Programming Abstractions

CS106B

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Topics:

- This time:
  - Starting with a dream: binary search in a linked list?
  - How our dream provided the inspiration for the BST
  - **Map** implemented as a Binary Search Tree (BST)
  - BST insert
  - Big-O analysis of BST

- *Next time:*
  - BST balance issues
  - Tree traversals
    - Pre-order, In-order, Post-order, Breadth-first
  - Applications of tree traversals

From last time: Binary Search in a Linked List?

EXPLORING A GOOD IDEA, FINDING WAY TO MAKE IT WORK

Recall our beautiful algorithm: binary search!

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- How long does it take us to find data in a sorted **array**?
  - **Use binary search!**
  - **O(logn):** awesome!!

- Big downside: O(N) insert, to keep the array sorted

Q. Can we do binary search on a linked list?

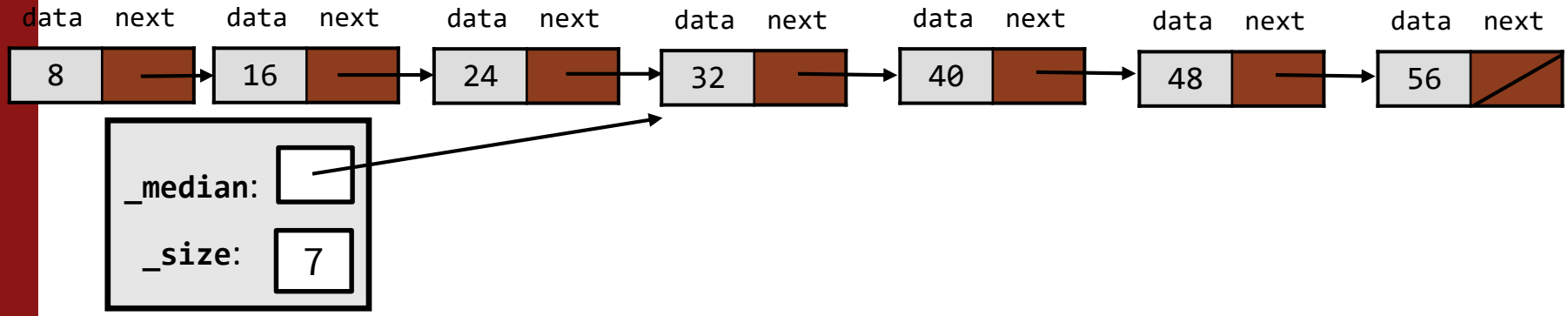
A. No.

- The nodes are spread all over memory, and we must follow "next" pointers one at a time to navigate (the treasure hunt).
- **Therefore cannot jump right to the middle.**
- **Therefore cannot do binary search.**
- **Find is O(N):** not terrible, but pretty bad compared to O(logn) or O(1)

Let's brainstorm a wild idea and then see if we can make it work

“What if…?”
The inspiration for Binary Search Trees

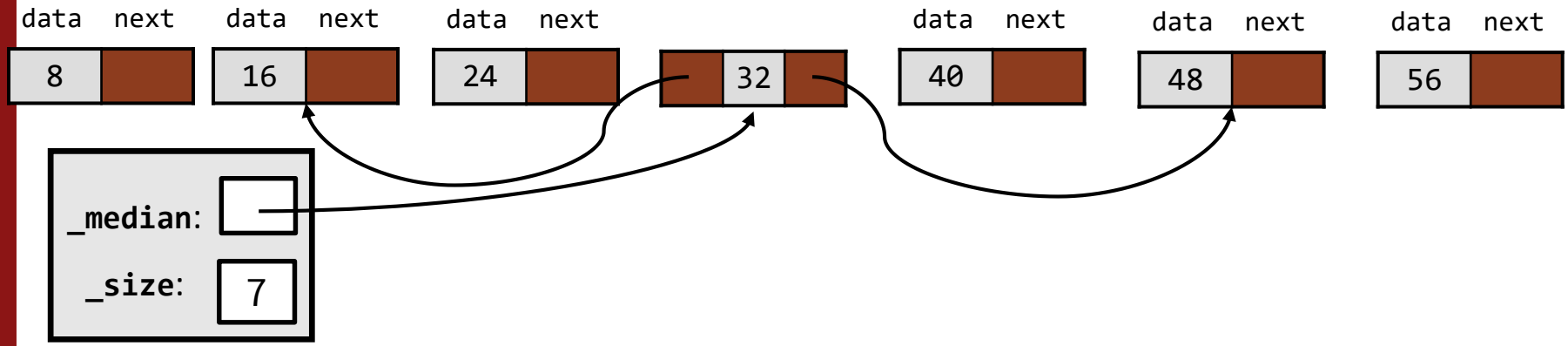
- What if…
- …instead of having a _front pointer in our linked list, we had a pointer to the element we want to look at first in binary search: the exact median/middle element?



- That would make the first step of our binary search **really** fast/easy!
- What about the next step? (and the front half of our list, lol)

“What if…?”
The inspiration for Binary Search Trees

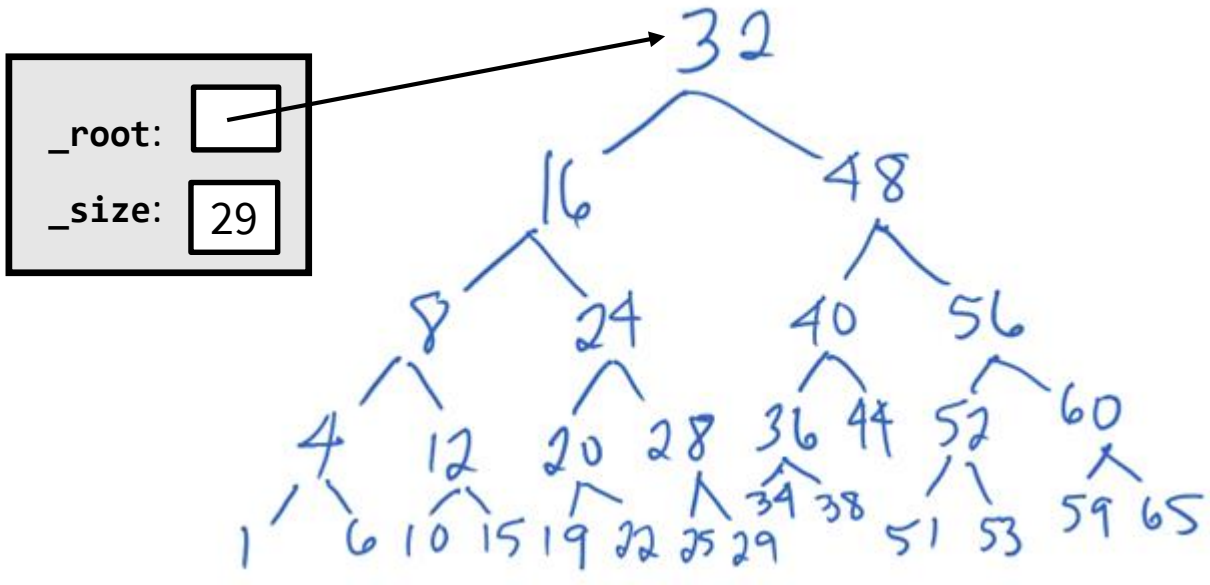
- What about the next step? (and the front half of our list, lol)
- Well, we could have the middle element point to the middle element of both the left half and the right half, so the 2nd step of our binary search is easy/fast too!



- Keep doing this until all elements have pointers to the middle of what remains to their left/right sides…voila!

An Idealized Binary Search Tree

- Our class will have a pointer to the median element*, and each element has pointers to the medians of everything to their left and right
  - ** actually it's hard to guarantee it will be the exact middle element, more on this, and lots more about Binary Search Trees, next time!*



Binary Search Trees

IMPLEMENTING THE **MAP INTERFACE** WITH BINARY SEARCH TREES

The centrality of search for Map interface

- The Map implementation should be **highly optimized** for finding a key amongst its collection of keys (to access the associated value)
  - Hence looking at binary search a moment ago—very fast search!
- One difference: a map has **keys** and **values**
- So remember our binary search array?

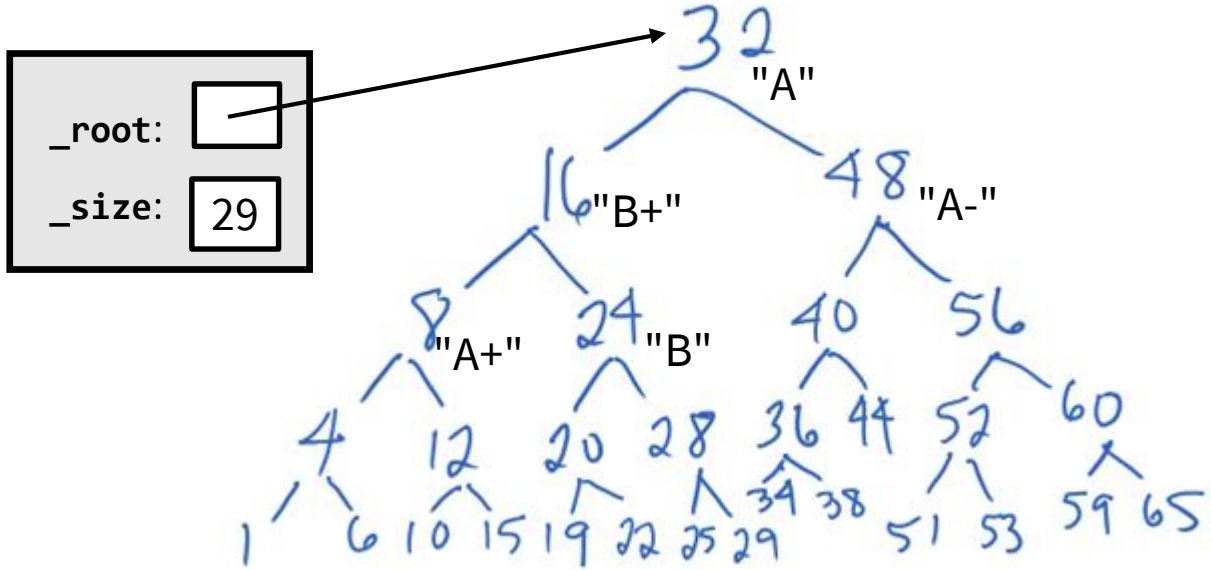
0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- Now imagine each number stored in here is a key, and has a value attached to it:

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95
"A"	“B+"	"A-"	“B"	“C+"	"A+"	"A-"	"A"	“B-"	"A"	"A"

The centrality of search for Map interface

- And each number stored in here is a key, and has a value attached to it (not all pictured)



Implementing Map interface with a Binary Search Tree (BST)

- Binary Search Tree is one option for implementing Map
  - C++'s Standard Template Library (STL) uses a Red-Black tree (a type of BST) for their map
  - Stanford library also uses a BST

- Another Map implementation is a hash table
  - We will talk about this later!
  - This is what Stanford's HashMap uses

TreeMap

THIS IS BASICALLY THE SAME AS STANFORD MAP.

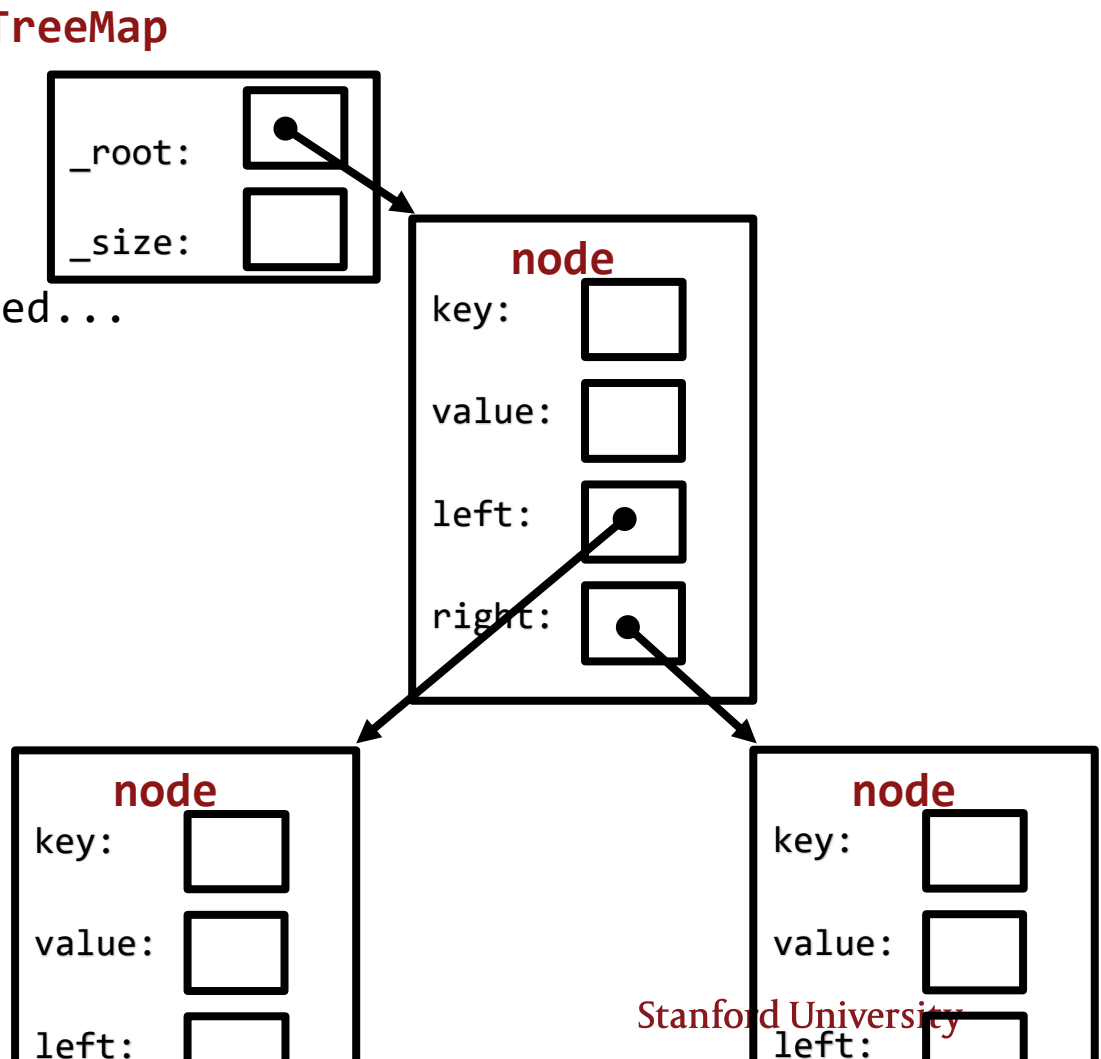
HERE IN CLASS WE'LL CALL IT TREEMAP JUST TO BE EXPLICIT ABOUT ITS IMPLEMENTATION.

tree-map.h

```
template <typename Key, typename Value>
class TreeMap {
public:
  TreeMap();
  ~TreeMap();

  bool isEmpty() const;
  int size() const;
  bool containsKey(const Key& key) const;
  void put(const Key& key, const Value& value);
  Value get(const Key& key) const;
  Value& operator[](const Key& key);
//...(continued on next slide)
```

tree-map.h

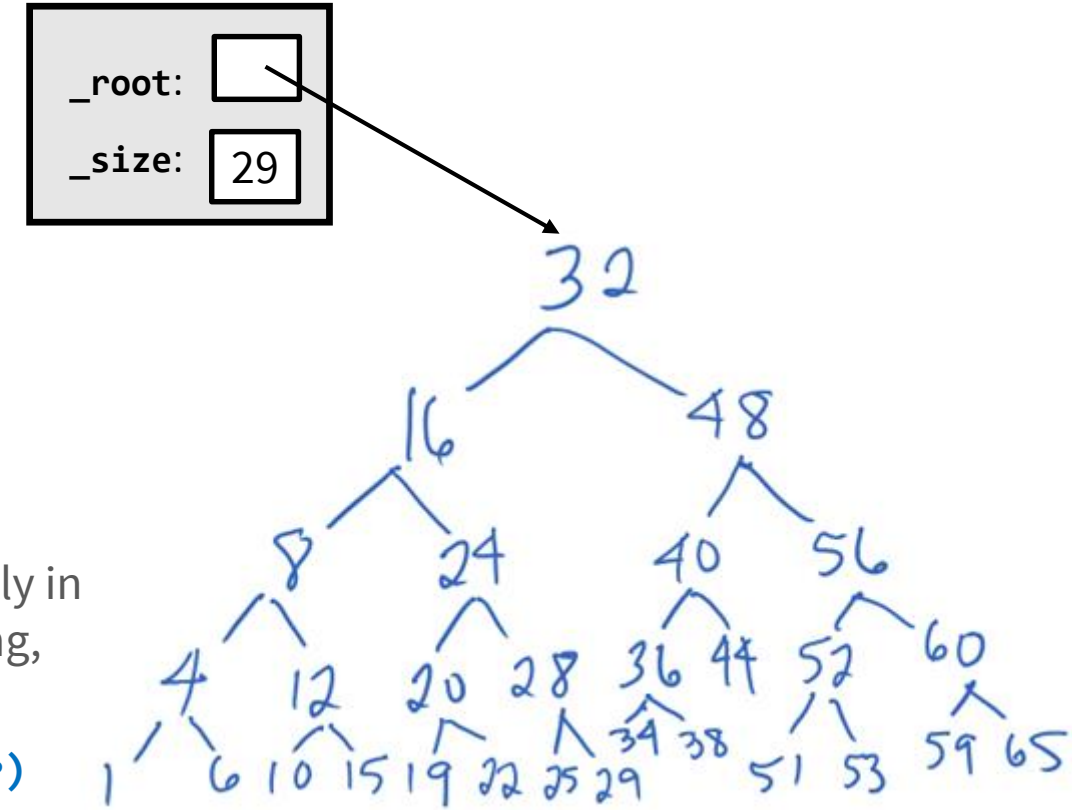
```
// class TreeMap continued...
private:
  struct node {
    Key   key;
    Value value;
    node* left;
    node* right;
  };
  int   _size;
  node* _root;

};
```



BST `put()`

Pretty simple!

- If key > node's key
  - Go right!
- If key < node's key
  - Go left!
- If there is nothing currently in the direction you are going, that's where you end up
- **Example:** `put(23, value)`



Question about our BST `put()` algorithm:

Pretty simple!

- If key > node's key
  - Go right!
- If key < node's key
  - Go left!

FAQ. What do we do if the key is equal to the node's key?

Stanford `Map` example:

```
Map<int, string> mymap;
mymap.put(5, "five");
mymap.put(5, "cinco");          // what should happen?
cout << mymap.get(5) << endl; // what should print?
```

BST `put()` algorithm:

- If key > node's key
  - Go right! (if doesn't exist—place here)
- If key < node's key
  - Go left! (if doesn't exist—place here)
- If key is equal, update value here.

BST `put()`

If key > node's key
  Go right! (if doesn't exist—place here)
If key < node's key
  Go left! (if doesn't exist—place here)
If key is equal, update value here.

Insert: 22, 9, 34, 18, 3

Your Turn: How many of these result in the same tree structure as above?

22, 34, 9, 18, 3

22, 18, 9, 3, 34

22, 9, 3, 18, 34

A. None of these
B. 1 of these
C. 2 of these
D. All of these

BST Big-O Performance

WHAT CAN WE EXPECT FROM A BST-BASED MAP?

Your Turn: What is the worst case cost for doing `containsKey()` in a BST?

A. O(1)
B. O(log n)
C. O(n)
D. O(n log n)
E. $O(n^2)$

What is the worst case cost for doing `containsKey()` in a BST *if the BST is balanced?*

O(logN)—awesome!

BSTs are great when balanced
BSTs are bad when unbalanced

- **…and Balance depends on order of insert** of elements…
- …but user controls this, not "us" (author of the Map class)…
- …no way for "us" (author of Map class) to ensure our Map doesn't perform terribly ☹ ☹

Your Turn: how many worst-case BSTs are there?

One way to create a bad BST is to insert the elements in *decreasing* order: 34, 22, 9, 3

That's not the only way…

How many **distinctly structured** BSTs are there that exhibit the worst case height (worst case is where height equals number of nodes) for a tree with the 4 nodes listed above?

A. 1-3
B. 4-5
C. 6-7
D. 8-9
E. More than 9

Bonus question: general formula for any BST of size n?

Extra bonus question (CS109): what is this as a fraction of all trees (i.e., probability of worst-case tree).

BST and Heap quick recap/cheat sheet

IT CAN BE EASY TO GET CONFUSED BETWEEN BST AND HEAP—HERE'S A QUICK GUIDE!

BST and Heap Facts (cheat sheet)

Heap (**Priority Queue**)

- Structure: must be "complete"
- Order: parent priority must be <= both children
  - This is for min-heap, opposite is true for max-heap
  - No rule about whether left child is > or < the right child
- Big-O: guaranteed log(n) enqueue and dequeue
- Operations: always add to end of array and then "bubble up"; for dequeue do "trickle down"

BST (**Map**)

- Structure: any valid binary tree
- Order: leftchild.key < self.key < rightchild.key
  - No duplicate keys
  - Because it's a Map, values go along for the ride w/keys
- Big-O: log(n) if balanced, but might not be balanced, then O(n)
- Operations: recursively repeat: start at root and go left if key < root, go right if key > root