Programming Abstractions

CS106B

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Today’s Agenda

- Analyzing ADT Implementations
  - Arrays
  - Binary Search Trees
- Hash tables
  - Hash functions
  - What makes a “good” hash function?
- Other uses of hashing

Analyzing ADT Implementations

Analyzing ADT Implementations

For all of our ADTs (`Vector`, `Set`, etc) our goal is to achieve fast

- Contains
- Add
- Remove

Implementing Set

- Let's use an array!
- We need dynamic memory (on the heap!)
- Let's focus on 2 versions: unsorted array and sorted array

Unsorted Array

Need to check if the element is contained in the array to ensure no duplicates

Contains

Add

Remove

Unsorted Array

Need to check if the element is contained in the array to ensure no duplicates

Contains	`O(n)`
Add	
Remove	

Unsorted Array

Need to check if the element is contained in the array to ensure no duplicates

Contains	`O(n)`
Add	`O(n)`
Remove	

Unsorted Array

Need to check if the element is contained in the array to ensure no duplicates

Contains	`O(n)`
Add	`O(n)`
Remove	`O(n)`

Sorted Array

Binary search speeds up lookups!

Contains

Add

Remove

Sorted Array

Binary search speeds up lookups!

Contains	`O(log(n))`
Add	
Remove	

Sorted Array

Still need to shift elements over 😕

Contains	O(log(n))
Add	O(n)
Remove	

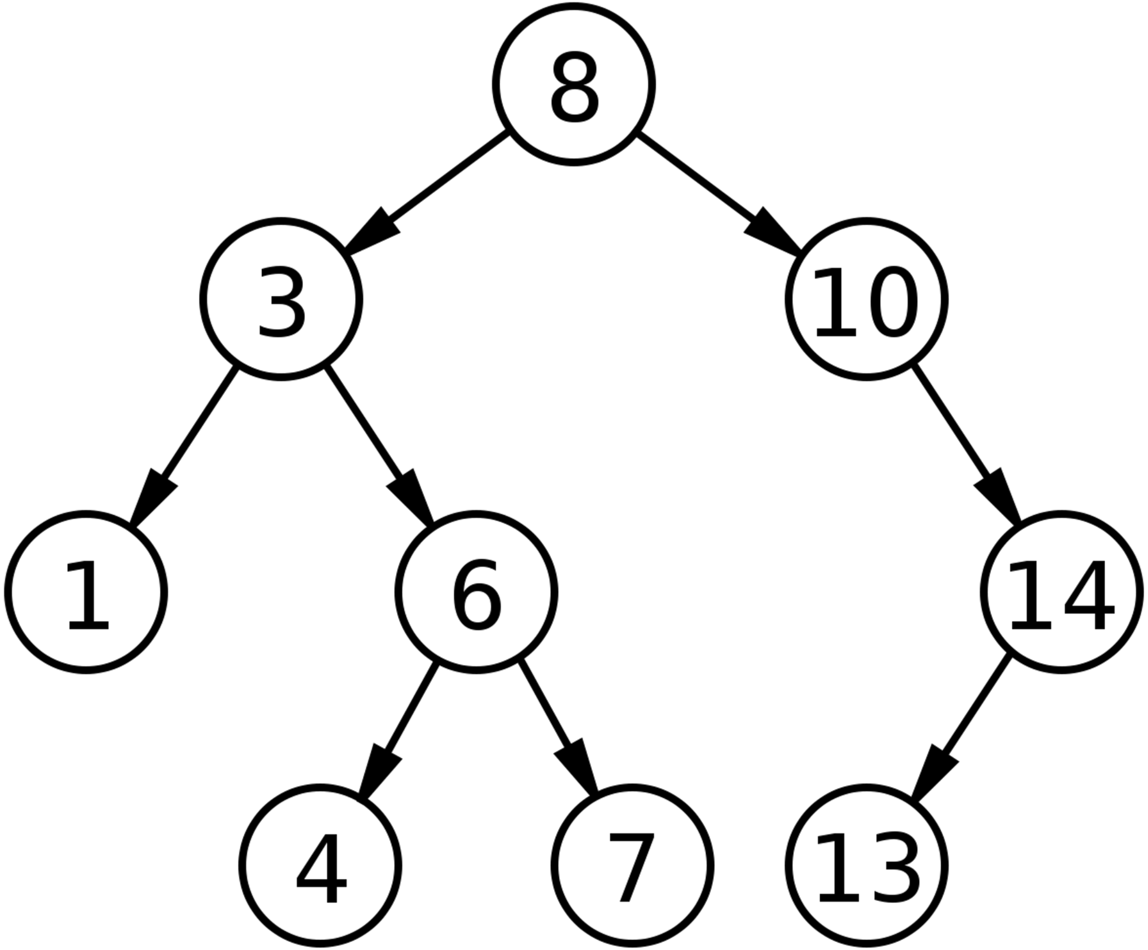
Sorted Array

Still need to shift elements over 😕

Contains	`O(log(n))`
Add	`O(n)`
Remove	`O(n)`

Next step for lookup-based structures...

Binary Search Trees 🌳


8
3
10
1
6
14
4
7
13

Stanford library `Map` and `Set` classes are backed by binary search trees

Binary Search Trees

Assuming a balanced binary search tree

Contains

Add

Remove

Binary Search Trees

Assuming a balanced binary search tree

Contains	`O(log(n))`
Add	
Remove	

Binary Search Trees

Assuming a balanced binary search tree

Contains	O(log(n))
Add	O(log(n))
Remove	

Binary Search Trees

Assuming a balanced binary search tree

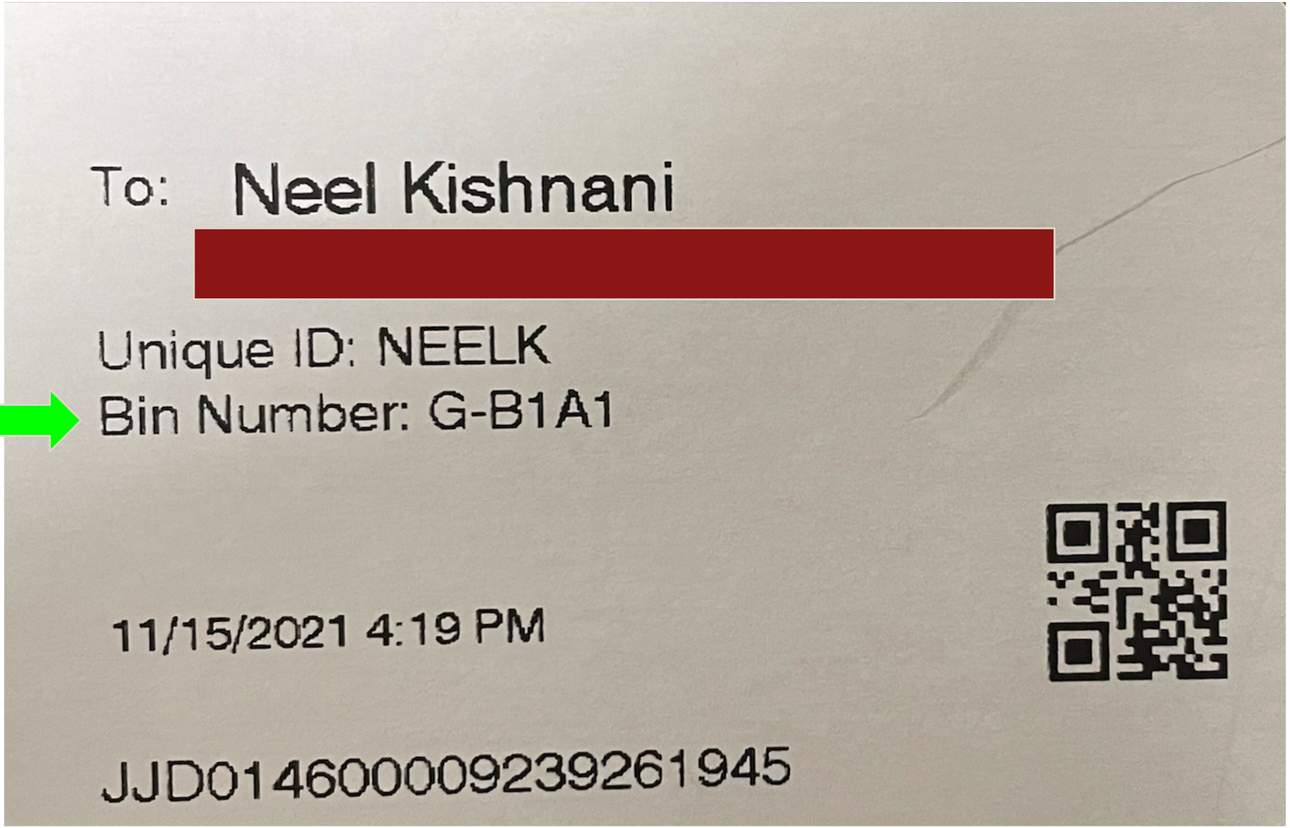
Contains	O(log(n))
Add	O(log(n))
Remove	O(log(n))

Can we do better than `O(log(n))`? 🤔

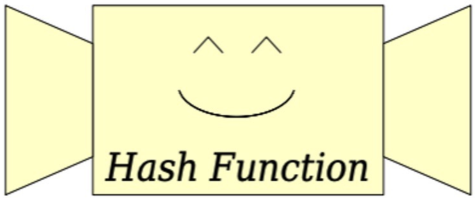

Student Package Center
UG2 Mail & Logistics
STANFORD HAIR

UG2 Package Center

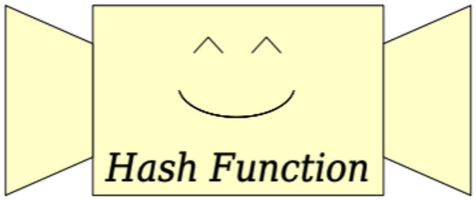
- The package center gets a lot of packages throughout the quarter
- They store packages by keeping a small number of **buckets** for groups of packages
- They have a **rule that assigns packages to buckets**
- When a student comes in to pick up their package, they know exactly which bucket to go to


To: Neel Kishnani
[redacted]
Unique ID: NEELK
Bin Number: G-B1A1
11/15/2021 4:19 PM
JJD014600009239261945

Let's introduce a special function called a hash function



We'll use this hash function to assign elements to buckets



Hash Functions

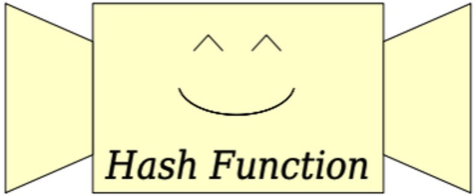
Important property:

The same input should produce the same output

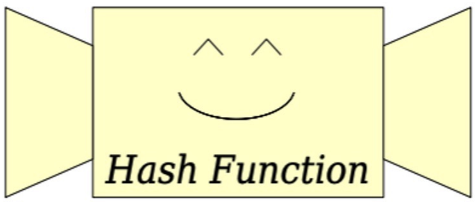
- Functions with this property are deterministic
- More on deterministic functions in CS103!

For the purposes of CS106B, assume our hash function returns an `int`

- The input can be of any type though! (`string, double, int,` etc.)



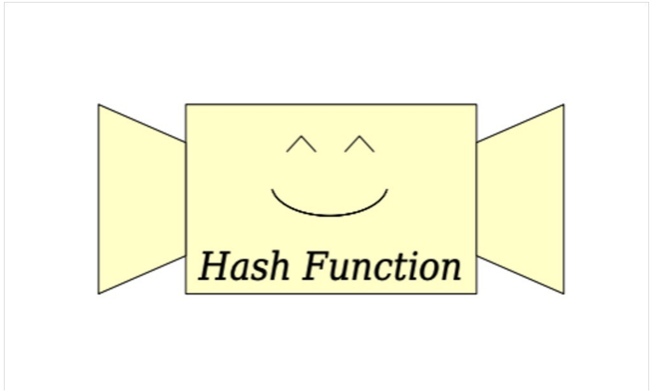
Input: 12



The output of a hash function is called a hash code!

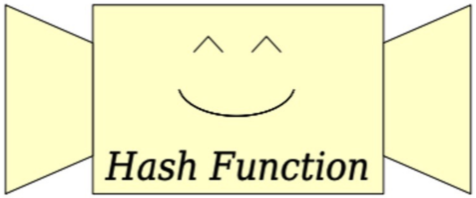
Input: 12

Hash Code: 106107



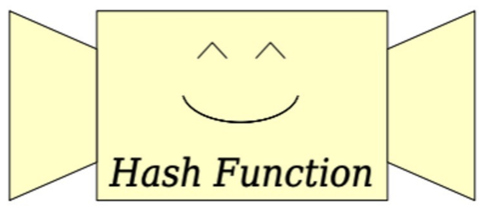
Input: `1016`

Hash Code:

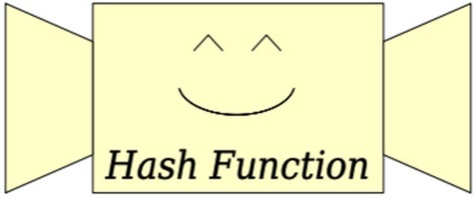


Input: `1016`

Hash Code: `309731`



Input: `12`



Input: `12`

Hash Code: `106107`

A new data structure 🪣

- Let’s go back to our array and treat each slot as a bucket for elements, just like the package center!

- We’ll assign each element we need to insert into a bucket and store it there

Use a **hash function** to assign elements to buckets 🪣

This data structure is called a

Hash Table

```
HashTable::HashTable() {
    // Initialize array of buckets
    _elements = new int[NUM_BUCKETS];
}
```

An idea for a hash function

Return the element itself!

```
int hash1(int elem) {
    return elem;
}
```

```
void HashTable::insert(int elem) {
    int bucket = hash1(elem);
    _elements[bucket] = elem;
}
```

Break

Logistics

- Assignment 7 is out now and due June 2nd
  - Huffman Coding!
  - Last assignment of the quarter – congrats!

- Final Exam:
  - **8:30-11:30AM on Friday June 9th**
  - Same format as midterm
  - Practice materials up on course website
  - Review session happening Sunday 2-4PM in Bishop

Resume

Our Buckets

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 3

Hash Function:

```
int hash1(int elem) {
     return elem;
}
```

Input: 3

Hash Code: 3

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 3

Hash Code: 3

The hash code is the bucket we put the element in

[0]	
[1]	
[2]	
[3]	3
[4]	

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 0

[0]	
[1]	
[2]	
[3]	3
[4]	

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 0

Hash Code: 0

[0]	0
[1]	
[2]	
[3]	3
[4]	

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 0

Hash Code: 0

Stanford University

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 17000

[0]	0
[1]	
[2]	
[3]	3
[4]	

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 17000

Hash Code: 17000

[0]	0
[1]	
[2]	
[3]	3
[4]	
...	
[17000]	17000

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: 17000

Hash Code: 17000

We need to enlarge our array - lots of wasted space!!

Issue #1

This hash function could lead to a **sparse hash table**

[0]	0
[1]	
[2]	
[3]	3
[4]	
	…
[17000]	17000

Hash Function:

```
int hash1(int elem) {
    return elem;
}
```

Input: -3

Issue #2

This hash function doesn't handle negative inputs

We want to limit the range of possible buckets

A better(?) hash function 💭

Let's use the `%` operator!

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

[0]	
[1]	
[2]	
[3]	
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 3

[0]
[1]
[2]
[3]
[4]

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 3

Hash Code: 3

[0]	
[1]	
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 3

Hash Code: 3

[0]	
[1]	
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 17000

[0]	
[1]	
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 17000

Hash Code: 0

Handles this large value!


```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 17000

Hash Code: 0

[0]	17000
[1]	
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: -6

[0]	17000
[1]	
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: -6

Hash Code: 1

[0]	17000
[1]	-6
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: -6

Hash Code: 1

Handles this negative value!

[0]	17000
[1]	-6
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 8

[0]	17000
[1]	-6
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 8

Hash Code: 3

[0]	17000
[1]	-6
[2]	
[3]	3
[4]	

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 8

Hash Code: 3

Hash Collisions

- Our hash function assigned two different elements to the same bucket
  - We call this a collision

- We have to decide what to do when collisions happen

- Idea: instead of having our array store `int`, let's have it a linked list
  - Each bucket will now be a `ListNode*`
  - When we have a collision, we can add the new element to the front of the list in `O(1)`

In the header file…

A double pointer! This means that each array element is a pointer. More in CS107!

```
private:
    ListNode** _elements;
```

In the cpp file…

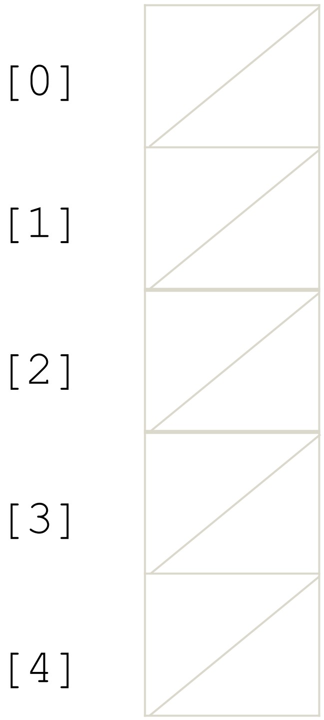
```
HashTable::HashTable() {
    // Initialize array of buckets
    _elements = new ListNode*[NUM_BUCKETS]();

}
```

Initialize each bucket to the `nullptr`

This is called a

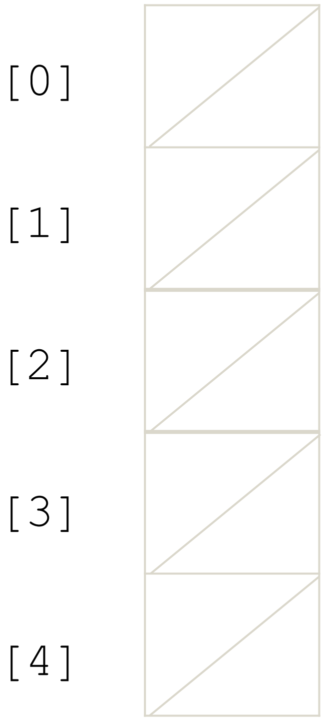
Chaining Hash Table

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```



Input: 2

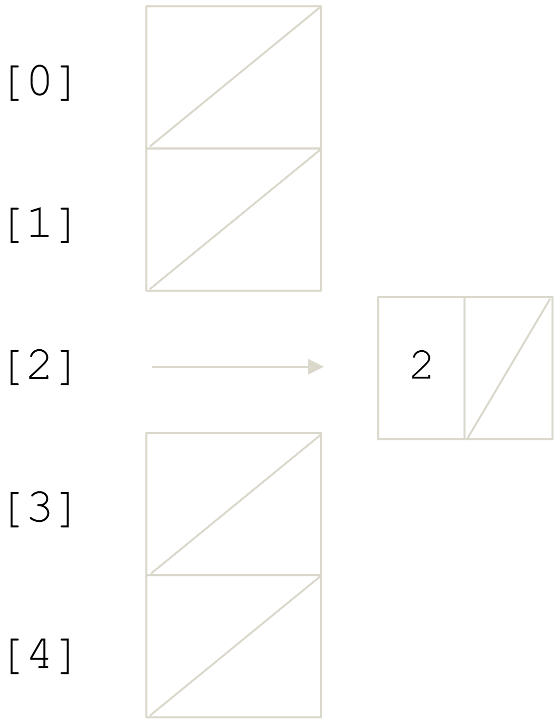
```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```



Input: 2

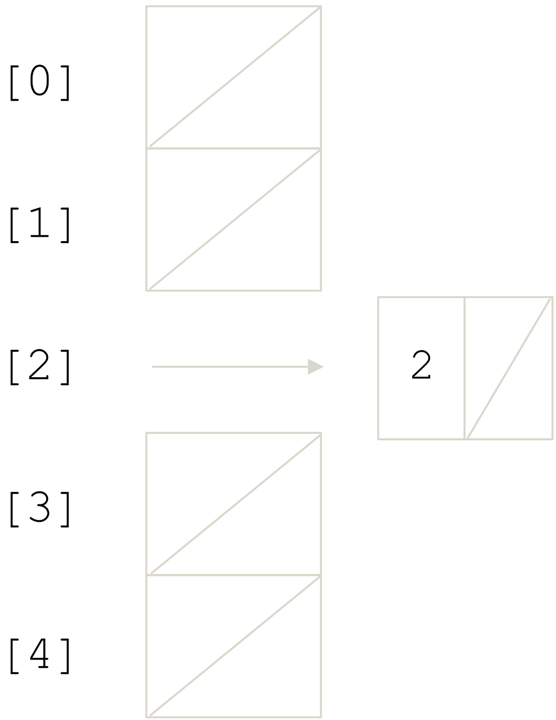
Hash Code: 2

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```



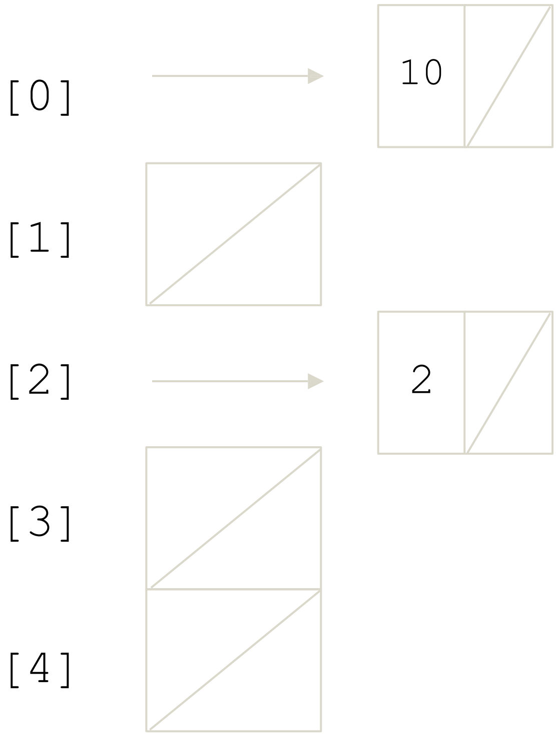
Input: 2

Hash Code: 2




```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

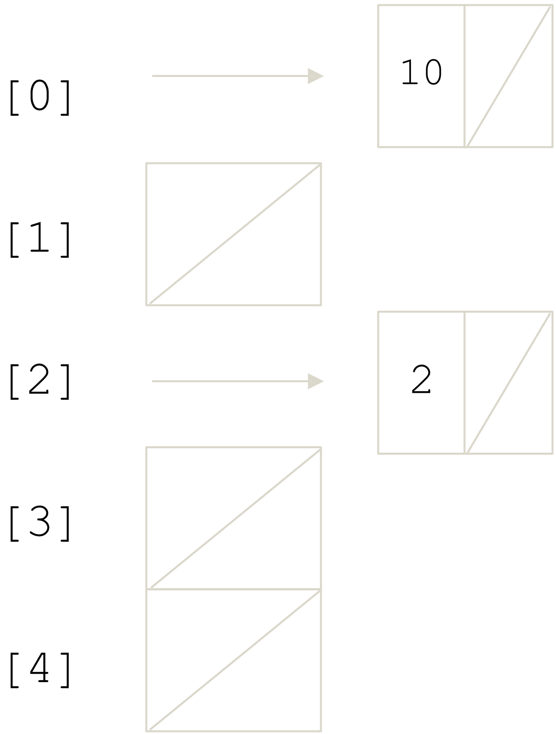
Input: 10




```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 10

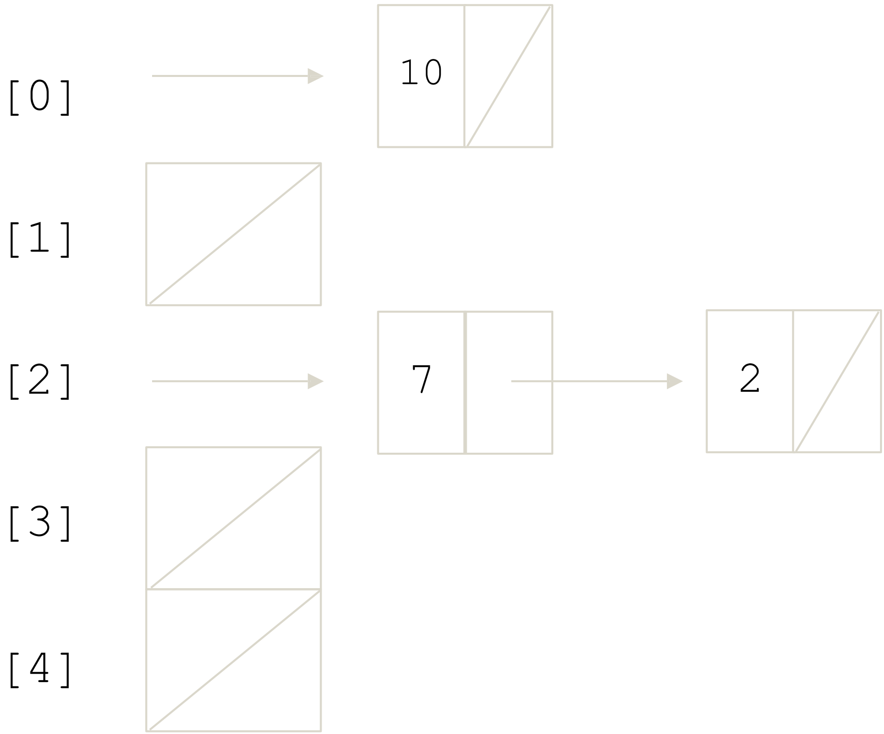
Hash Code: 0




```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```

Input: 7

```
int hash2(int elem) {
    return abs(elem) % numBuckets;
}
```



Input: 7

Hash Code: 2

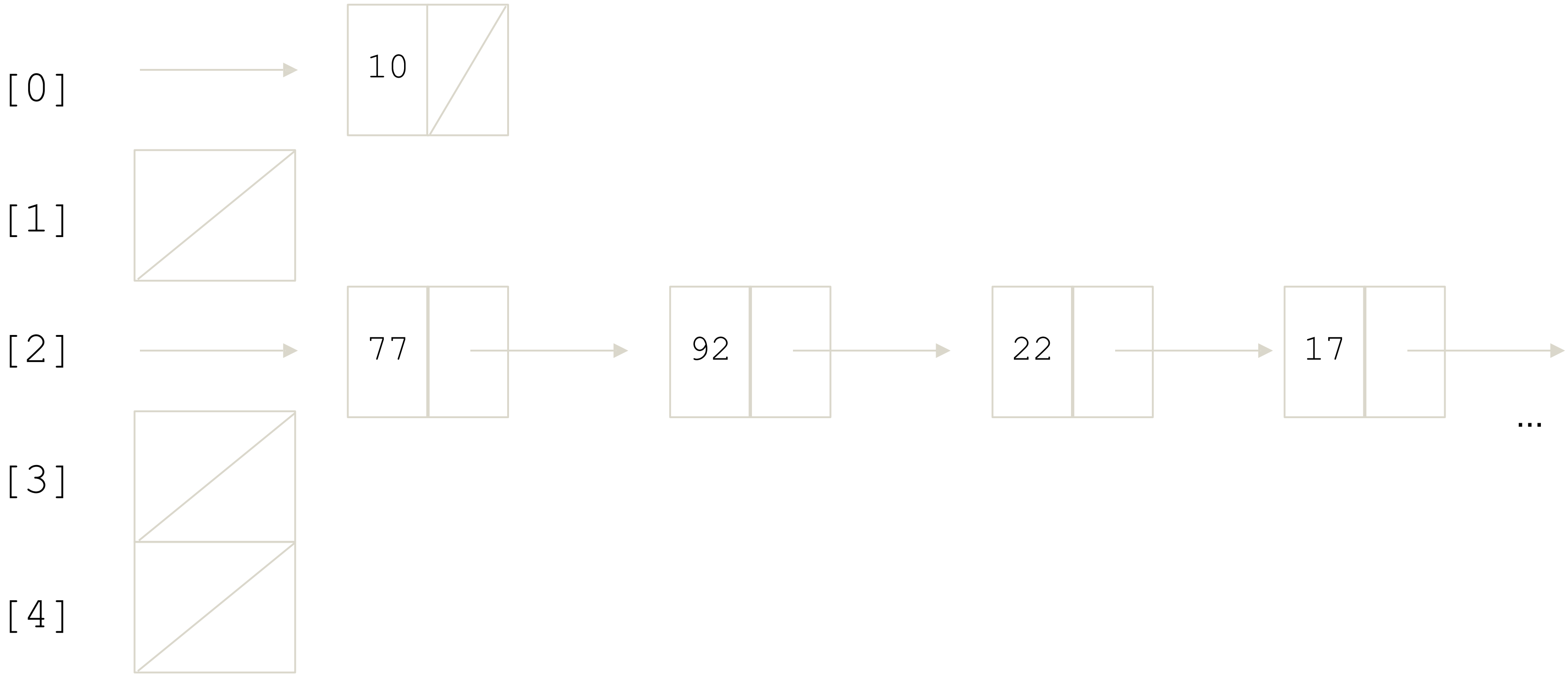
Inserting into this chaining hash table is

O(1)

```
void HashTable::insert(int elem) {
    if (contains(elem)) return;
    int bucket = hash2(elem);
    ListNode* front = _elements[bucket];

    // Create new front of list, tack previous onto end
    ListNode* cur = new ListNode{elem, front};
    _elements[bucket] = cur;
}
```

Say you got the following elements as inputs next:

`17, 22, 92, 77`


[0]
10
[1]
[2]
77
92
22
17
...
[3]
[4]

With several collisions, our `contains` and remove will be

`O(n)`

Where n is the number of elements in the relevant bucket

Our goal is to get a “good” hash function that:

- Distributes elements evenly (“spread”)
- Maintains a reasonable load factor

Load Factor

- The average number of elements in each bucket
  - If the load factor is low: wasted space
  - If the load factor is high: slow operations

- The load factor of a hash table with `n` elements and `b` buckets is:

$$\frac{n}{b}$$

Good Hash Functions

- There's tons of research in designing hash functions

- Beyond the scope of this class
  - CS161, CS166, CS265

HashSet

Assuming we have a good hash function

Contains

Add

Remove

HashSet

Assuming we have a good hash function

Contains	O(n/b)
Add	
Remove	

HashSet

Assuming we have a good hash function

Contains	O(n/b)
Add	O(n/b)
Remove	

HashSet

Assuming we have a good hash function

Contains	O(n/b)
Add	O(n/b)
Remove	O(n/b)

With `b` chosen to be close to `n`, we can approximate `O(1)` contains, add, and remove

That’s just about as good as we can do! ✅

The Stanford library `HashSet` and `HashMap` are implemented with hash tables!

HashMap	
clear()	O(N)
containsKey(key)	O(1)
equals(map)	O(N)
firstKey()	O(1)
get(key)	O(1)
isEmpty()	O(1)
keys()	O(N)
lastKey()	O(1)
mapAll(fn)	O(N)
put(key, value)	O(1)
remove(key)	O(1)

HashSet	
add(value)	O(1)
clear()	O(N)
contains(value)	O(1)
difference(otherSet)	O(N)
equals(set)	O(N)
first()	O(1)
intersect(otherSet)	O(N)
isEmpty()	O(1)
isSubsetOf(otherSet)	O(N)
isSupersetOf(otherSet)	O(N)
last()	O(1)
mapAll(fn)	O(N)
remove(value)	O(1)

Other uses of hash functions

Hash Functions

- Broadly, hash functions map a value to a unique integer value

- Presents in several CS domains

- The magic of hash functions:
  - They can take in any value and boil it down to a unique number
  - Images, ADTs, files, etc.

- Thought question: how would you hash a string?
  - Length?
  - ASCII representation?
  - What about an image?

Hash Functions

Goal: different values should produce very different hash codes

User table (bcrypt)

Username	Password
alice	$2b$10$aQNe4MK0HDhrkus8GZGQL.Nj11nsx12VTMTDBkykiL/jRbb.fJuGC
bob	$2b$10$TSbaMNCCq6.xNkDVszwwh09Fpb.eeW6aUSIFzGkPoQrs5RahskOUO
charlie	$2b$10$.5KcQQNEfnkPBYxeiqS2ZeePXLT5J30HG7zngfesyGuc0js37X41e
dakotah	$2b$10$l8n7ZLsq13ygE0m3cQ8oEuBjPnGcGBUA4zvJhnsKgyDEZdEd2EFXa

CS145: Data Management and Data Systems

1

Big Scale

Roadmap

Hashing

Sorting

Hashing-Sorting solves “all” known data scale problems :=)

+ Boost with a few patterns -- Cache, Parallelize, Pre-fetch

Note
Works for Relational, noSQL
(e.g. mySQL, postgres, BigQuery, BigTable, MapReduce, Spark)

Cryptographic Hash Functions

- Hash functions used in a security context
- One-way function: can't reverse
- Collision resistant
- Most popular: SHA-256
- More in CS155, CS 253, CS255

END