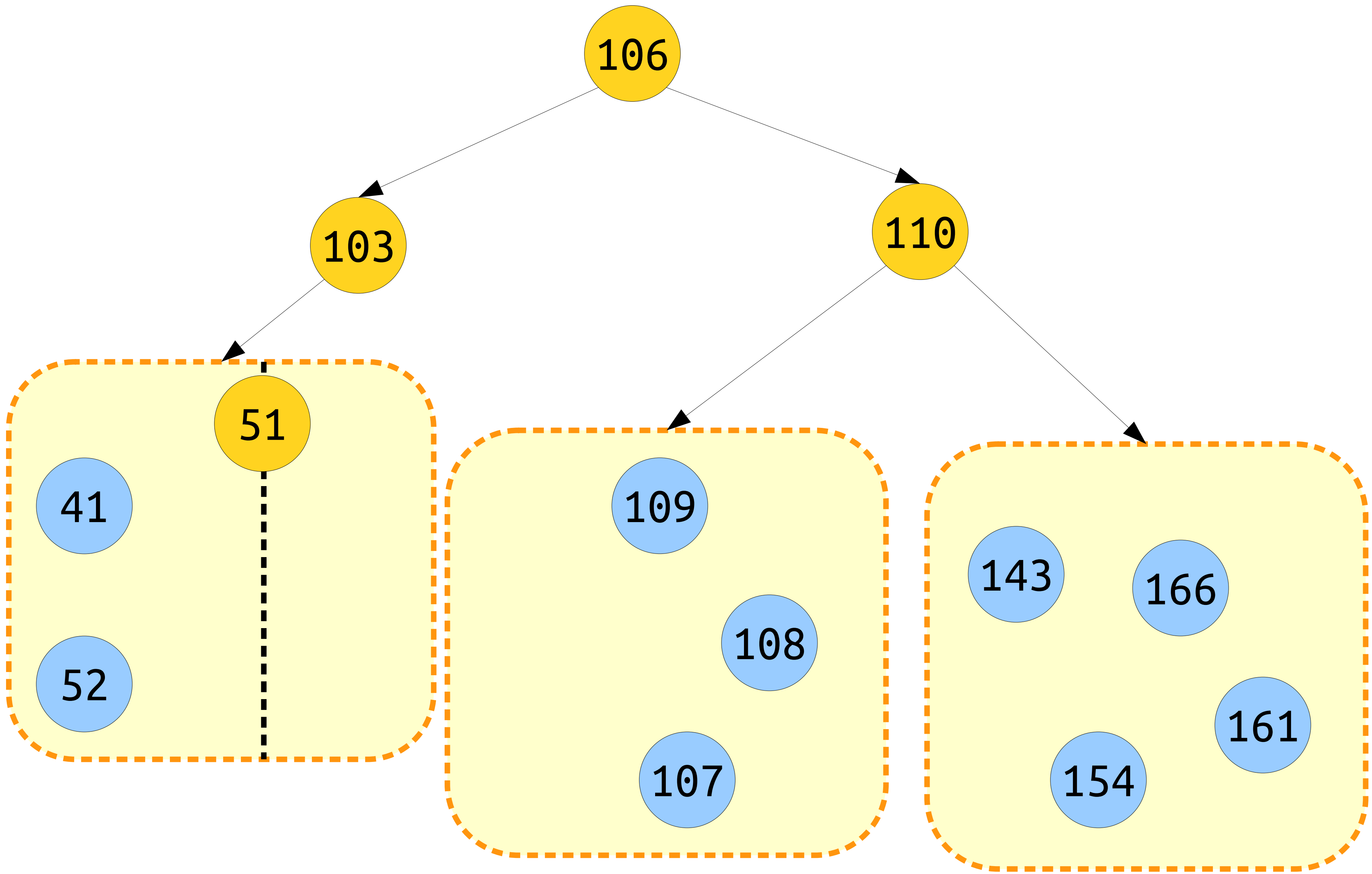
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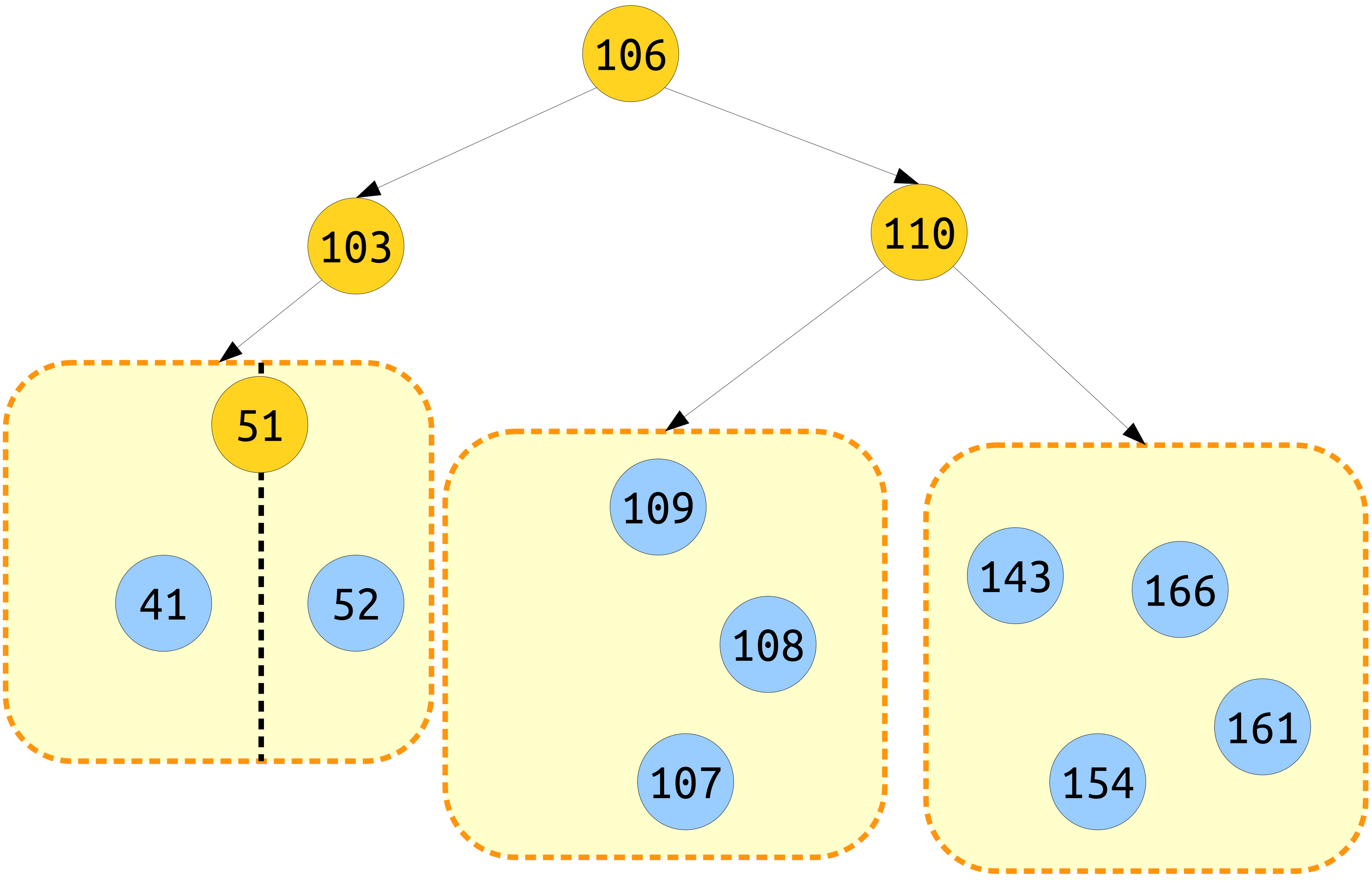
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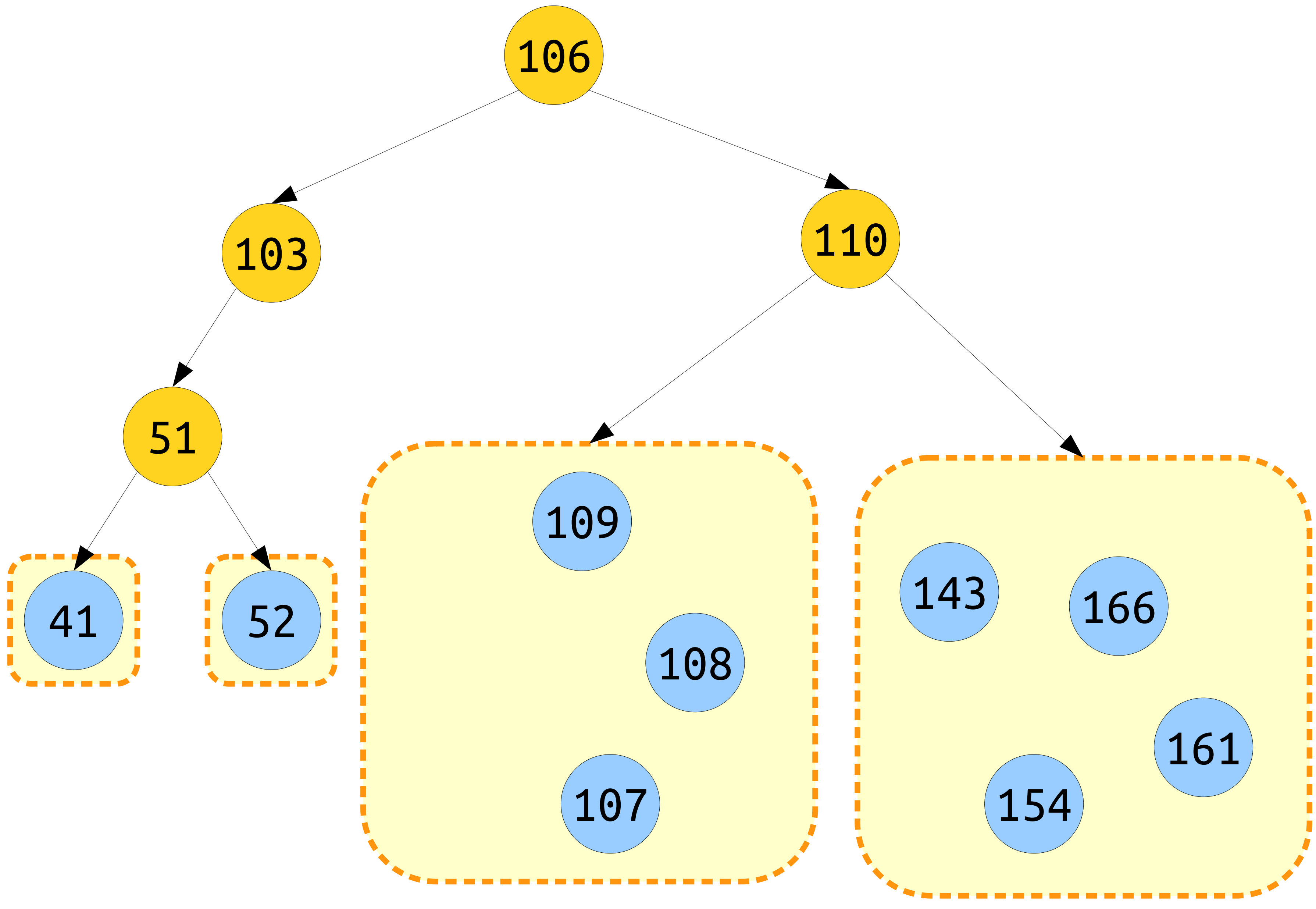
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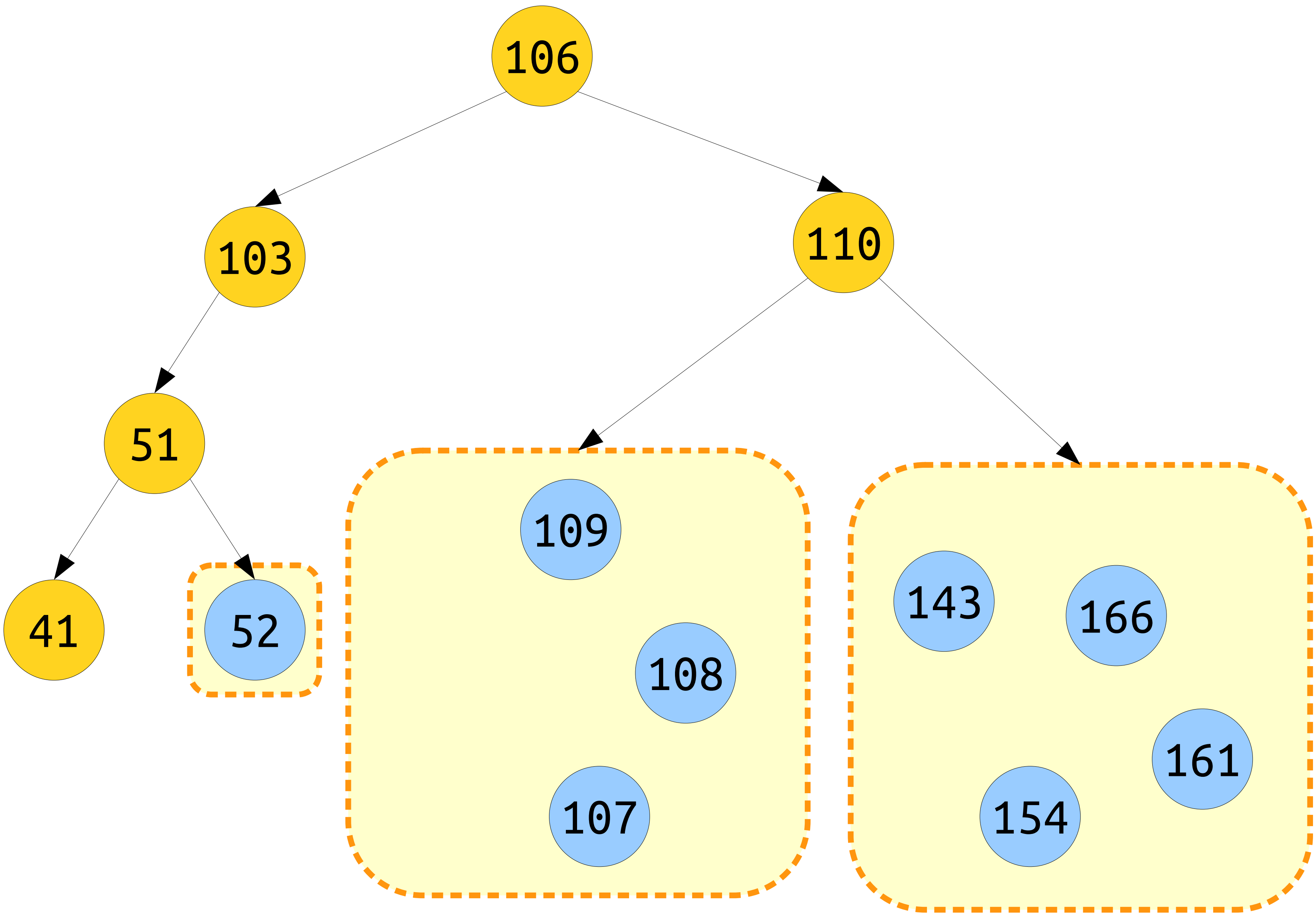
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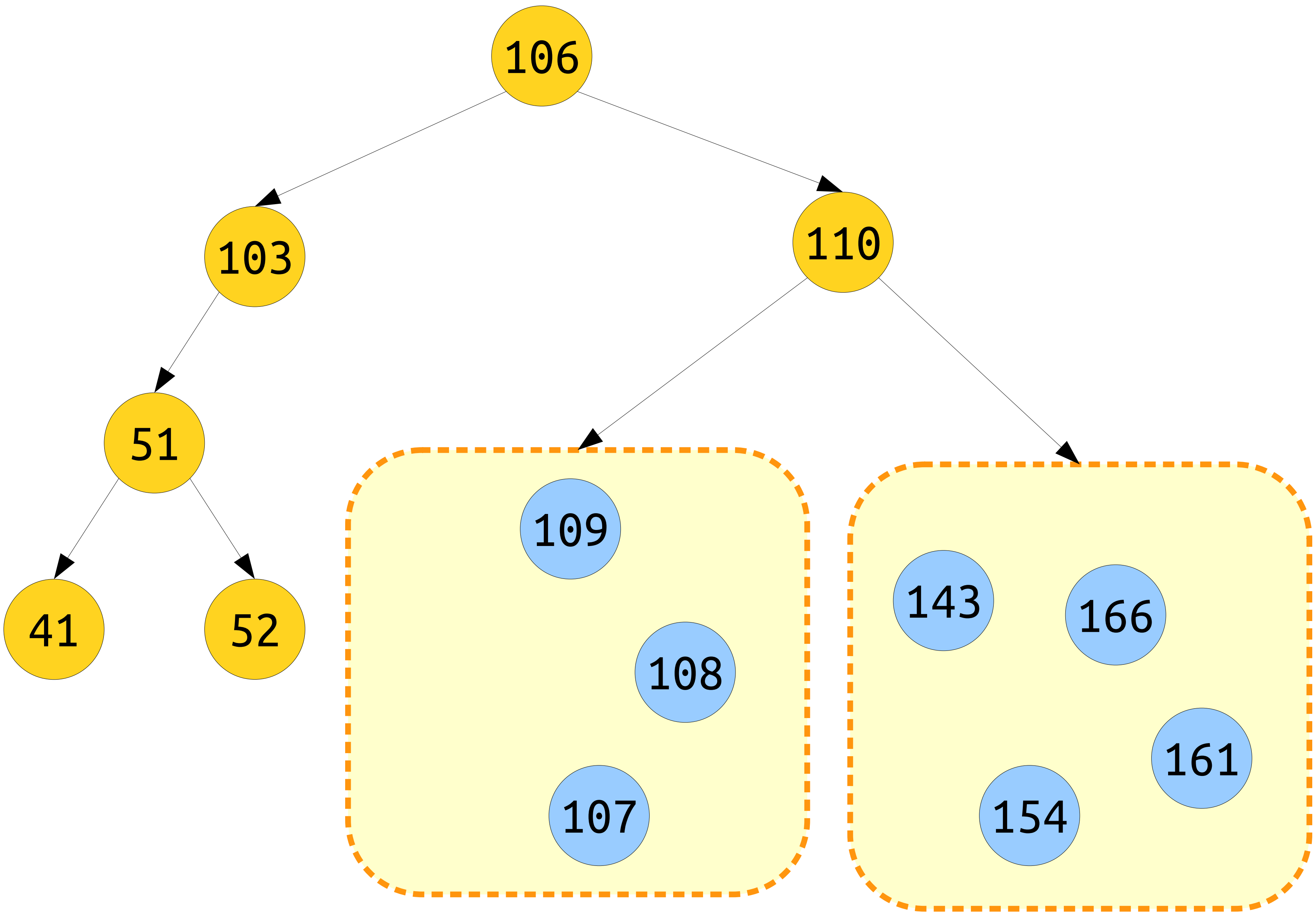
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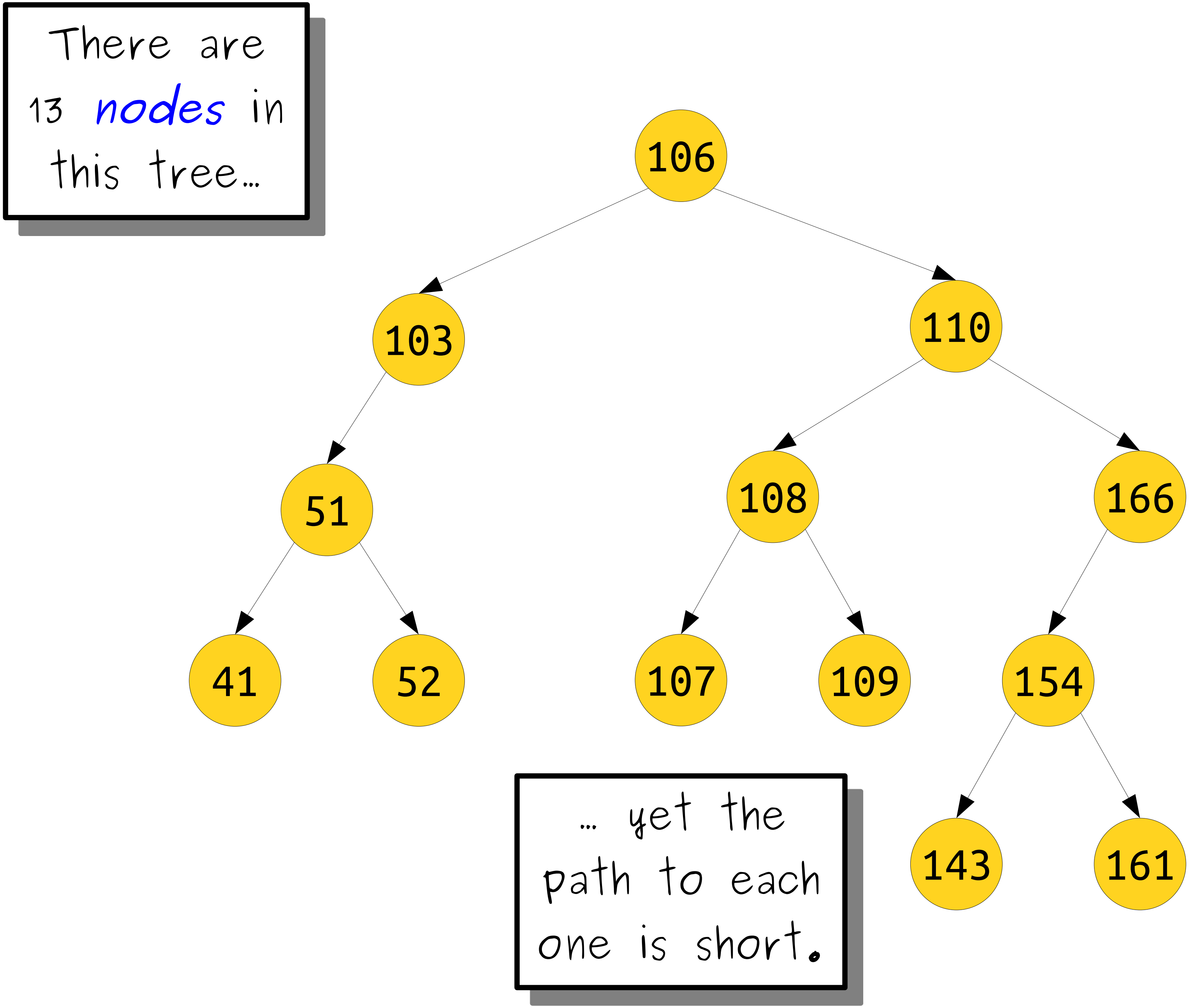
There are 13 *nodes* in this tree...
106
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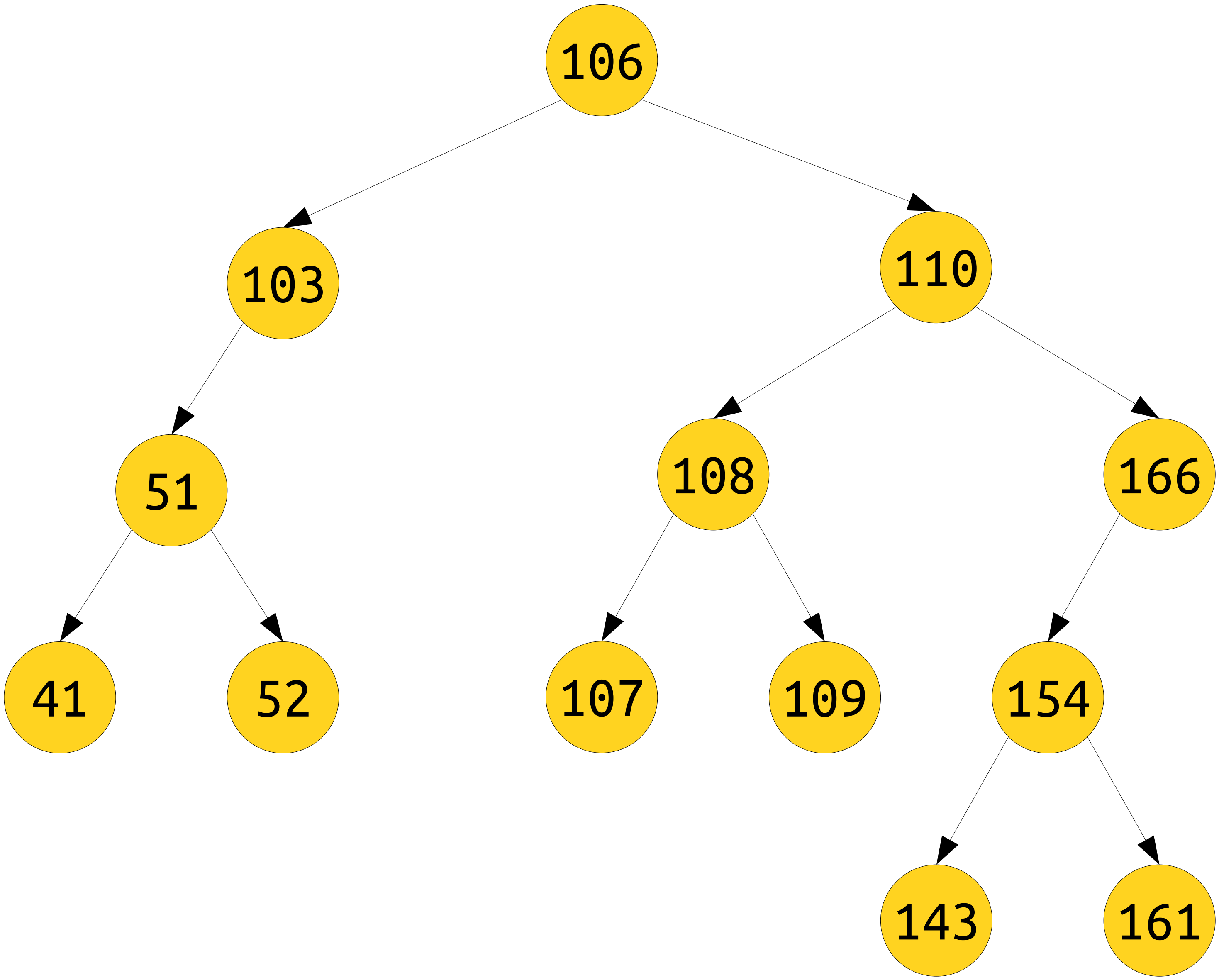
... yet the path to each one is short.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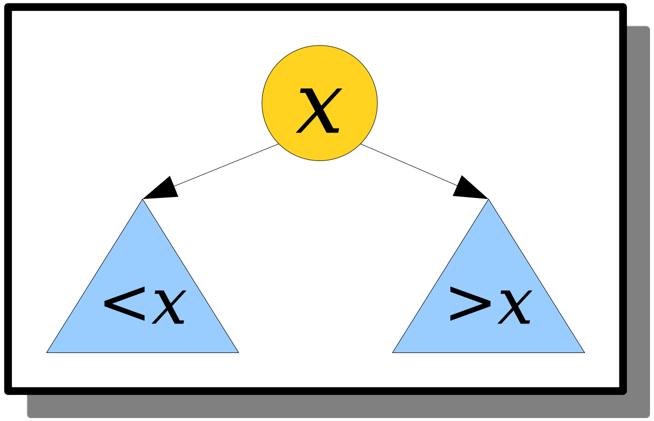
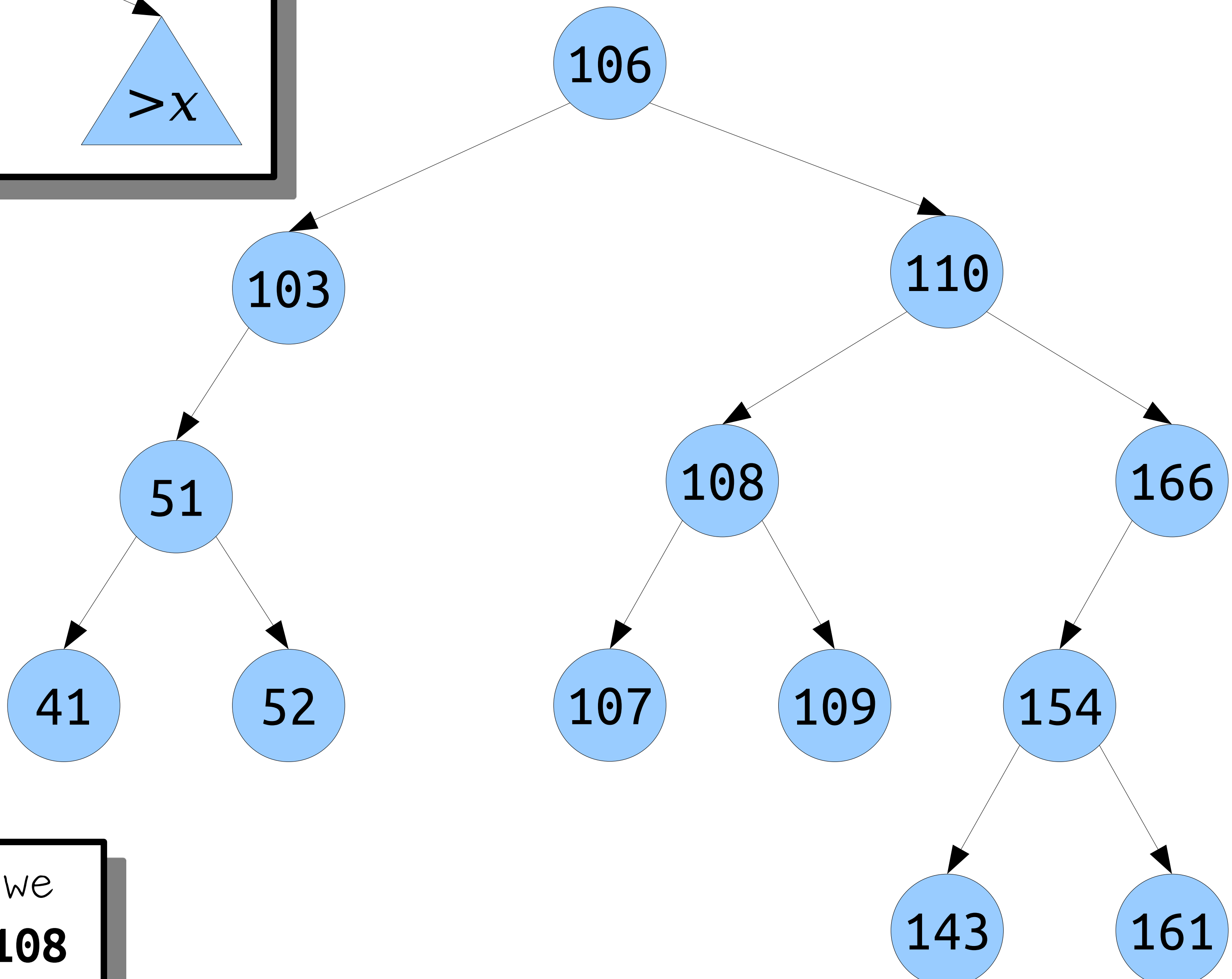
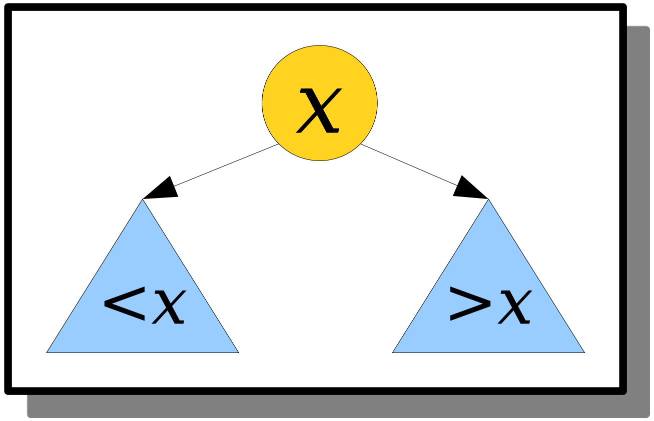
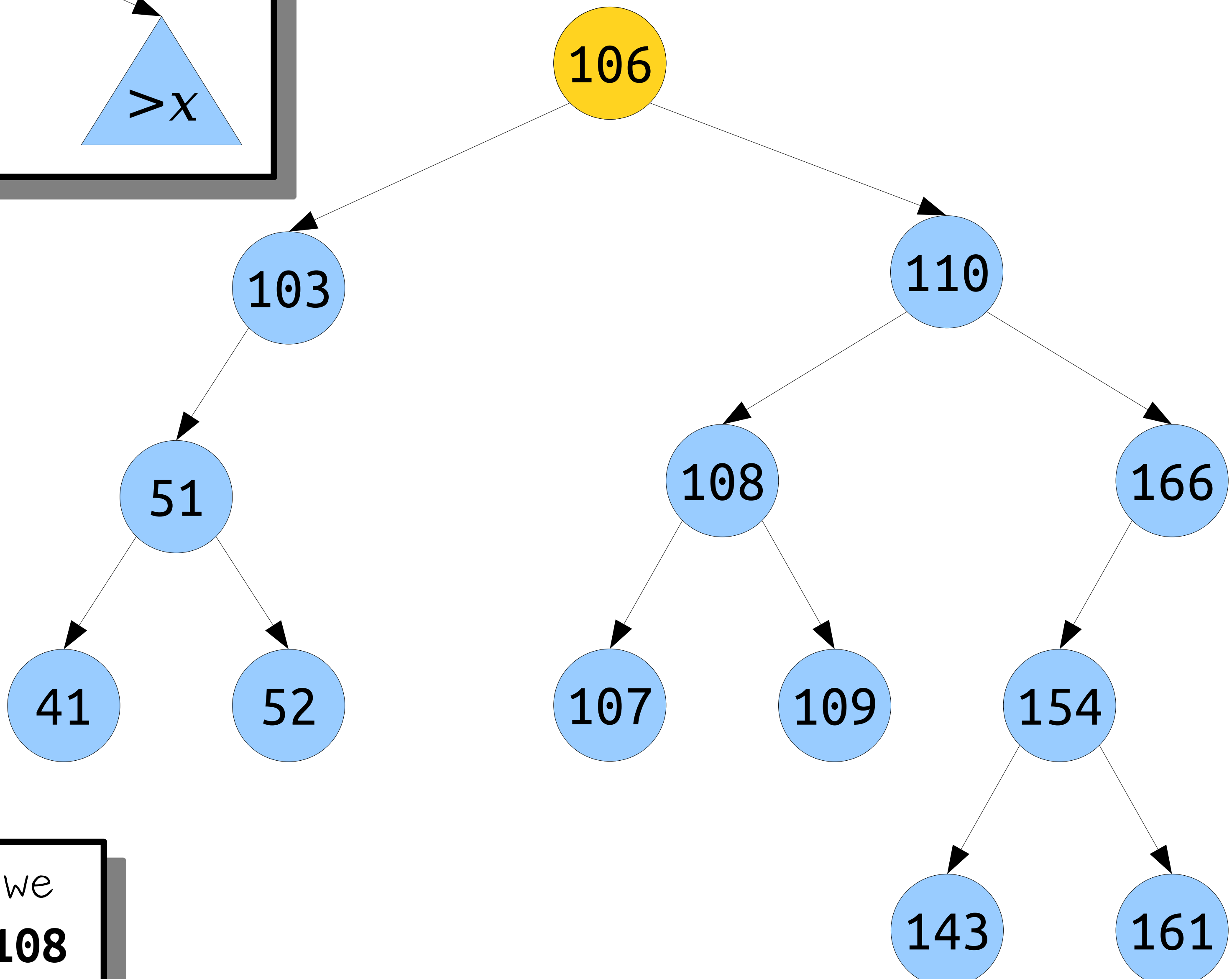




How can we check if **108** is in this tree?





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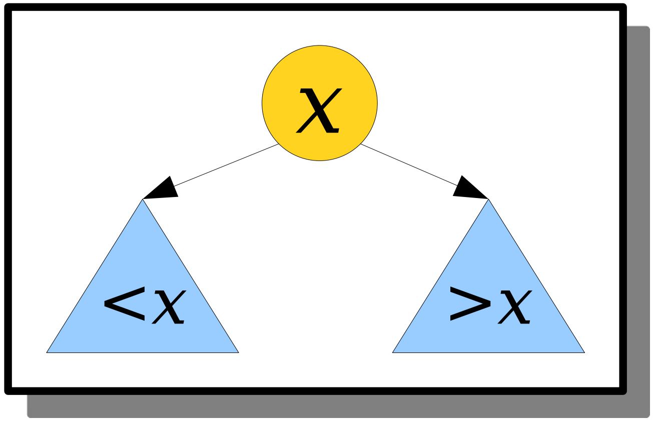

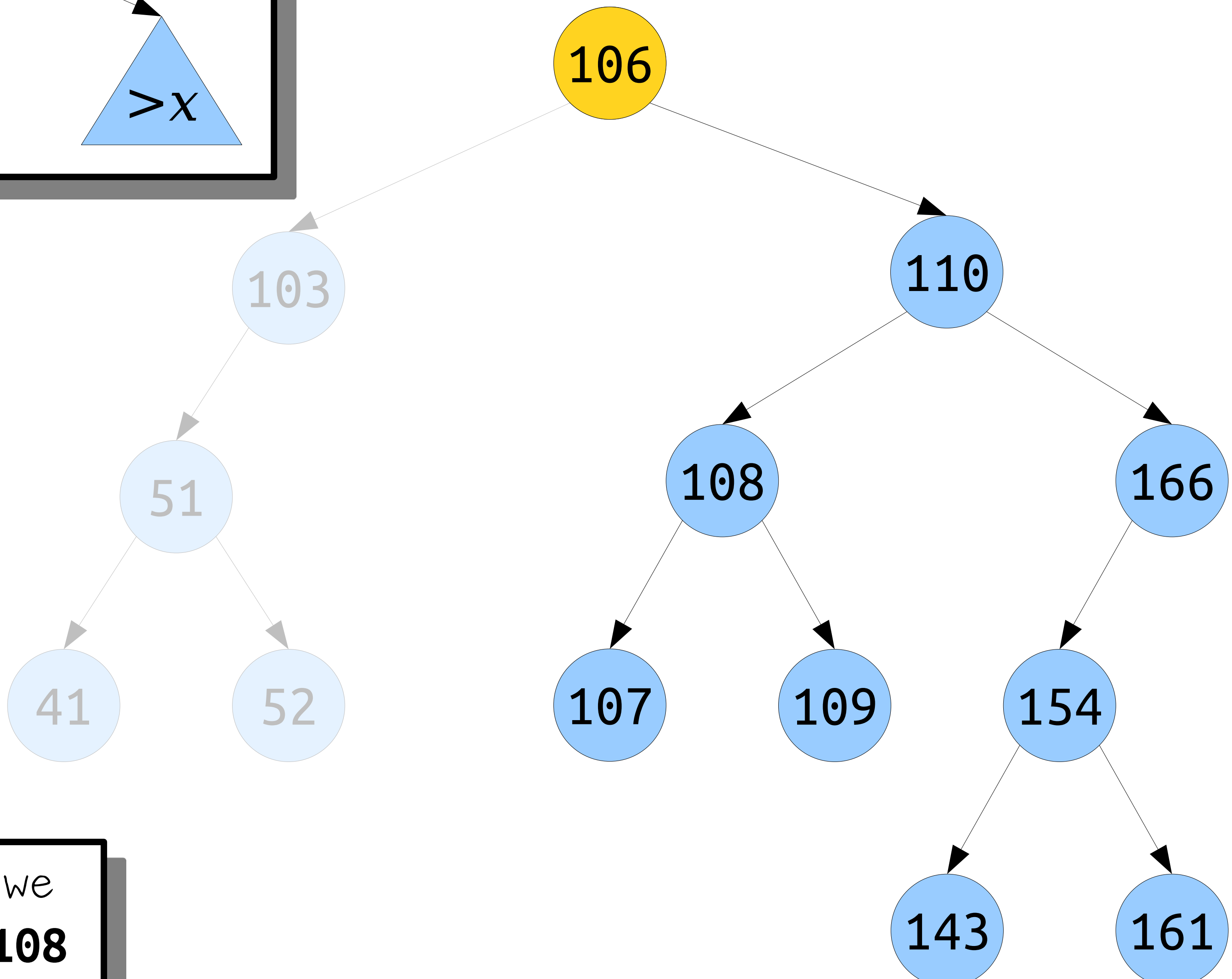

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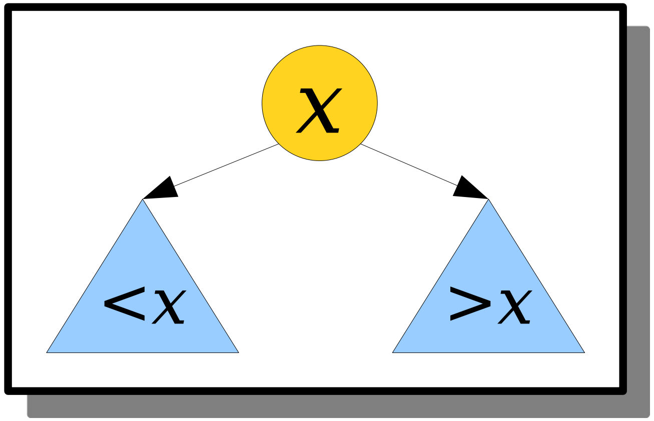

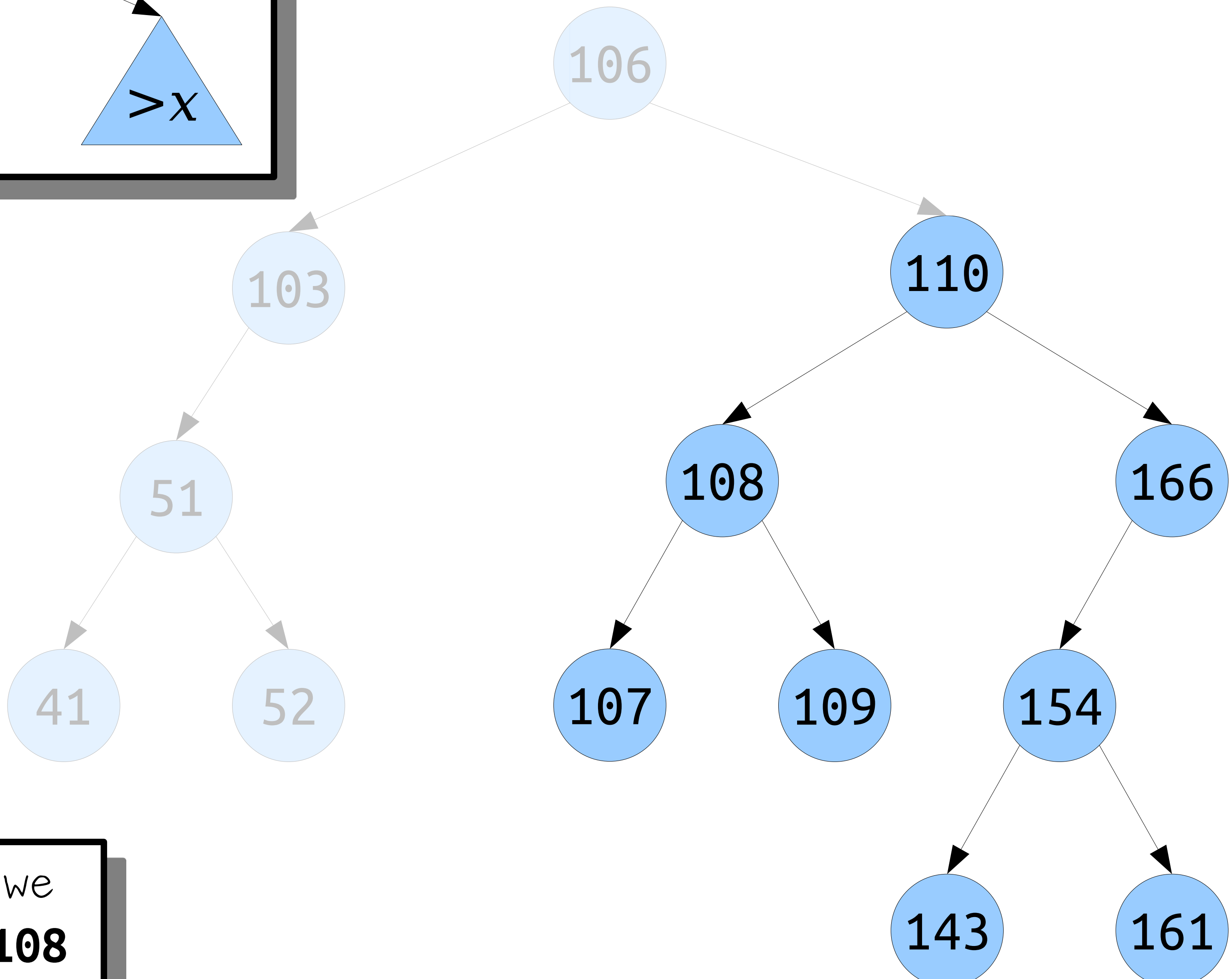

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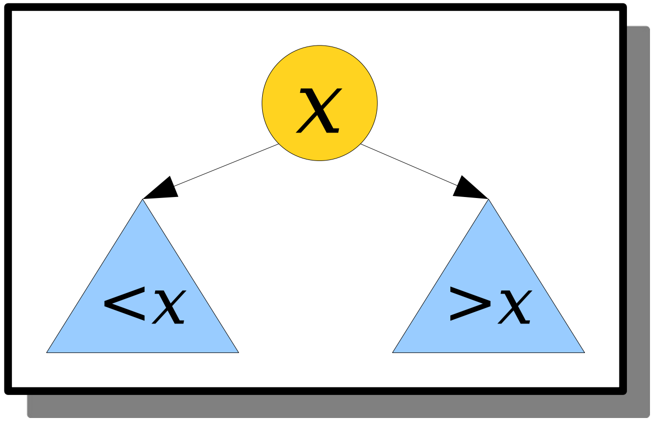

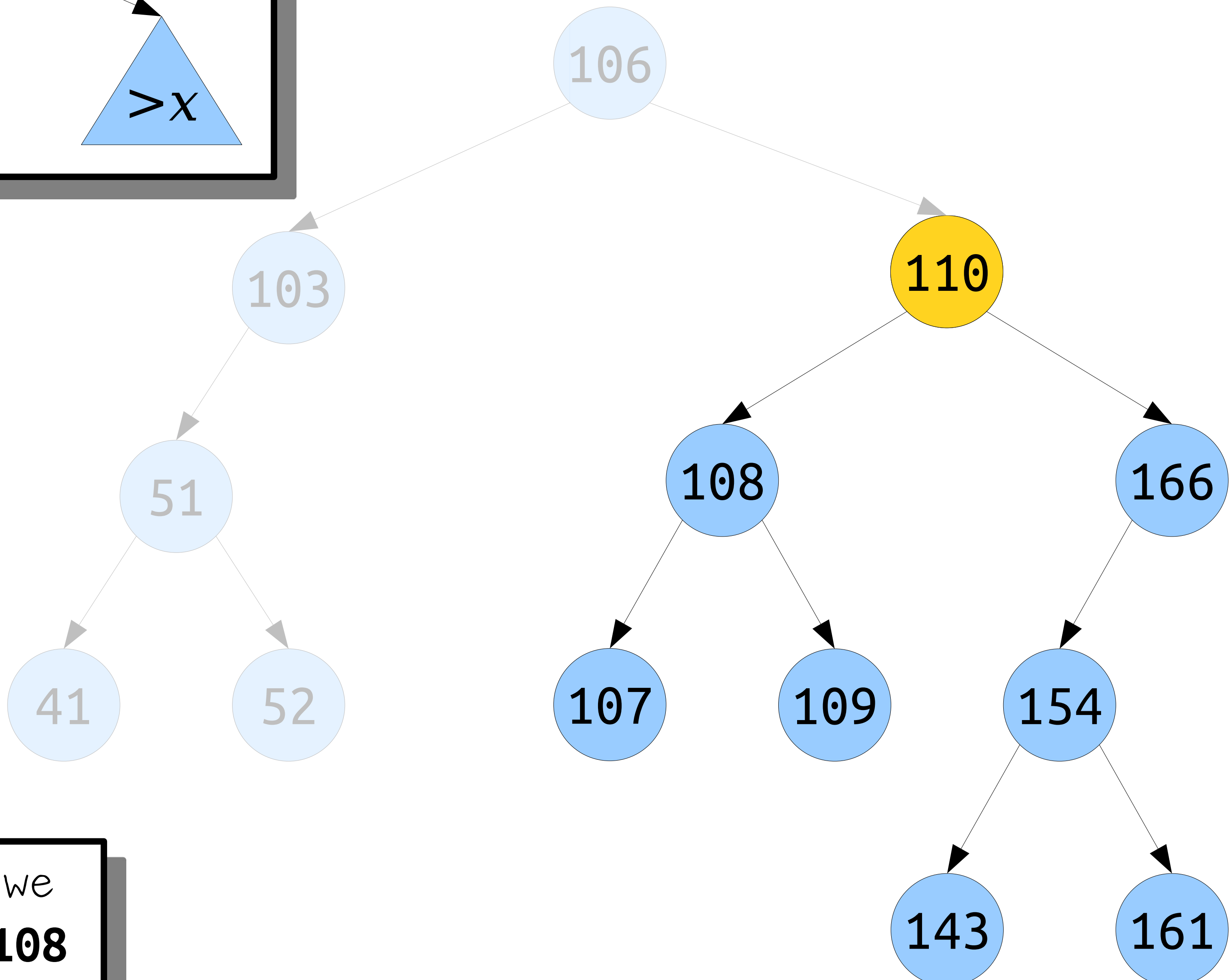

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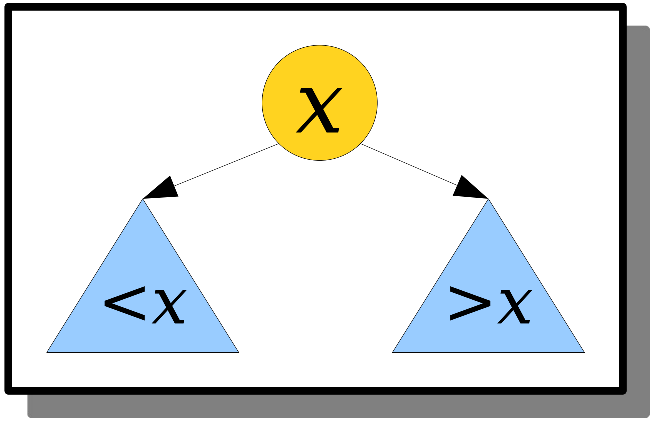

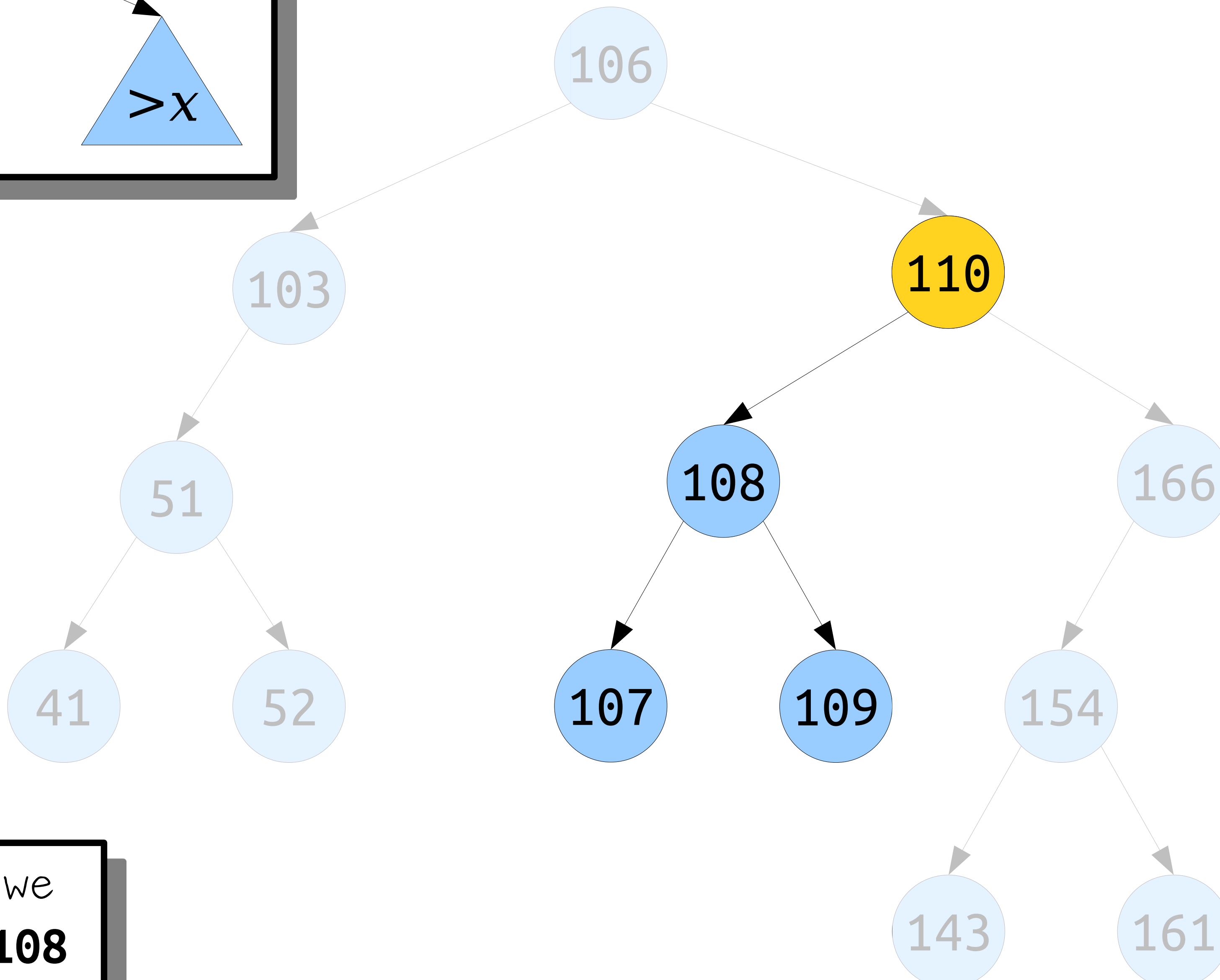

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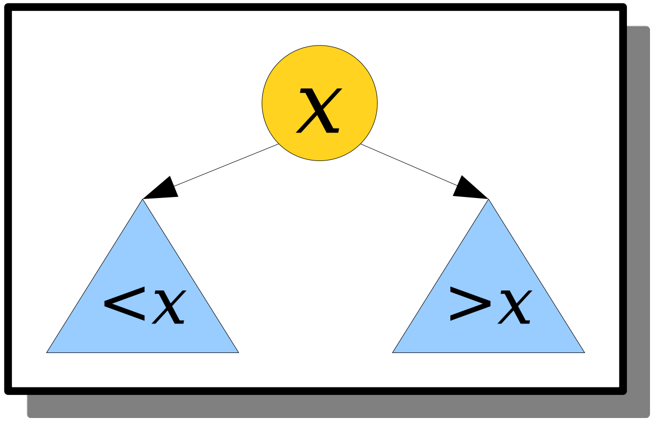

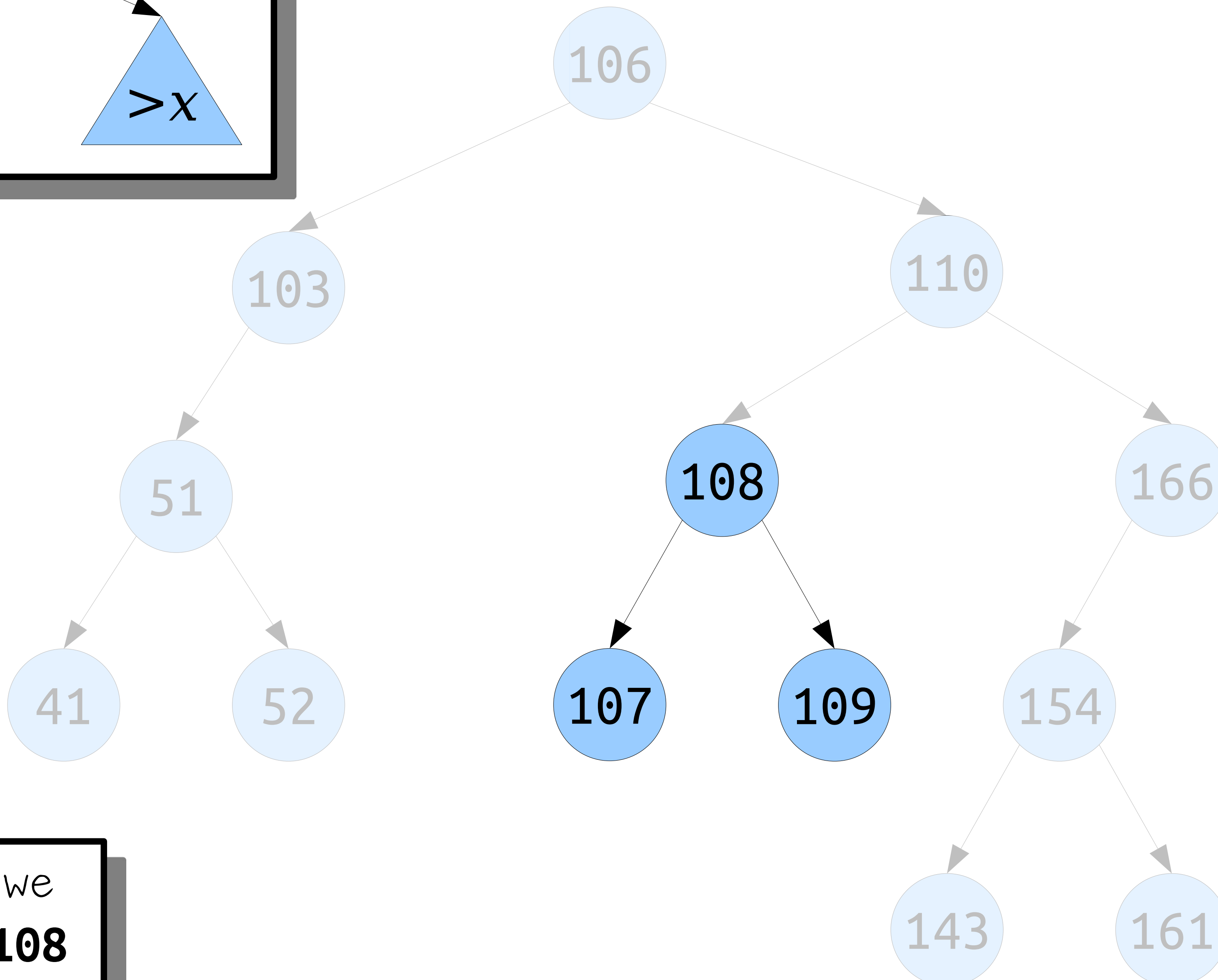

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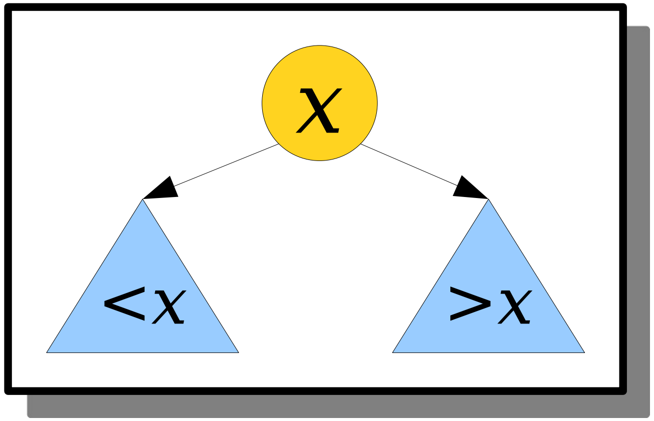

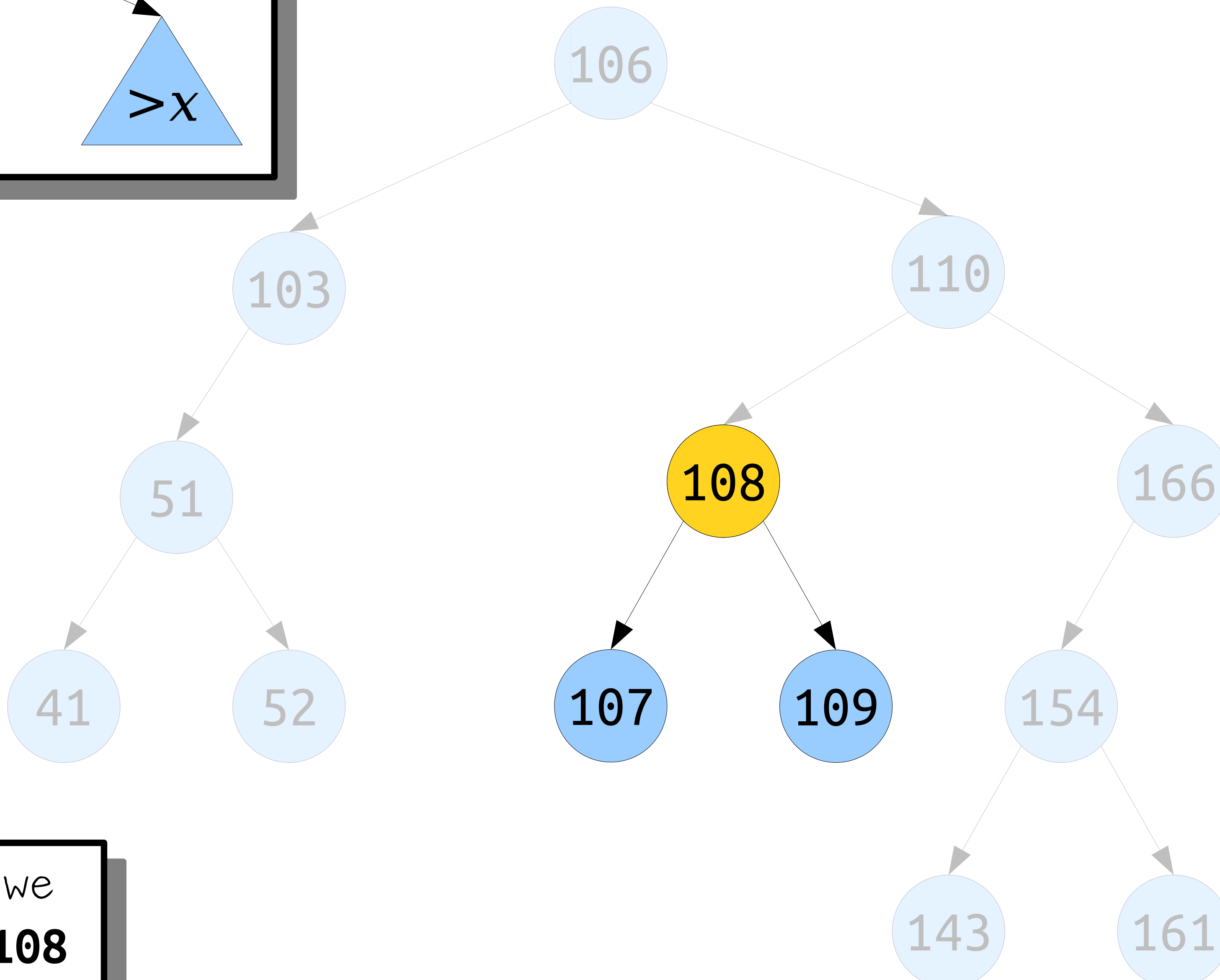

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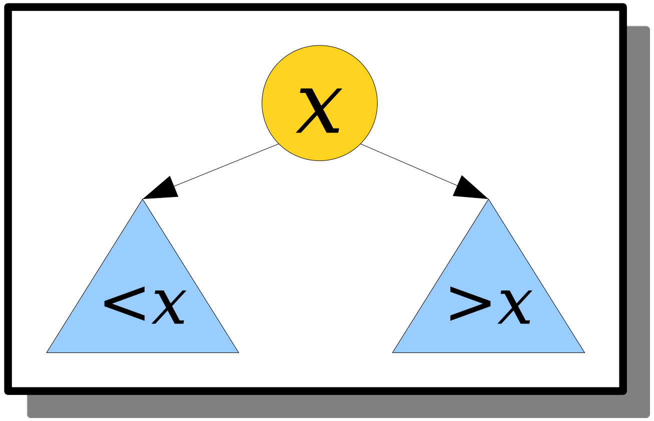

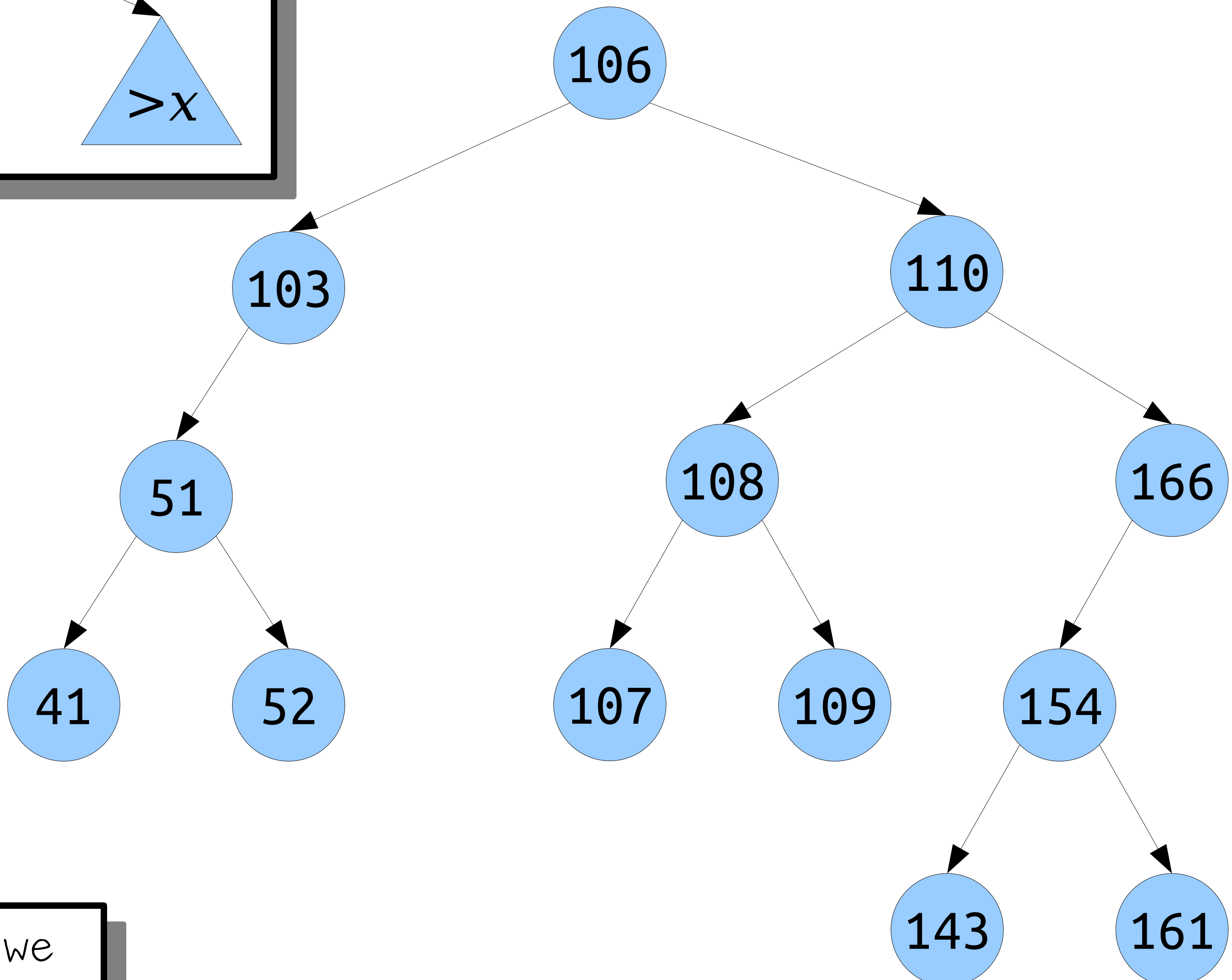

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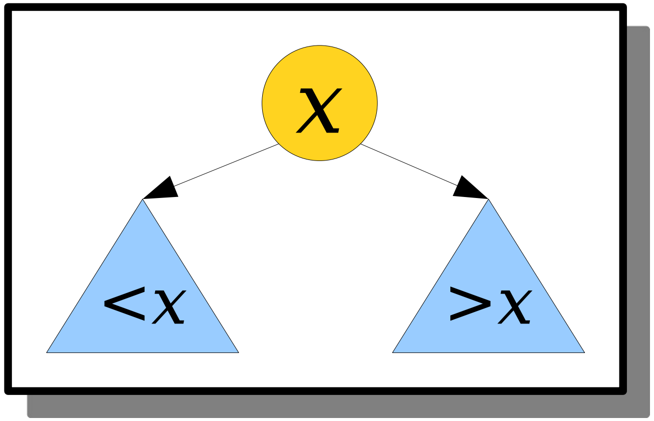
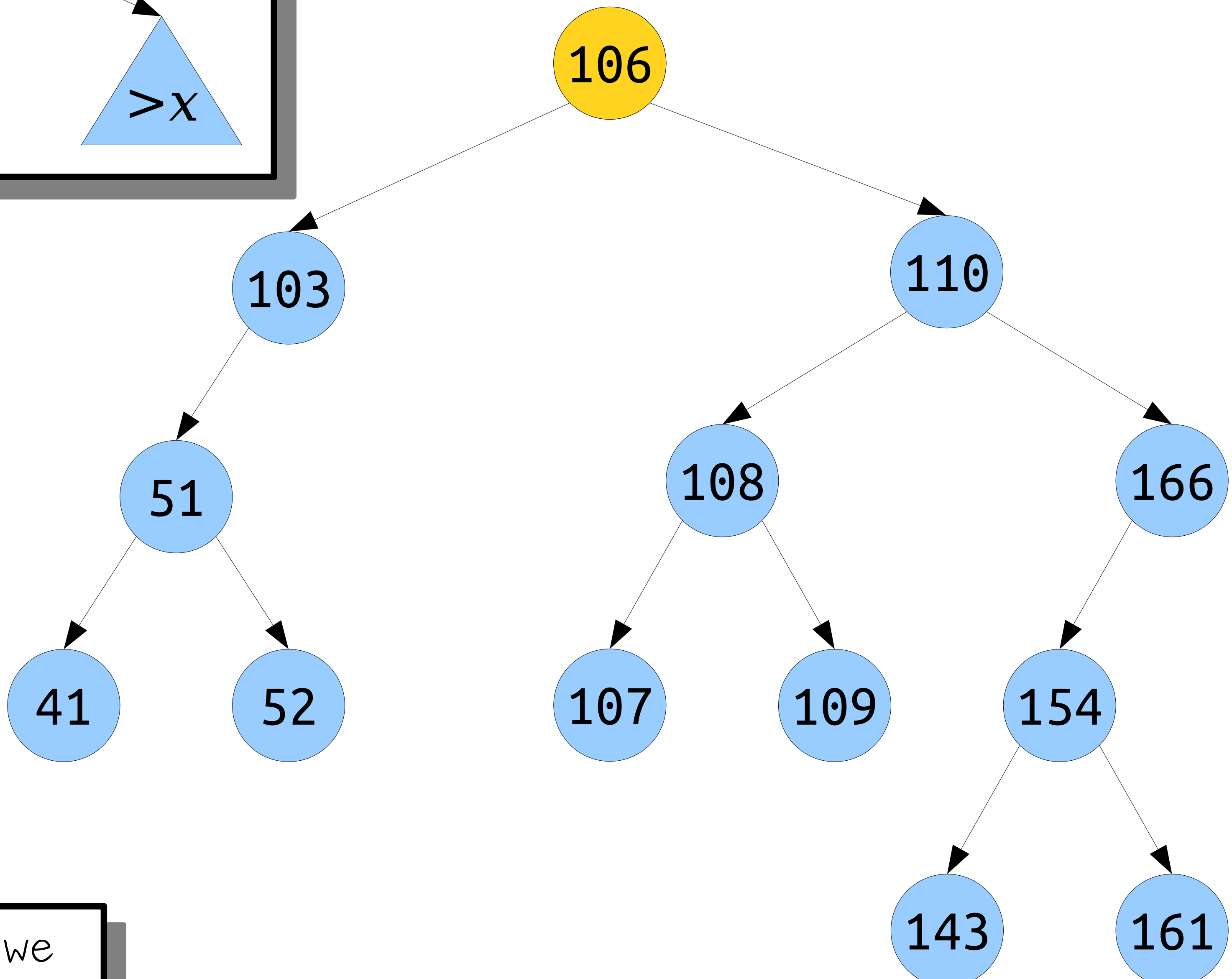
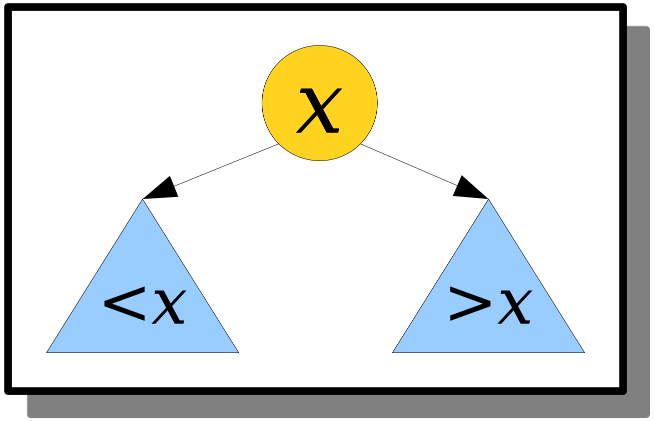
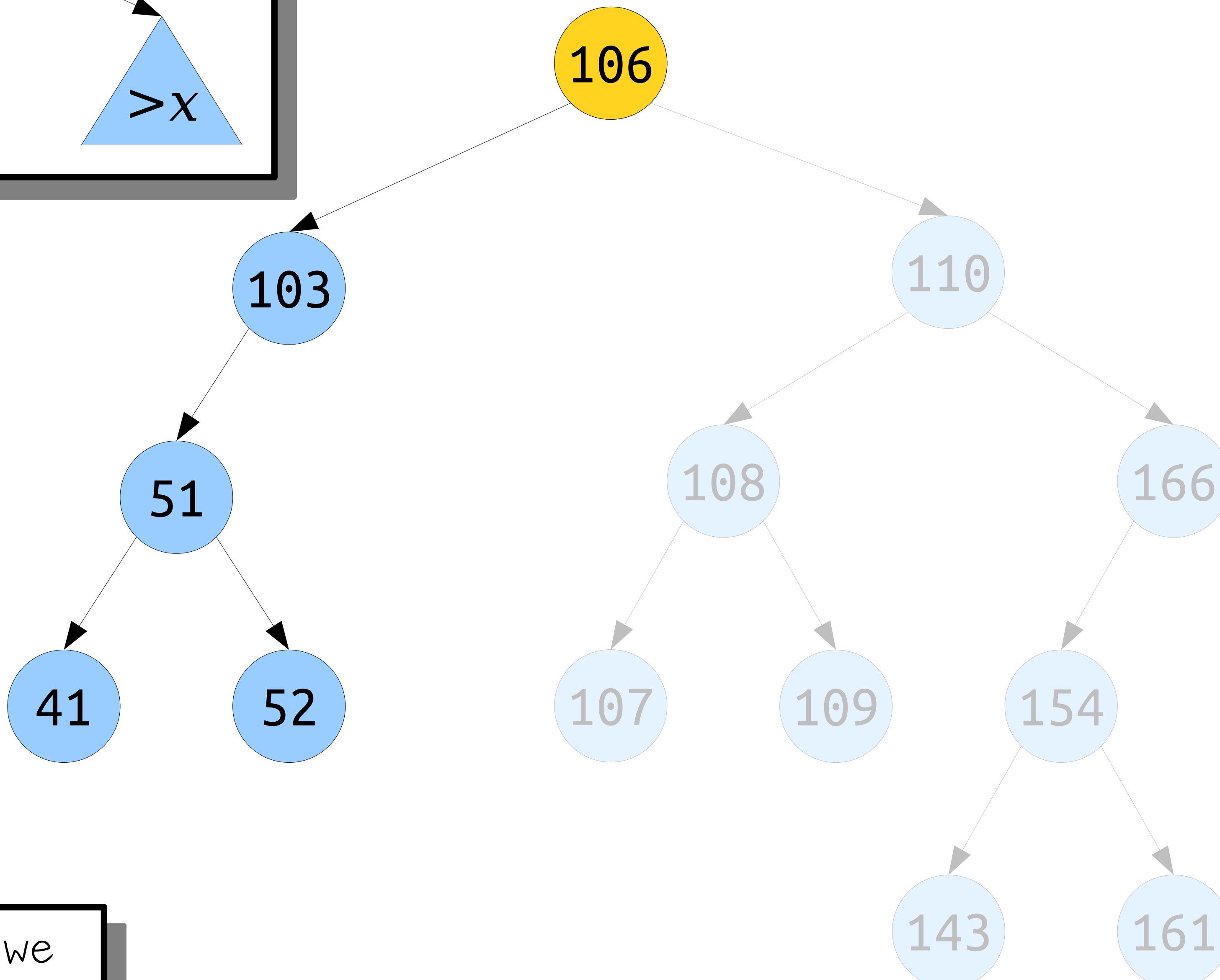




How can we check if **83** is in this tree?





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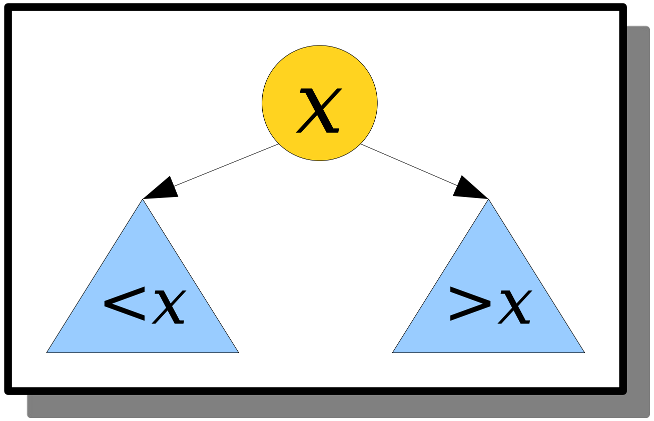
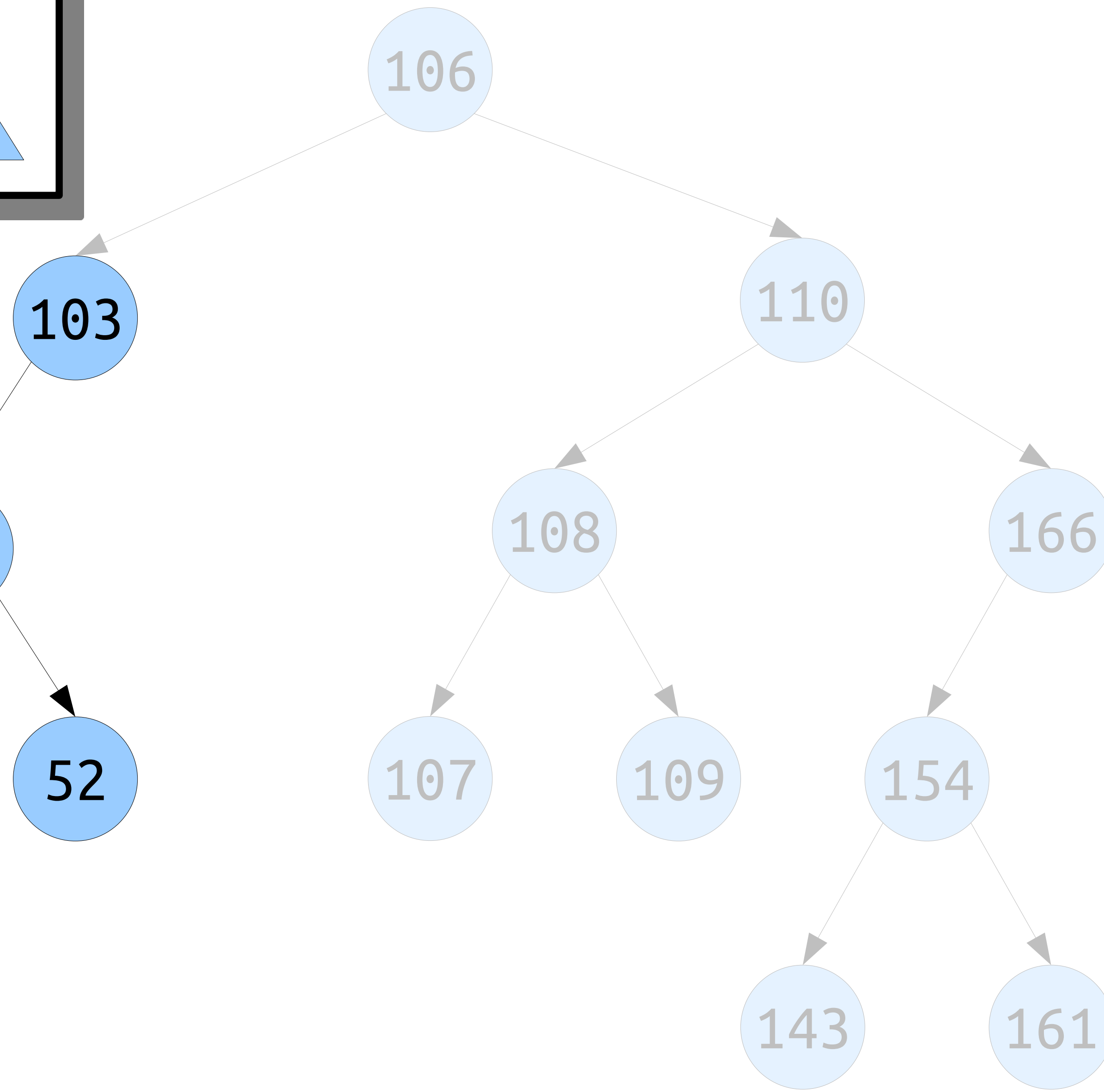
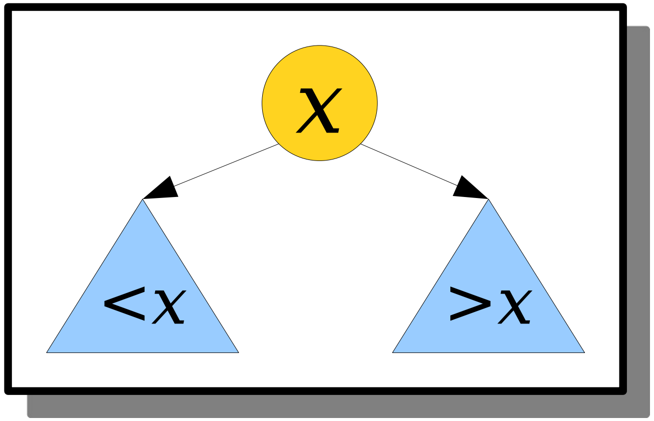
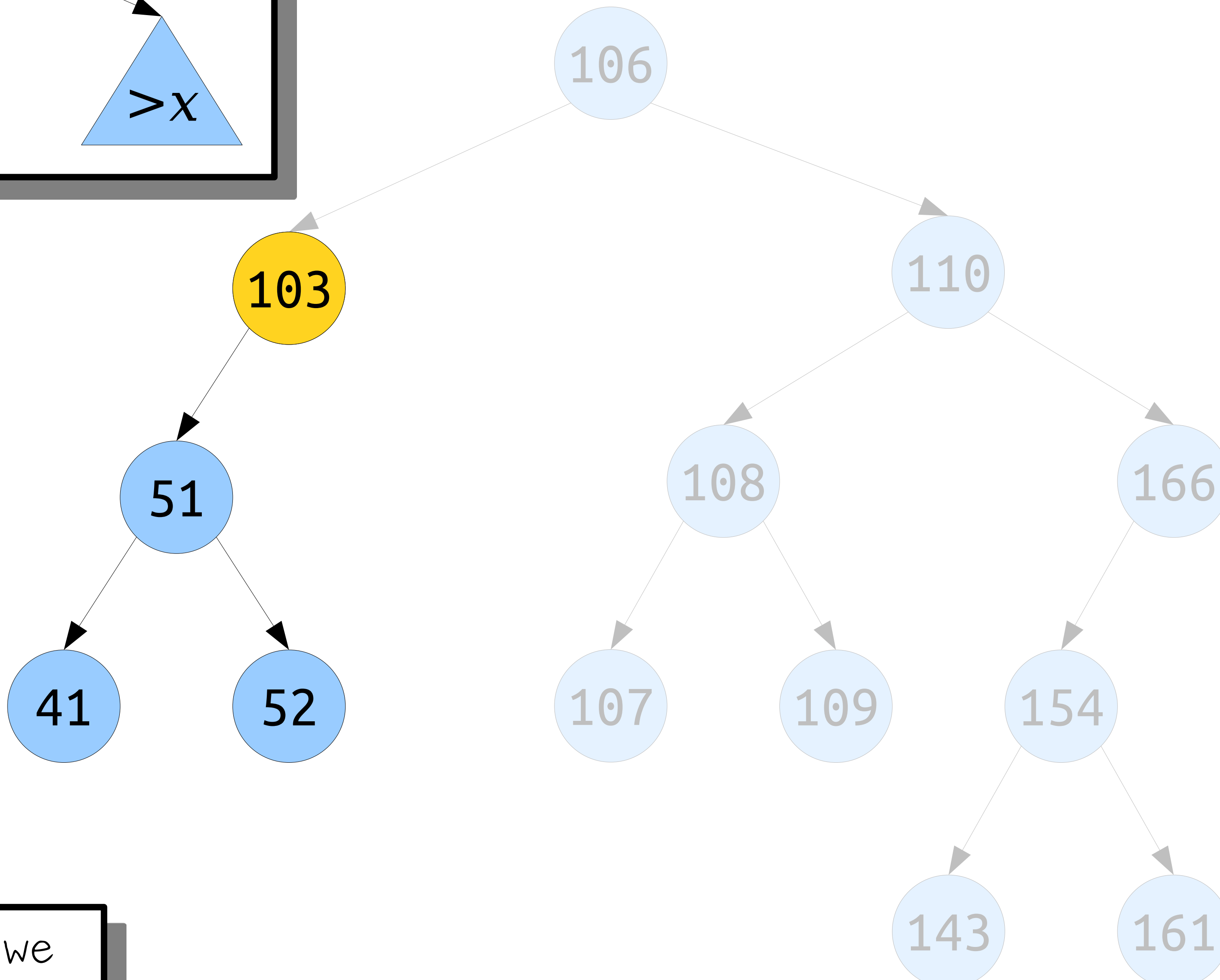


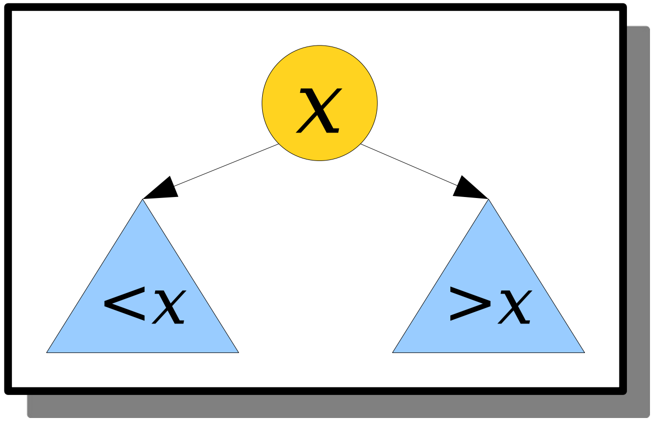


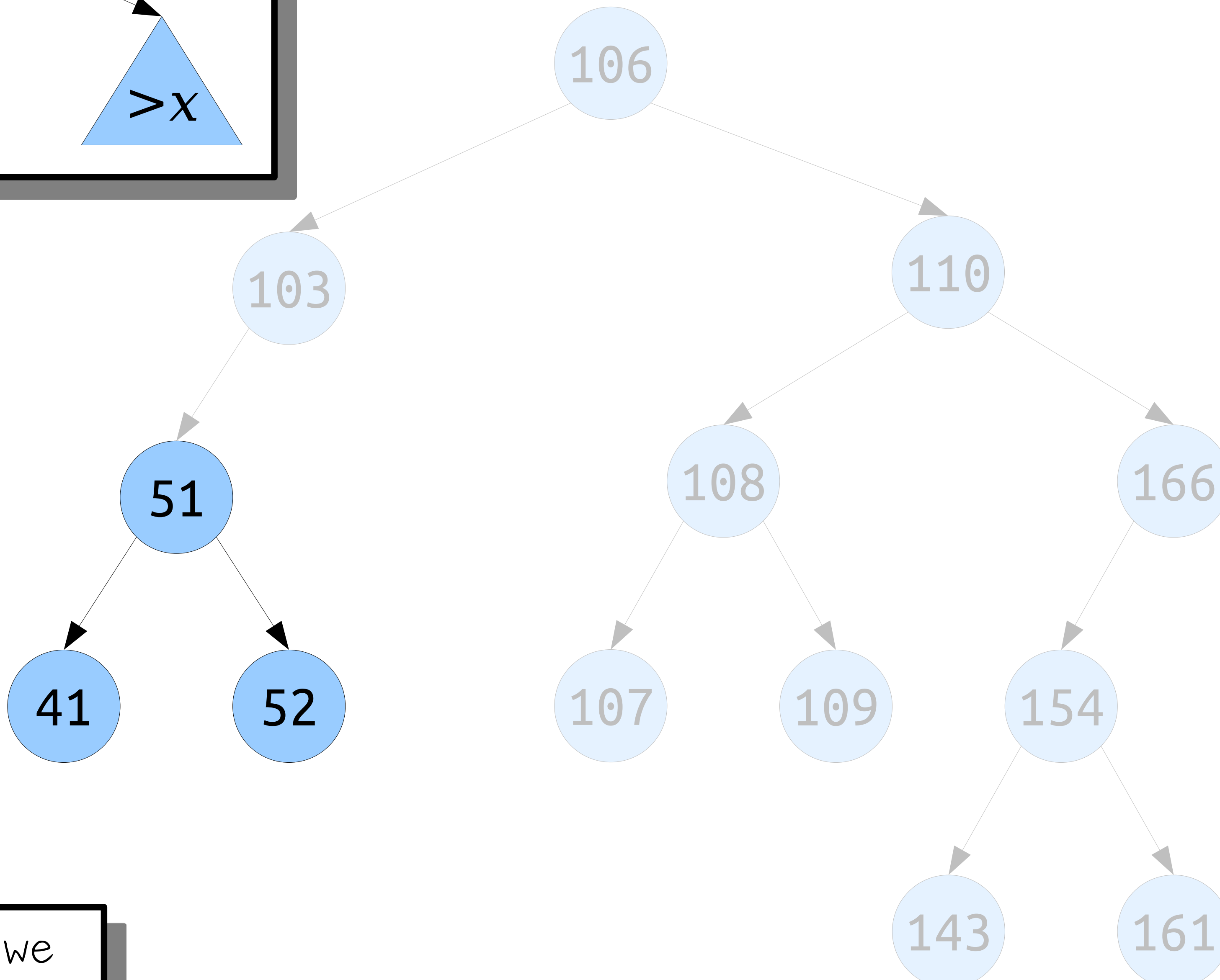
How can we check if **83** is in this tree?



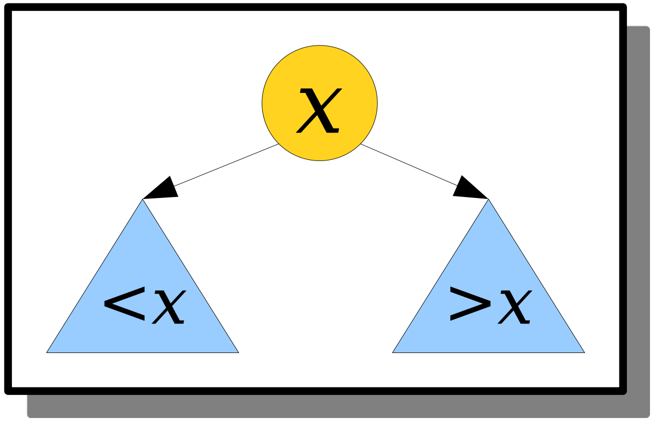


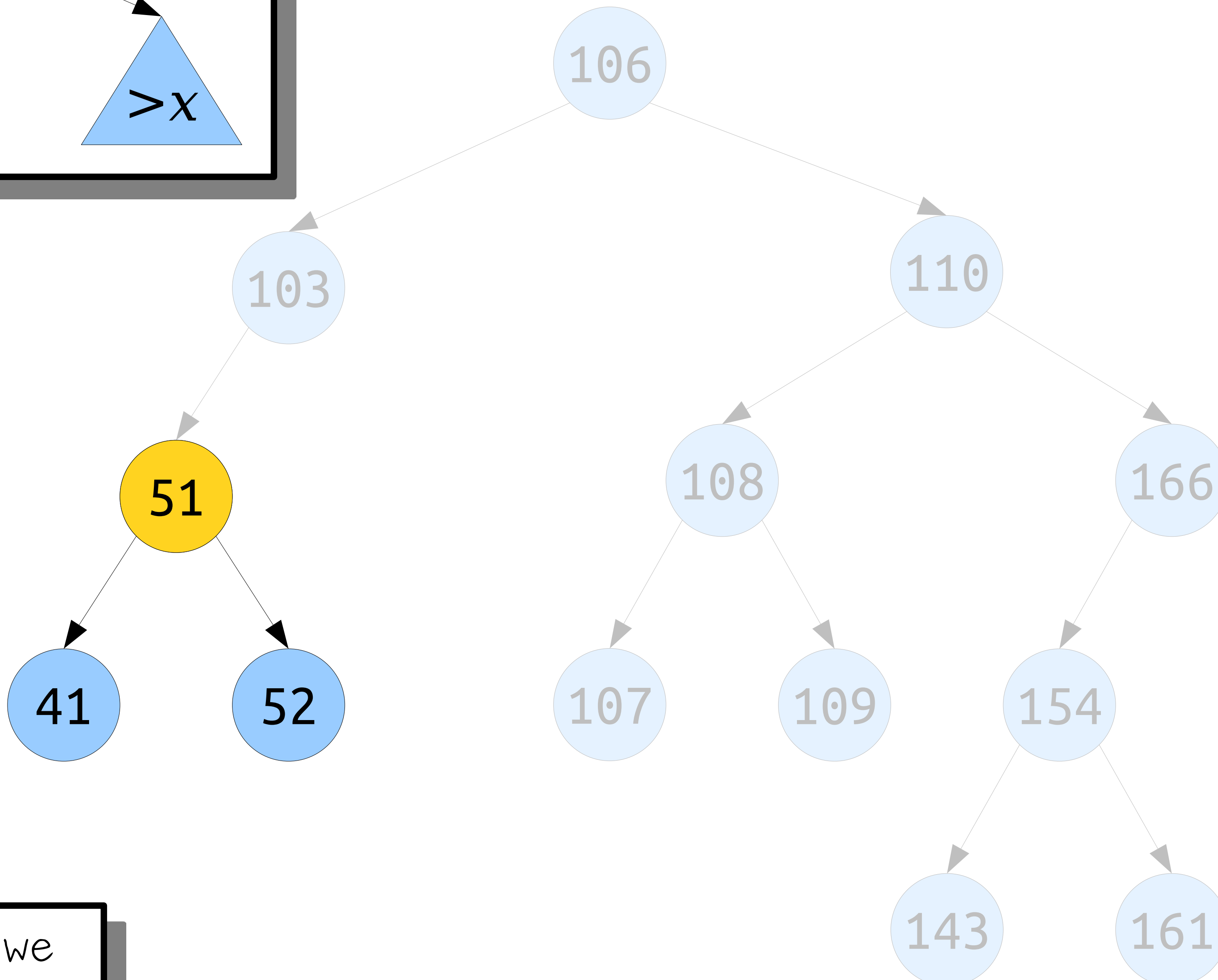
How can we check if **83** is in this tree?



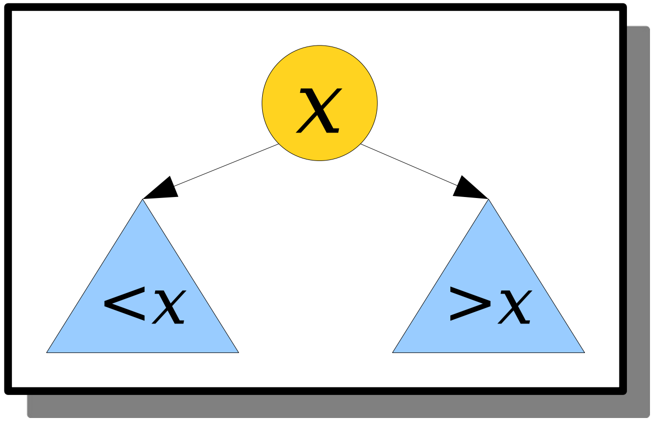


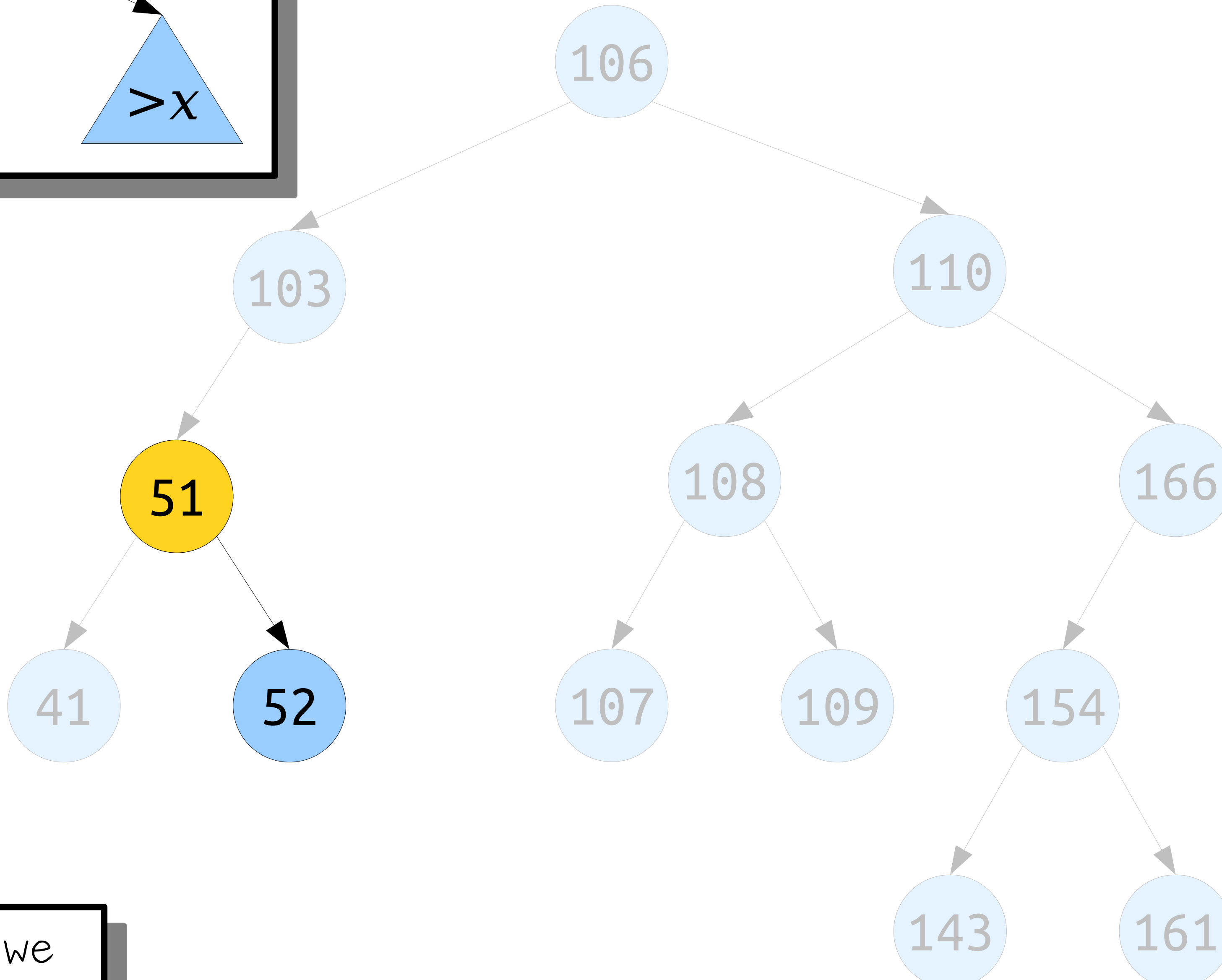
How can we check if **83** is in this tree?



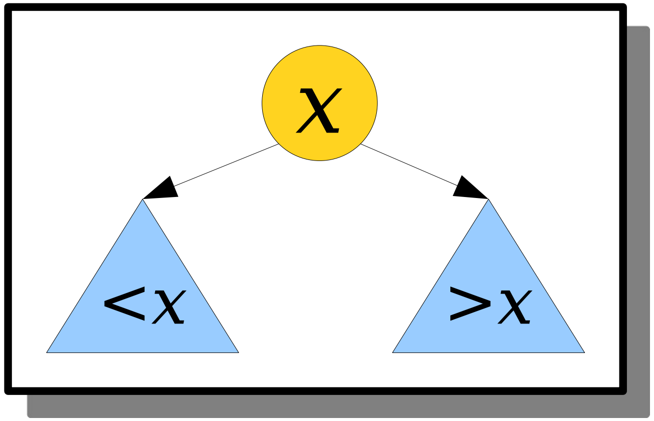


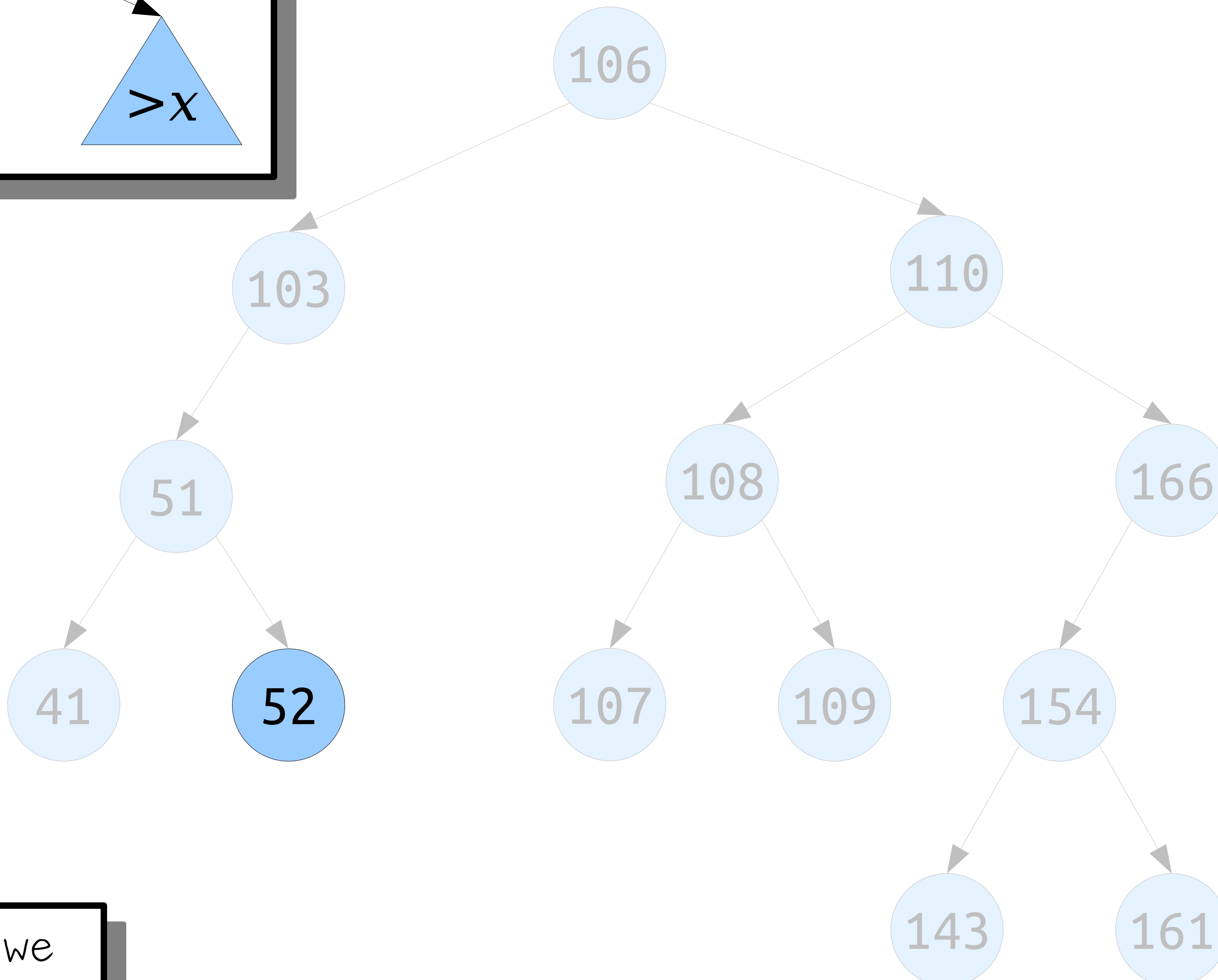
How can we check if **83** is in this tree?



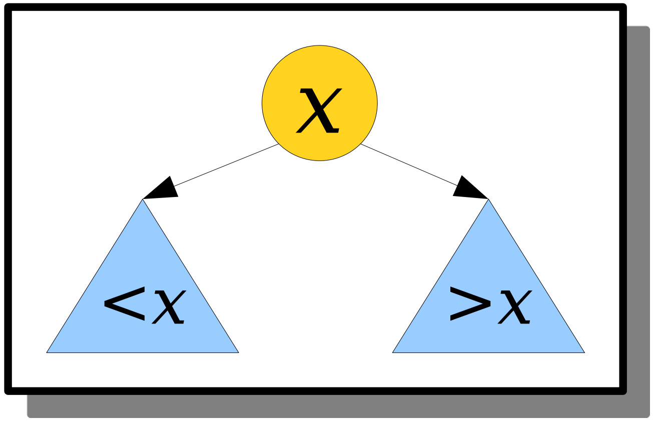


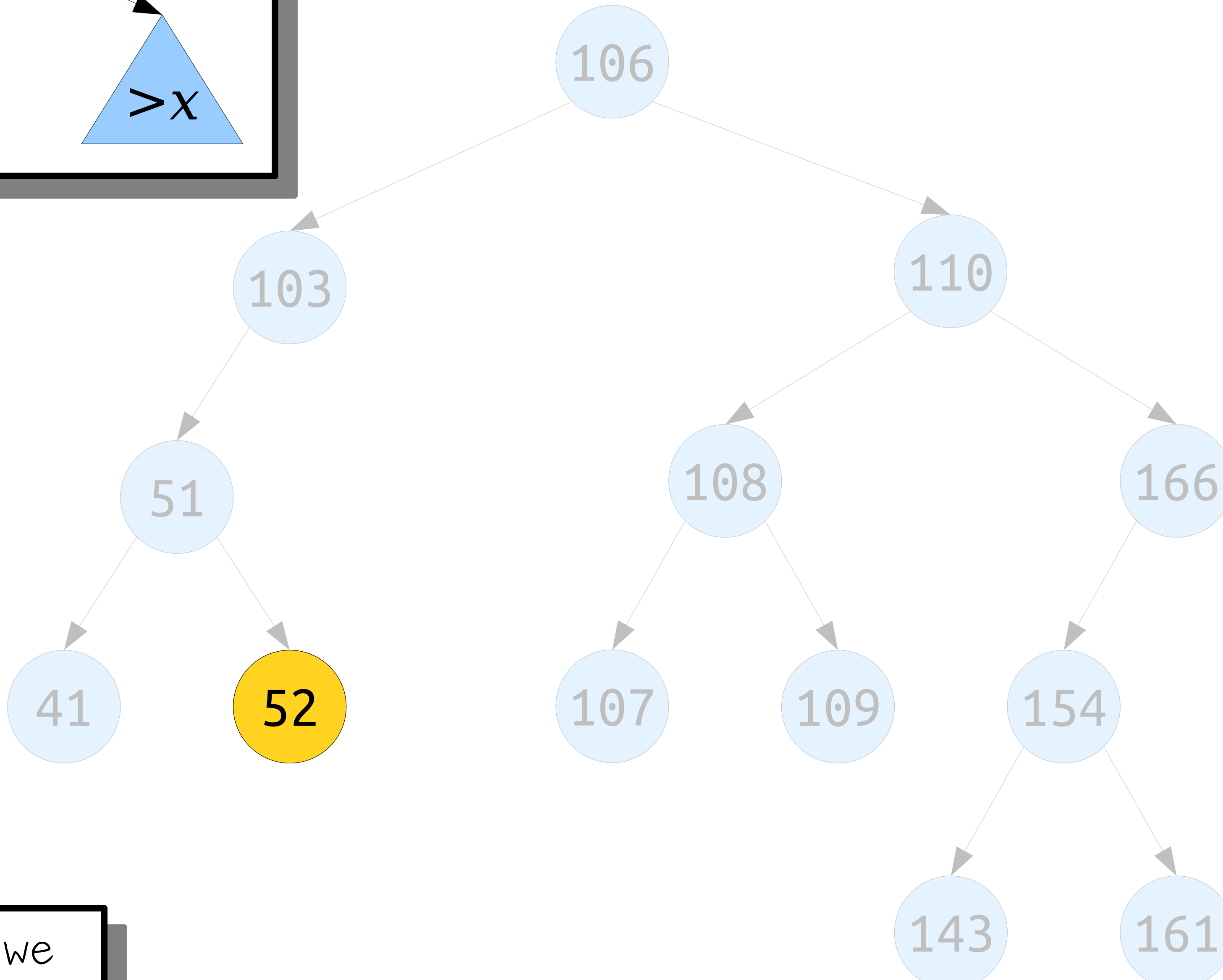
How can we check if **83** is in this tree?



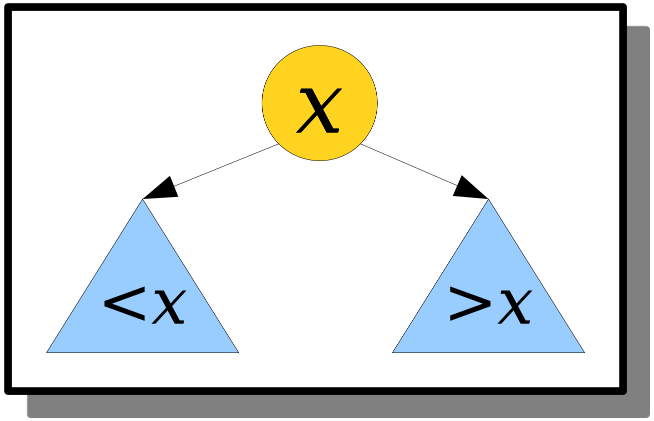


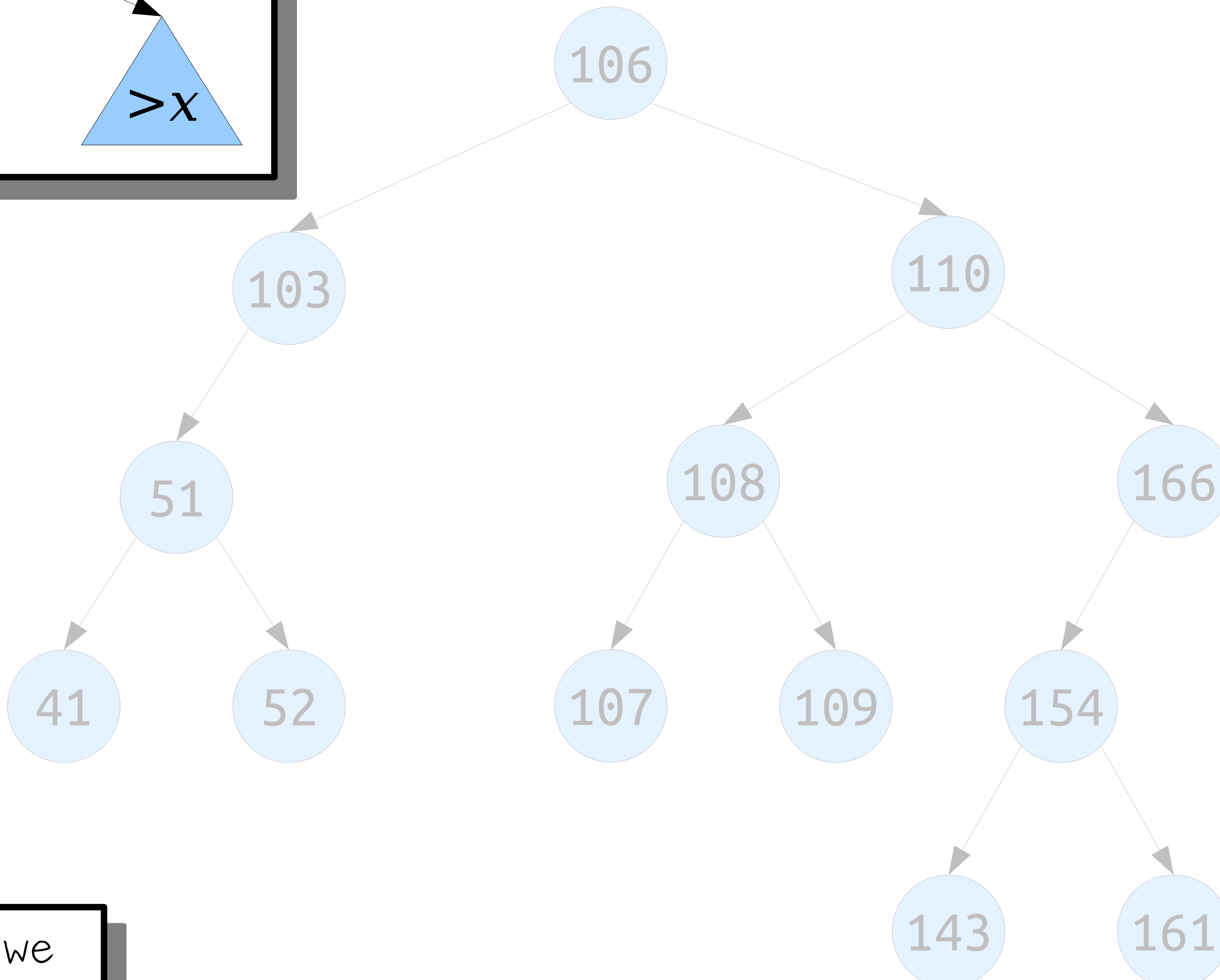
How can we check if **83** is in this tree?





How can we check if **83** is in this tree?

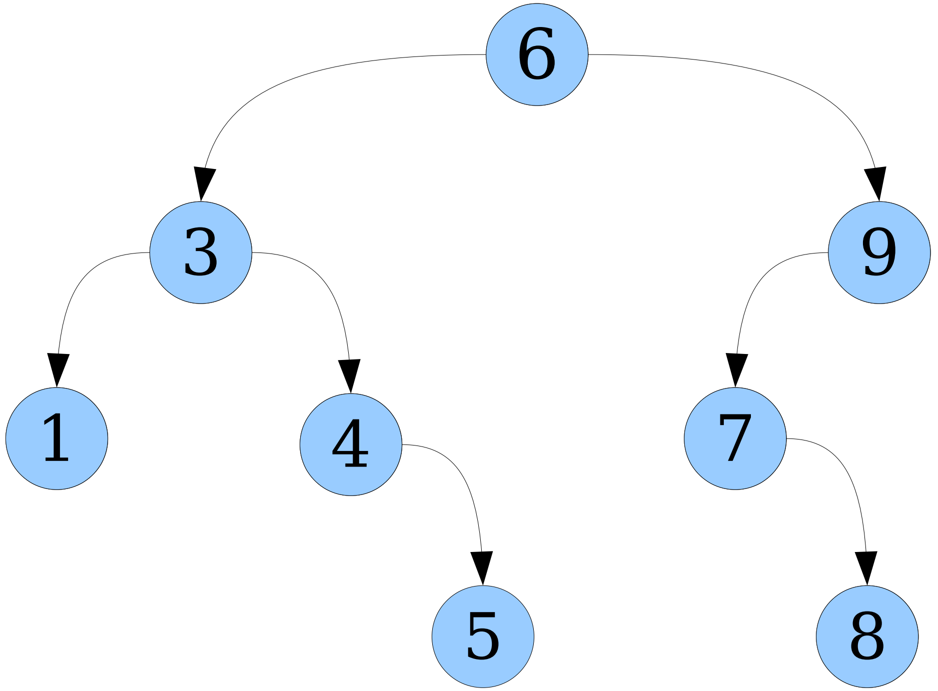


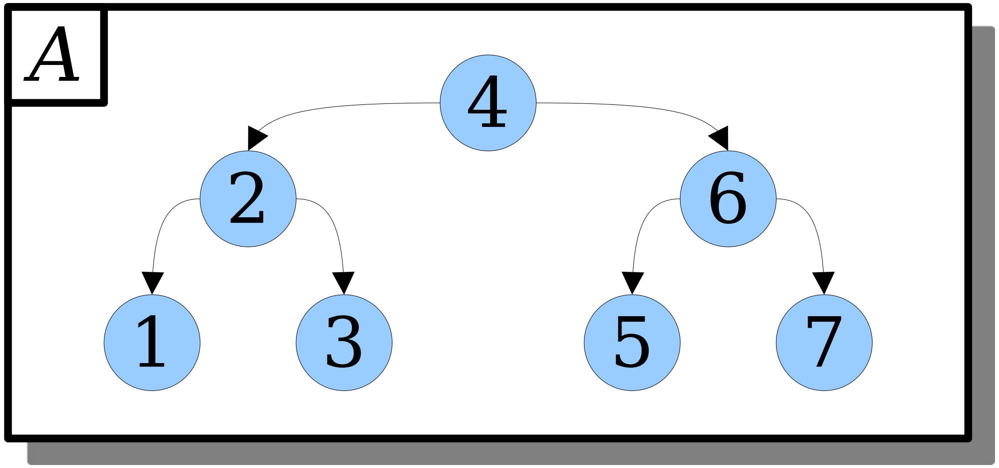


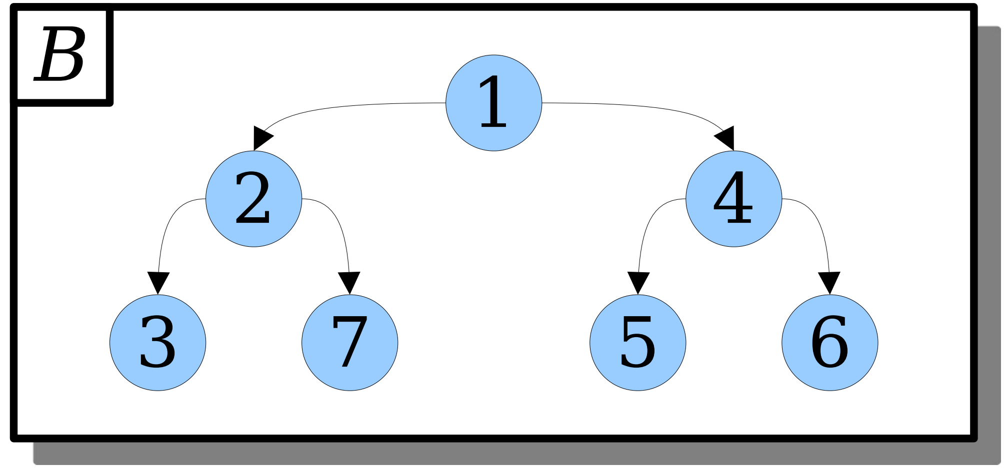
How can we check if **83** is in this tree?

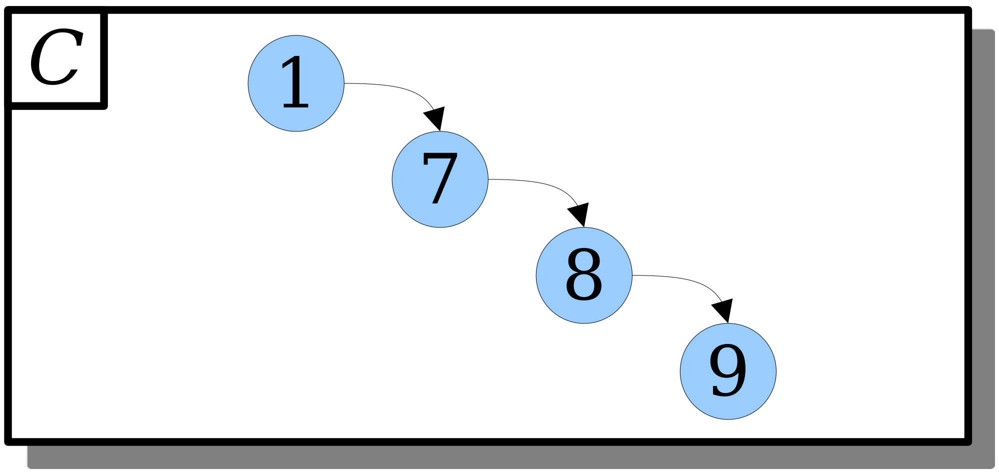
Binary Search Trees

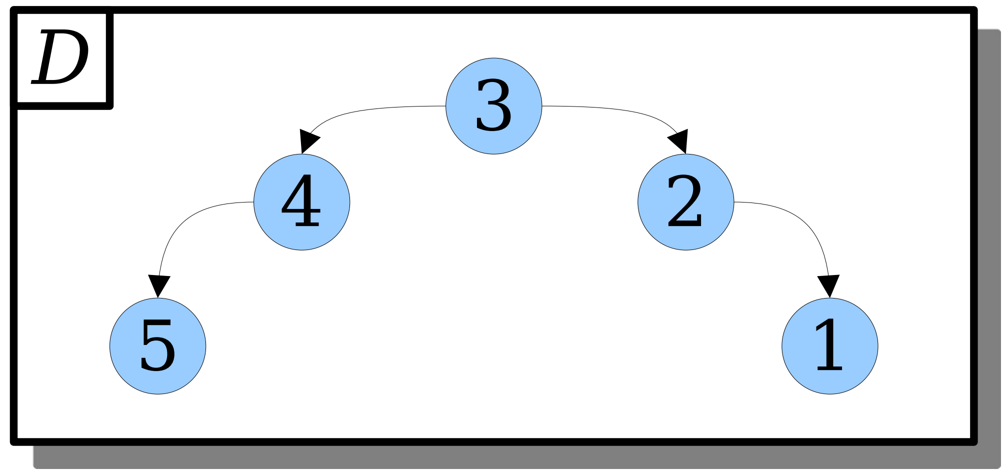
- The data structure we have just seen is called a ***binary search tree*** (or ***BST***).
- The tree consists of a number of ***nodes***, each of which stores a value and has zero, one, or two ***children***.
- All values in a node's left subtree are ***smaller*** than the node's value, and all values in a node's right subtree are ***greater*** than the node's value.

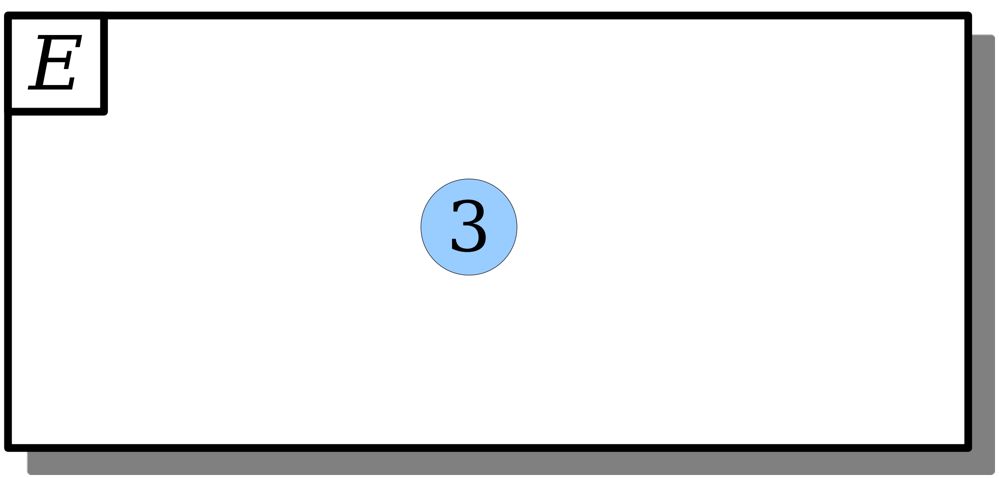






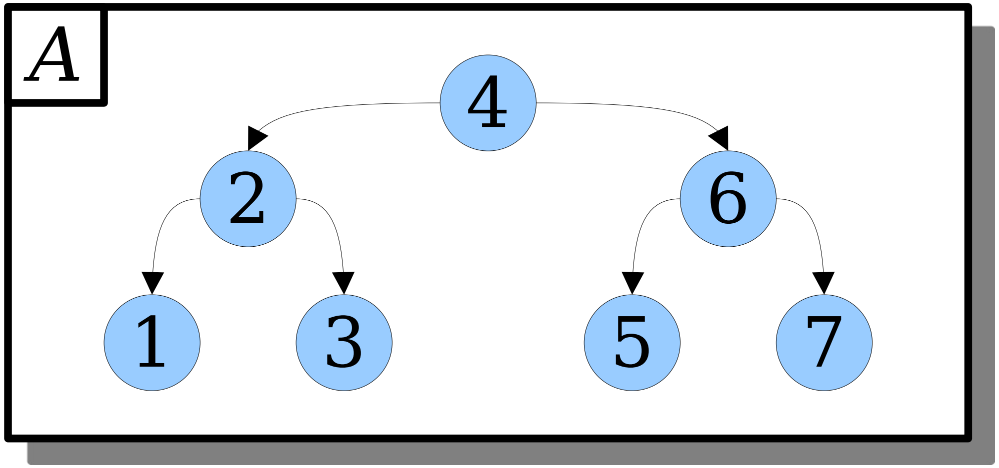


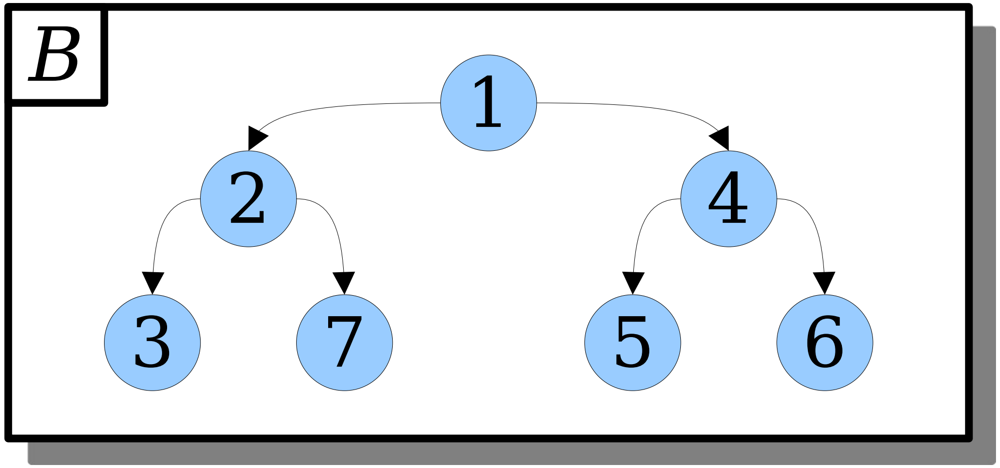


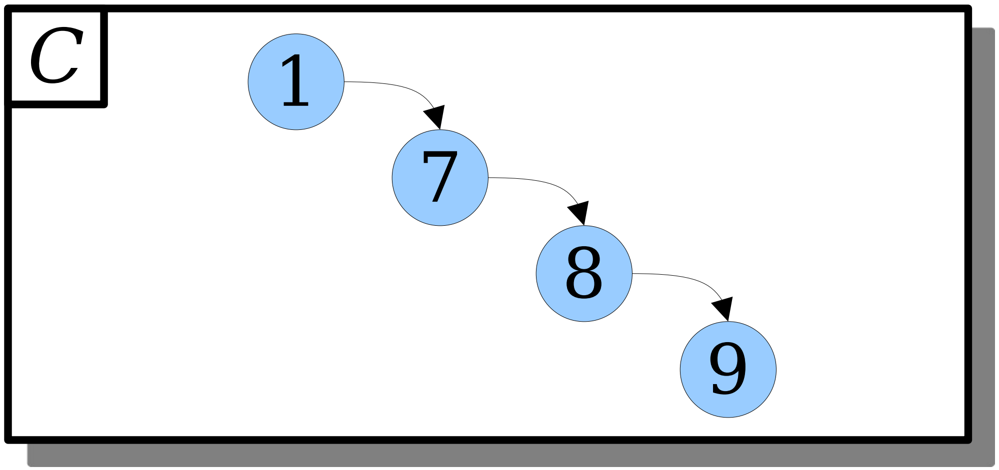


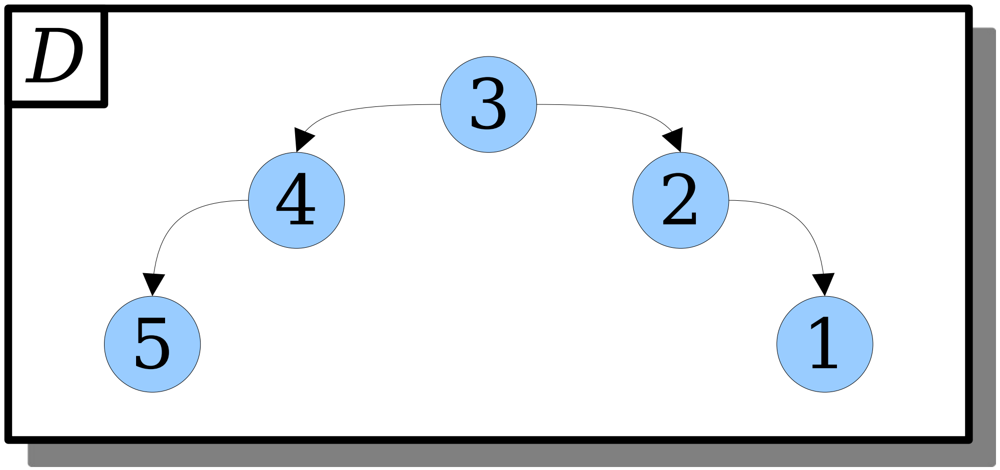
Which of these are BSTs?
Which are binary heaps?

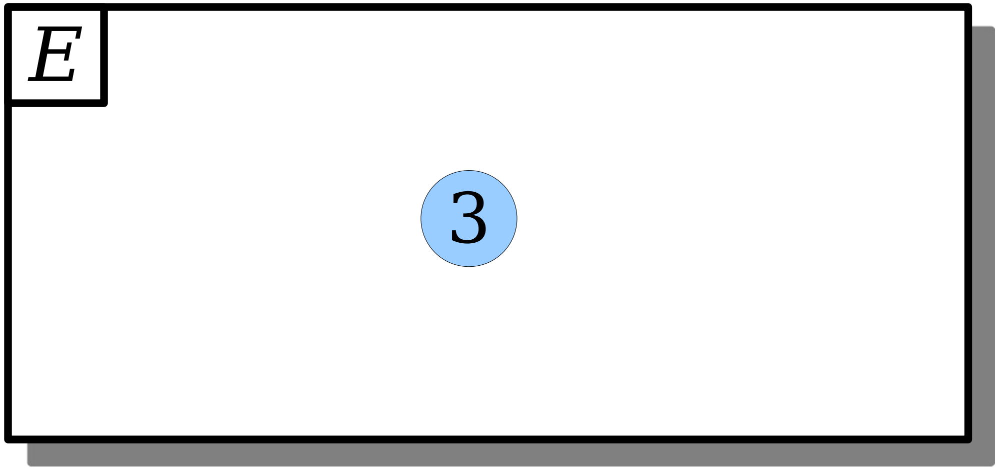
Formulate a hypothesis!





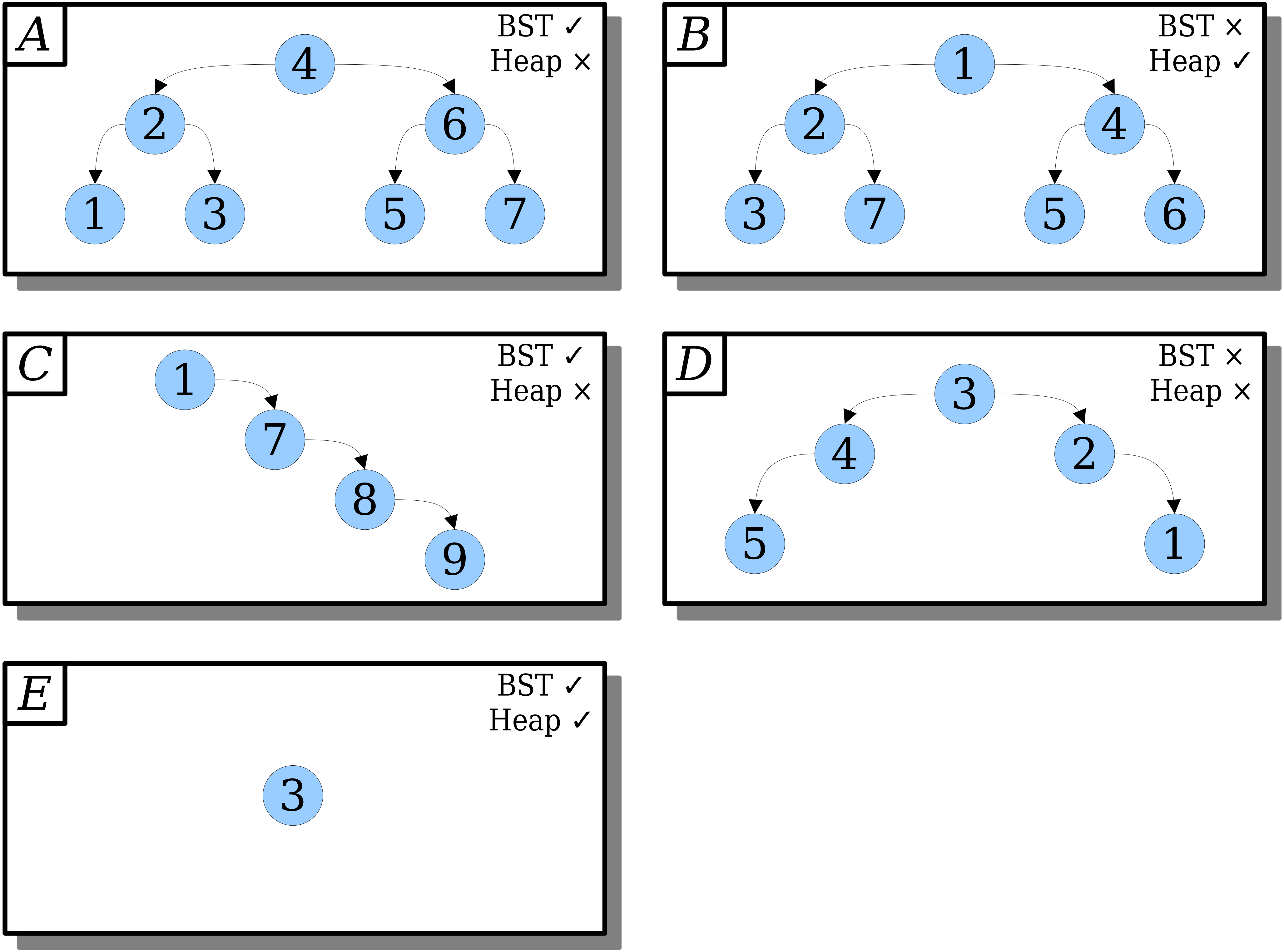






Which of these are BSTs?
Which are binary heaps?

Chat with your neighbors!

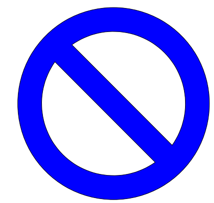

A
BST ✓
Heap ×
4
2
6
1
3
5
7
B
BST ×
Heap ✓
1
2
4
3
7
5
6
C
BST ✓
Heap ×
1
7
8
9
D
BST ×
Heap ×
3
4
2
5
1
E
BST ✓
Heap ✓
3

A Binary Search Tree Is Either…

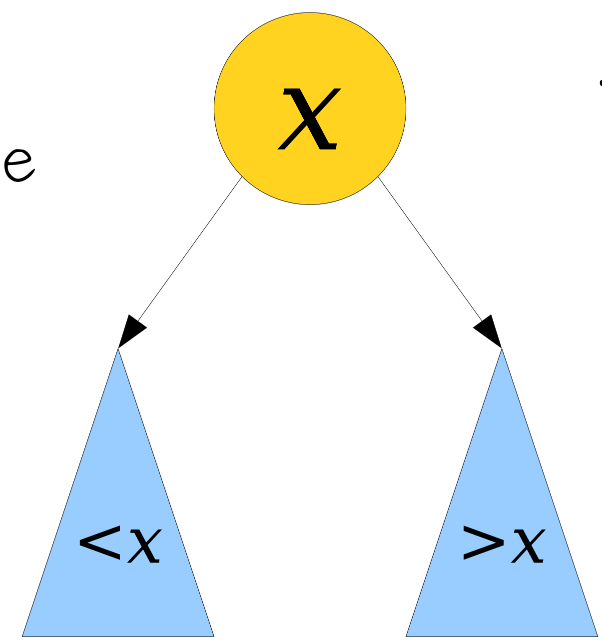
an empty tree,
represented by
`nullptr`

A Binary Search Tree Is Either…

an empty tree,
represented by
`nullptr`, or…



… a single node,
whose left subtree
is a BST of
smaller values …



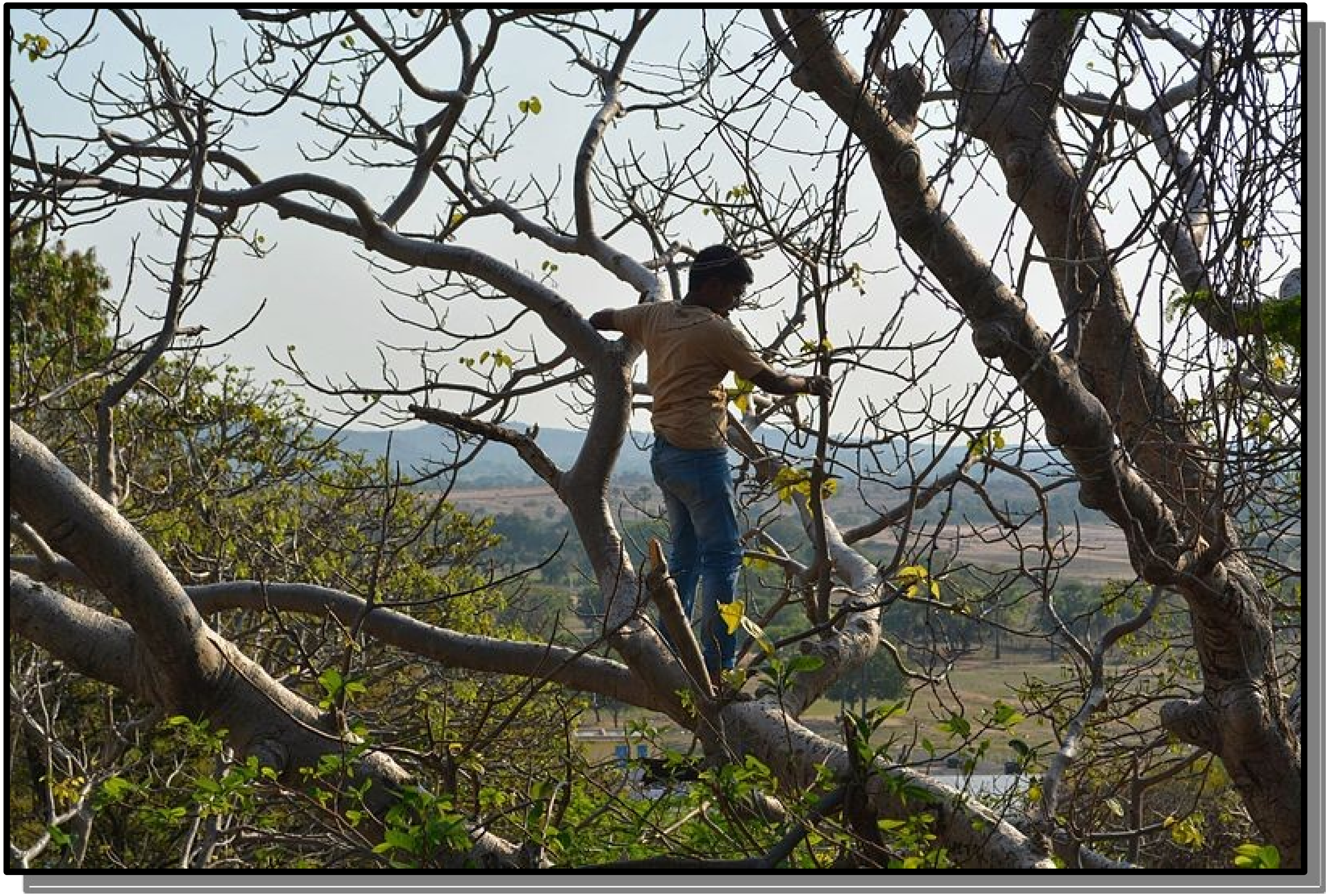
… and whose right
subtree is a BST
of larger values.

Binary Search Tree Nodes

```
struct Node {
    Type value;
    Node* left;  // Smaller values
    Node* right; // Bigger values
};
```

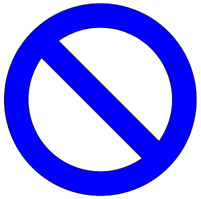
Kinda like a linked list, but with two pointers instead of just one!

Searching Trees



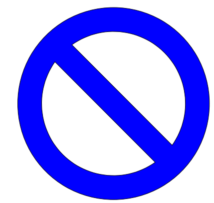
A Binary Search Tree Is Either…

an empty tree,
represented by
`nullptr`



A Binary Search Tree Is Either…

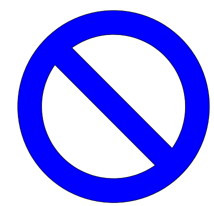
an empty tree, represented by `nullptr`



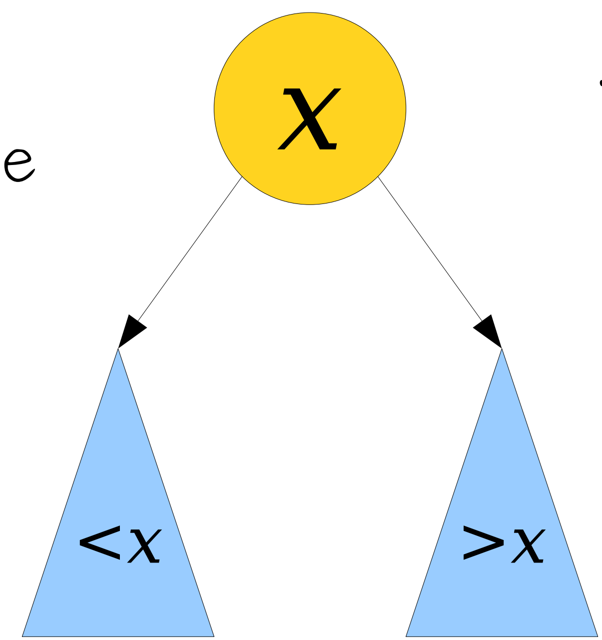
If you're looking for something in an empty BST, it's not there! Sorry.

A Binary Search Tree Is Either…

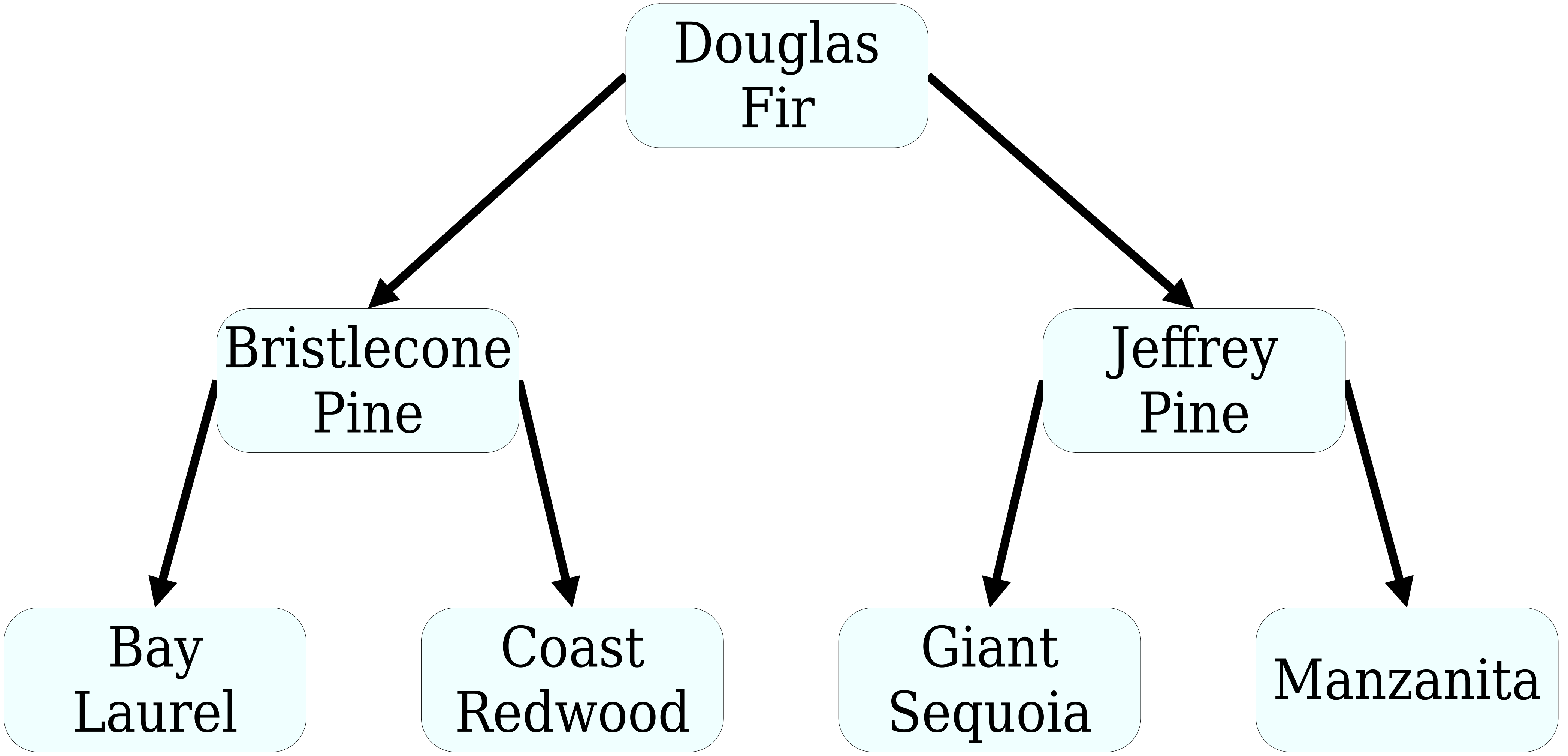
an empty tree,
represented by
`nullptr`, or…



… a single node,
whose left subtree
is a BST of
smaller values …



… and whose right
subtree is a BST
of larger values.



Douglas Fir
Bristlecone Pine
Jeffrey Pine
Bay Laurel
Coast Redwood
Giant Sequoia
Manzanita

Good exercise:

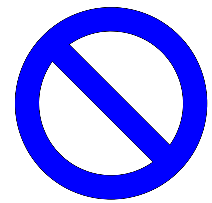
Rewrite this function iteratively!

Walking Trees

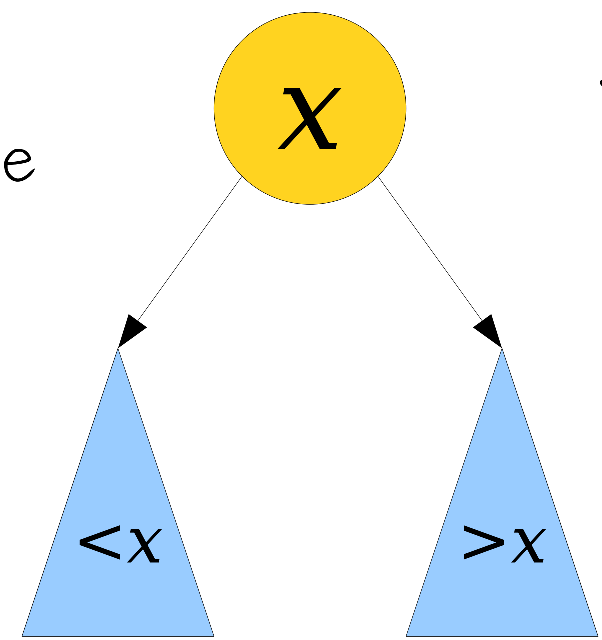
Print all the values in a BST,
in sorted order.

A Binary Search Tree Is Either…

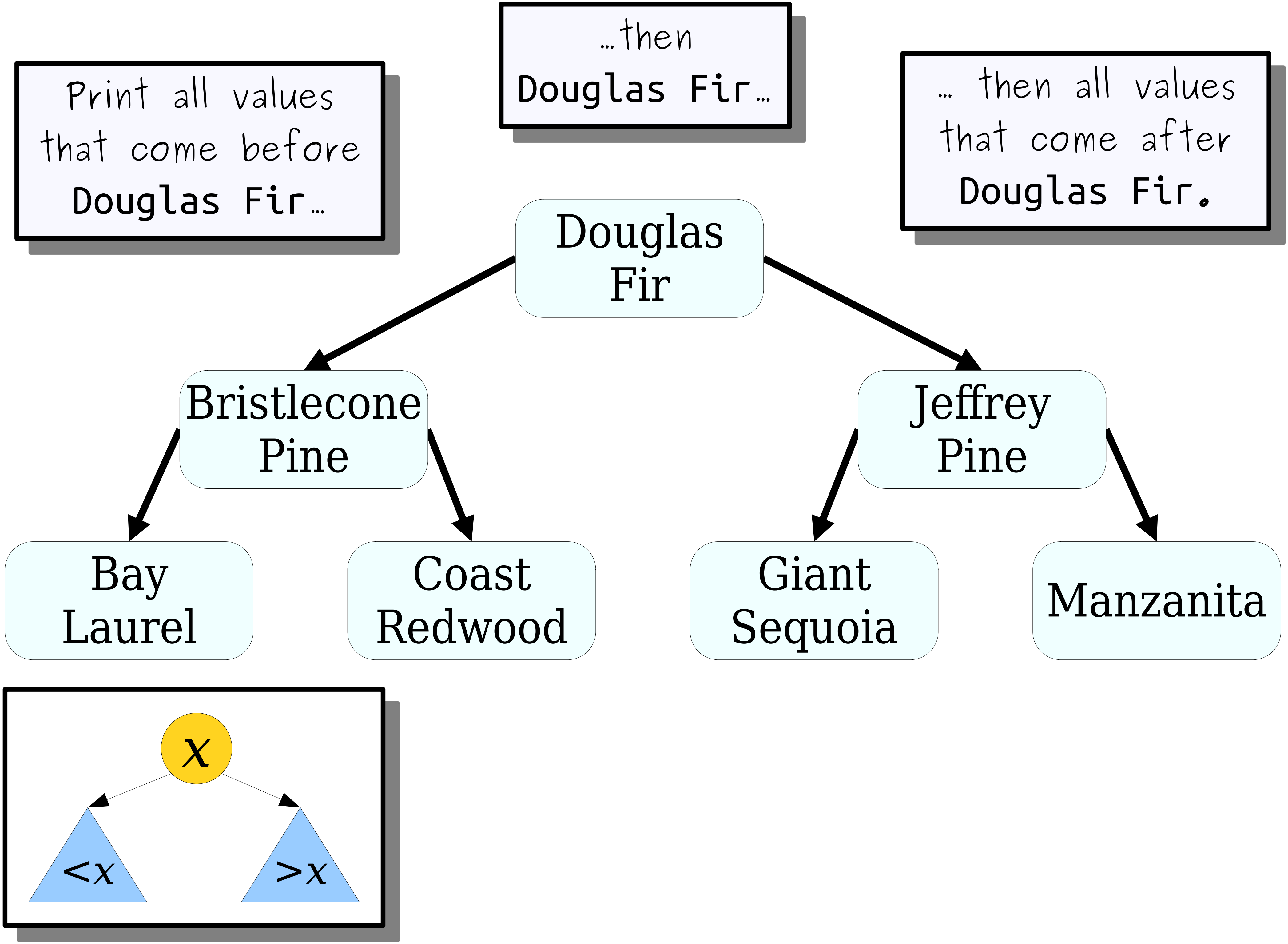
an empty tree,
represented by
`nullptr`, or…



… a single node,
whose left subtree
is a BST of
smaller values …



… and whose right
subtree is a BST
of larger values.



Print all values that come before Douglas Fir…
…then Douglas Fir…
… then all values that come after Douglas Fir.
Douglas Fir
Bristlecone Pine
Jeffrey Pine
Bay Laurel
Coast Redwood
Giant Sequoia
Manzanita
x
<x
>x

Inorder Traversals

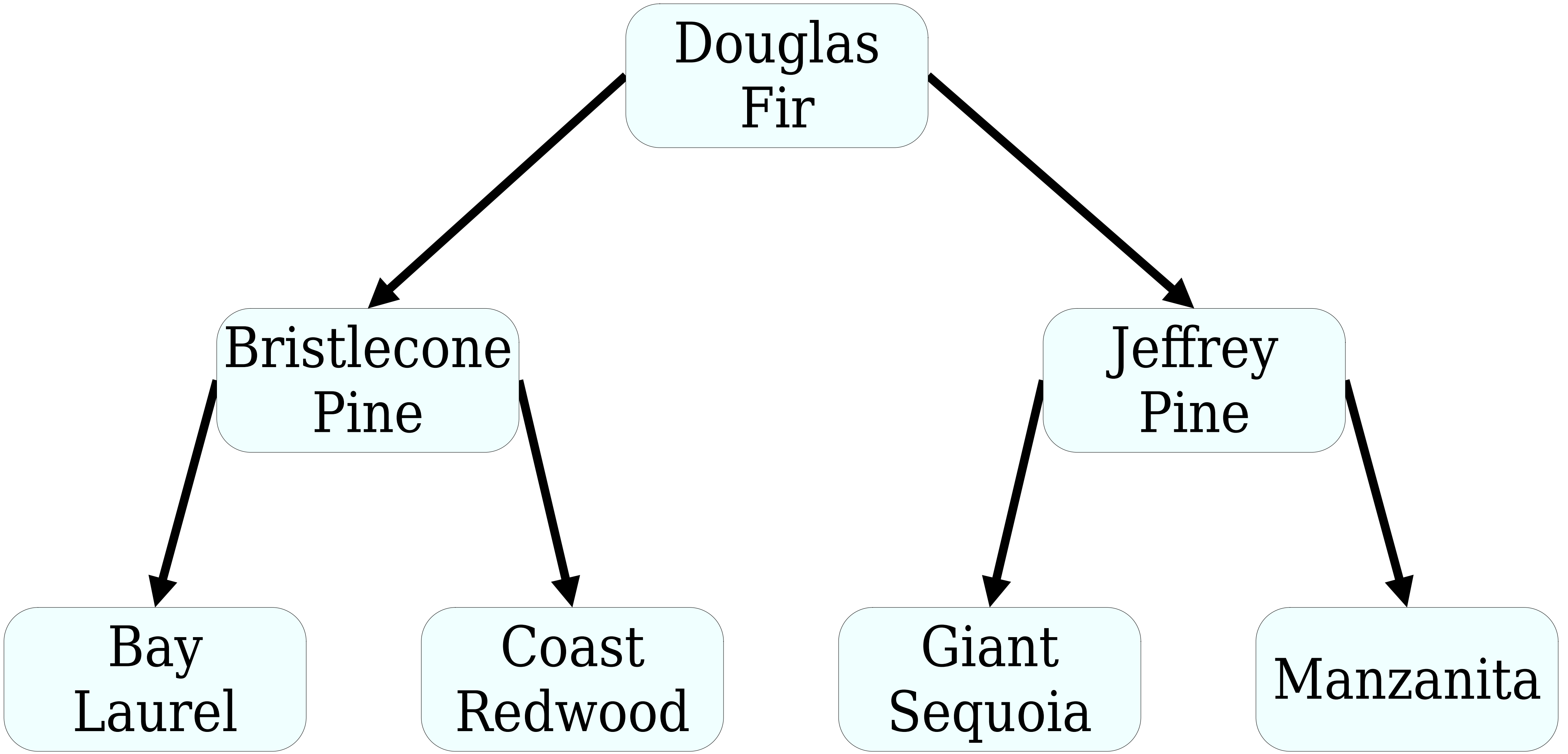
- The particular recursive pattern we just saw is called an ***inorder traversal*** of a binary tree.
- Specifically:
    - Recursively visit all the nodes in the left subtree.
    - Visit the node itself.
    - Recursively visit all the nodes in the right subtree.

What will happen if we swap these two lines?

Formulate a hypothesis!

What will happen if we swap these two lines?

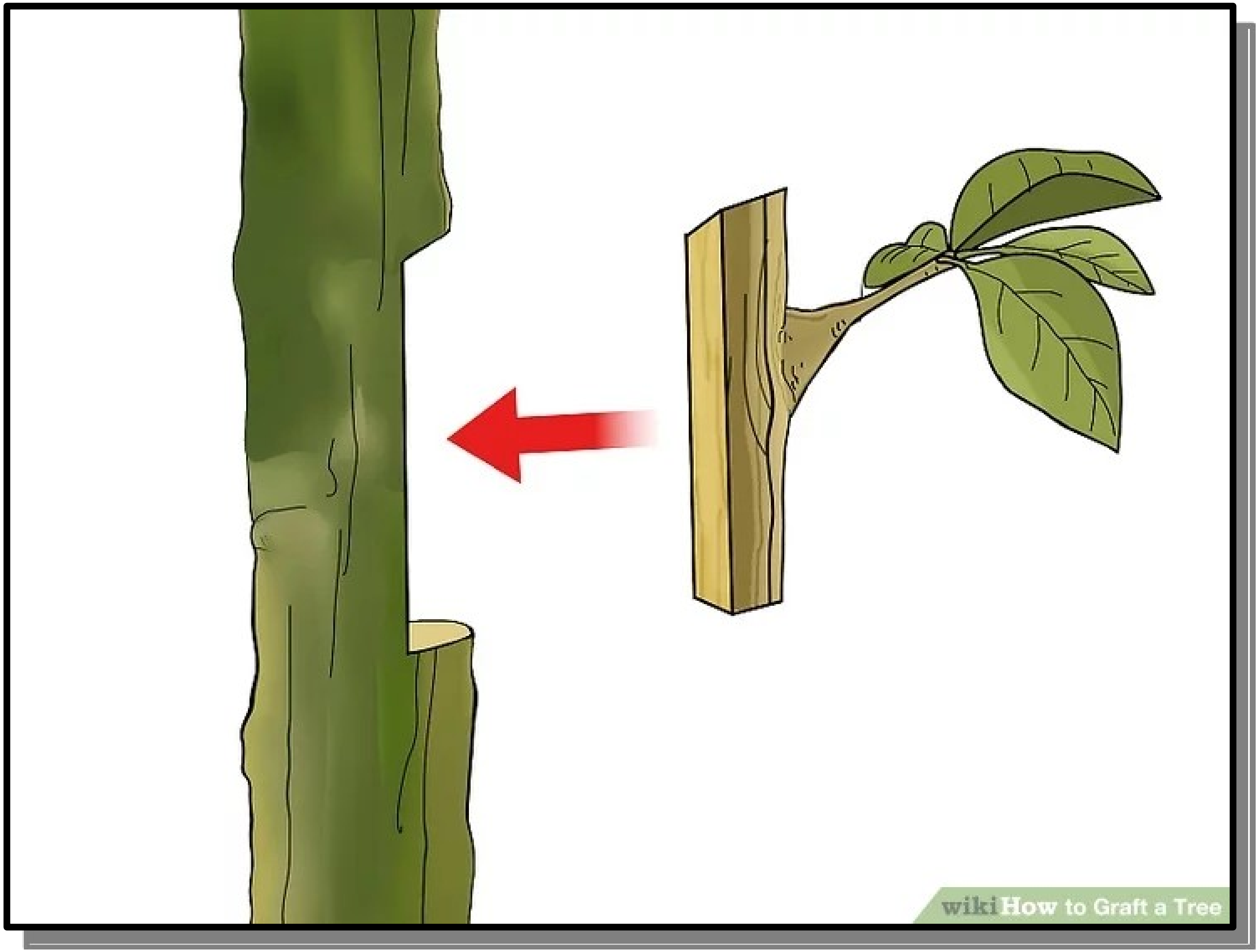
Discuss with your neighbor!



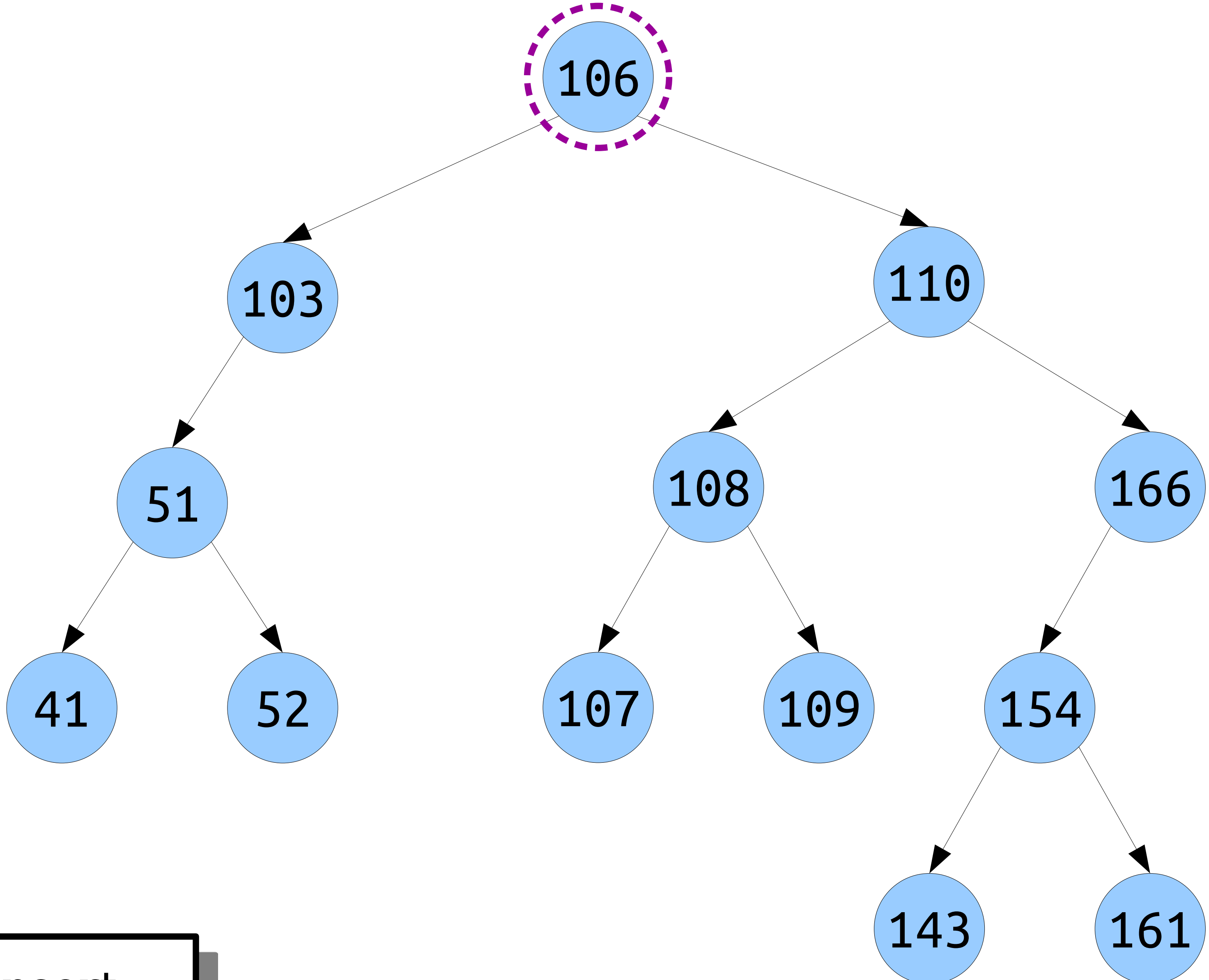
Douglas Fir
Bristlecone Pine
Jeffrey Pine
Bay Laurel
Coast Redwood
Giant Sequoia
Manzanita

Challenge problem:
Rewrite this function iteratively!

Adding to Trees



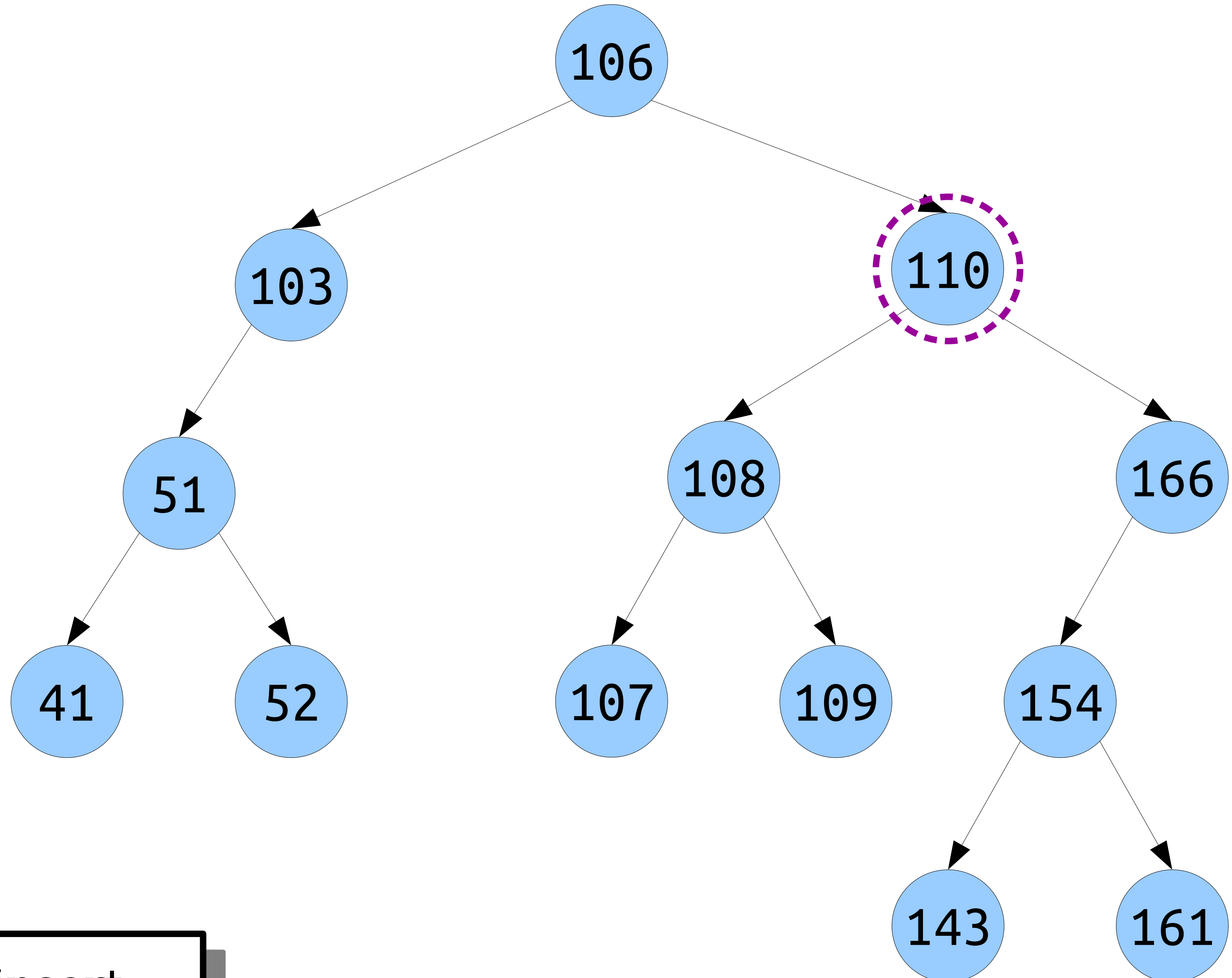
Thanks, WikiHow!



Let’s insert

147

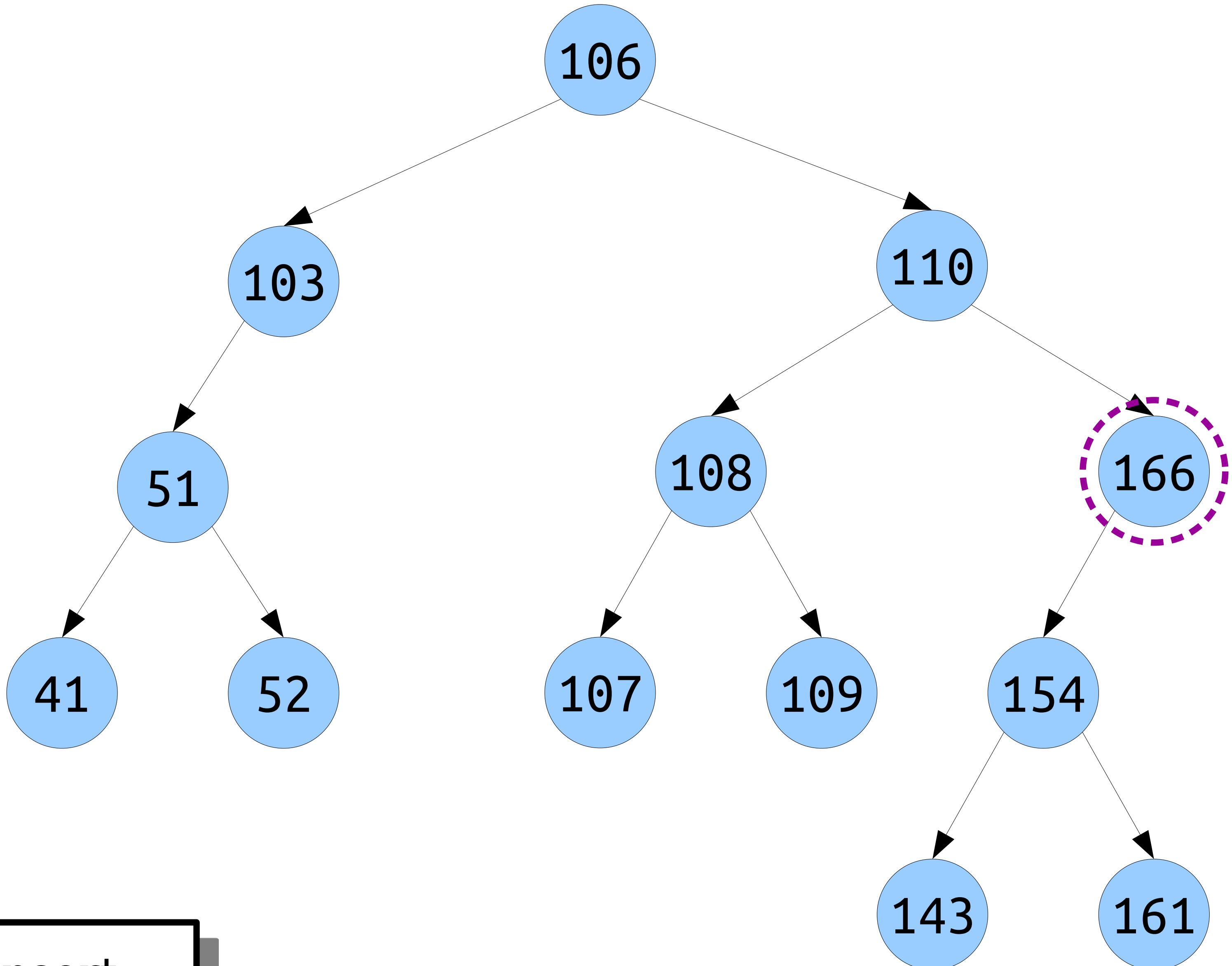
into this tree.



Let’s insert

147

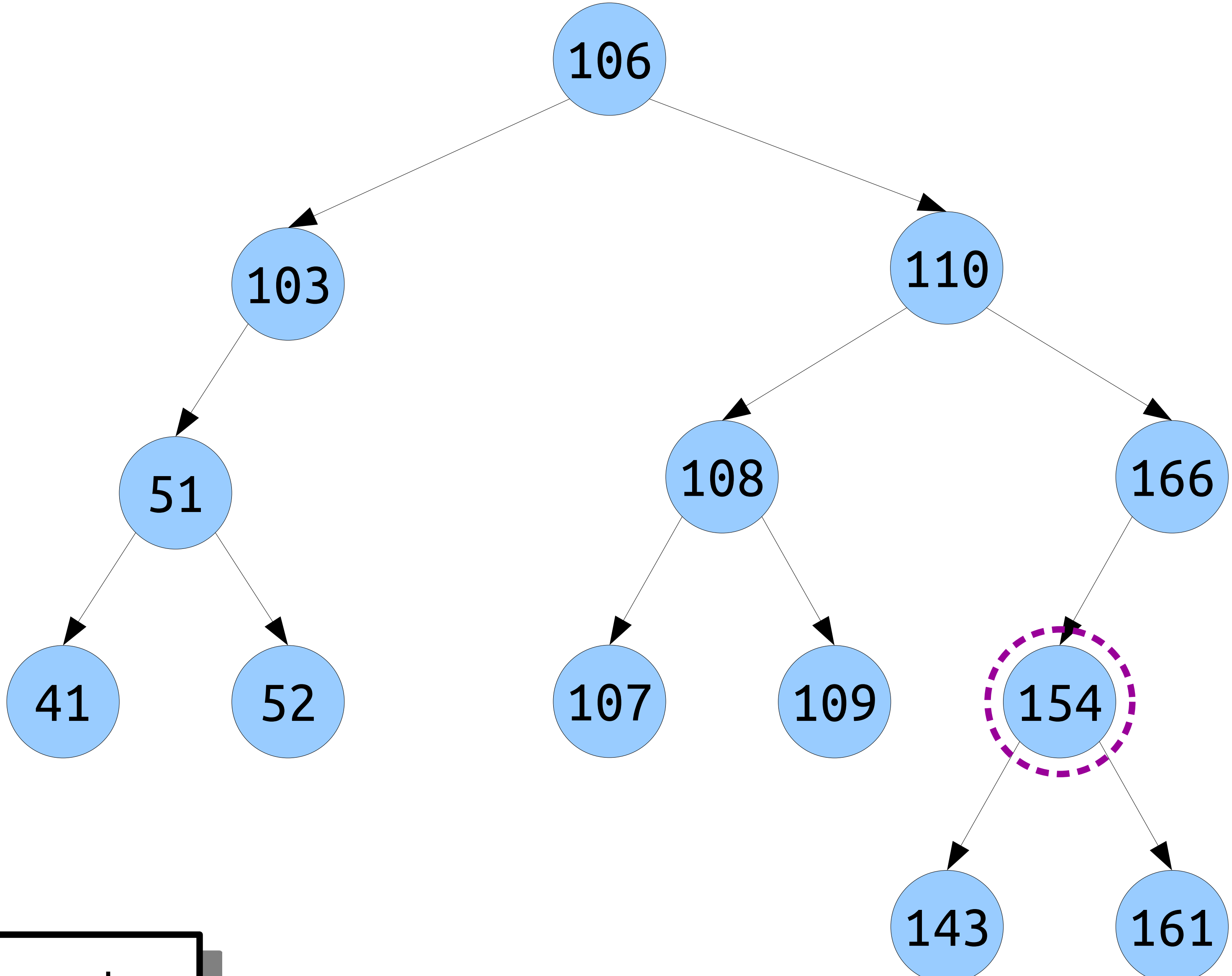
into this tree.



Let’s insert

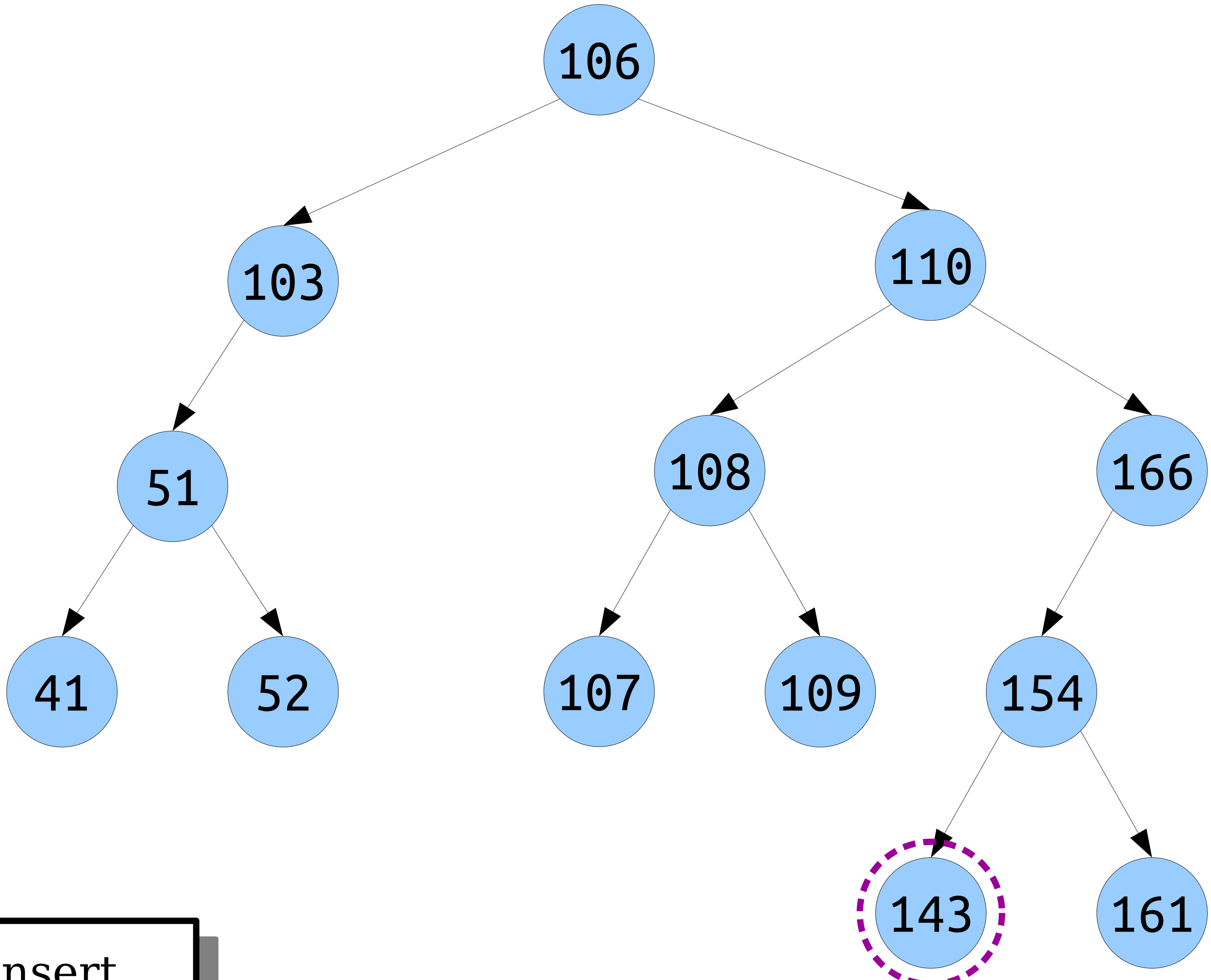
147

into this tree.



Let’s insert

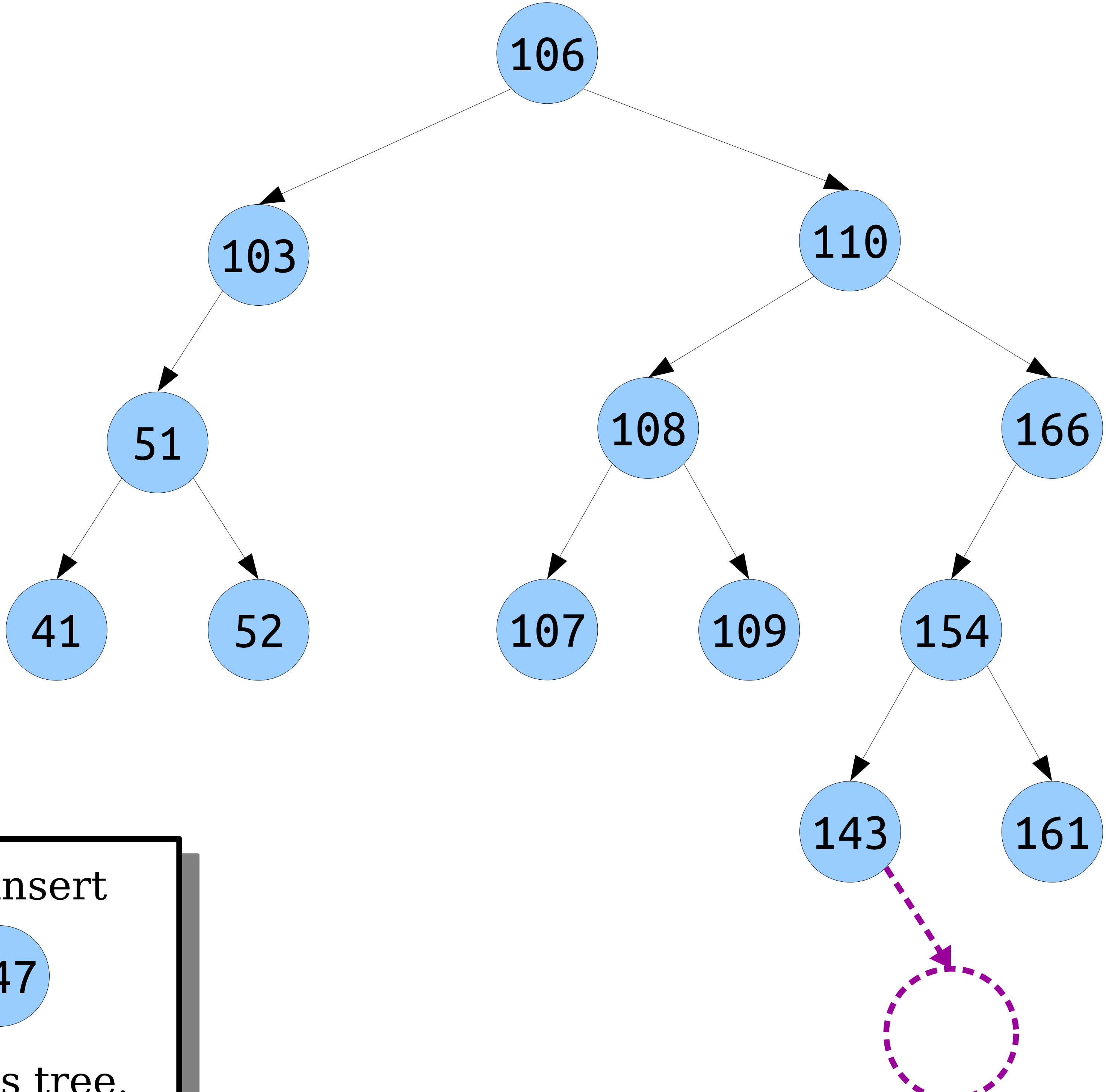
into this tree.



Let’s insert

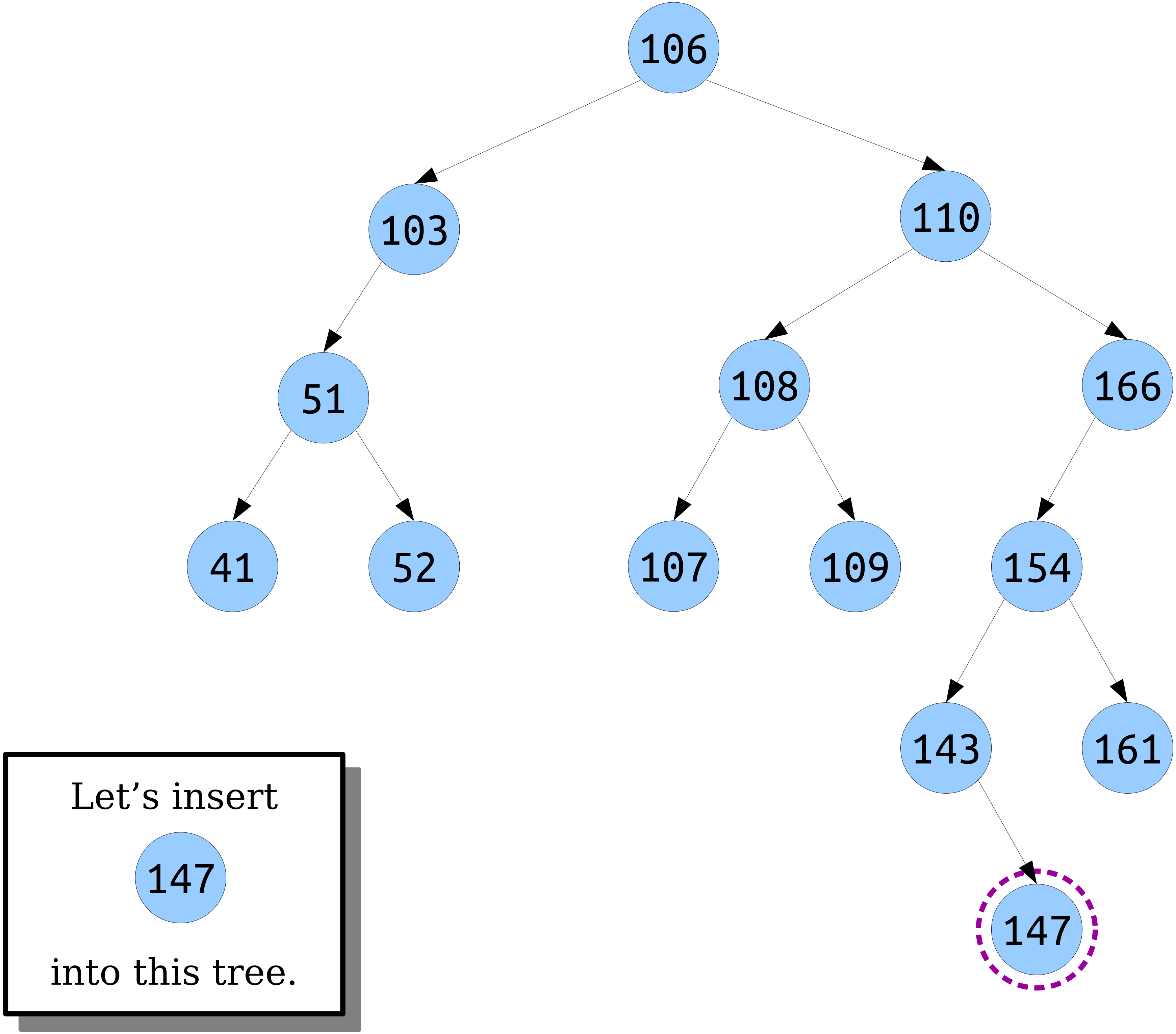
147

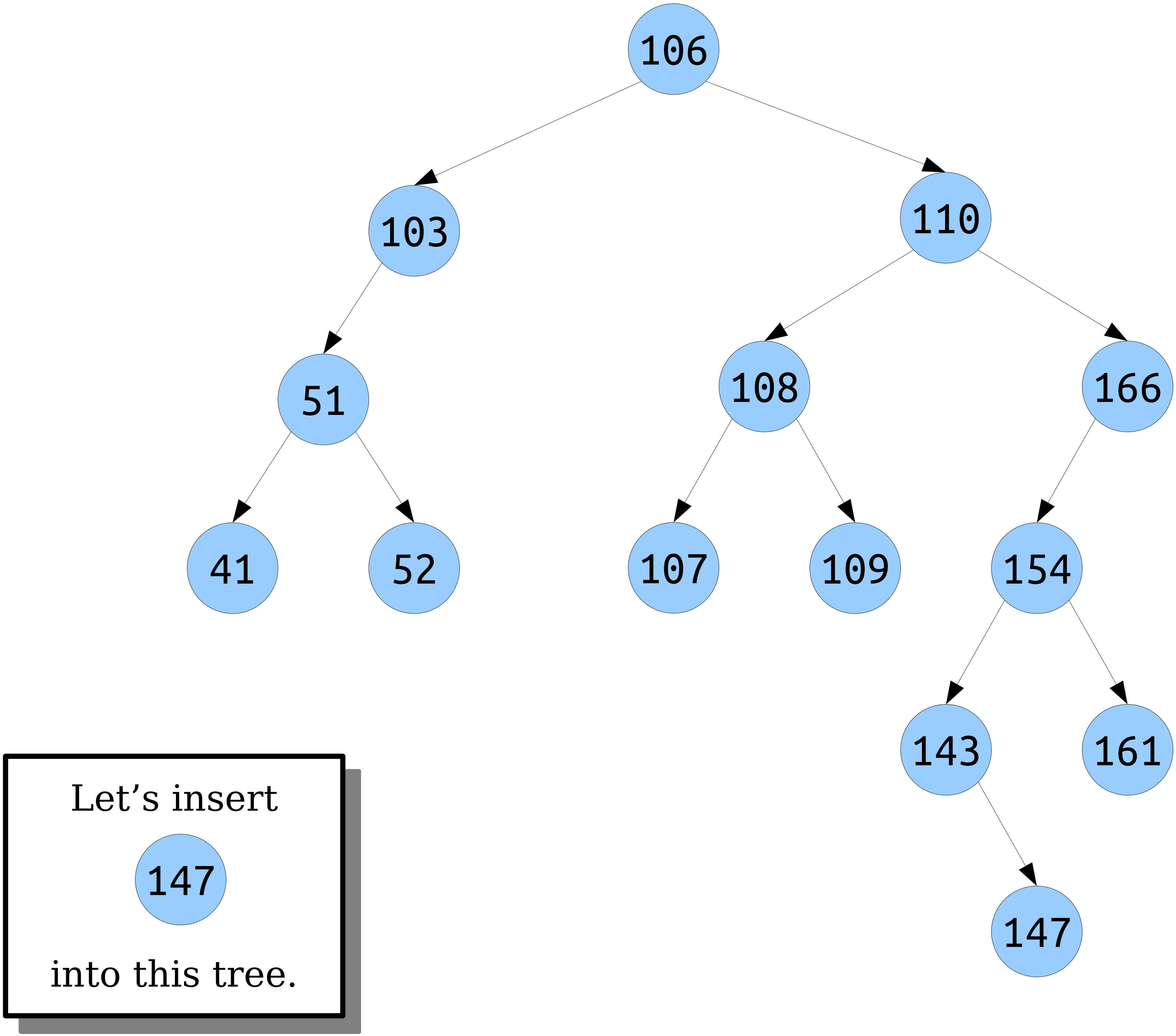
into this tree.



Let's insert

into this tree.


106
103
110
51
108
166
41
52
107
109
154
143
161
147
Let's insert
147
into this tree.


106
103
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51
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166
41
52
107
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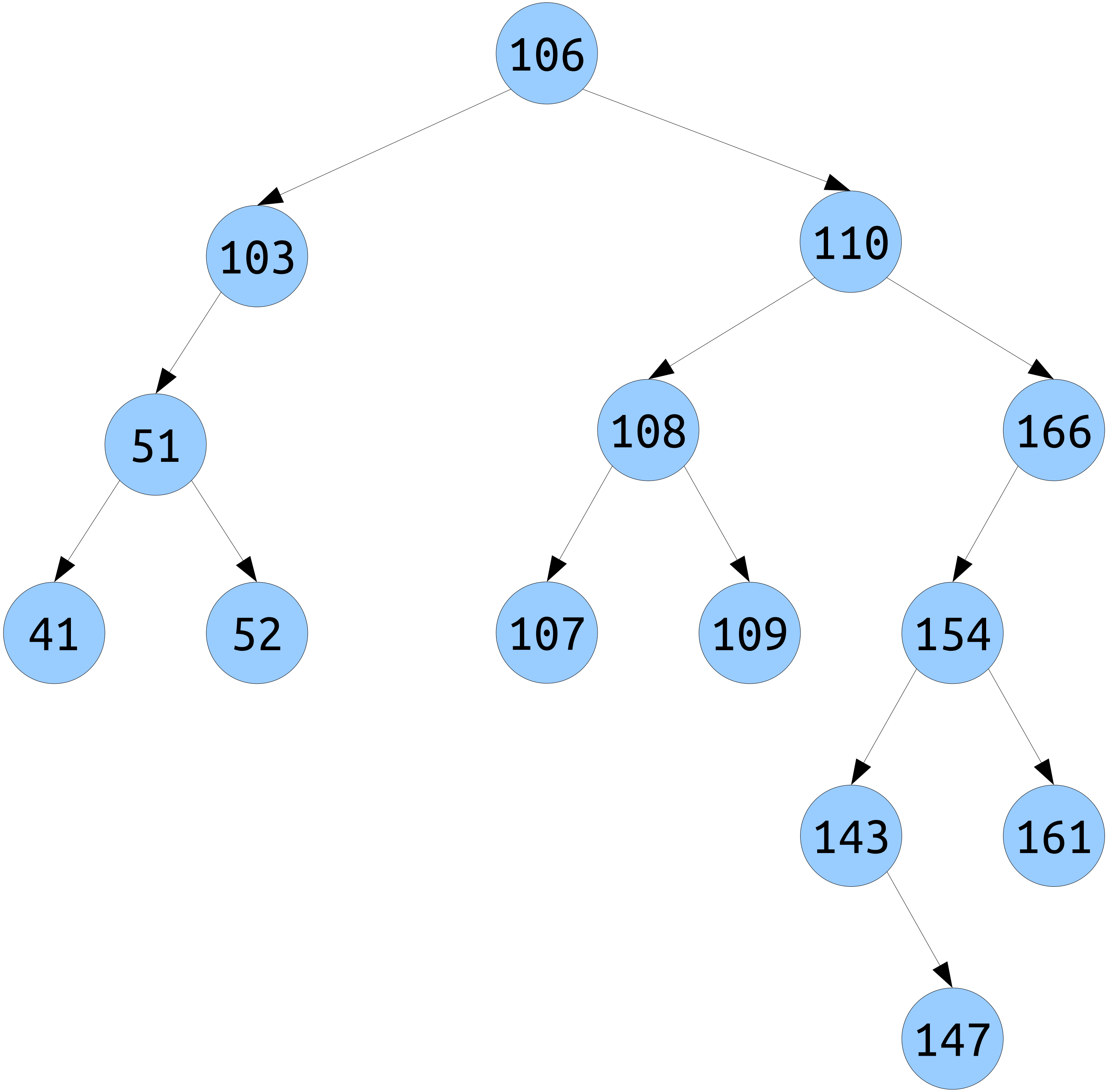
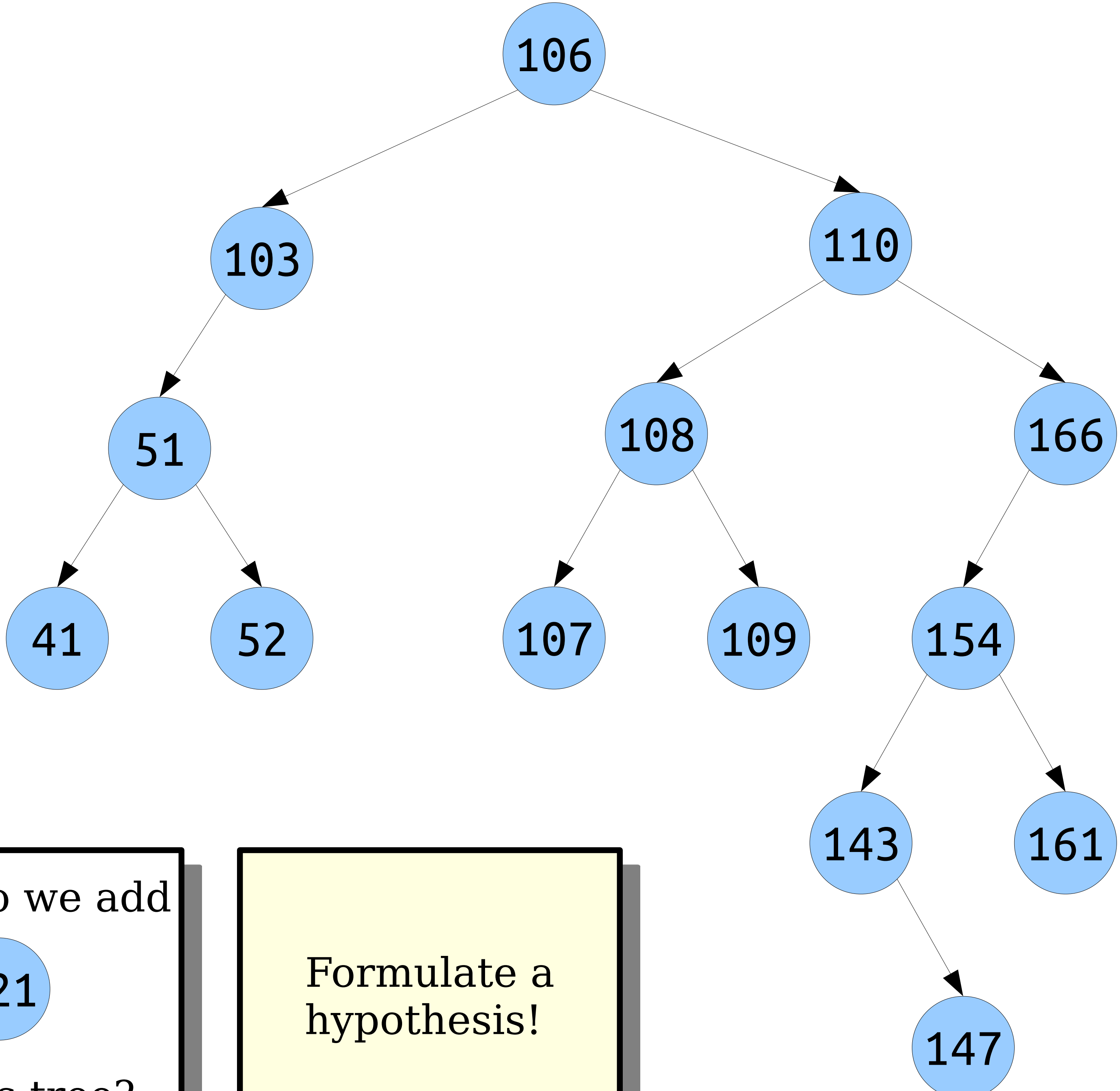
154
143
161
147
Let’s insert
147
into this tree.


106
103
110
51
108
166
41
52
107
109
154
143
161
147

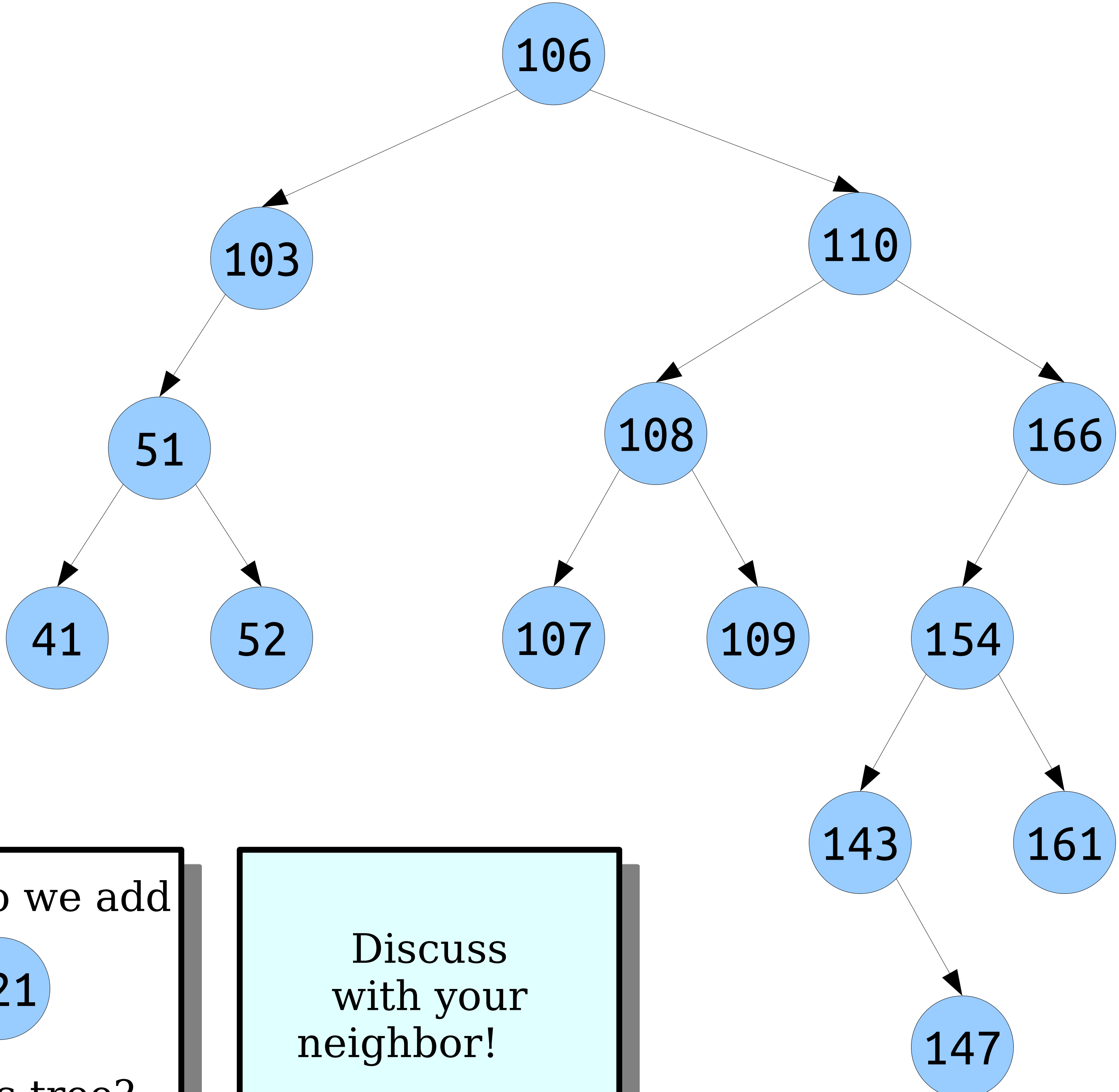


Where do we add

221

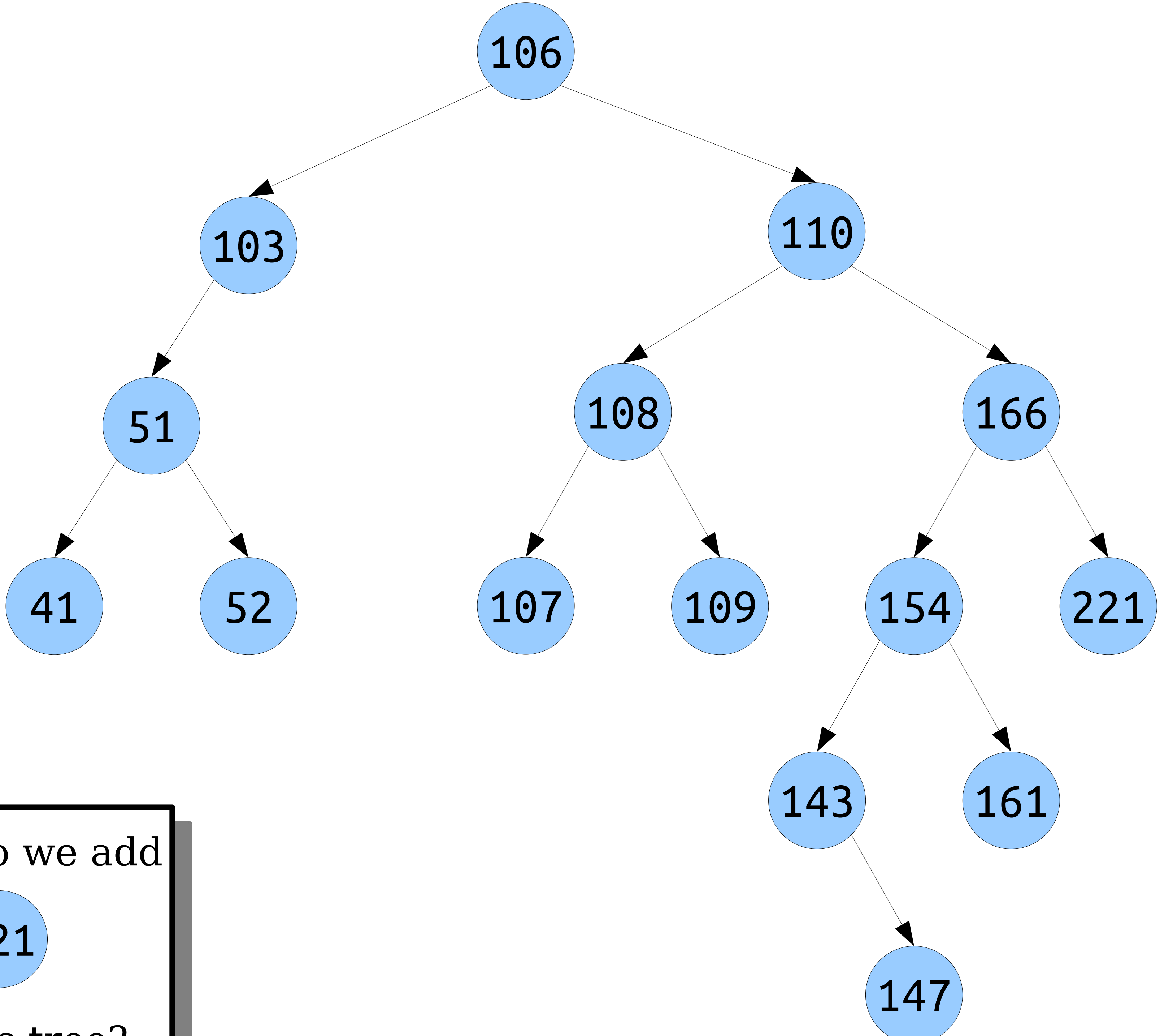
into this tree?

Formulate a hypothesis!



Where do we add 221 into this tree?

Discuss with your neighbor!



Where do we add

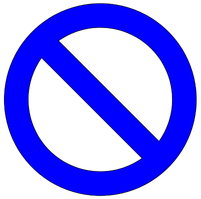
221

into this tree?

Let's Code it Up!

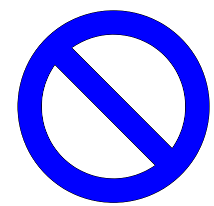
A Binary Search Tree Is Either…

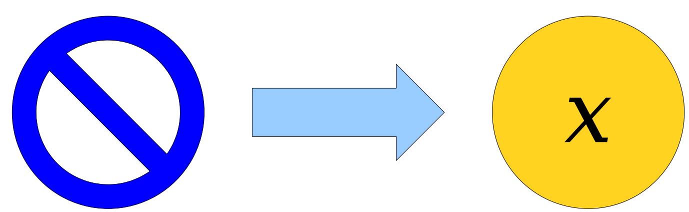
an empty tree,
represented by
`nullptr`



A Binary Search Tree Is Either…

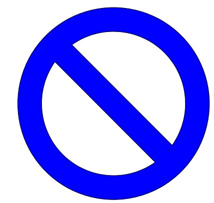
an empty tree,
represented by
nullptr



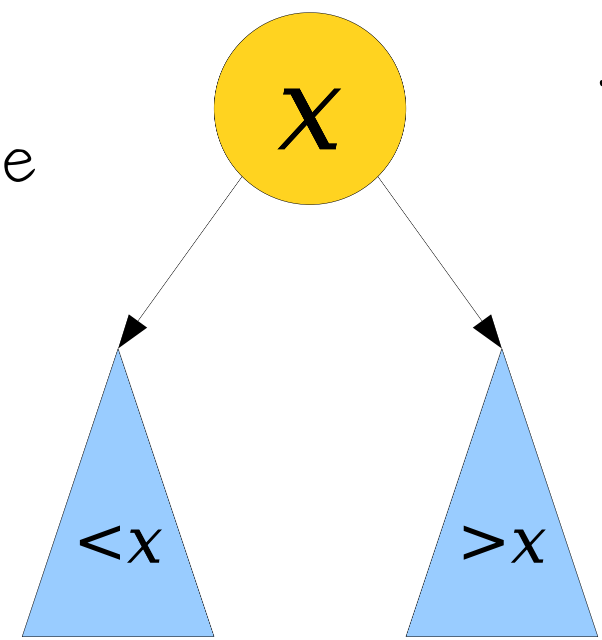


A Binary Search Tree Is Either…

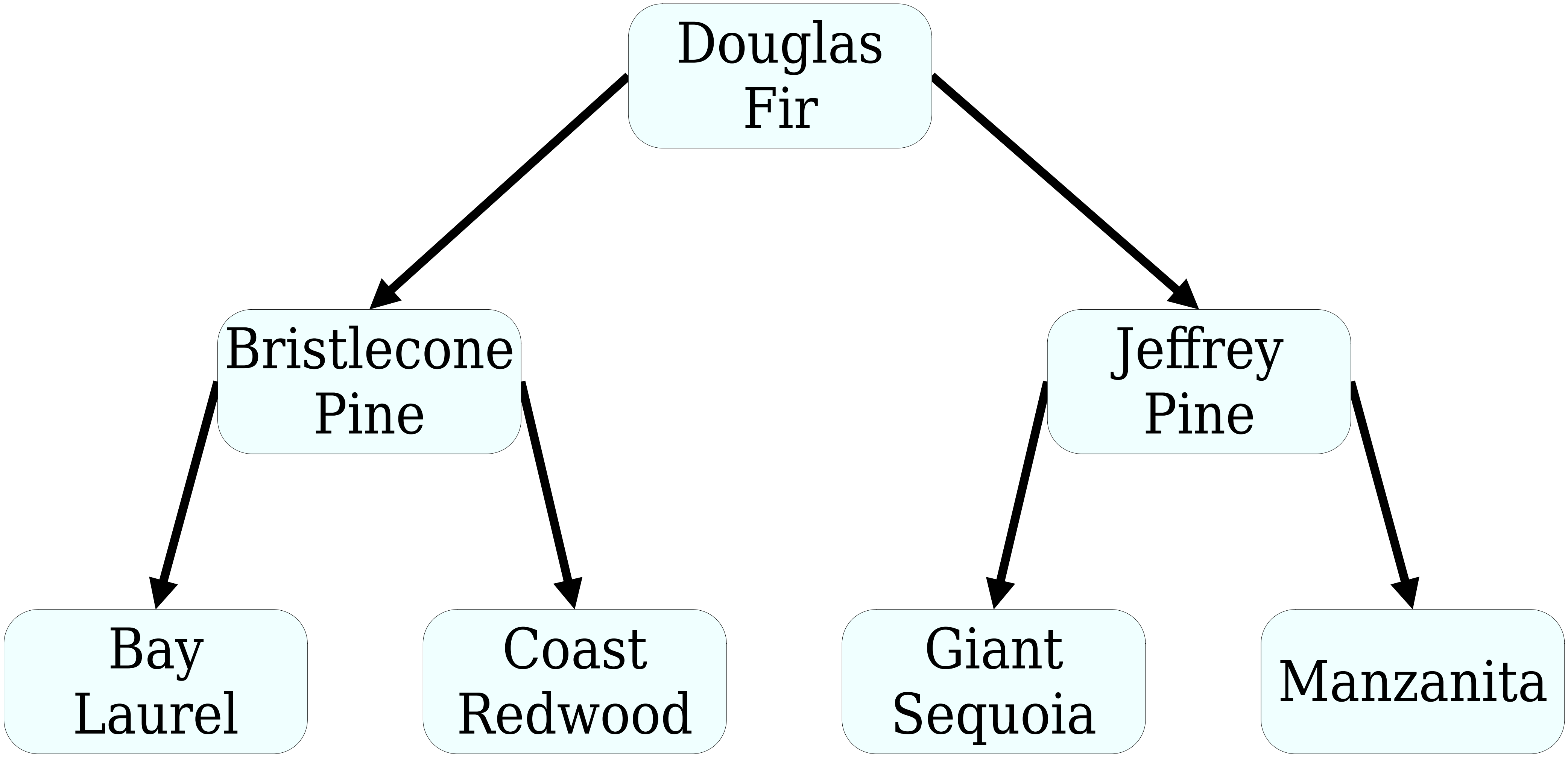
an empty tree, represented by `nullptr`, or…



… a single node, whose left subtree is a BST of smaller values …



… and whose right subtree is a BST of larger values.


Douglas Fir
Bristlecone Pine
Jeffrey Pine
Bay Laurel
Coast Redwood
Giant Sequoia
Manzanita

Your Action Items

- ***Read Chapter 16.1 – 16.2.***
  - There’s a bunch of BST topics in there, along with a different intuition for how they work.
- ***Work on Assignment 8.***
  - Slow and steady progress is the name of the game here!

Next Time

- ***Tree Heights***
  - Many trees can hold the same keys. How do we compare them?
- ***Freeing Trees***
  - Reclaiming memory in a tree.
- ***Range Searches***
  - Quickly finding all values in a range.