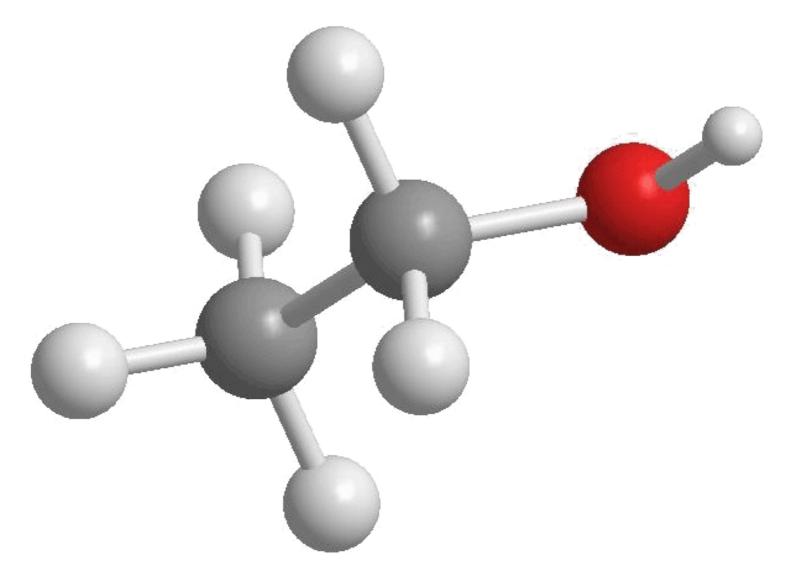
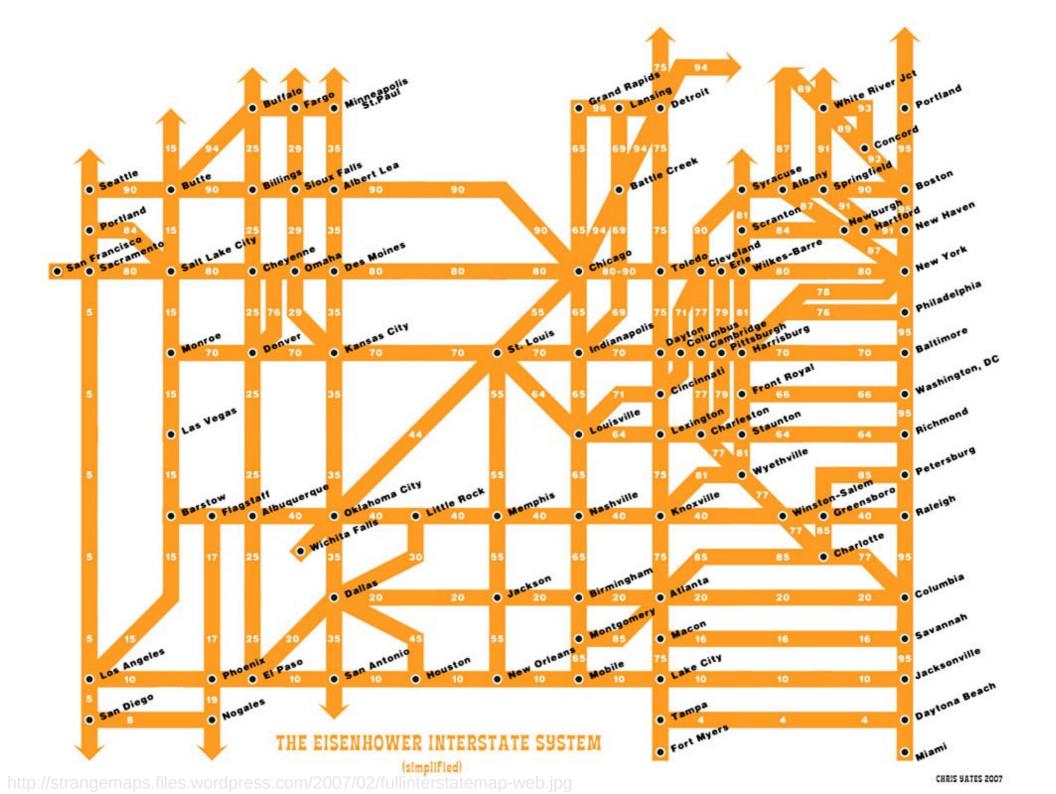
Graphs

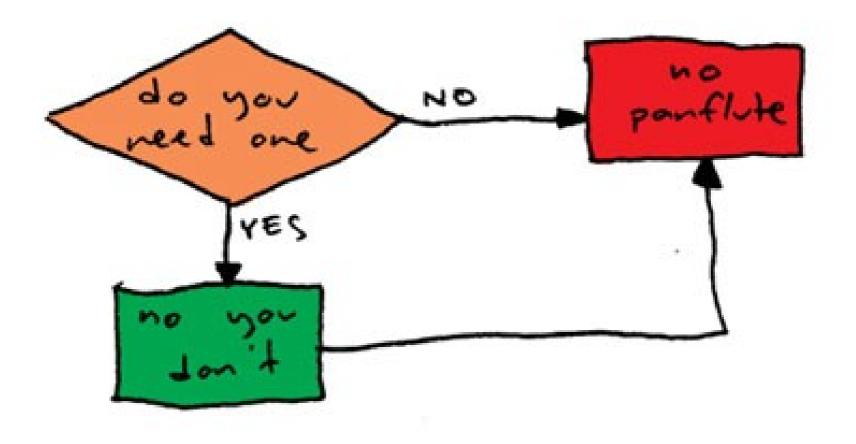
A Social Network 0 0 0 0 0 0 0 0 0 0 0 0

Chemical Bonds

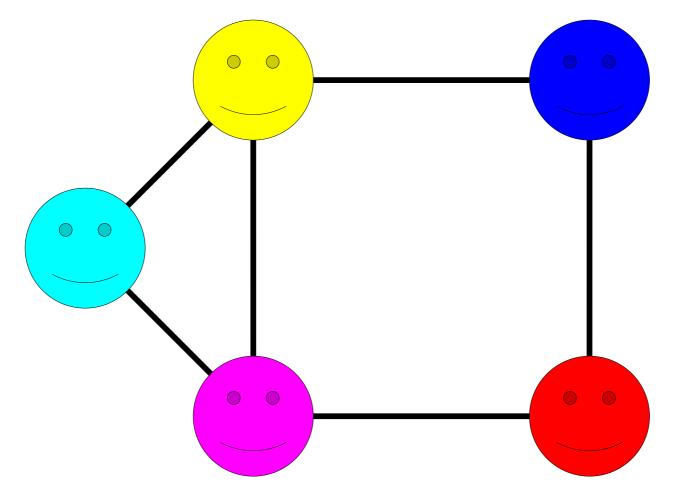


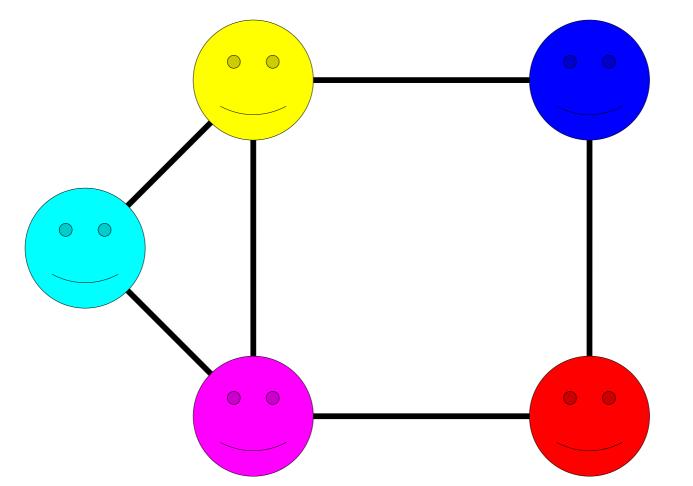


PANFLUTE FLOWCHART

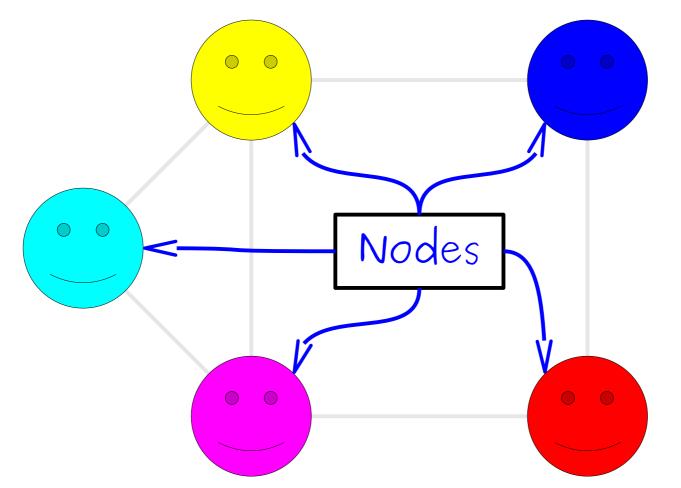


http://www.toothpastefordinner.com/

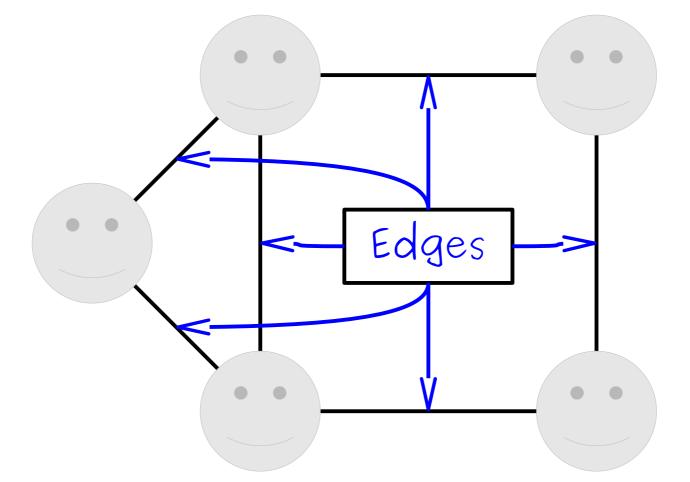




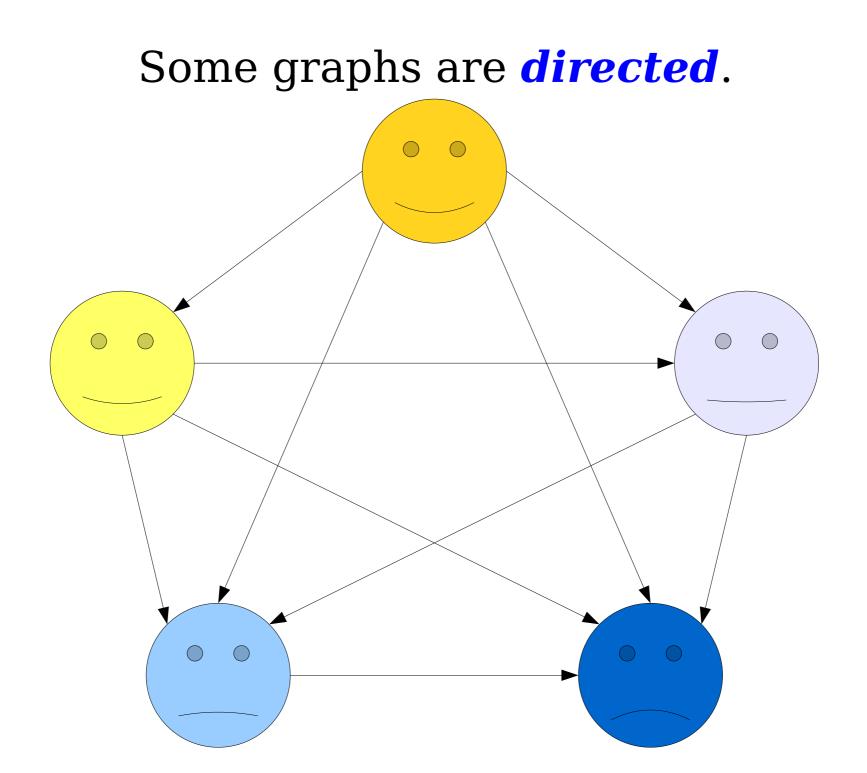
A graph consists of a set of *nodes* connected by *edges*.



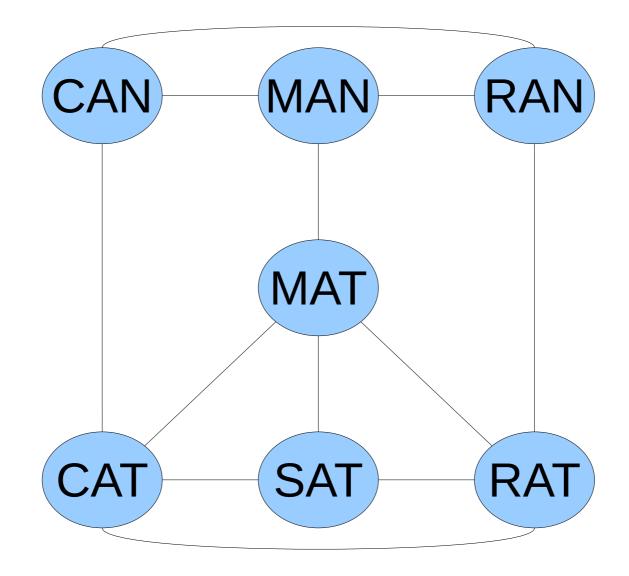
A graph consists of a set of *nodes* connected by *edges*.



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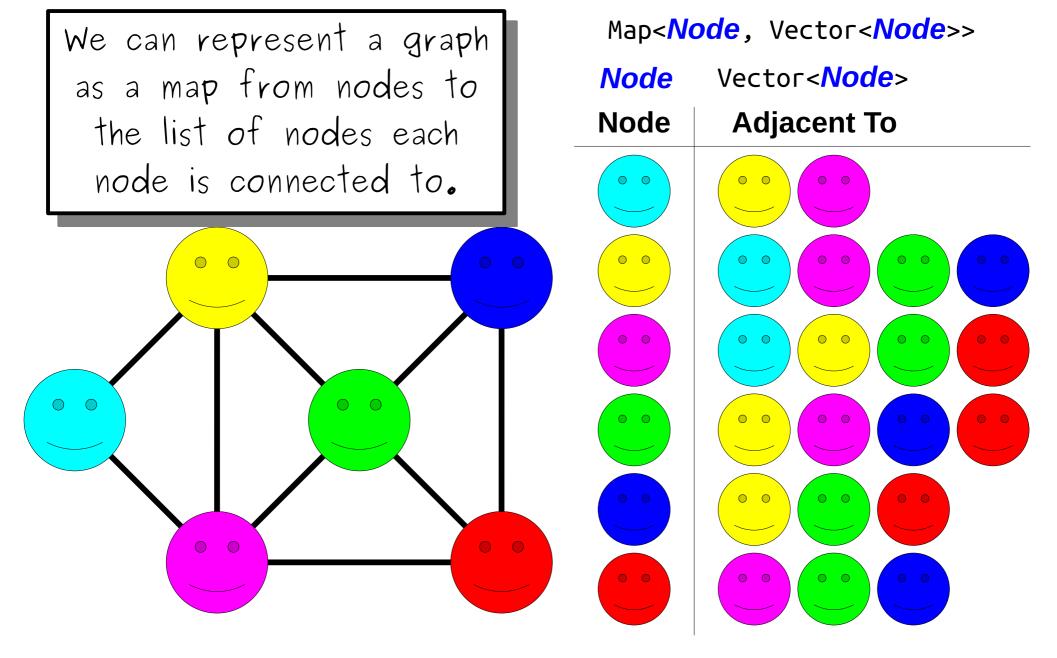


Some graphs are *undirected*.



How can we represent graphs in C++?

Representing Graphs



Representing Graphs

 The approach we just saw is called an adjacency list in comes in a number of different forms:

Map<string, Vector<string>>
Map<string, Set<string>>
Vector<Vector<int>>

• The core idea is that we have some kind of mapping associating each node with its outgoing edges.

Representing Graphs

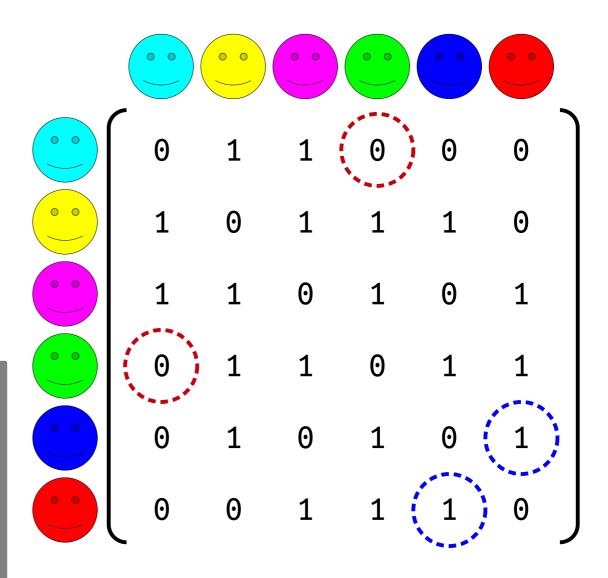
The approach we just saw is called an *adjacency list* in comes in a number of different forms:

Map<string, Vector<string>> Map<string, Set<string>> Vector<Vec Question to ponder: Where have you seen this before? Its outgoing edges.

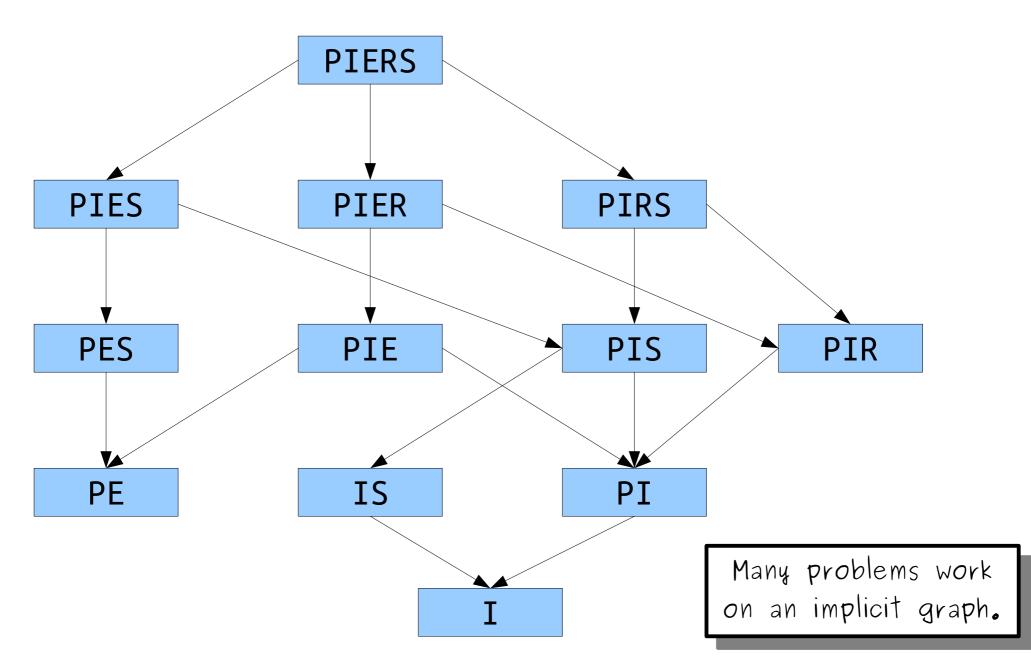
Other Graph Representations

This representation is called an adjacency matrix.

For those of you in Math 51: if A is an adjacency matrix for a graph G, what is the significance of the matrix A²?



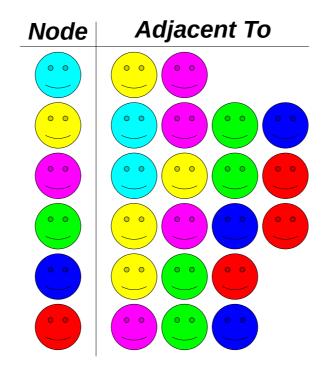
Other Representations



You'll find graphs just about everywhere you look.

They're an *extremely* versatile and powerful abstraction to be aware of.

Going forward, unless stated otherwise, assume we're using an *adjacency list*.

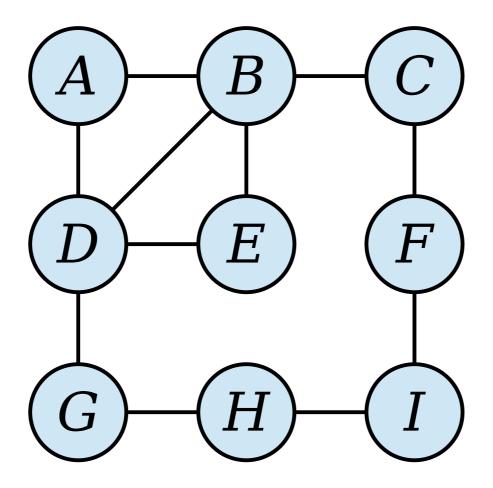


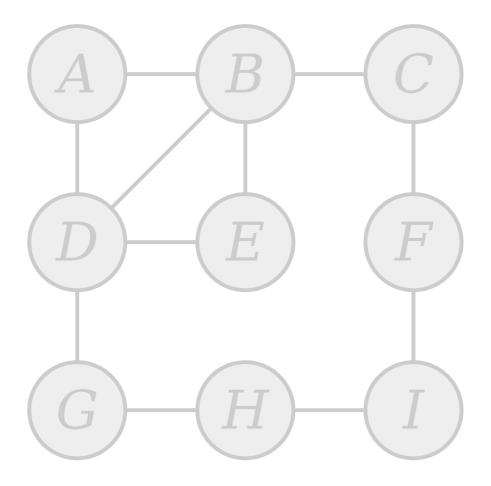
Traversing Graphs

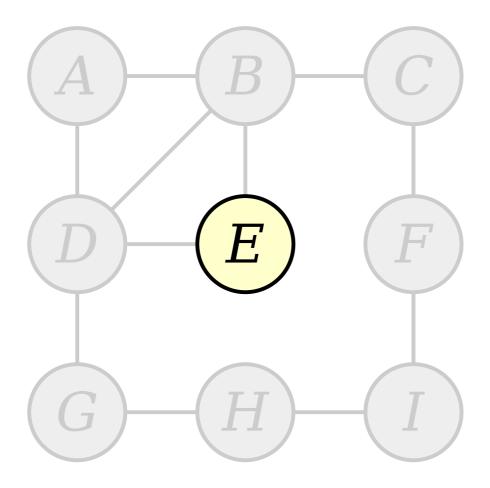
Iterating over a Graph

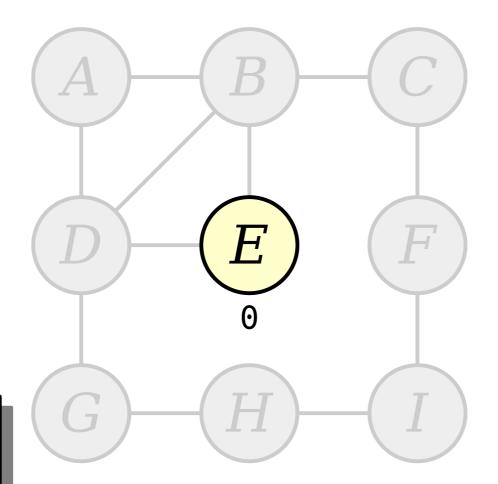
- In a singly-linked list, there's pretty much one way to iterate over the list: start at the front and go forward!
- In a binary search tree, there are many traversal strategies:
 - An *inorder traversal* that produces all the elements in sorted order.
 - A *postorder traversal* is used to delete all the nodes in the BST.
- There are *many* ways to iterate over a graph, each of which have different properties.

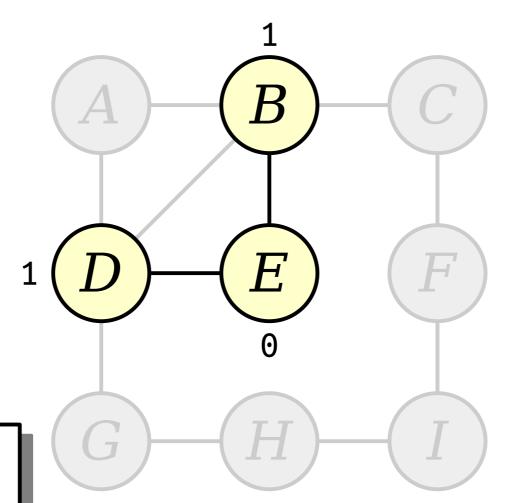
One Search Strategy

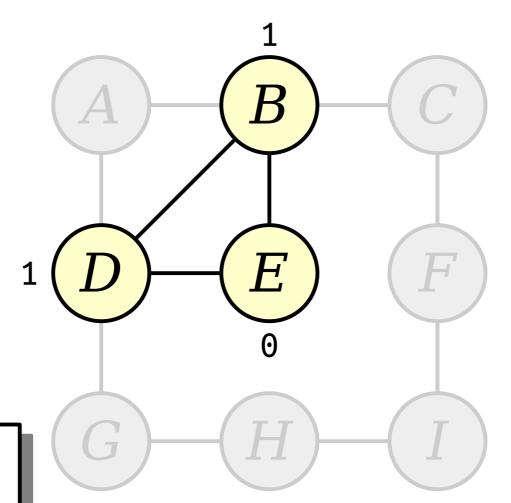


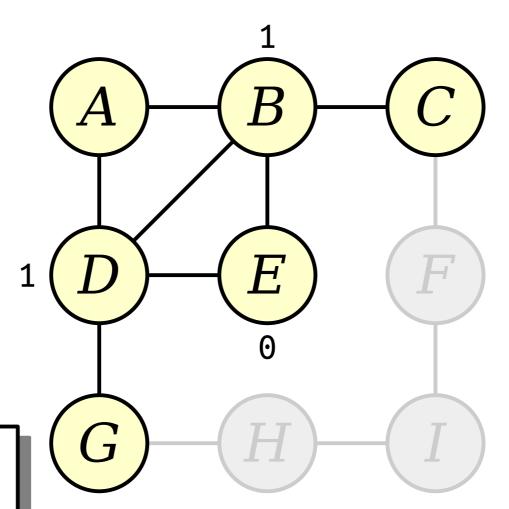


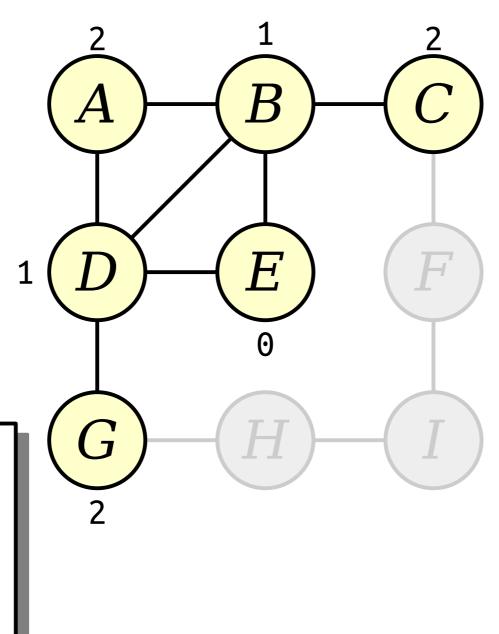


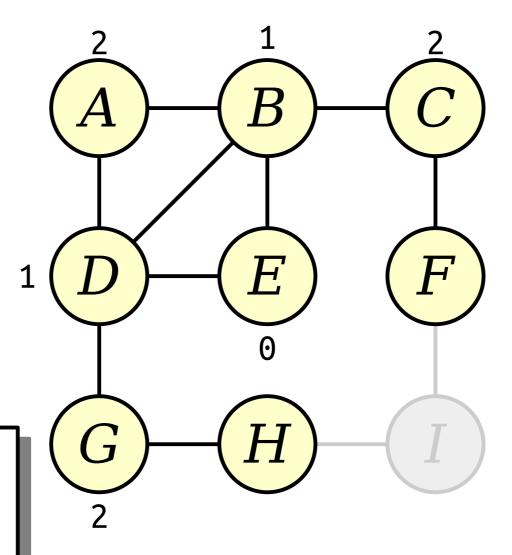


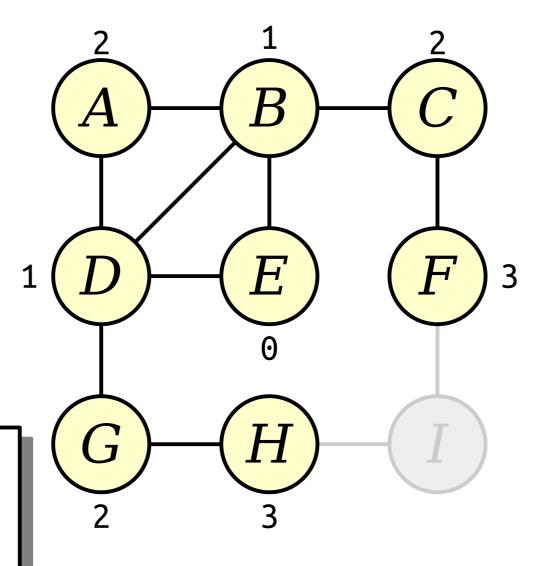


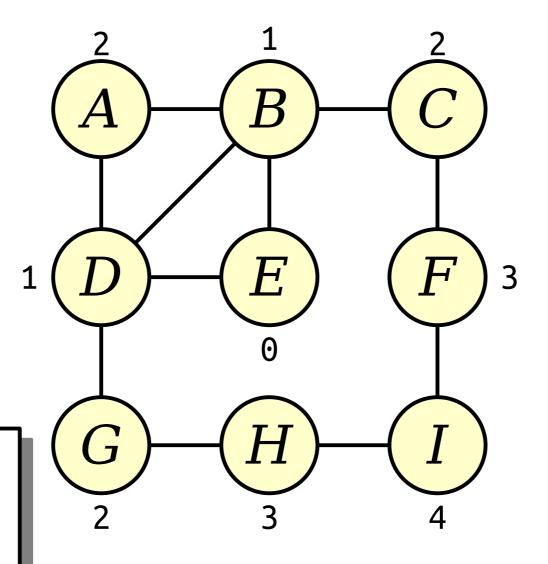




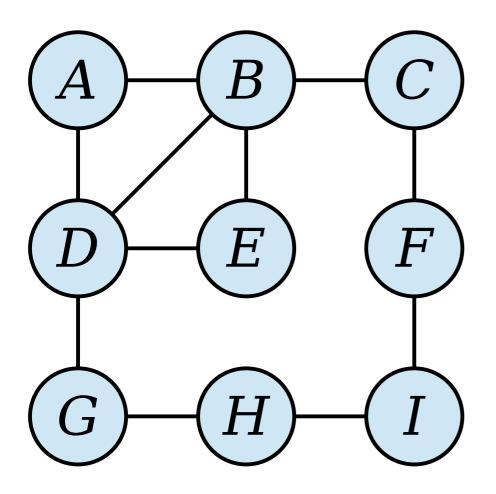


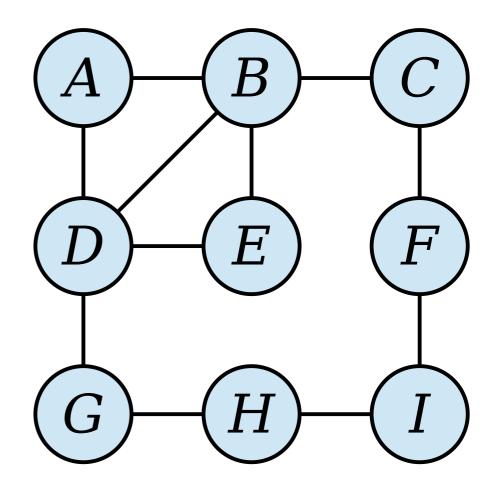


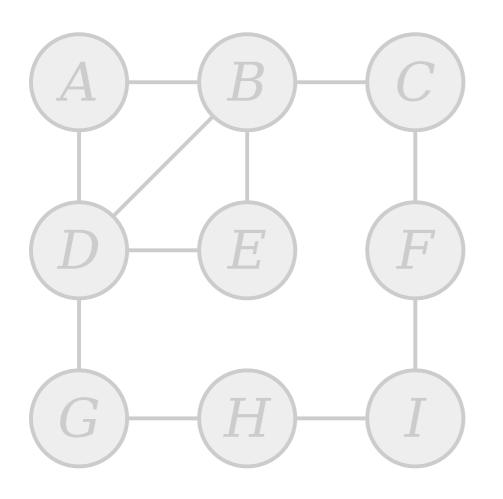


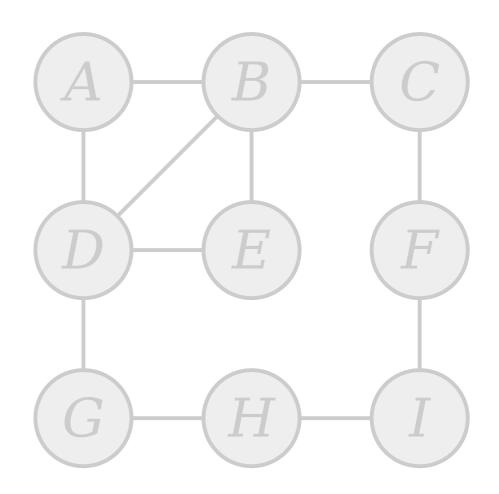


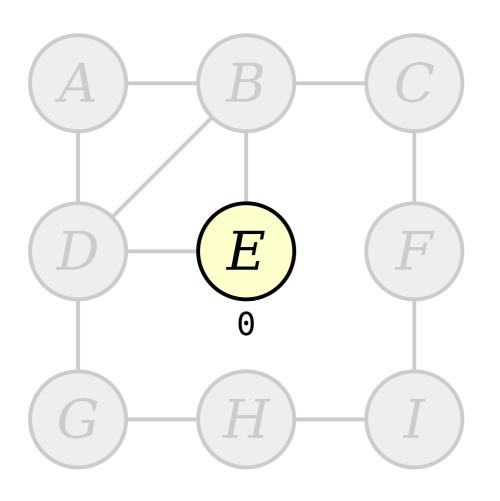
Implementing this Idea

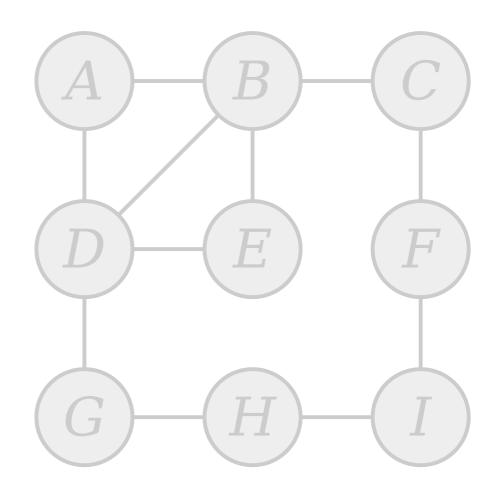


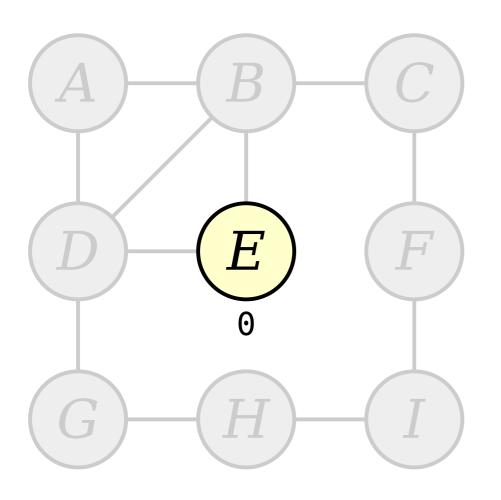


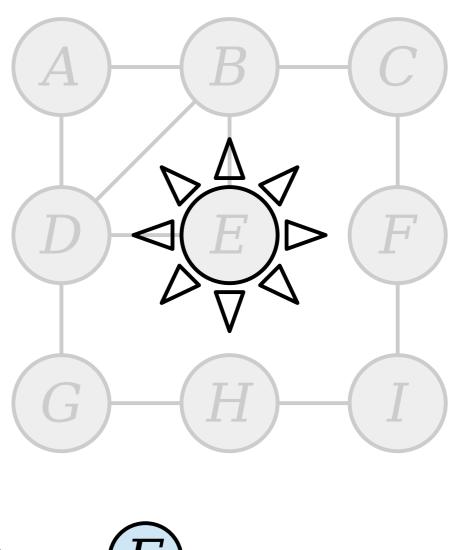






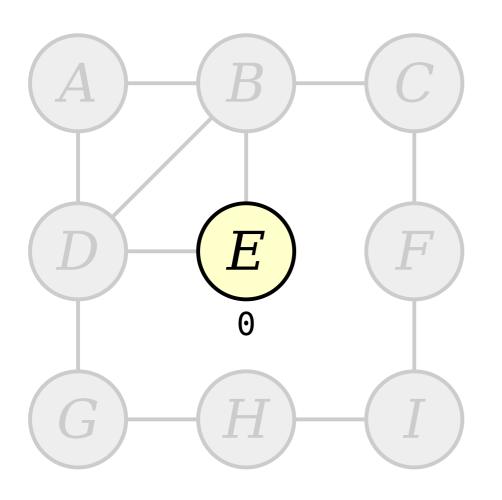


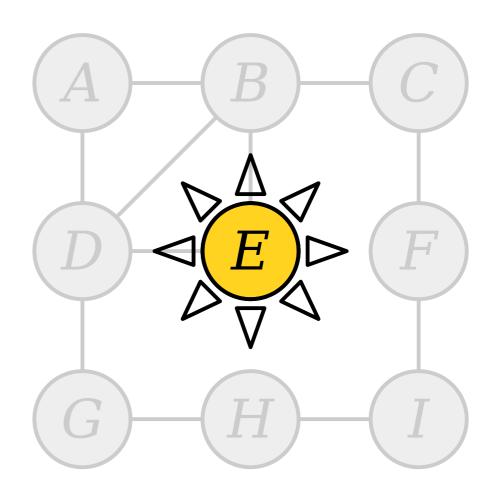




Queue: (E)

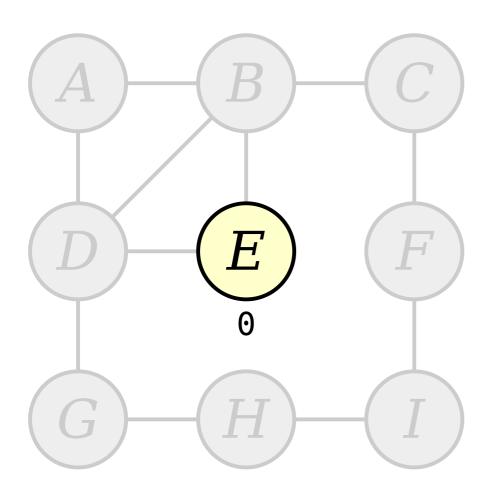
Visit nodes in ascending order of distance from the start node E.

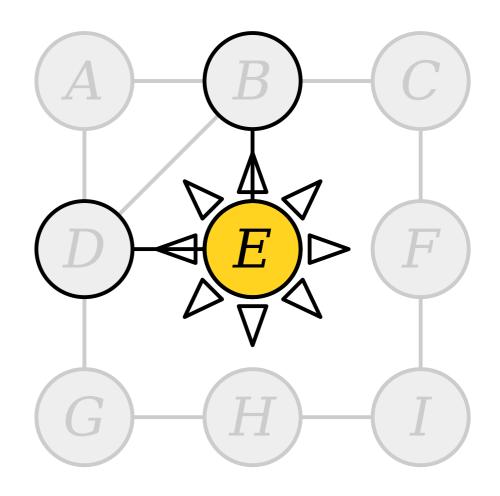




Queue:

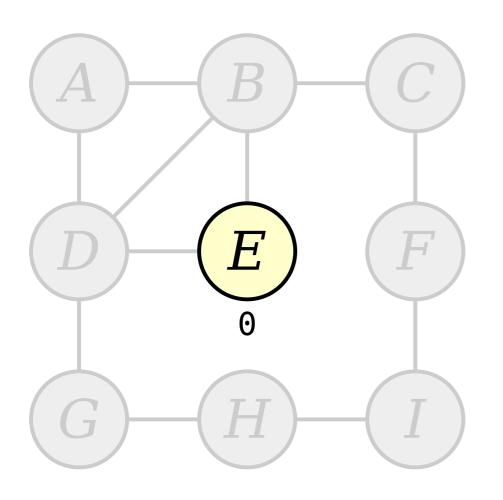
Visit nodes in ascending order of distance from the start node E.

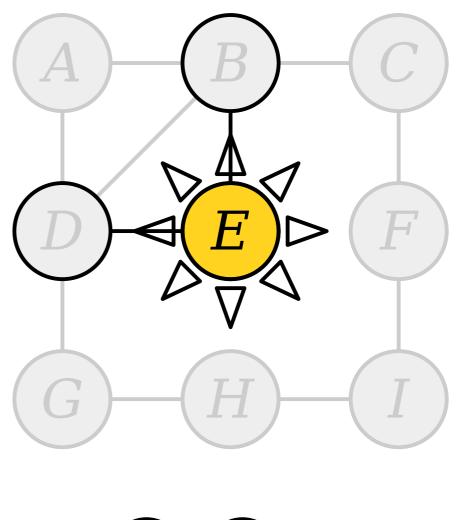




Queue:

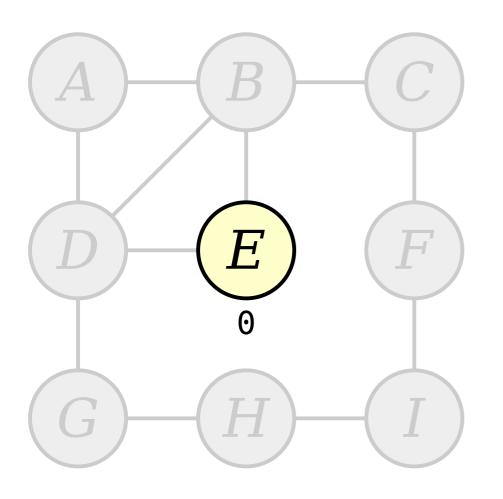
Visit nodes in ascending order of distance from the start node E.

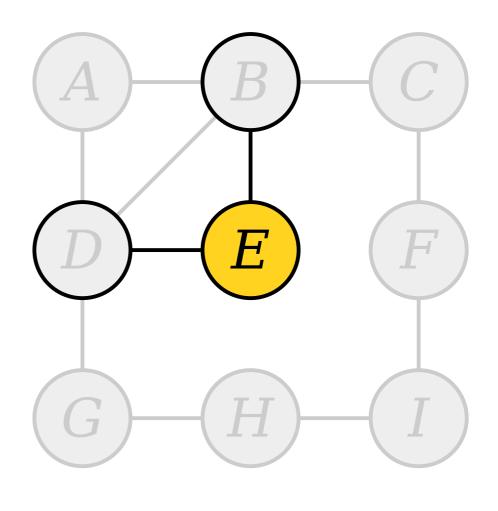




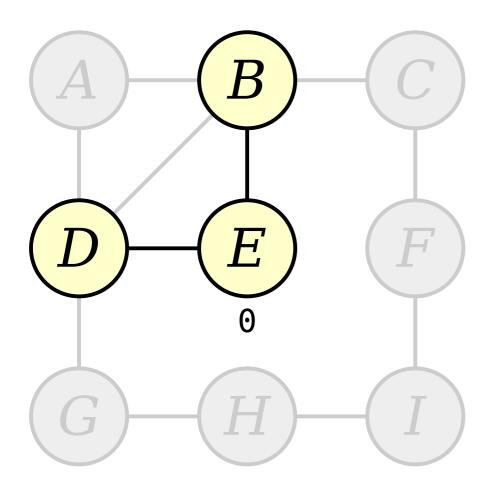
Queue: D B

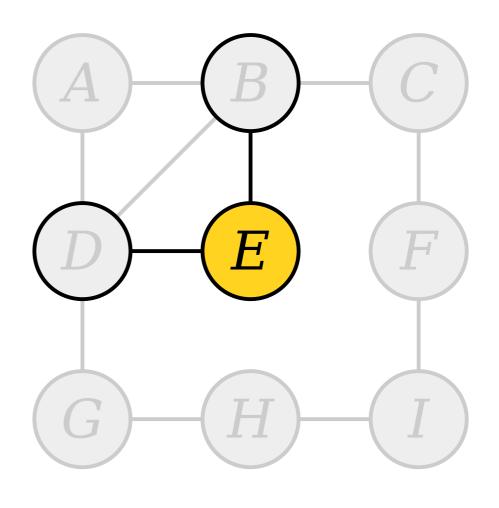
Visit nodes in ascending order of distance from the start node E.



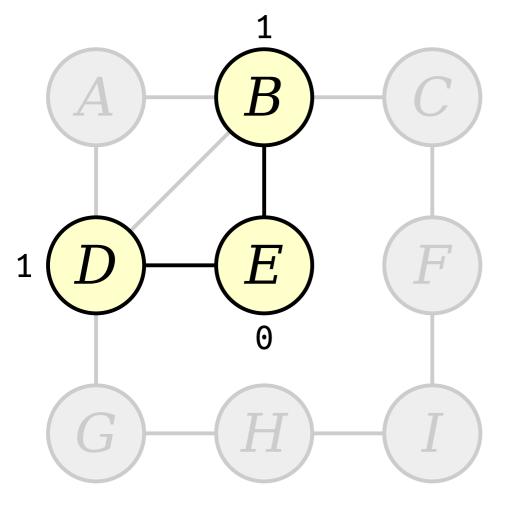


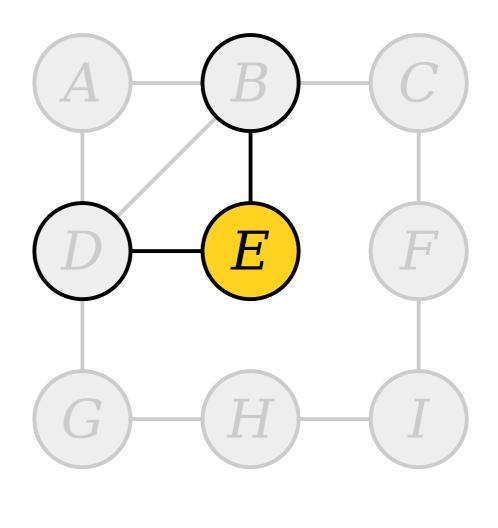






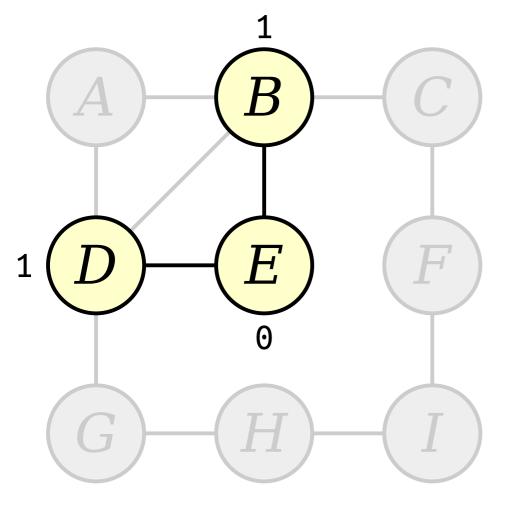


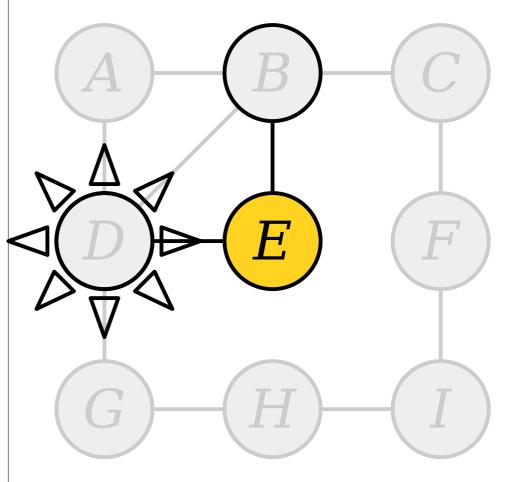


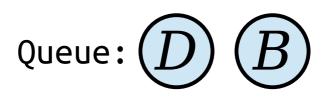


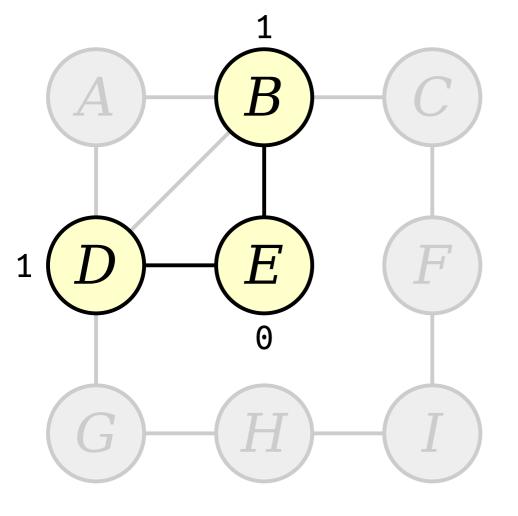
Queue: D B

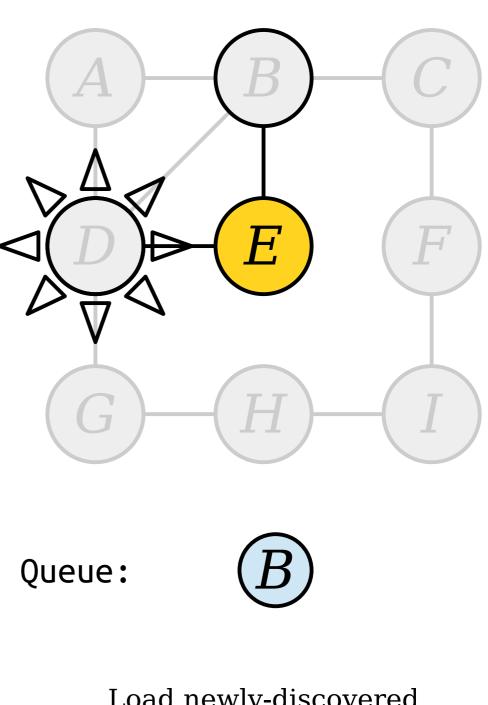
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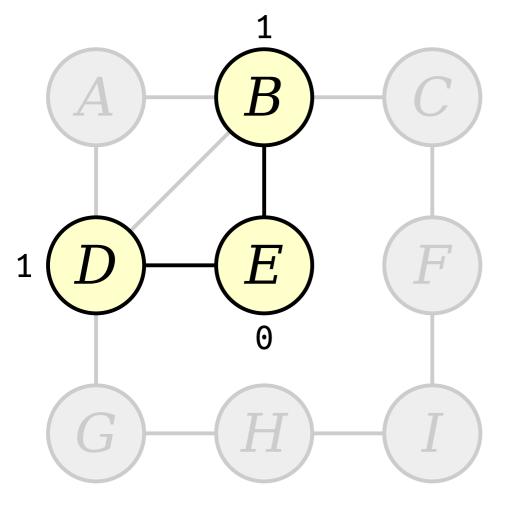


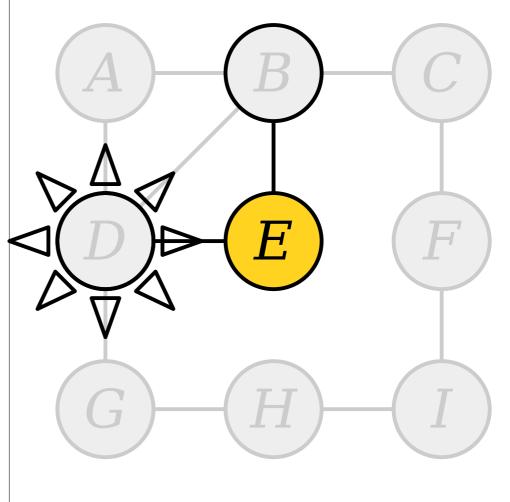




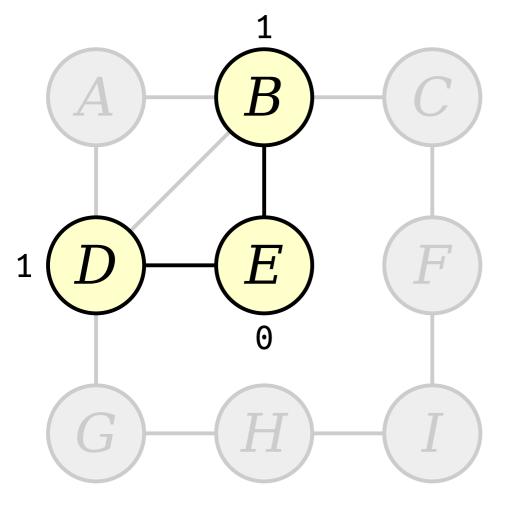


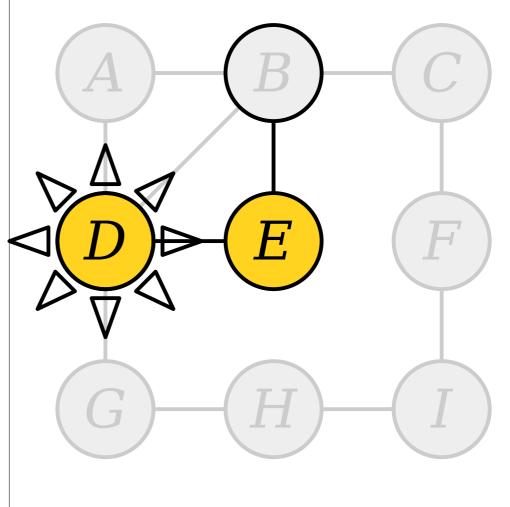




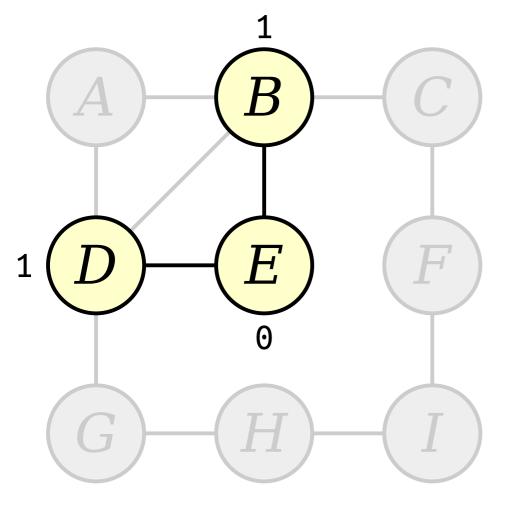


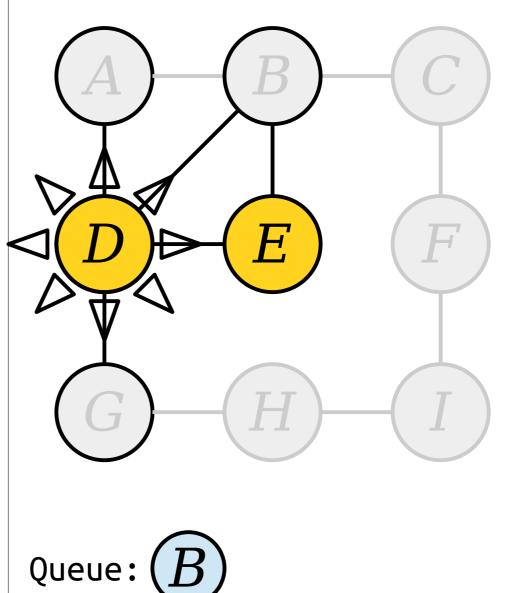


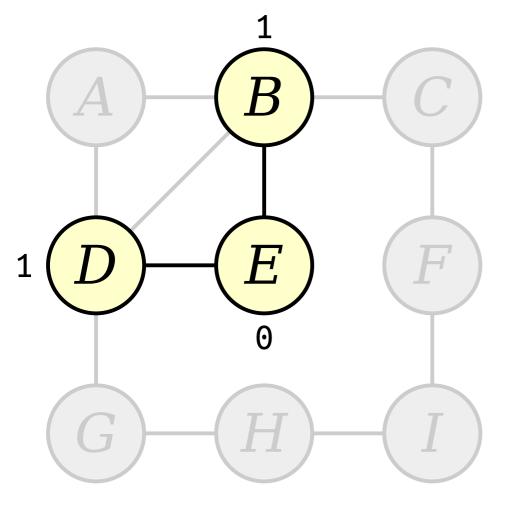


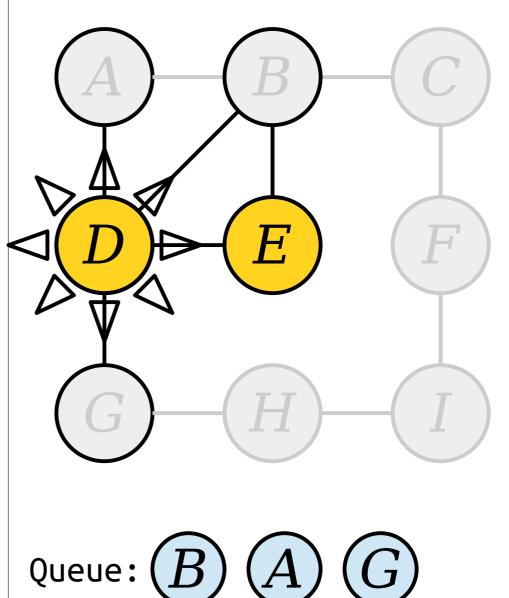


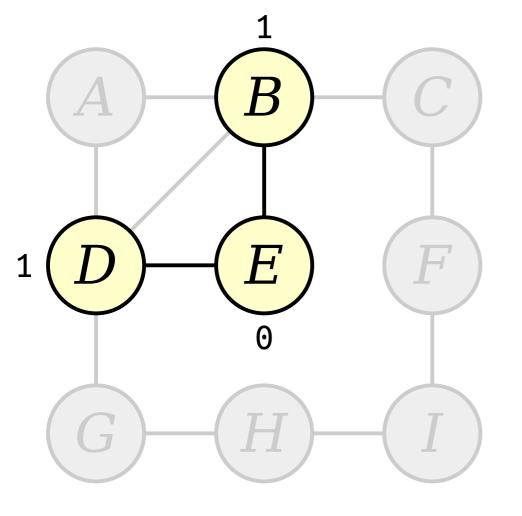


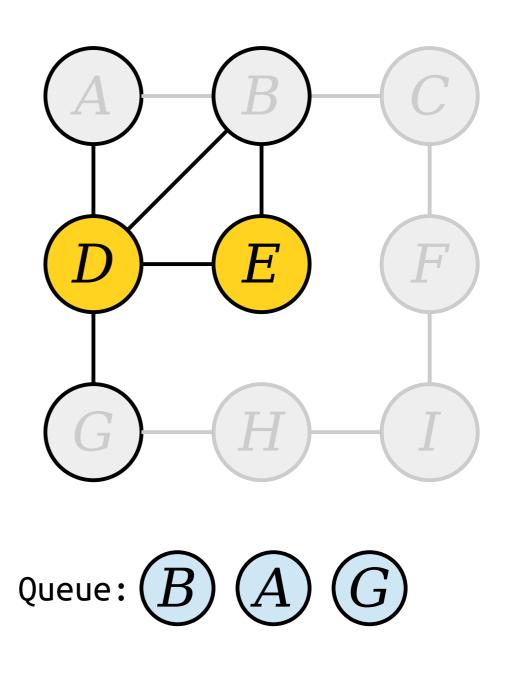


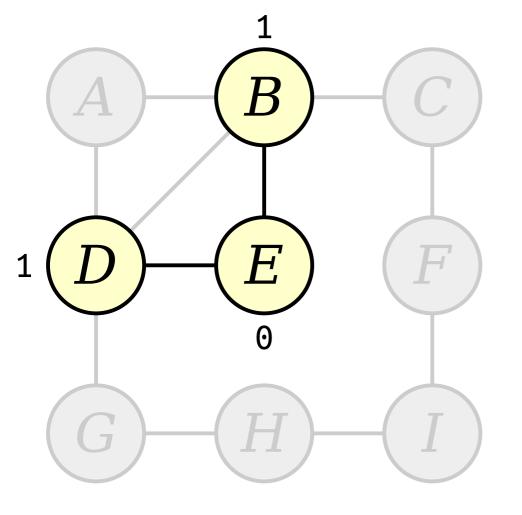


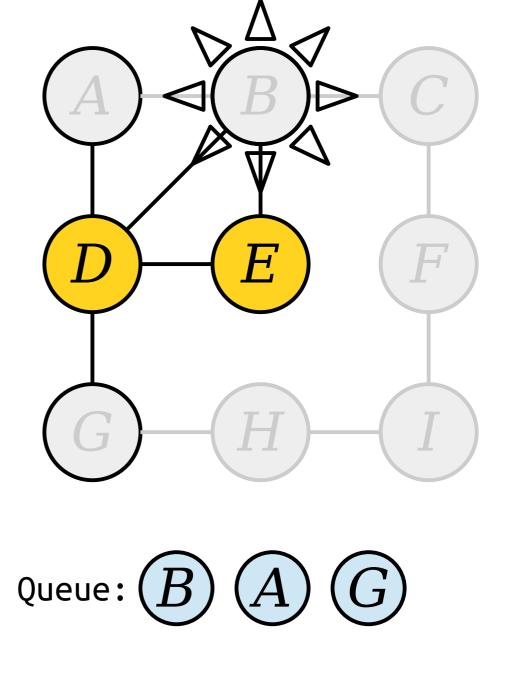


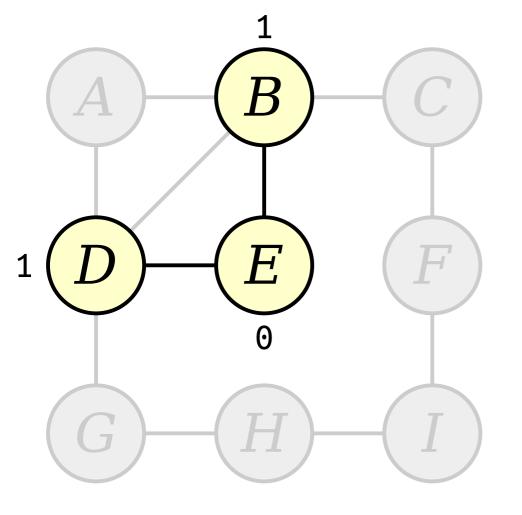


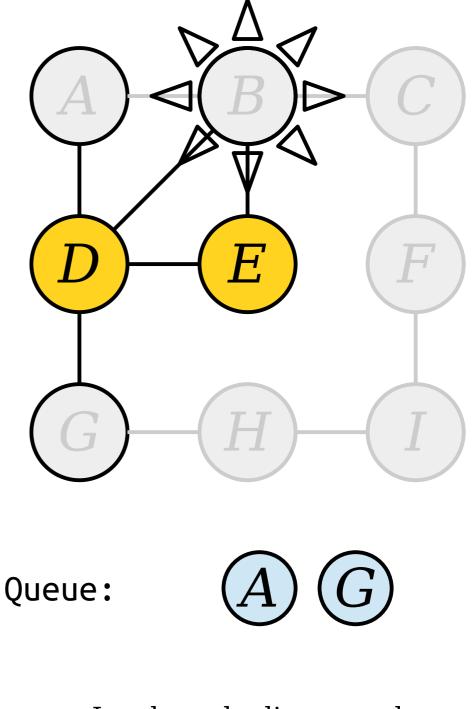


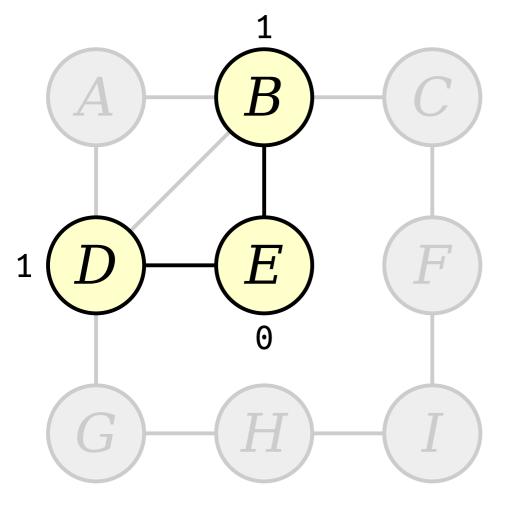


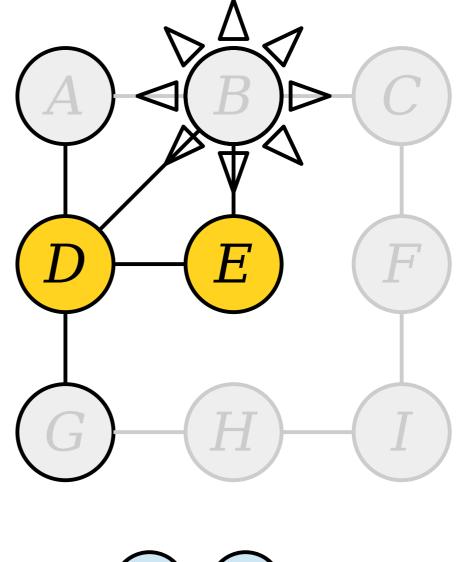






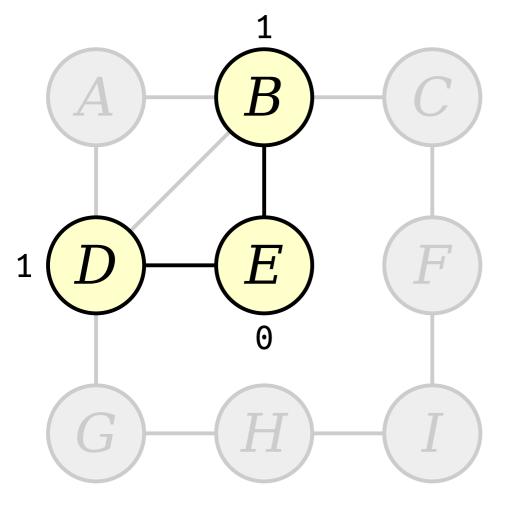


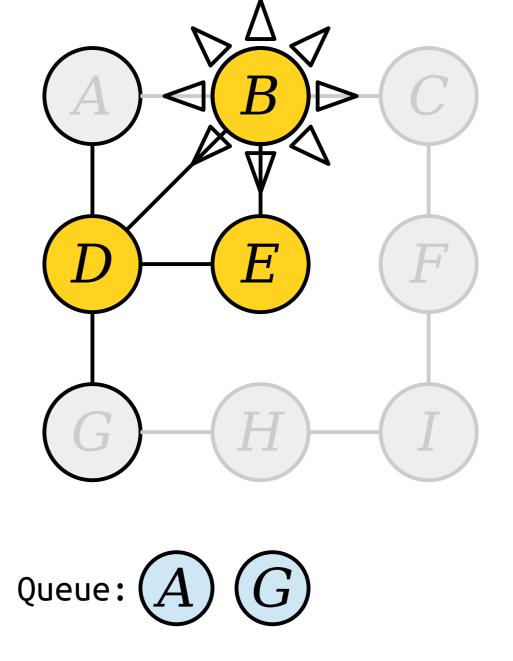


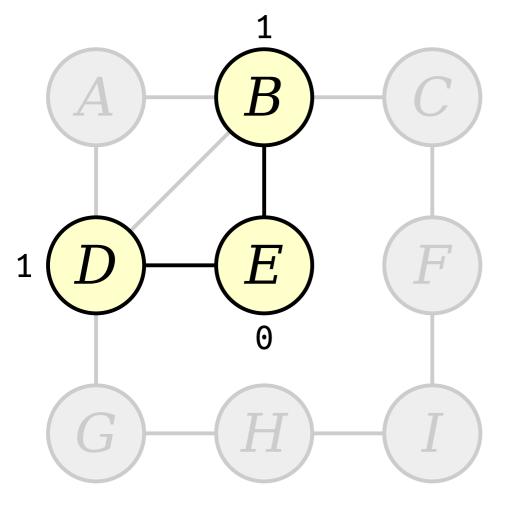


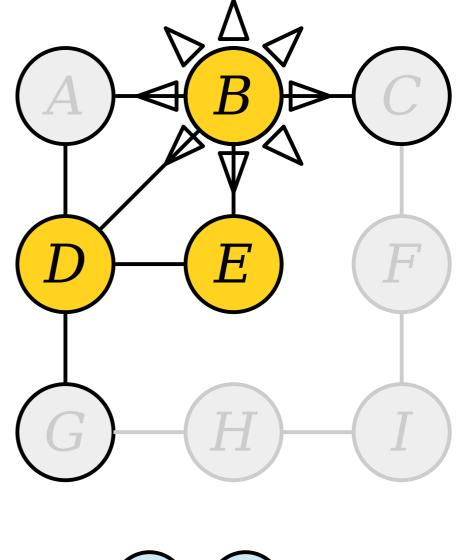
Queue: (A) (G)

Visit nodes in ascending order of distance from the start node E.



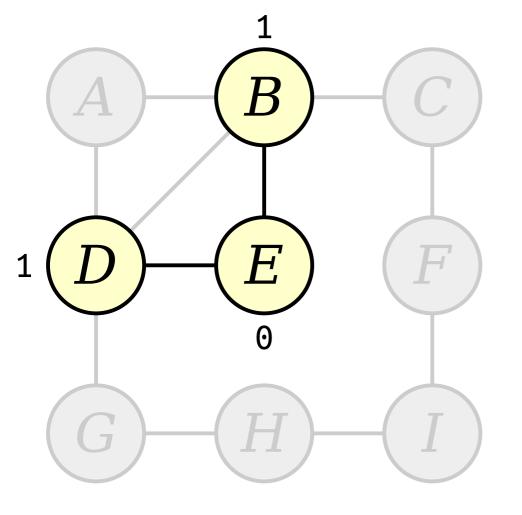


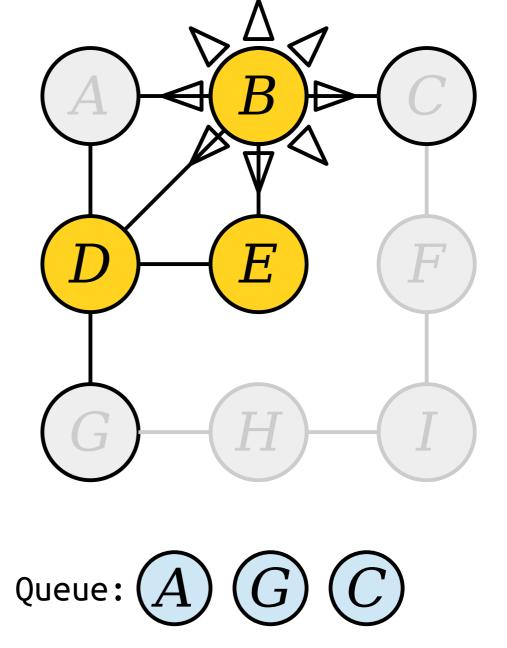


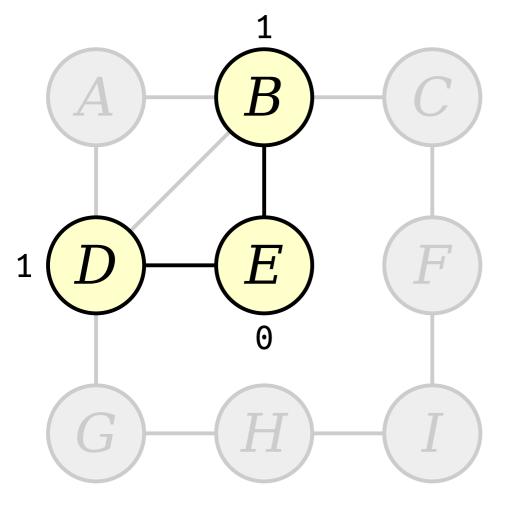


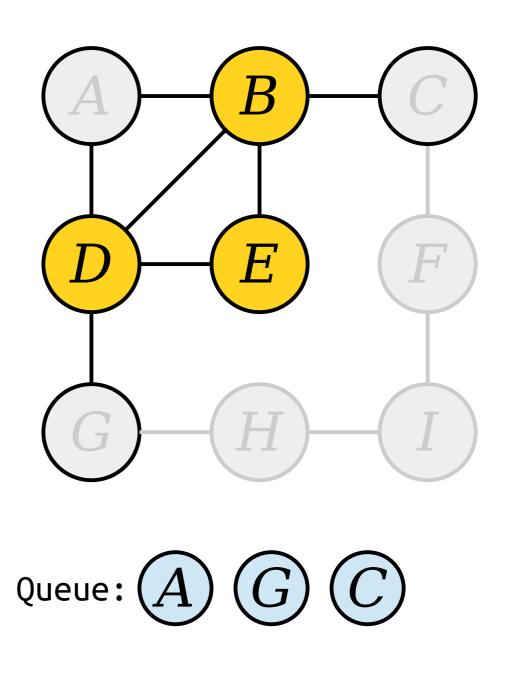
Queue: (A) (G)

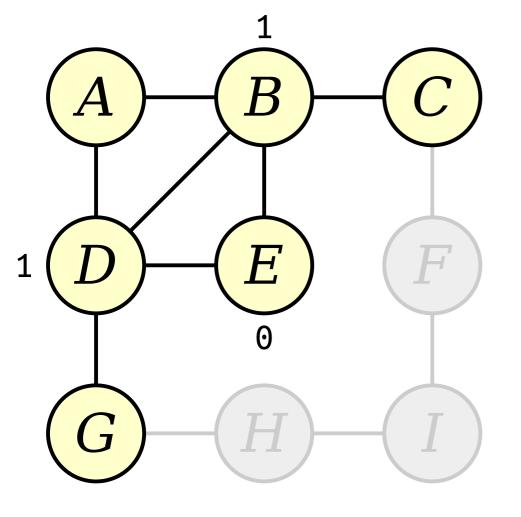
Visit nodes in ascending order of distance from the start node E.

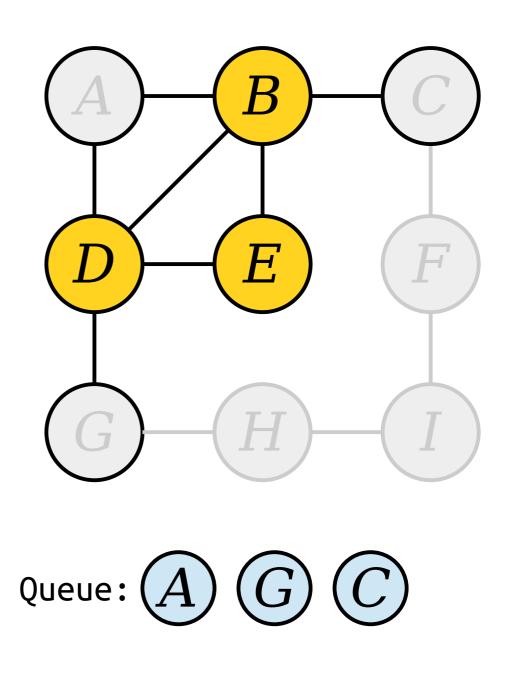


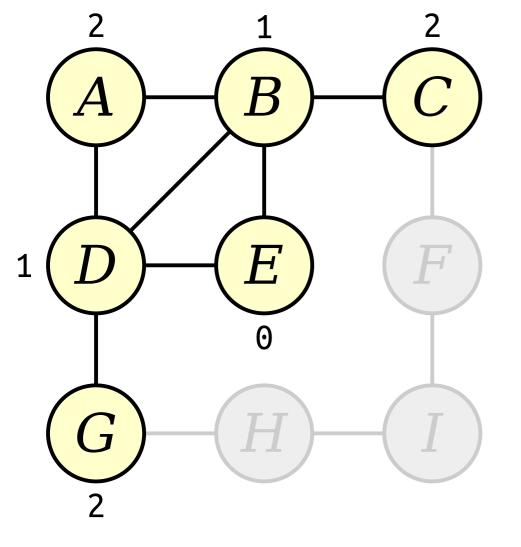


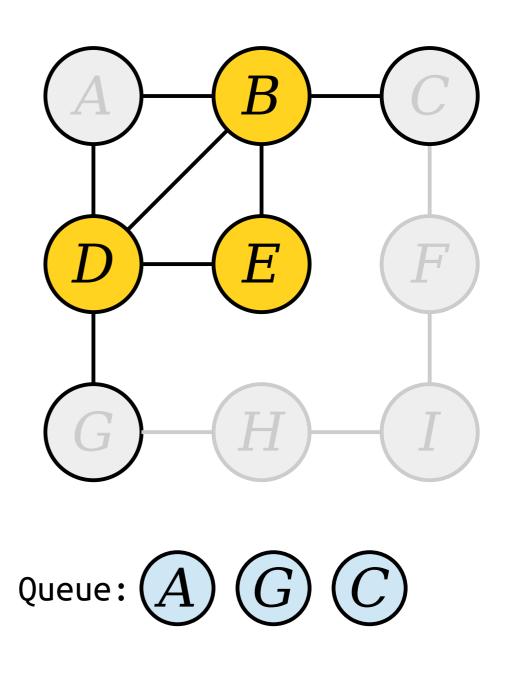


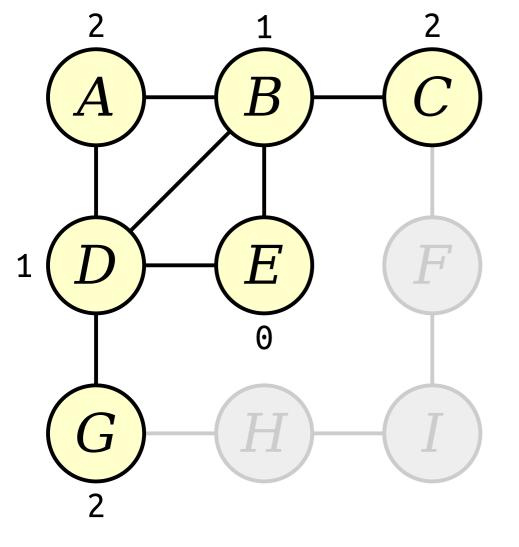


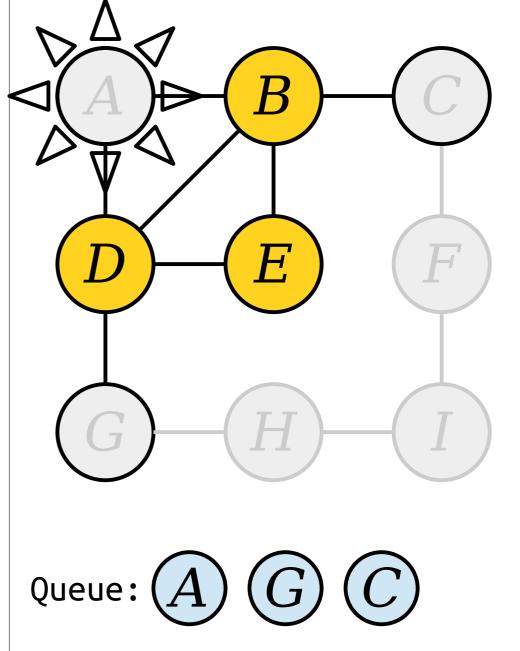


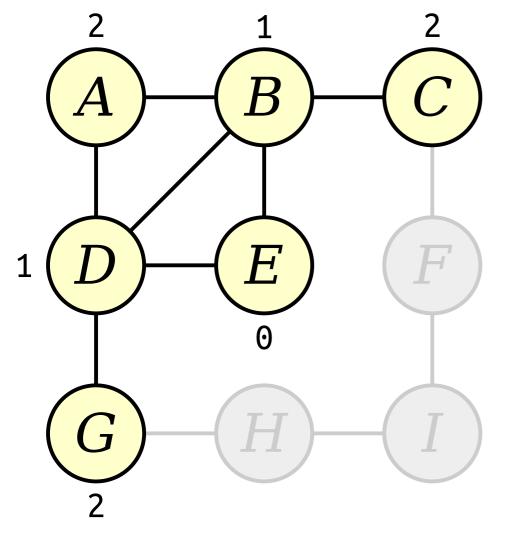


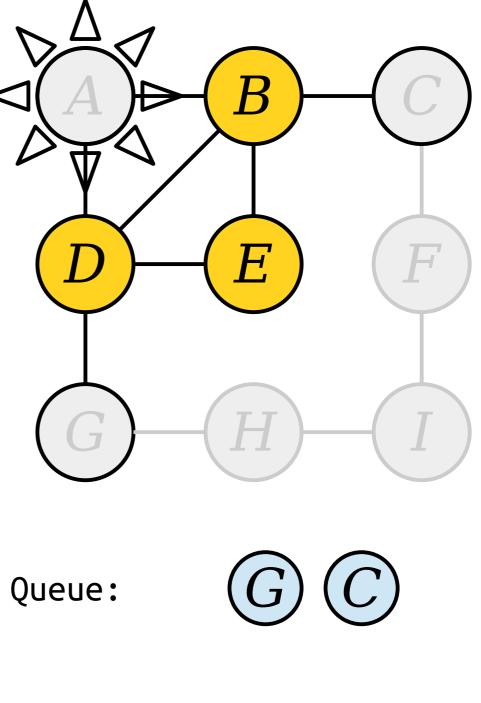


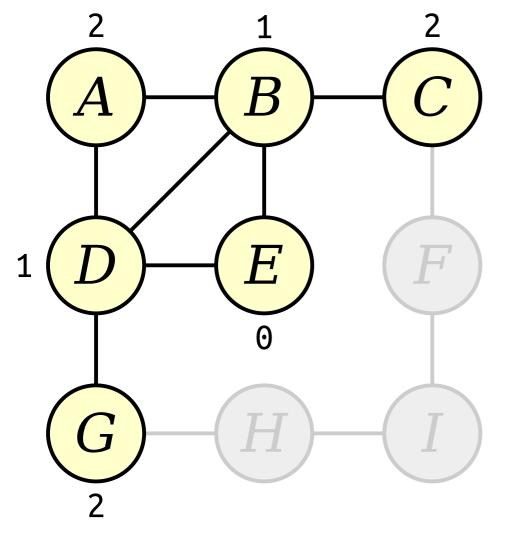


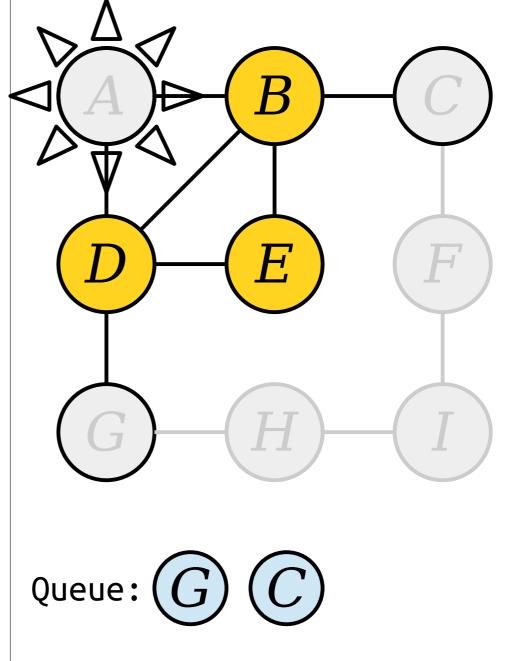


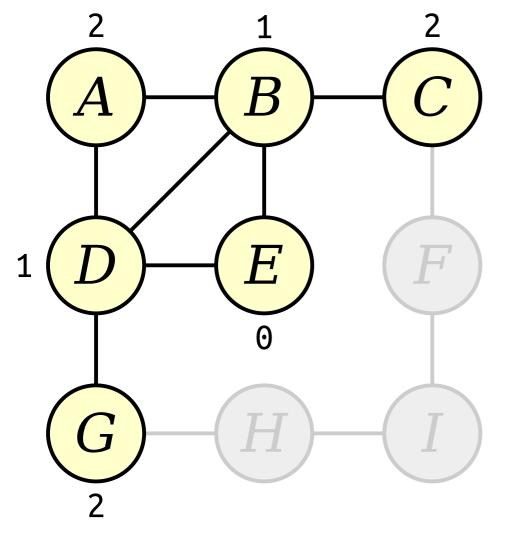


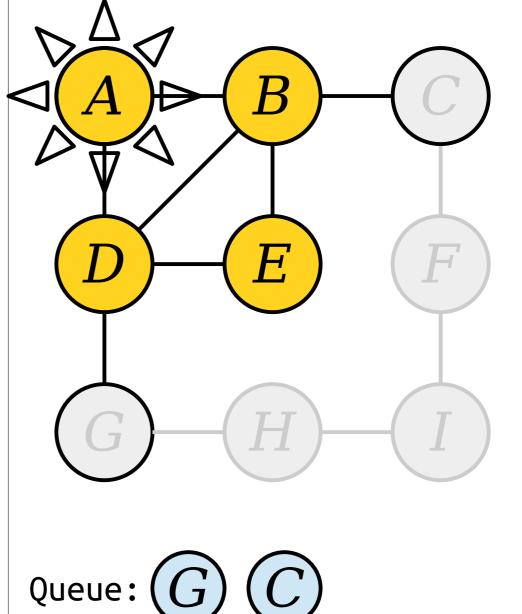


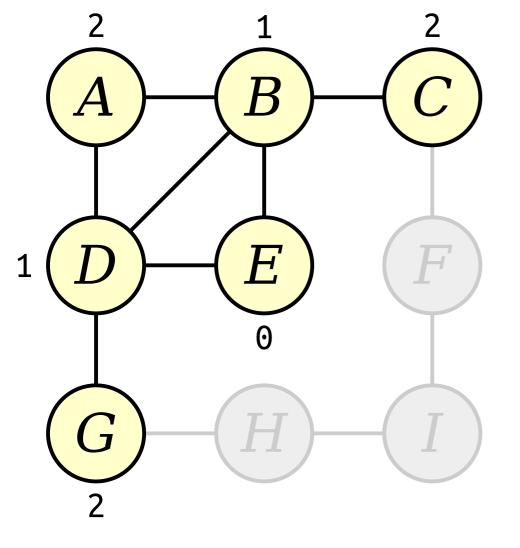


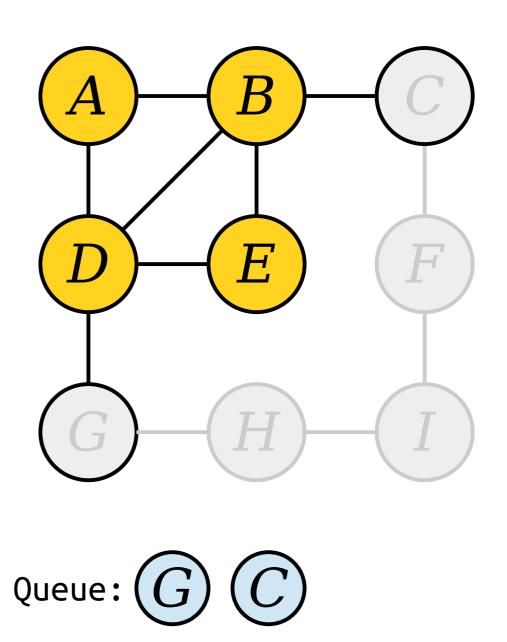


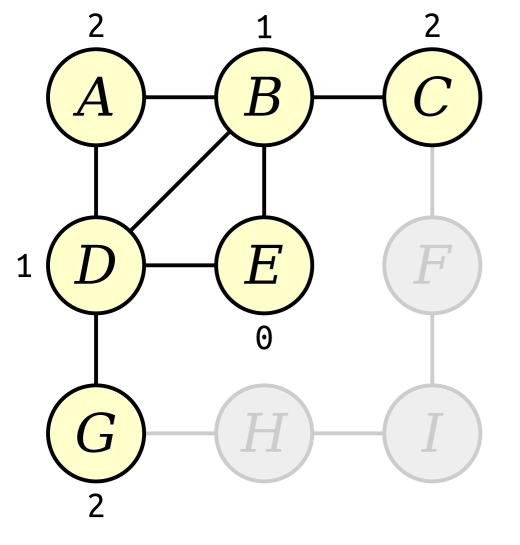






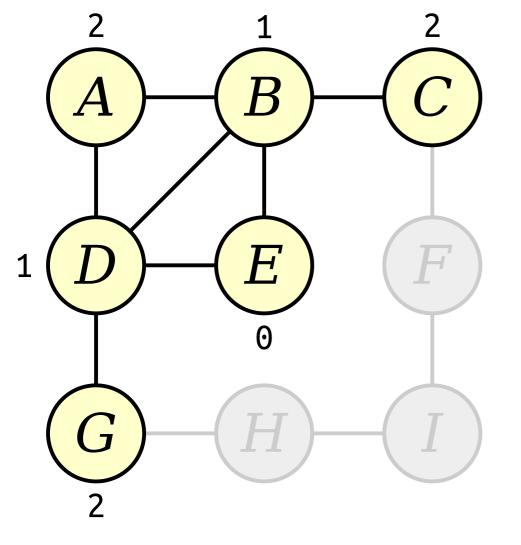


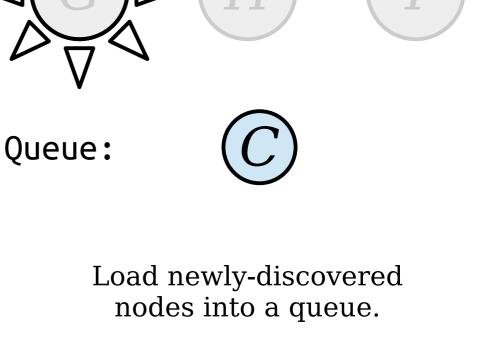




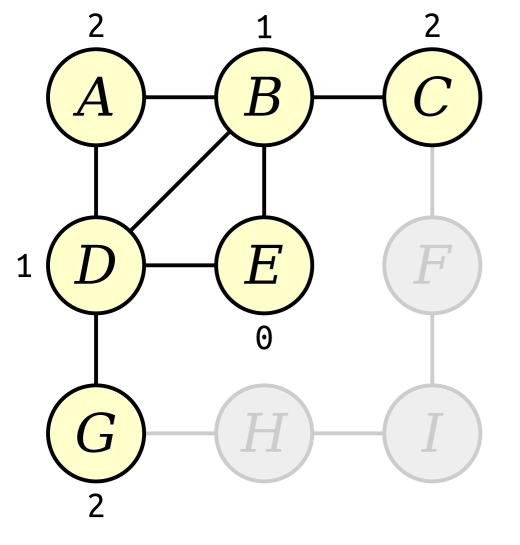
Queue: G C Load newly-discovered nodes into a queue.

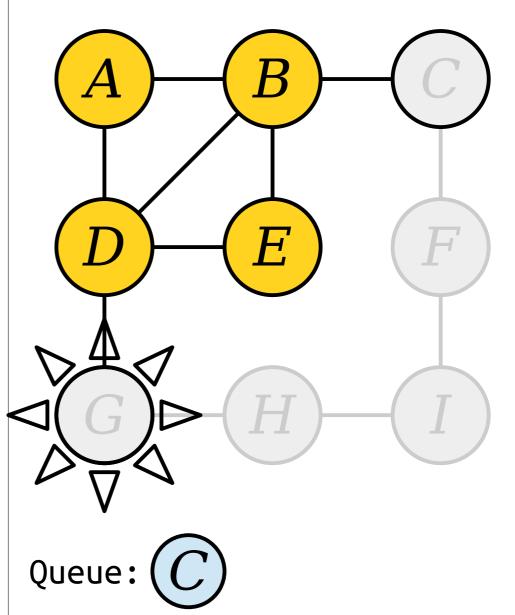
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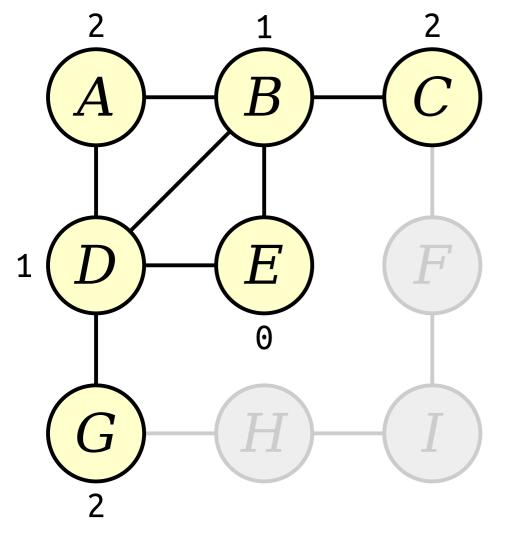


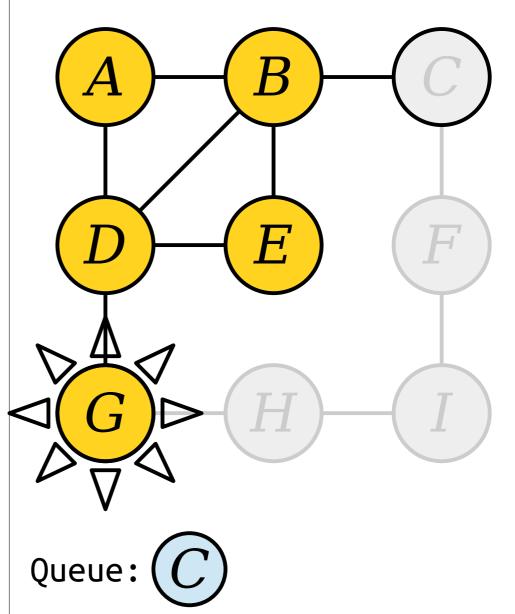


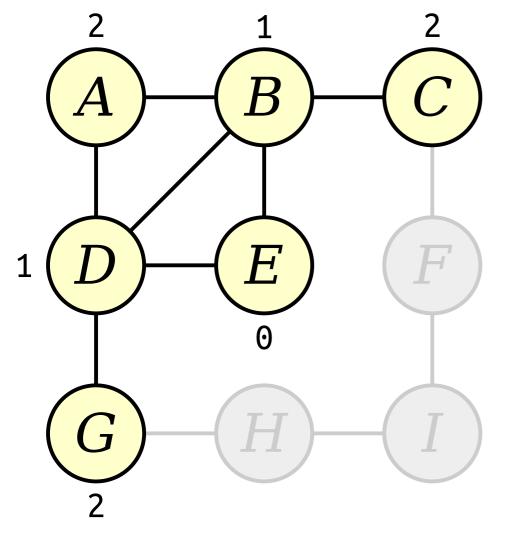
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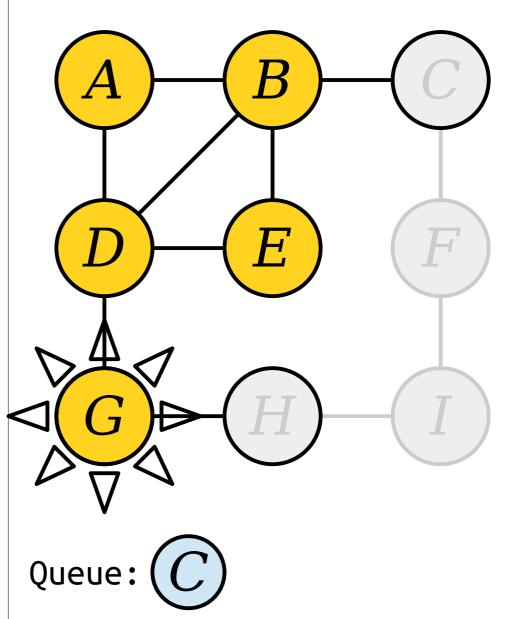


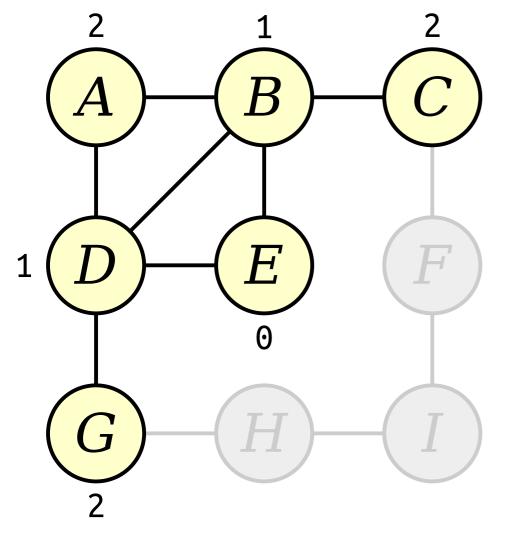


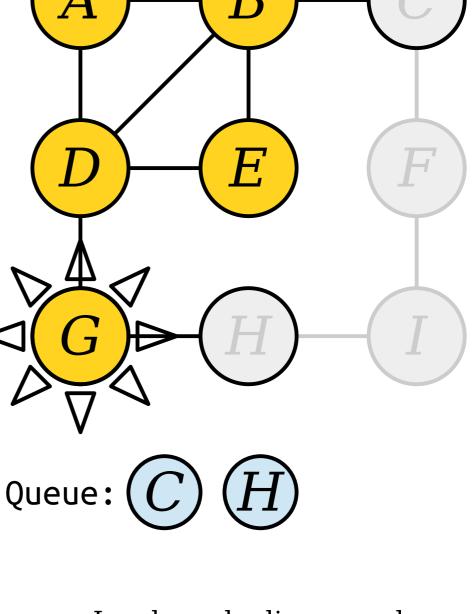


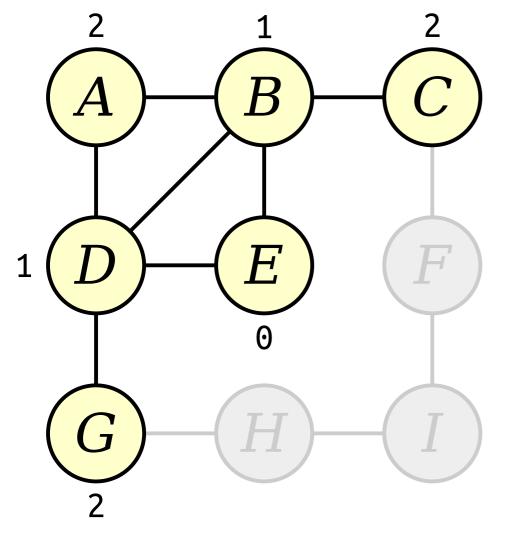


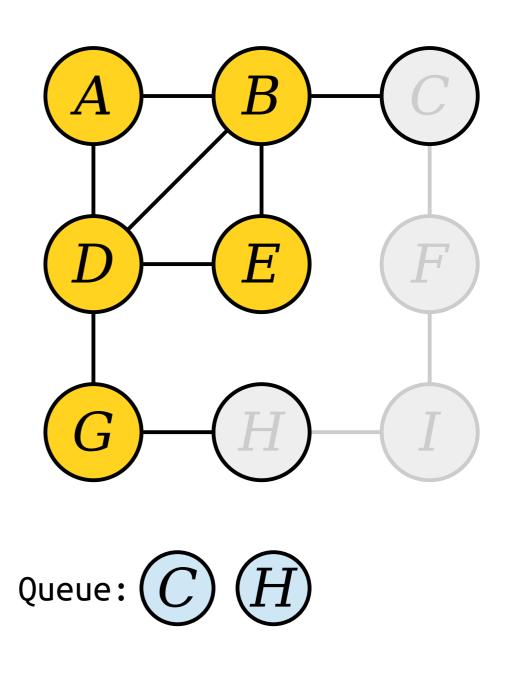


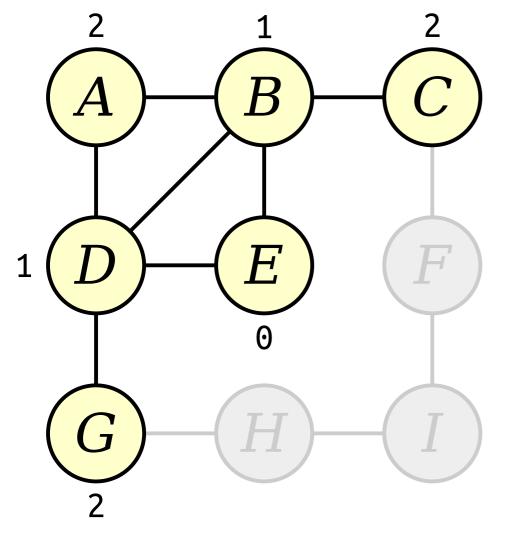


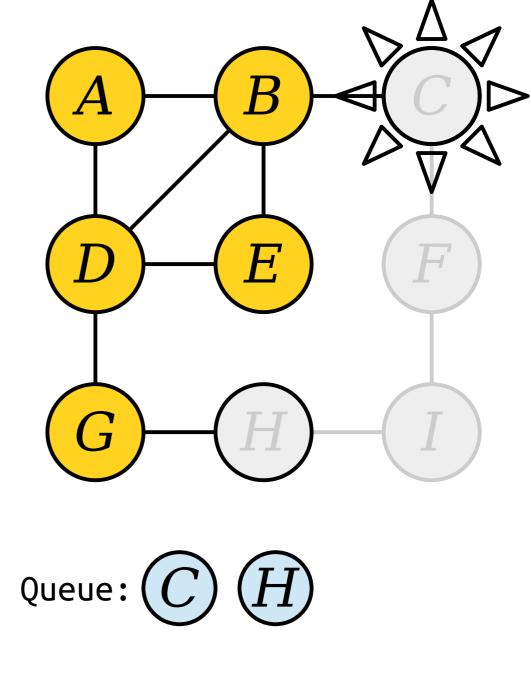


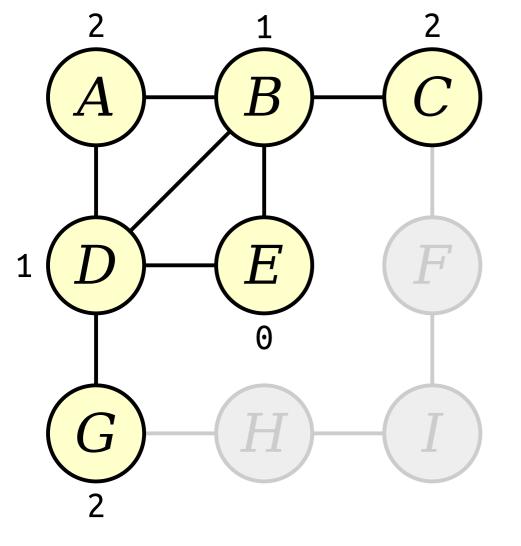


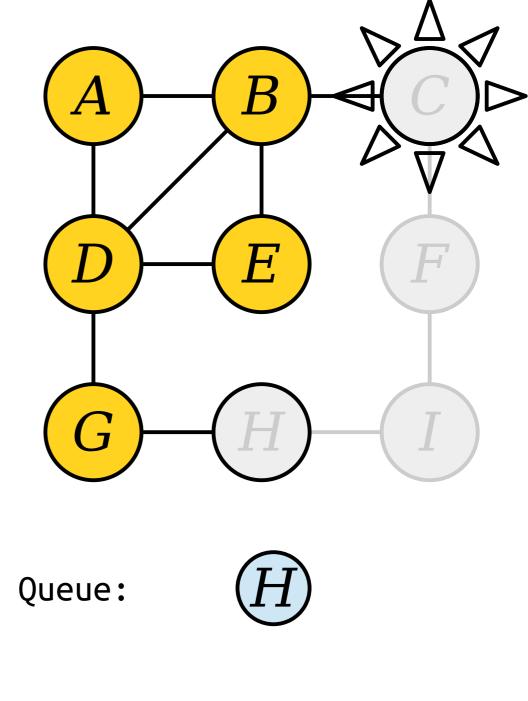


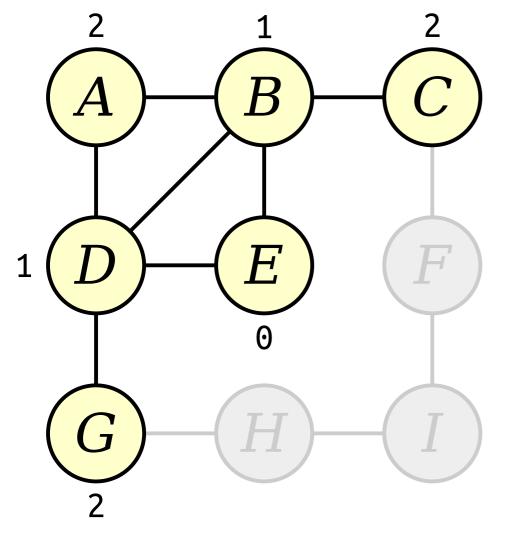


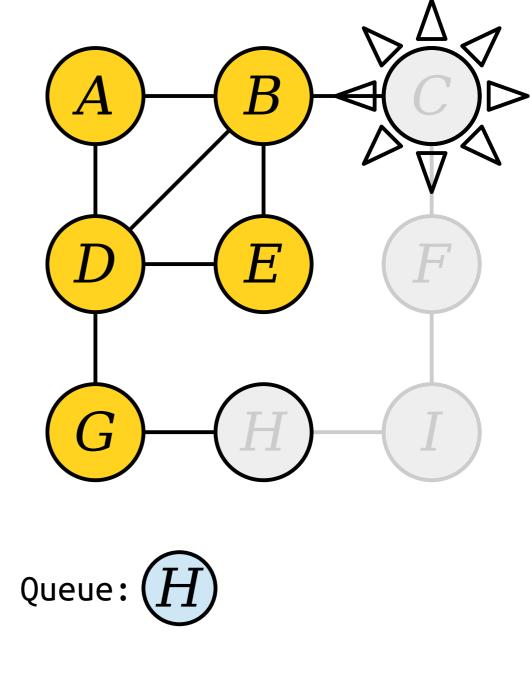


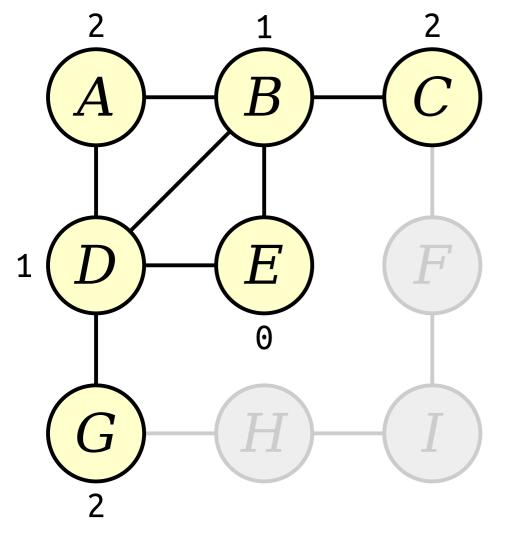


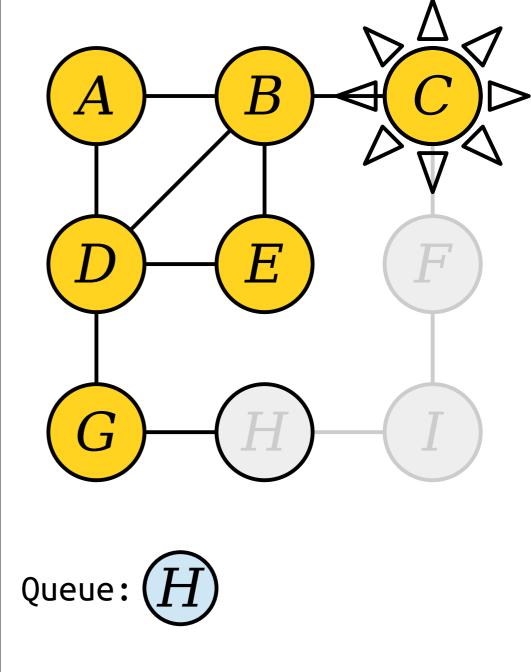


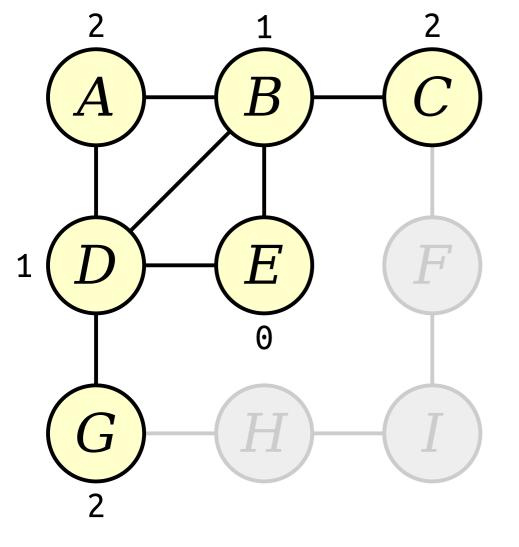


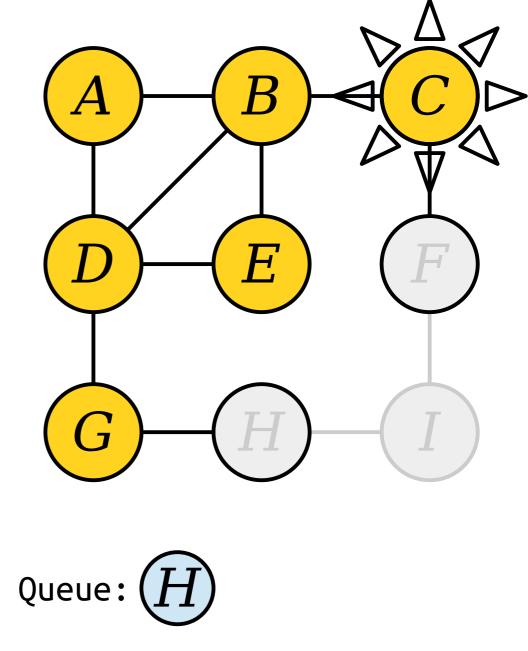


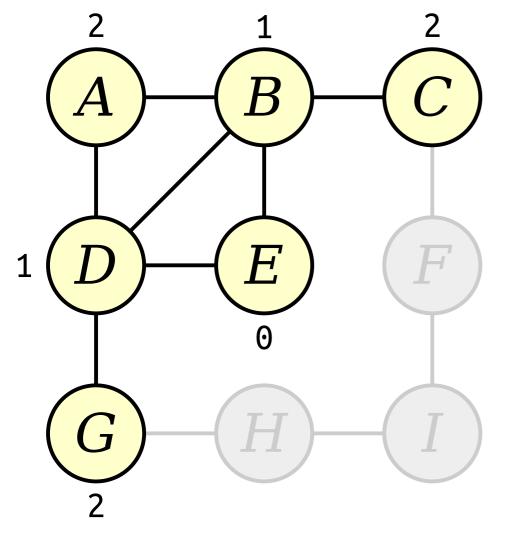


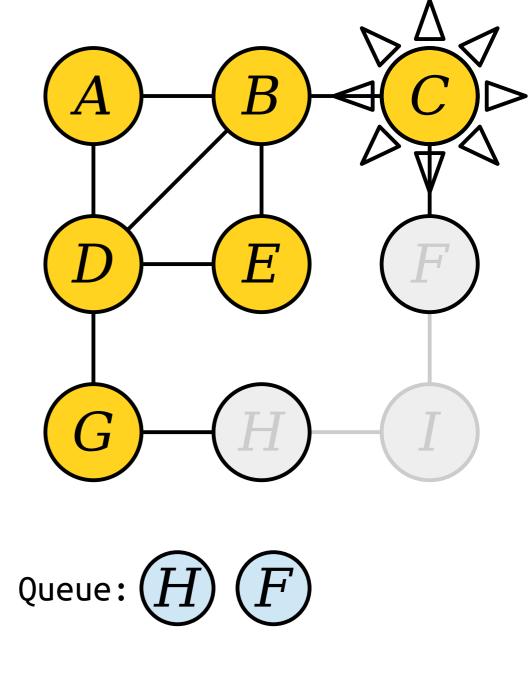


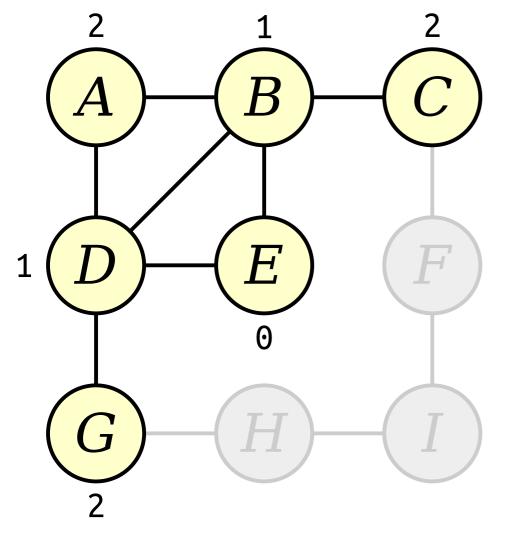


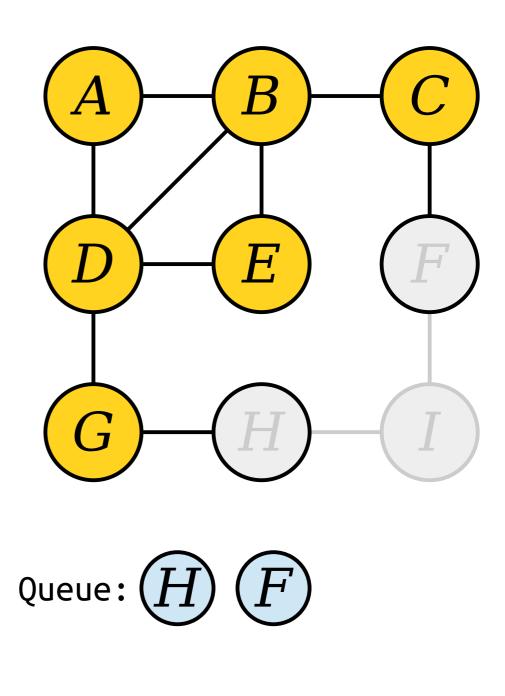


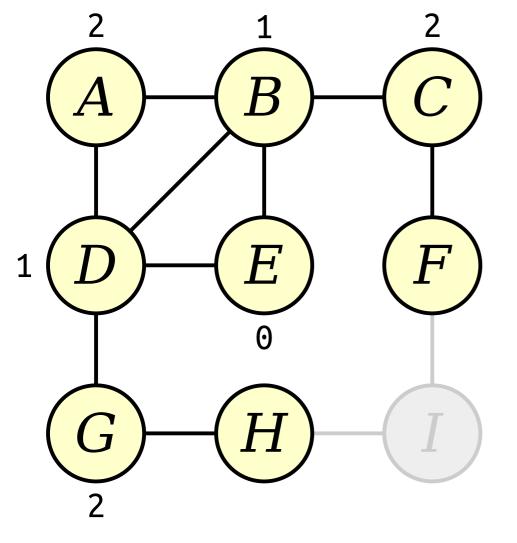


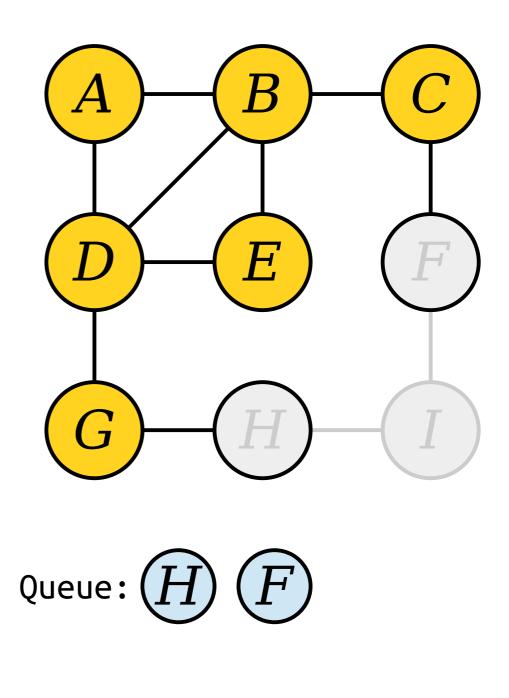


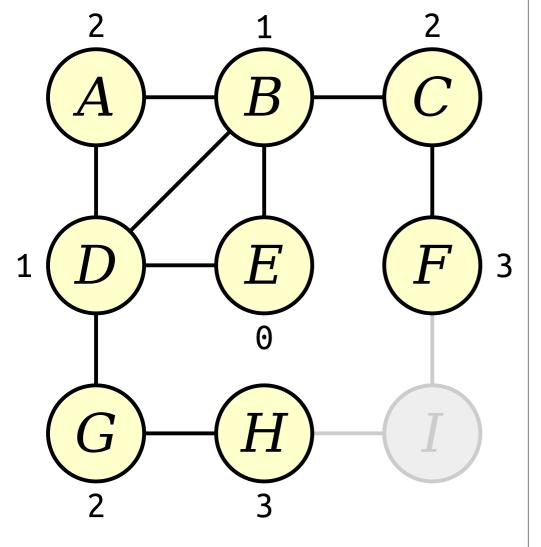






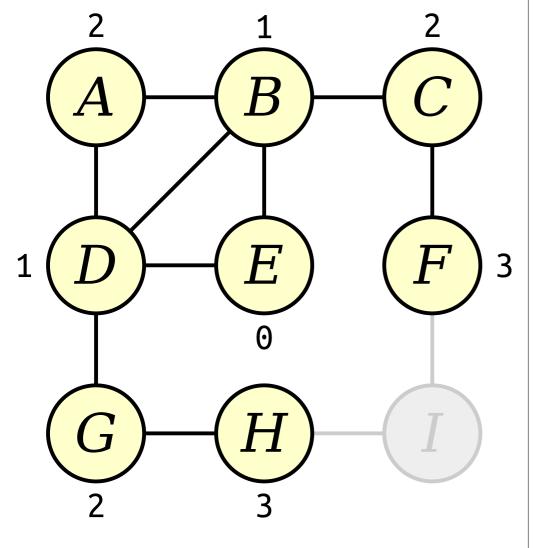


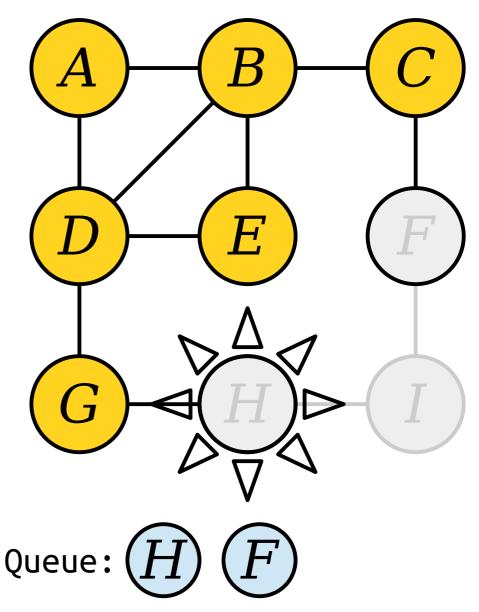


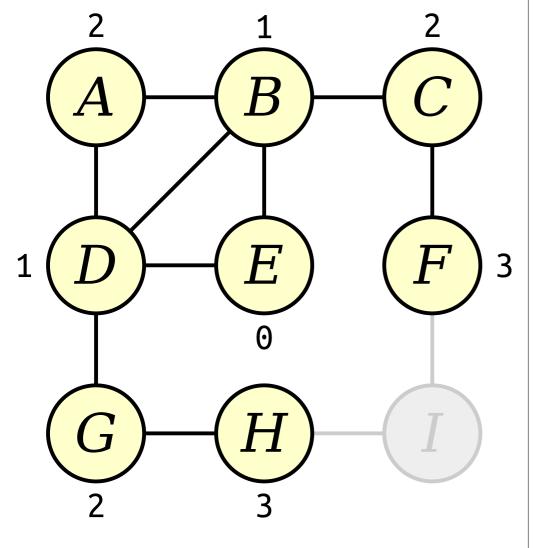


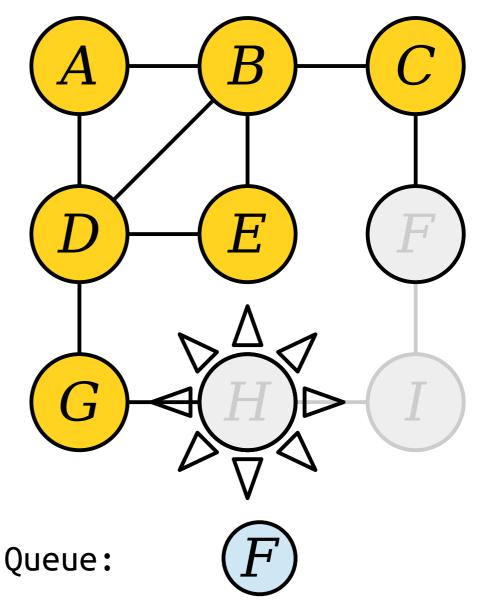
H. (\cdot) Queue: F

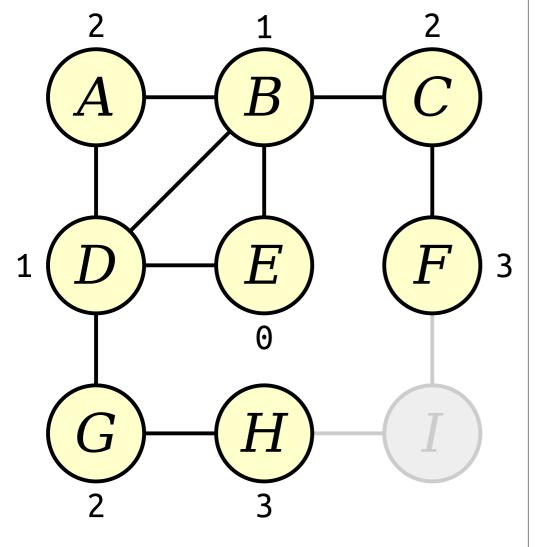
Visit nodes in ascending order of distance from the start node E.



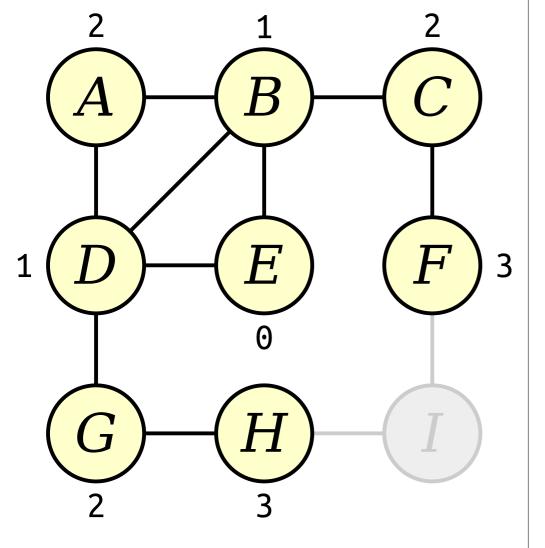


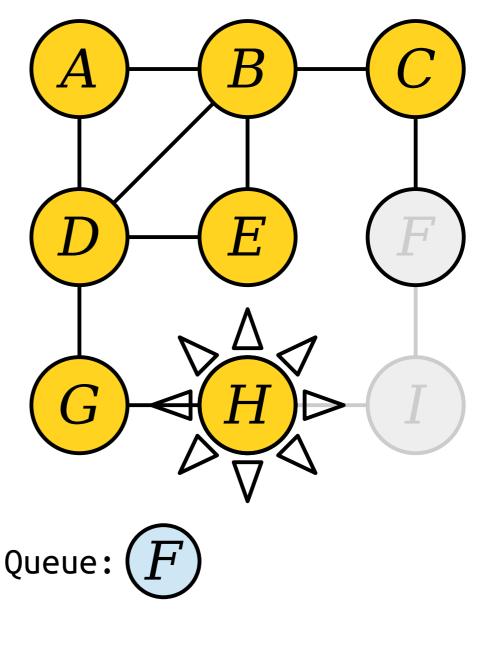


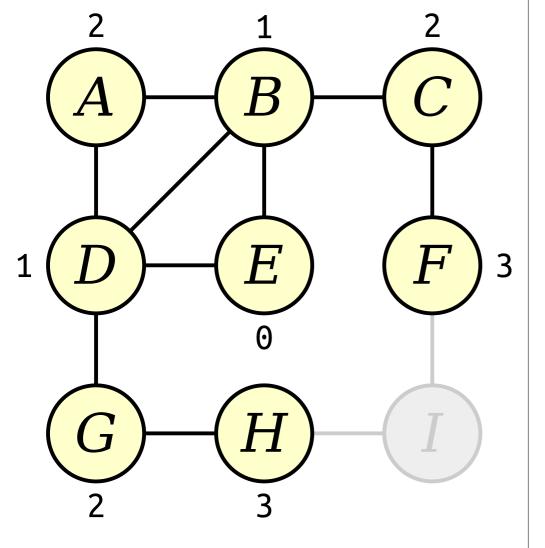


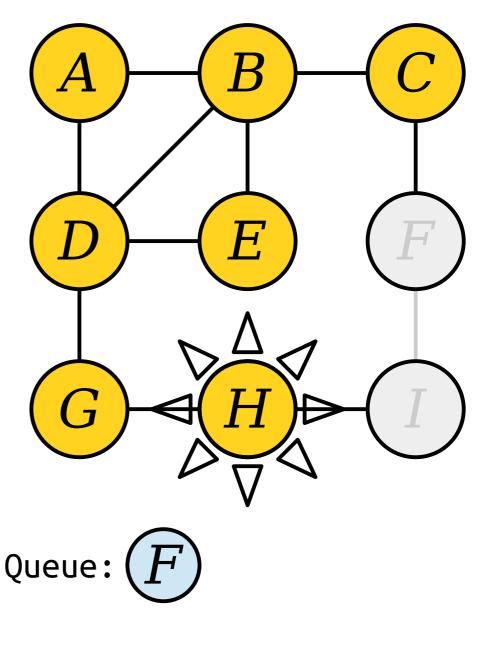


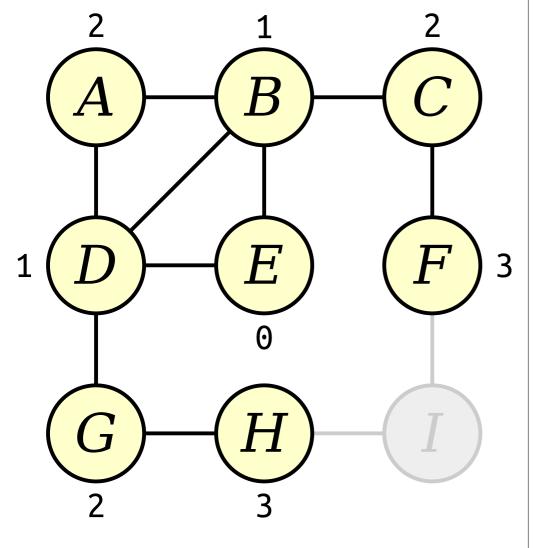
H. (7 Queue: Load newly-discovered nodes into a queue.

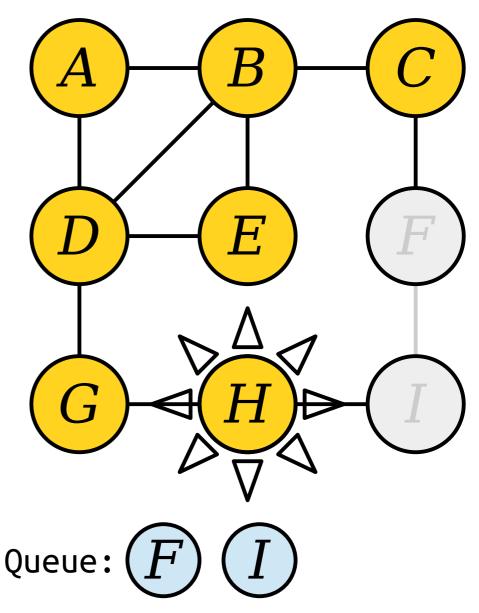


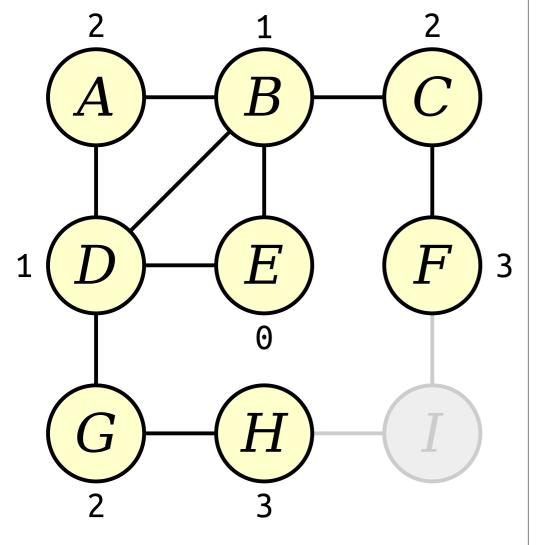






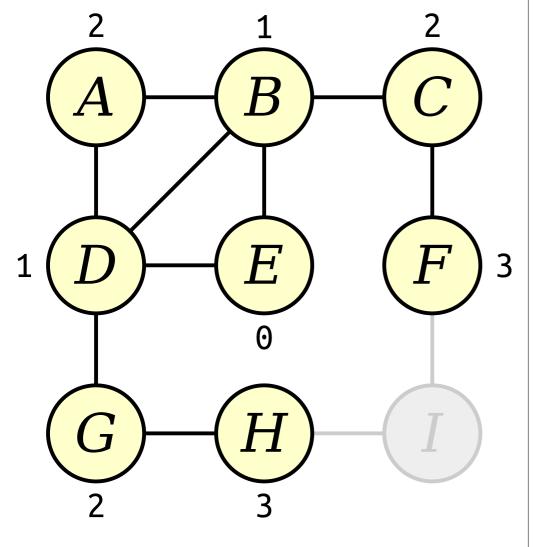


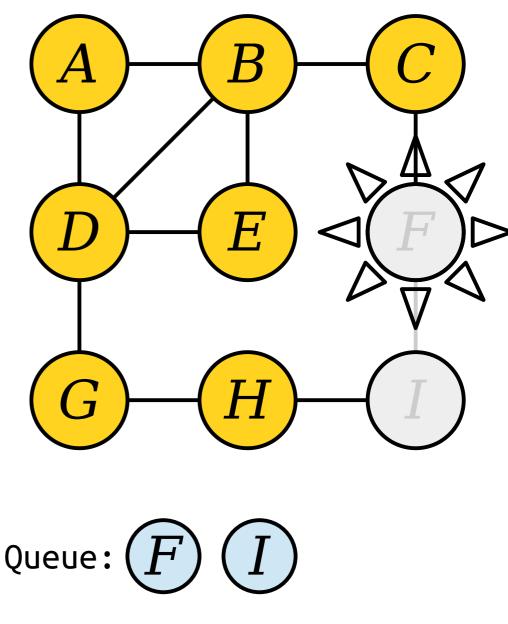


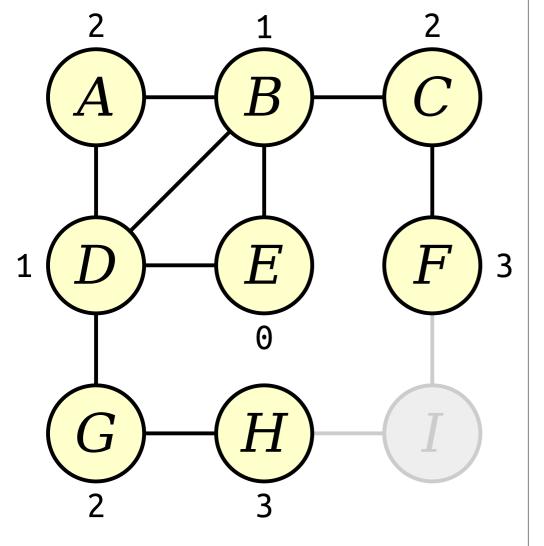


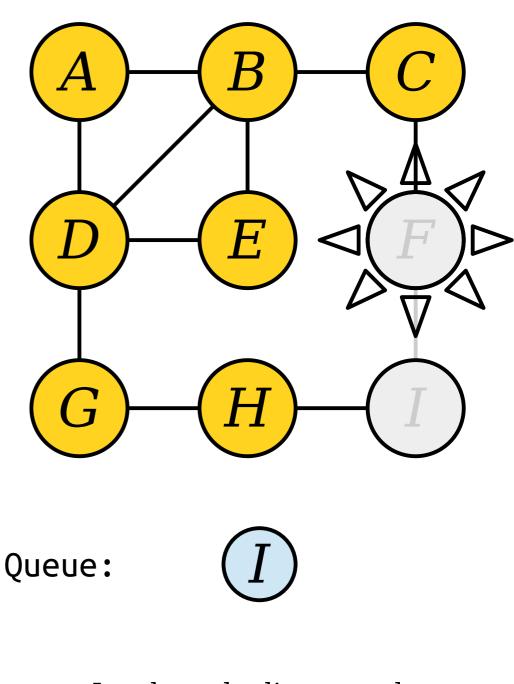
H. G Queue: I

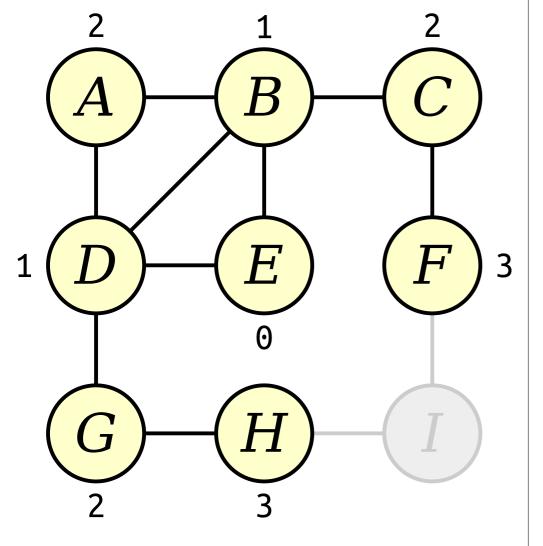
Visit nodes in ascending order of distance from the start node E.

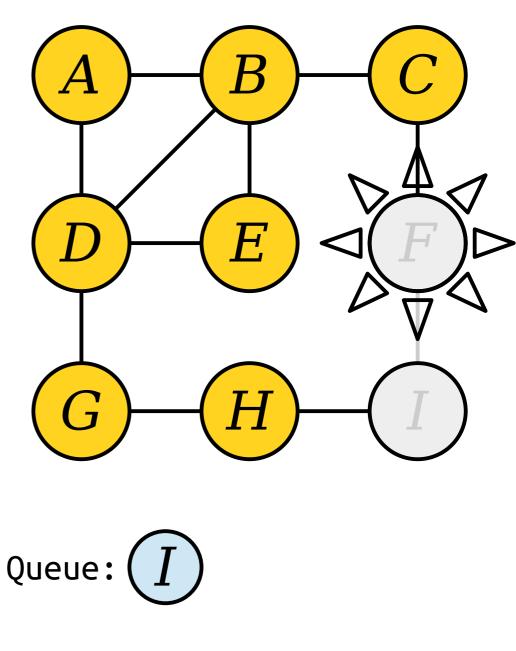


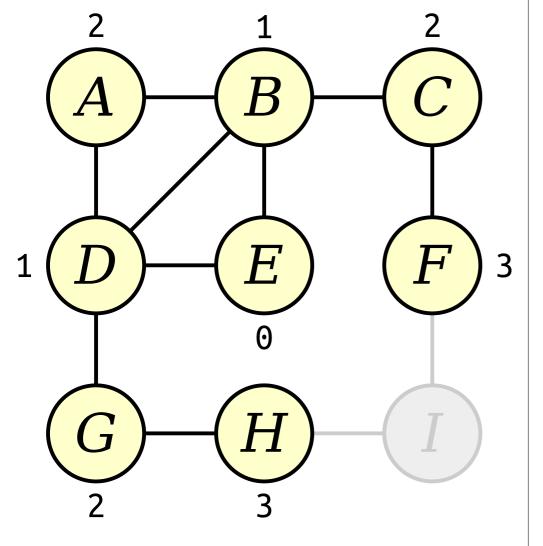


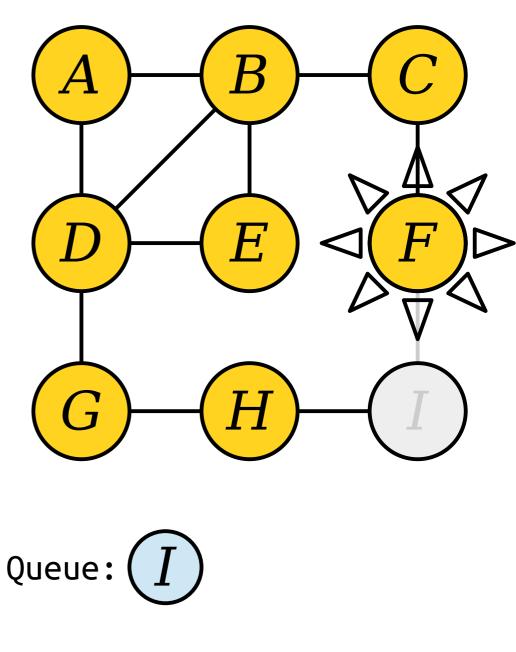


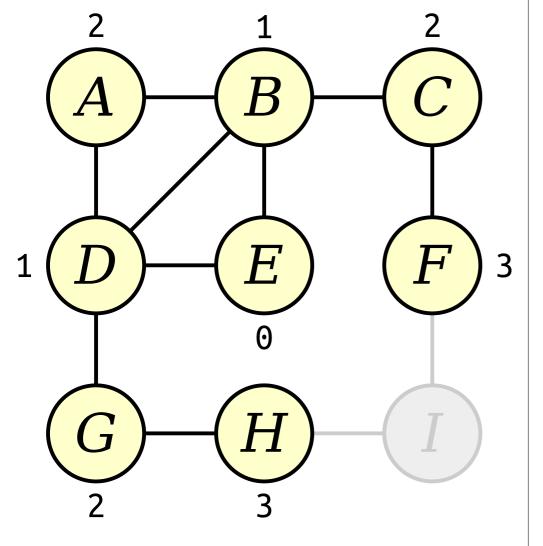


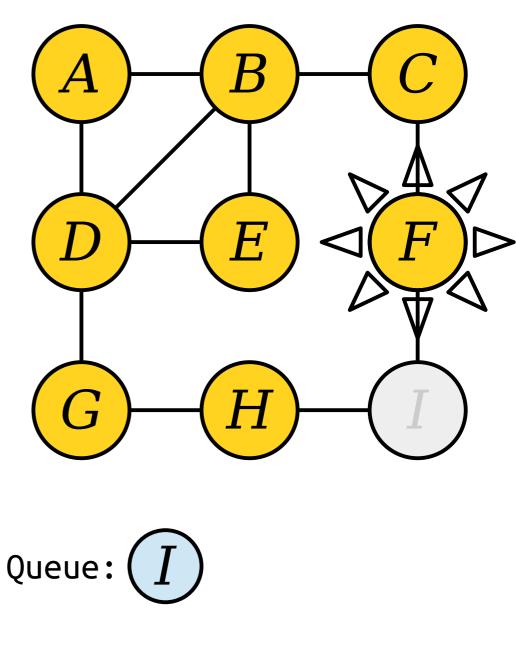


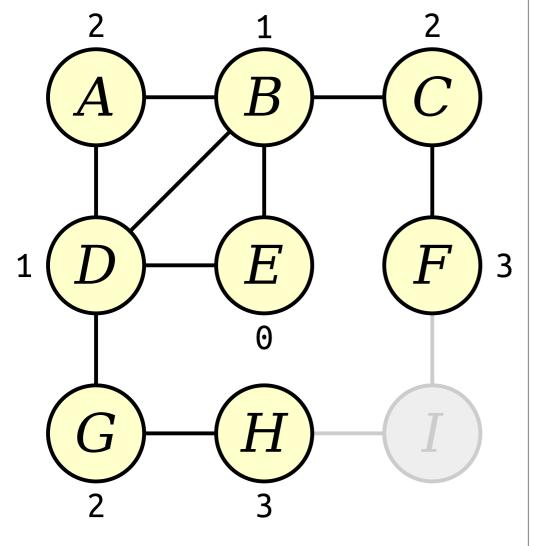


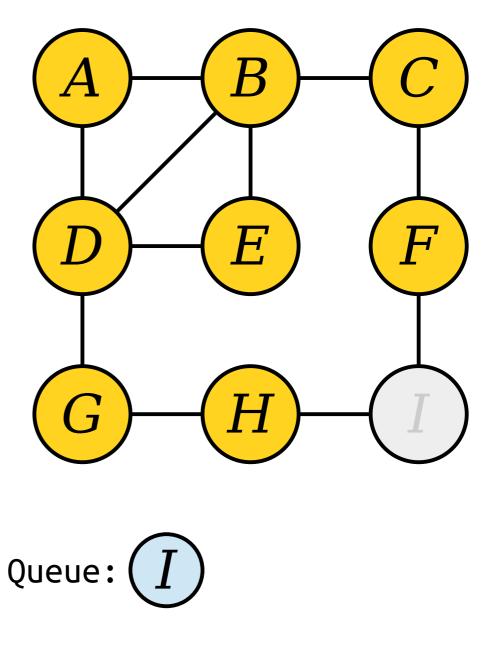


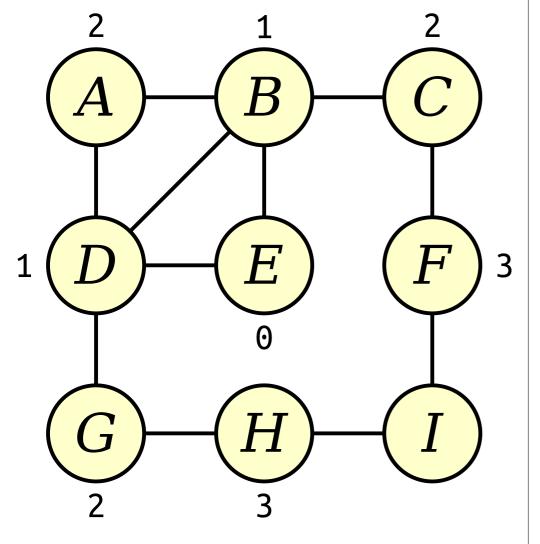


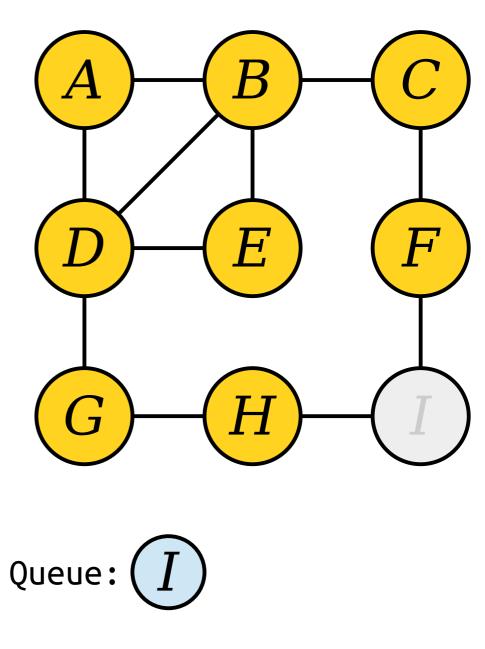


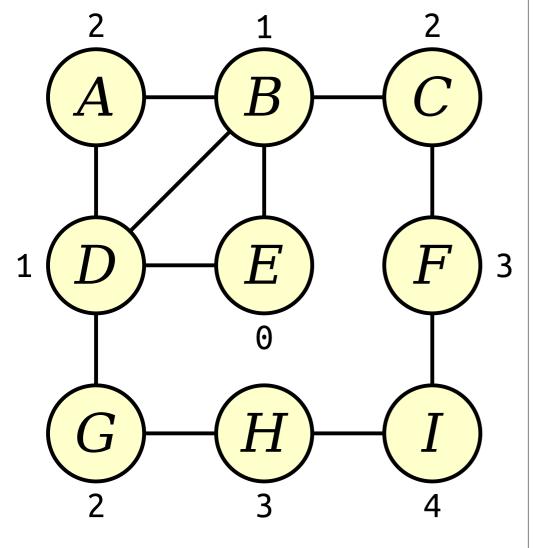


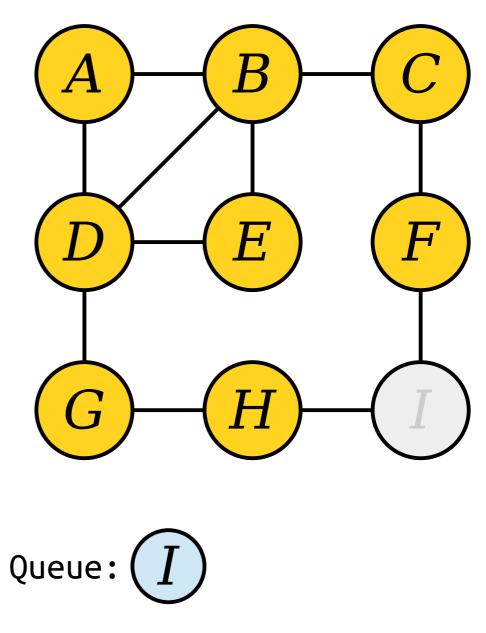


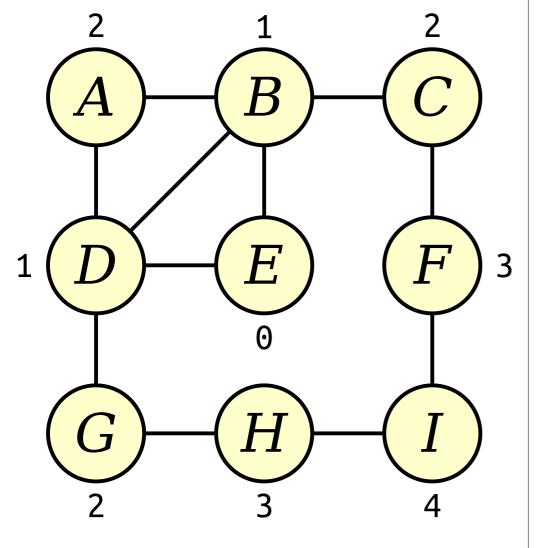


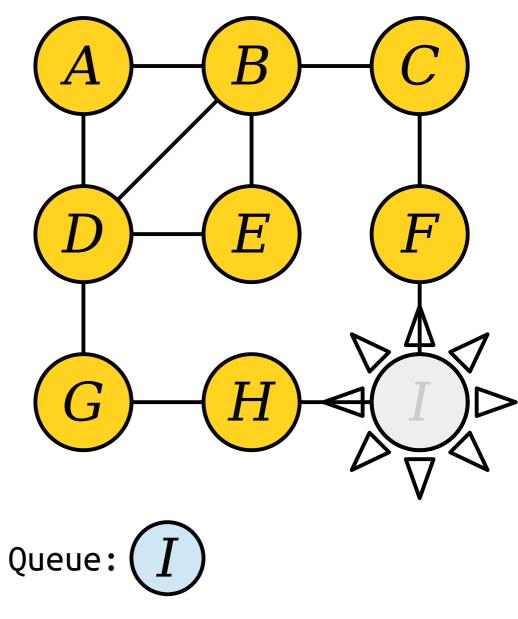


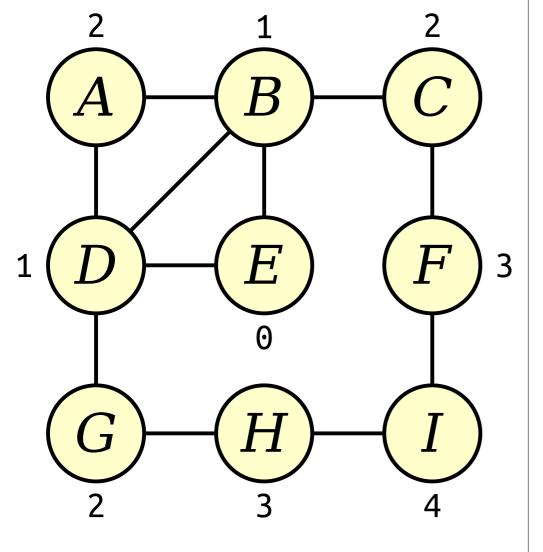


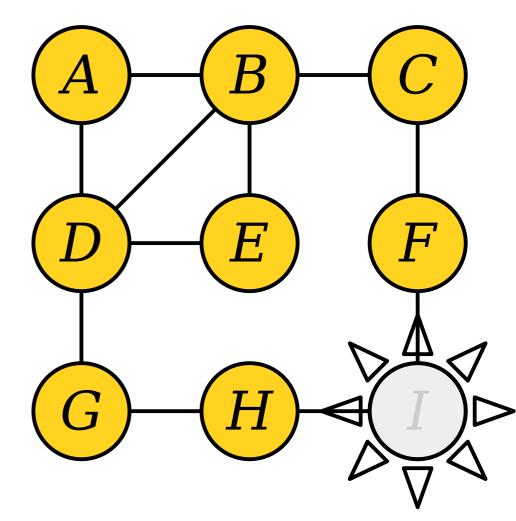






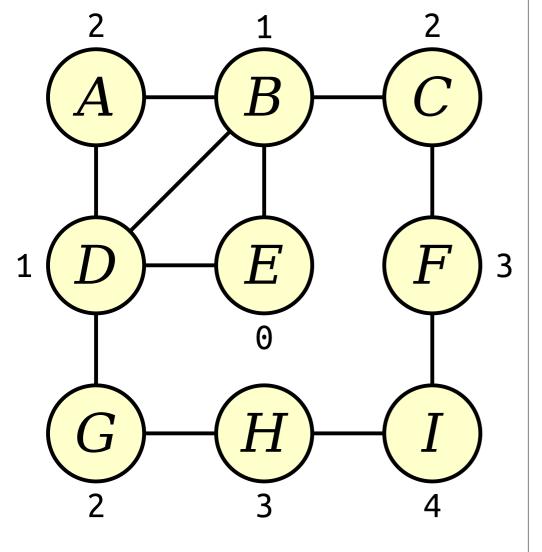


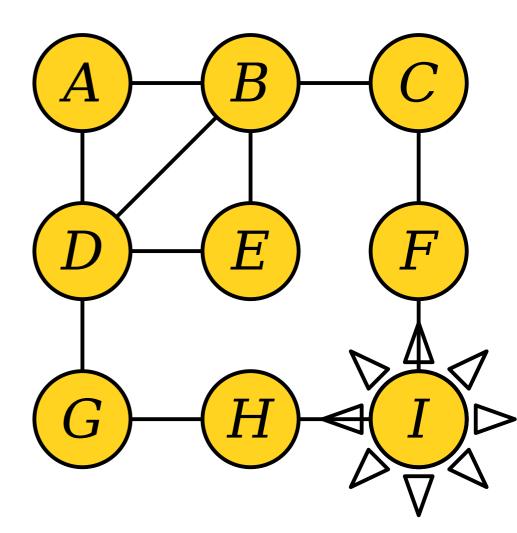




Queue:

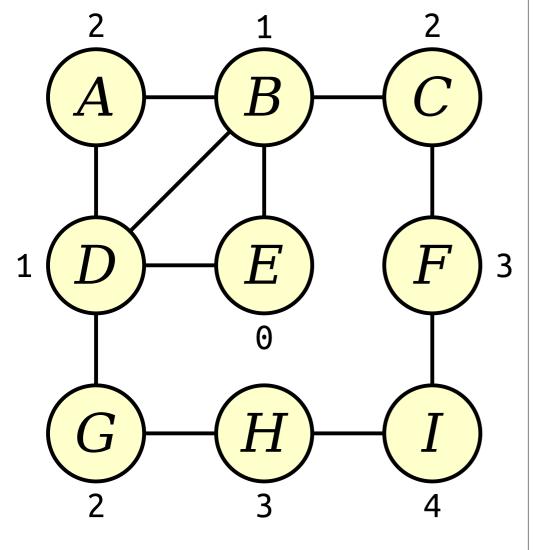
Visit nodes in ascending order of distance from the start node E.

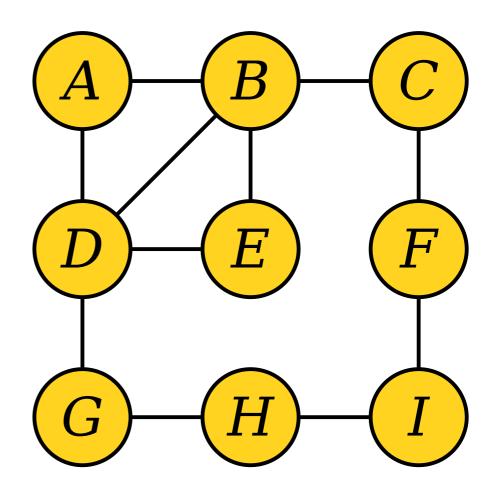




Queue:

Visit nodes in ascending order of distance from the start node E.





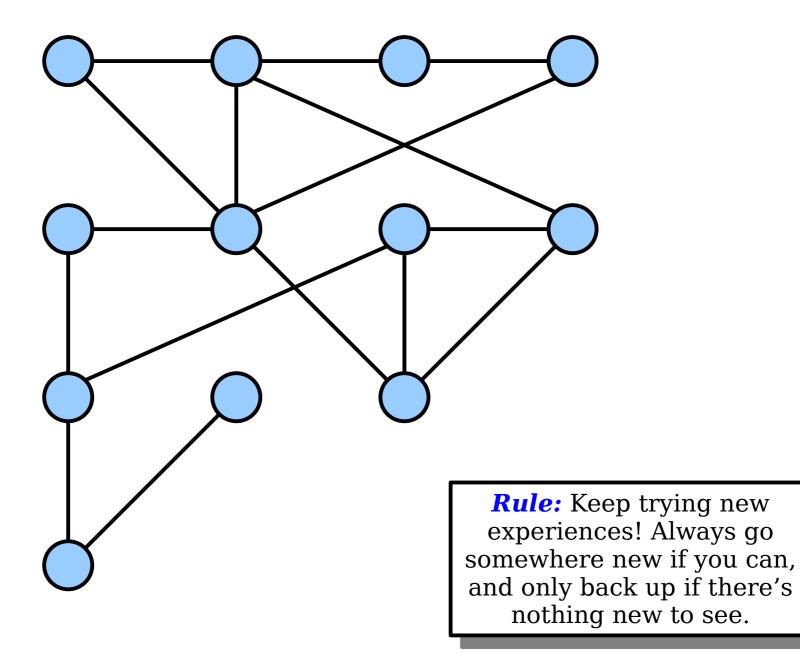
Queue:

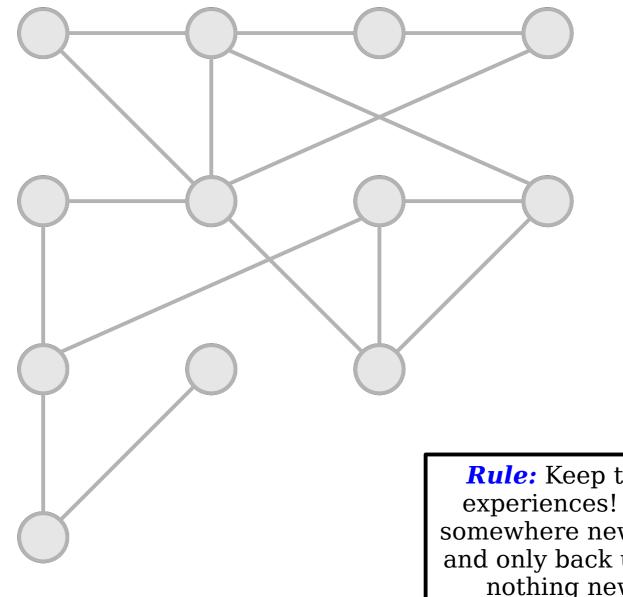
Visit nodes in ascending order of distance from the start node E.

Breadth-First Search

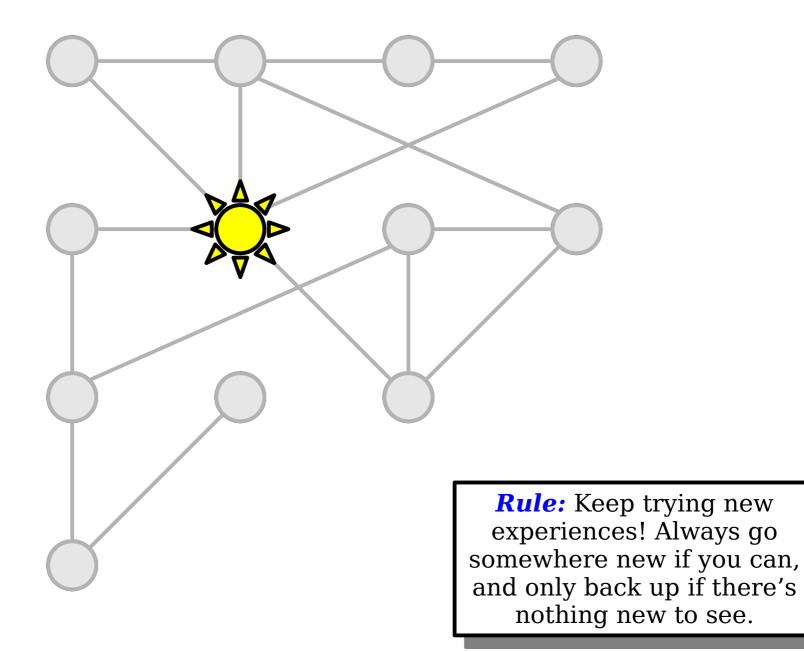
- The Queue-based search strategy we just saw is called *breadth-first search* (or just *BFS* for short).
- In pseudocode:

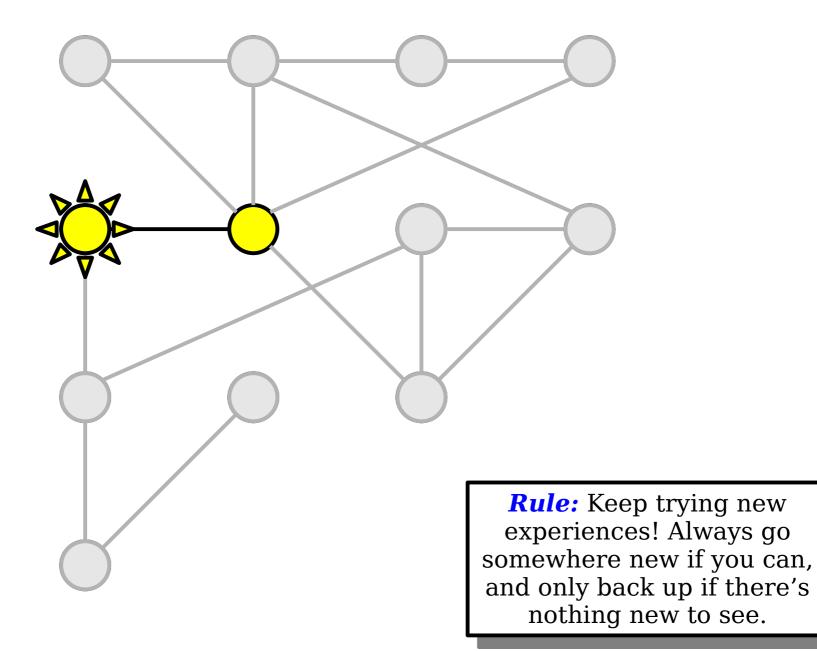
```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
        dequeue a node curr.
        process the node curr.
        for each node adjacent to curr:
            if that node has never been enqueued:
                enqueue that node.
}
```

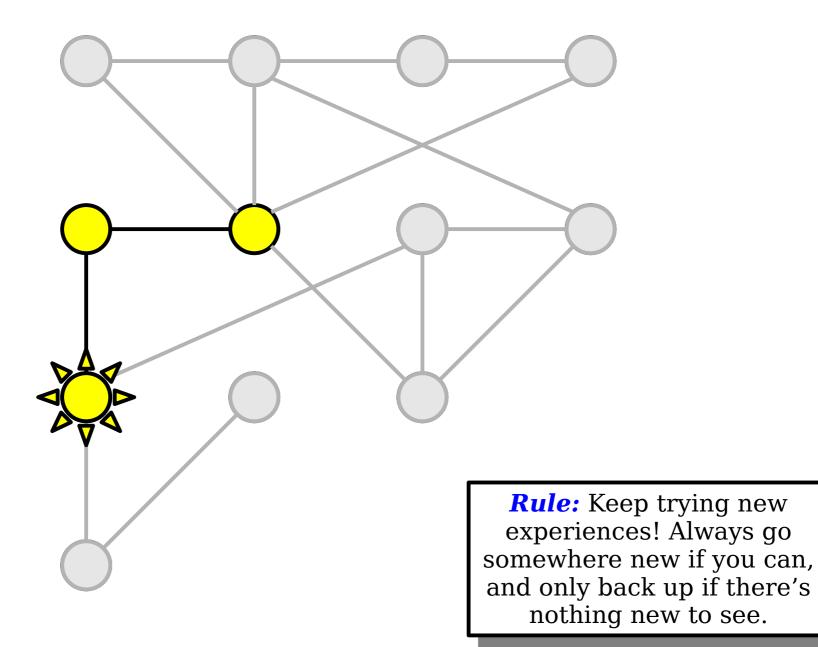


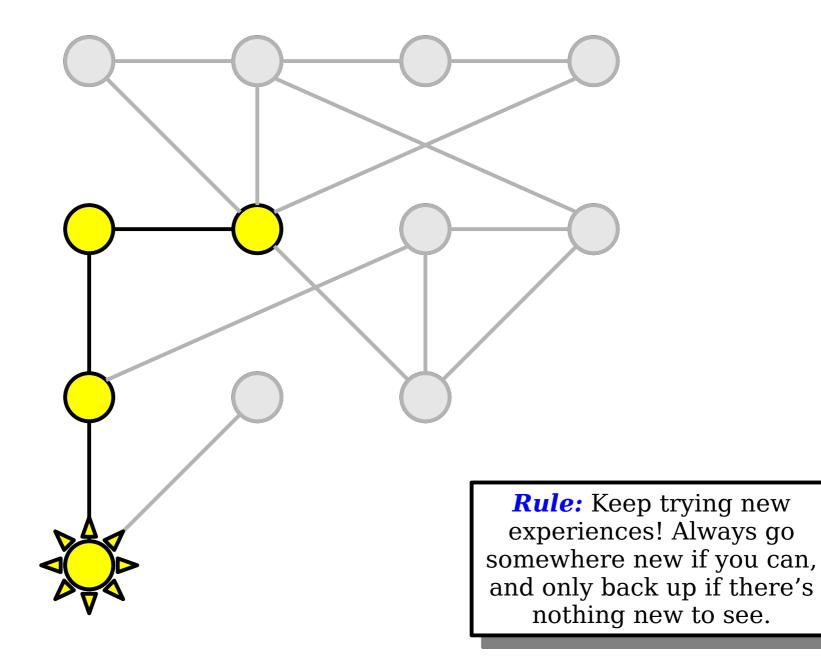


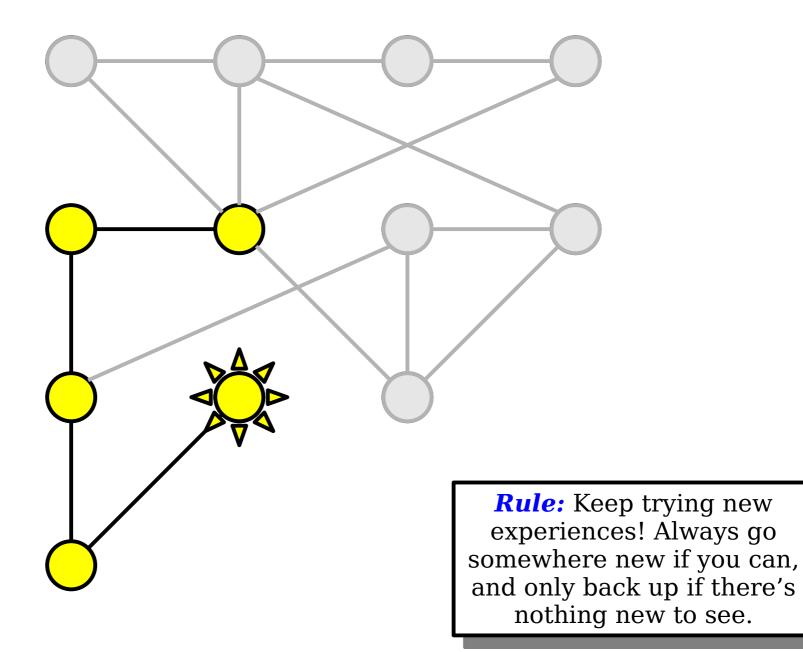
Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

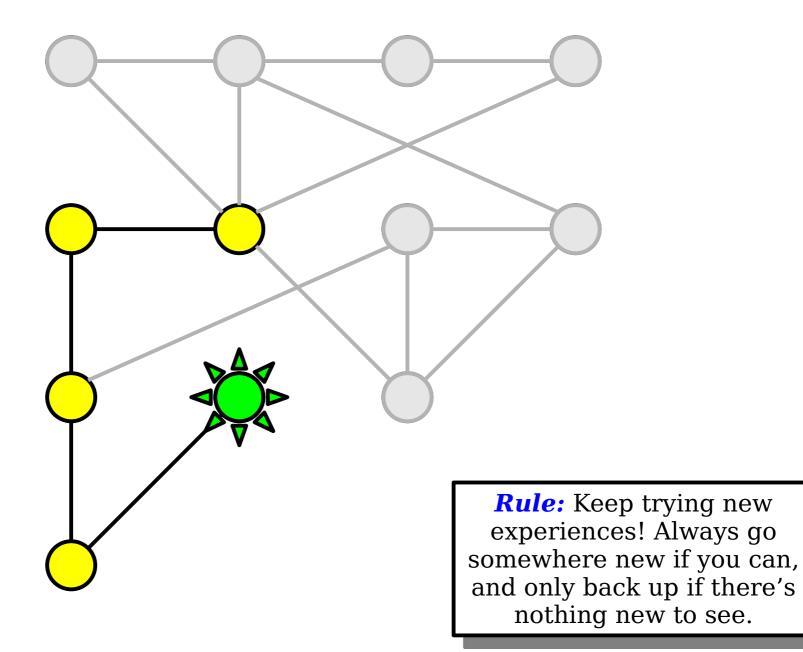


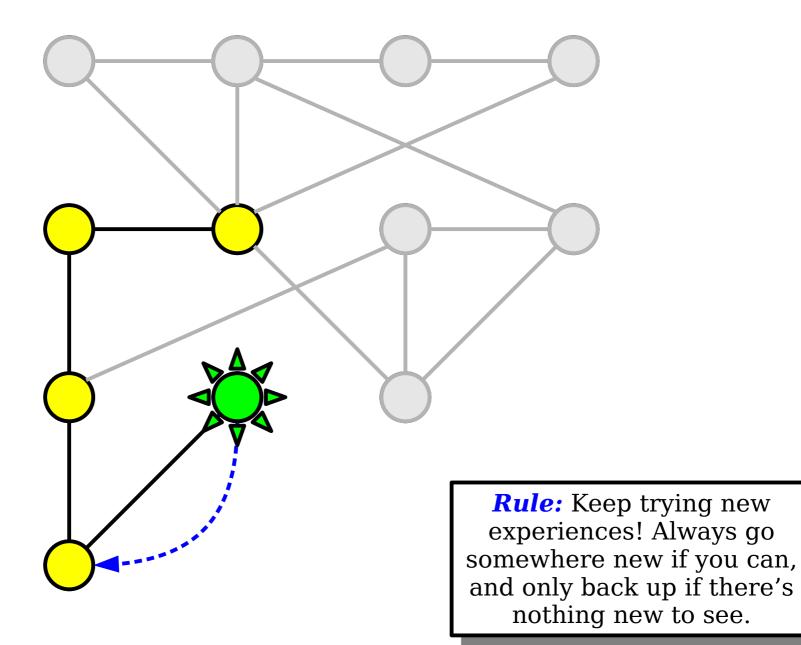


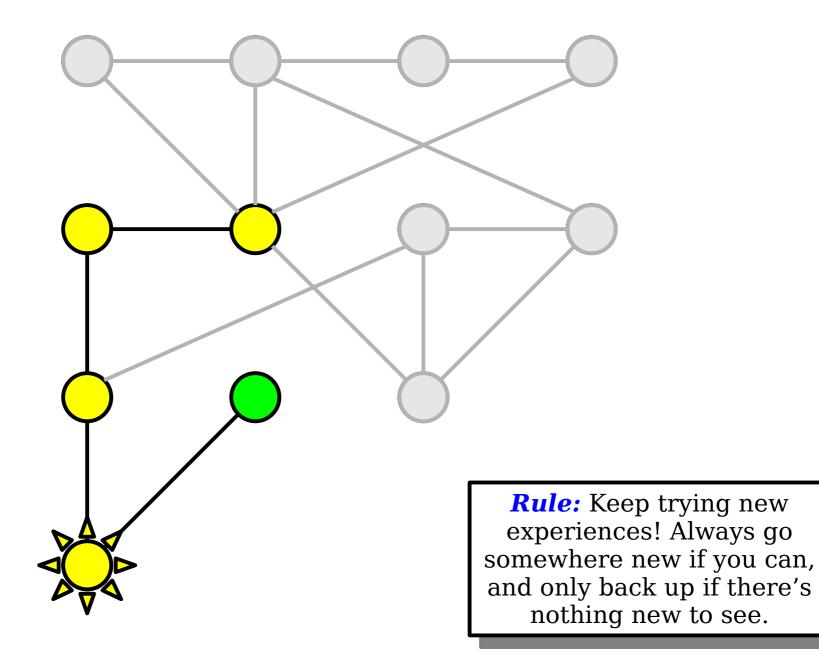


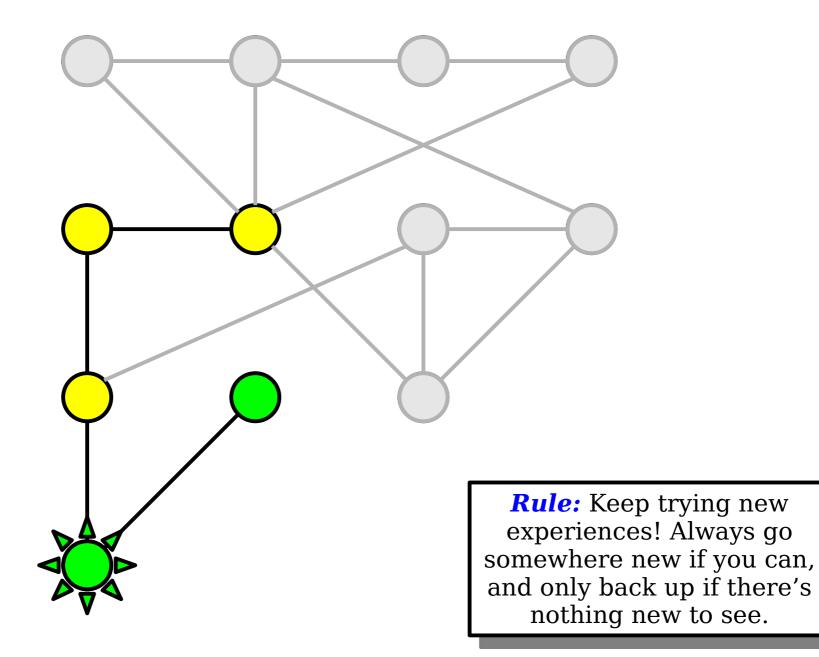


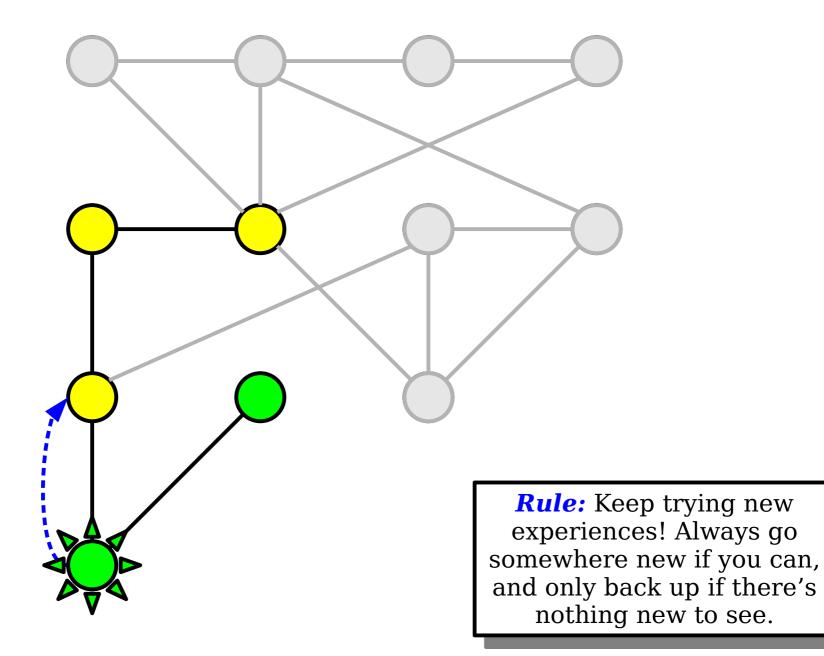


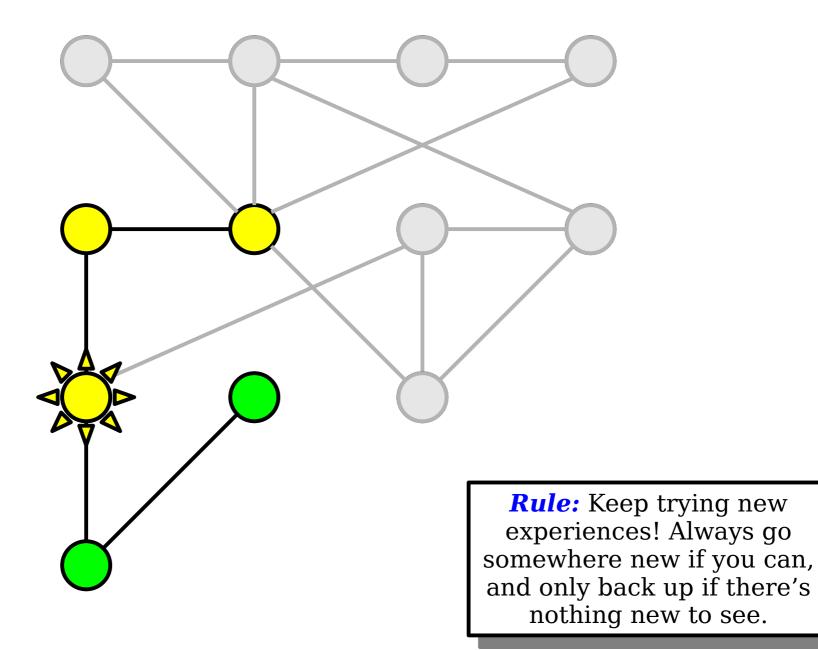


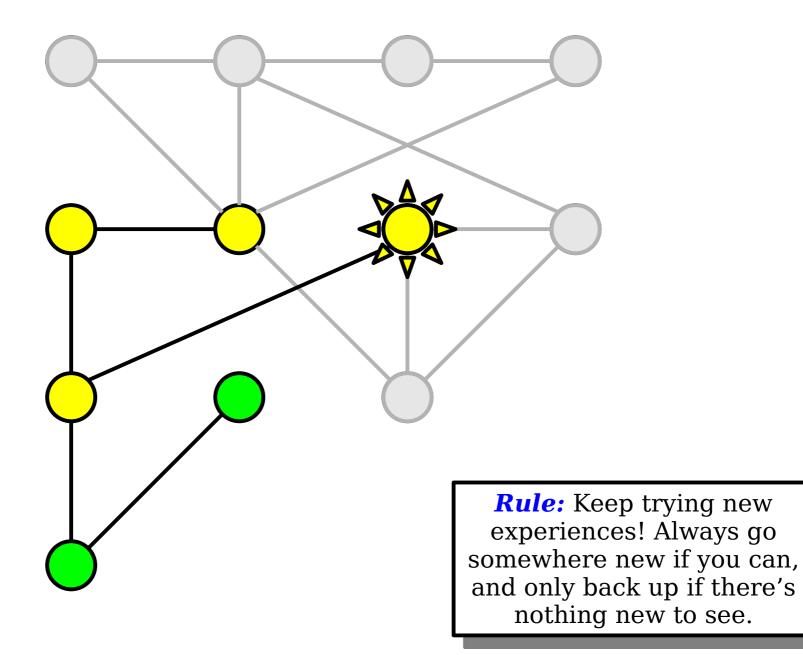


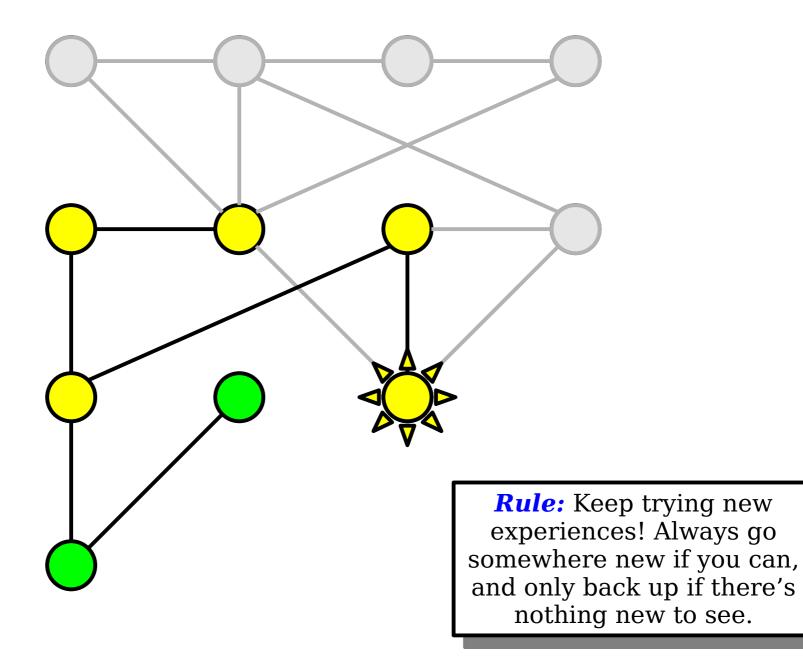


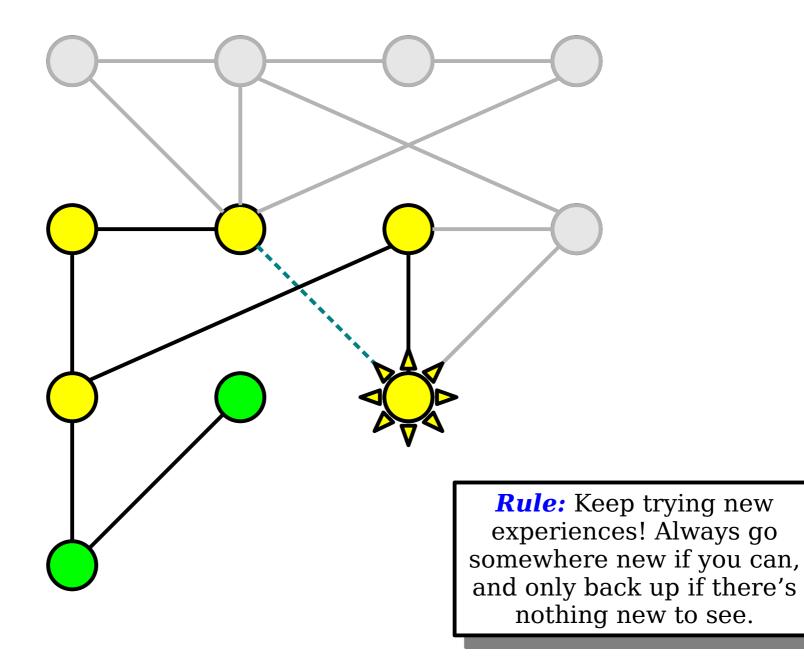


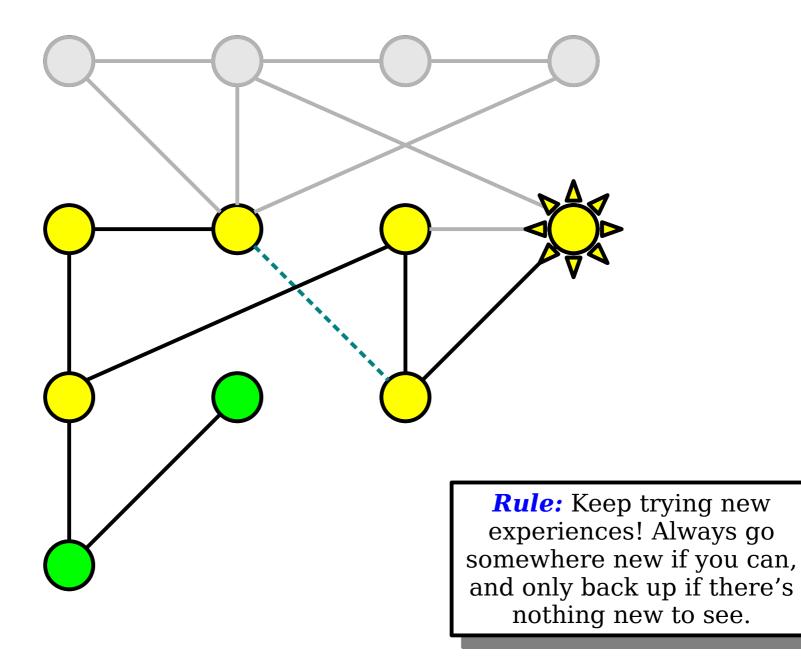


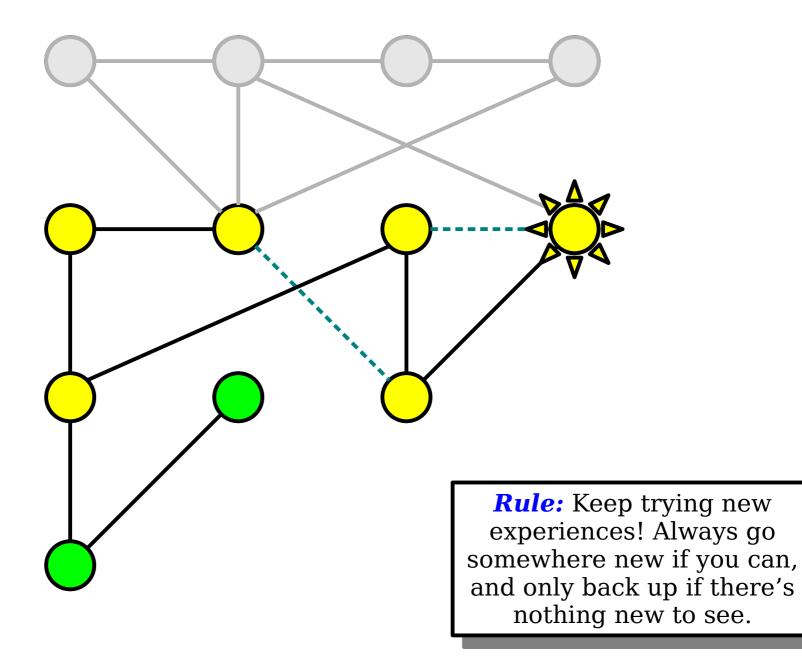


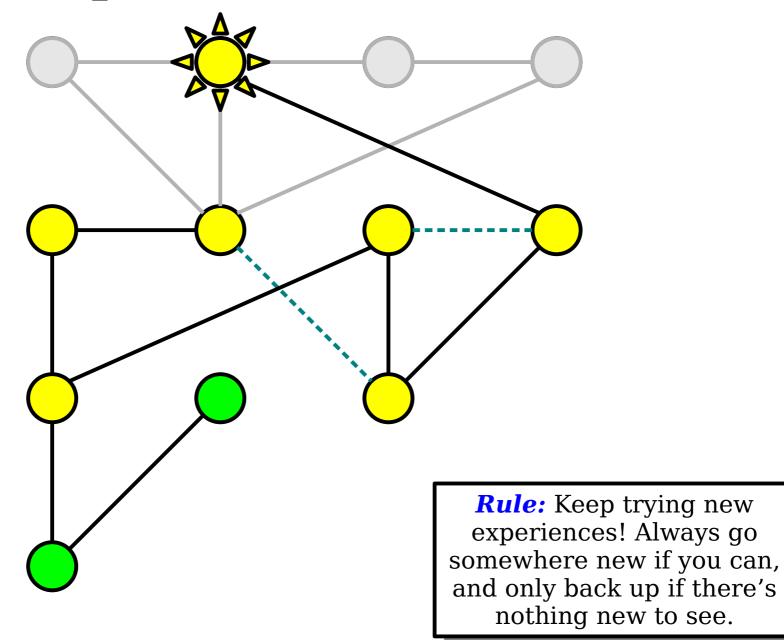


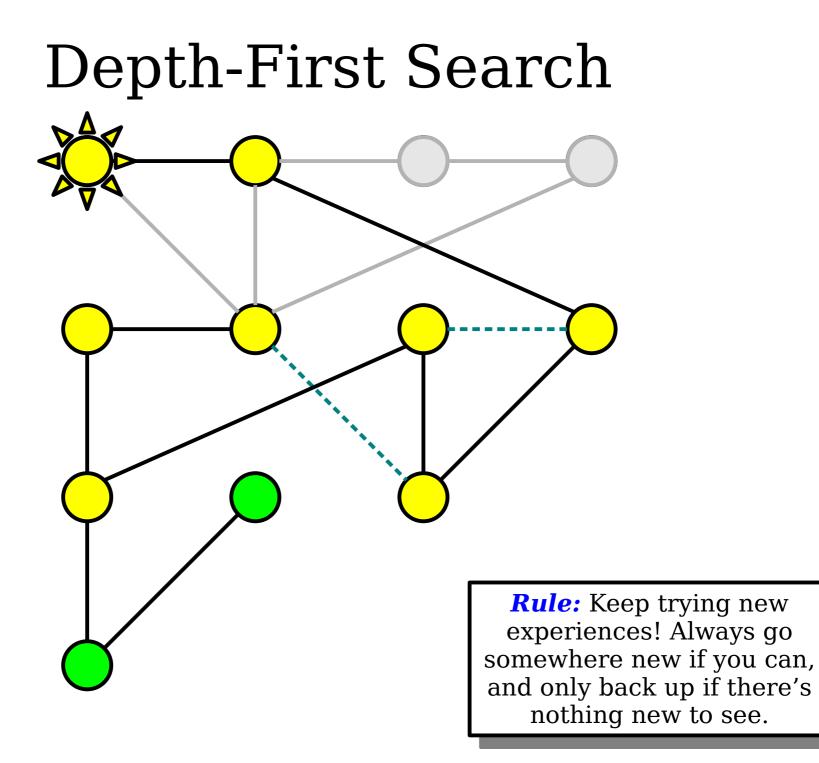


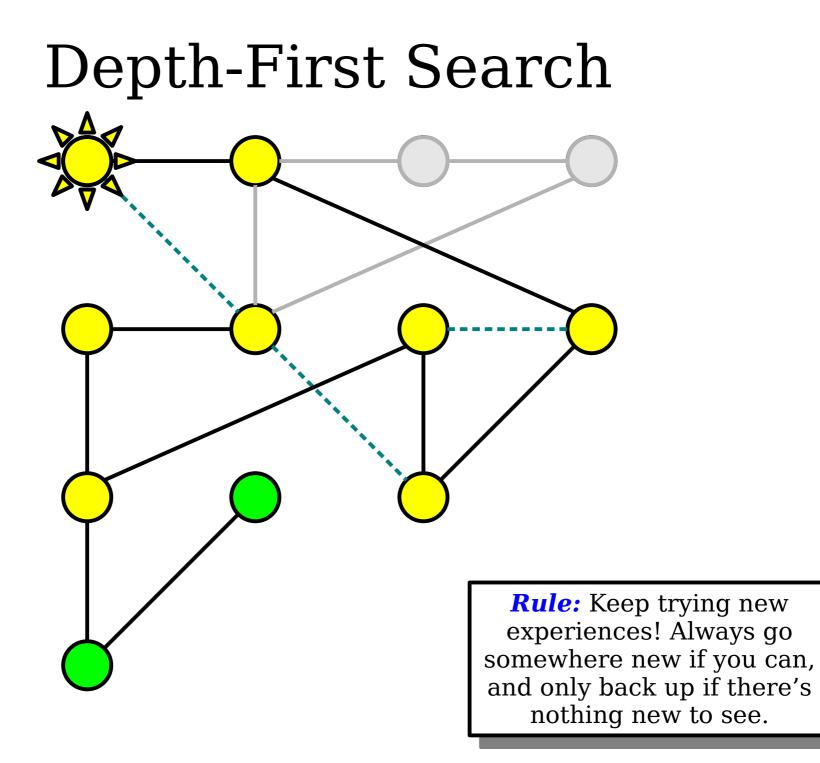


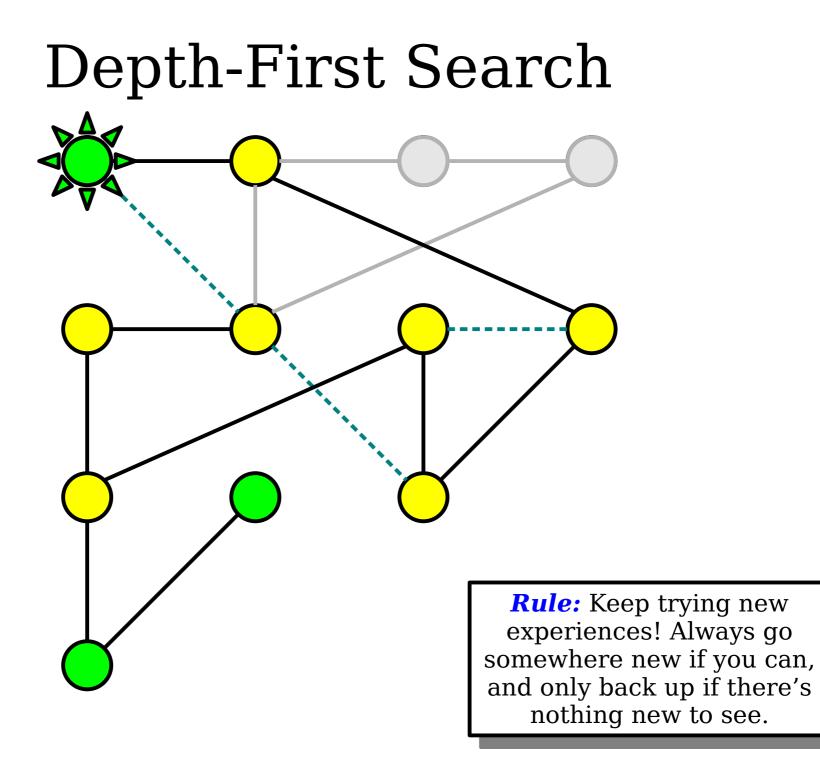


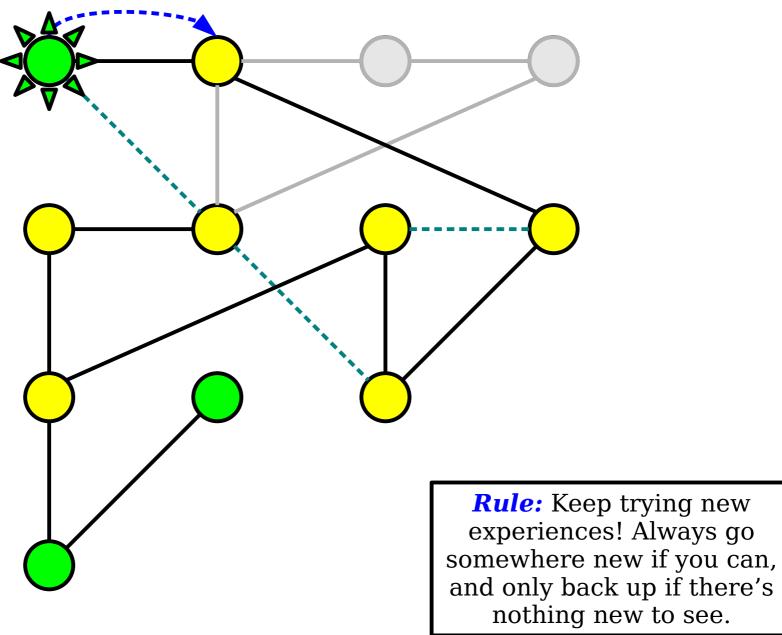


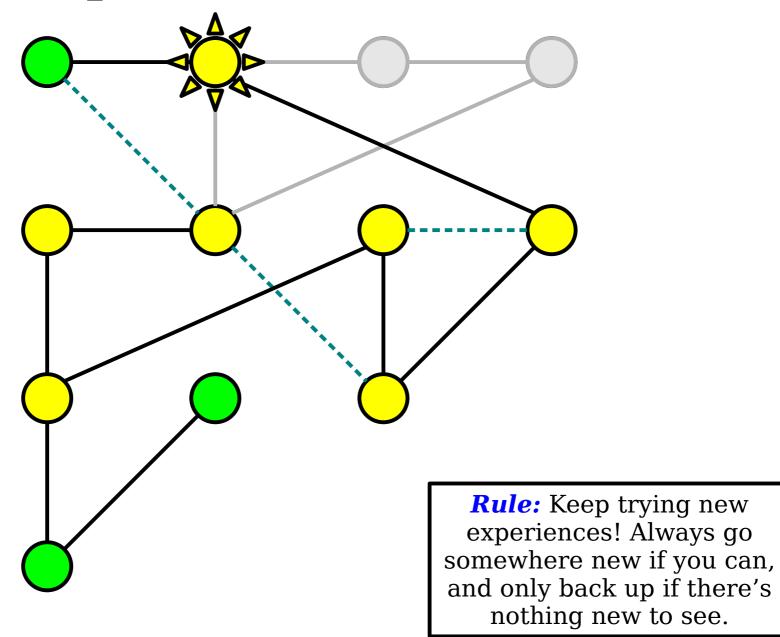


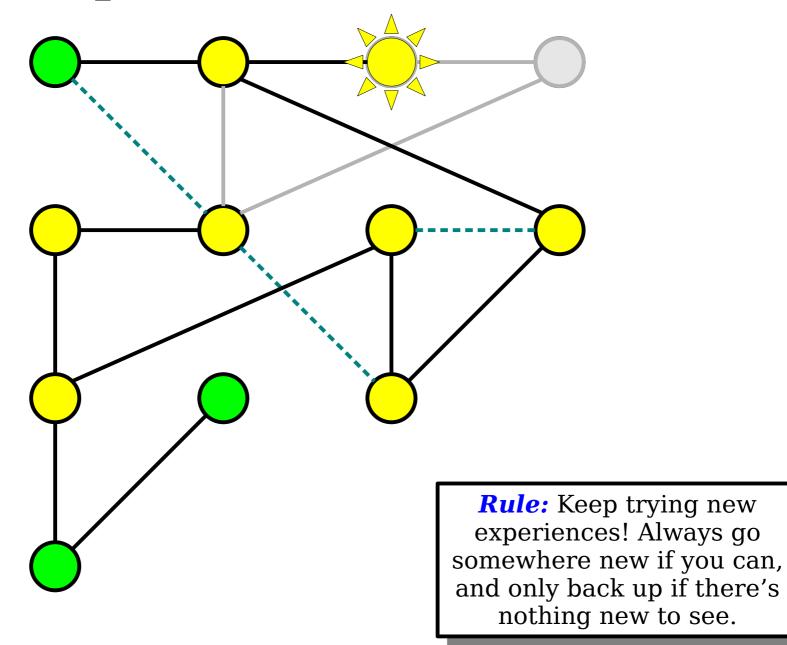


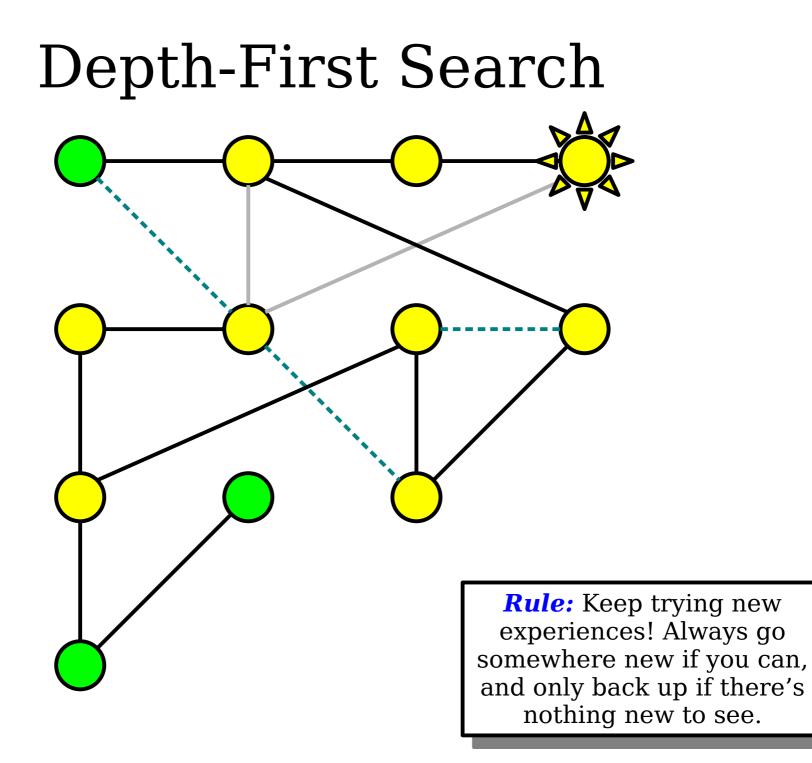


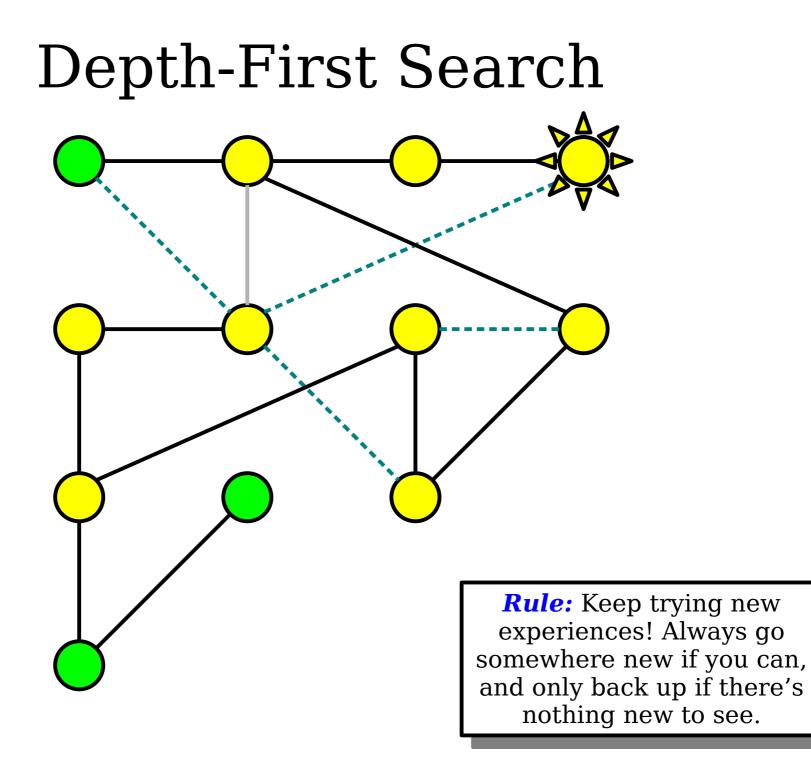


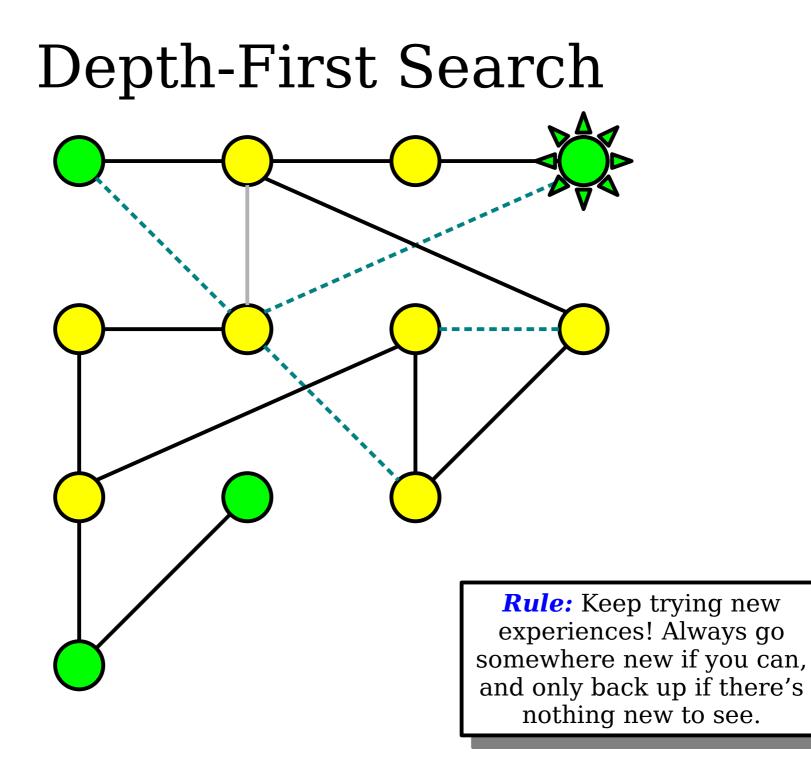


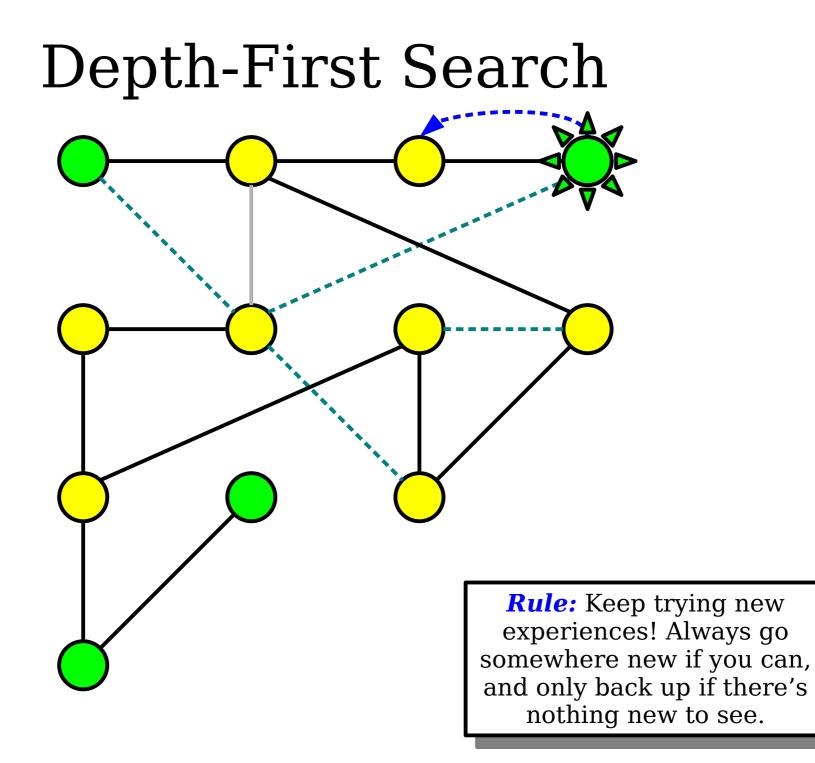


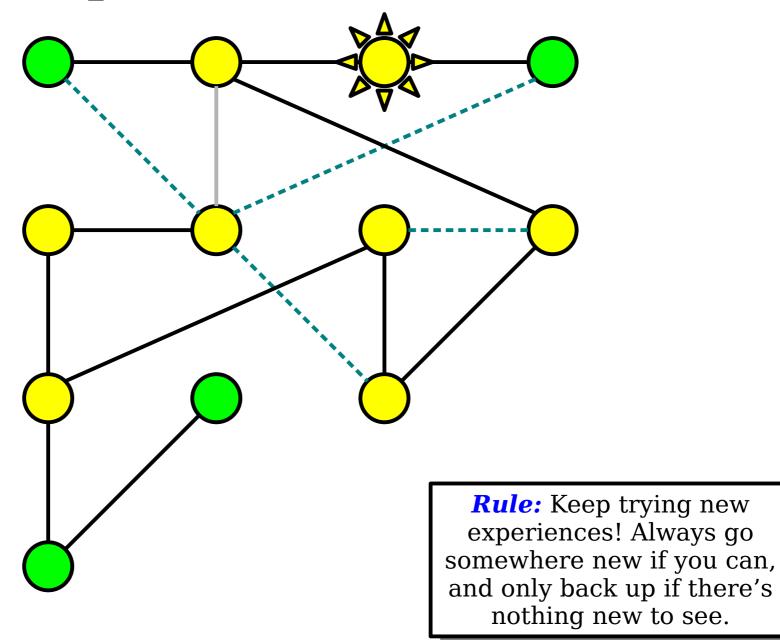


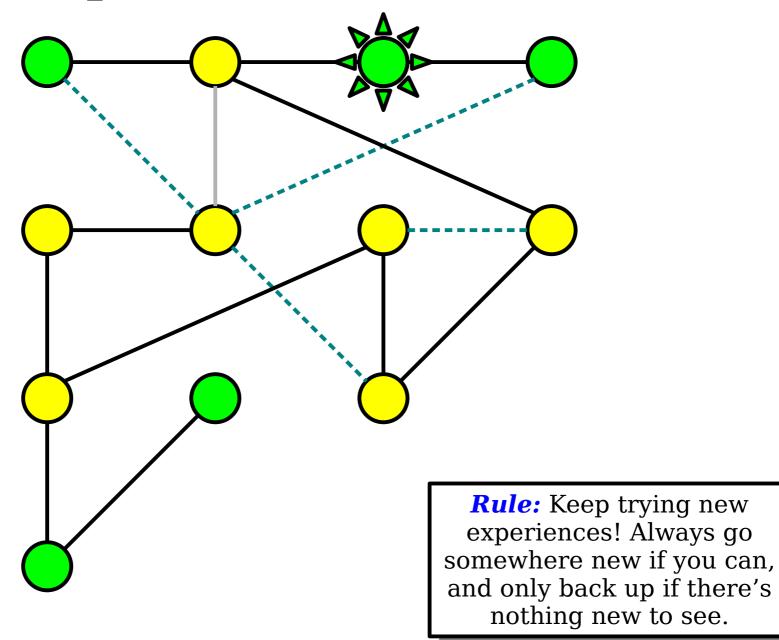


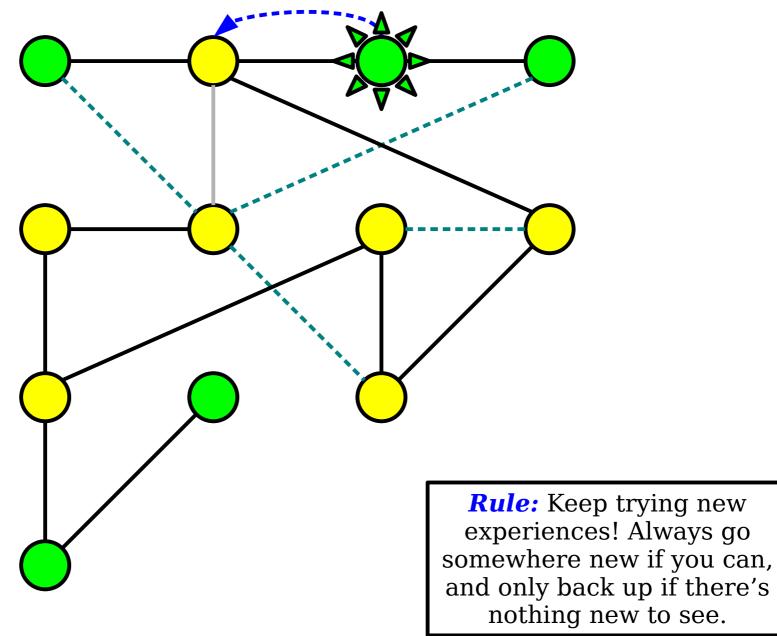


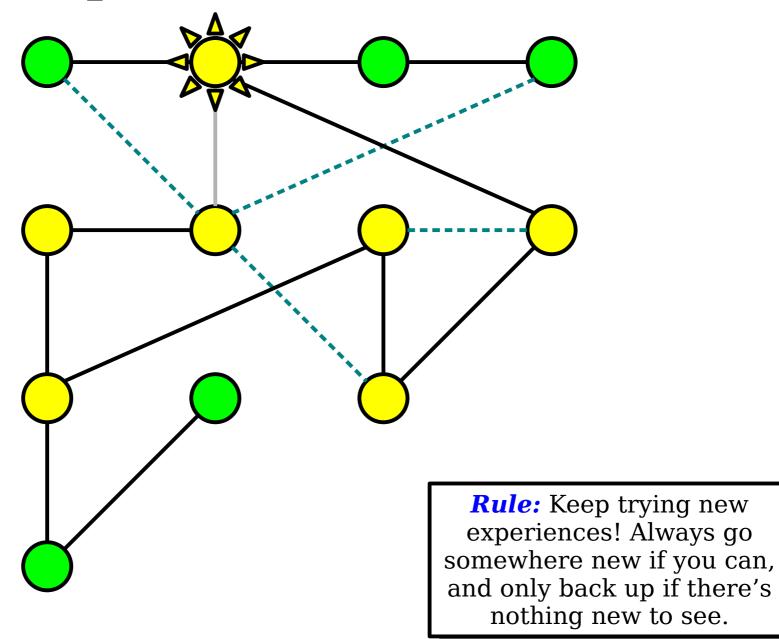


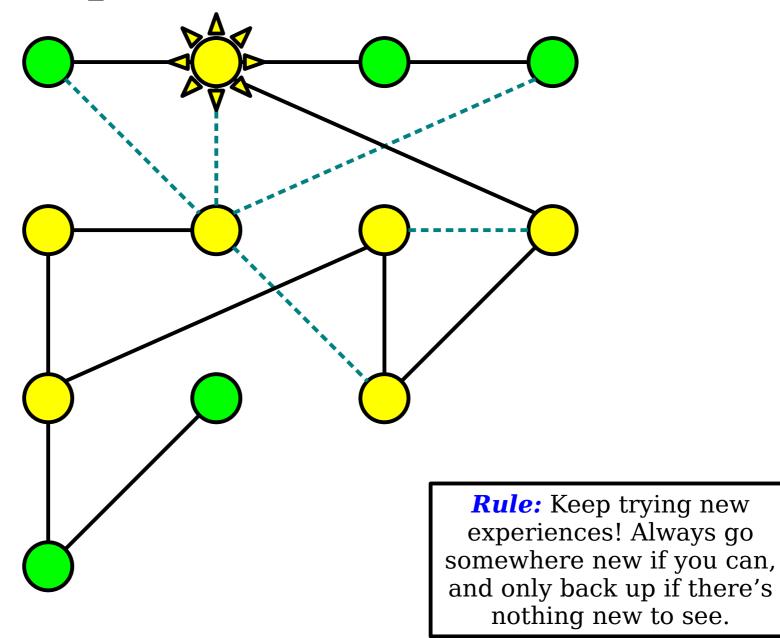




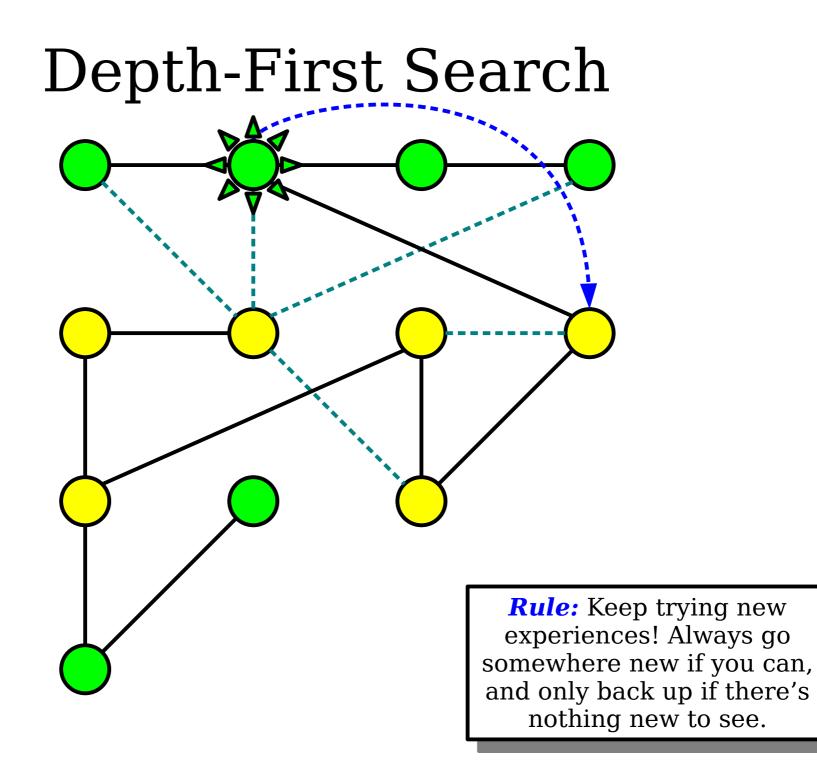


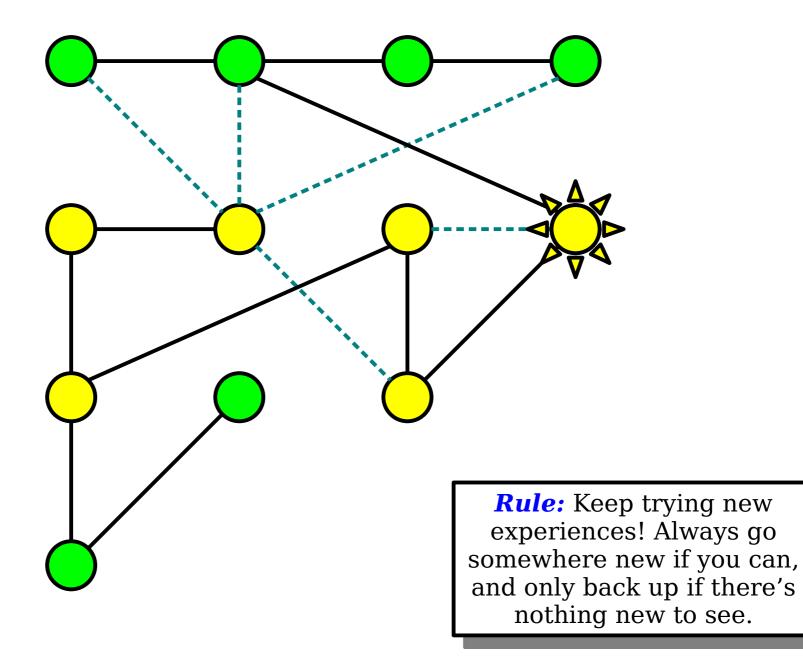


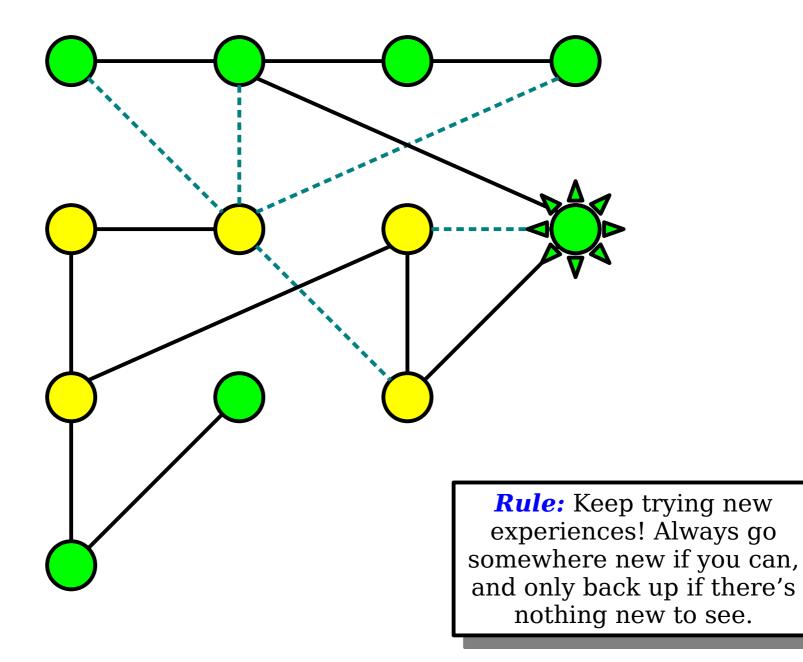


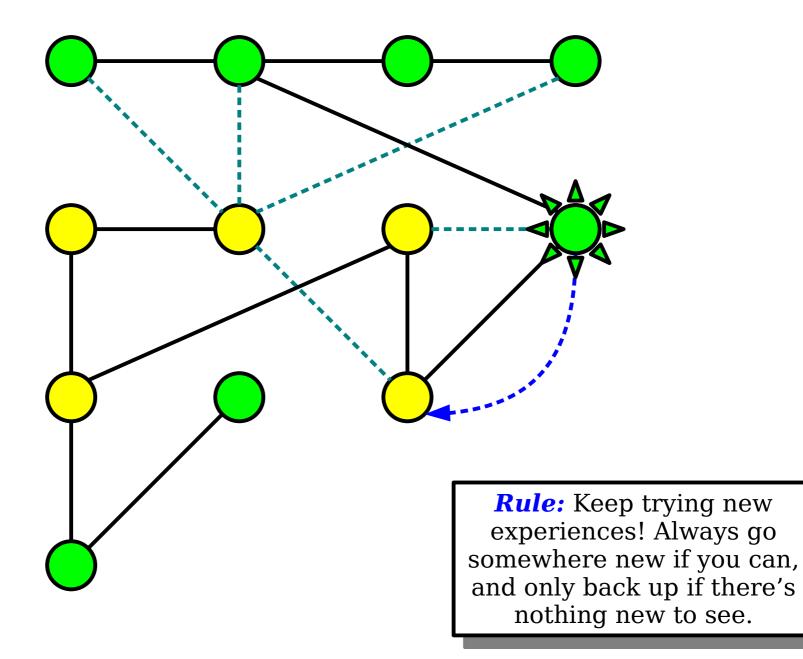


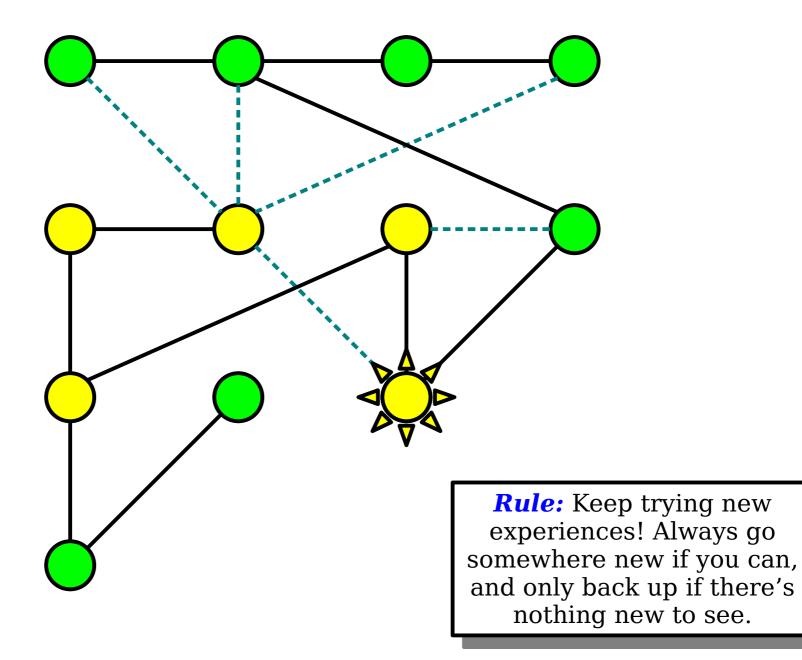
Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

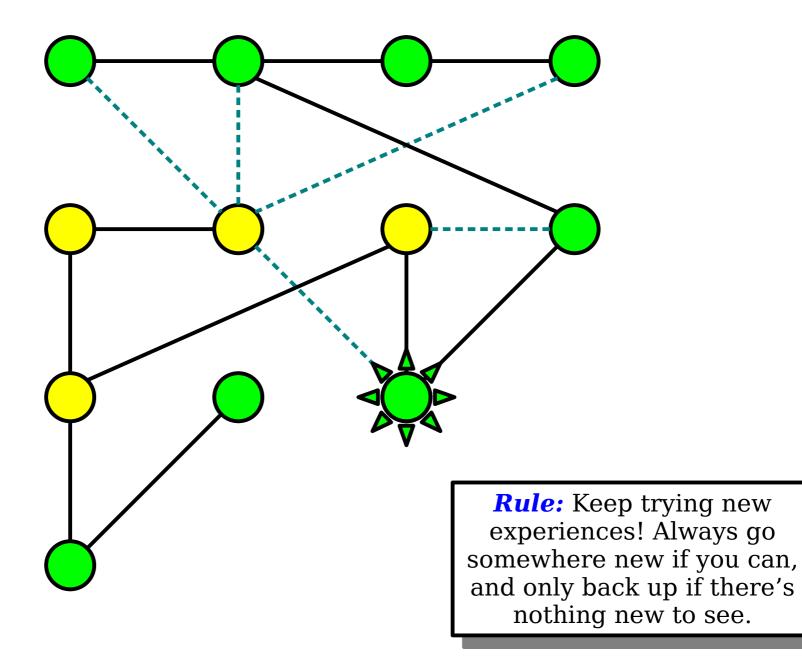


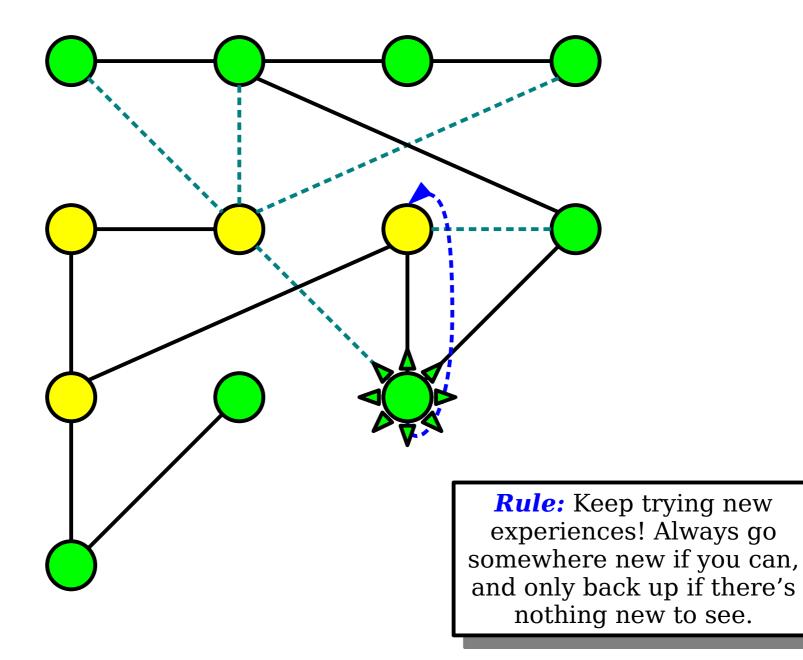


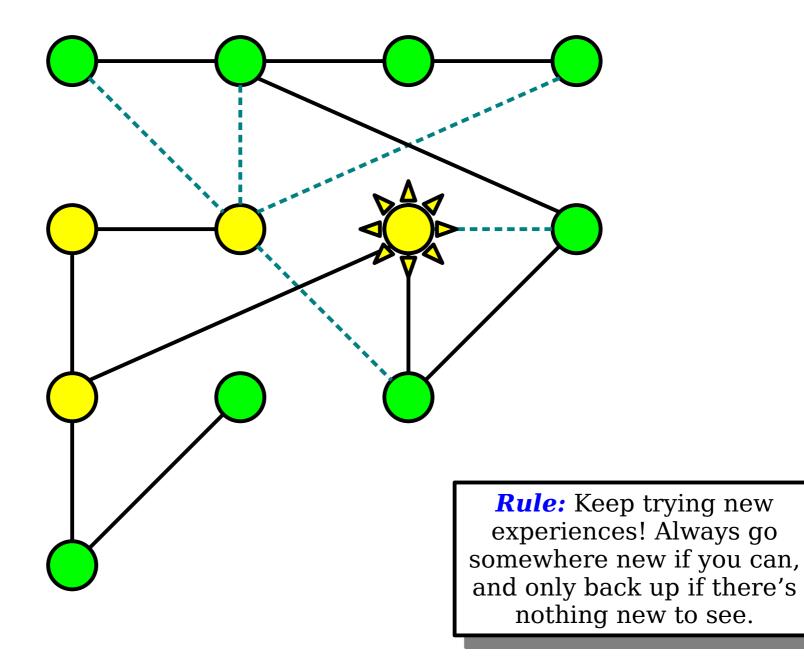


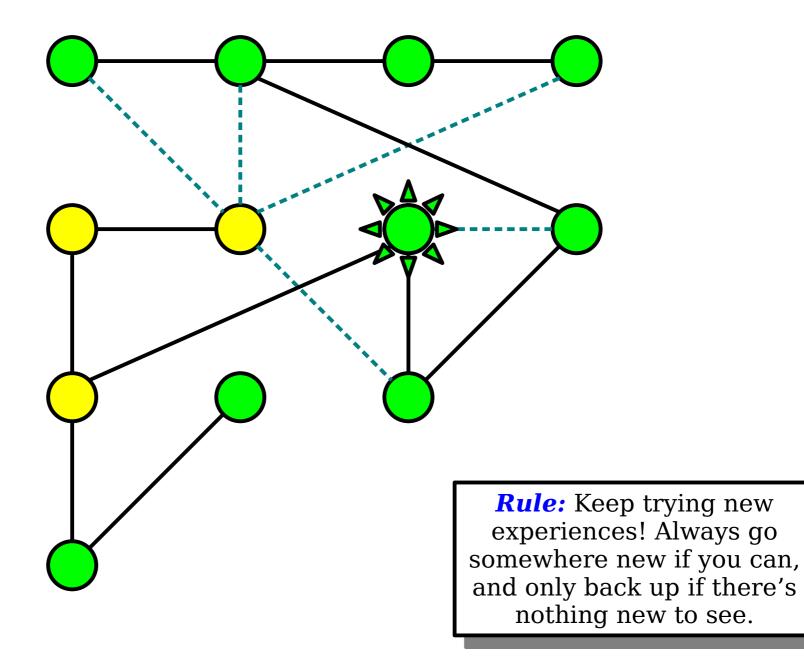


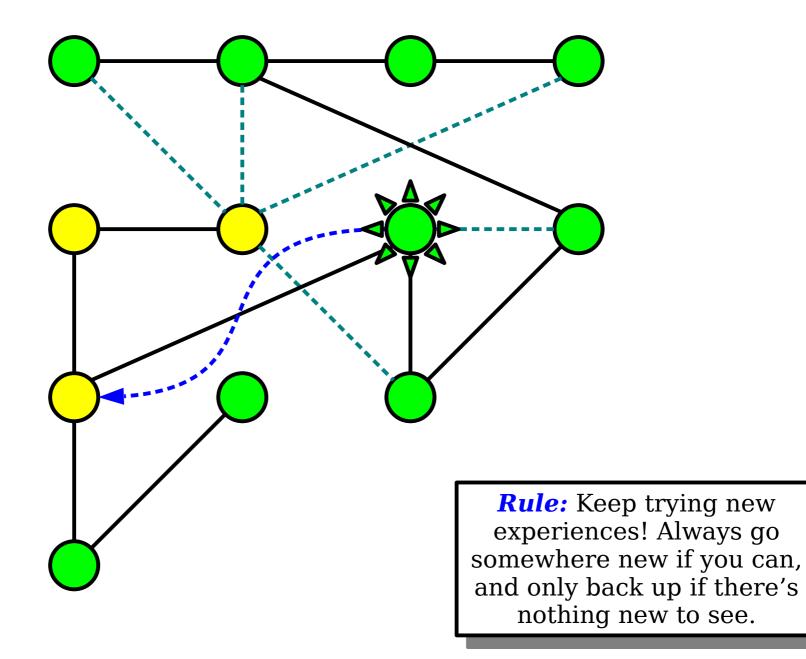


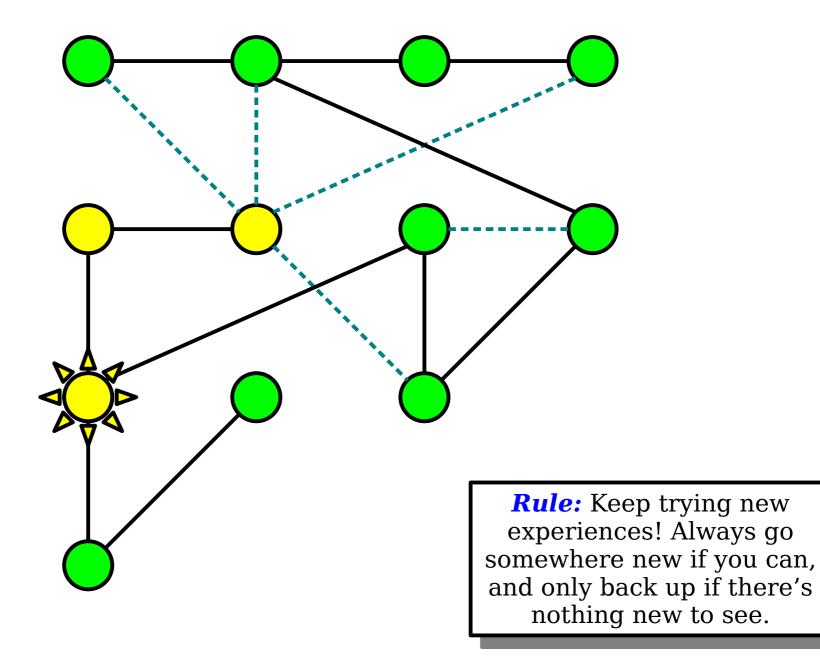


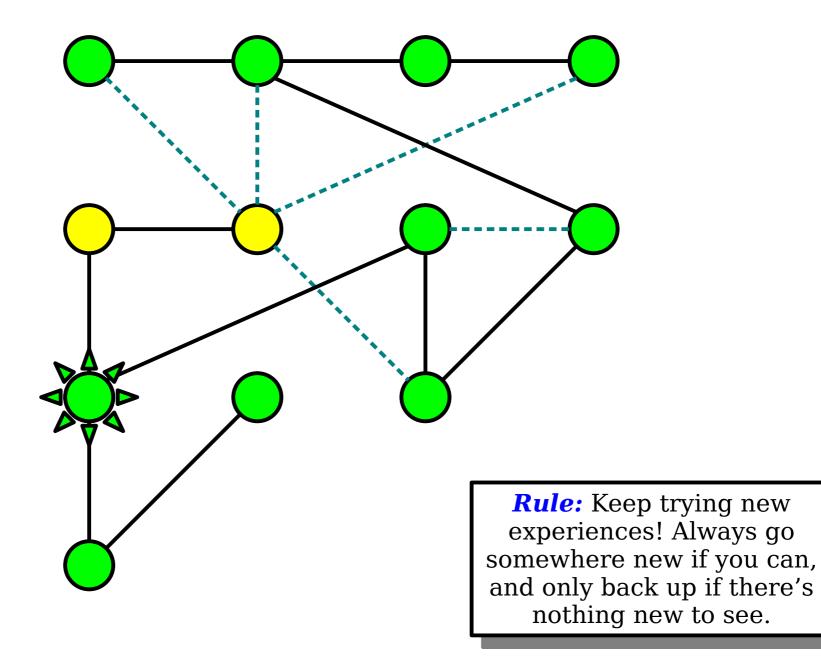


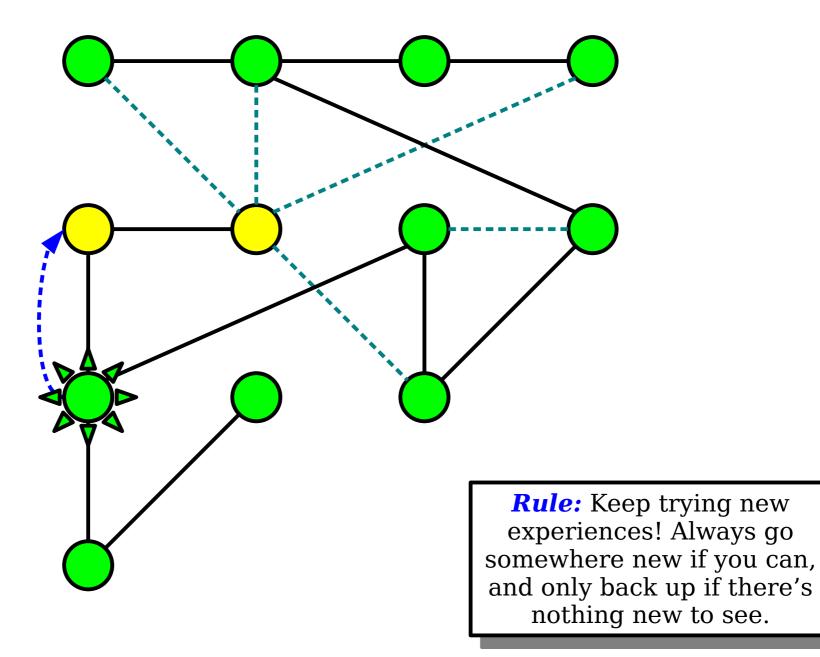


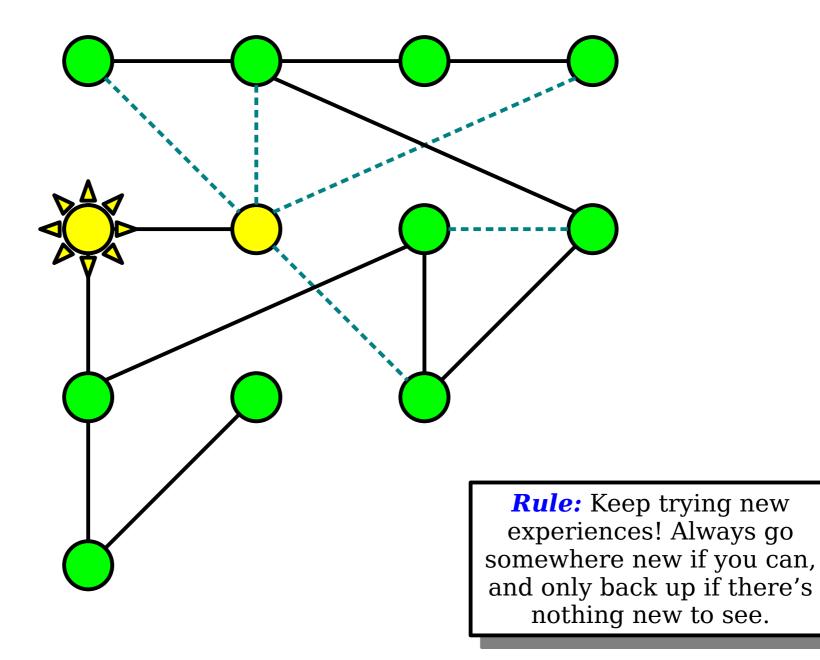


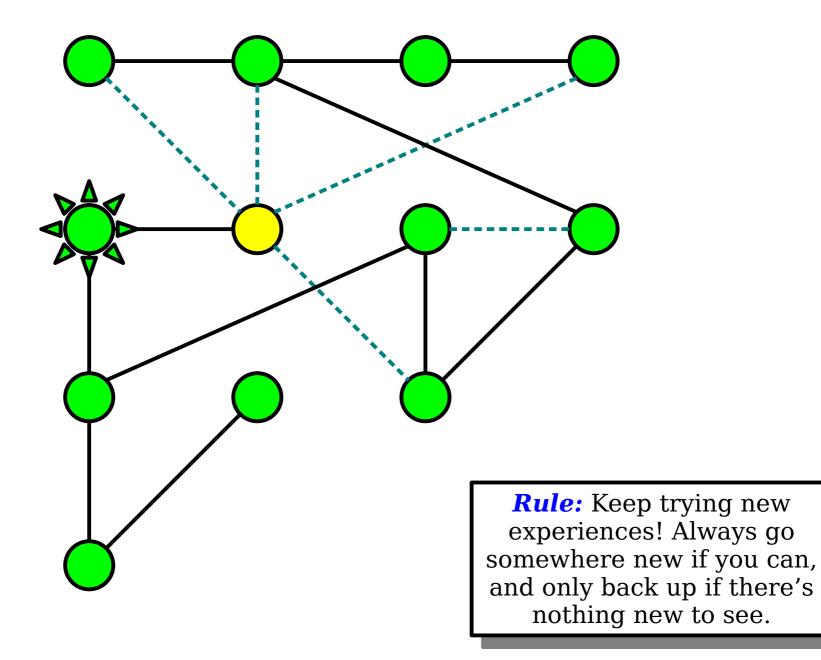


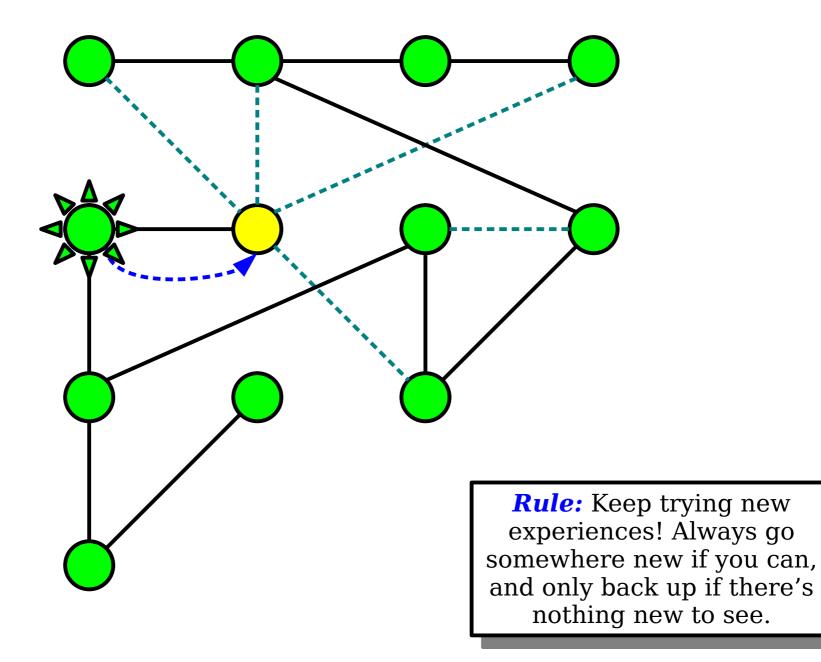


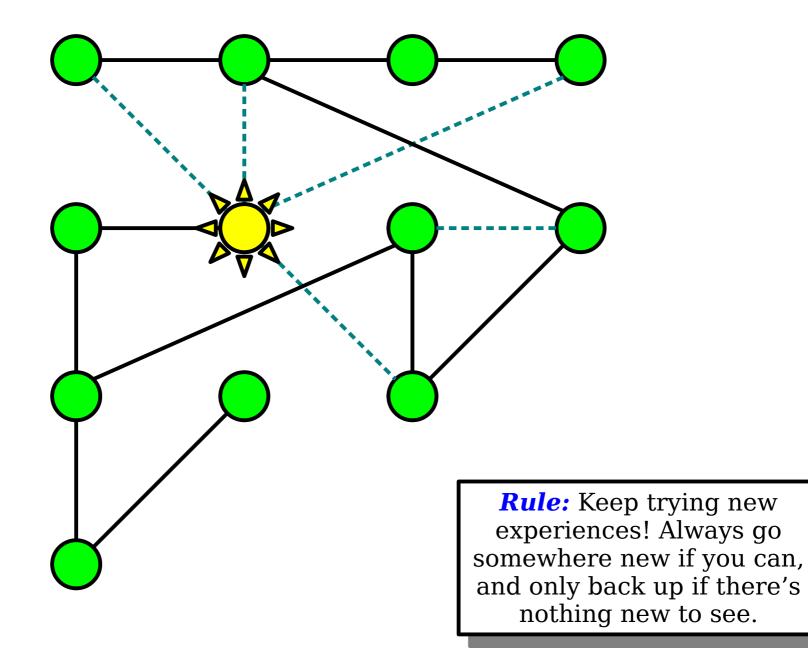


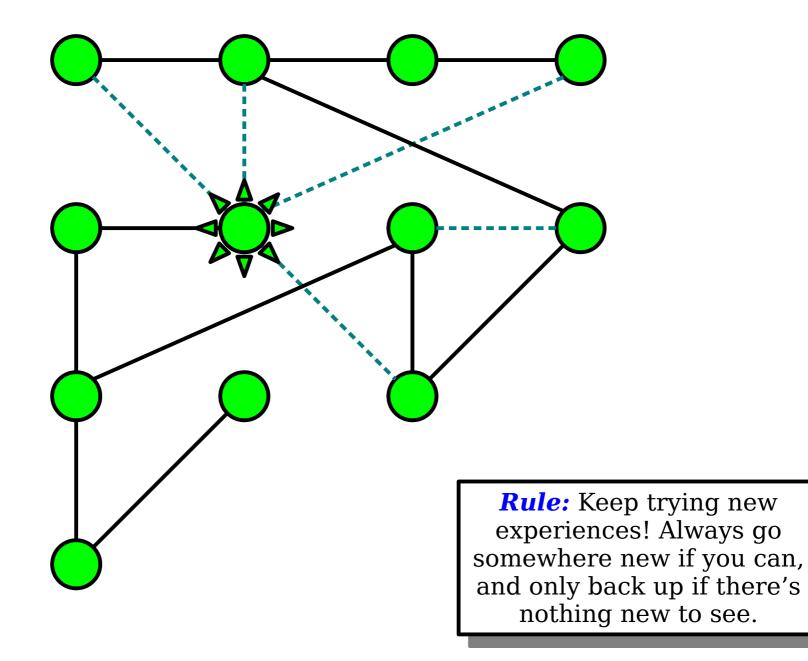


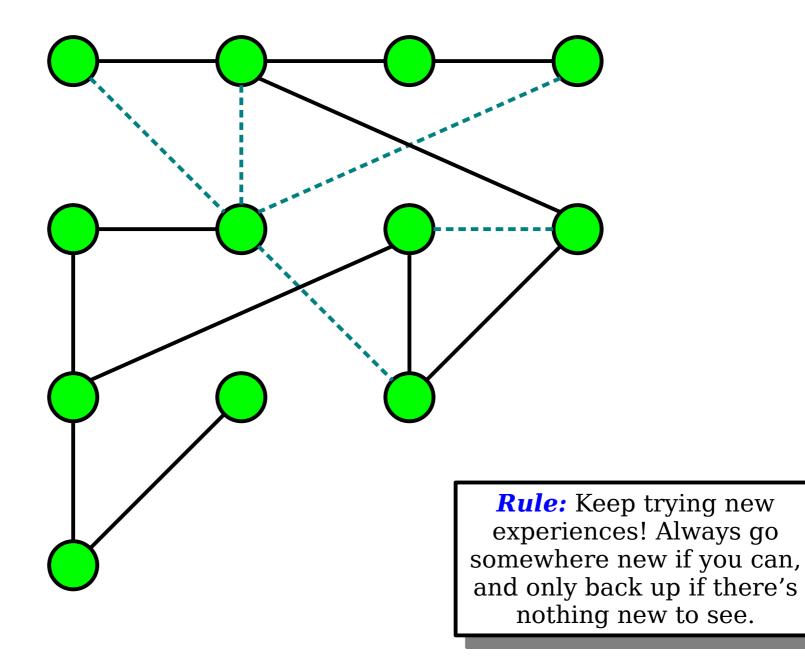




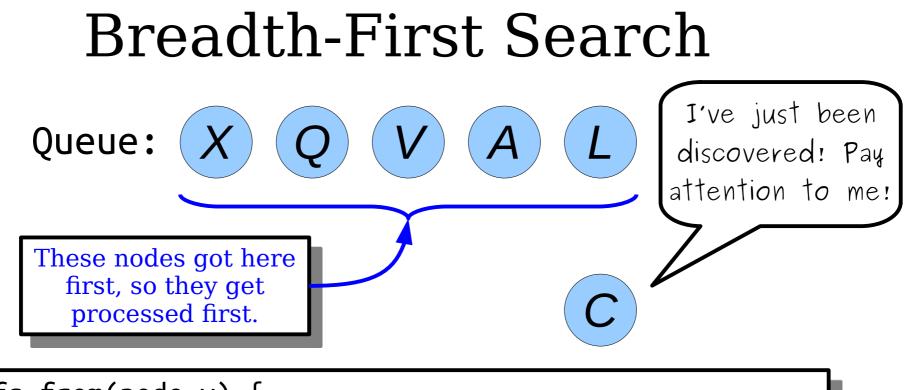




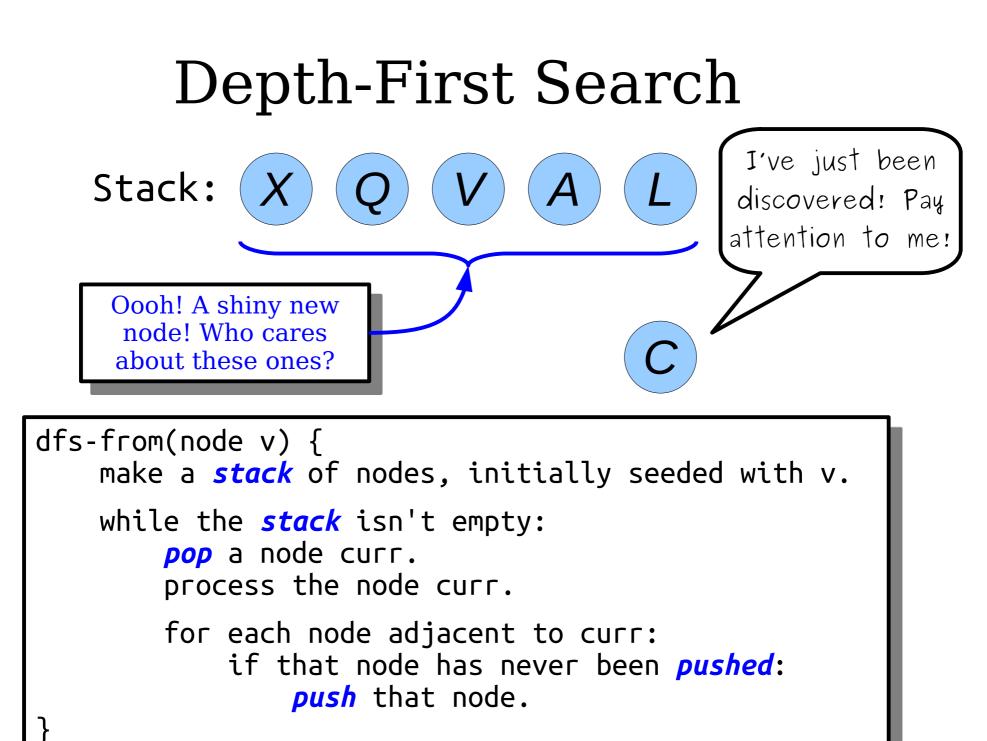




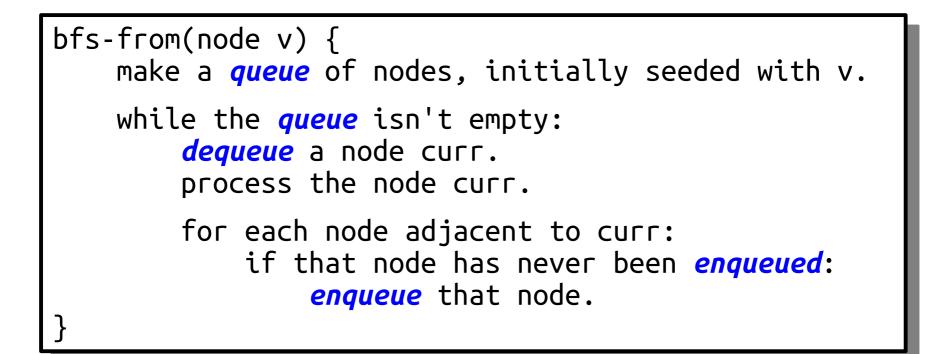
How can we implement this?



```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
        dequeue a node curr.
        process the node curr.
        for each node adjacent to curr:
            if that node has never been enqueued:
                enqueue that node.
}
```

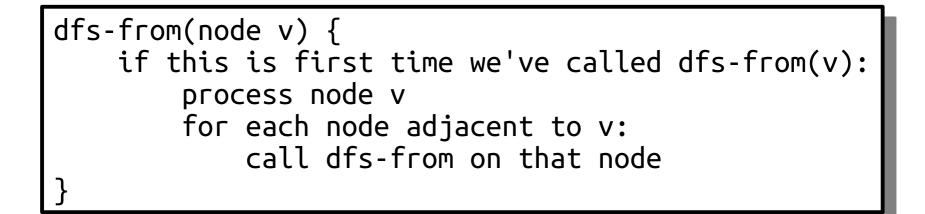


For Comparison



For Comparison

```
dfs-from(node v) {
   make a stack of nodes, initially seeded with v.
   while the stack isn't empty:
        pop a node curr.
        process the node curr.
        for each node adjacent to curr:
            if that node has never been pushed:
                push that node.
}
```



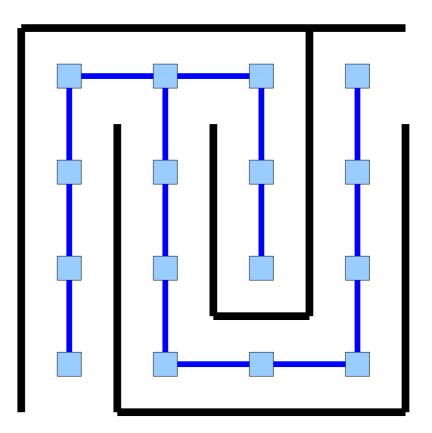
When you see a stack-based algorithm, think recursion!

BFS and DFS

- Running BFS or DFS from a node in a graph will visit the same set of nodes, but probably in a different order.
- BFS will visit nodes in increasing order of distance.
- DFS does visit nodes in *some* interesting order, but not order of distance.
 - Take CS161 for more details!

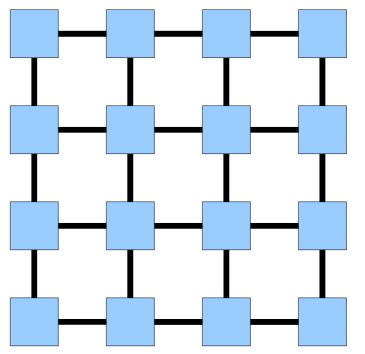
A Whimsical Application

Mazes as Graphs



Creating a Maze with DFS

• Create a *grid graph* of the appropriate size.



- Starting at any node, run a depth-first search, choosing neighbor orderings at random.
- The resulting DFS tree is a maze with one solution.

Next Time

- Minimum Spanning Trees
 - How to wire an electrical grid cheaply.
- Applications of MSTs
 - Data clustering, computational biology, and more!