## Graphs

## A Social Network



## Chemical Bonds




PANFLUTE FLOWCHART


A graph is a mathematical structure for representing relationships.

A graph is a mathematical structure for representing relationships.

A graph is a mathematical structure for representing relationships.

A graph is a mathematical structure for representing relationships.


A graph consists of a set of nodes connected by edges.

A graph is a mathematical structure for representing relationships.


A graph consists of a set of nodes connected by edges.

A graph is a mathematical structure for representing relationships.

A graph consists of a set of nodes connected by edges.

## Some graphs are directed.



## Some graphs are undirected.



## How can we represent graphs in $\mathrm{C}++$ ?

## Representing Graphs

We can represent a graph as a map from nodes to the list of nodes each node is connected to.


Node Vector<Node>
Node Adjacent To


## Representing Graphs

- The approach we just saw is called an adjacency list in comes in a number of different forms:

> Map<string, Vector<string>> Map<string, Set<string>> Vector<Vector<int>>

- The core idea is that we have some kind of mapping associating each node with its outgoing edges.


## Representing Graphs

## The approach we just saw is called an adjacency list in comes in a number of different forms:

Map<string, Vector<string>>
Map<string, Set<string>>
 its outgoing edges.

## Other Graph Representations



This representation is called an adjacency matrix.
For those of you in Math 51: if $A$ is an adjacency matrix for a graph $G$, what is the significance of the matrix $A^{2}$ ?

## Other Representations



Many problems work on an implicit graph.

## You'll find graphs just about everywhere you look.

They're an extremely versatile and powerful abstraction to be aware of.

Going forward, unless stated otherwise, assume we're using an adjacency list.


## Traversing Graphs

## Iterating over a Graph

- In a singly-linked list, there's pretty much one way to iterate over the list: start at the front and go forward!
- In a binary search tree, there are many traversal strategies:
- An inorder traversal that produces all the elements in sorted order.
- A postorder traversal is used to delete all the nodes in the BST.
- There are many ways to iterate over a graph, each of which have different properties.

One Search Strategy




> Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.

Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


Core idea: Find everything one hop away from the start, then two hops away, then three hops away, etc.


## Implementing this Idea



Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


## Queue:

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


## Queue:

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (D) (B)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (D) (B)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (D) (B)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


## Queue:

## (B)

Load newly-discovered nodes into a queue.


Queue: $B$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queue: (B)


Load newly-discovered nodes into a queue.


Queue: (B)

Visit nodes in ascending order of
Load newly-discovered distance from the start node $E$. nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (B) (A) (G)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (B) (A) (C)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (B) (A) (G)

Load newly-discovered nodes into a queue.


## Queue:

Visit nodes in ascending order of distance from the start node $E$.



Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G) (C)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G) (C)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G) (C)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (A) (G) (C)

Load newly-discovered nodes into a queue.


## Queve: (A) (G) (C)

Load newly-discovered nodes into a queue.


## Queue:



Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


## Queve: (C) (C)

Load newly-discovered nodes into a queue.


## Queue:



Load newly-discovered nodes into a queue.


## Queue: $C$

Load newly-discovered nodes into a queue.


## Queue: $C$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Queve: (C) (H)

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Load newly-discovered nodes into a queue.


## Queue:

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Queue: $H$

Load newly-discovered nodes into a queue.


Queue: $H$

Load newly-discovered nodes into a queue.


Queue: $H$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Load newly-discovered nodes into a queue.


Queve: (H) F)

Load newly-discovered nodes into a queue.


Queve: (H) F)

Load newly-discovered nodes into a queue.



Queve: $(H)(F$

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: $F$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: $F$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queue: (F)

Load newly-discovered nodes into a queue.


Queve: $(F)$

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.




Visit nodes in ascending order of distance from the start node $E$.

## Queue:



Load newly-discovered nodes into a queue.






Queve: (I)

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.


## Queue: $I$

Load newly-discovered nodes into a queue.


Visit nodes in ascending order of distance from the start node $E$.


Queve: (I)

Load newly-discovered nodes into a queue.




Queue:

Visit nodes in ascending order of distance from the start node $E$.

Load newly-discovered nodes into a queue.

## Breadth-First Search

- The Queue-based search strategy we just saw is called breadth-first search (or just BFS for short).
- In pseudocode:

```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
    dequeue a node curr.
    process the node curr.
    for each node adjacent to curr:
    if that node has never been enqueued:
                enqueue that node.
}
```


## Depth-First Search

## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.





## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search <br>  <br> Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



## Depth-First Search



## Depth-First Search



Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## Depth-First Search



## How can we implement this?

## Breadth-First Search



```
I've just been discovered: Pay attention to me:
```

```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
        dequeue a node curr.
        process the node curr.
        for each node adjacent to curr:
        if that node has never been enqueued:
                                    enqueue that node.

\section*{Depth-First Search}

```

dfs-from(node v) {
make a stack of nodes, initially seeded with v.
while the stack isn't empty:
pop a node curr.
process the node curr.
for each node adjacent to curr:
if that node has never been pushed:
push that node.
}

```

\section*{For Comparison}
```

bfs-from(node v) {
make a queue of nodes, initially seeded with v.
while the queue isn't empty:
dequeue a node curr.
process the node curr.
for each node adjacent to curr:
if that node has never been enqueued:
enqueue that node.

## For Comparison

```
dfs-from(node v) {
        make a stack of nodes, initially seeded with v.
        while the stack isn't empty:
            pop a node curr.
            process the node curr.
            for each node adjacent to curr:
                if that node has never been pushed:
                push that node.
```

dfs-from(node v) {
if this is first time we've called dfs-from(v):
process node v
for each node adjacent to v:
call dfs-from on that node

```

When you see a stack-based algorithm, think recursion!

\section*{BFS and DFS}
- Running BFS or DFS from a node in a graph will visit the same set of nodes, but probably in a different order.
- BFS will visit nodes in increasing order of distance.
- DFS does visit nodes in some interesting order, but not order of distance.
- Take CS161 for more details!

A Whimsical Application

\section*{Mazes as Graphs}


\section*{Creating a Maze with DFS}
- Create a grid graph of the appropriate size.

- Starting at any node, run a depth-first search, choosing neighbor orderings at random.
- The resulting DFS tree is a maze with one solution.

\section*{Next Time}
- Minimum Spanning Trees
- How to wire an electrical grid cheaply.
- Applications of MSTs
- Data clustering, computational biology, and more!```

