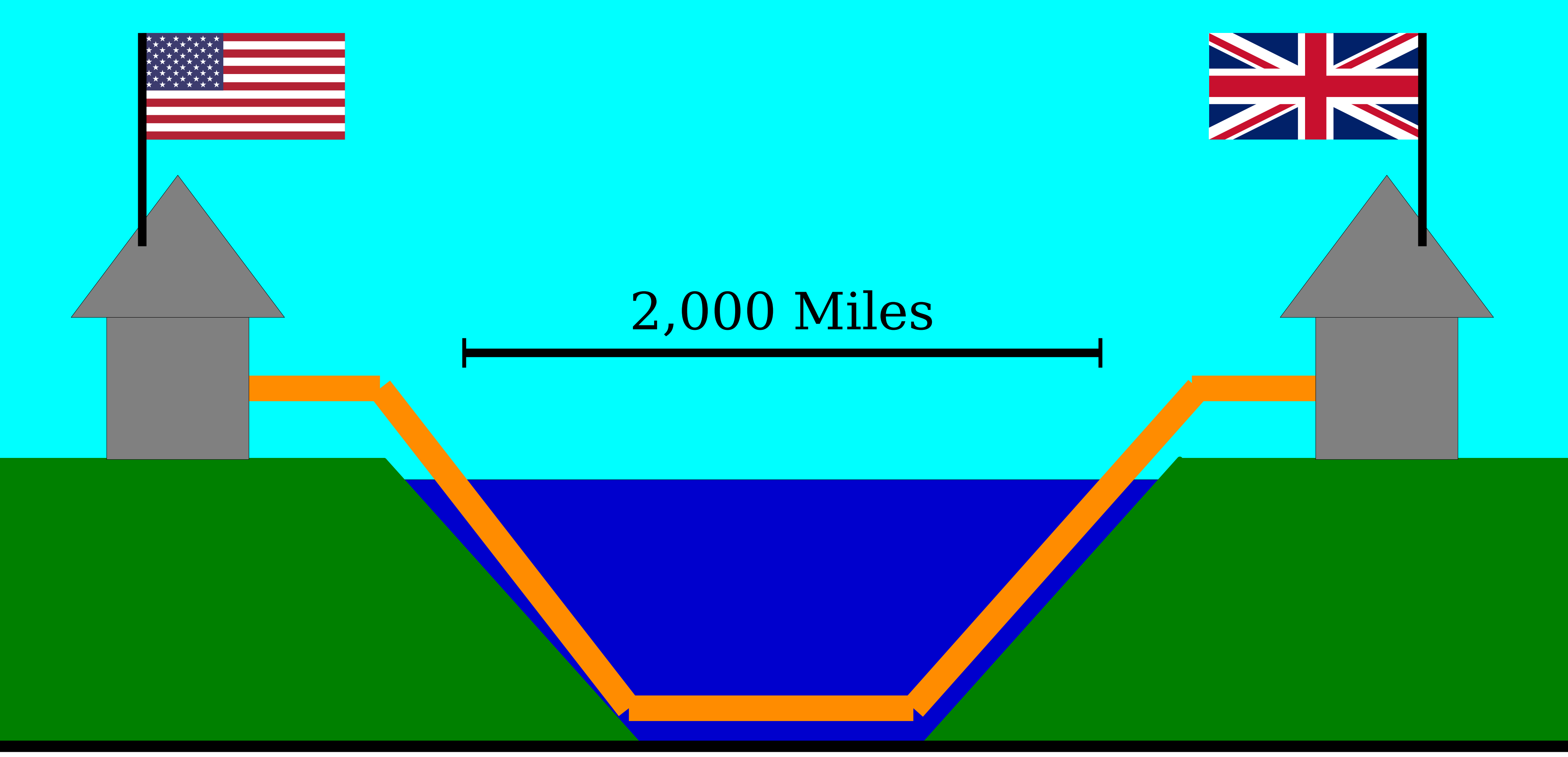
Beyond Data Structures

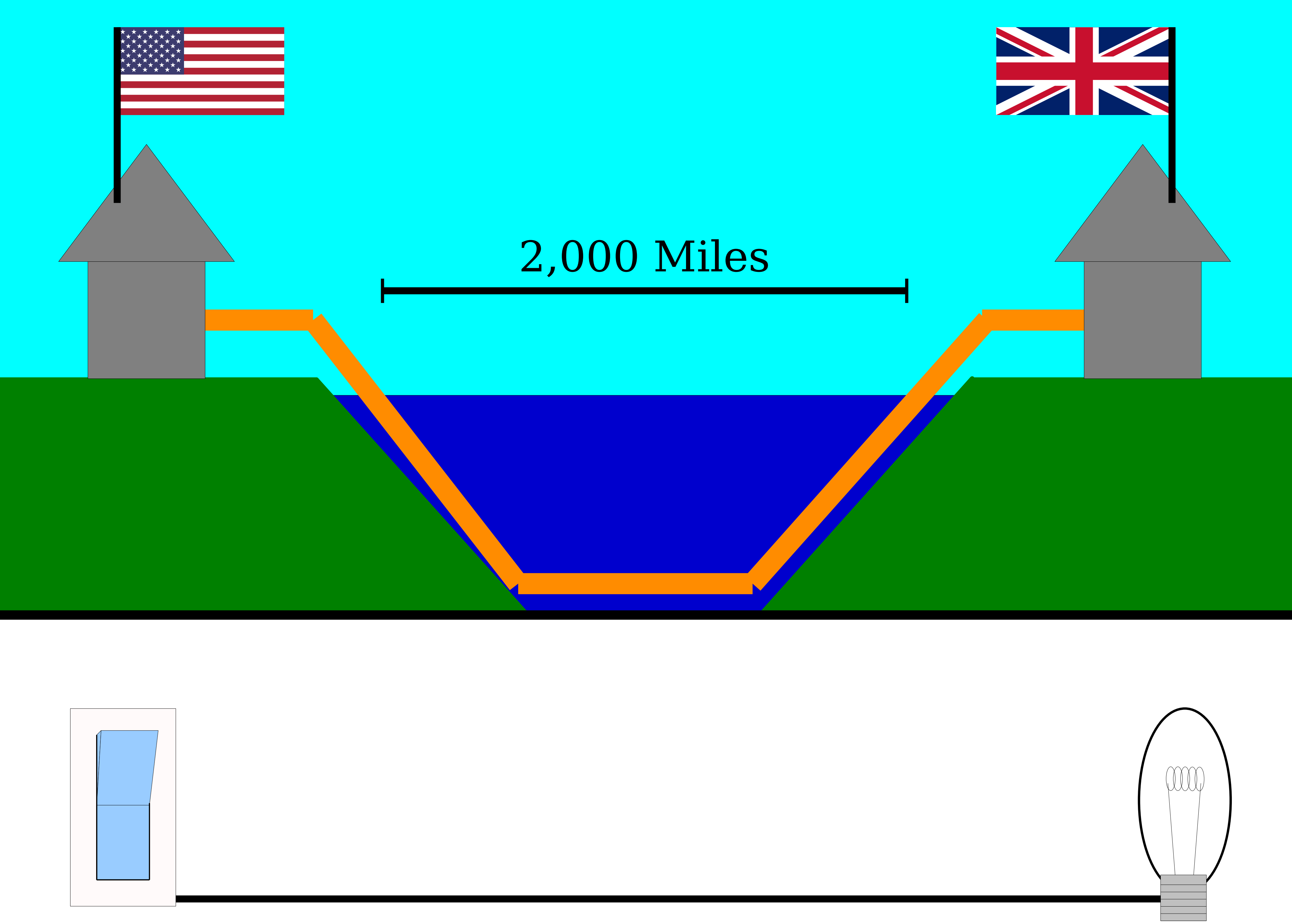
Outline for Today

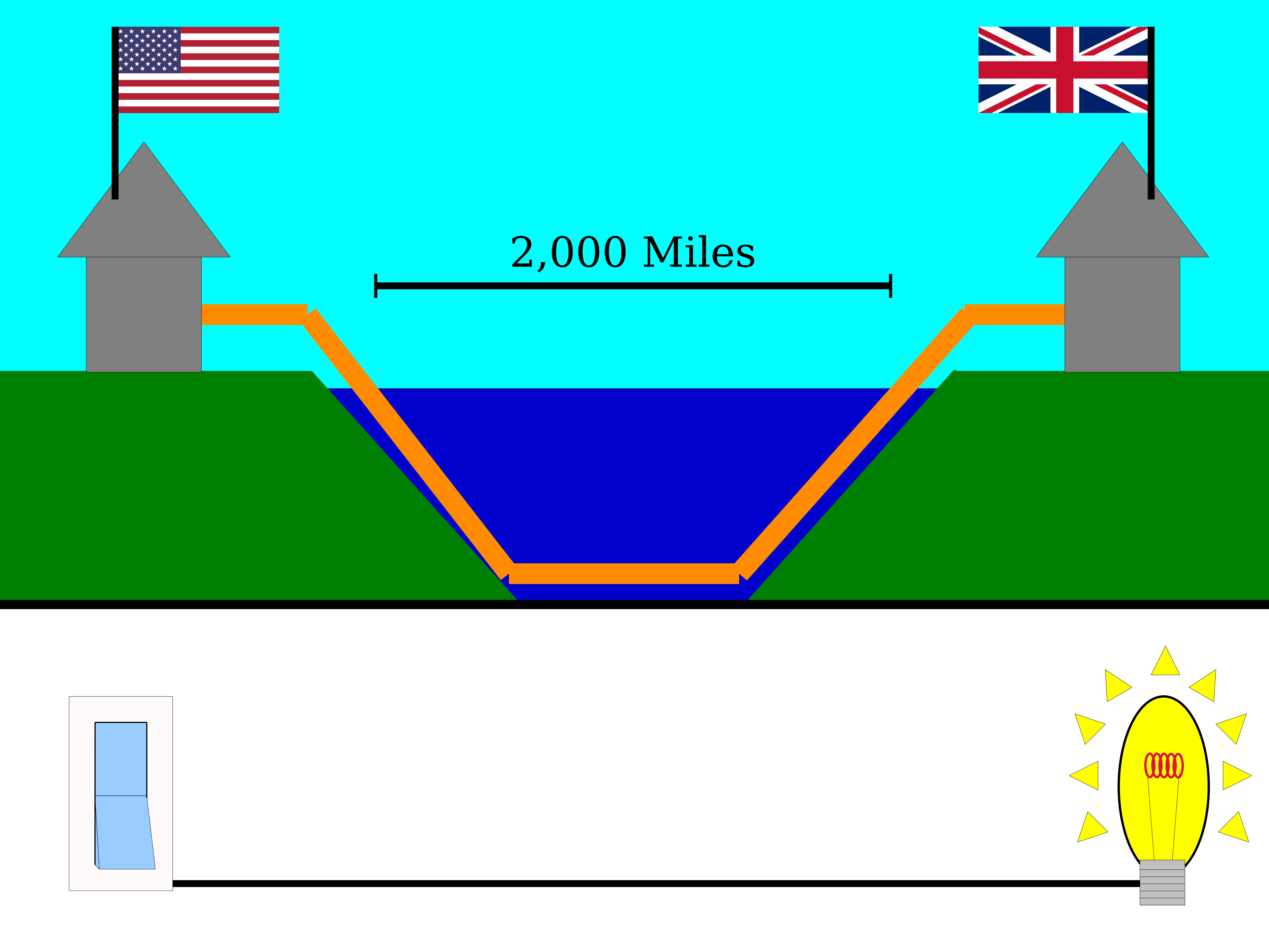
- ***Bits and Bytes***
  - Representing things with 0s and 1s.
- ***Data Compression***
  - Reducing transmission requirements.
- ***Prefix-Free Codes***
  - A clever space-saving trick.
- ***Huffman Coding***
  - Finding good prefix-free codes.

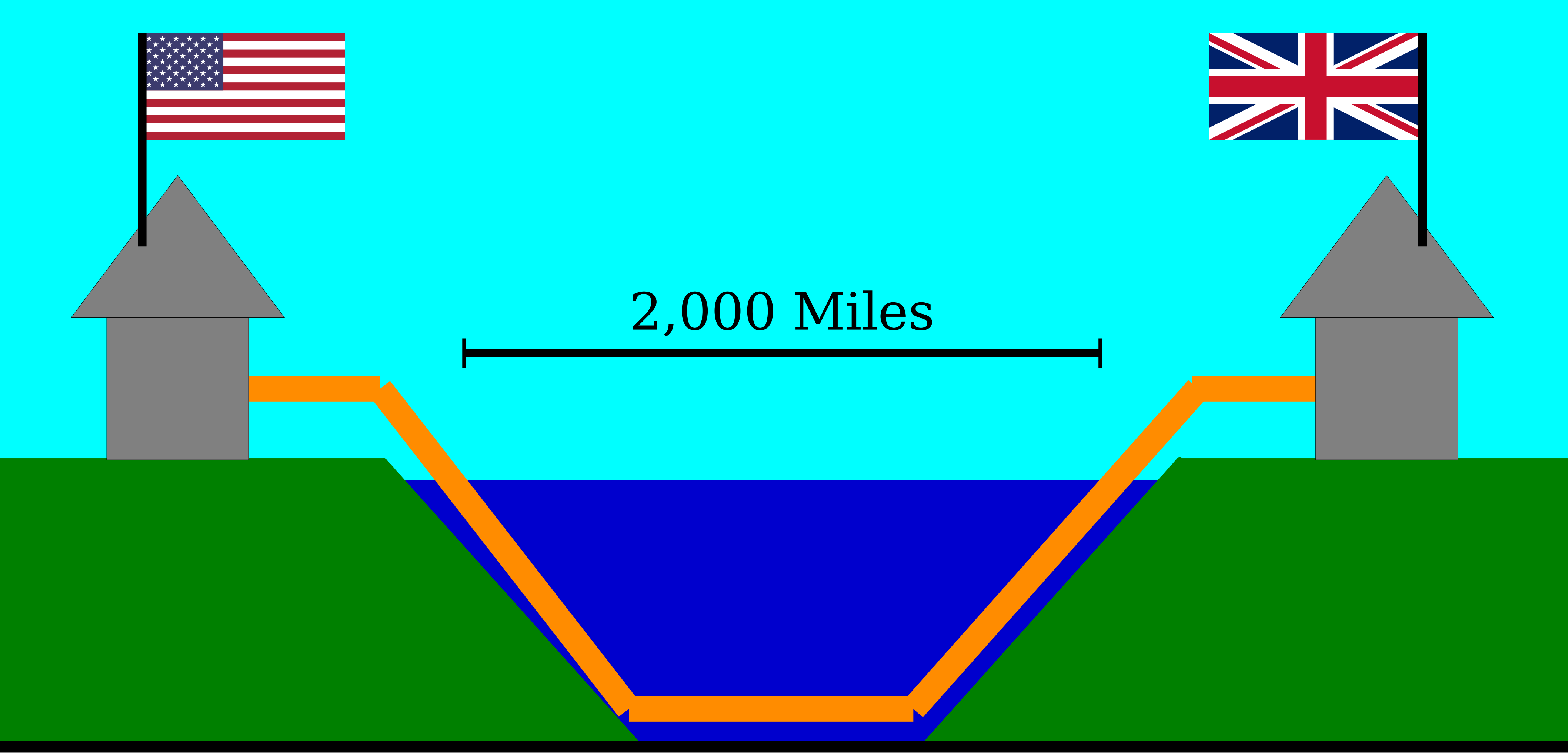
Bits and Bytes

19th Century Data Transmission


2,000 Miles


2,000 Miles


2,000 Miles


2,000 Miles

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

-•• •• -•- -•• •• -•-

A	•-	H	••••	O	---	V	•••-
B	-•••	I	••	P	•--•	W	•--
C	-•-•	J	•---	Q	--•-	X	-••-
D	-••	K	-•-	R	•-•	Y	-•--
E	•	L	•-••	S	•••	Z	--••
F	••-•	M	--	T	-		
G	--•	N	-•	U	••-		

What is the title of this slide?

Formulate a hypothesis!

–•• •• –•– –•• •• –•–

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

What is the title of this slide?

Discuss with your neighbors!

D I K D I K

–•• •• –•– –•• •• –•–

A	•–	H	••••	O	–––	V	•••–
B	–•••	I	••	P	•––•	W	•––
C	–•–•	J	•–––	Q	––•–	X	–••–
D	–••	K	–•–	R	•–•	Y	–•––
E	•	L	•–••	S	•••	Z	––••
F	••–•	M	––	T	–		
G	––•	N	–•	U	••–		

20th Century Data Transmission

It's All Bits and Bytes

- Digital data is stored as sequences of `0`s and `1`s.
    - They're usually encoded by magnetic orientation on small (10nm!) metal particles or by trapping electrons in nanoscale gates.
- A single `0` or `1` is called a ***bit***.
- A group of eight bits is called a ***byte***.

`00000000`, `00000001`, `00000010`,
`00000011`, `00000100`, `00000101`, …

- There are $2^8 = 256$ different bytes.
    - ***Great practice:*** Write a function to list all of them!

Representing Text

- We think of strings as being made of characters representing letters, numbers, emojis, etc.
- Internally to the computer, everything is just a series of bits.
- To bridge the gap, we need to agree on some way of representing characters as sequences of bits.
- ***Idea:*** Assign each character a sequence of bits called a ***code***.

ASCII

- Early (American) computers needed some standard way to send output to their (physical!) printers.
- Since there were fewer than 256 different characters to print (1960’s America!), each character was assigned a one-byte value.
- This initial code was called ***ASCII***. It still lives on in a modified form as ***UTF-8***, which you saw on Assignment 2.
- For example, the letter `A` is represented by the byte `01000001` (65). You can still see this in C++:

```
cout << int('A') << endl; // Prints 65
```

0100100001000101010000000101000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

Formulate a hypothesis!

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

010010000100010101000001010001000

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

Chat with your neighbors!

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

0100100001000101010000000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H 0100010101000000101000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H 01000101010000000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E 0100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E 0100000101000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E A 01000100

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E A 01000100

- Here’s a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

H E A D

- Here's a small segment from the ASCII encodings for characters.
- What is the title of this slide?

character	*code*
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000

An Observation

- In ASCII, every character has exactly the same number of bits in it.
- Any message with n characters will use up exactly $8n$ bits.
  - Space for `CS106BLECTURE`: 104 bits.
  - Space for `COPYRIGHTABLE`: 104 bits.
- ***Question:*** Can we reduce the number of bits needed to encode text?

KIRK'S DIKDIK

A Different Encoding

- ASCII uses one byte per character. There are 256 possible bytes.
- If we're specifically writing the string **`KIRK'S DIKDIK`**, which has only seven different characters, using full bytes is wasteful.
- Here's a three-bit encoding we can use to represent the letters in **`KIRK'S DIKDIK`**.
- This uses 37.5% as much space as what ASCII uses.

character	*code*
K	000
I	001
R	010
'	011
S	100
␣	101
D	110

000	001	010	000	011	100	101	110	001	000	110	001	000
K	I	R	K	'	S	␣	D	I	K	D	I	K

Where We're Going

- Storing data using the ASCII encoding is portable across systems, but is not ideal in terms of space usage.
- Building custom codes for specific strings might let us save space.
- ***Idea:*** Use this approach to build a ***compression algorithm*** to reduce the amount of space needed to store text.

The Key Idea

- If we can find a way to

    give all characters a bit pattern,

    that both the sender and receiver know about, and

    that can be decoded uniquely,

  then we can represent the same piece of text in multiple different ways.
- ***Goal:*** Find a way to do this that uses *less space* than the standard ASCII representation.

Exploiting Redundancy

- Not all letters have the same frequency in **KIRK'S DIKDIK**.
- Here's the frequencies of each letter.
- So far, we've given each letter codes of the same length.
- ***Key Question:*** Can we give shorter encodings to more common characters?

character	*frequency*
K	4
I	3
D	2
R	1
'	1
S	1
␣	1

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111000001000010

0	1	01	0	10	11	100	00	1	0	00	1	0
K	I	R	K	'	S	␣	D	I	K	D	I	K

A First Attempt

character	*code*
K	**0**
I	**1**
D	**00**
R	**01**
'	**10**
S	**11**
␣	**100**

01010101110000100010

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111000010010

0	1	01	0	10	11	100	00	1	0	00	1	0
K	I	R	K	'	S	␣	D	I	K	D	I	K

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111000010010

01	01	01	01	1	10	0	00	10	0	0	10
R	R	R	R	I	'	K	D	'	K	K	'

A First Attempt

character	*code*
K	0
I	1
D	00
R	01
'	10
S	11
␣	100

0101010111 0

01	01	01	01	1	10	0	00	10	0	0	10
R	R	R	R	I	'	K	D	'	K	K	'

The Problem

- If we use a different number of bits for each letter, we can't necessarily uniquely determine the boundaries between letters.
- We need an encoding that makes it possible to determine where one character stops and the next starts.
- Is this possible? If so, how?

Prefix-Free Codes

- A ***prefix-free code*** is an encoding system in which no code is a prefix of another code.
- Here's a sample prefix code for the letters in **KIRK'S DIKDIK**.

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

`1001001100001101110011101101110110`

10	**01**	**001**	**10**	**000**	**1101**	**1100**	**111**	**01**	**10**	**111**	**01**	**10**
K	**I**	**R**	**K**	**'**	**S**	**␣**	**D**	**I**	**K**	**D**	**I**	**K**

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

1001001100001101110011101101110110

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10010011000011011100111011011110110

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

100100110000110111001110110111010110

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

100100110000110111001110110111011 0

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

100100110000110111001110110111011 0

10
K

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10
K

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10**0**100110000110111001110110111011 0

10
K

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

10**01**0011000011011100111011011110110

10
K

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10**01**001100001101110011101101110110

10	01
K	I

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10	01
K	I

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10010011000011011100111011011110110

10	01
K	I

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**00**110000110111001110110111011 0

10	01
K	I

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**001**10000110111001110110110110

10	01
K	I

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001**001**10000110111001110110111011O

10	01	001
K	I	R

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10	01	001
K	I	R

Prefix-Free Codes

- Using this prefix code, we can represent **`KIRK'S DIKDIK`** as the sequence

  `1001001100001101110011101101110110`

- This uses just 34 bits, compared to our initial 104. Wow!
- But where did this code come from? How could you come up with codes like this for other strings?

character	code
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

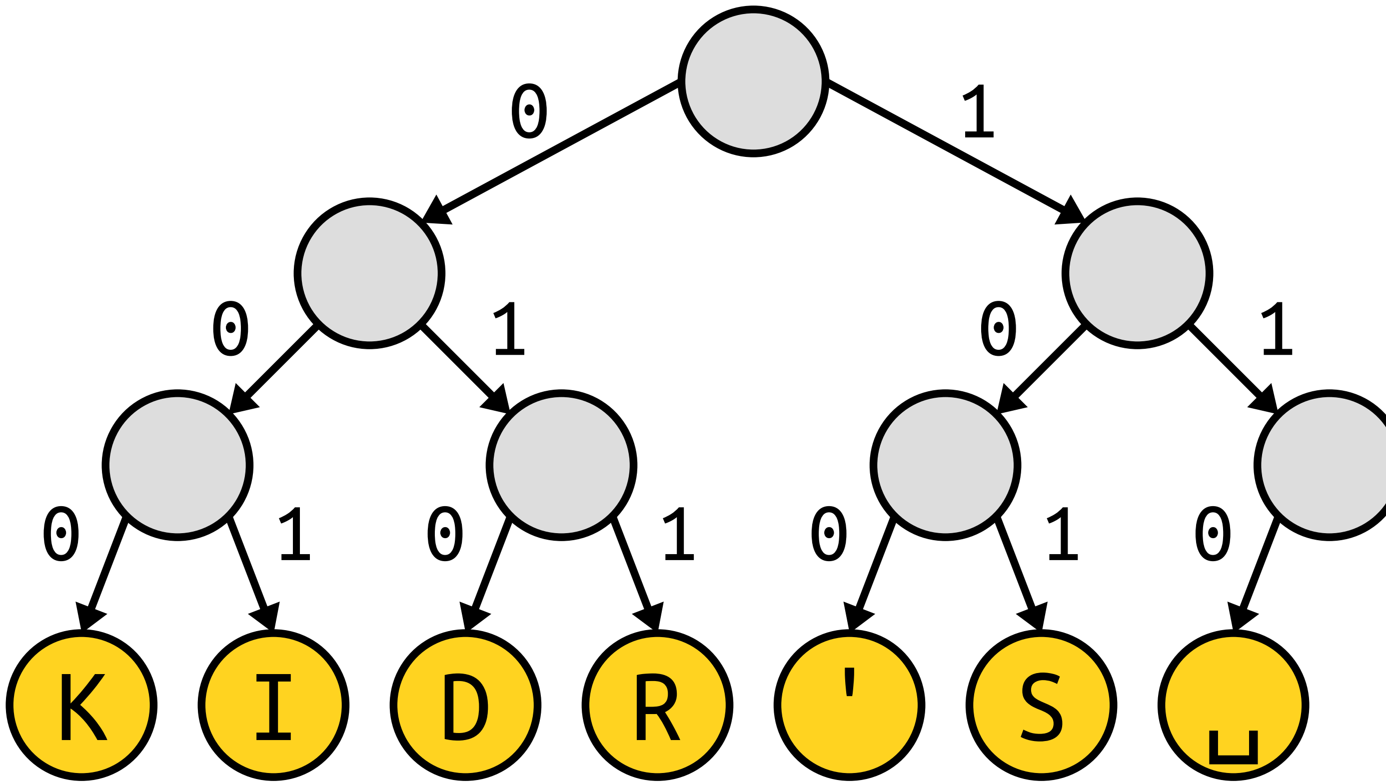
character	code
K	1111110
I	111110
D	11110
R	1110
'	110
S	10
␣	0

1111110111110111011111101101001111011111011111101111011111101111110

How do you find a “good” prefix-free code?

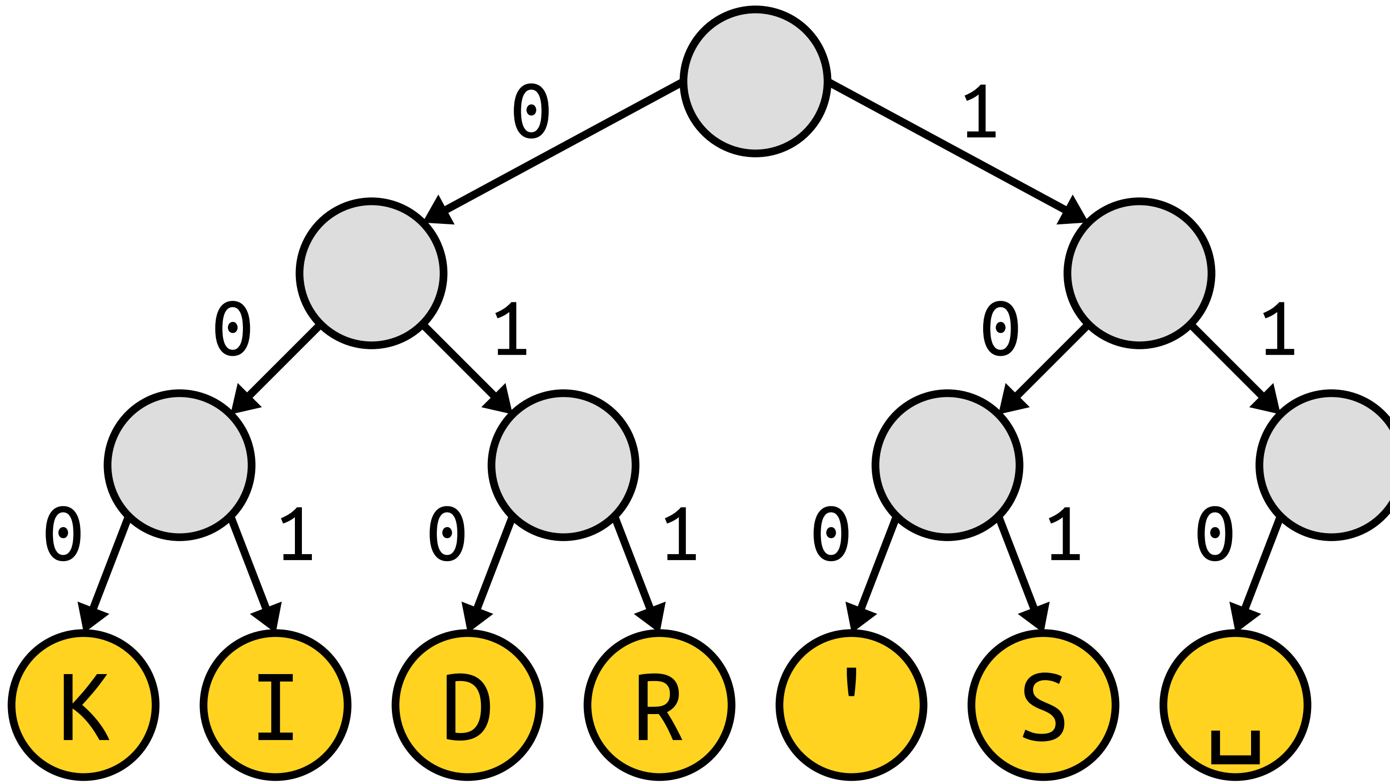
The Main Insight

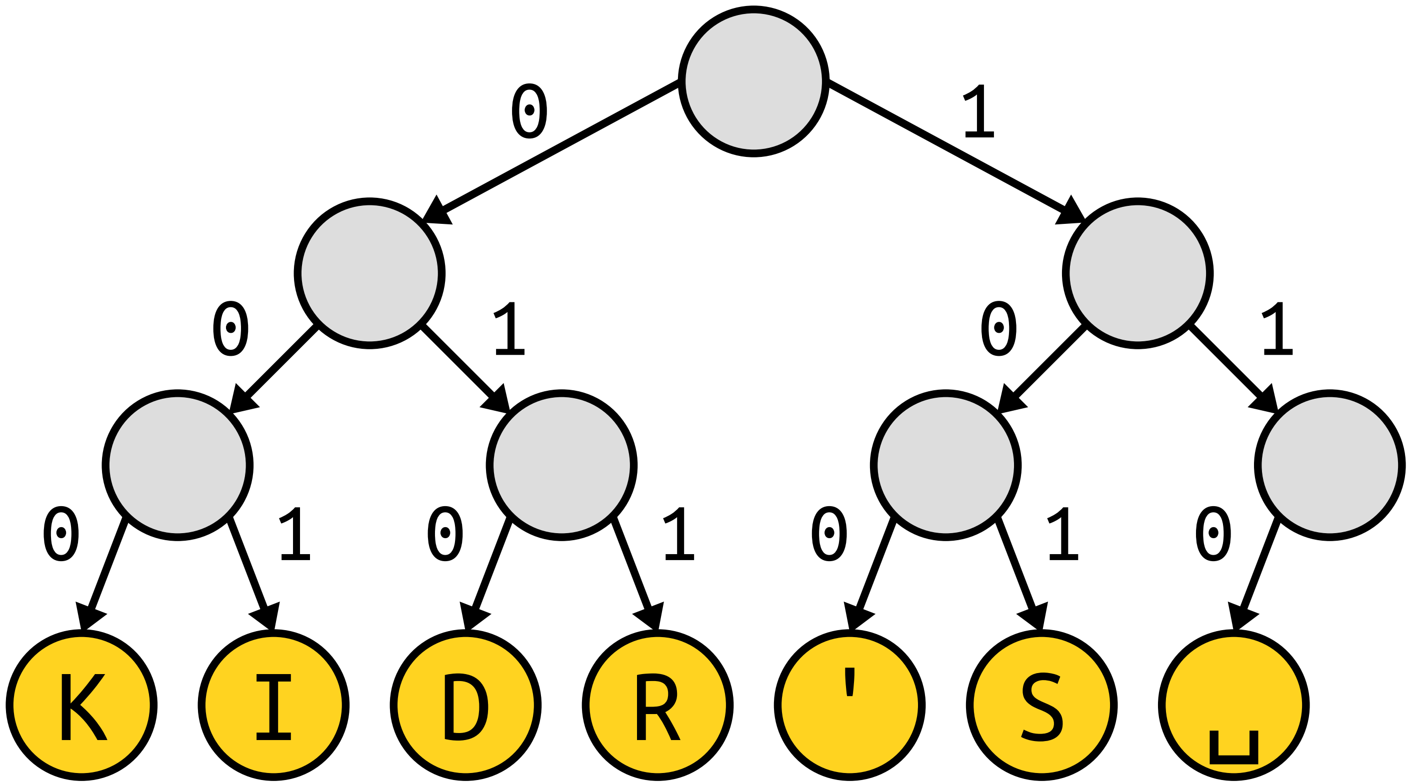
character	*code*
K	**000**
I	**001**
D	**010**
R	**011**
'	**100**
S	**101**
␣	**110**



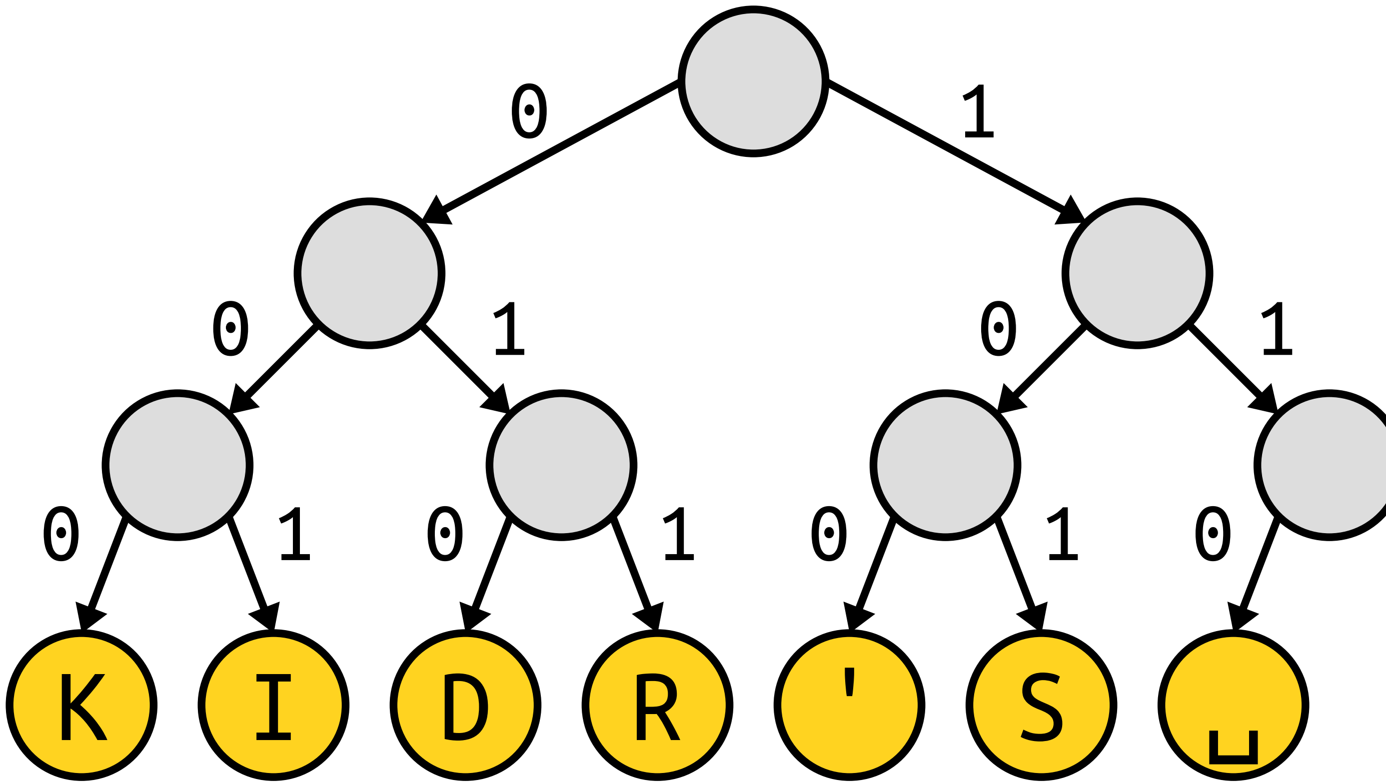
This special type of binary tree is called a ***coding tree***.

character	*code*
K	**000**
I	**001**
D	**010**
R	**011**
'	**100**
S	**101**
␣	**110**




0
1
0
1
0
1
0
1
0
1
0
1
0
K
I
D
R
'
S
␣

101000001

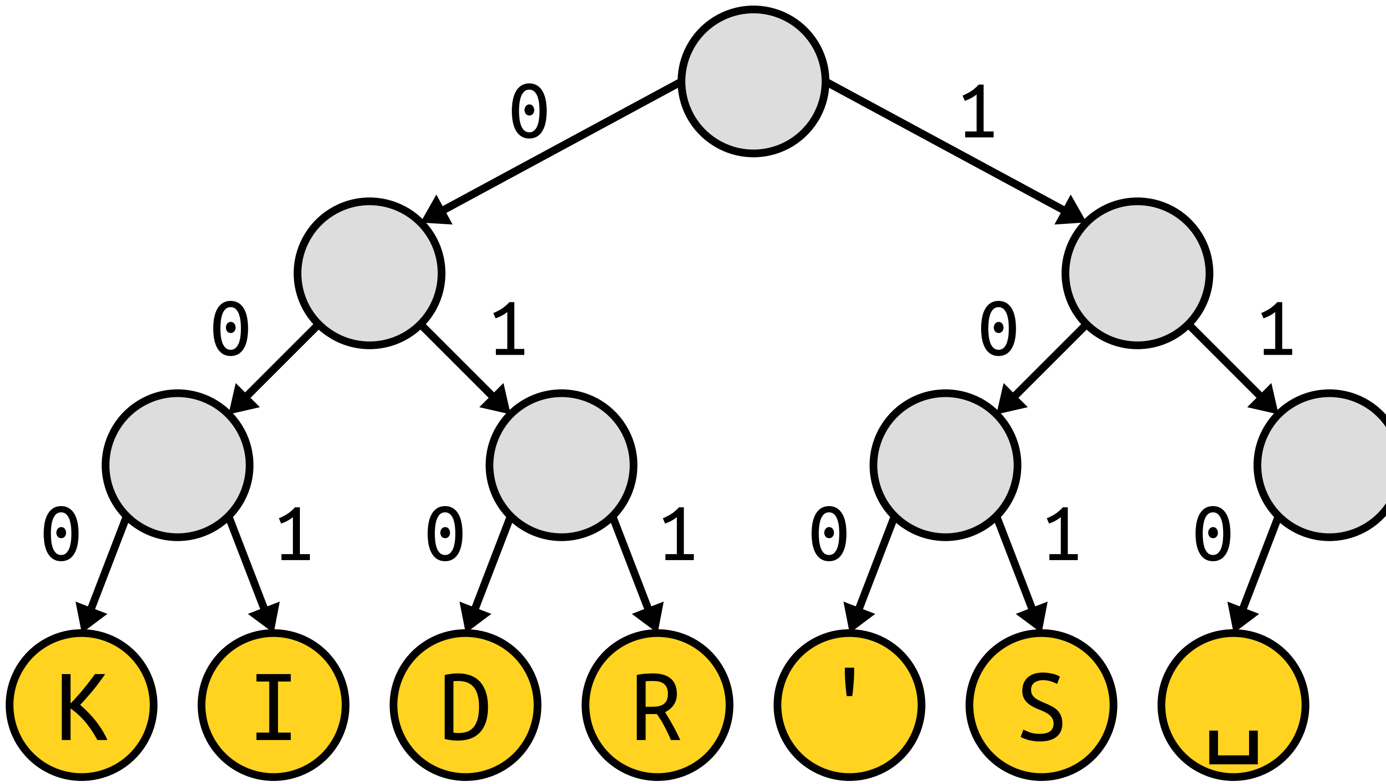


What is the title of this slide?

Formulate a hypothesis!

101000001

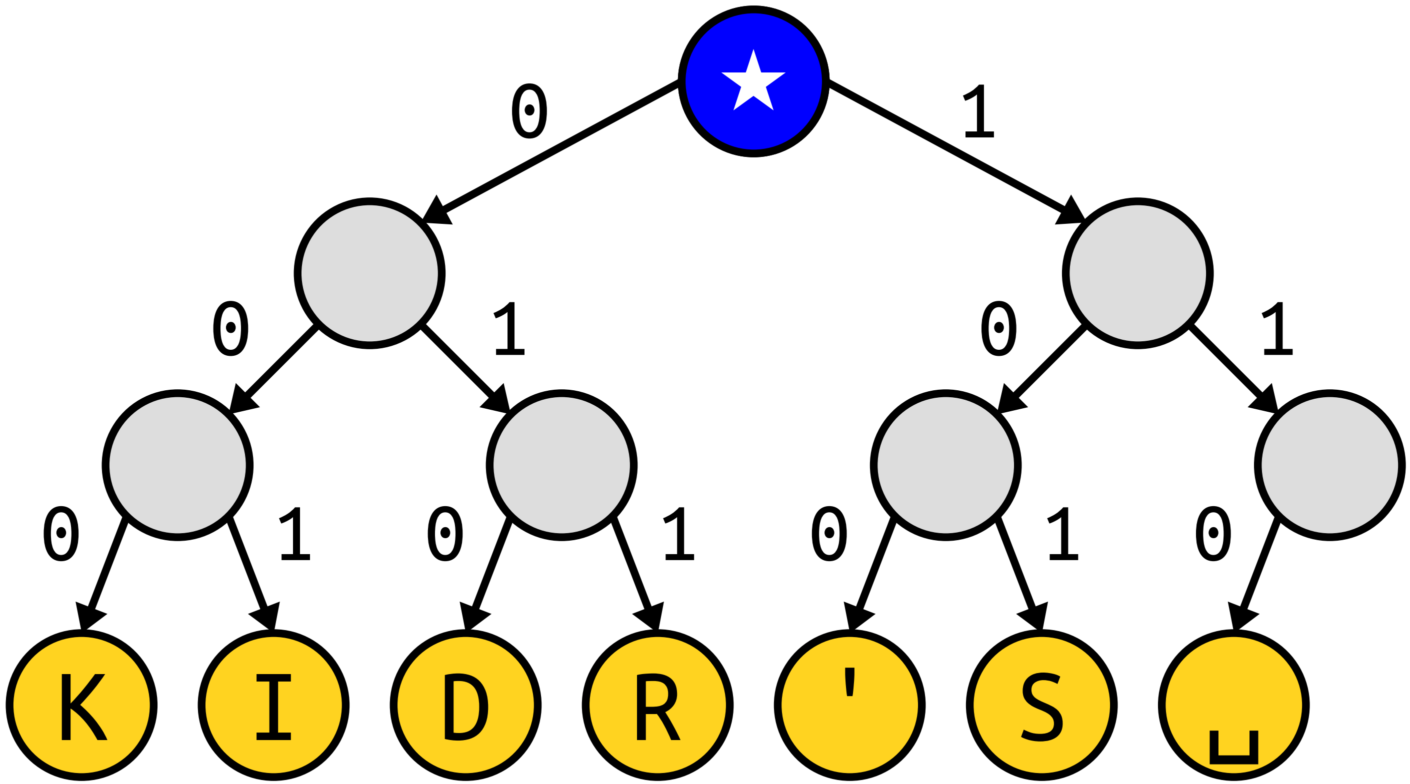
What is the title of this slide?



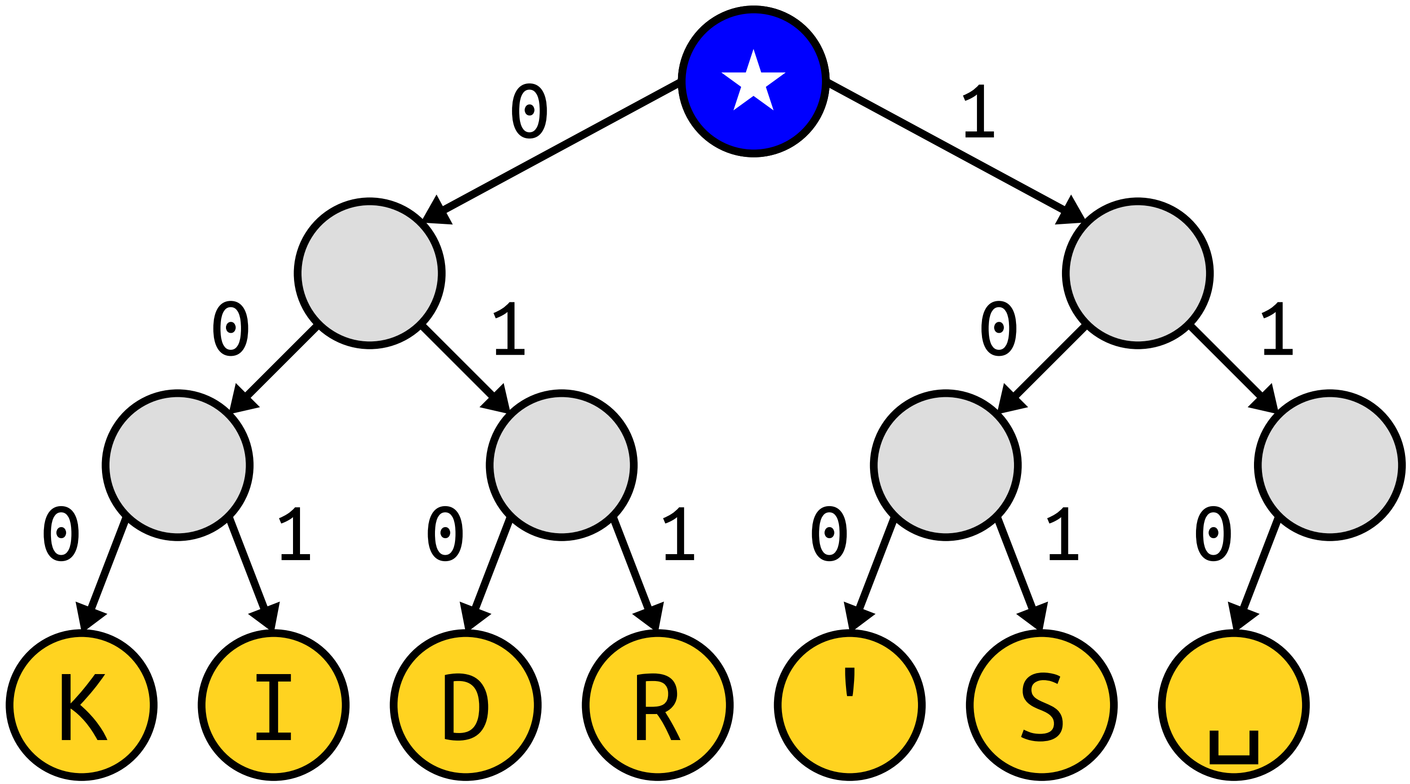
Chat with your neighbors!

101000001

What is the title of this slide?



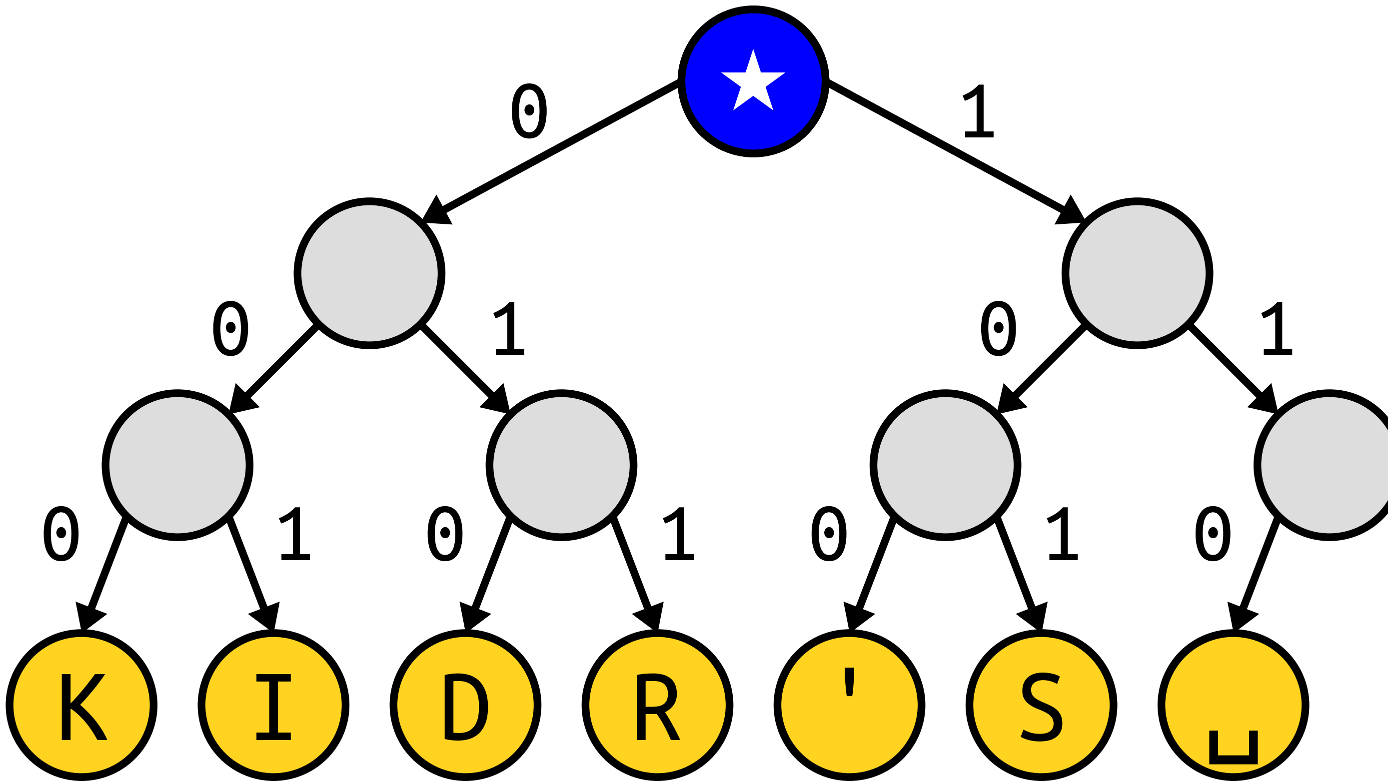
101000001



What is the title of this slide?

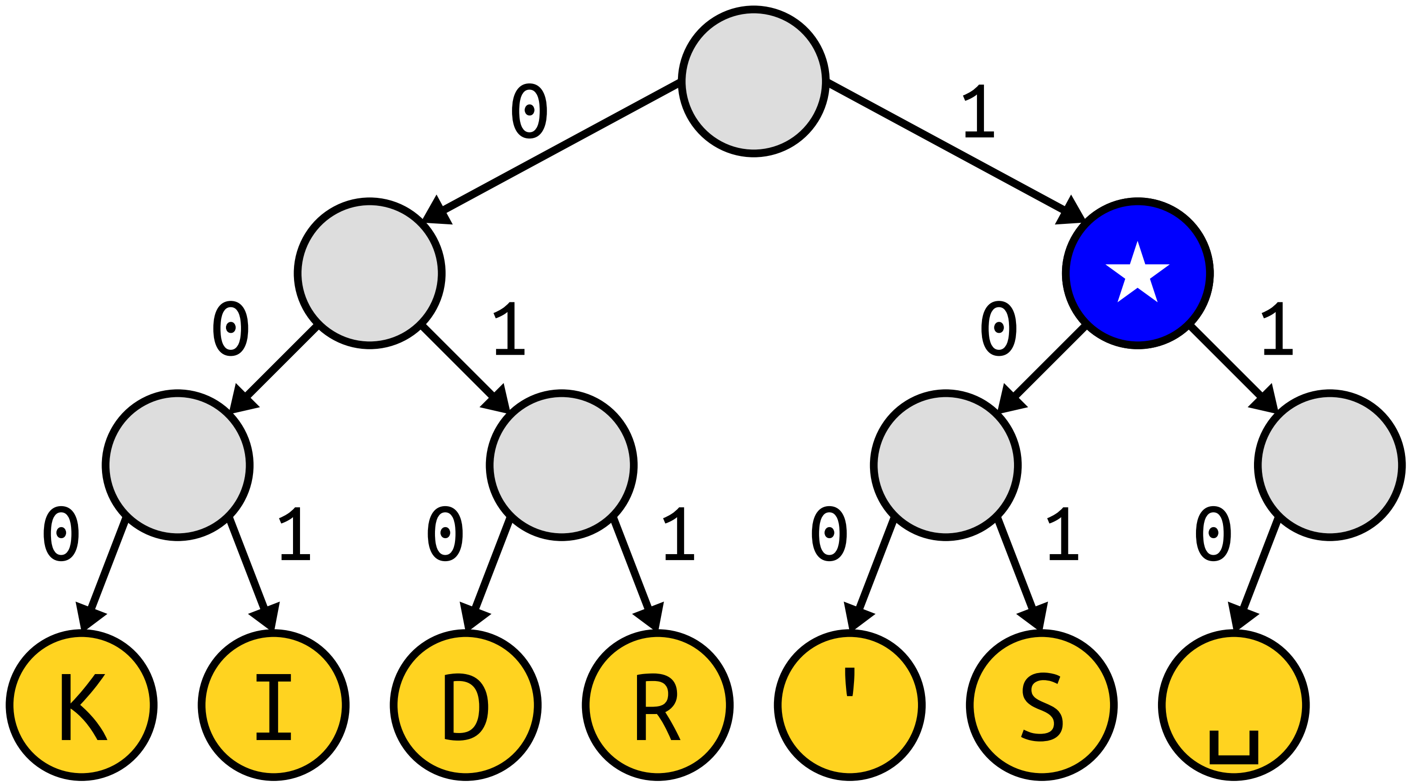
101000001

What is the title of this slide?

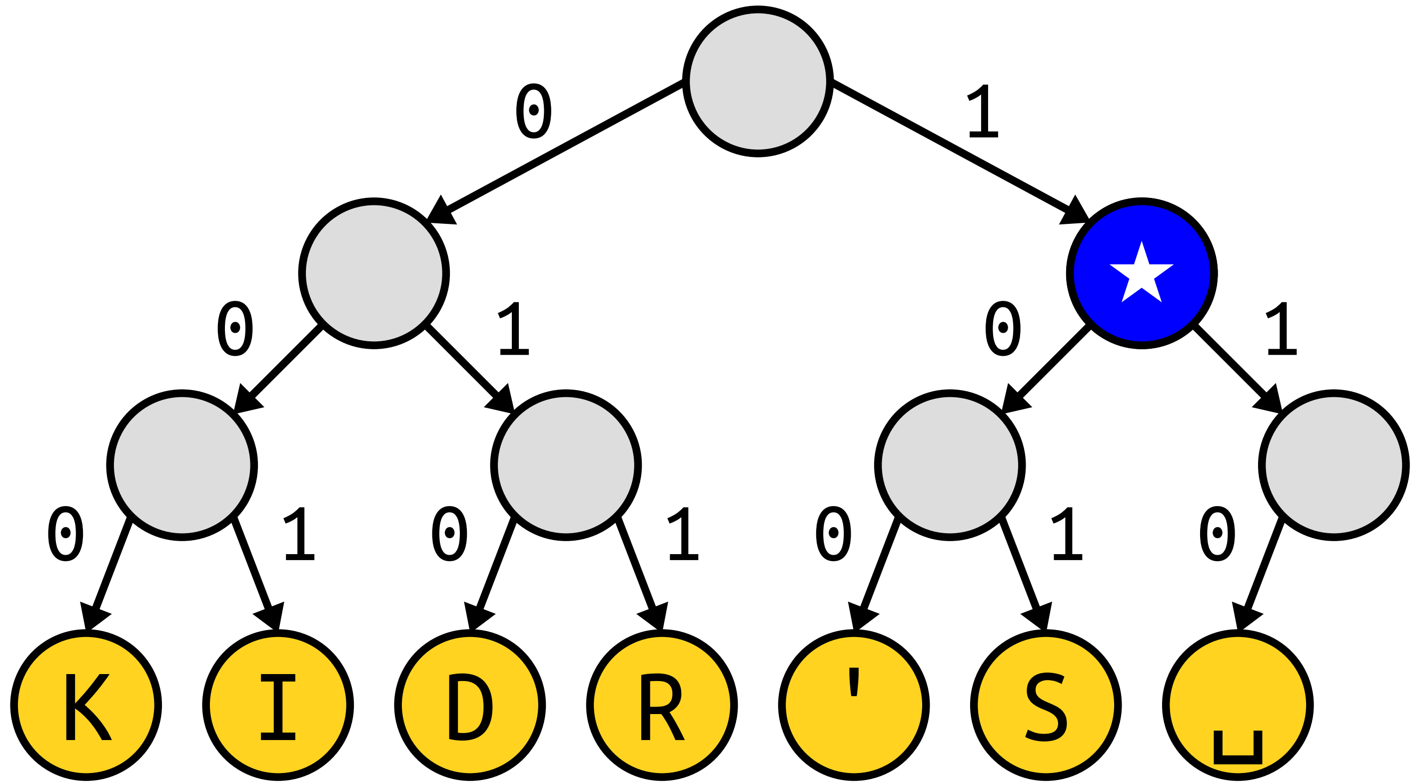


10100000001

What is the title of this slide?



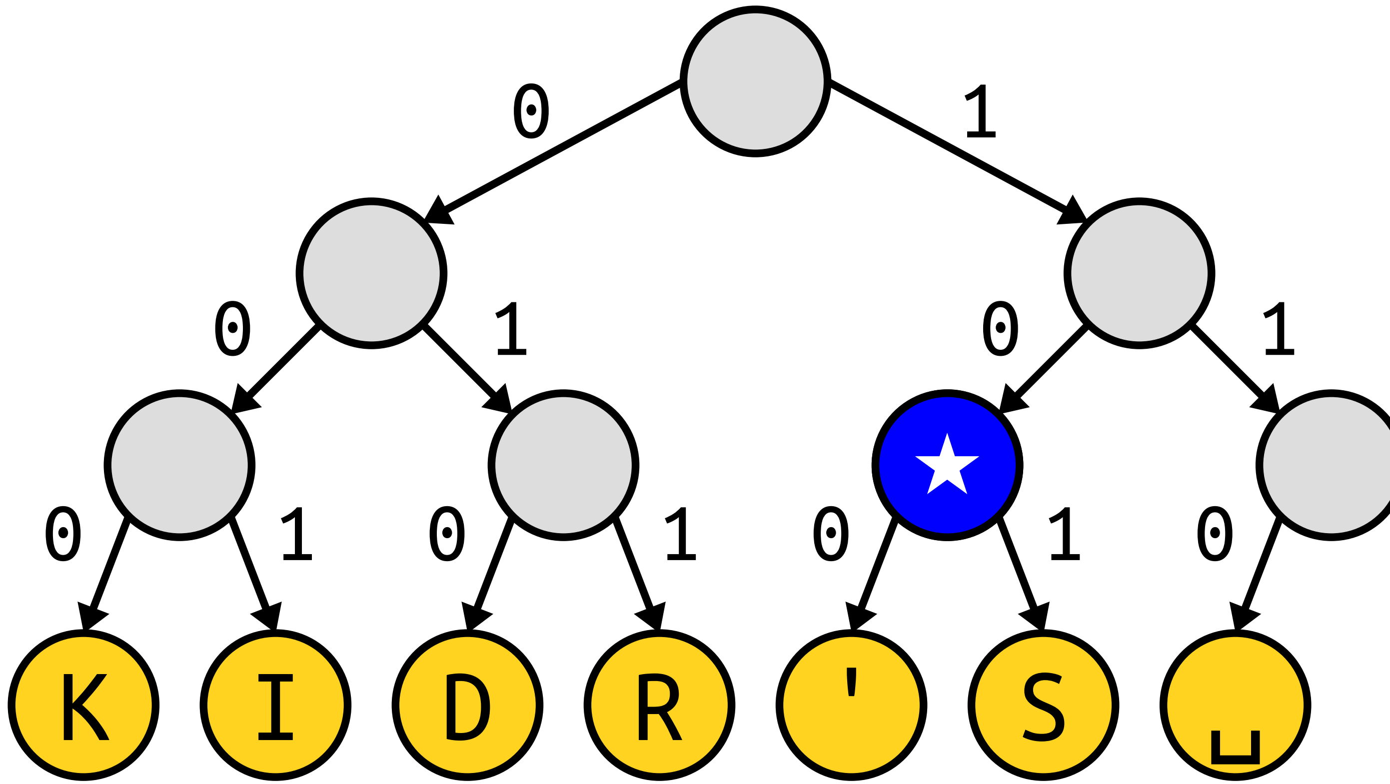
101000001



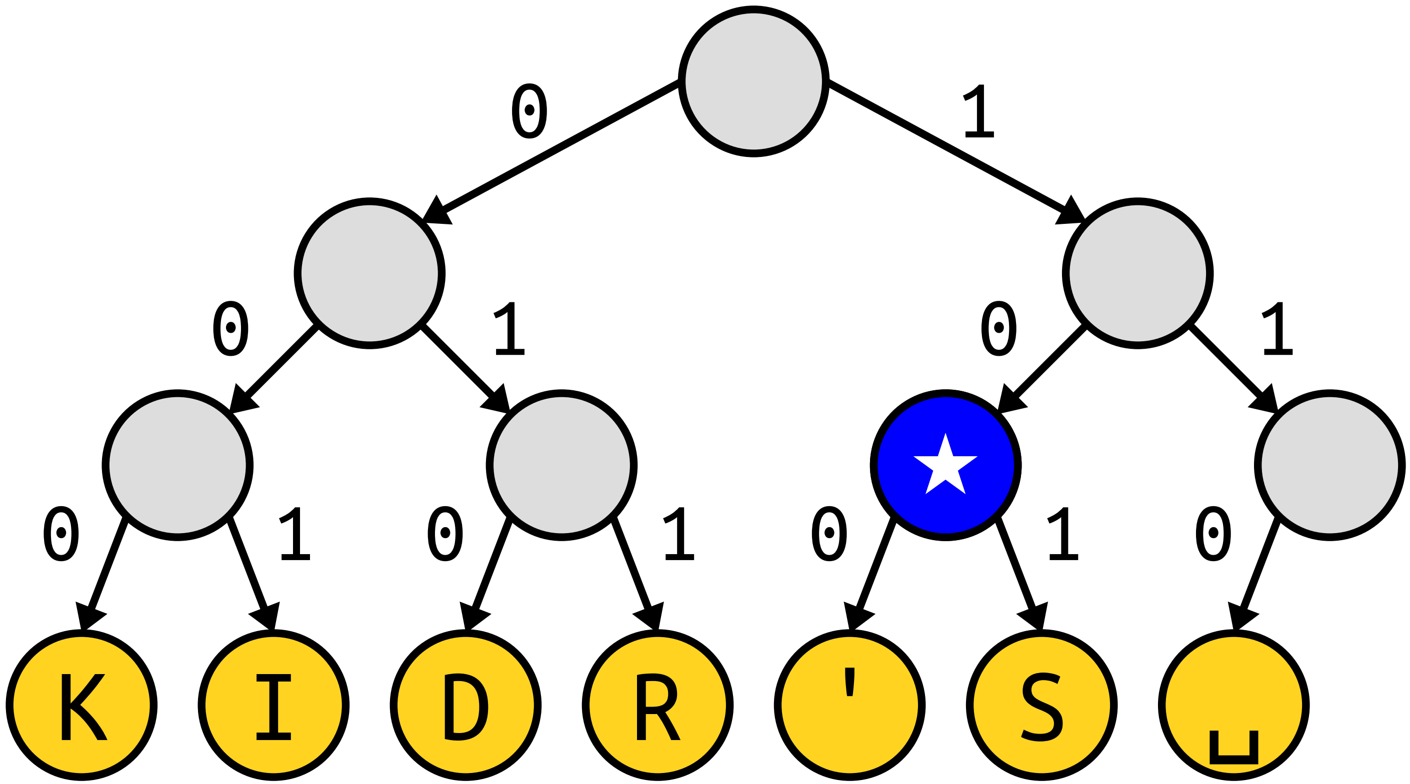
What is the title of this slide?

1010000001

What is the title of this slide?



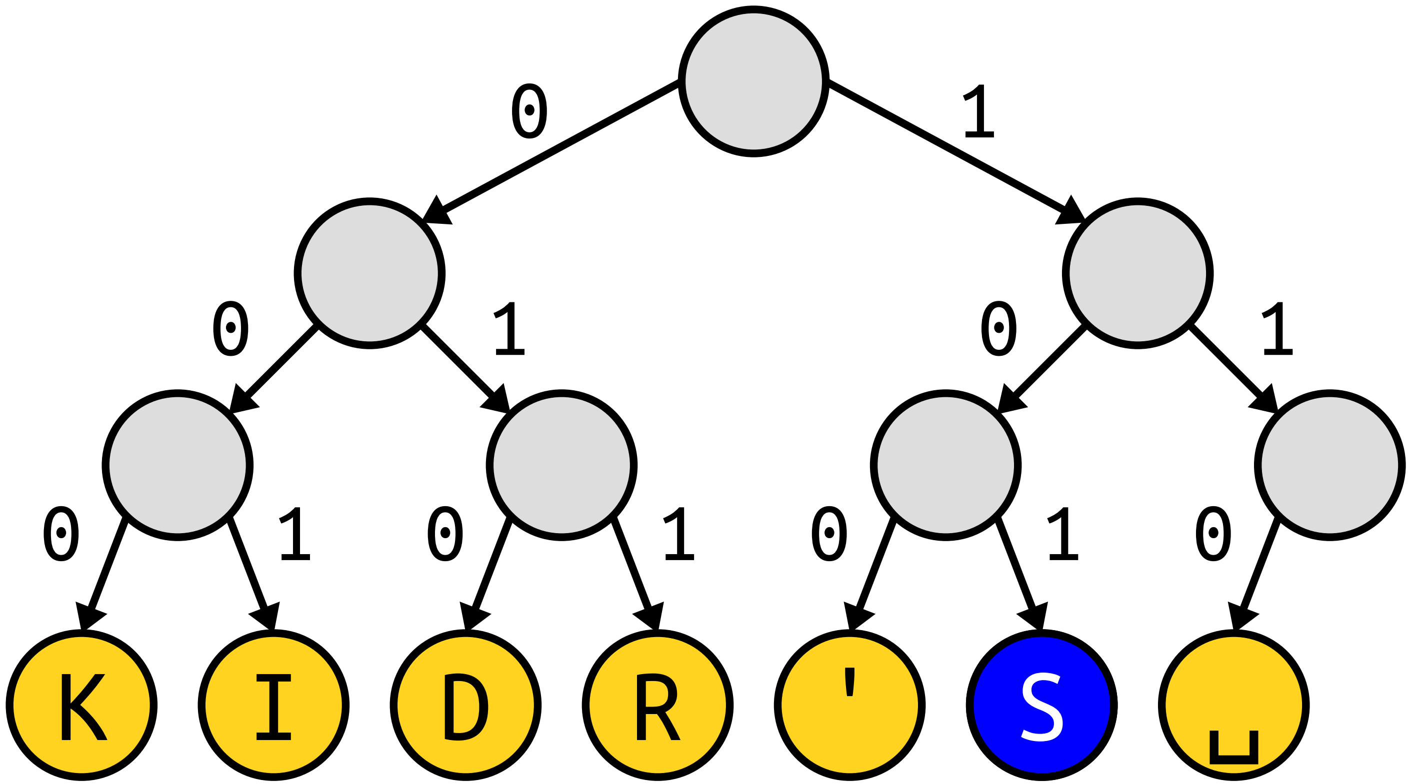
101000001



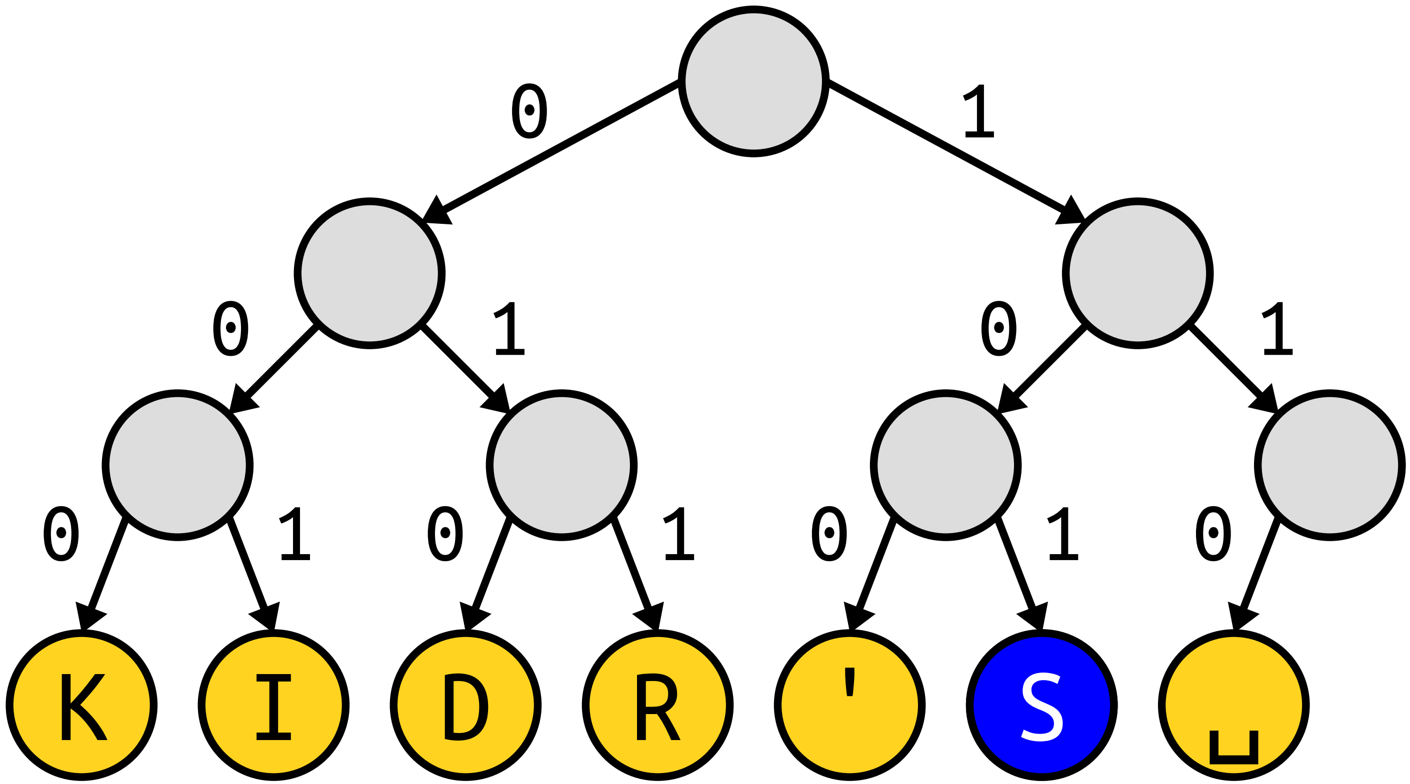
What is the title of this slide?

101000001

What is the title of this slide?



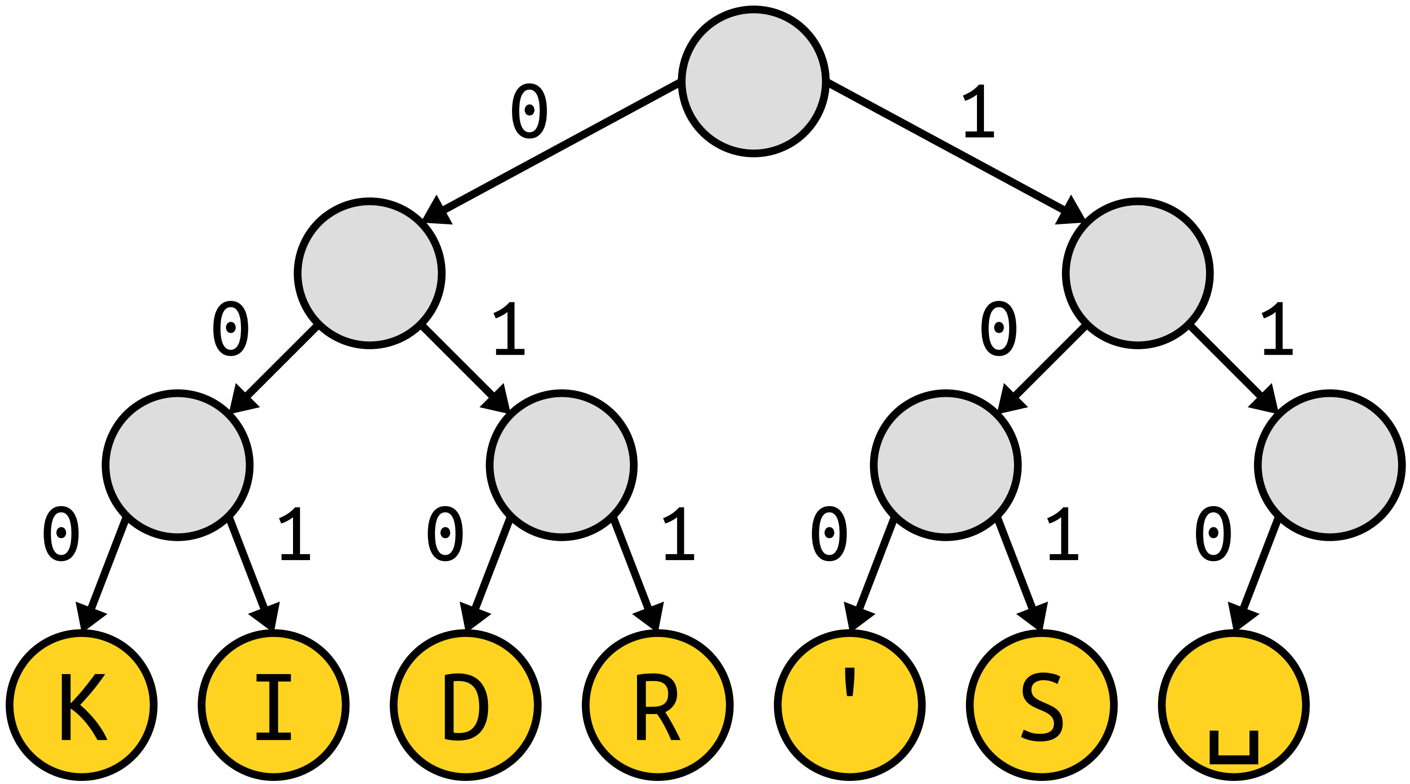
S 000001



What is the title of this slide?

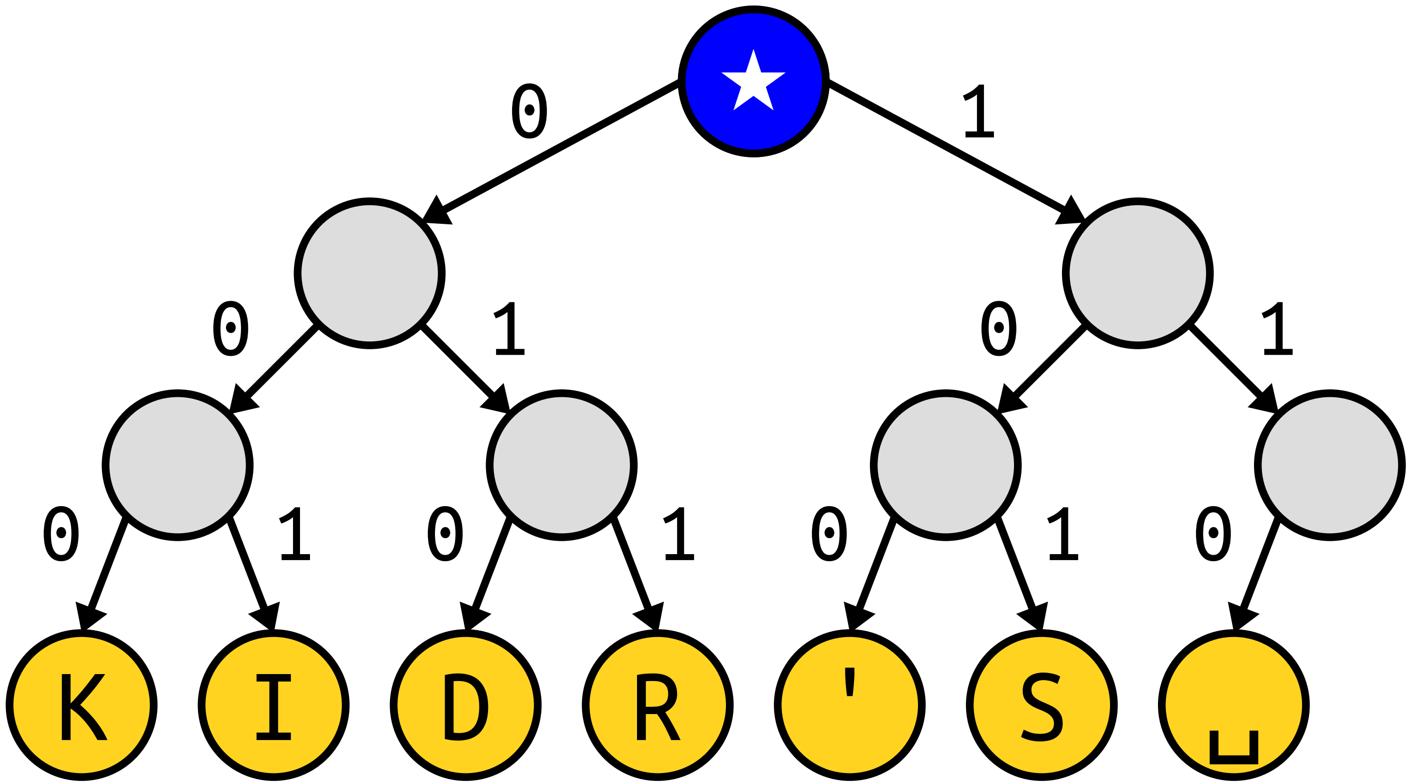
S 000001

What is the title of this slide?



S 000001

What is the title of this slide?



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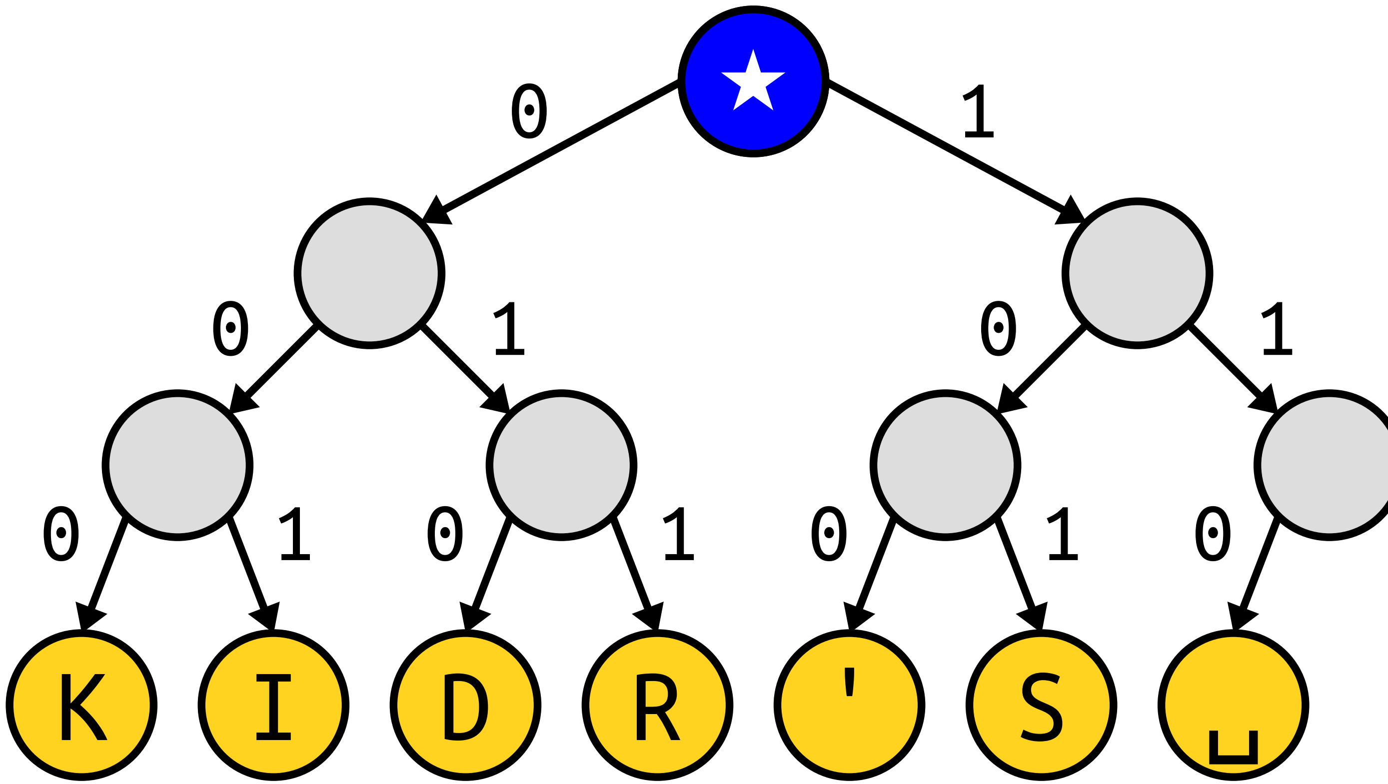

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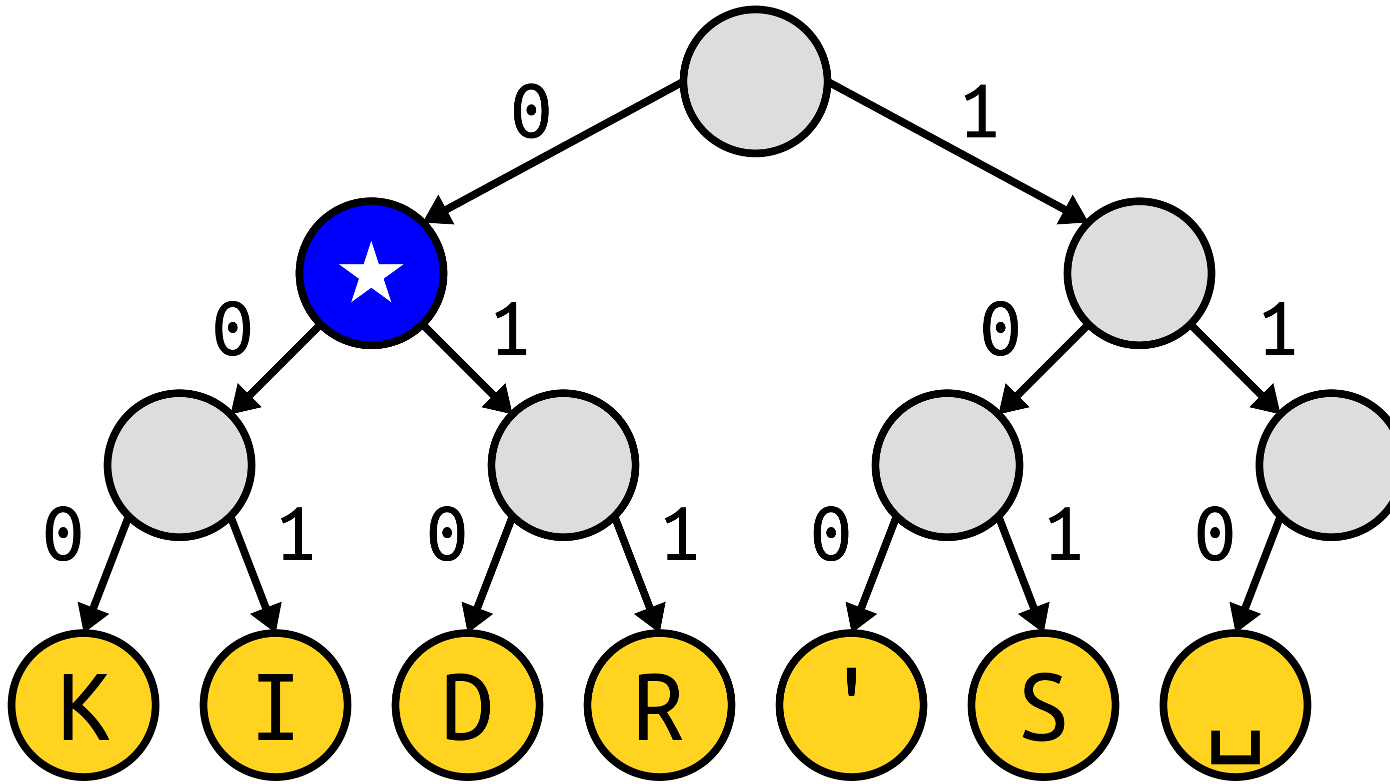

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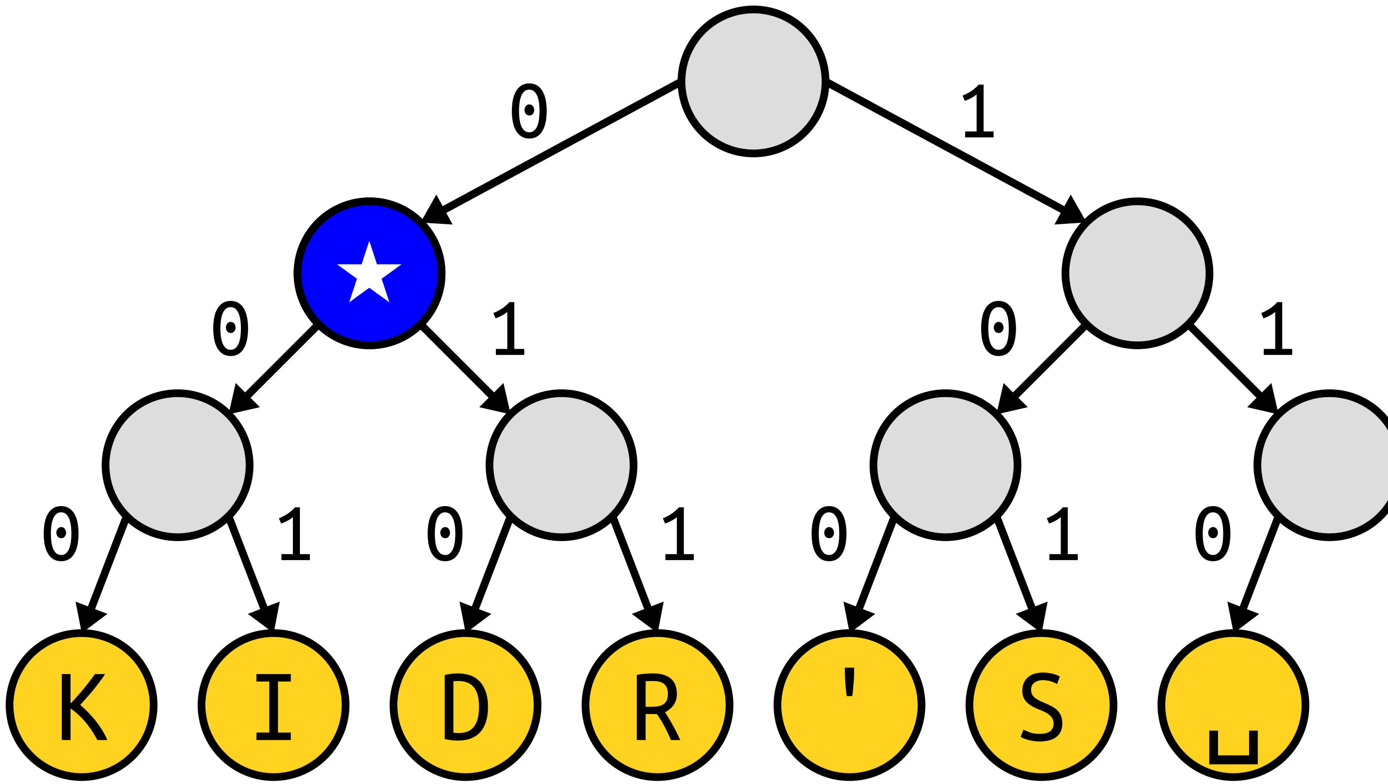

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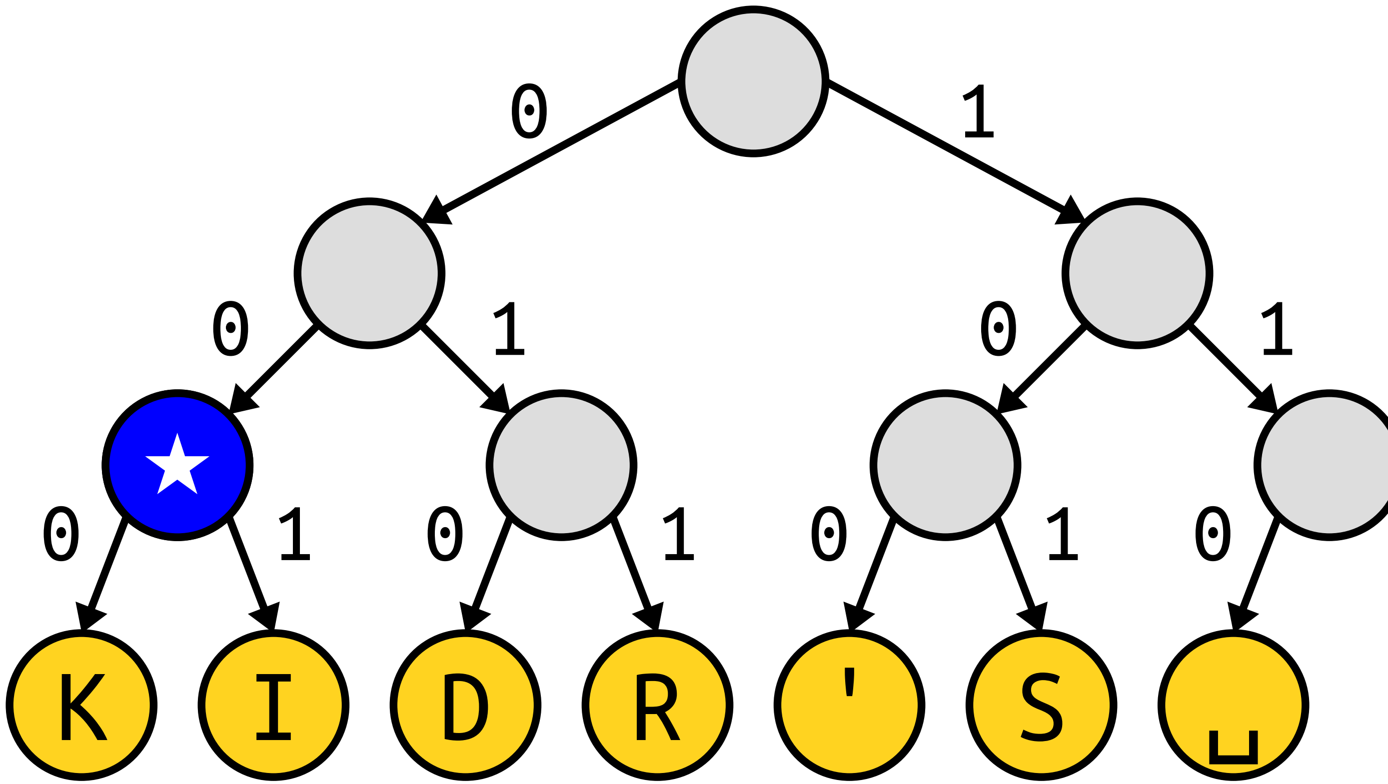

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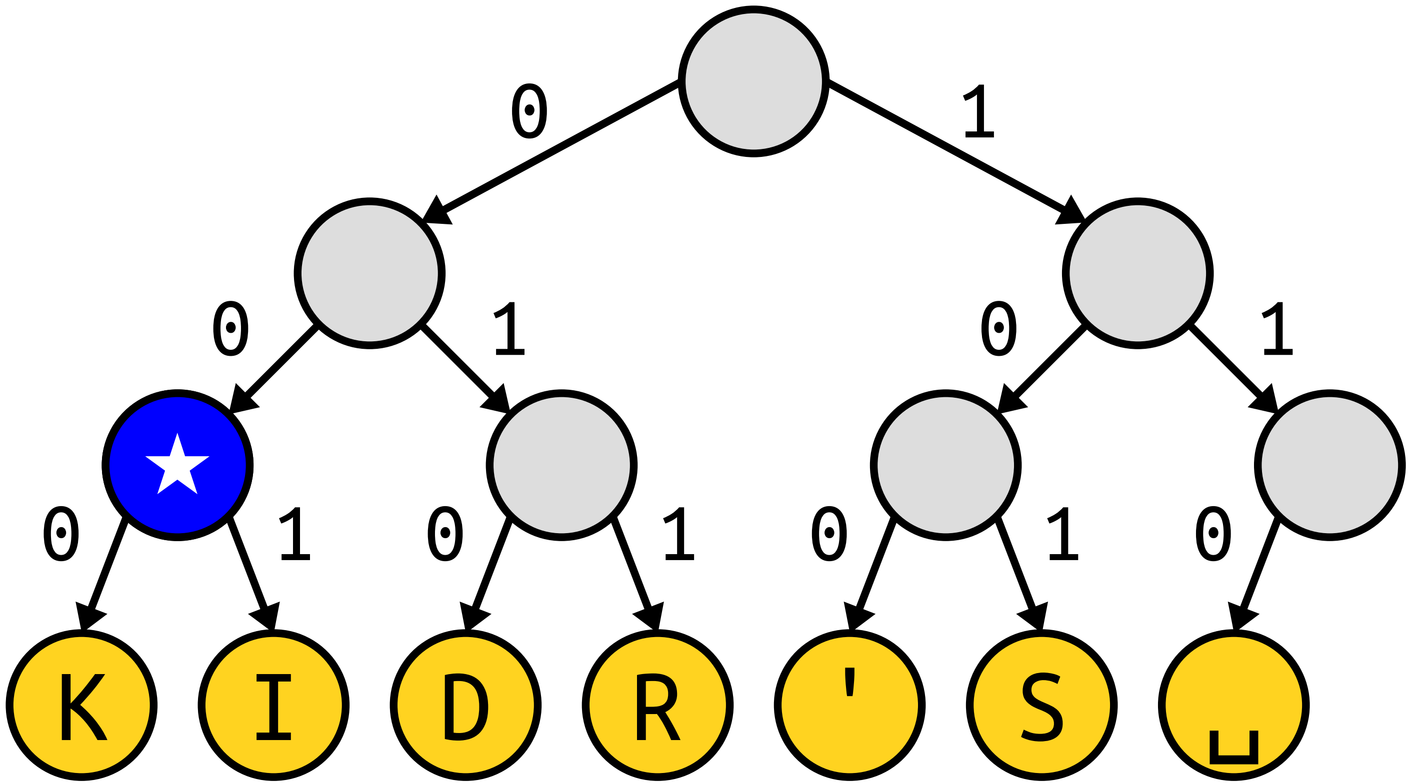

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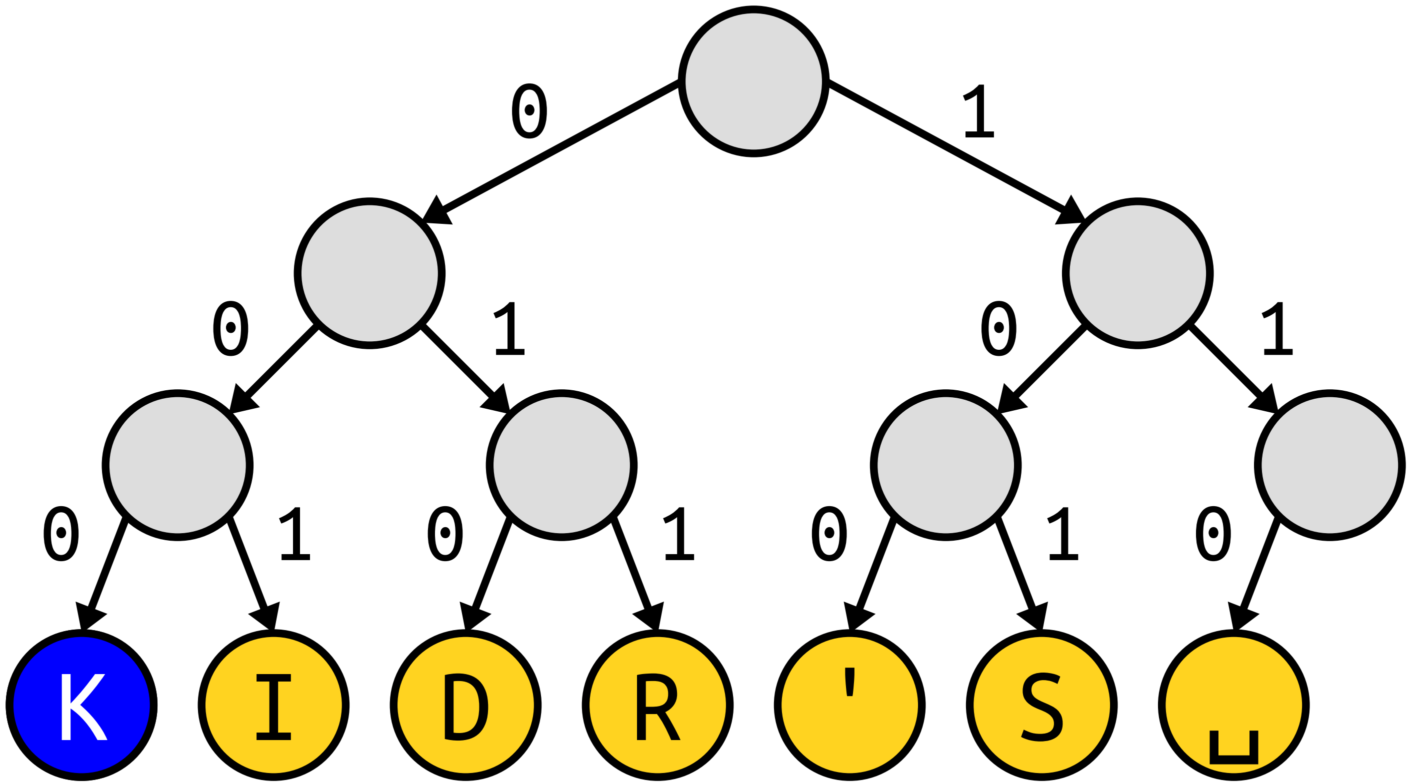

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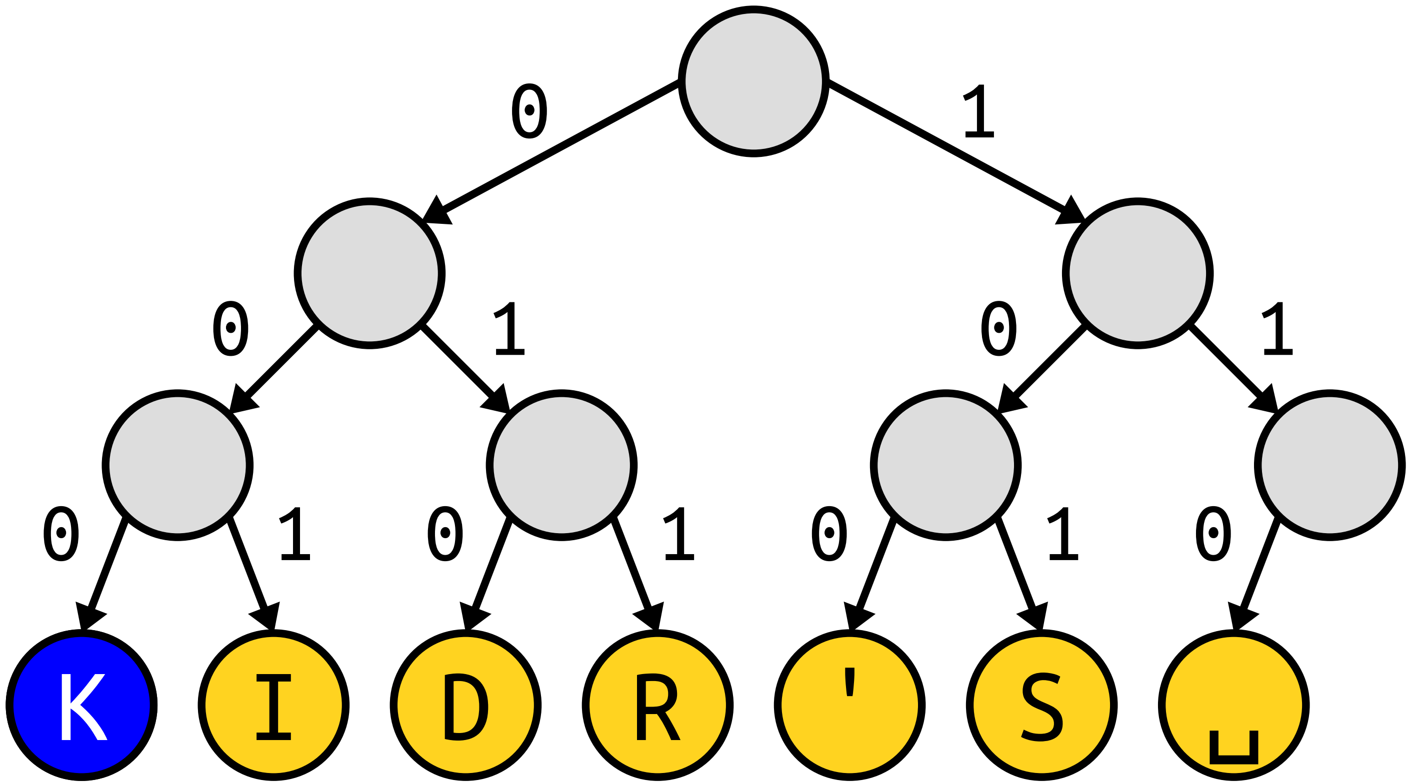

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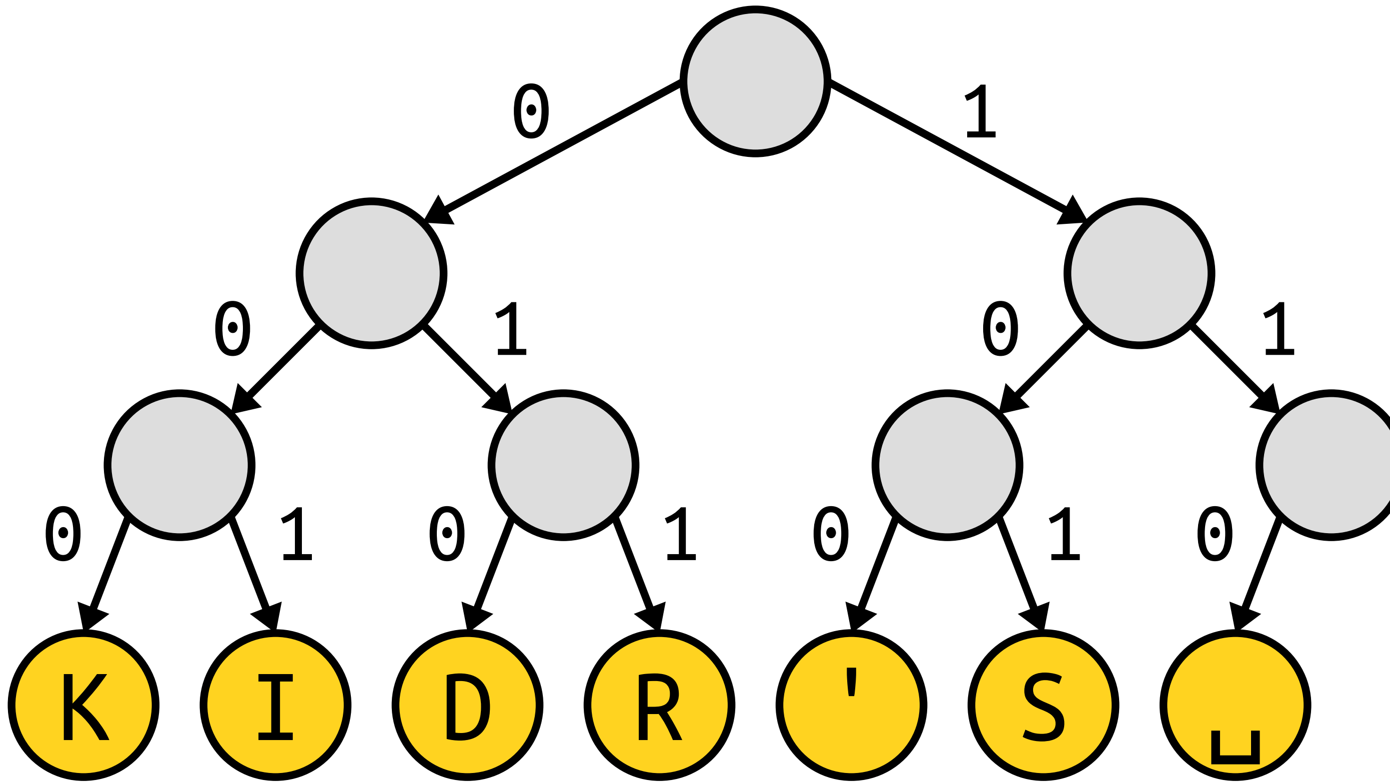

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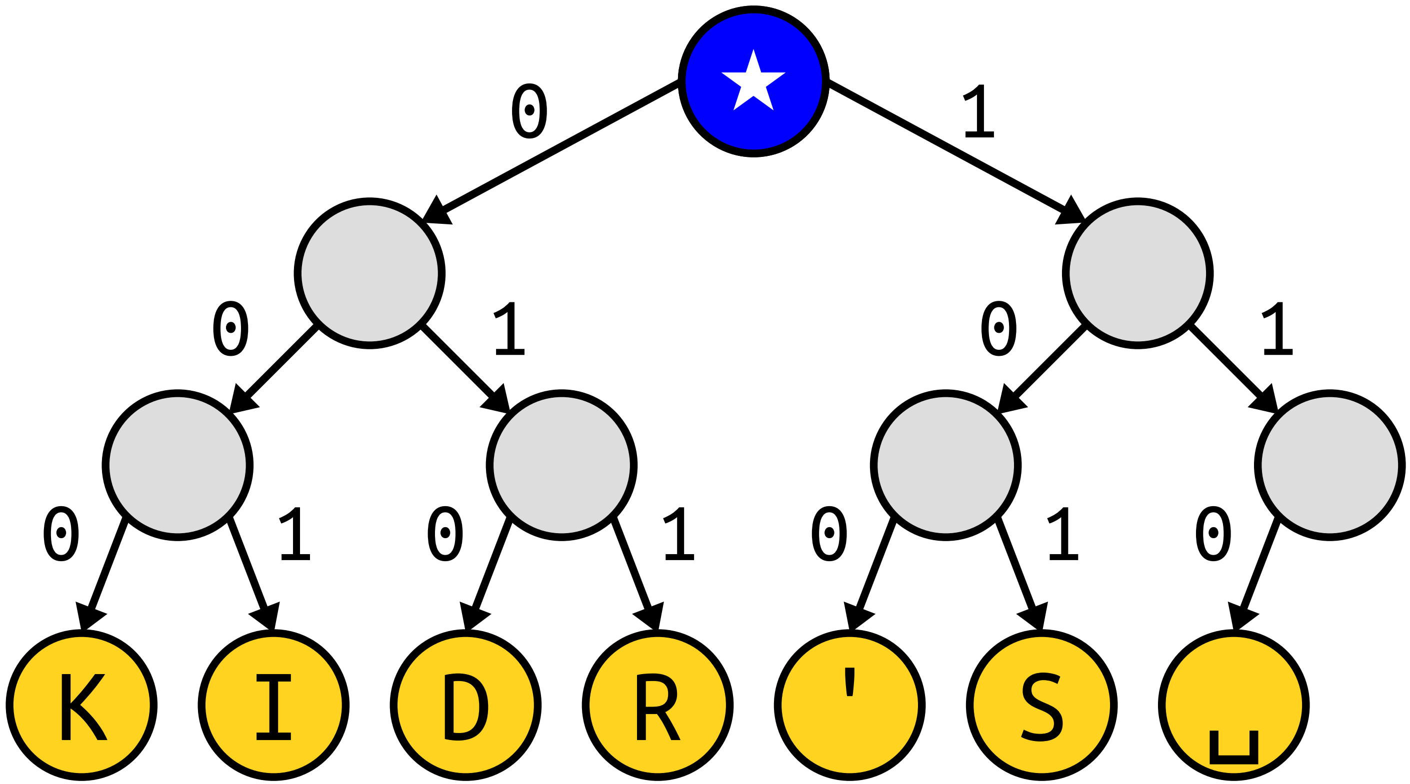

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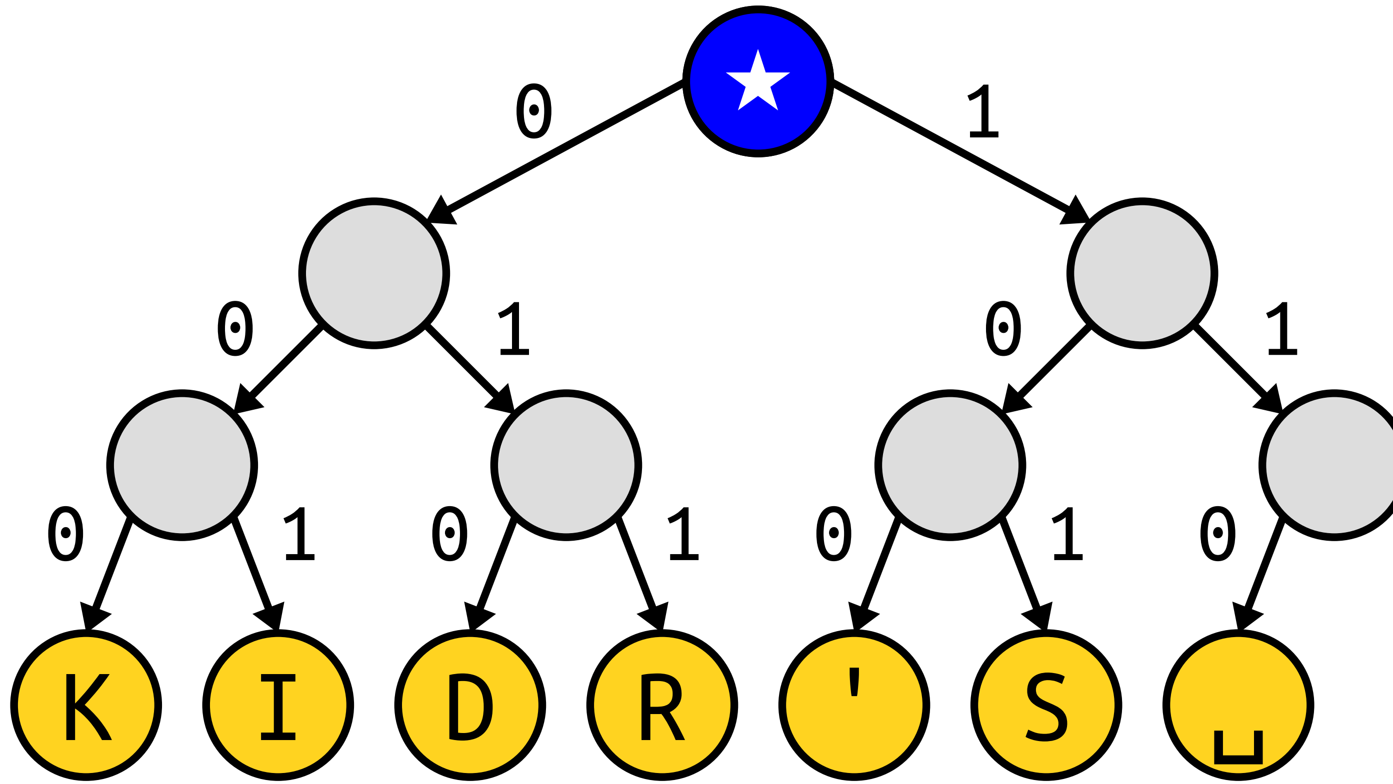

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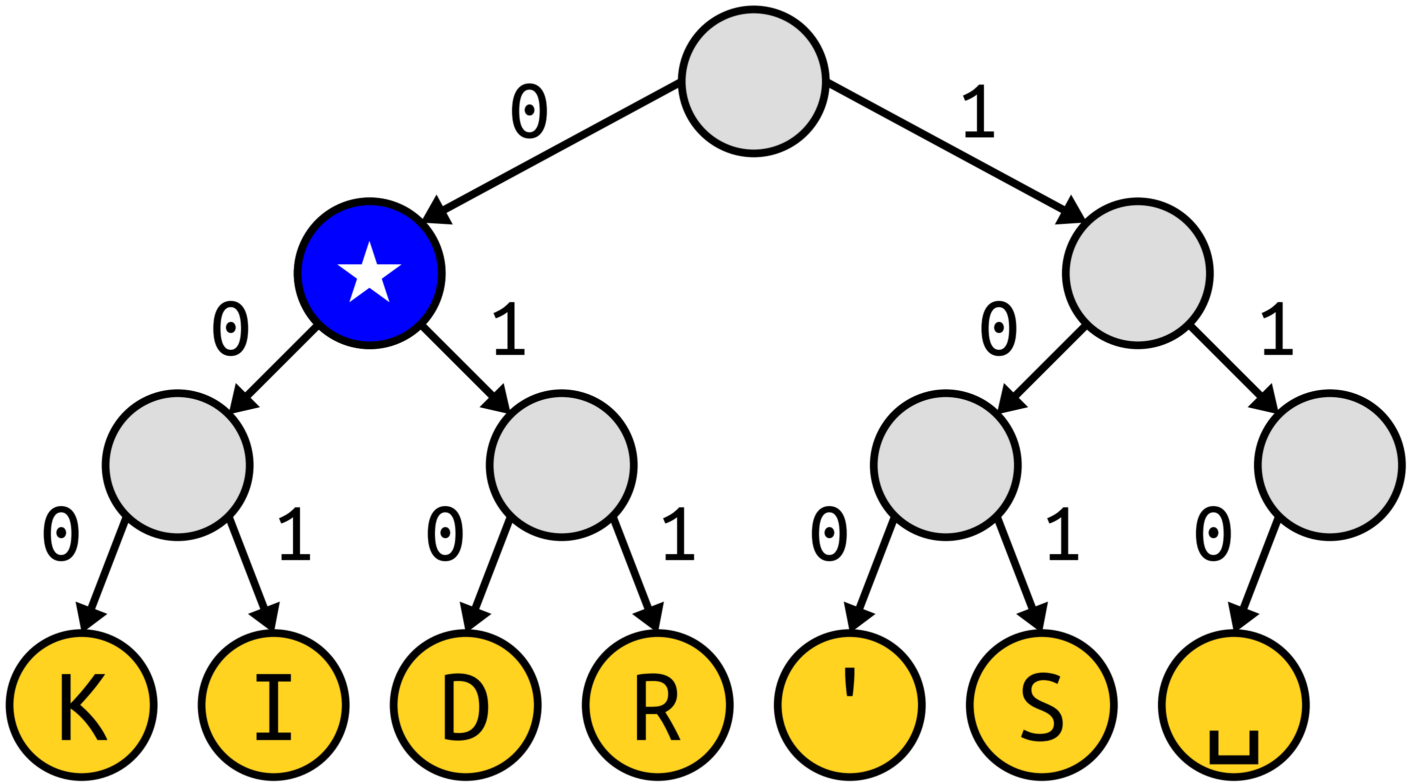

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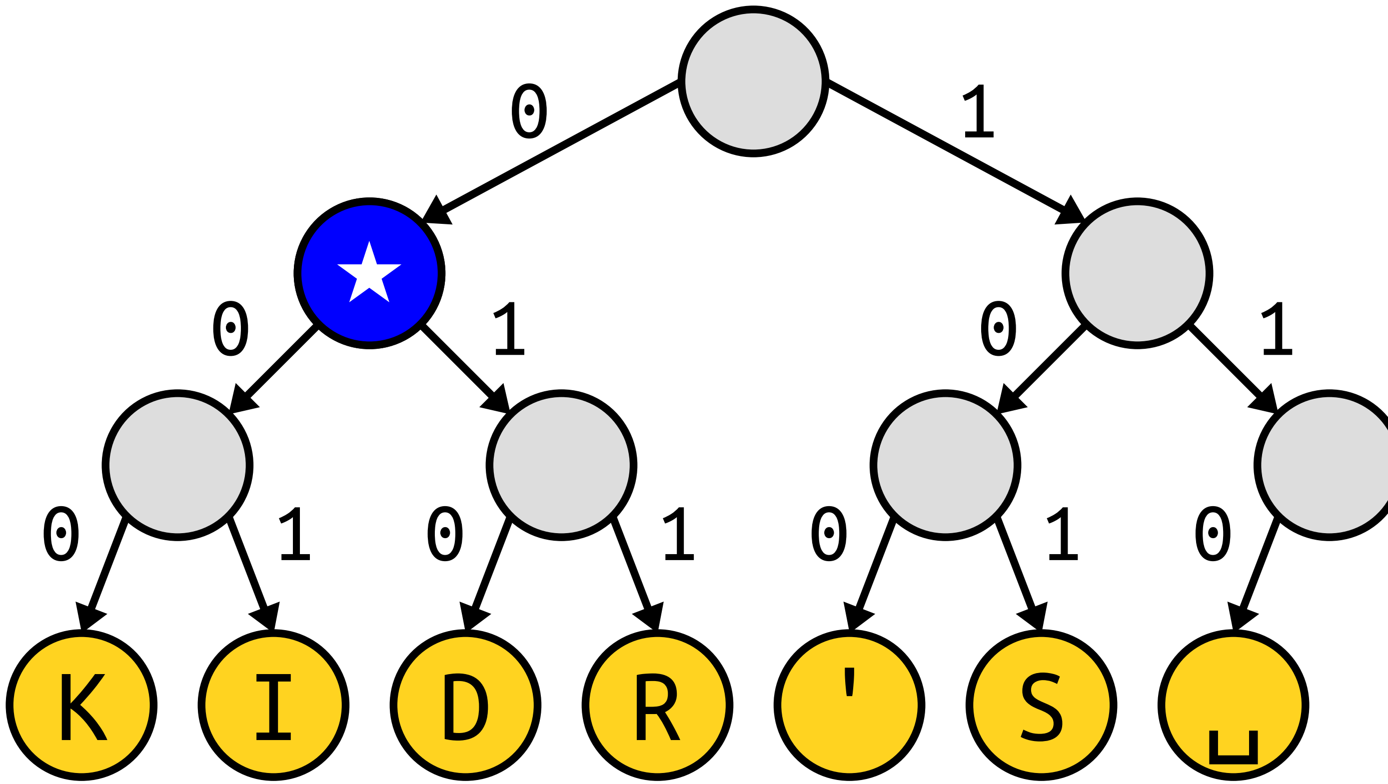

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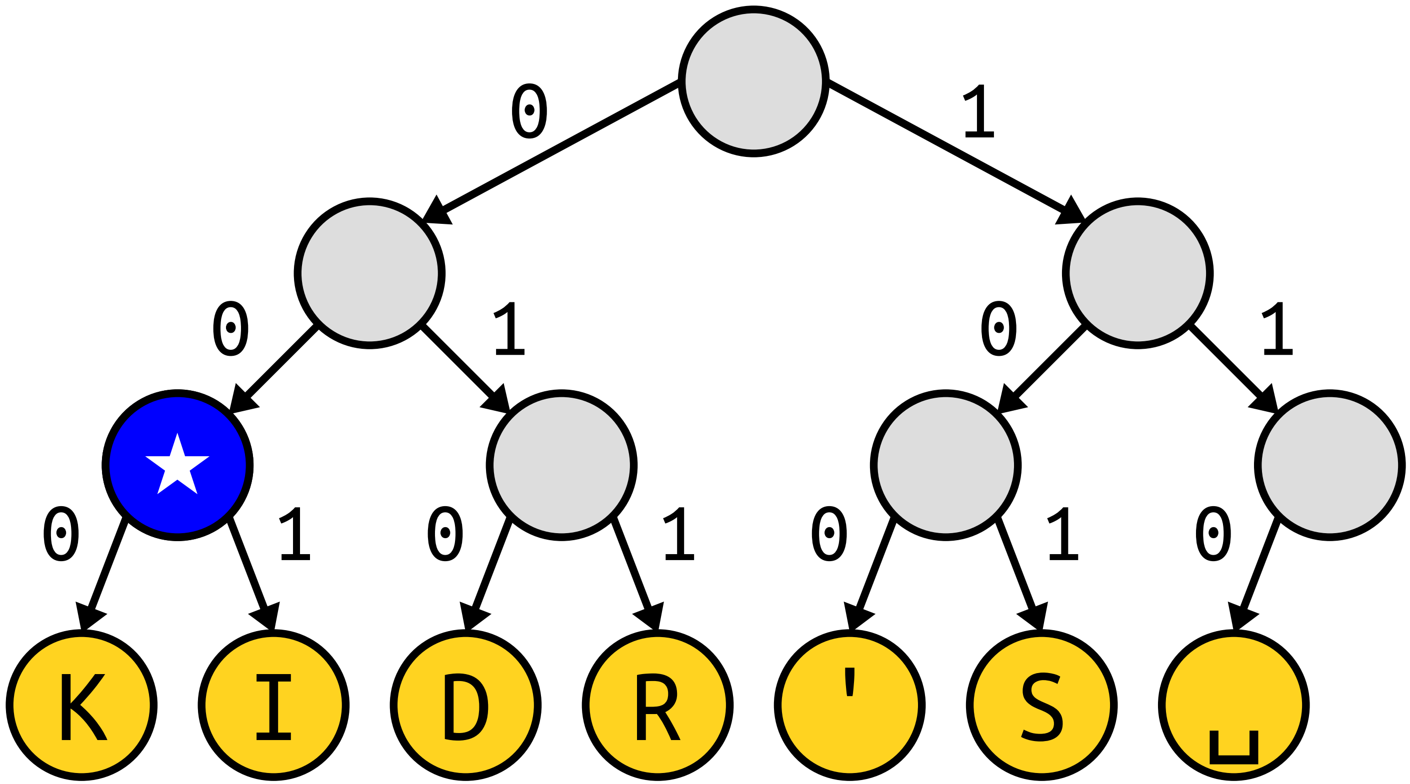

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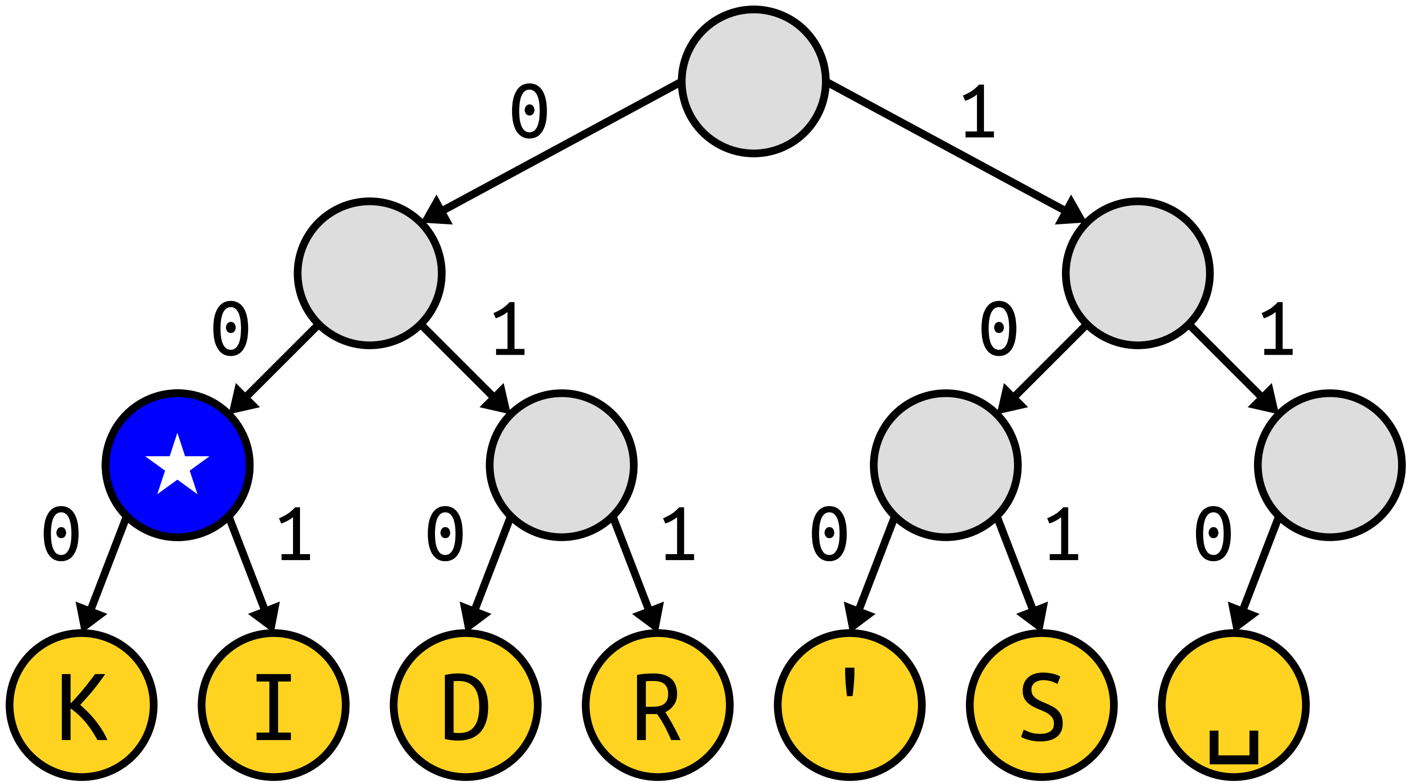

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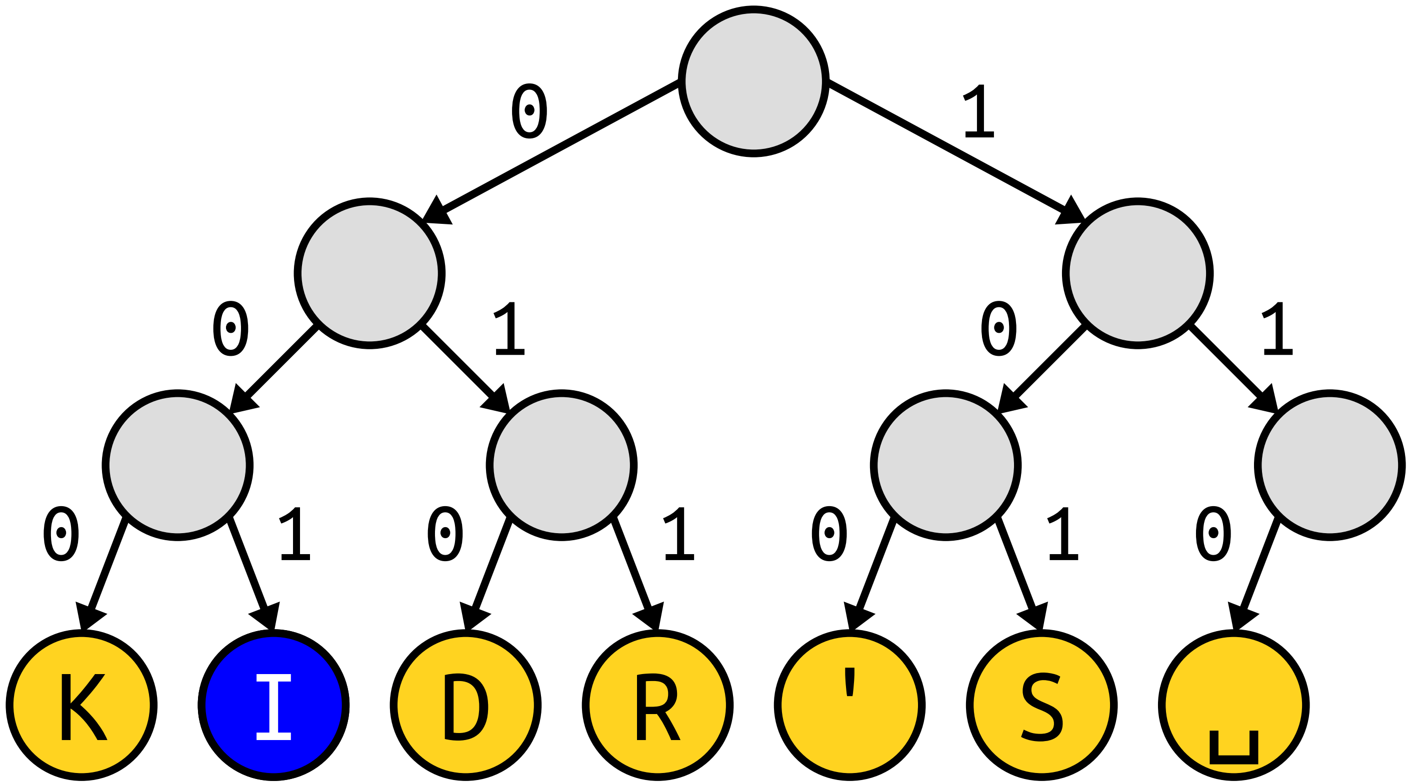

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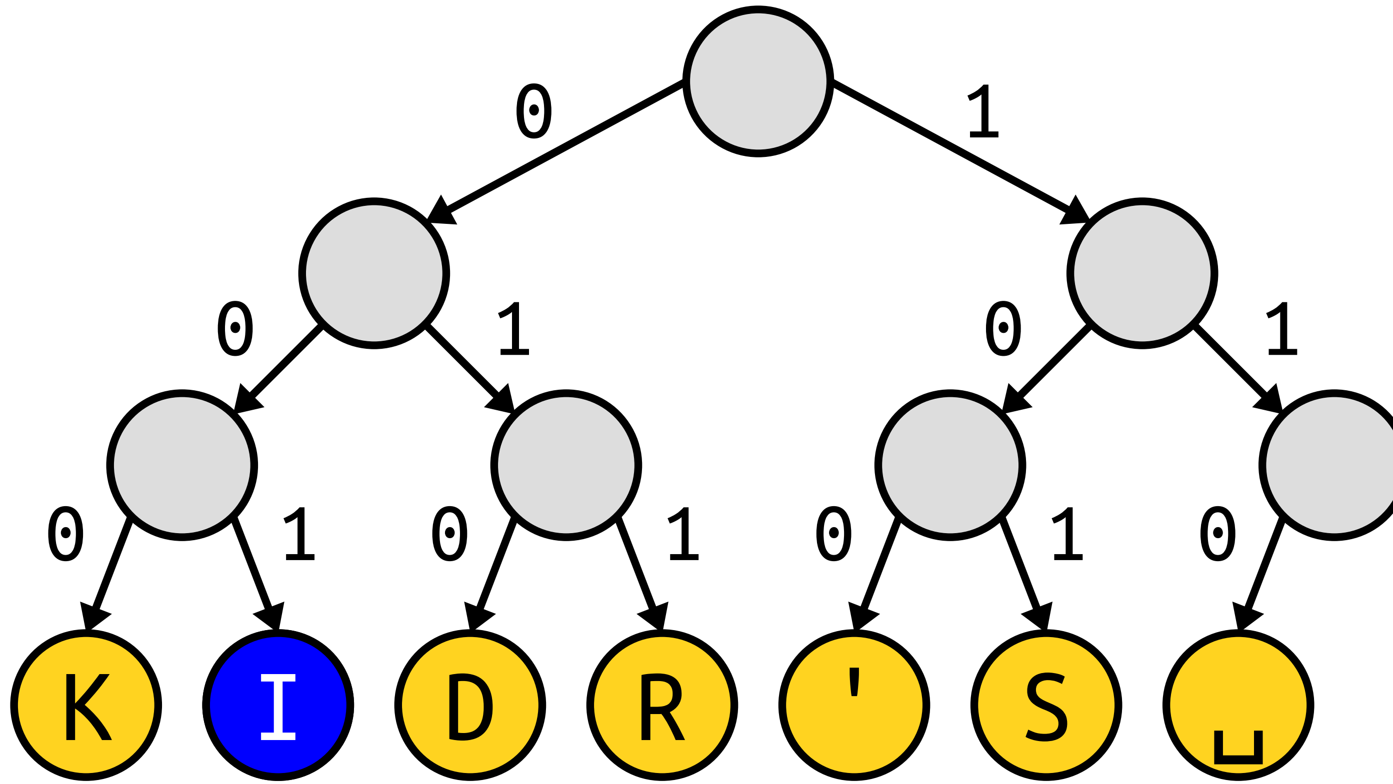

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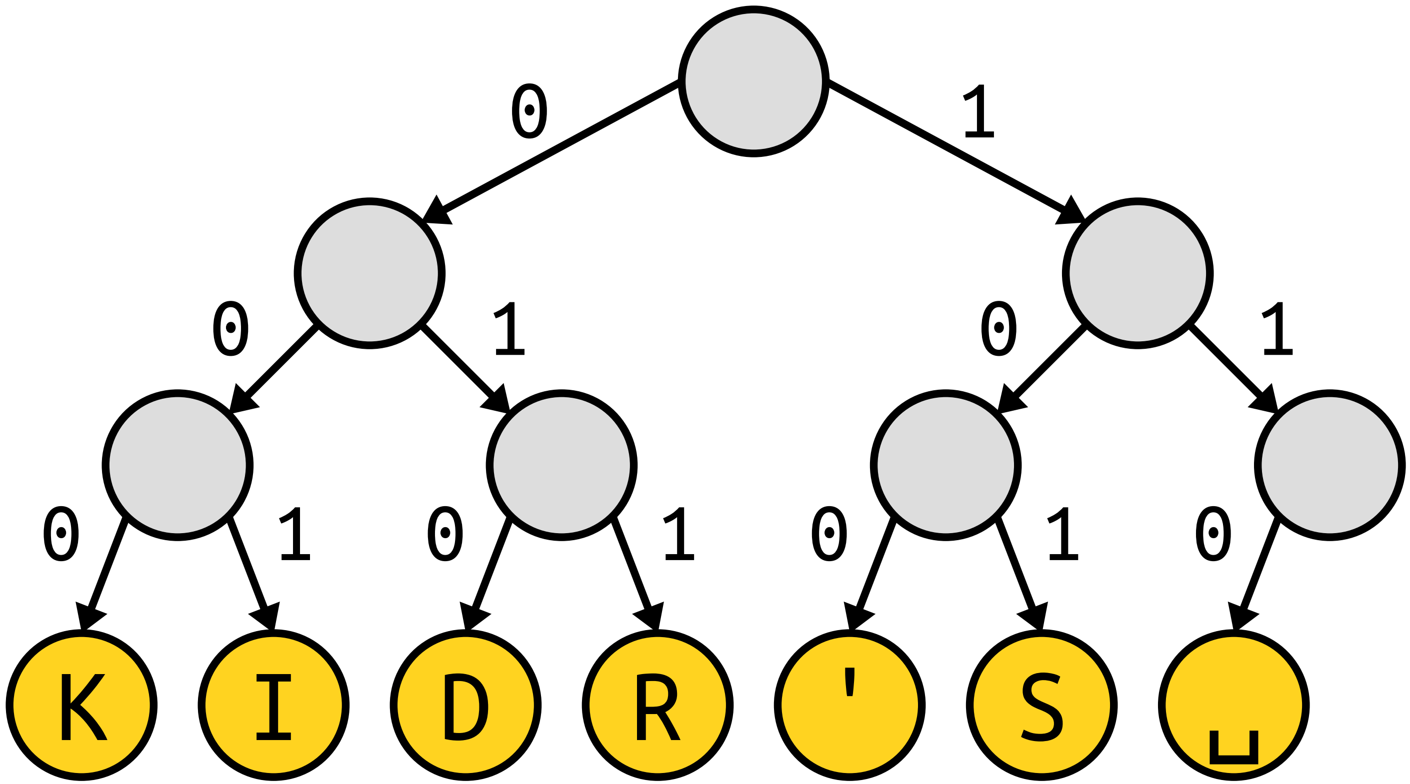

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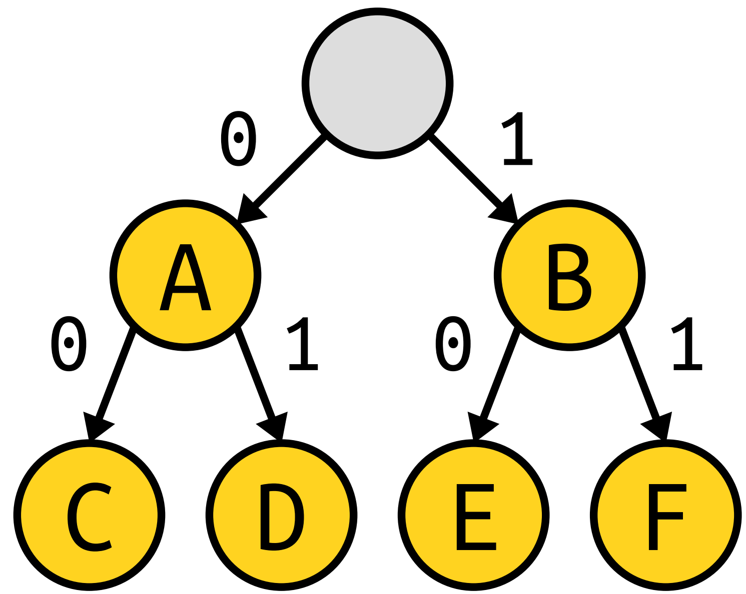
Coding Trees

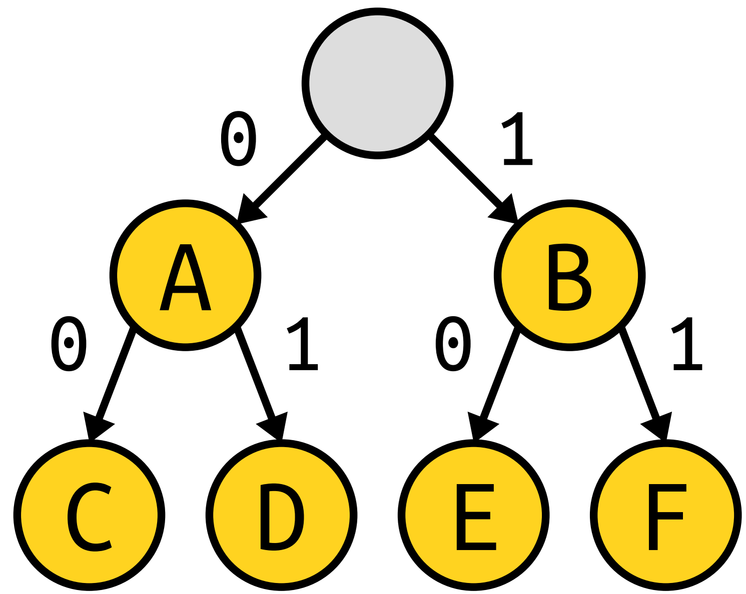
- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?



Formulate a hypothesis!

Coding Trees

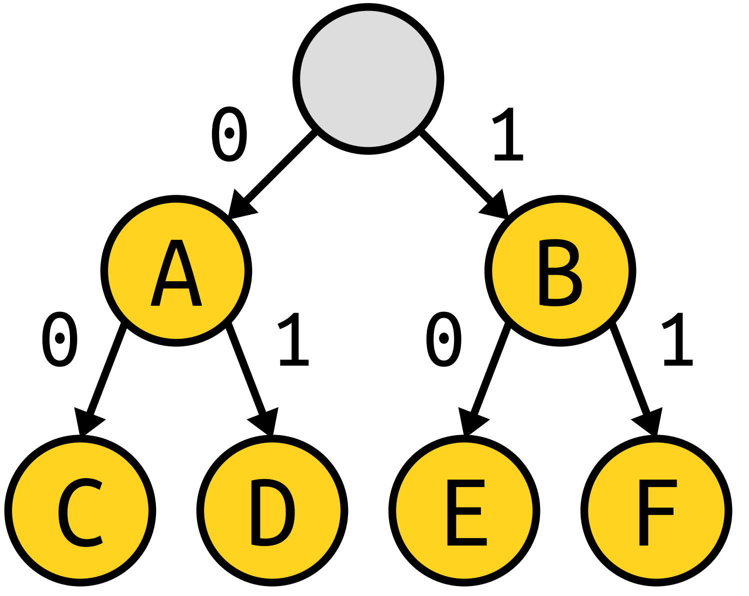
- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?



Discuss with your neighbors!

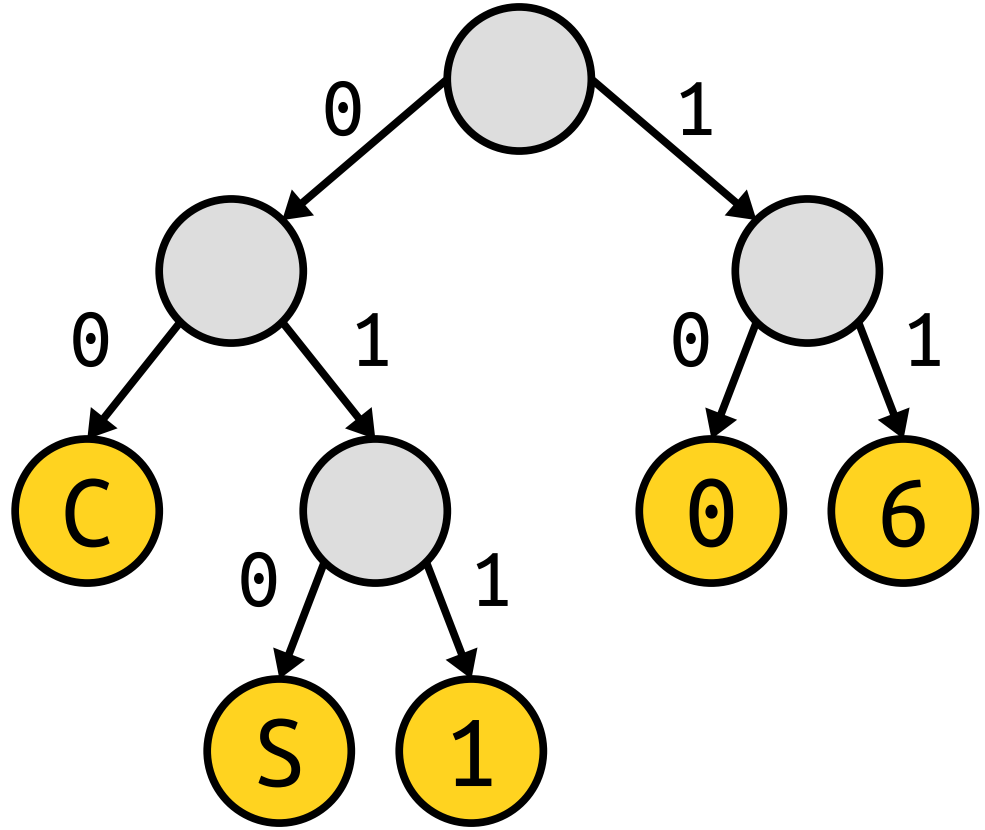
Coding Trees

- Not all binary trees will work as coding trees.
- Why is the one to the right not a valid coding tree?
- ***Answer:*** It doesn't give a prefix-free code. The code for `A` is a prefix for the codes for `C` and `D`, and the code for `B` is a prefix of the codes for `E` and `F`.



Coding Trees

- A coding tree is valid if all the letters are stored at the ***leaves***, with internal nodes just doing the routing.
- ***Goal:*** Find the best coding tree for a string.



How do we find the best
coding tree for a piece of text?

Time-Out for Announcements!

Assignment 9

- Assignment 8 was due today at 10:30AM.
  - Feel free to use the grace period if you need to and submit tomorrow at 10:30AM.
  - If you'll need more time than that, email Neel and me and ask for an extension. It's okay. We understand.
- Assignment 9 goes out today.
  - Implement the techniques from this lecture!
  - See how much space-saving is available!
  - YEAH Hours run today at 5:30PM; check EdStem for the Zoom link.

Extra Credit Practice Final Exam

- We’ve posted a practice final exam on Gradescope.
- If you submit answers to all of the problems – regardless of whether those answers are working solutions or just a blank text file – we’ll give you a bonus point on the final exam.
- The deadline to submit is next Friday, March 11th at 10:30AM, right when the final exam goes out.

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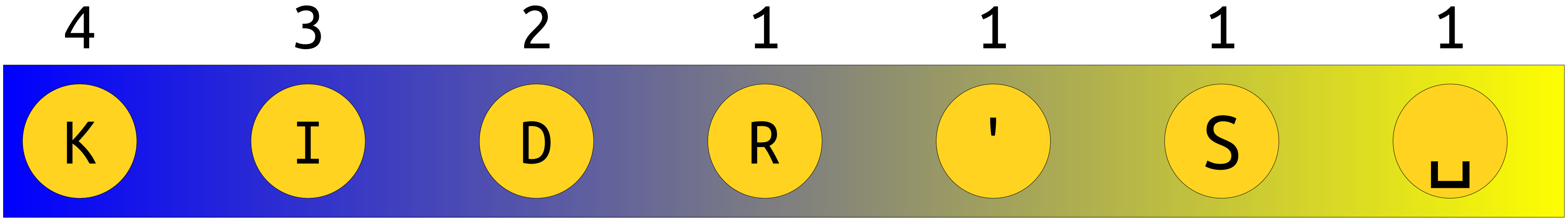
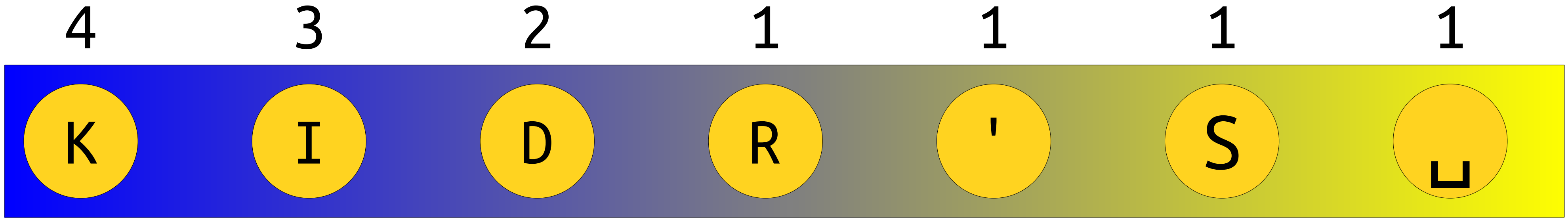
How do we find the best coding tree for a piece of text?

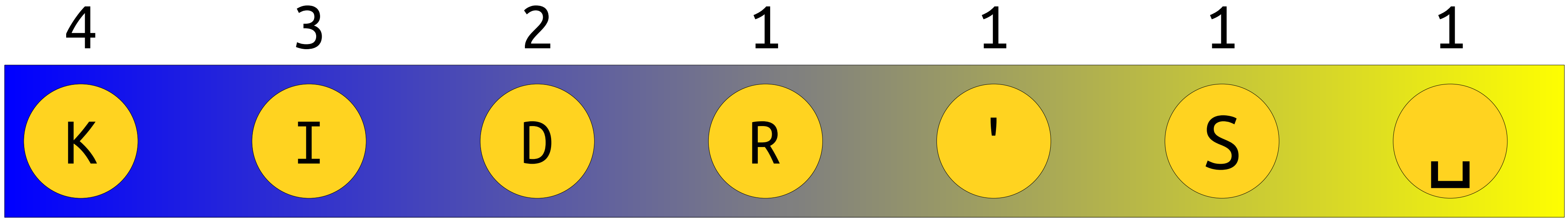
Huffman Coding



character	*frequency*
K	**4**
I	**3**
D	**2**
R	**1**
'	**1**
S	**1**
␣	**1**



Right now, we have all the leaves of the tree. We now need to build the tree around them.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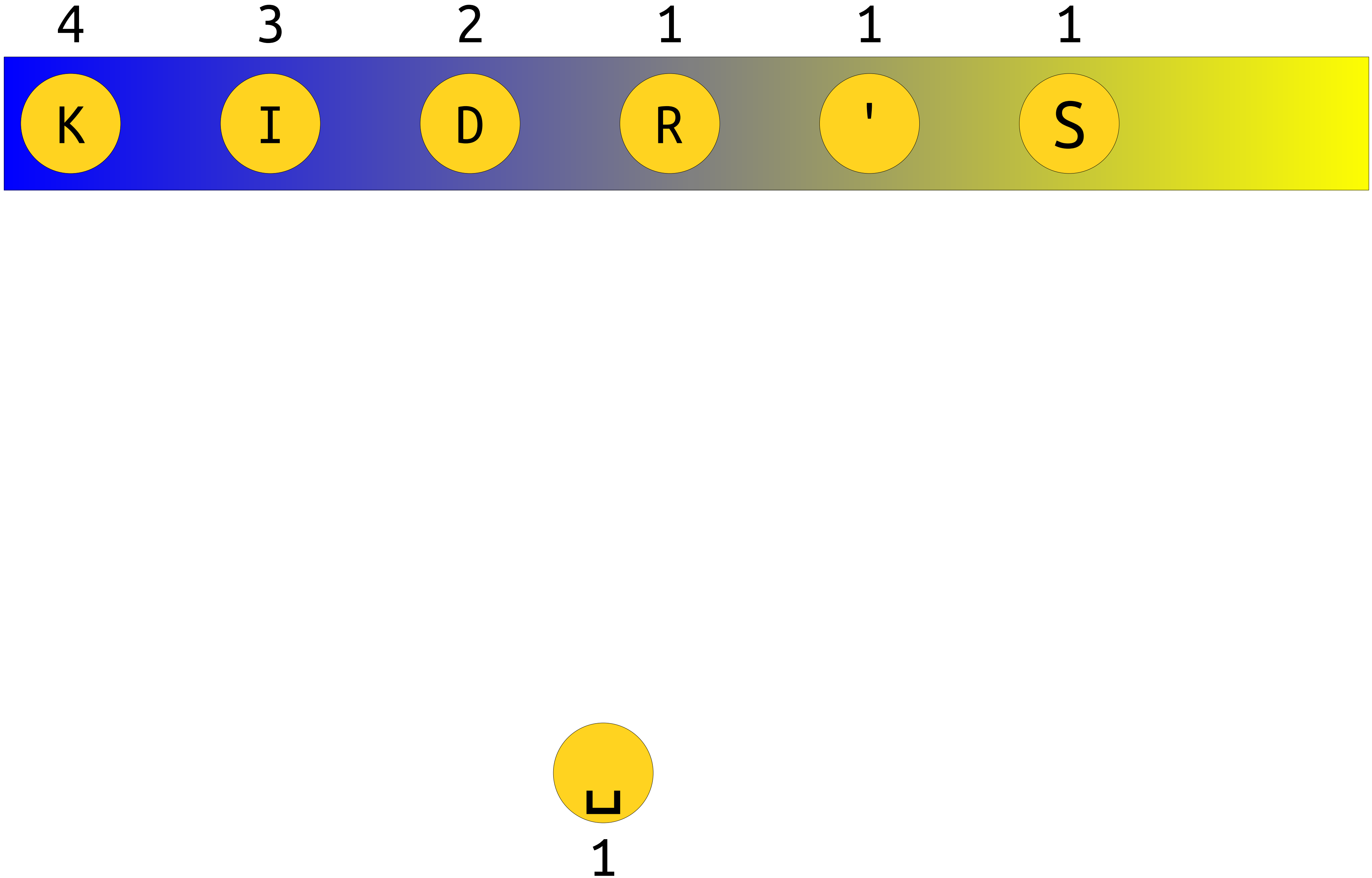
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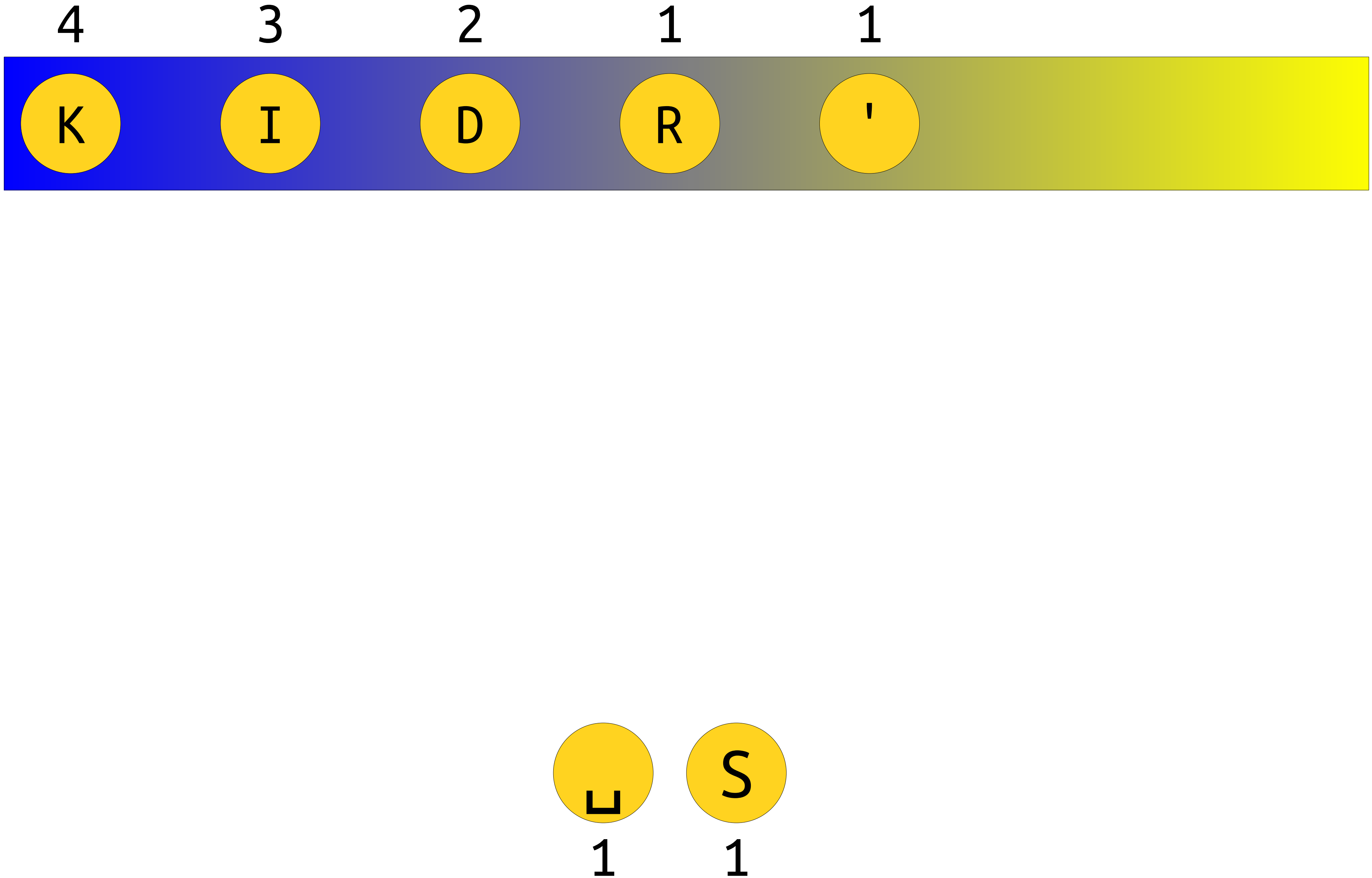
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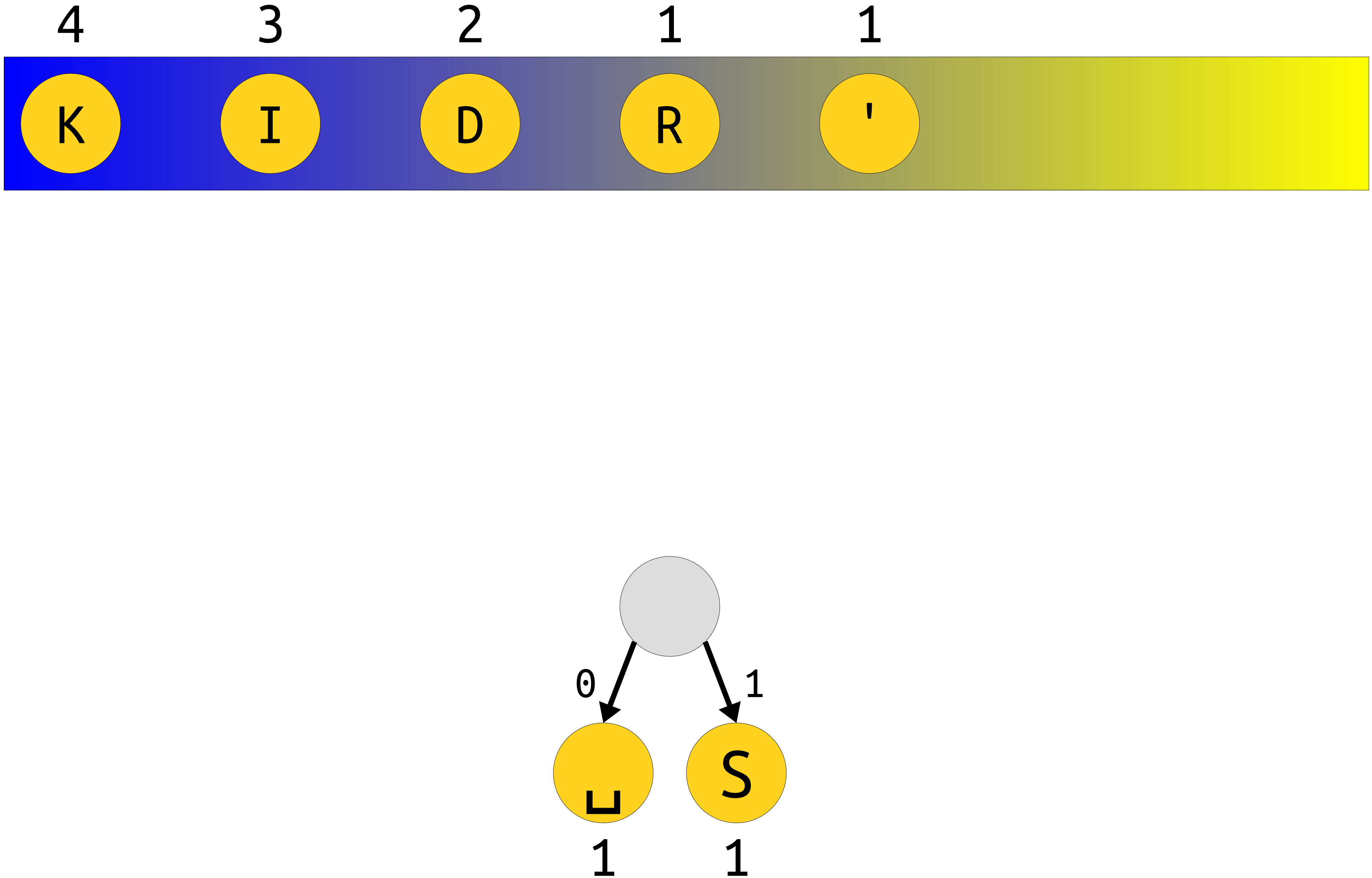
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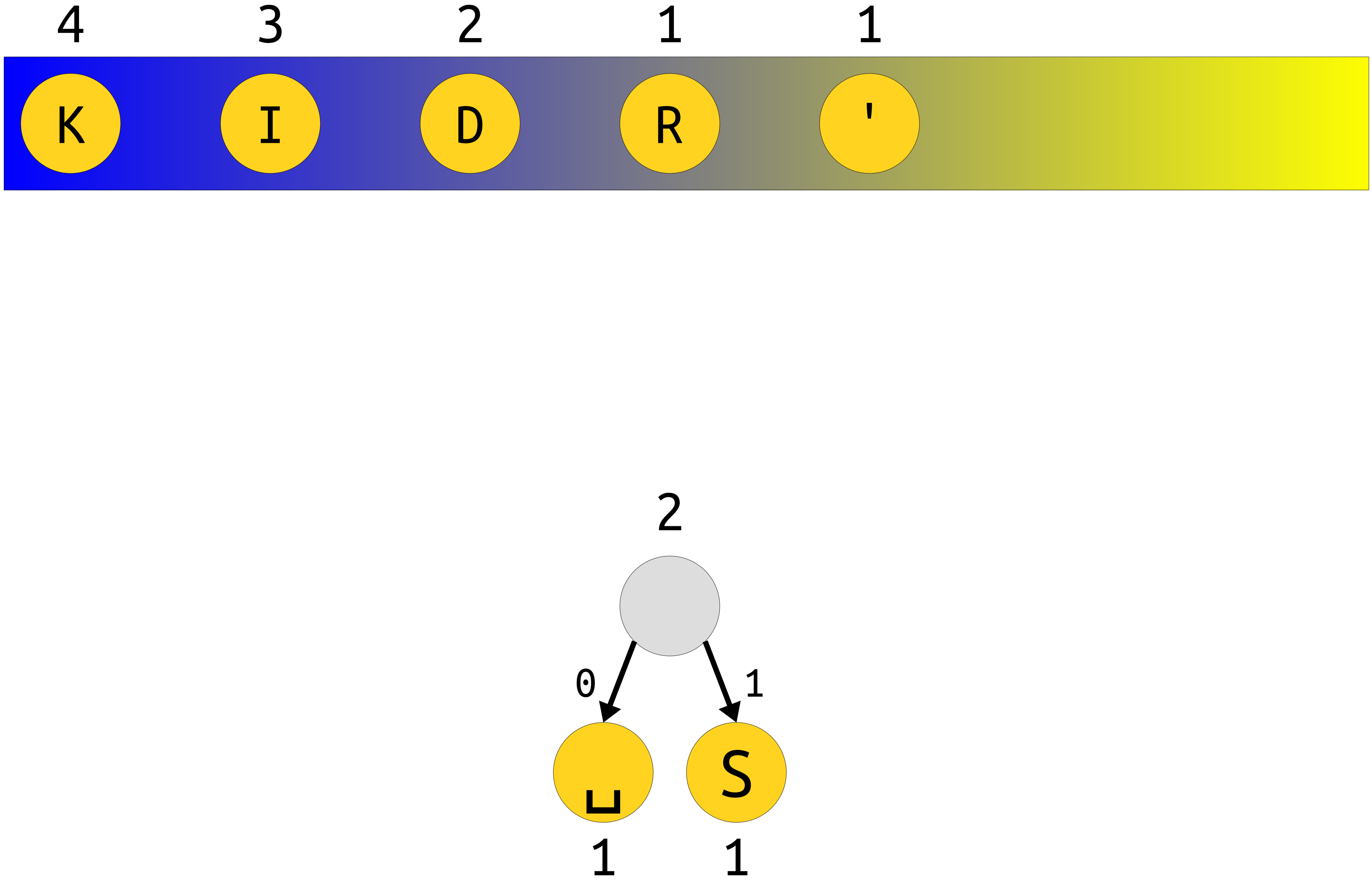
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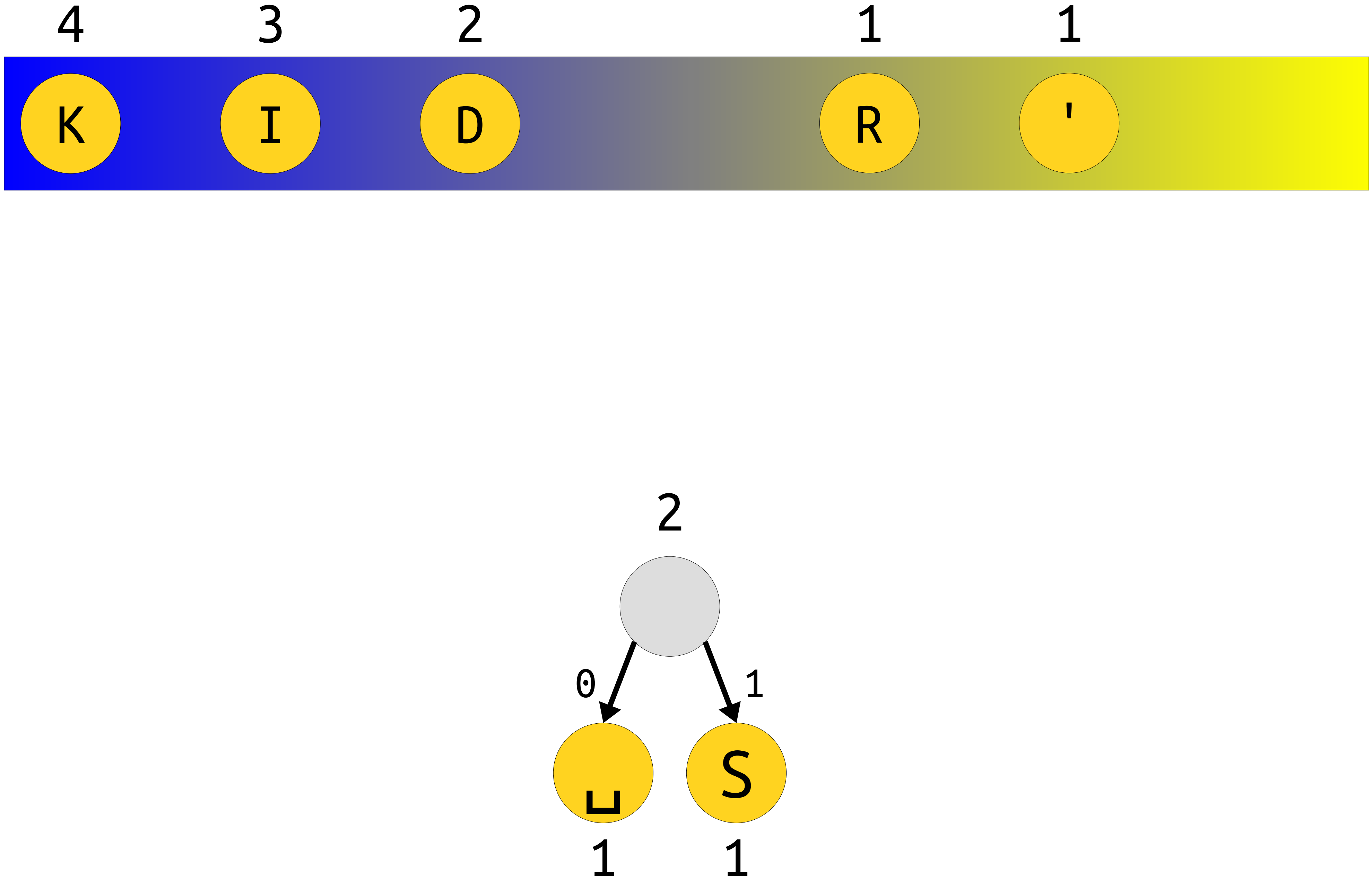
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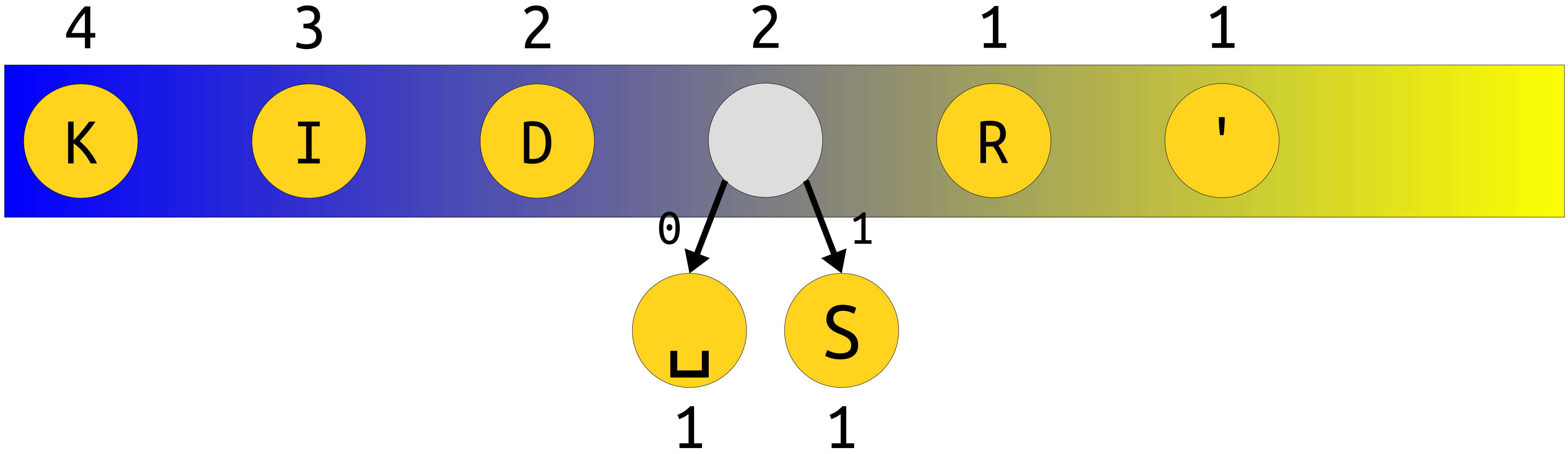
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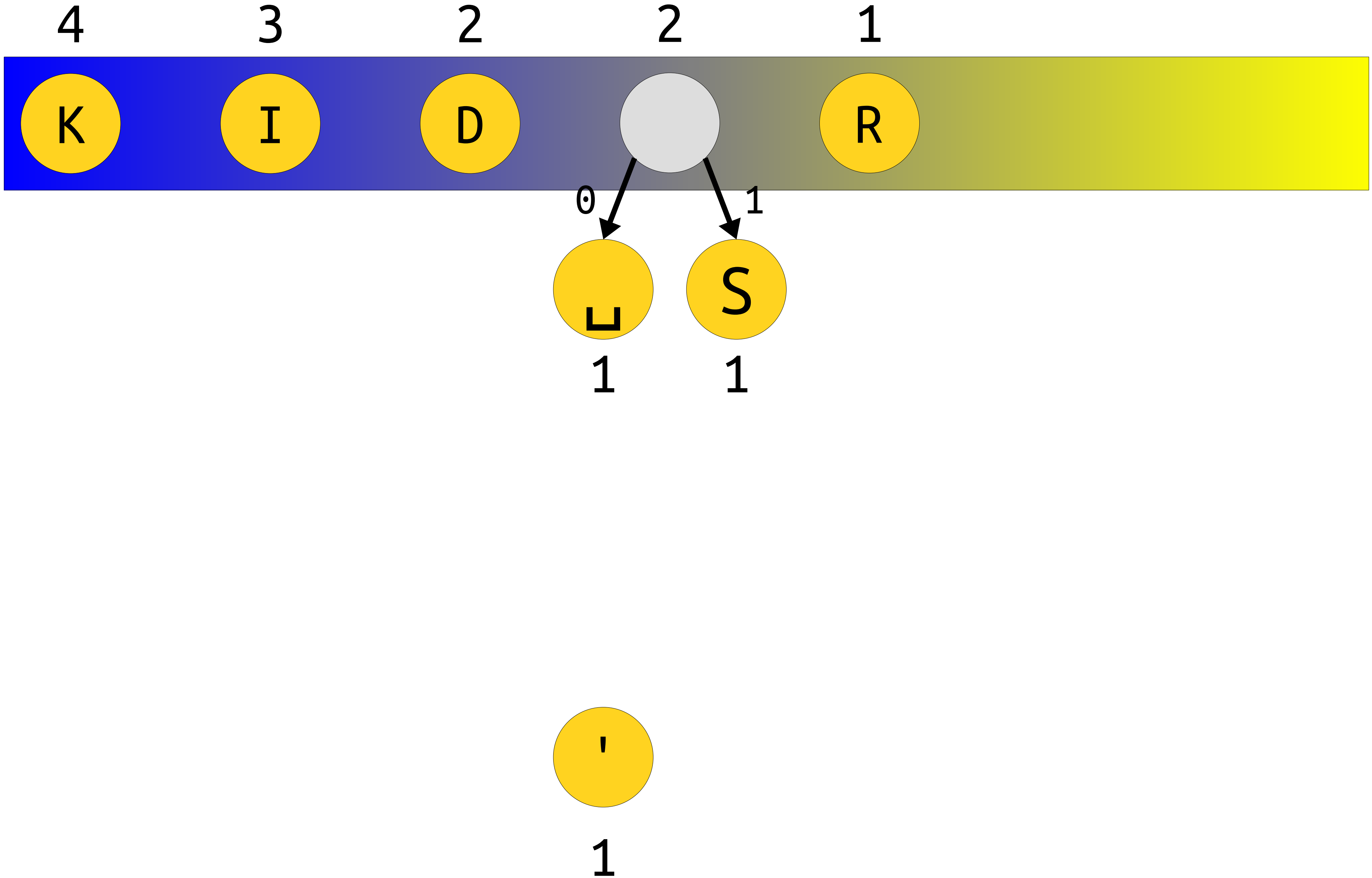

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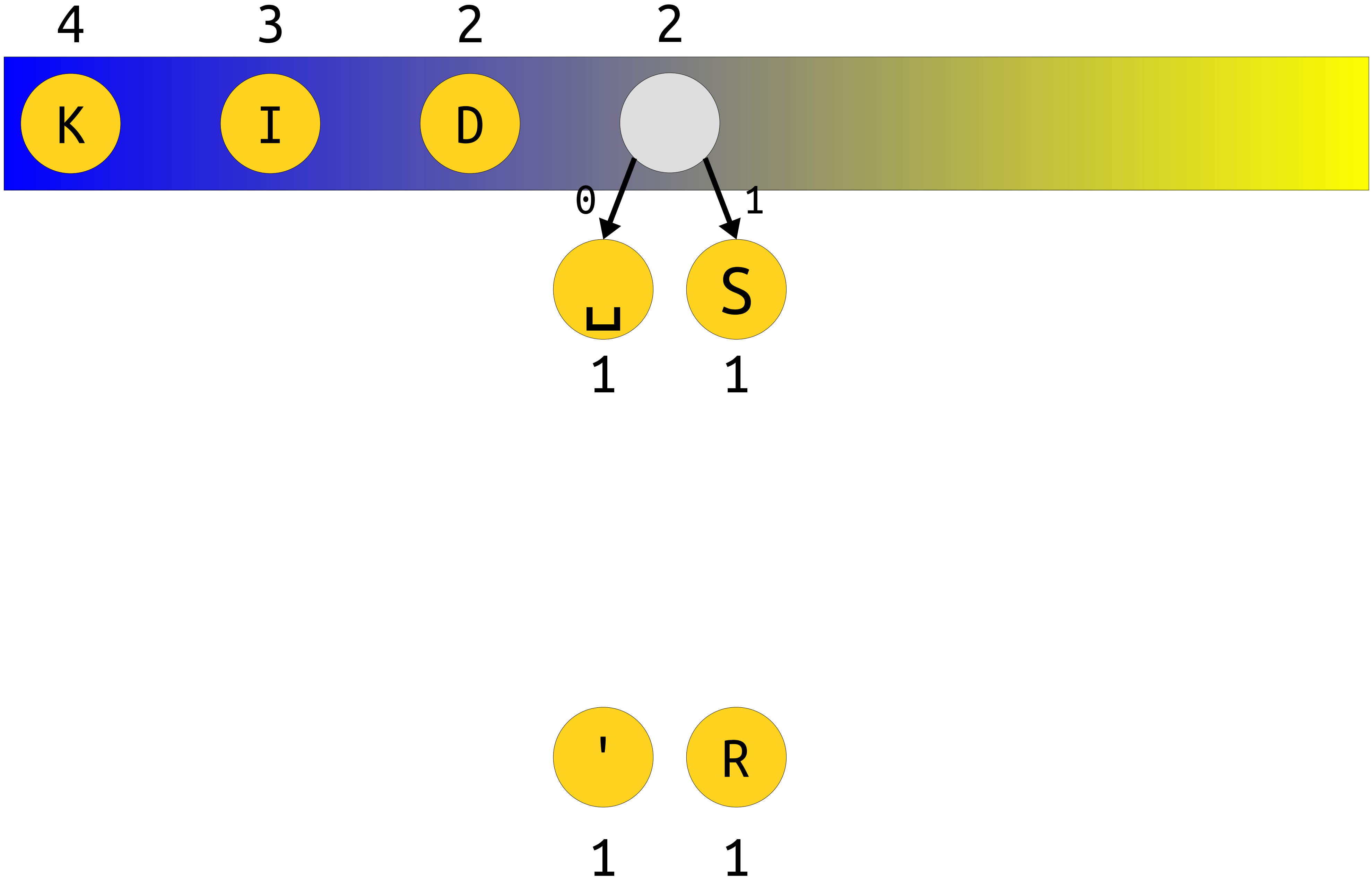

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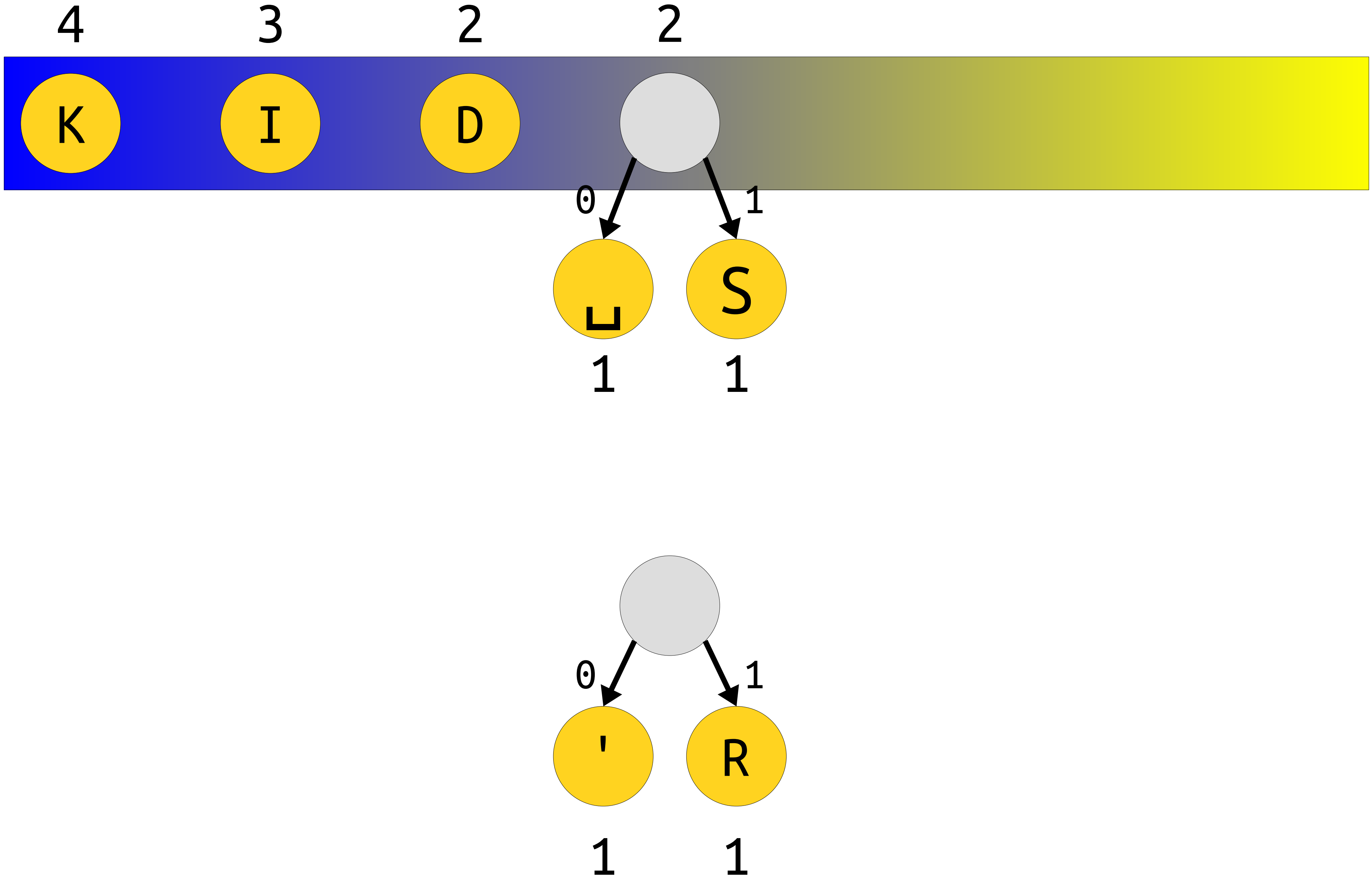

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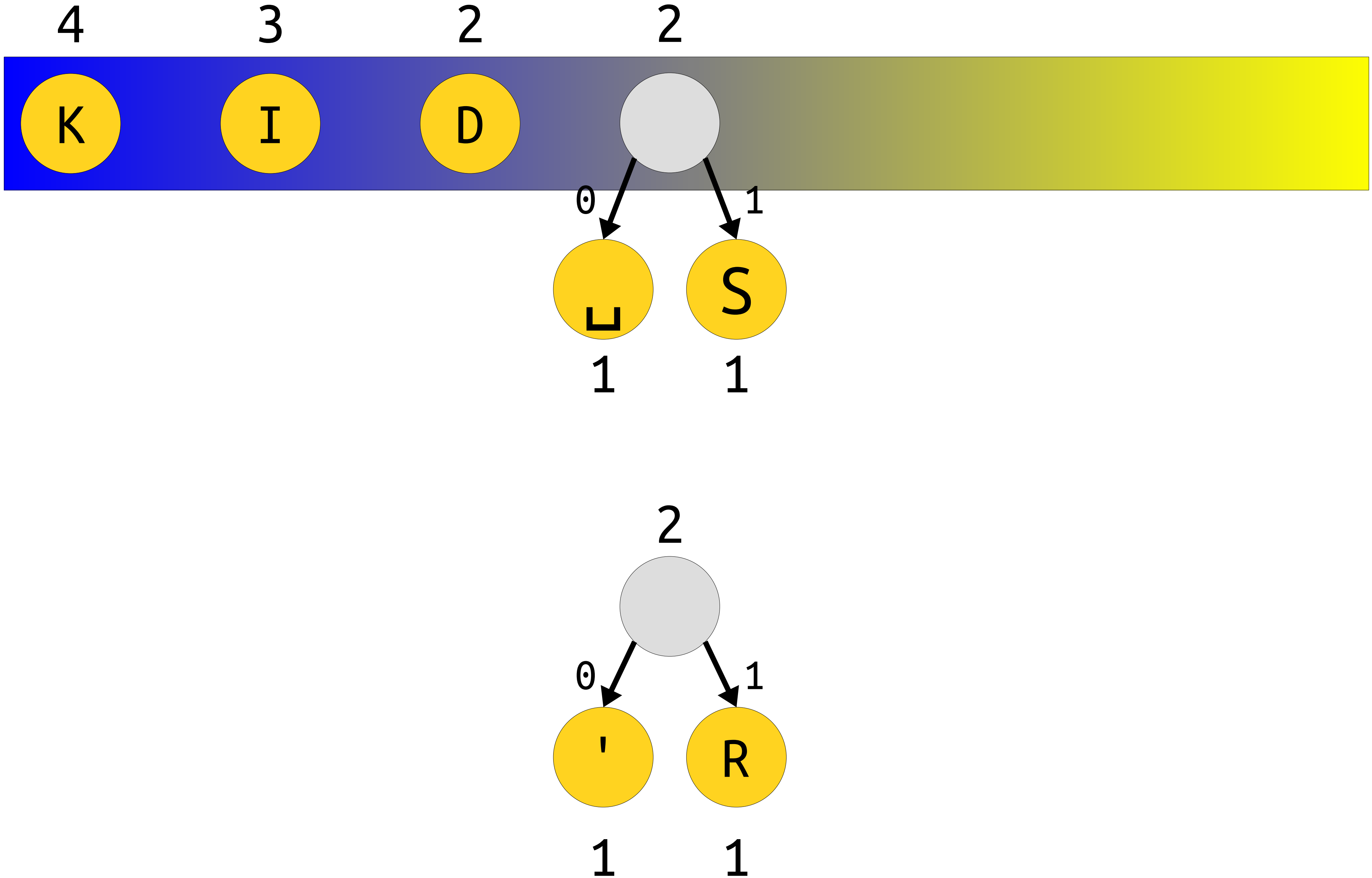

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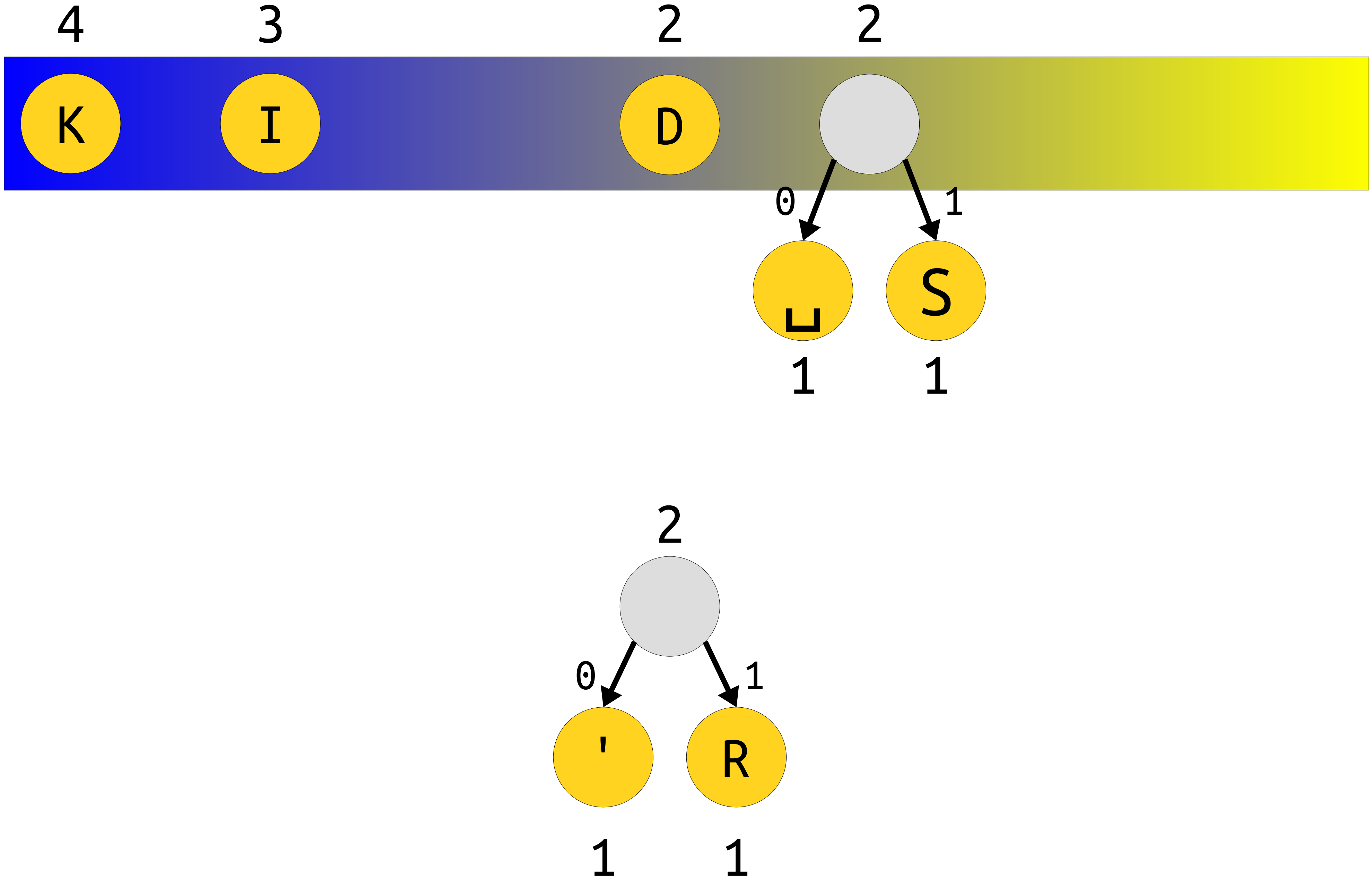
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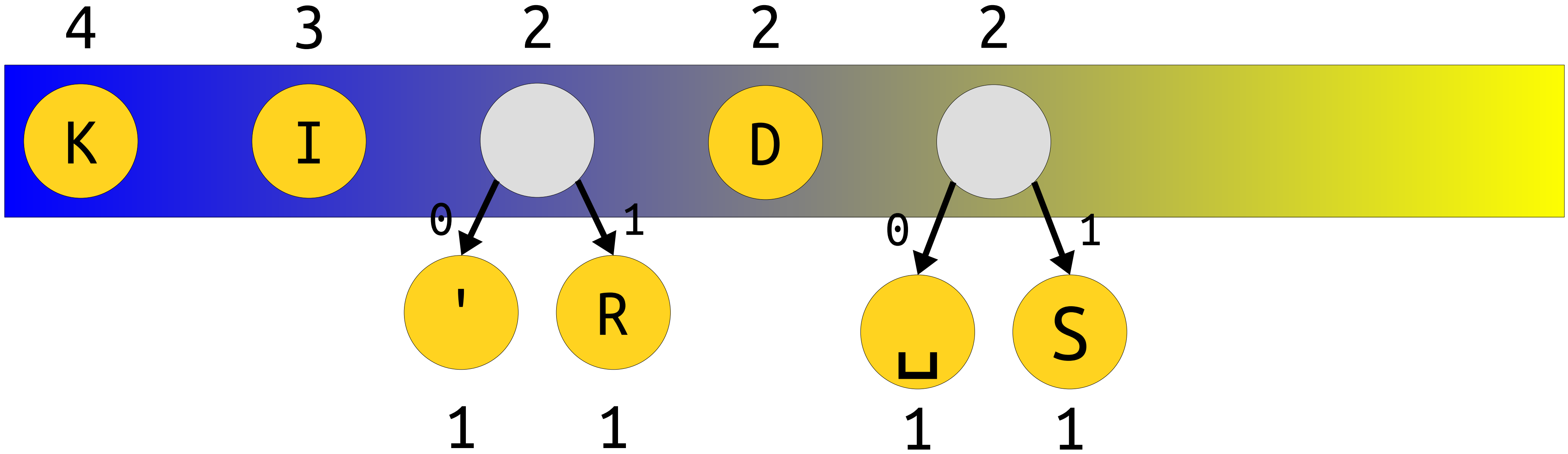

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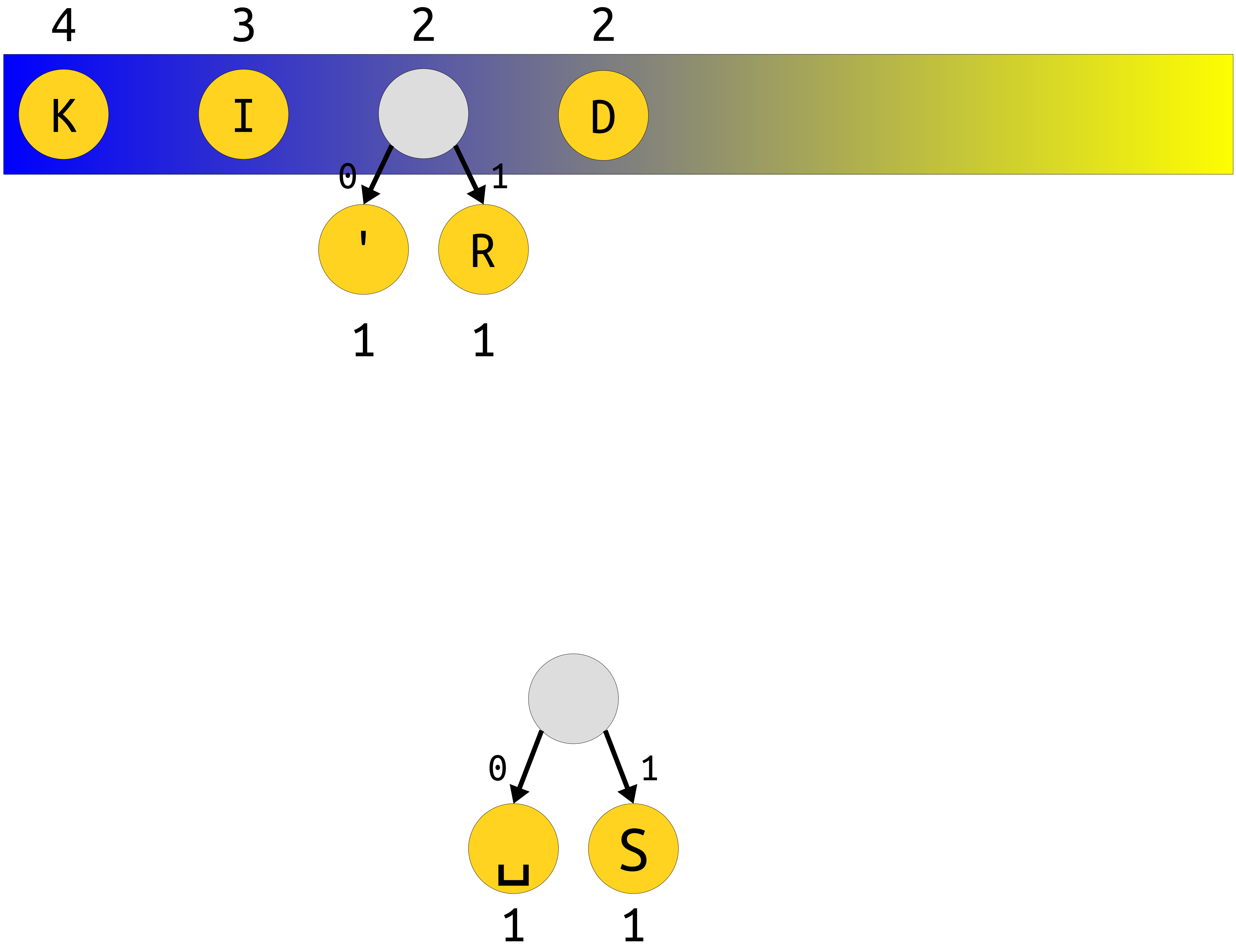

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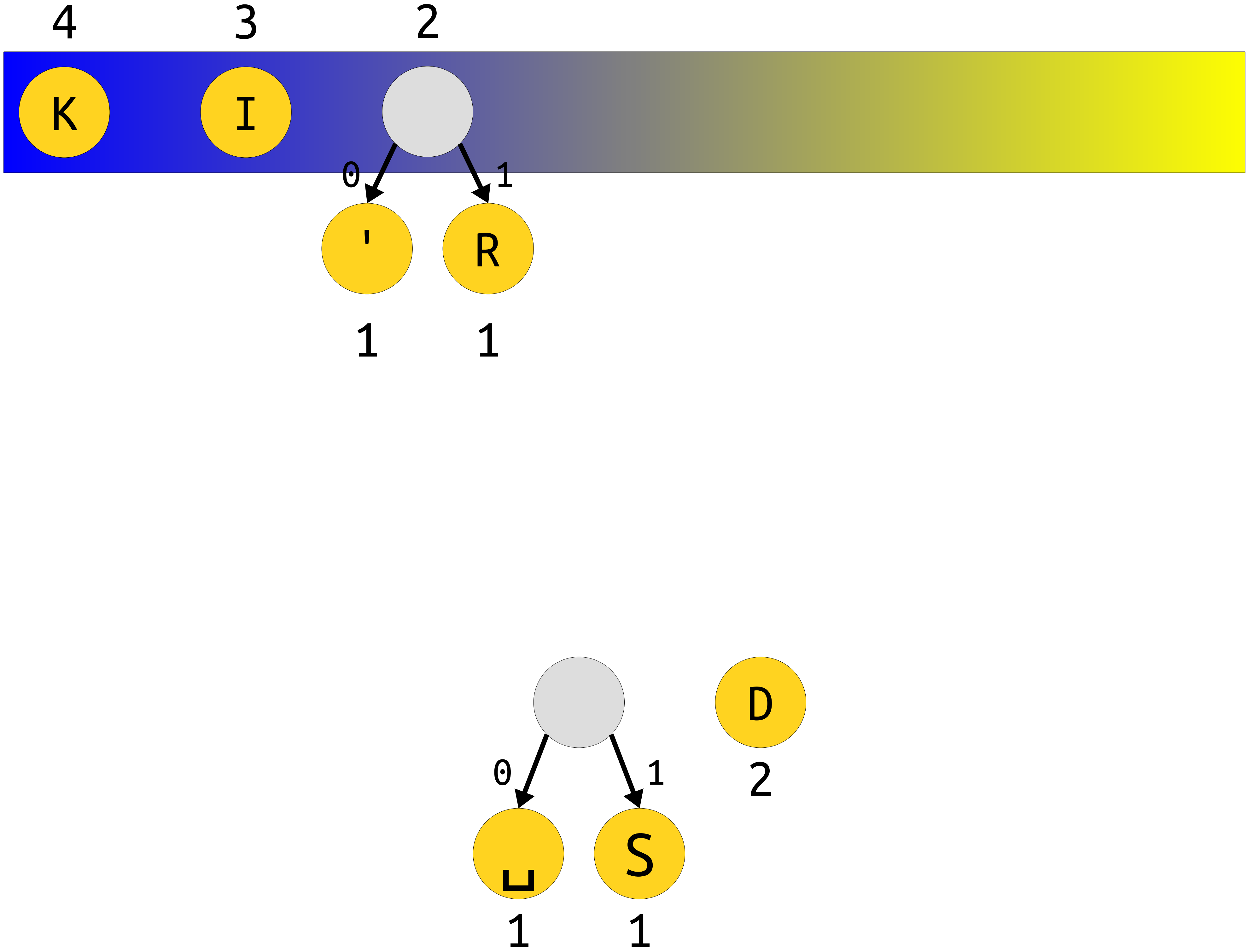
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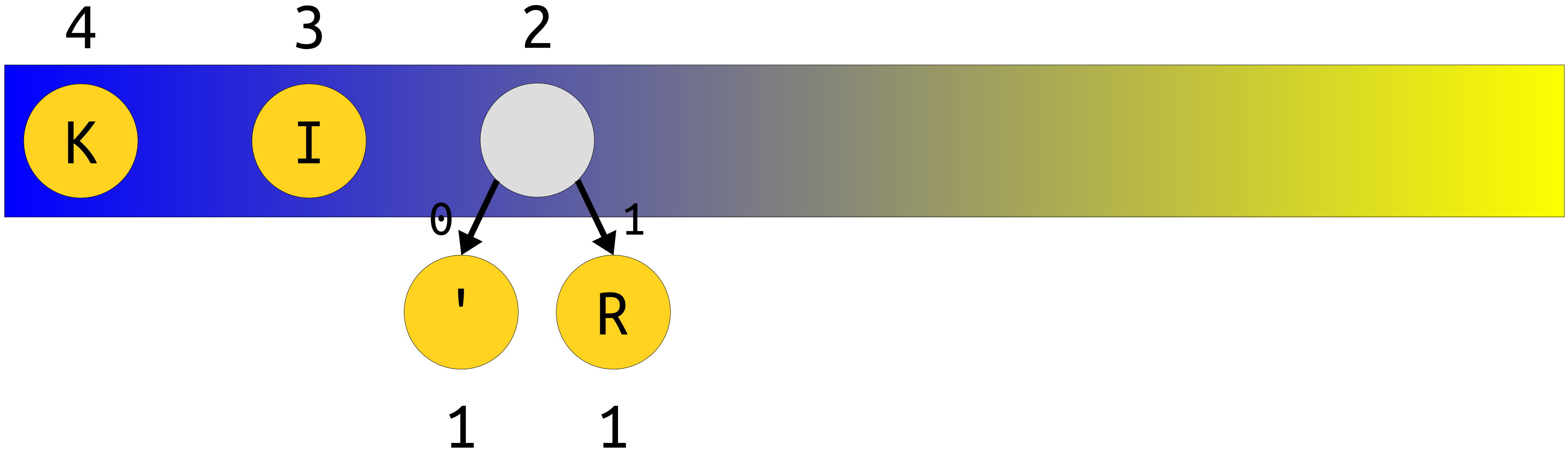
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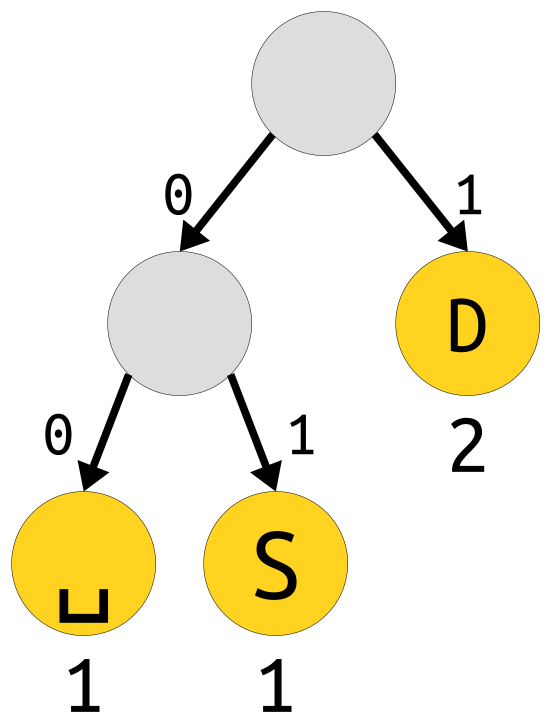
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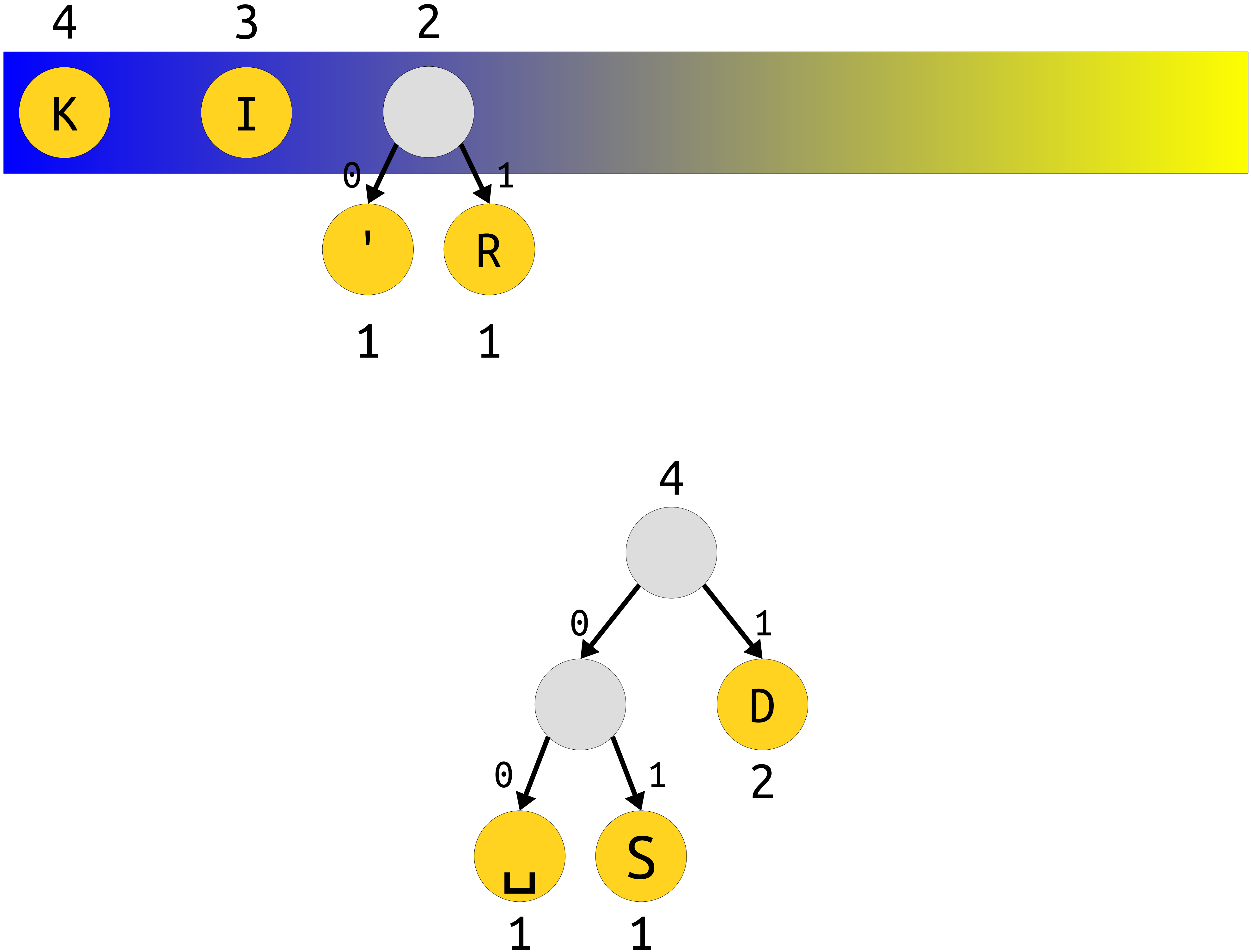

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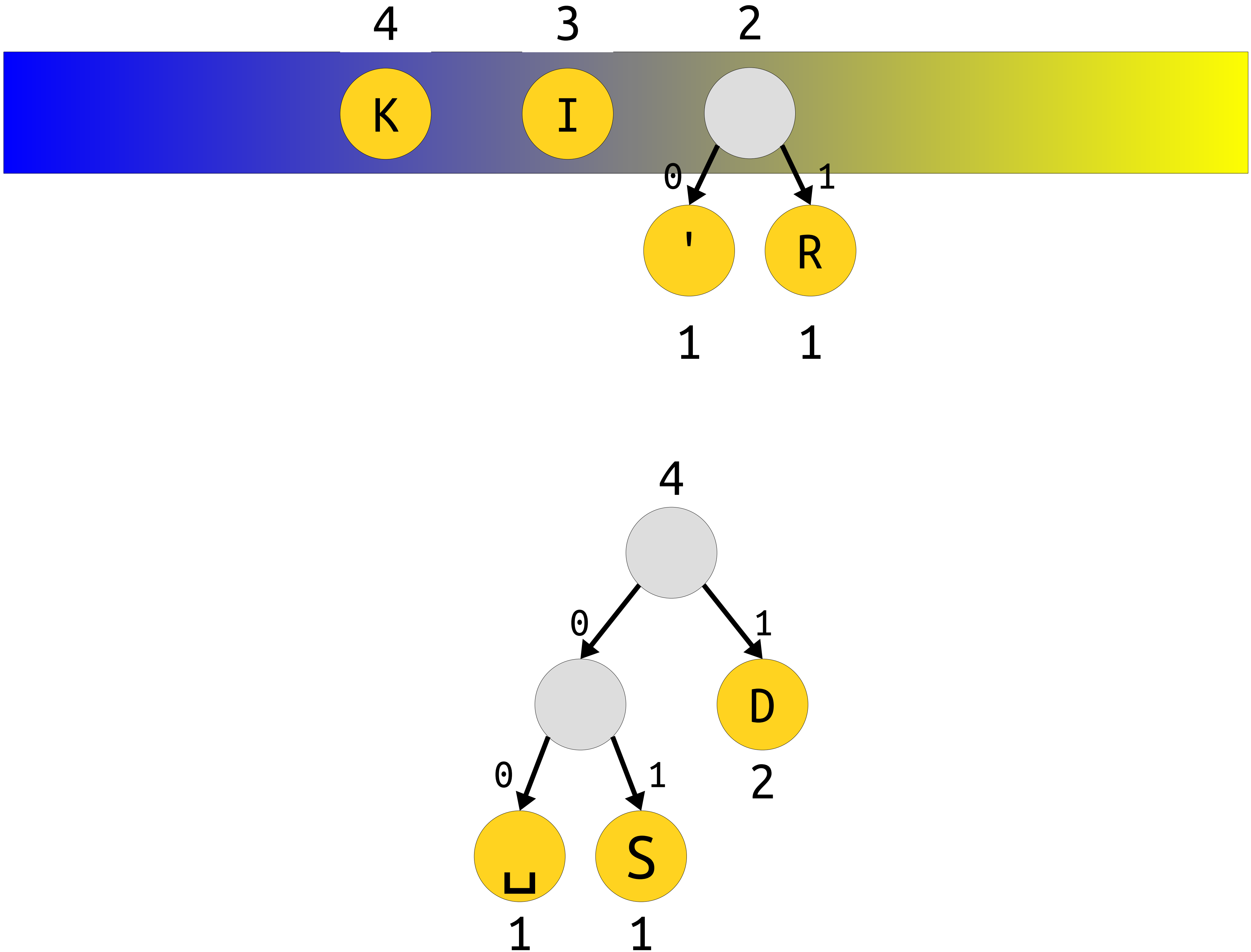

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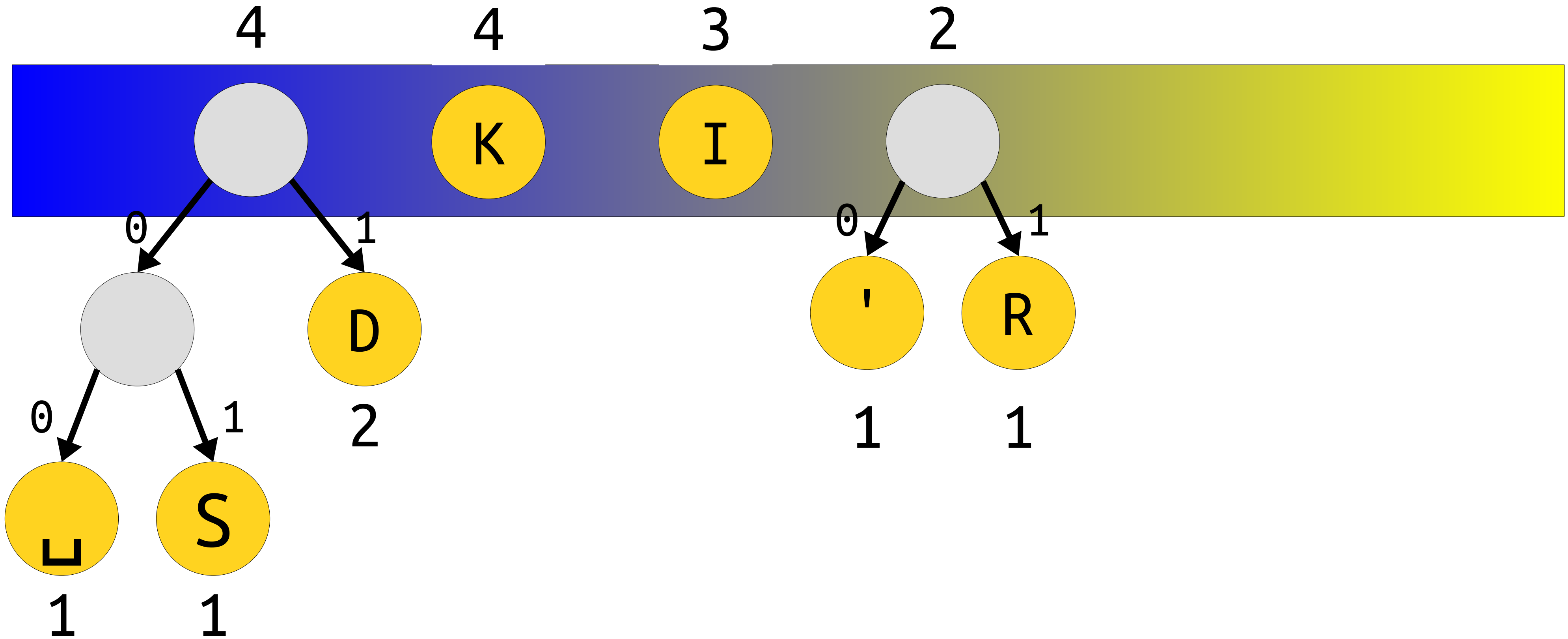
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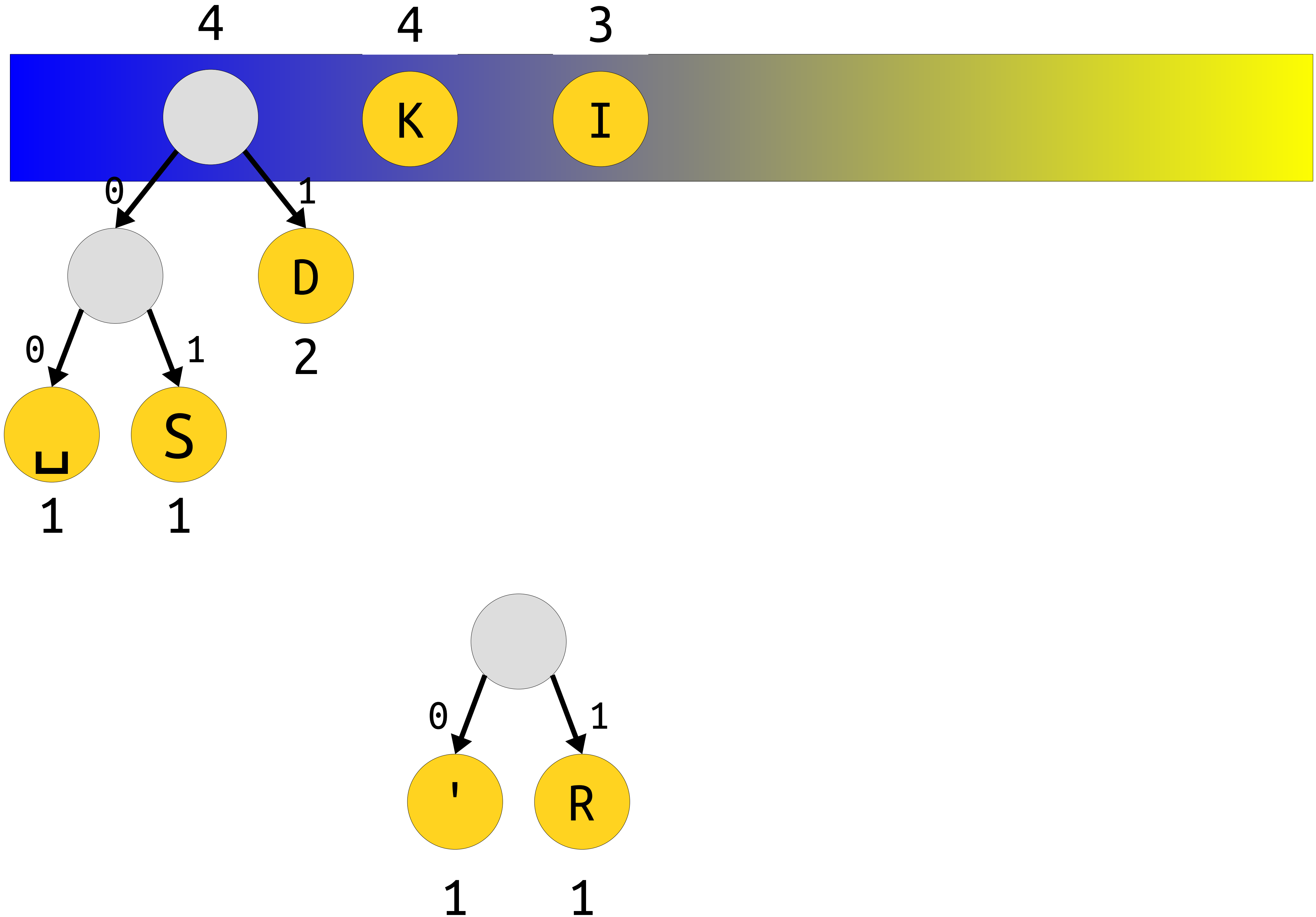
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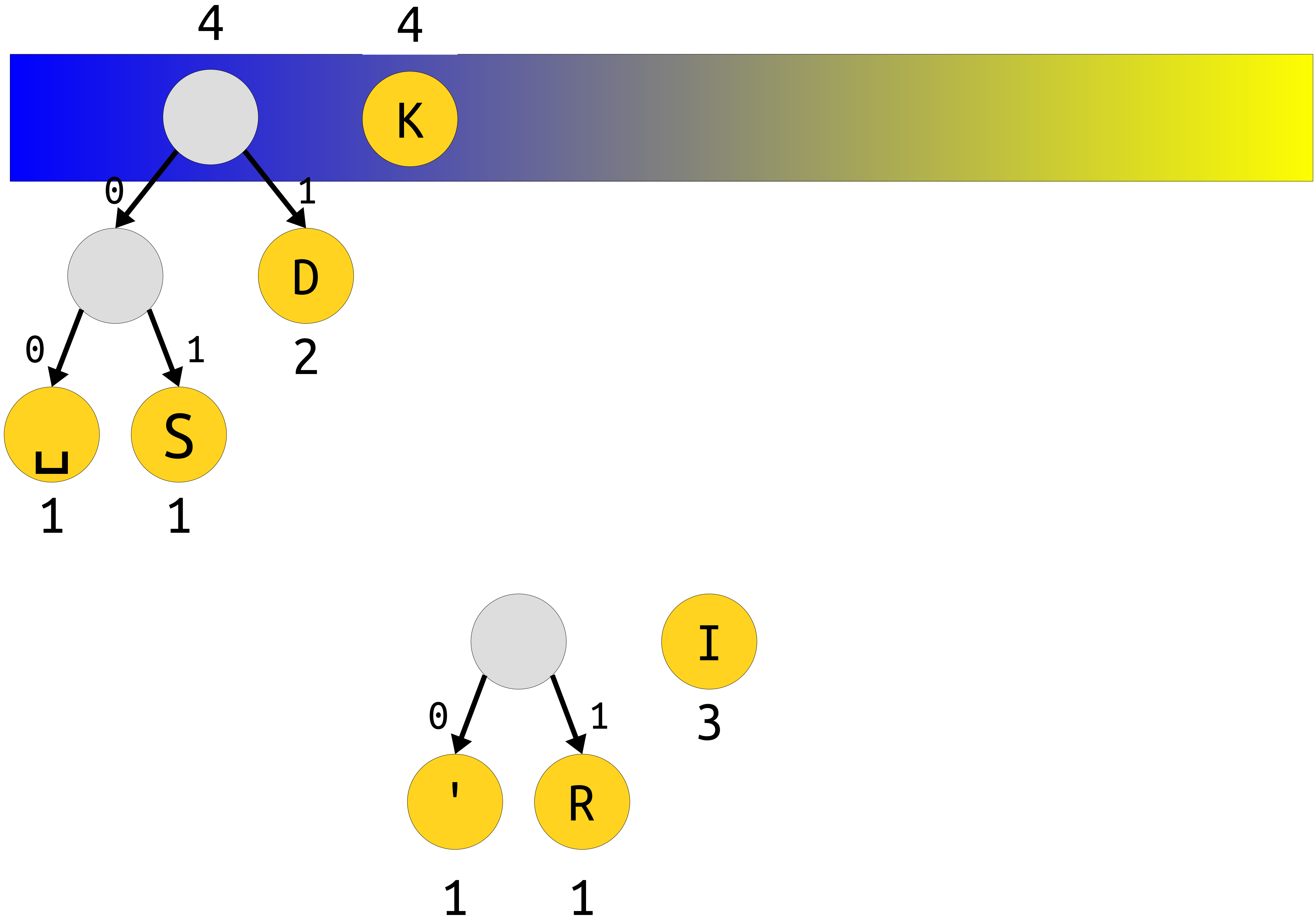
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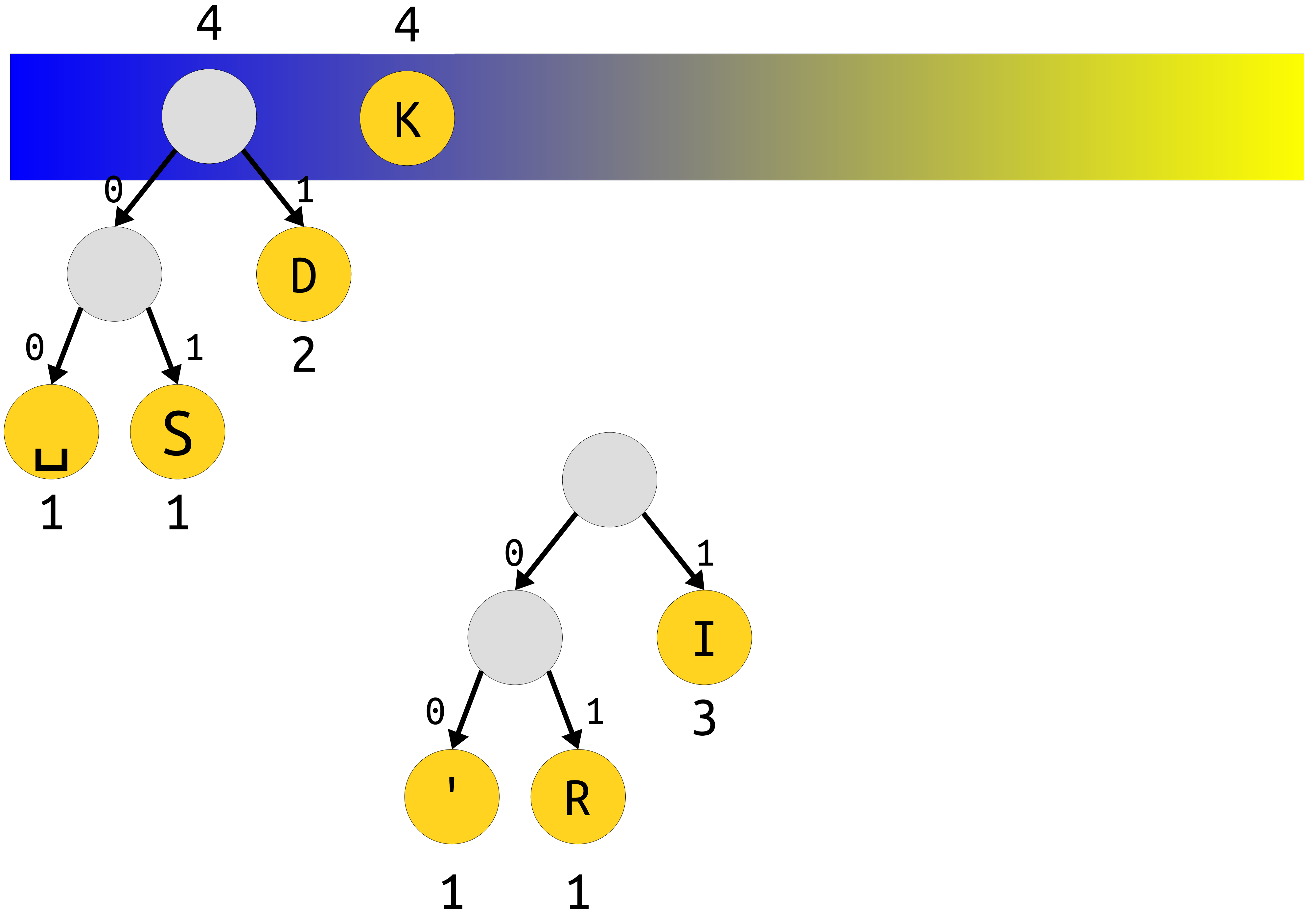

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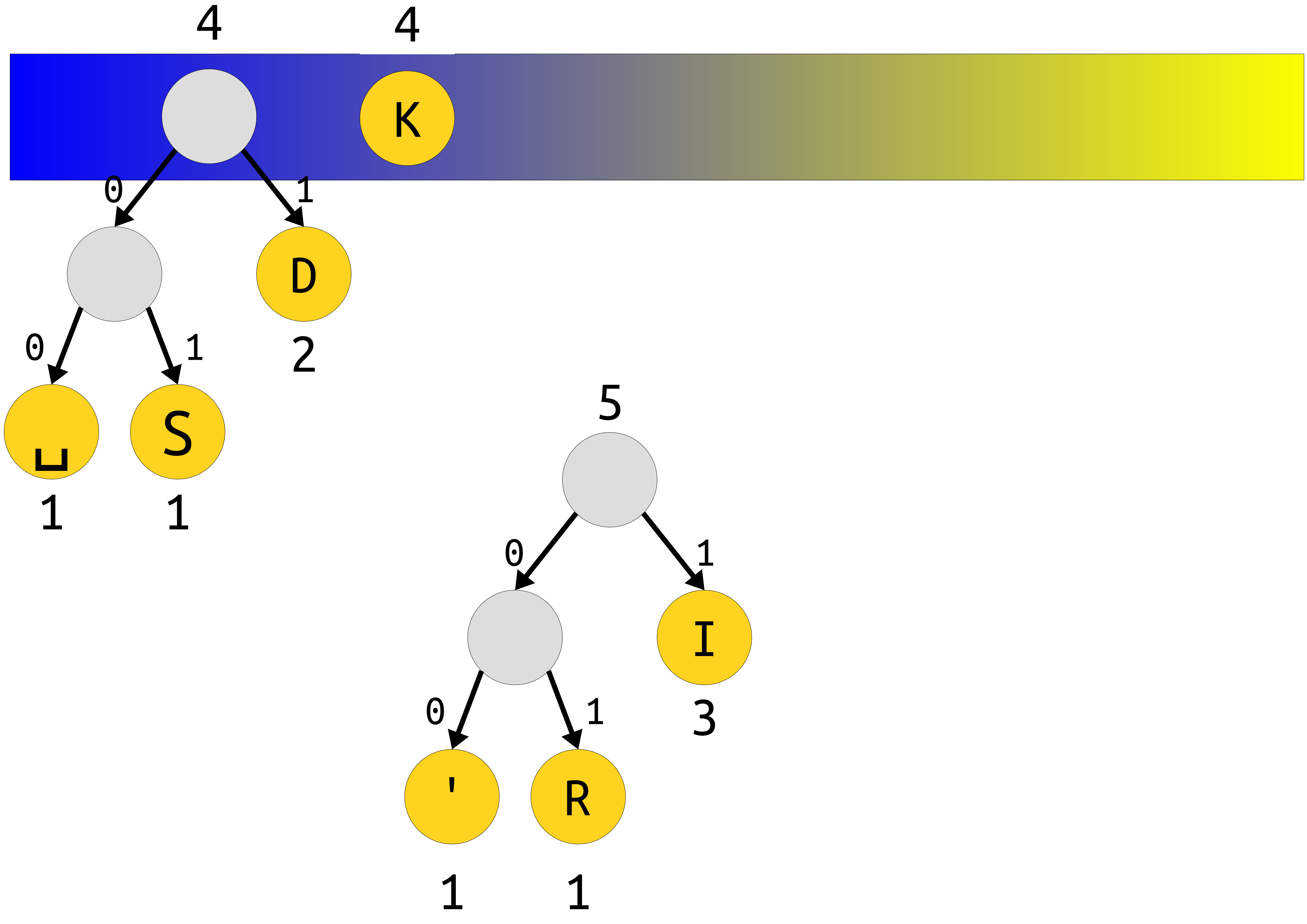

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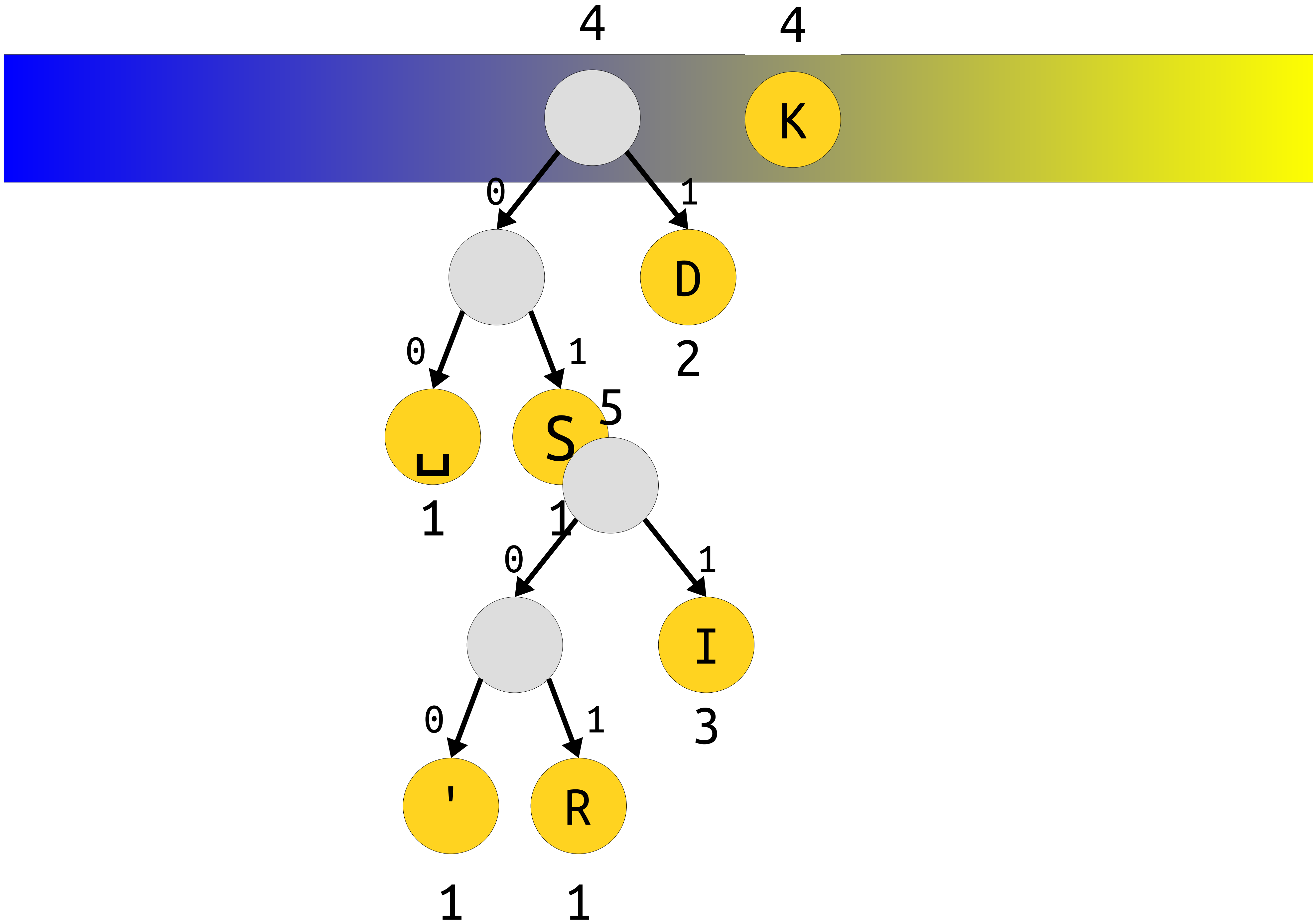
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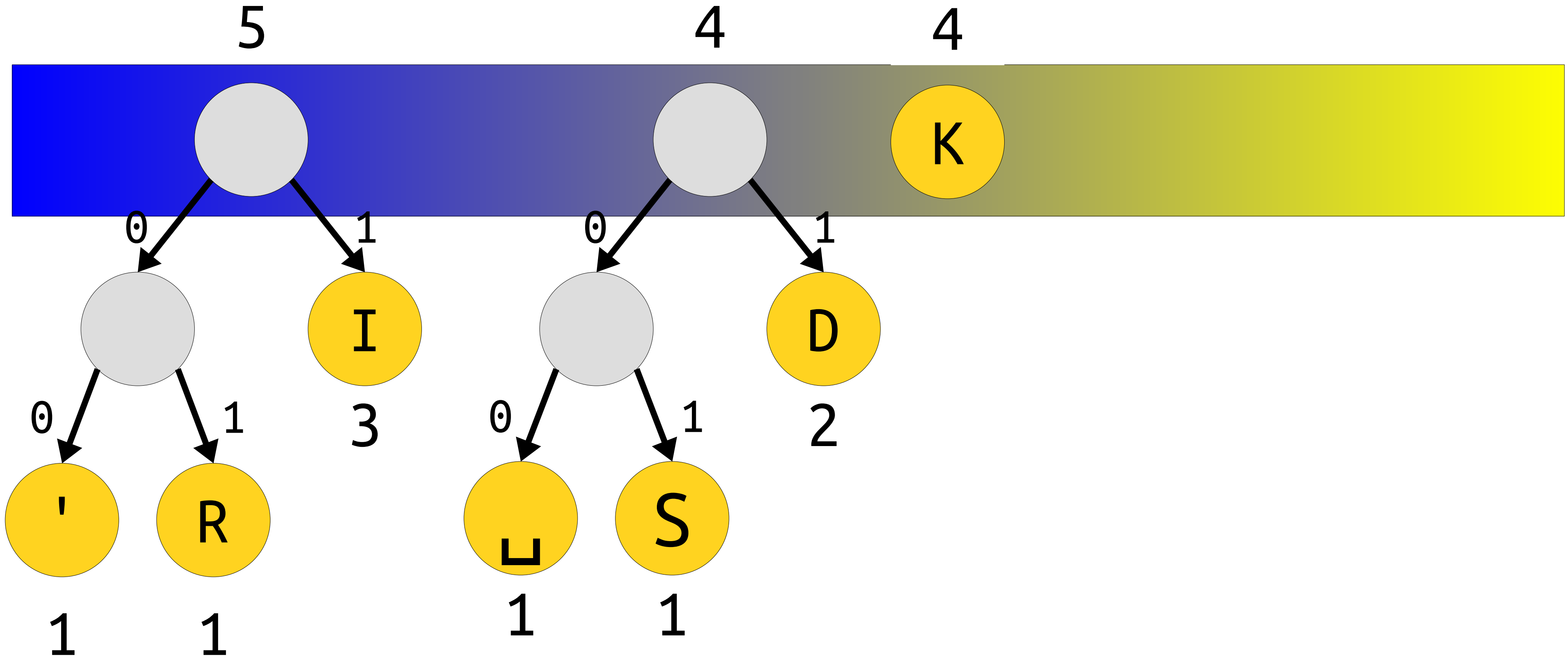

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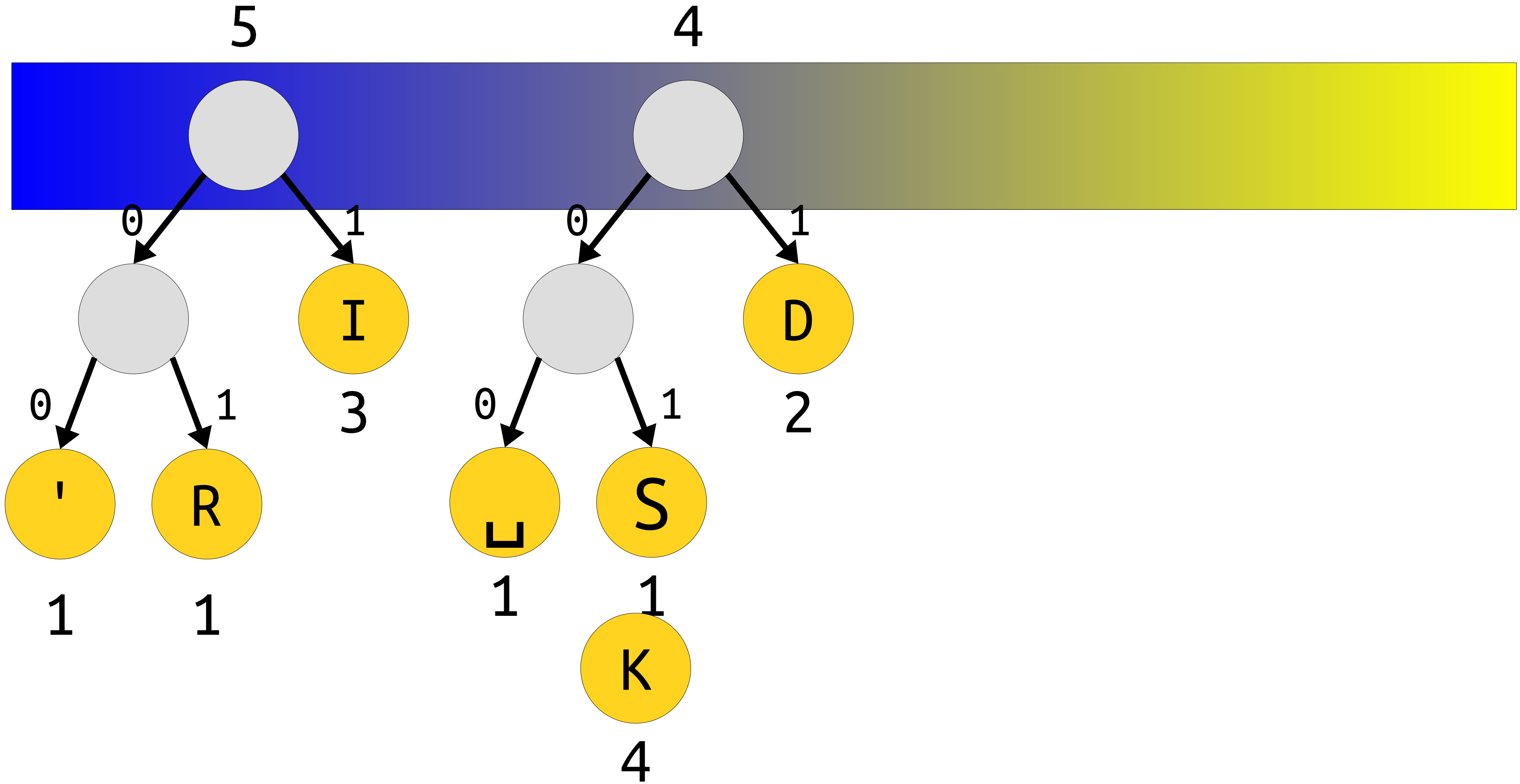
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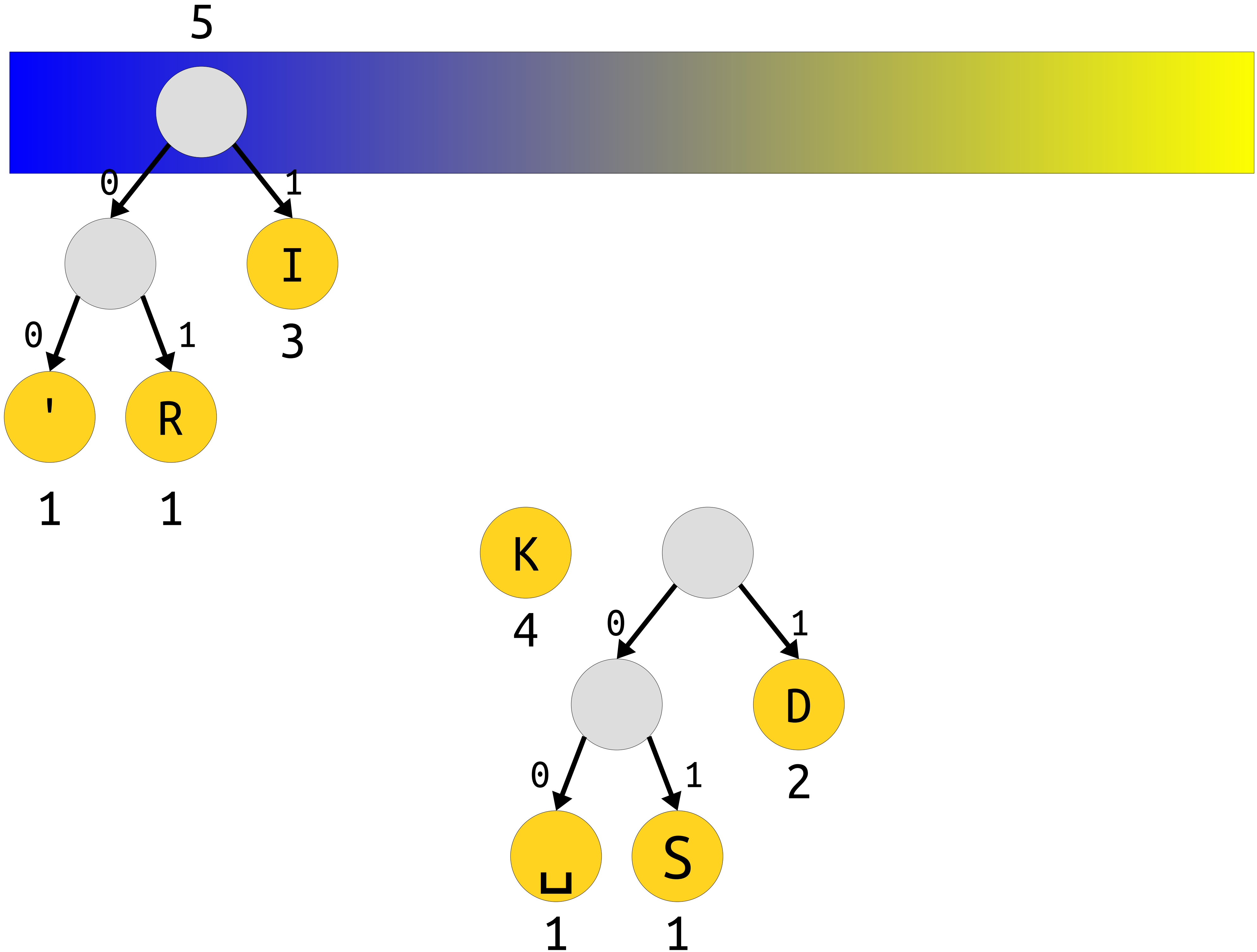

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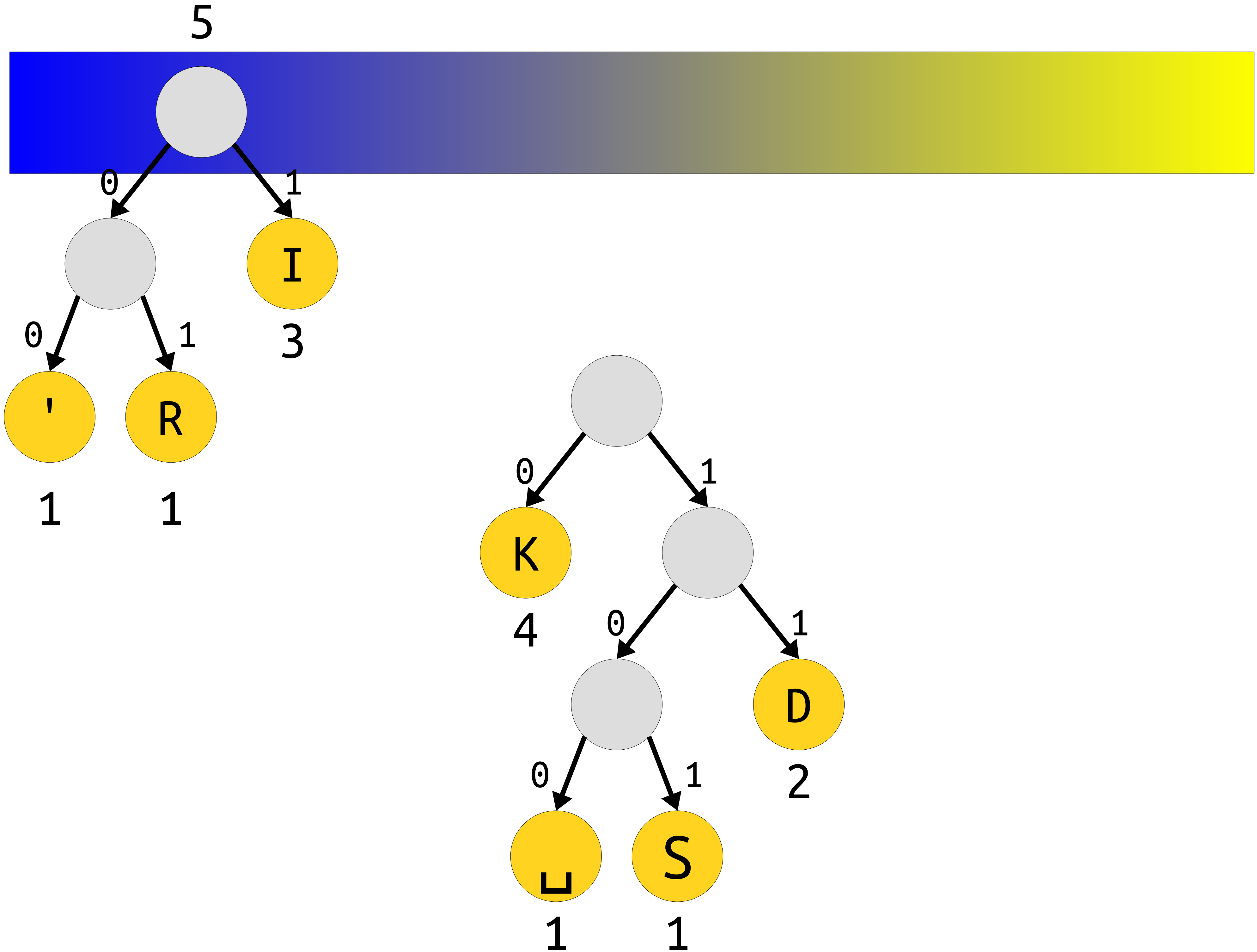
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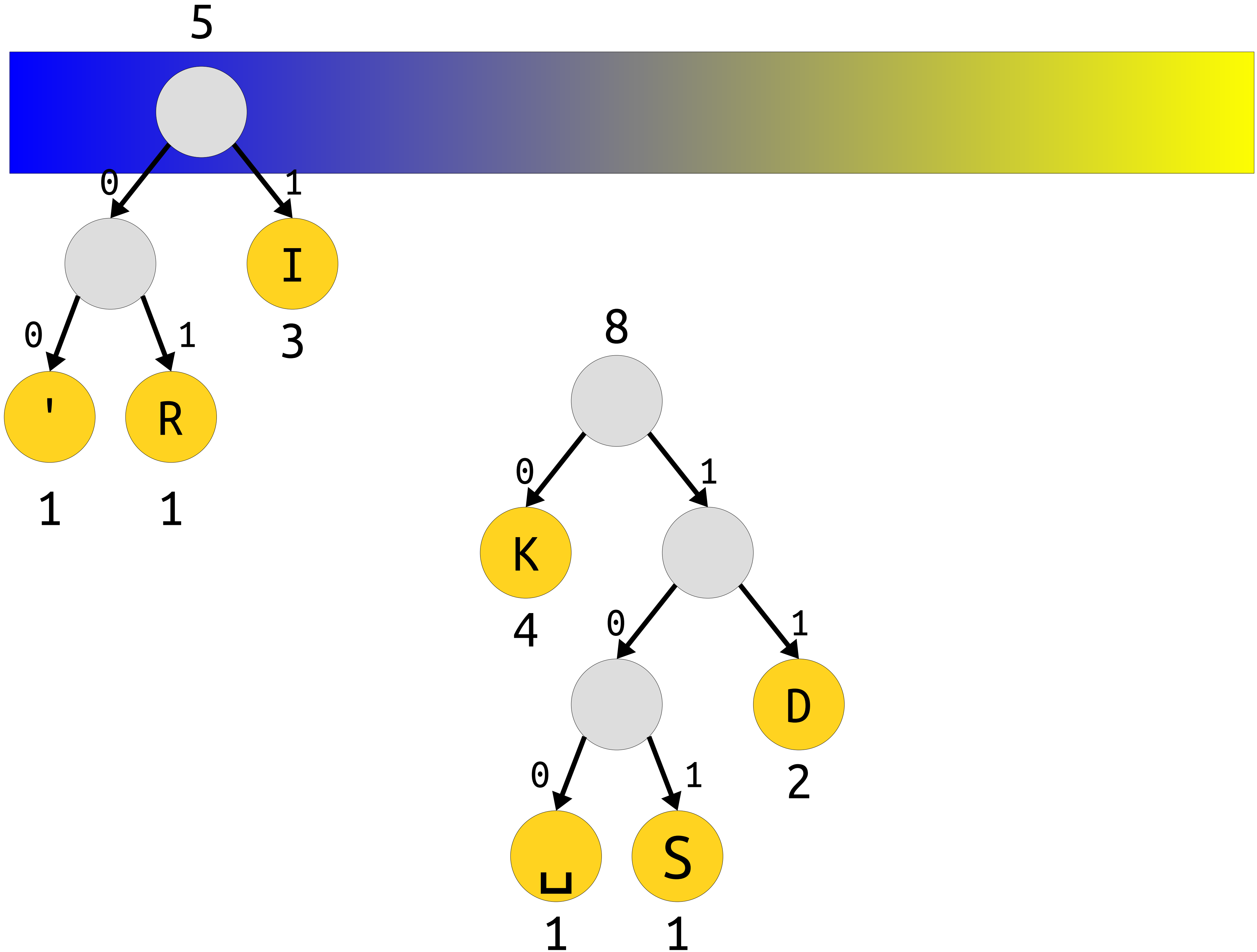
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'
R
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K
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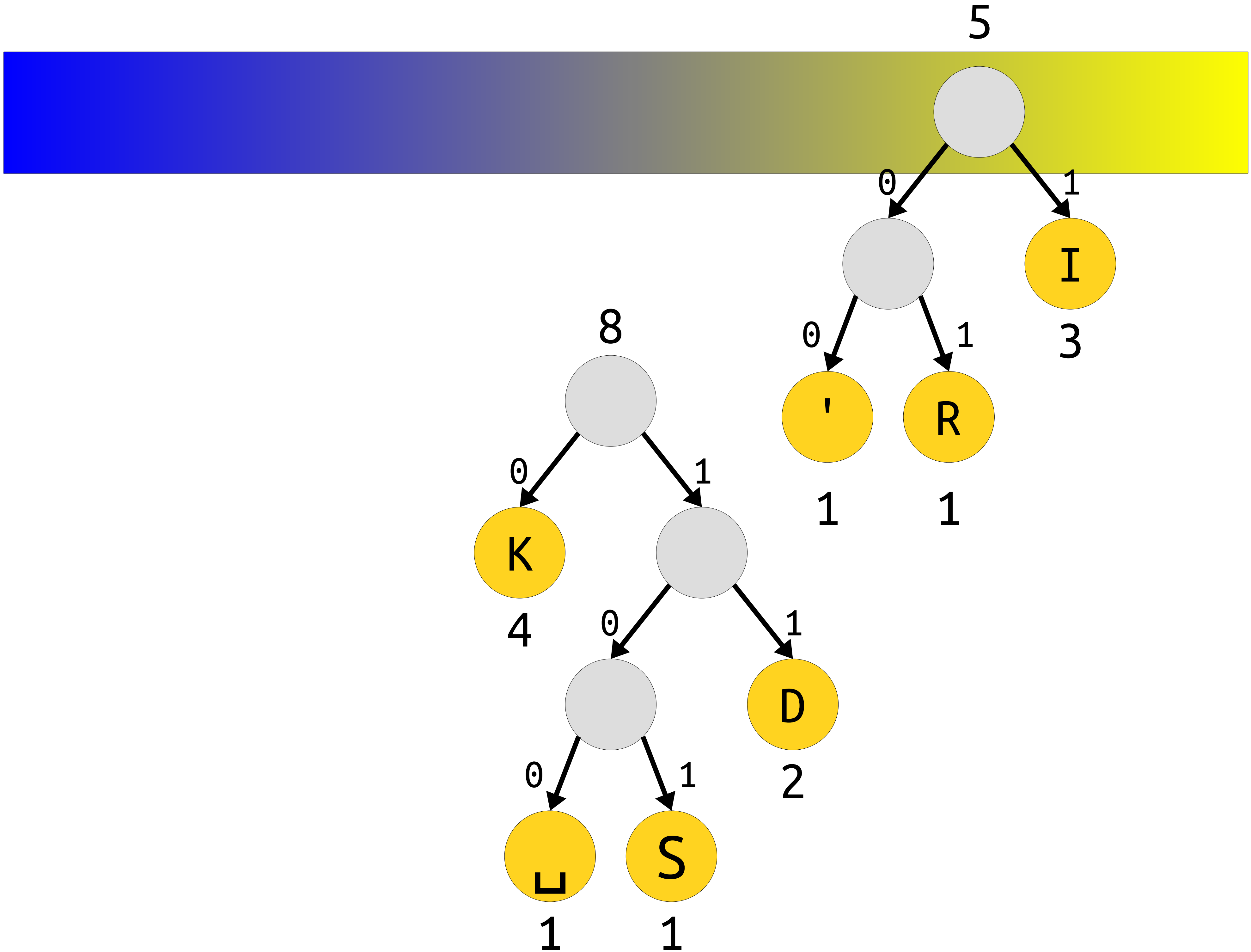

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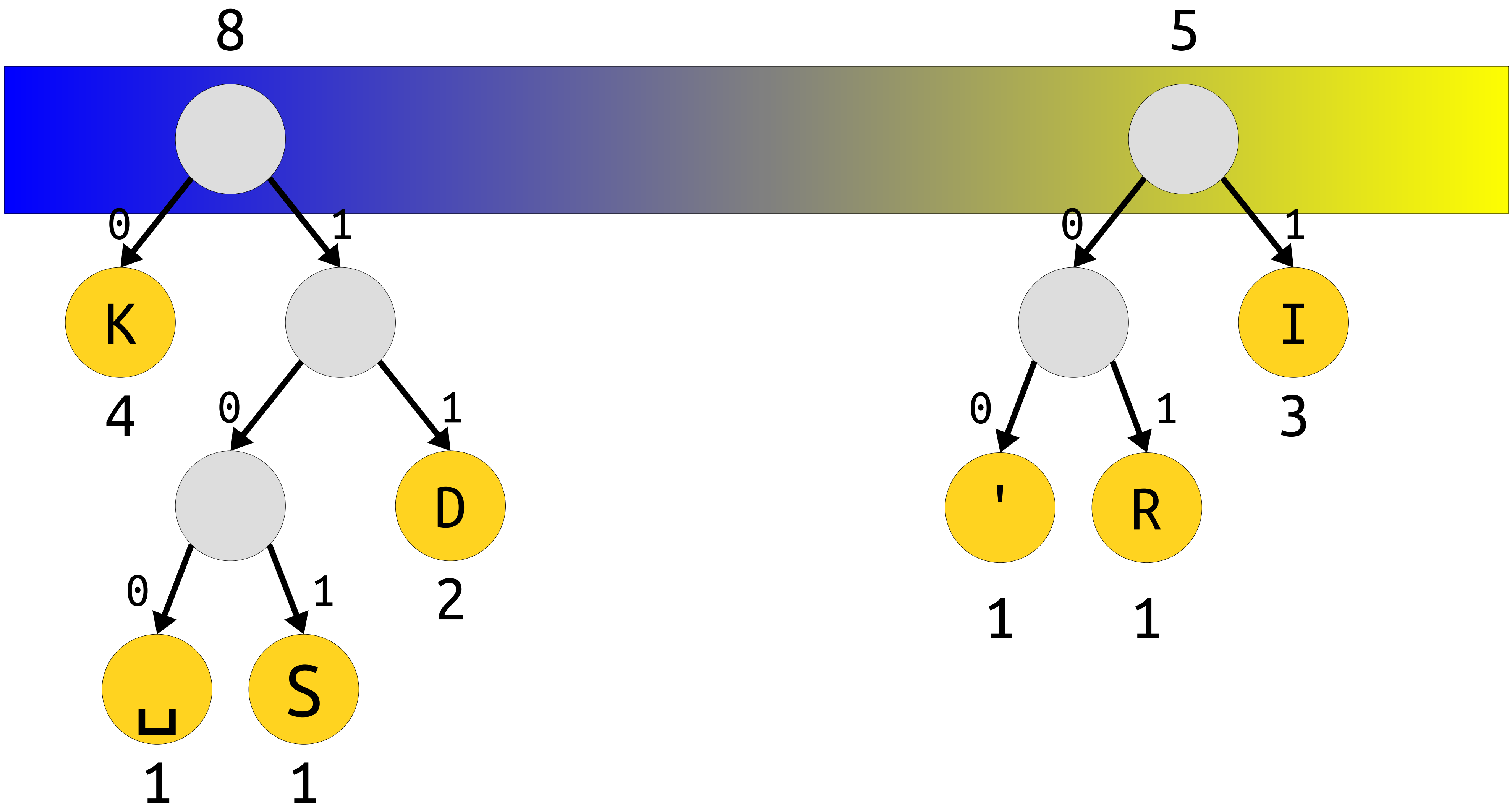
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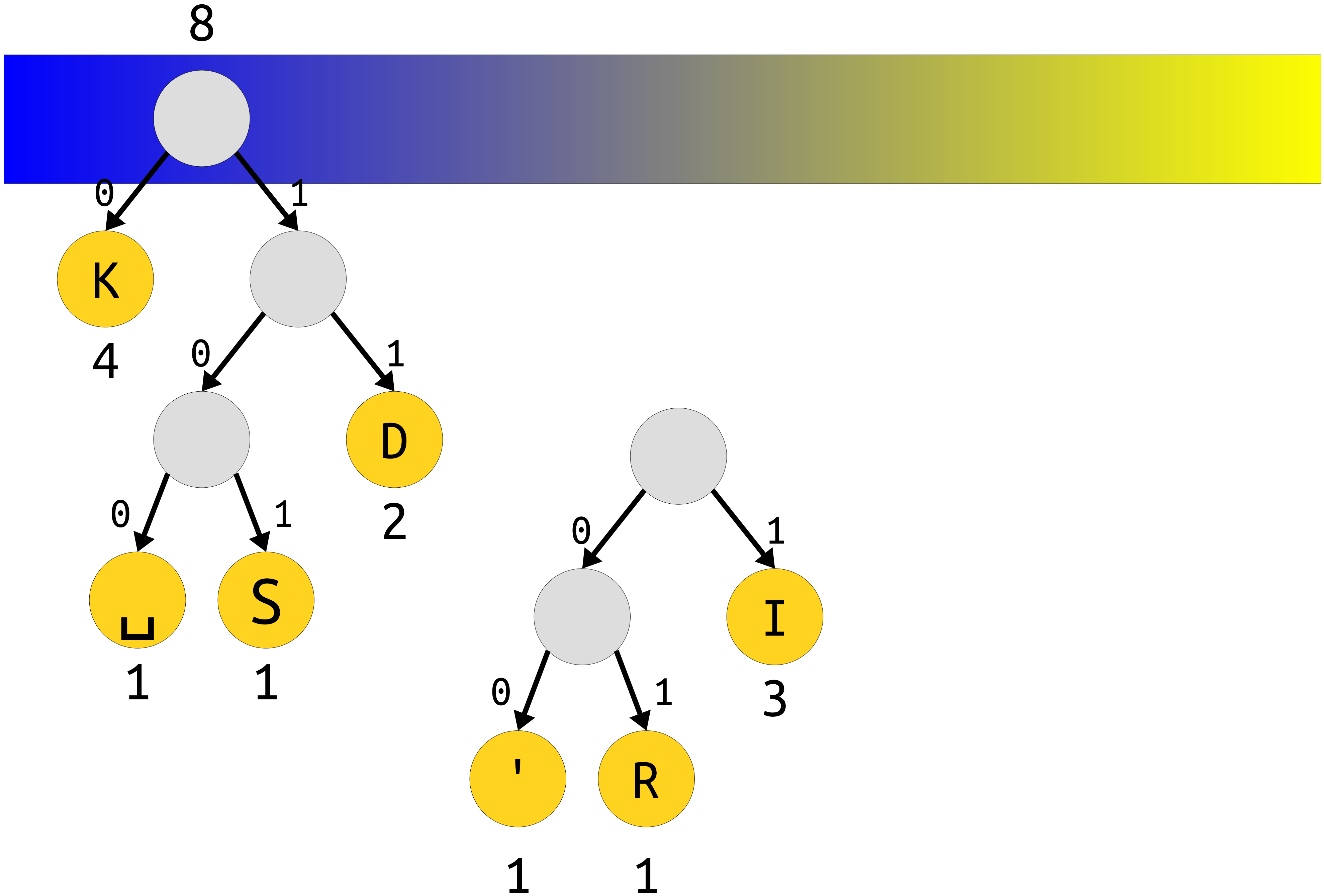

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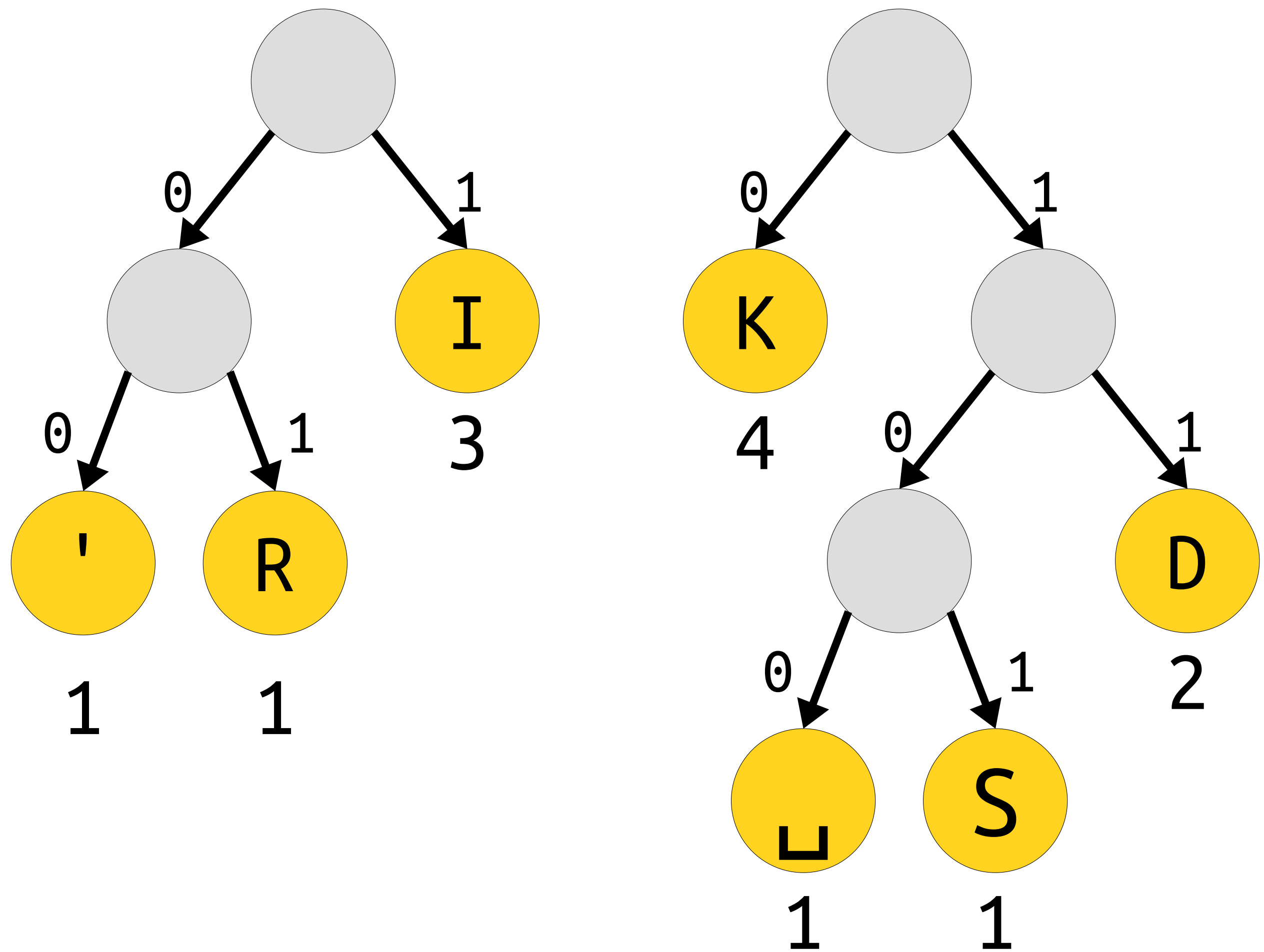
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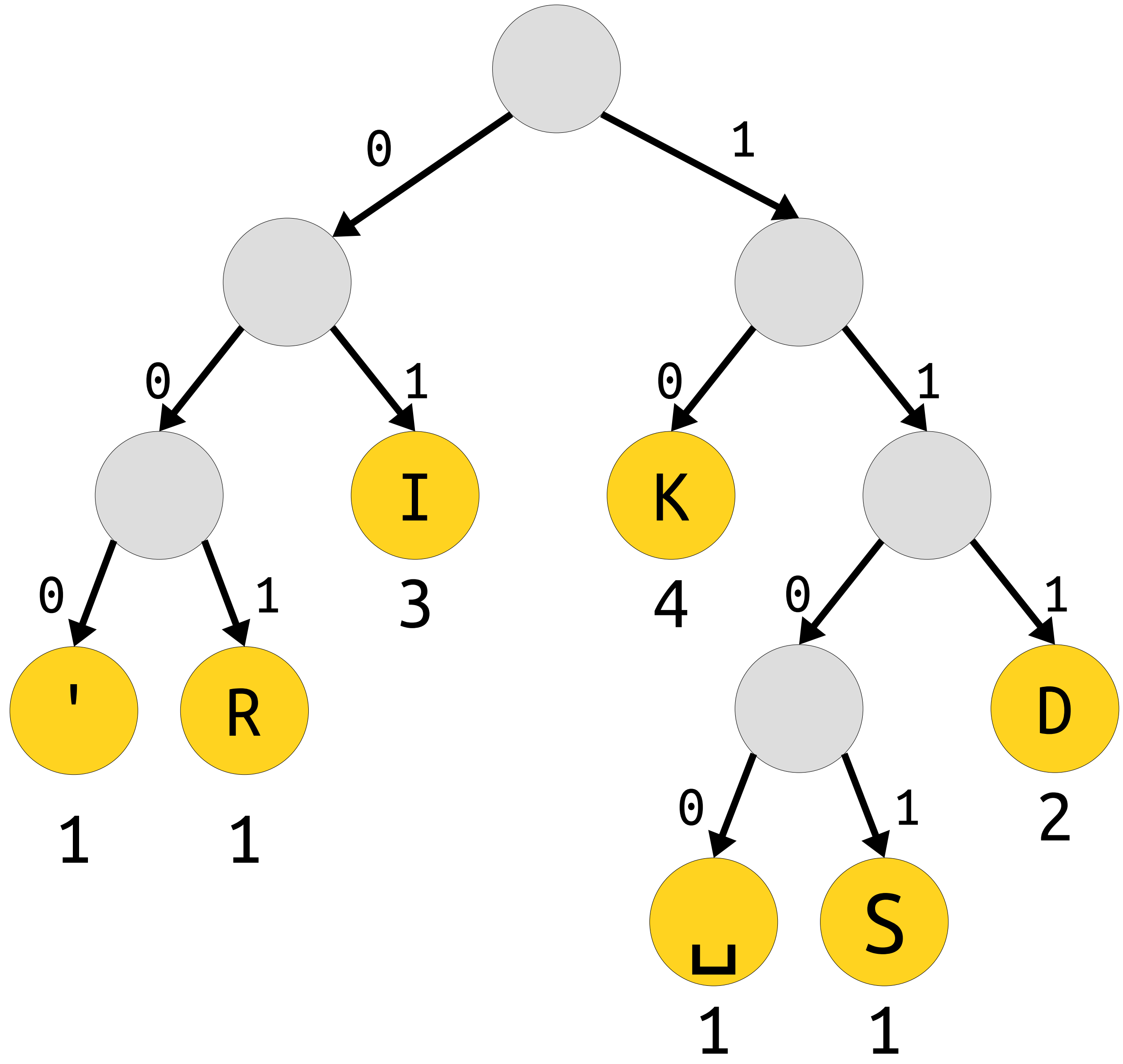
0
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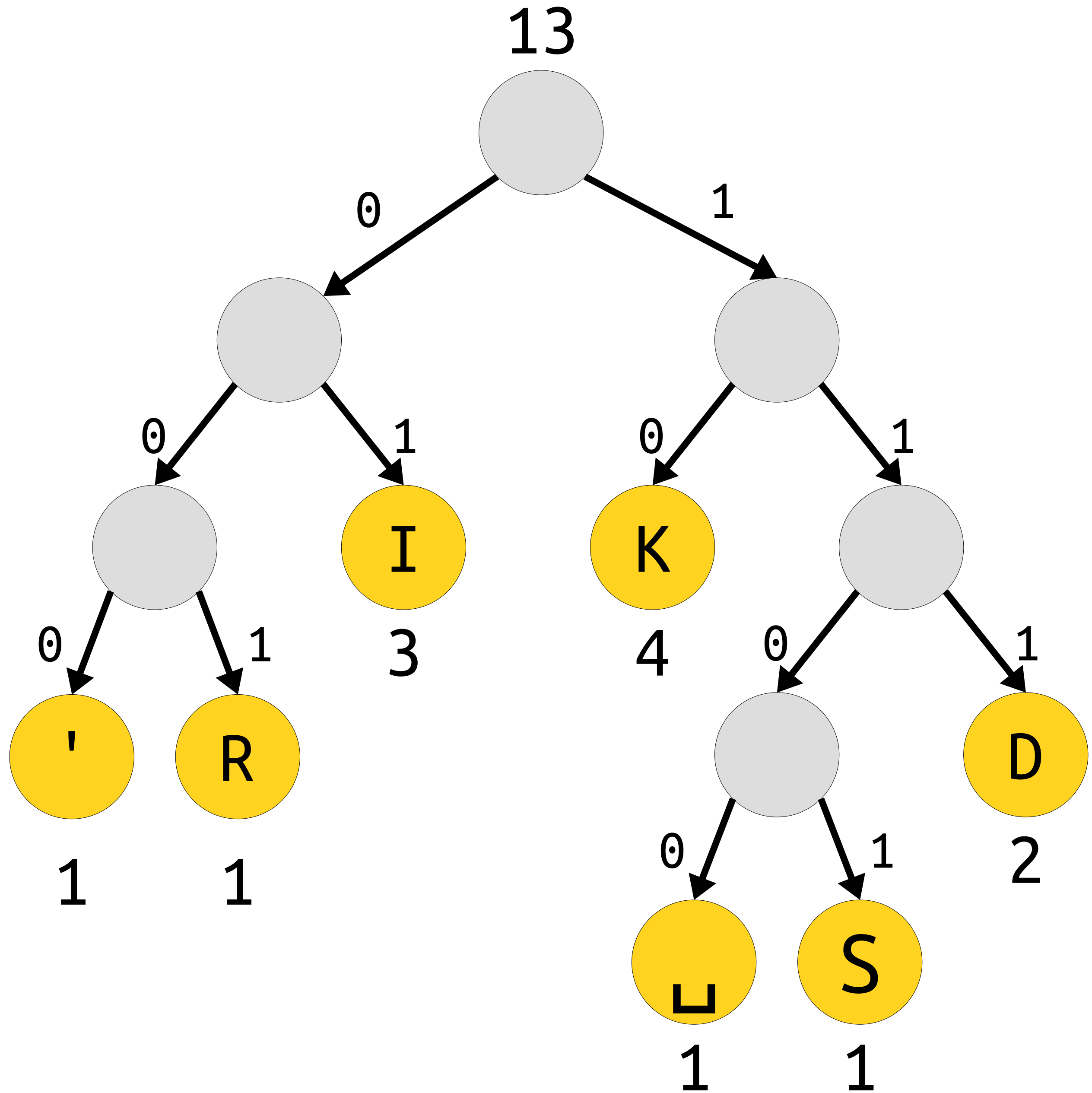
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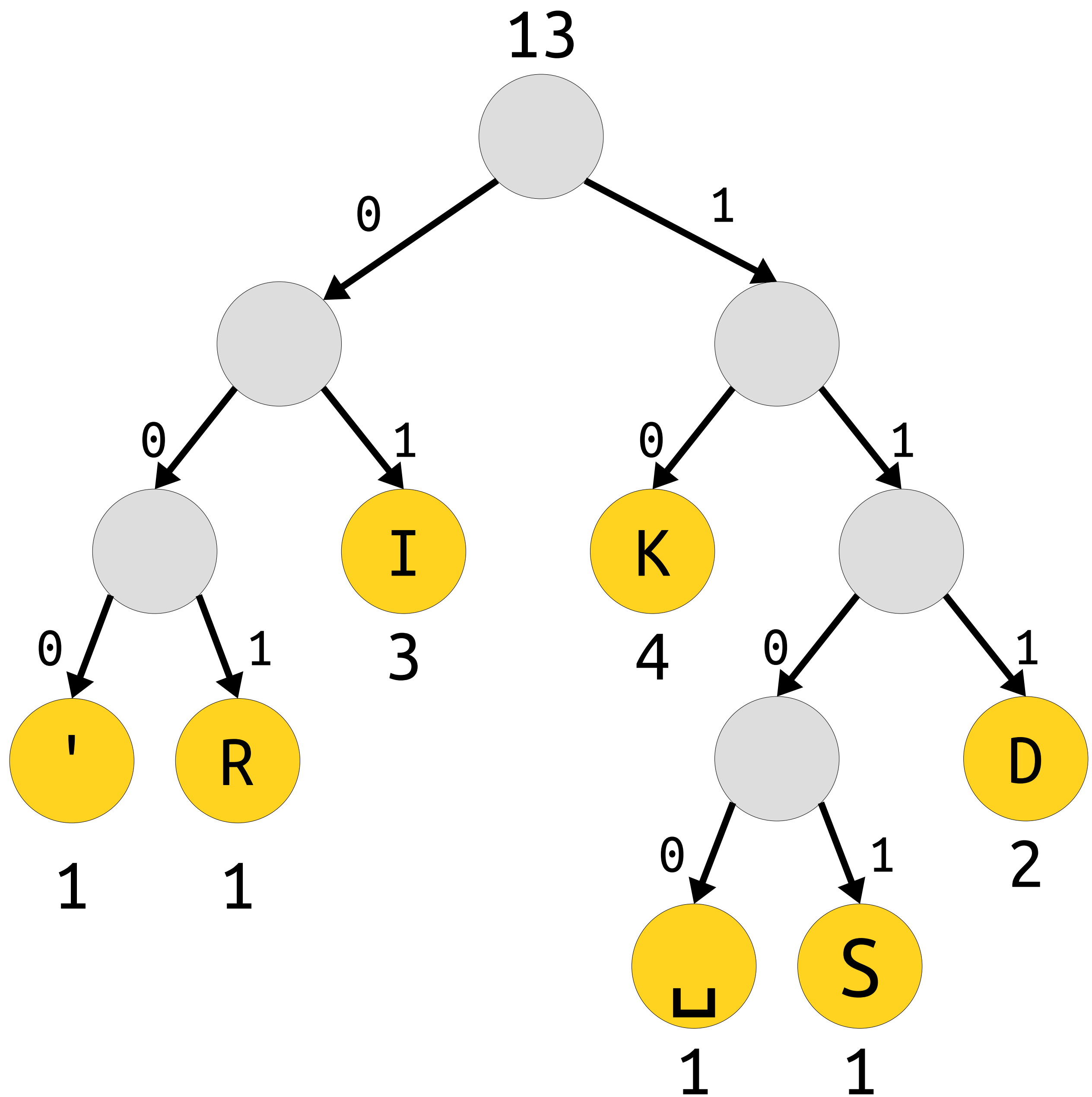
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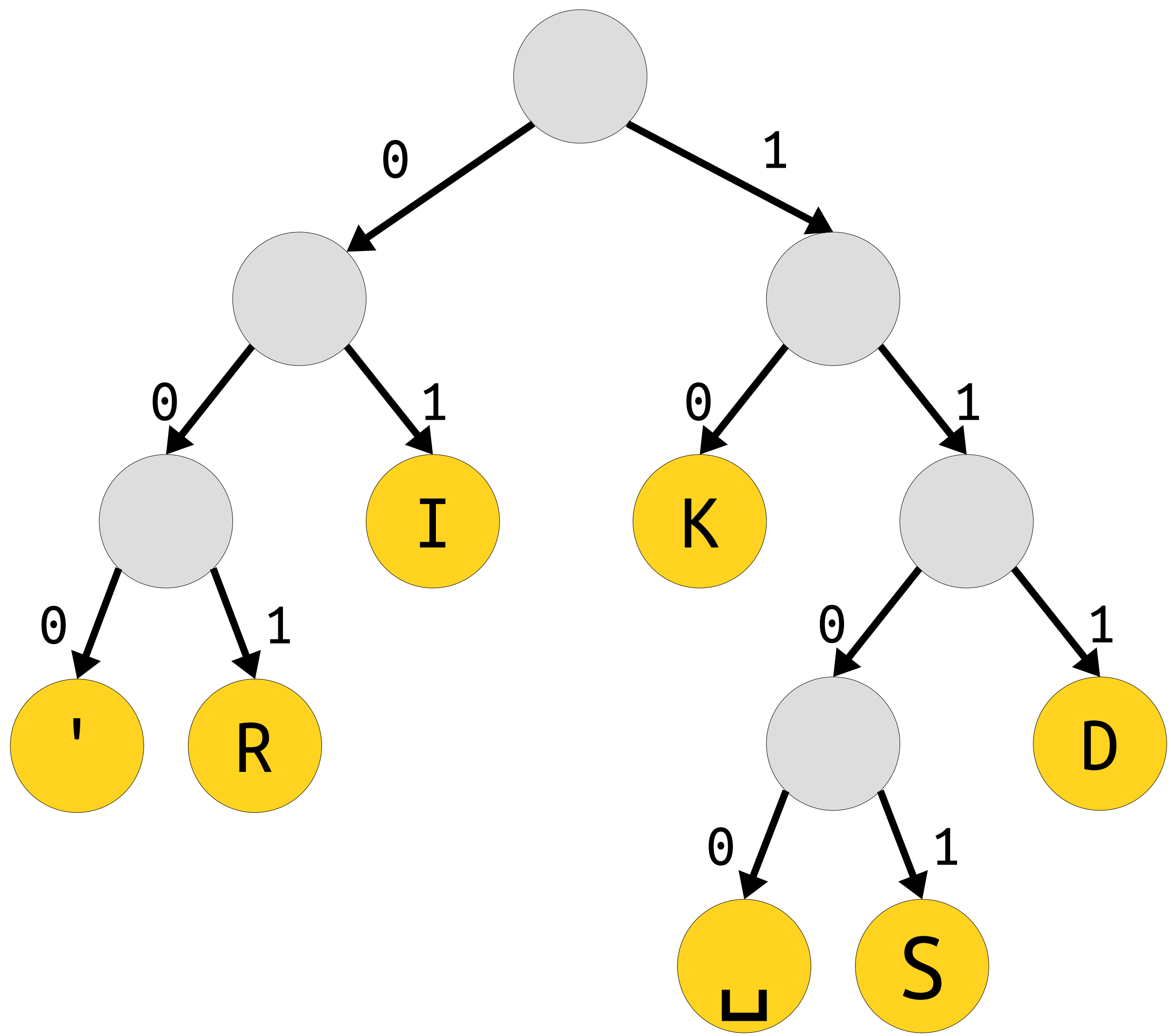
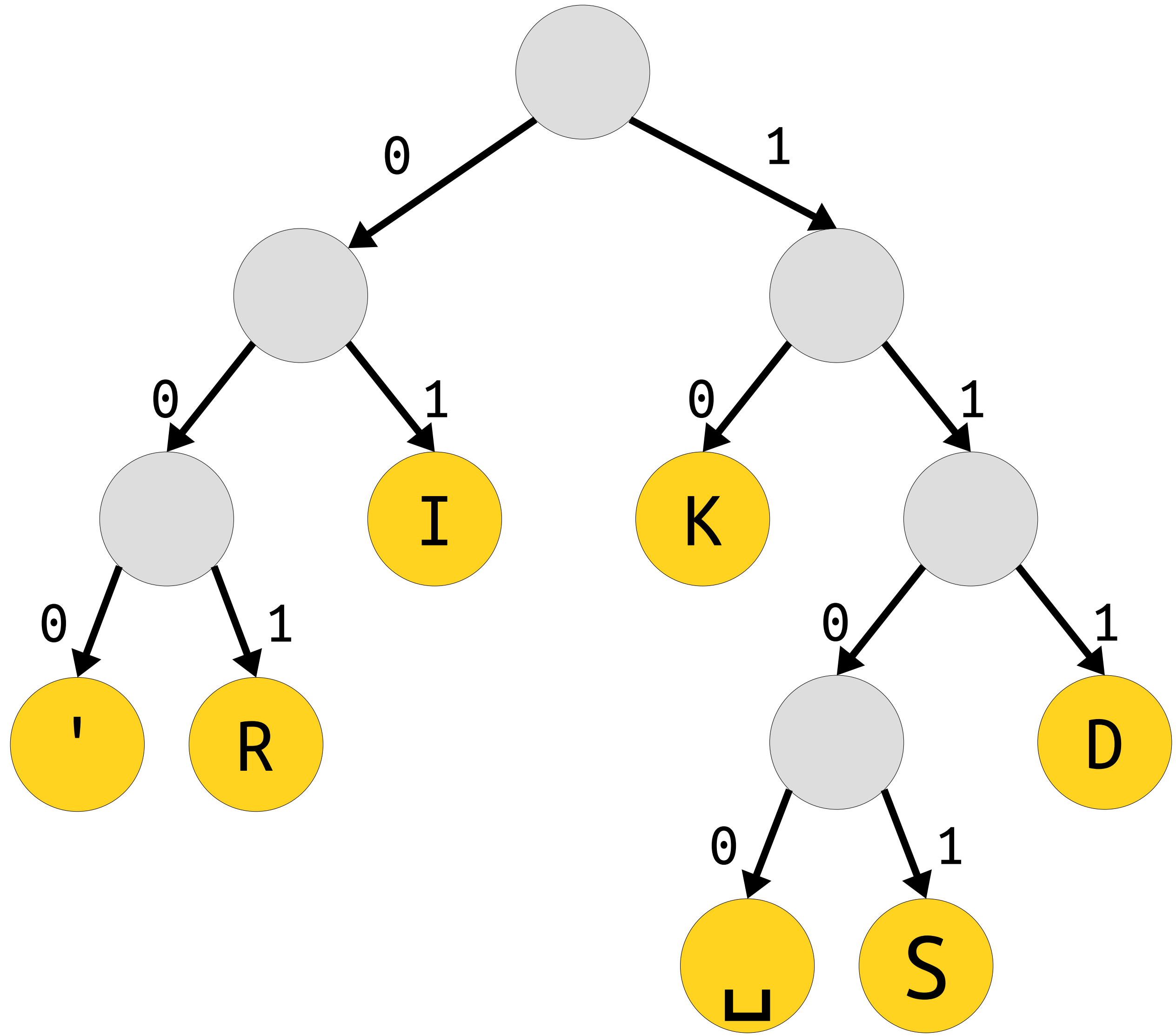

0
1
0
1
0
1
I
K
0
1
'
R
0
1
D
0
1
␣
S

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**



★ Huffman Coding ★

- Create a priority queue that holds partial trees.
- Create one leaf node per distinct character in the input string. The weight of that leaf is the frequency of the character. Add each to the priority queue.
- While there are two or more trees in the priority queue:
  - Dequeue the two lowest-priority trees.
  - Combine them together to form a new tree whose weight is the sum of the weights of the two trees.
  - Add that tree back to the priority queue.

An Important Detail

Prefix-Free Codes

character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix-Free Codes

character	*code*
K	10
I	01
D	111
R	001
'	000
S	1101
␣	1100

1001001100001101110011101101110110

10	01	001	10	000	1101	1100	111	01	10	111	01	10
K	I	R	K	'	S	␣	D	I	K	D	I	K

Prefix-Free Codes

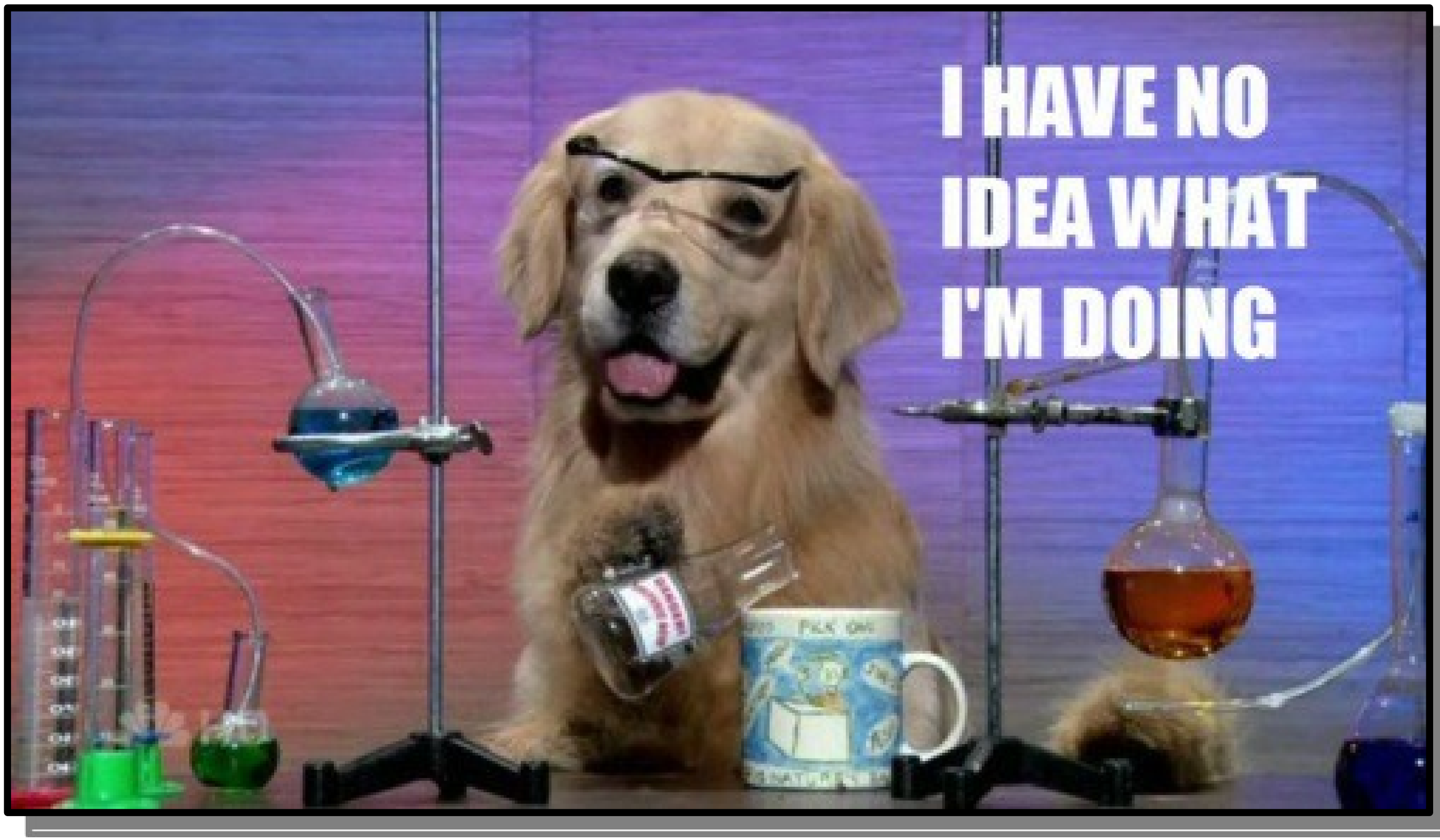
character	*code*
K	**10**
I	**01**
D	**111**
R	**001**
'	**000**
S	**1101**
␣	**1100**

1001001100001101110011101101110110

Prefix-Free Codes

```
1001001100001101110011101101110110
```

Prefix-Free Codes



110110

Transmitting the Tree

- In order to decompress the text, we have to remember what encoding we used!
- ***Idea:*** Prefix the compressed data with a header containing information to rebuild the tree.

Encoding Tree	110111001011101111000100110101011110…

- This might increase the total file size!
- ***Theorem***: There is no compression algorithm that can always compress all inputs.
  - ***Proof:*** Take CS103!

Summary of Huffman Coding

- Prefix-free codes can be modeled as binary trees with characters in the leaves.
- Huffman coding assembles an encoding tree by repeatedly combining the lowest-frequency trees together until only one tree remains.
- We need to send the encoding table with the compressed message for it to be decoded, which can increase file sizes.

More to Explore

- ***Kolmogorov Complexity***
  - What’s the theoretical limit to compression techniques?
- ***Adaptive Coding Techniques***
  - Can you change your encoding system as you go?
- ***Shannon Entropy***
  - A mathematical bound on Huffman coding.
- ***Binary Tries***
  - Other applications of trees like these!

Next Time

- ***Graphs***
  - Representing networks of all sorts.
- ***Graph Searches***
  - A new perspective on some earlier ideas.

Appendix: ***UTF-8***

Beyond ASCII

- ASCII was invented in 1960s America, when the main concern was storing English text.
- It’s completely inadequate for storing the rich breadth of characters that actually get used across the whole world in the 2020s.
- What are we using now?

Unicode

- ***Unicode*** is a system for representing glyphs and symbols from all languages and disciplines.
- One of the most common encodings is ***UTF-8***, which uses sequences of bytes to represent any one individual character.
- The basic idea:
  - UTF-8 is a prefix code, so less common characters like and [illegible] use more bits than common characters like e and 你.
  - UTF-8 encodings are always a full multiple of 8 bits long, making it easier for computers to work one byte at a time.
  - UTF-8 is backwards-compatible with ASCII, so any text encoded with ASCII is also valid UTF-8.

UTF-8

Option 1

0ddddddd

Option 2

110ddddd	10dddddd

Option 3

1110dddd	10dddddd	10dddddd

Option 4

11110ddd	10dddddd	10dddddd	10dddddd

UTF-8

11110000100111111001010110001100

UTF-8

```
11110000 10011111 10010101 10001100
```

UTF-8

11110000	10011111	10010101	10001100
11110000	10011111	10010101	10001100

UTF-8

11110000	10011111	10010101	10001100
11110**000**	10**011111**	10**010101**	10**001100**

UTF-8

11110000	10011111	10010101	10001100
11110**000**	10**011111**	10**010101**	10**001100**

000011111010101001100

UTF-8

11110000	10011111	10010101	10001100
11110**000**	10**011111**	10**010101**	10**001100**

000011111010101001100