Programming Abstractions

CS106B

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Topics:

- Priority Queue ADT
  - Heap data structure implementation
    - What are binary trees?
    - What are heaps?
    - How do we do enqueue/dequeue operations on heaps?

Priority Queue

Emergency Department waiting room operates as a priority queue: patients are sorted according to priority (urgency), not “first come, first serve” (in computer science, “first in, first out” or FIFO).

Contents of one element of a Priority Queue

- Individual elements of our priority queue will have two pieces to them:
  - An integer indicating the **priority** of this element
    - We will use smaller number means higher priority, but could be done either way
  - A "**payload**" of whatever the actual element data is
    - Examples:
      - **a class MedicalRecord** that has many fields and is the patient's entire medical history
      - **a string** that is the name of a student waiting in the Lair queue (in a world where Lair is based on urgency of request, rather than FIFO)
      - etc.

6	"SooMin"	13	"Diego"	15	"Muhammad"	22	"Sasha"

Two priority queue implementation options

0		1		2		3		4	
22	"Sasha"	6	"SooMin"	15	"Muhammad"	13	"Diego"		

Unsorted array

- Always enqueue new element *at the end of the array*
- Dequeue by searching entire array for highest-priority item, then removing it, and (if needed) scooting elements over to fill in the gap

0		1		2		3		4	
22	"Sasha"	15	"Muhammad"	13	"Diego"	6	"SooMin"		

Sorted array

- Always enqueue new elements *where they go* in priority-sorted order, with the highest-priority item at the end of the array
- Dequeue by taking the last element of the array

Priority queue implementations

Unsorted array

Enqueue is FAST

- Just throw it in the array at the back
- **O(1)**

Dequeue/peek is SLOW

- Hard to find item the highest priority item—could be anywhere
- Might need to scoot over elements to fill gap
- **O(N)**

0		1		2		3		4	
22	"Sasha"	6	"SooMin"	15	"Muhammad"	13	"Diego"		

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Priority queue implementations

Sorted array

Enqueue is SLOW

- Need to step through the array to find where item goes in priority-sorted order
- If proper place is in the front/middle, need to scoot over other elements to make room
- **O(N)**

Dequeue/peek is FAST

- Easy to find item you are looking for (last in array)
- No need to scoot over elements when removing last
- **O(1)**

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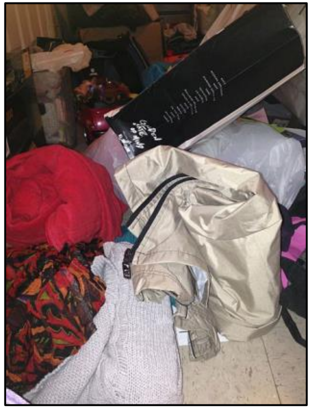
0		1		2		3		4	
22	"Sasha"	15	"Muhammad"	13	"Diego"	6	"SooMin"		

Would be great if we could get the best of both…

Fast enqueue *and* fast dequeue/peek



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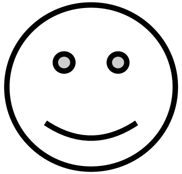
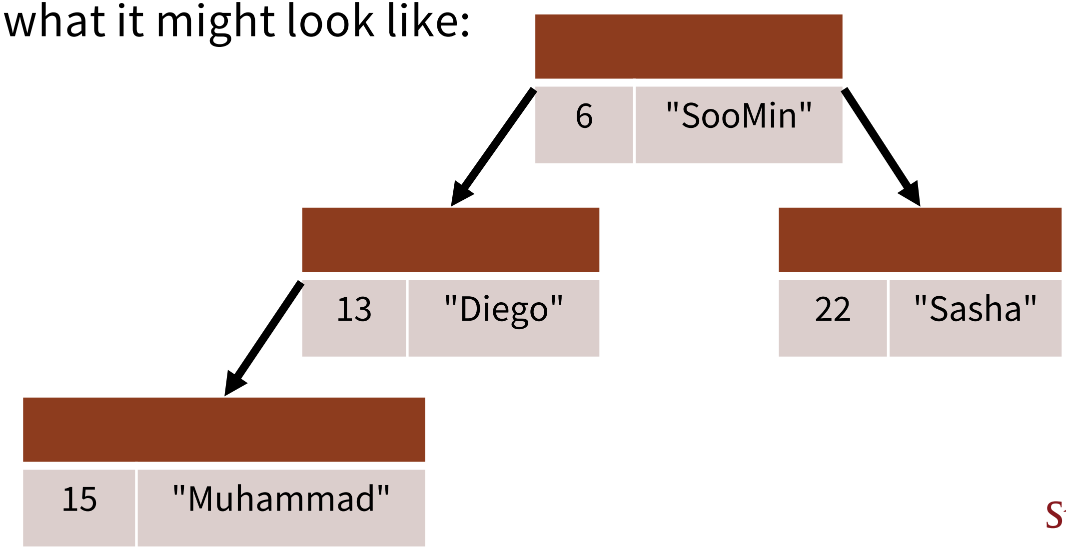


Fast enqueue Fast dequeue/peek

Binary heap for our priority queue

- Instead of storing our priority queue nodes entirely sorted or entirely unsorted, we will store them *partially-sorted*.
- The partial sorting will still be stored in an array, but it's best to imagine it as what we call a "tree" in computer science (computer science trees are upside-down for some reason ¯_(ツ)_/¯)
- Here's what it might look like:



Binary trees

Before we delve into how to construct a binary heap, let's take a step back and introduce computer science binary trees generally

A binary tree

“In computer science, a **binary tree** is a tree data structure in which each node has at most two child nodes, usually distinguished as "left" and "right."”

(Thanks, Wikipedia!)

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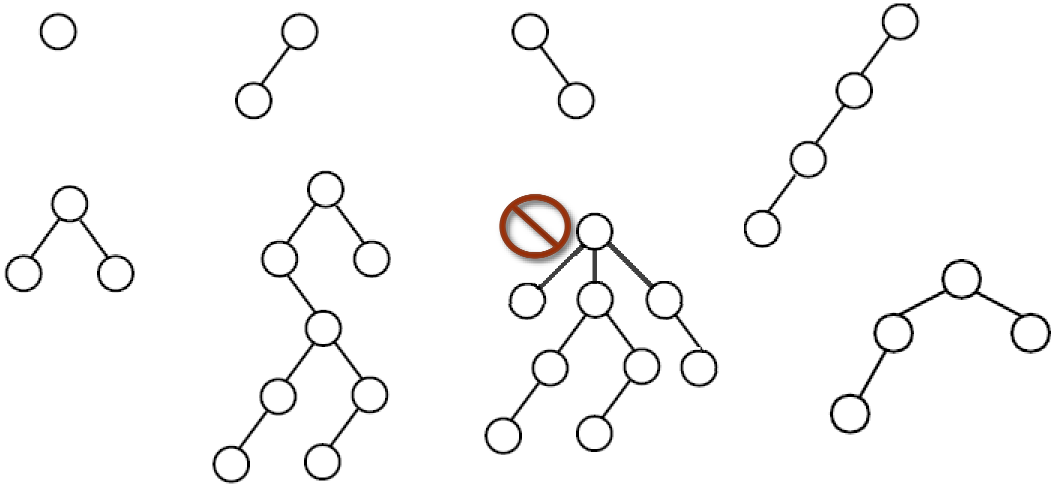
How many of these are valid binary trees?



Heaps!

Binary Heaps*

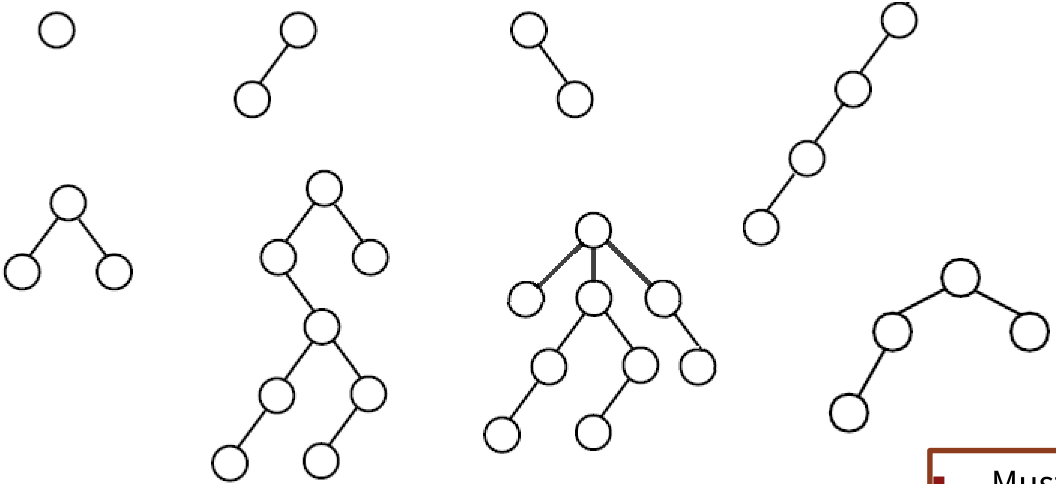
Binary heaps are **one kind** of binary tree

They have a few special restrictions, in addition to the usual binary tree:

- Must be **complete**
  - No "gaps"—nodes are filled in left-to-right on each level (row) of the tree
- Ordering of data must obey **heap property**
  - Min-heap version: a parent's priority is always ≤ both its children's priority
  - Max-heap version: a parent's priority is always ≥ both its children's priority

** There are other kinds of heaps as well. For example, binomial heap is an extra-fun one!*

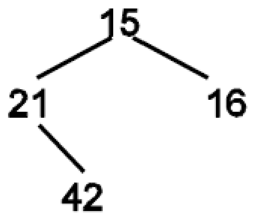
How many of these could be valid binary heaps?

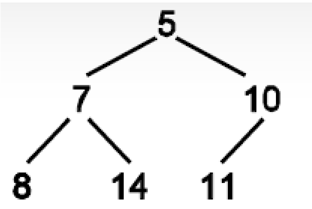


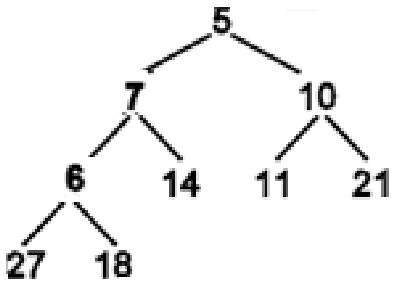
A. 0-1
B. 2
C. 3
D. 4
E. 5-8

- Must be a valid **binary tree**
- Must be **complete**
- Ordering of data must obey **heap property**

How many of these are valid min-binary-heaps?





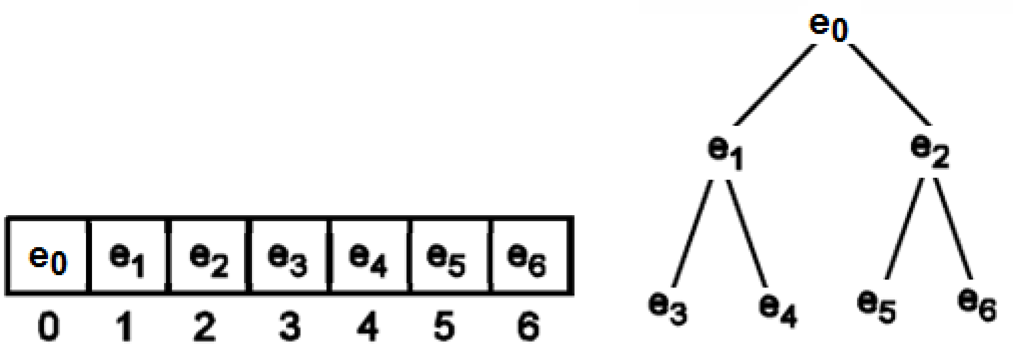


- Must be a valid **binary tree**
- Must be **complete**
- Ordering of data must obey **heap property**

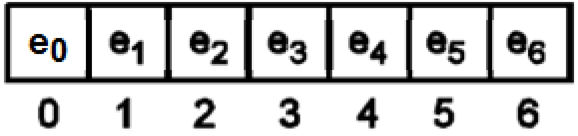
Binary heap in an array

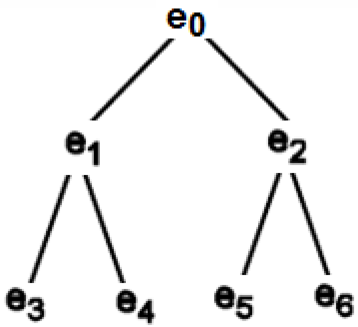
Binary heap in an array

- Because of the special constraint that they must be **complete**, binary heaps fit nicely into an **array**
  - *As we'll see in later lectures, this is <u>not</u> true of some other kinds of tree data structures, and we'll use a different approach for those*



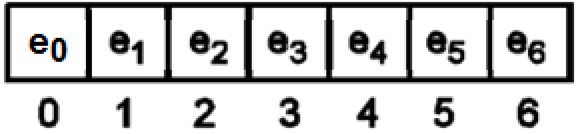
Heap in an array

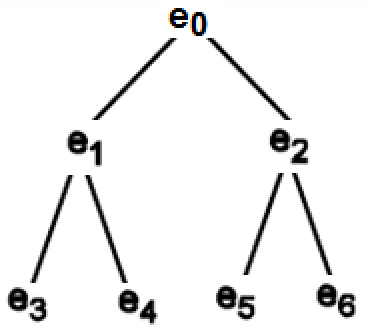




- Q: The parent of the node found in array index **`i`** is found where?
  - A. In array index **`i - 2`**
  - B. In array index **`i / 2`**
  - C. In array index **`(i - 1)/2`**
  - D. In array index **`2i`**
  - E. Somewhere else
  - › For now, assume that the node in array index **`i`** has a parent.
  - › In your code, of course you'll want to be careful not to go up past the top of the tree.

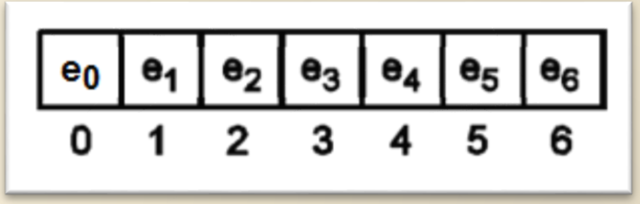
Heap in an array

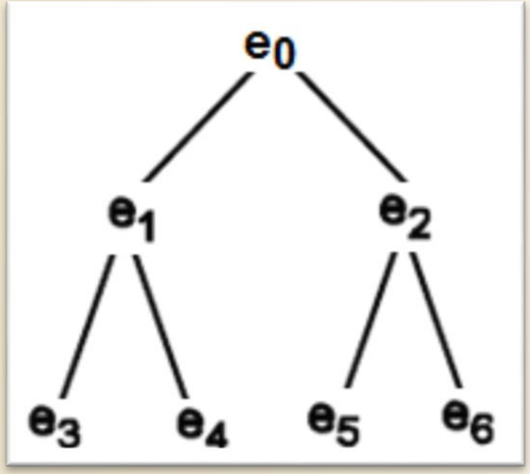




- Q: Write an expression for the array index where we find the right child of the node in array index **`i`**.
  - For now, assume that the node in array index **`i`** has a right child.
  - In your code, of course you'll want to be careful not to go past the ends of the tree.

Fact summary: Binary heap in an array



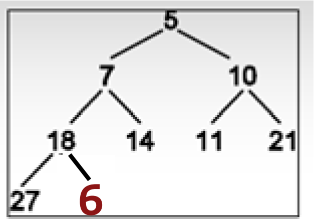


- For tree of height **`h`**, array length is **`2^h-1`**
- For a node in array index **`i`**:
  - Parent is at array index: **`(i – 1)/2`**
  - Left child is at array index: **`2i + 1`**
  - Right child is at array index: **`2i + 2`**

Bookmark this slide!

Binary heap enqueue and dequeue

Binary heap enqueue example (insert 6 + "bubble up")

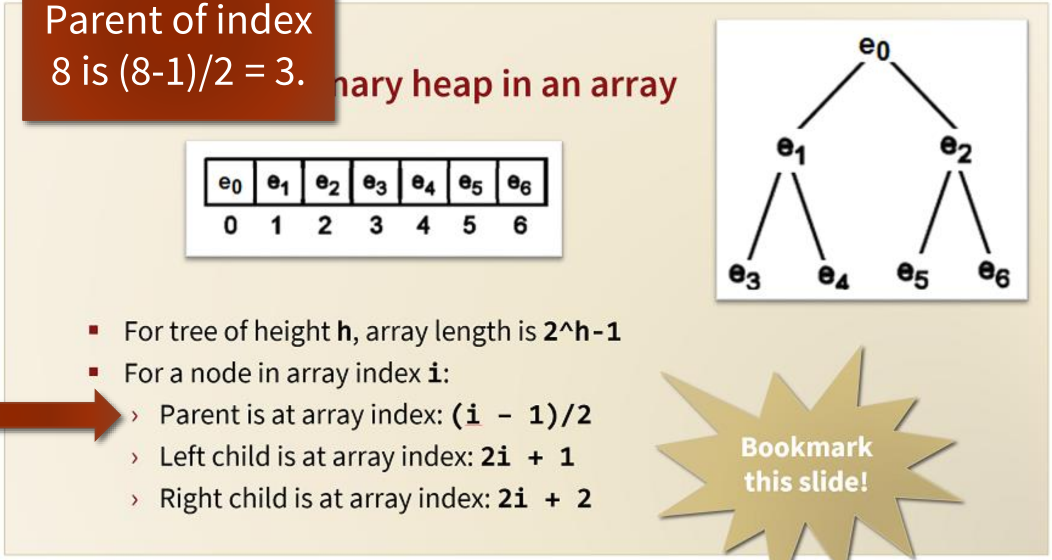


We can tell by looking at this tree visualization that the 6 doesn't go here—but remember in the code all you have is the array. How do we tell there?

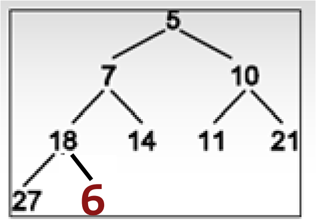
Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	18	14	11	21	27	6	?	…	?

Parent of index 8 is (8-1)/2 = 3.

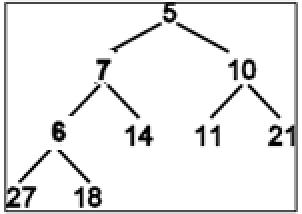


Binary heap enqueue example (insert 6 + “bubble up”)



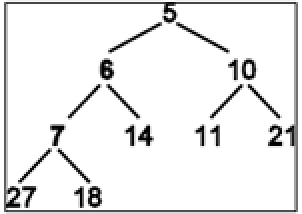
Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	18	14	11	21	27	6	?	…	?



Size=9, Capacity=15

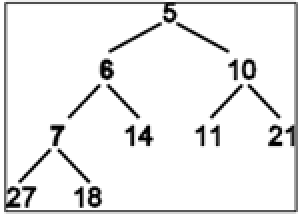
0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	6	14	11	21	27	18	?	…	?



Size=9, Capacity=15

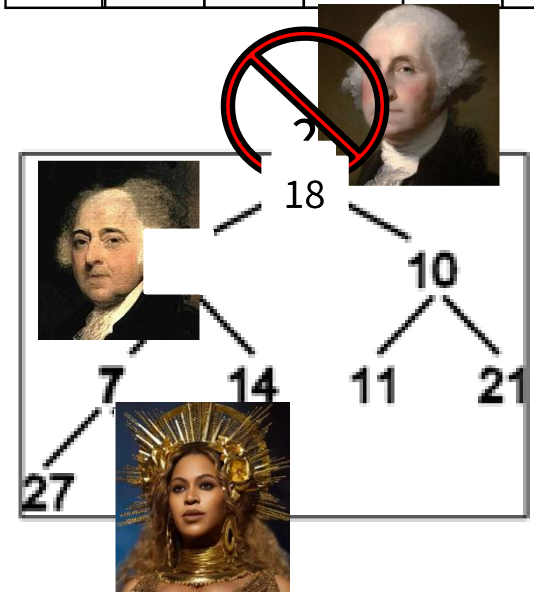
0	1	2	3	4	5	6	7	8	9	…	14
5	6	10	7	14	11	21	27	18	?	…	?

Binary heap dequeue (delete min)



Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
18	6	10	7	14	11	21	27		…	…	…



Dequeue and “trickle-down” algorithm summary

1. Remove the min element (the one in the root node—index 0) and that’s the value you’re going to return

- There’s now a “gap”—so the heap no longer follows the structural requirement that it be “complete”

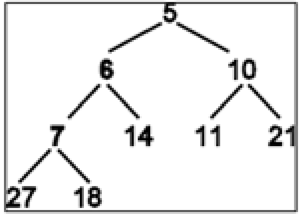
2. Promote the *last* element into the root node (index 0) position

- We have now immediately restored the “complete” property, but…
- …we have likely broken the “heap ordering” property!

3. “Trickle down” the new root element until the heap ordering property is restored

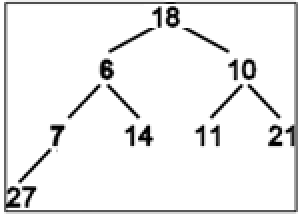
- Pick the <u>smaller</u> value of the left and right children of this element, and swap downward with that smaller one (i.e., you might trickle-down left, and you might trickle-down right, depending on which is smaller!)
- Repeat step 3 as needed (until it is smaller than <u>both</u> left and right children)

Binary heap dequeue (delete min + “trickle down”)



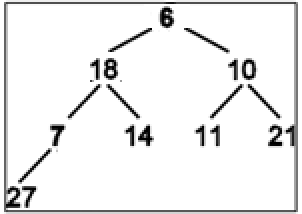
Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	6	10	7	14	11	21	27	18	?	…	?



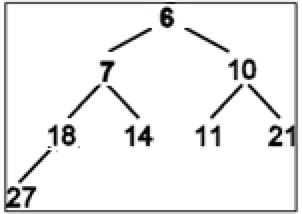
Size=**8**, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
18	6	10	7	14	11	21	27	18	?	…	?



Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
6	**18**	10	7	14	11	21	27	18	?	…	?



Size=8, Capacity=15

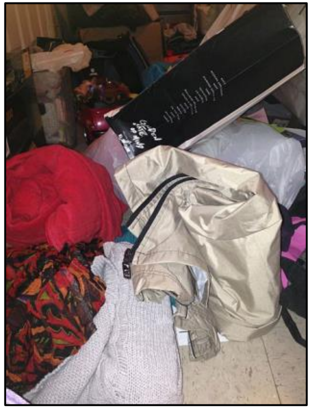
0	1	2	3	4	5	6	7	8	9	…	14
6	**7**	10	**18**	14	11	21	27	18	?	…	?

Summary analysis

Comparing our priority queue options

Would be great if we could get the best of both…

Fast enqueue *and* fast dequeue/peek



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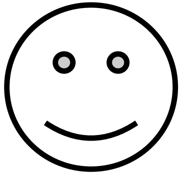

Fast enqueue

Fast dequeue/peek

Review: priority queue implementation options performance

Unsorted array

- Enqueue new element in back: O(1)
- Dequeue by searching list and scooting over: O(N)

Sorted array

- Always enqueue in sorted order: O(N)
- Dequeue from back: O(1)

Binary heap

- Enqueue + “bubble up”: O(logN)
- Dequeue + “trickle down”: O(logN)

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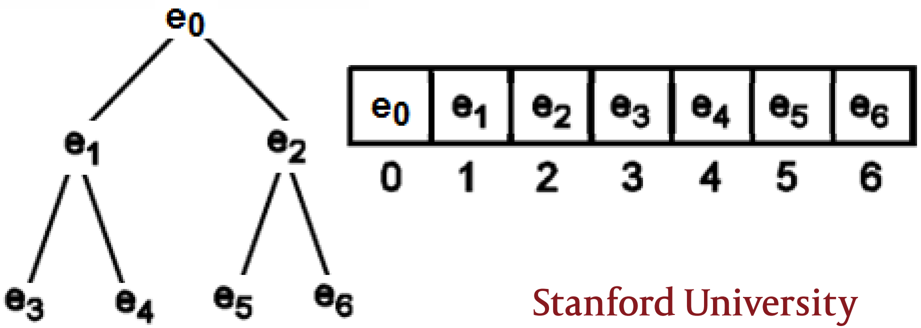
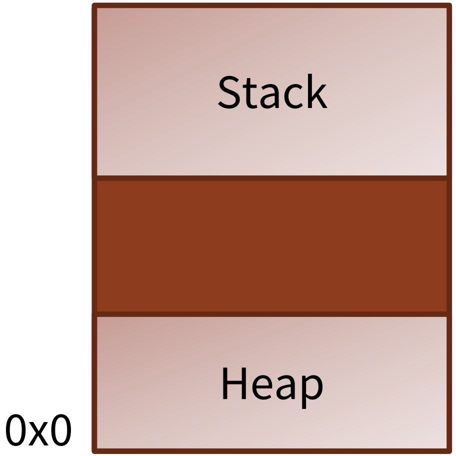


Final aside on terminology

Aside: Binary Heap, not to be confused with Heap memory!

- The Stack section of memory **<u>is</u>** a Stack like the ADT
- The Heap section of memory **<u>has nothing to do</u>** with the Heap structure.



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Stack ADT

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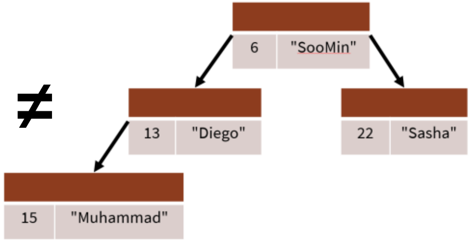

Heap data structure

- Probably just happened to reuse the same word ☹

Stanford University

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