

Merge Sort

(break down the vector, then merge the pieces back together)

0	1	2	3	4	5	6	7
10	18	2	14	3	12	1	8

Slides by **Sean Szumlanski**
for **CS106B**, Programming Abstractions

Autumn 2023

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Do you agree we can merge two **sorted** vectors into **one** sorted vector in **linear** time?

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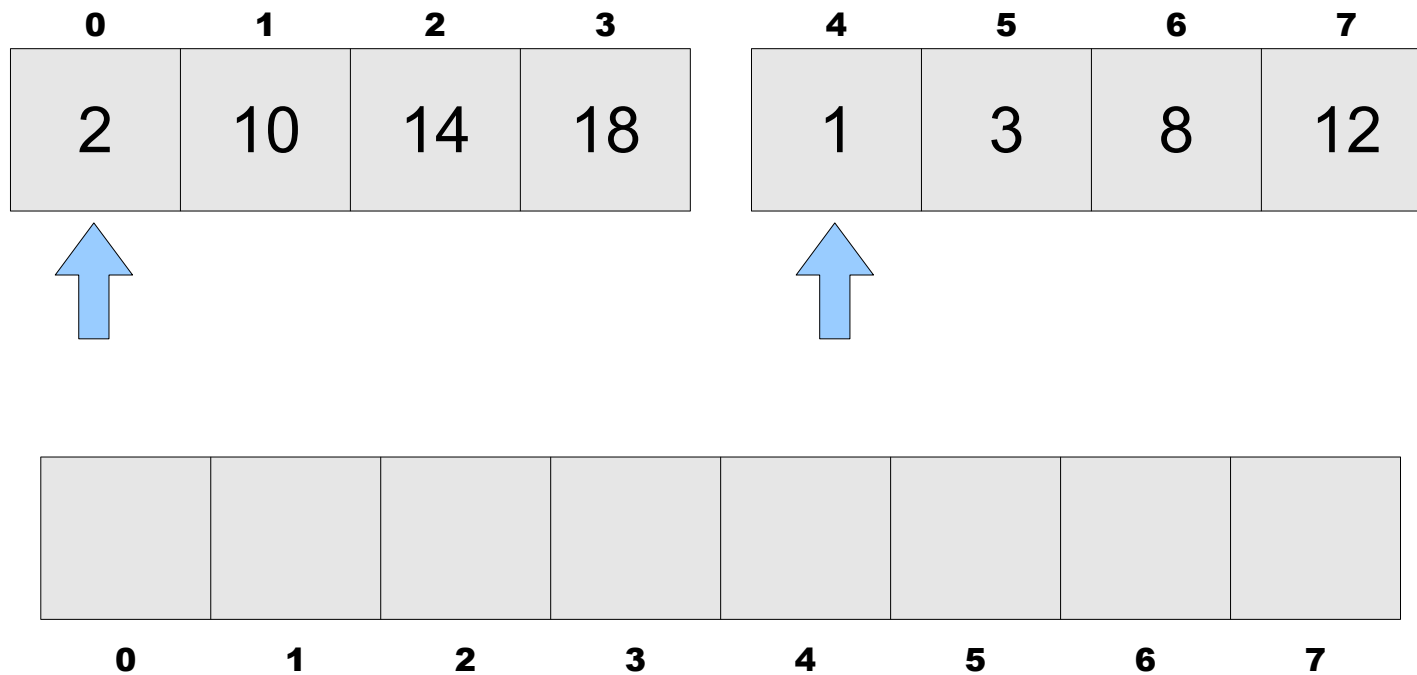
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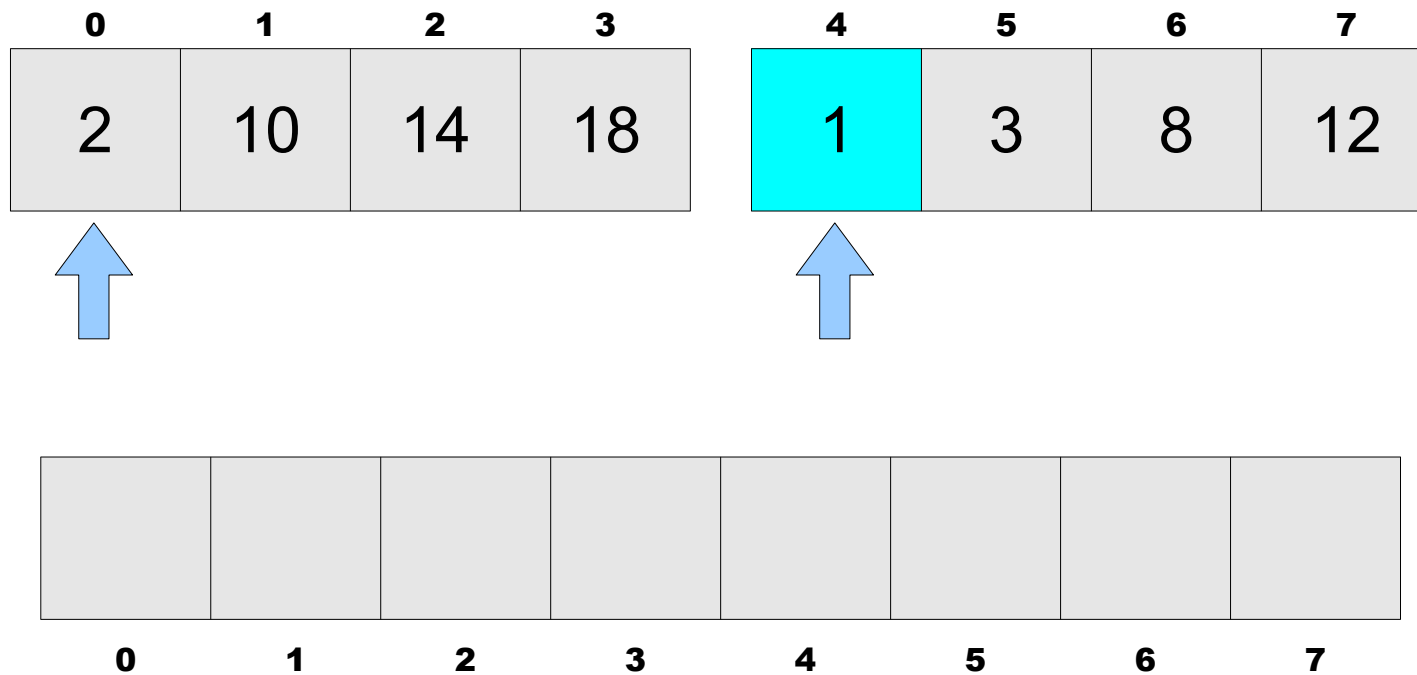
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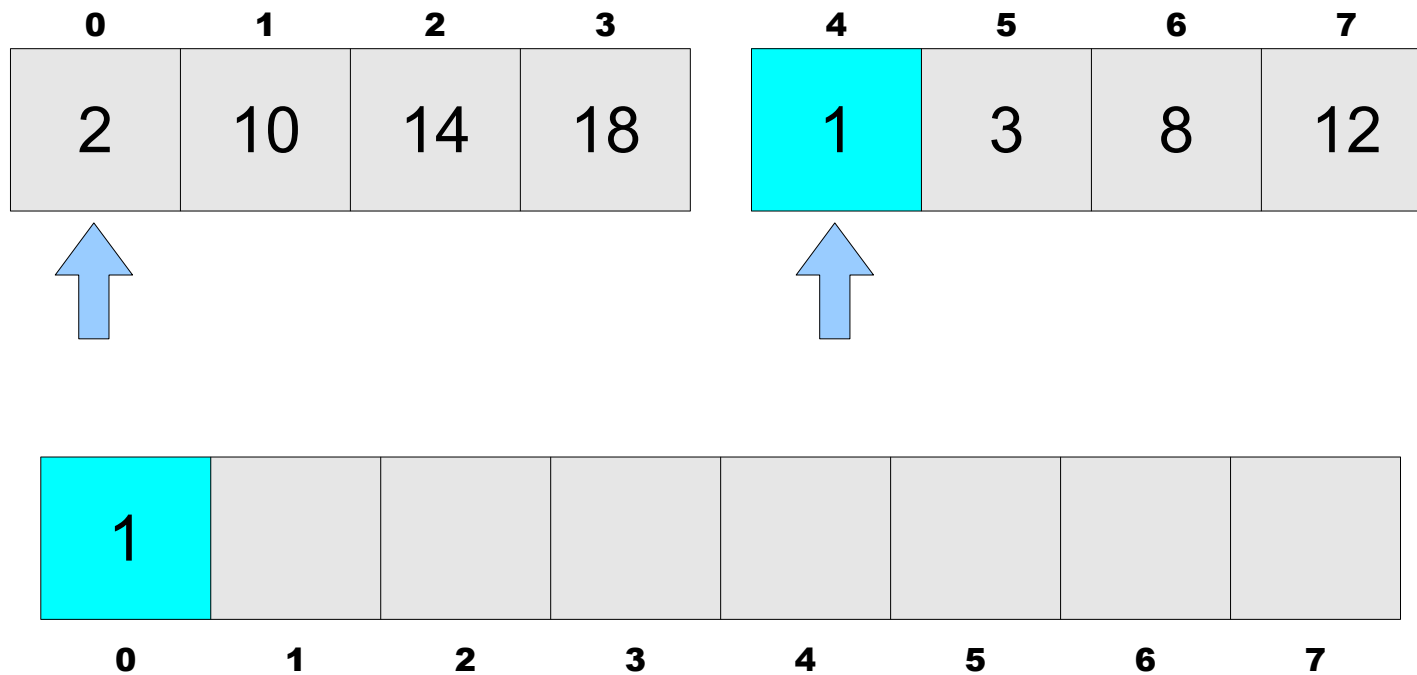
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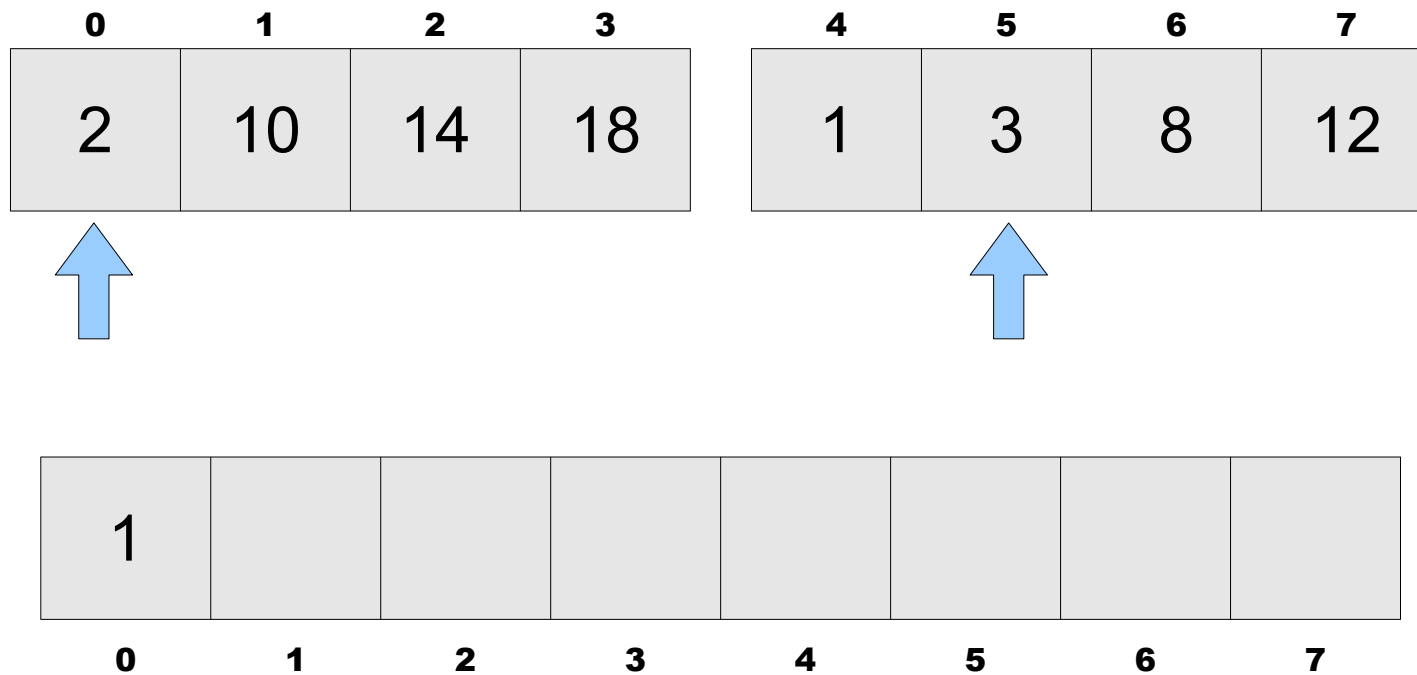
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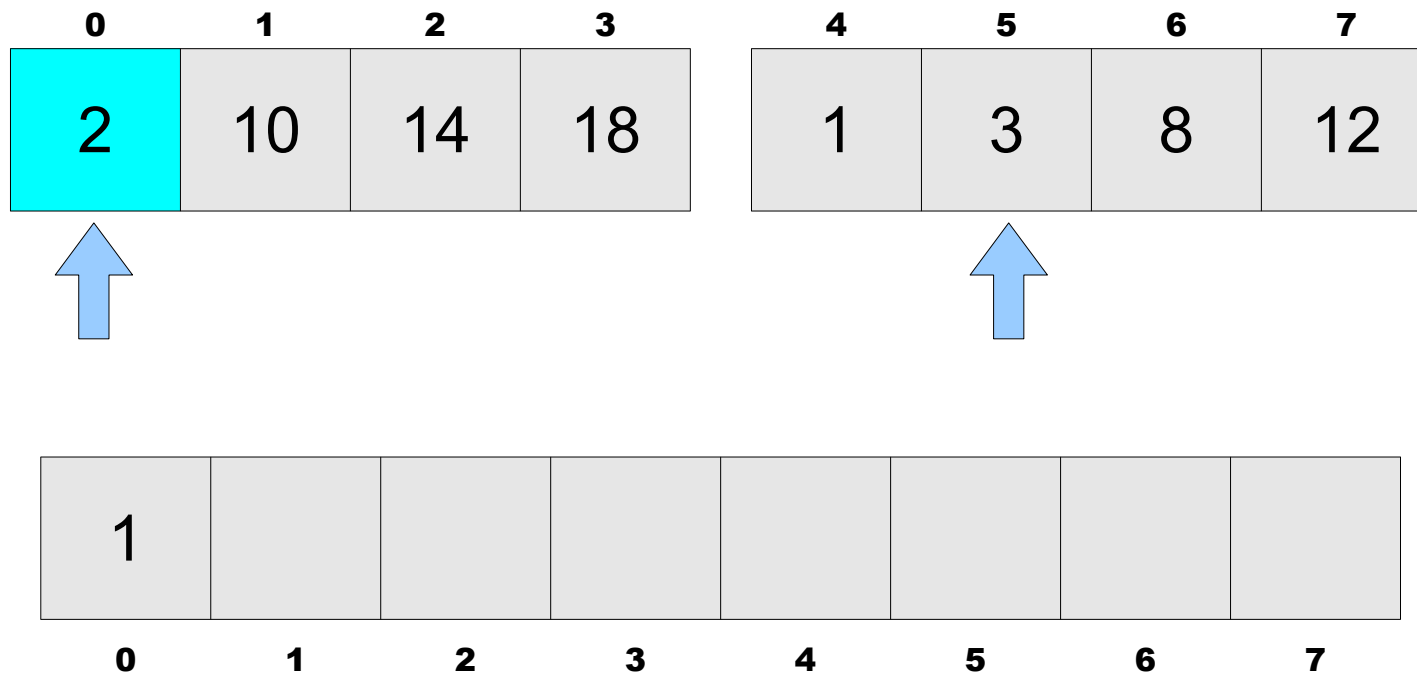
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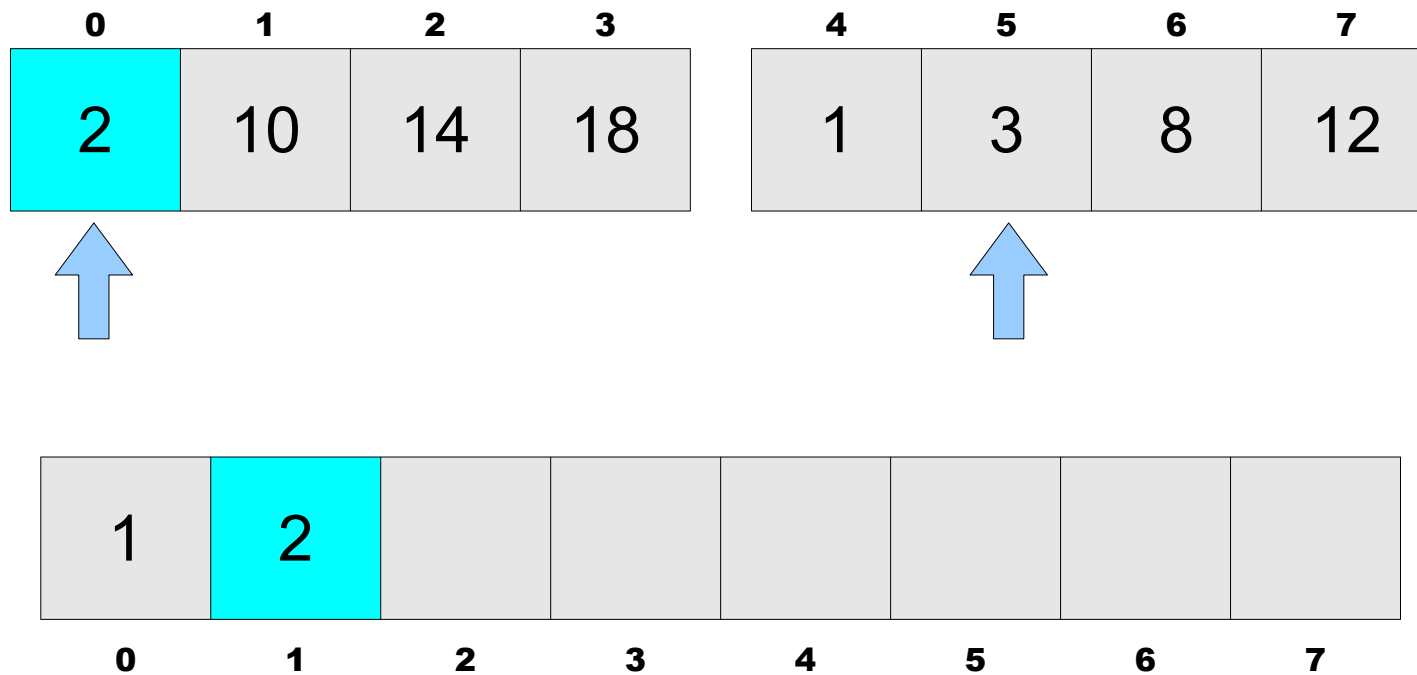
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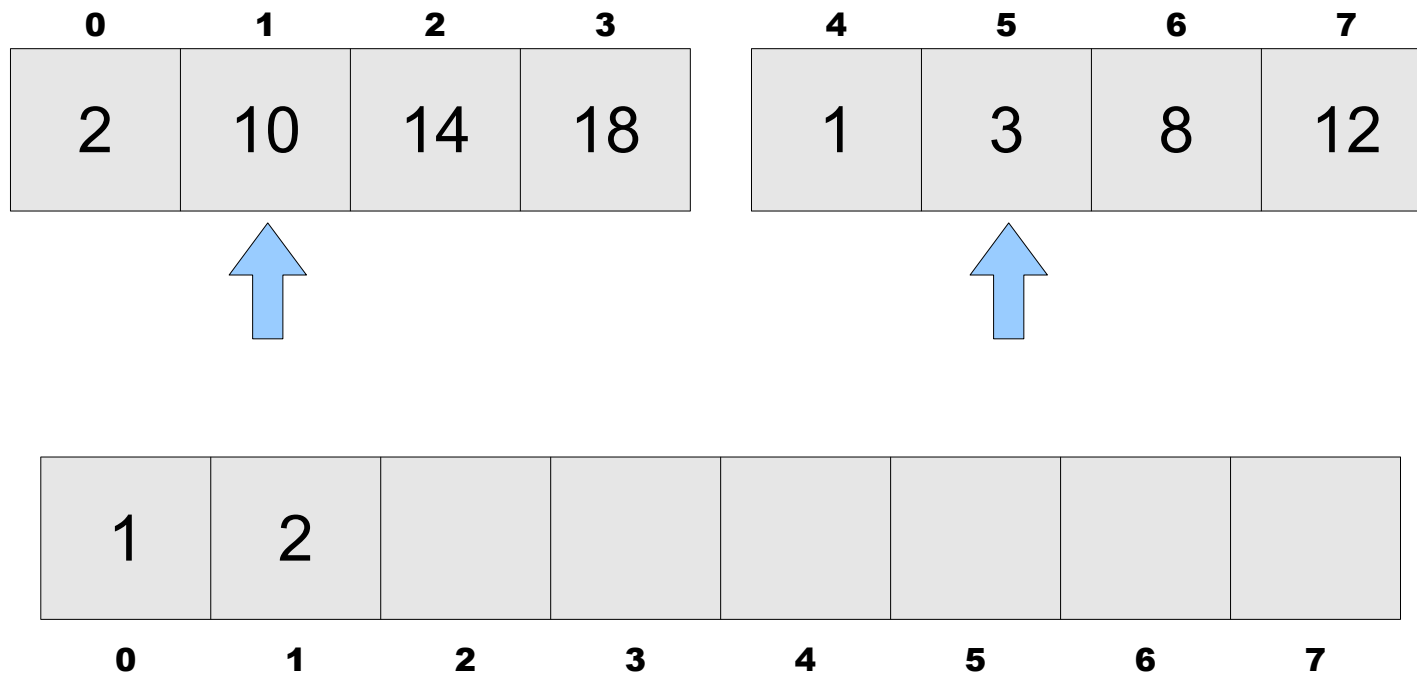
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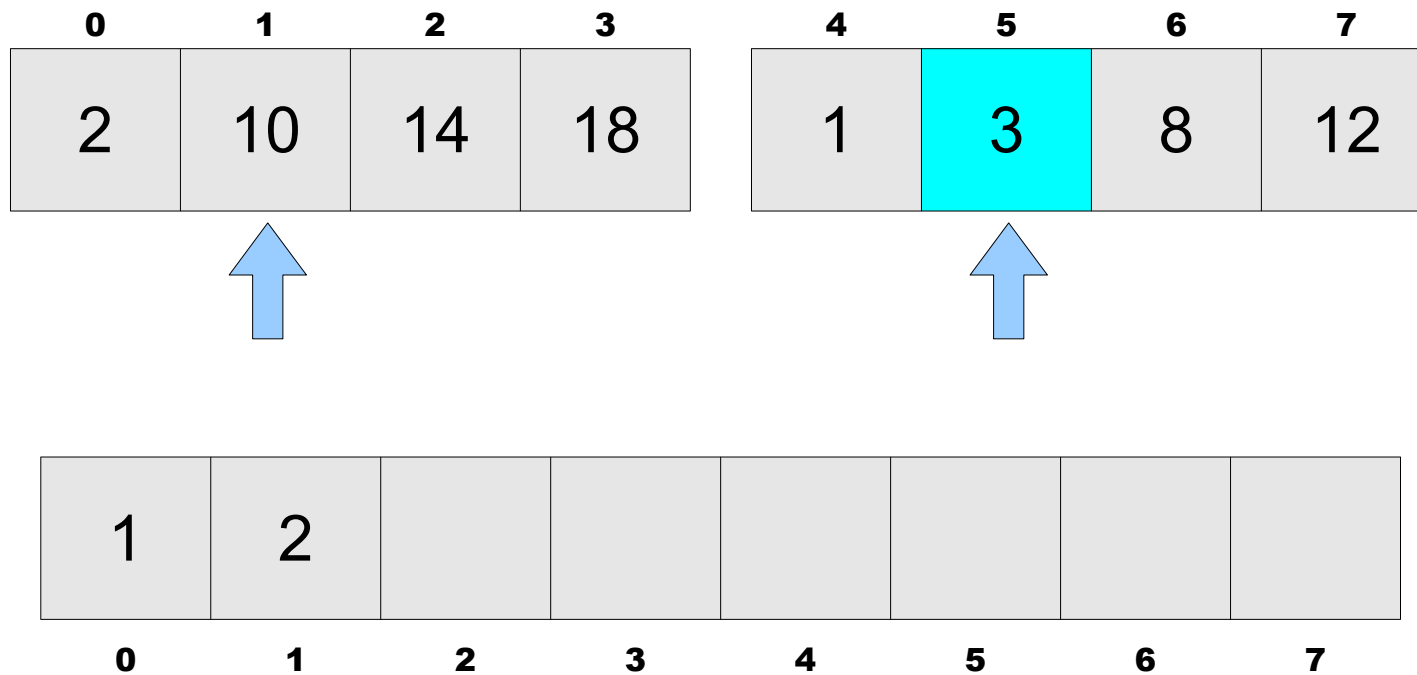
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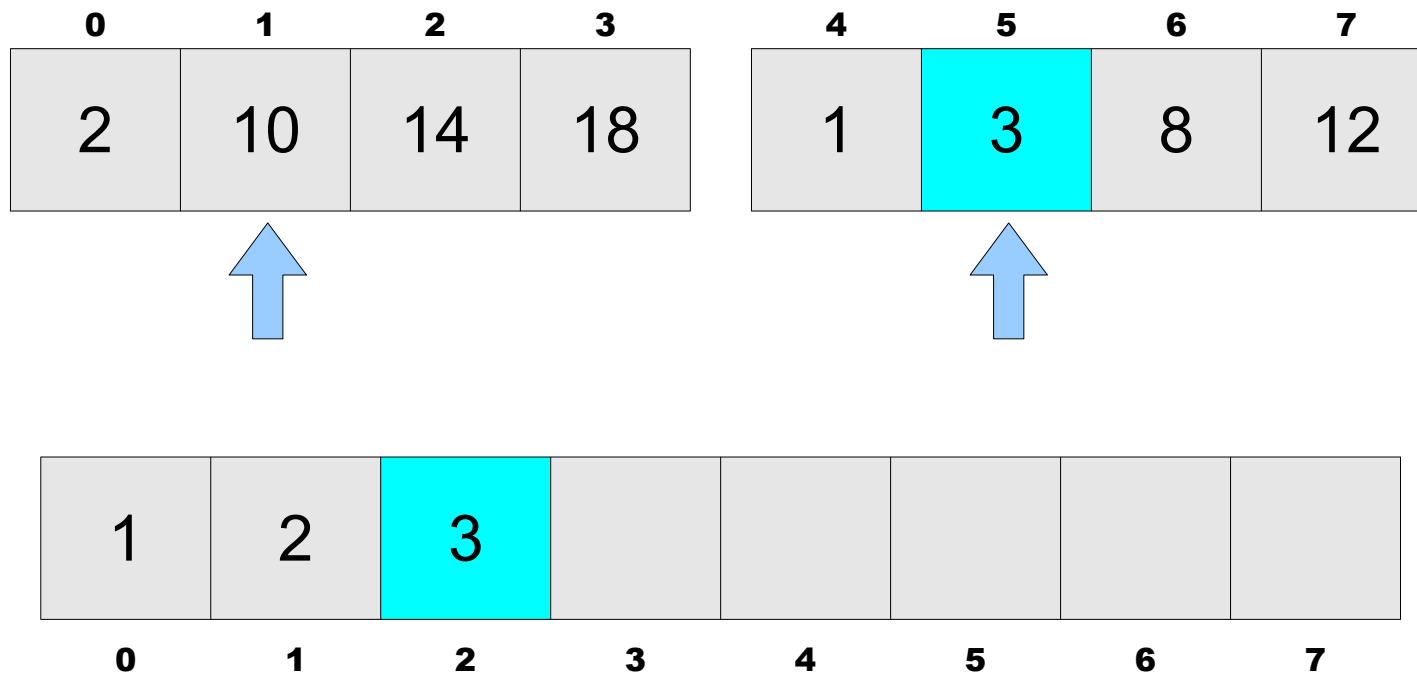
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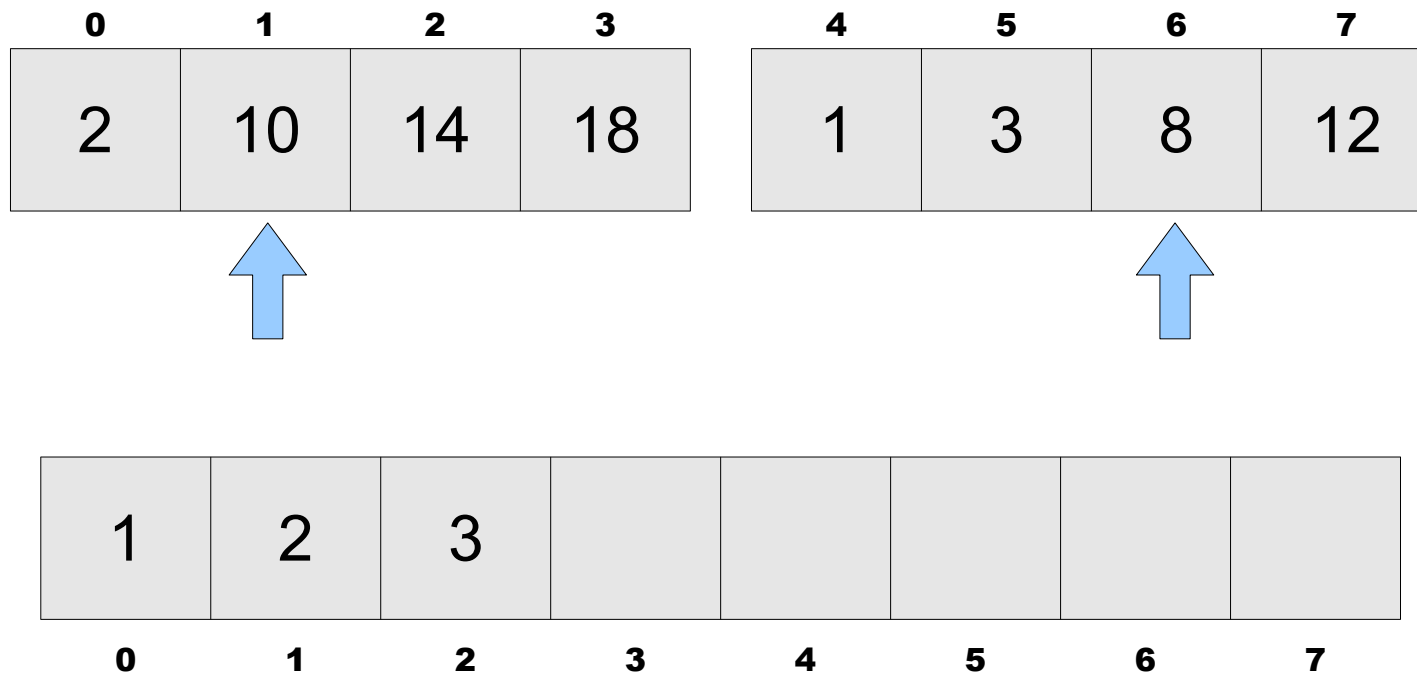
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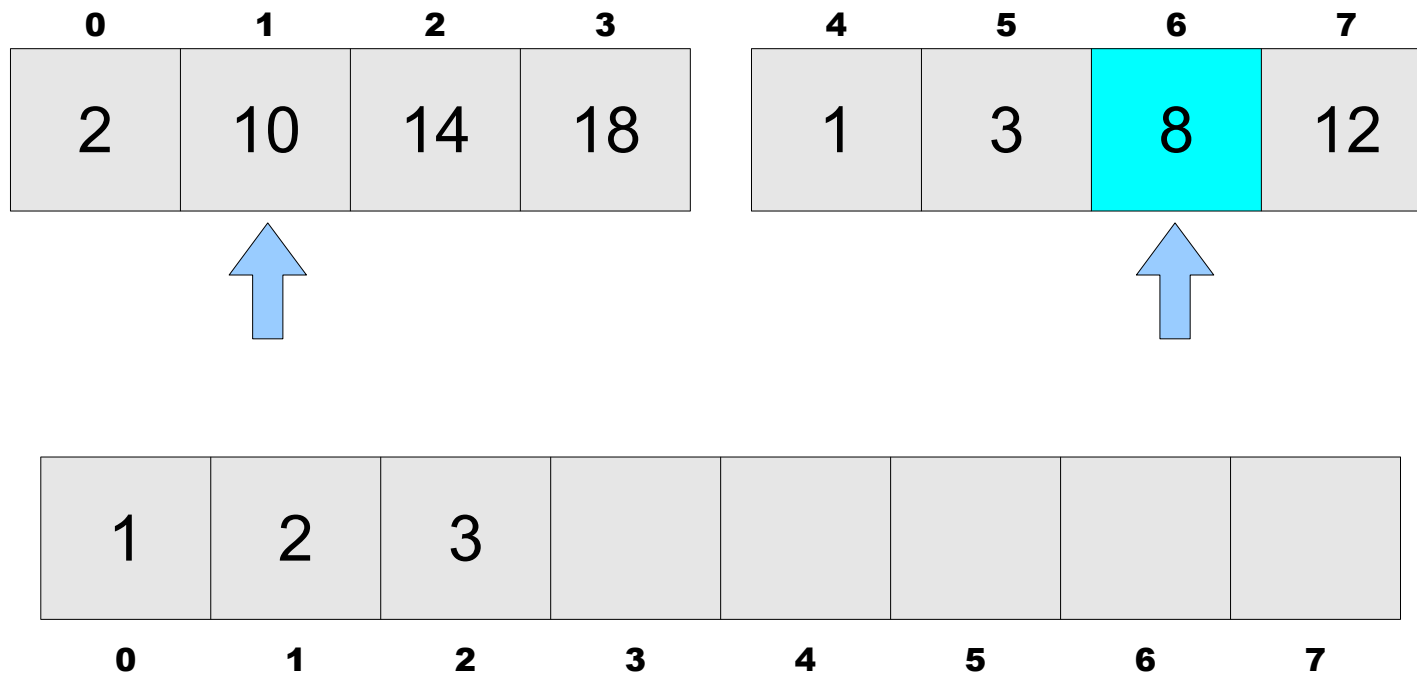
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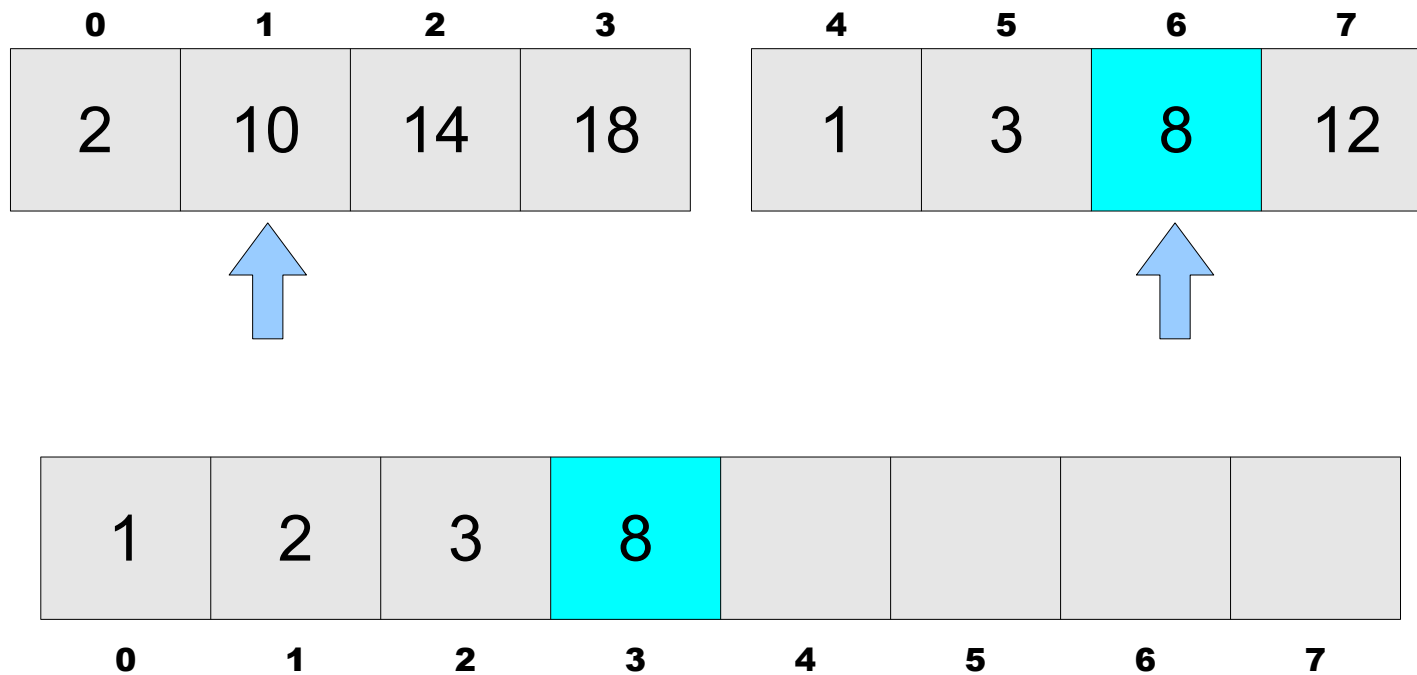
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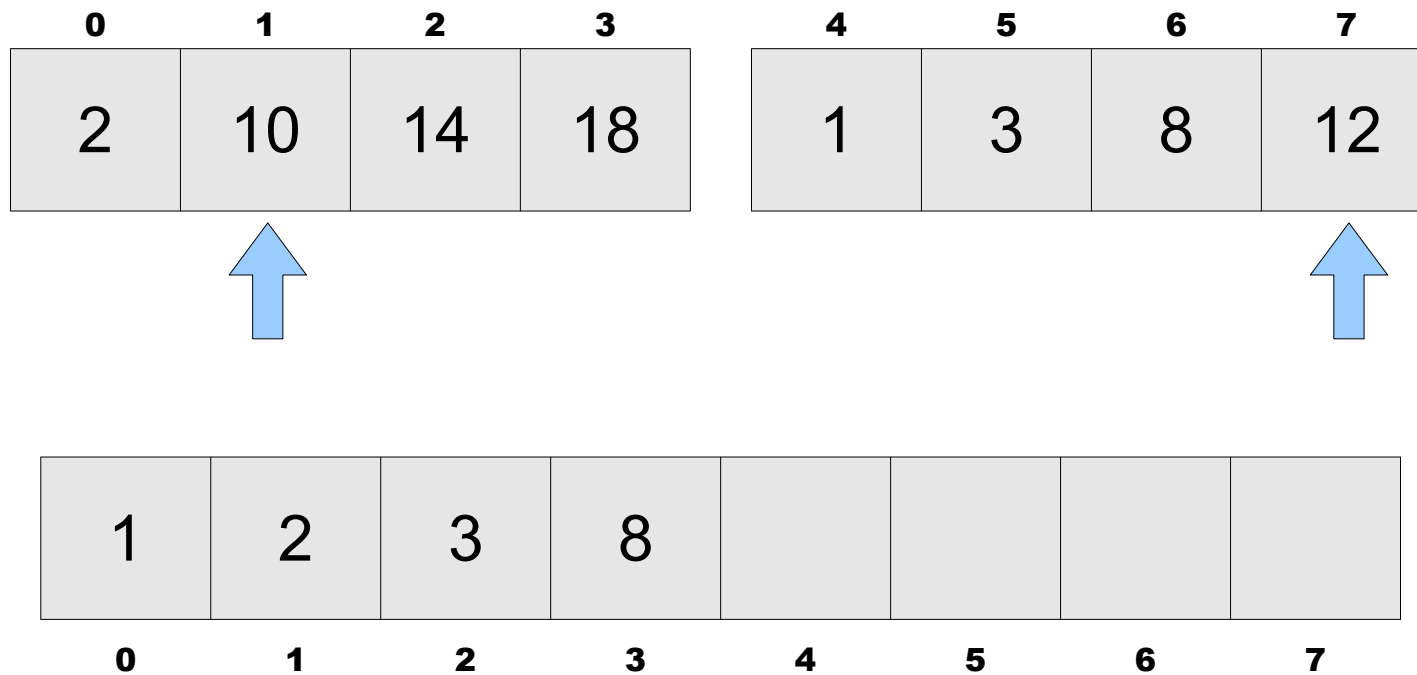
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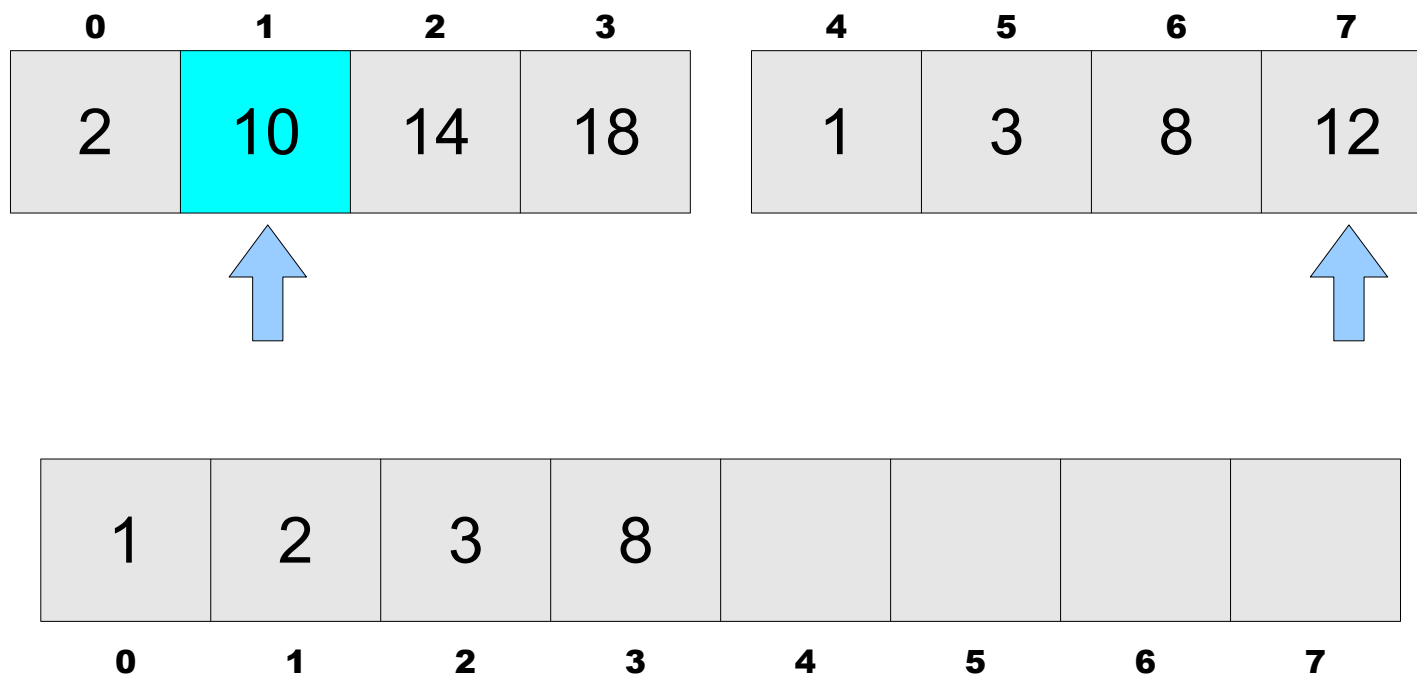
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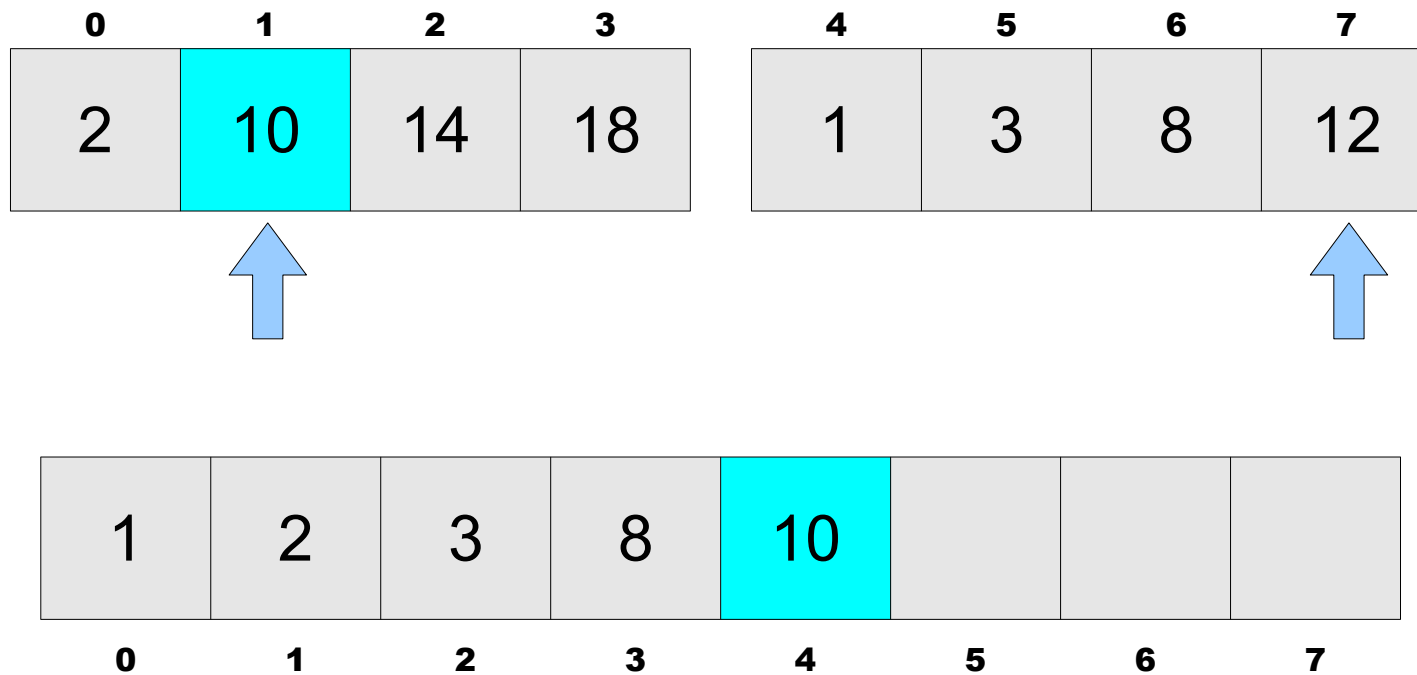
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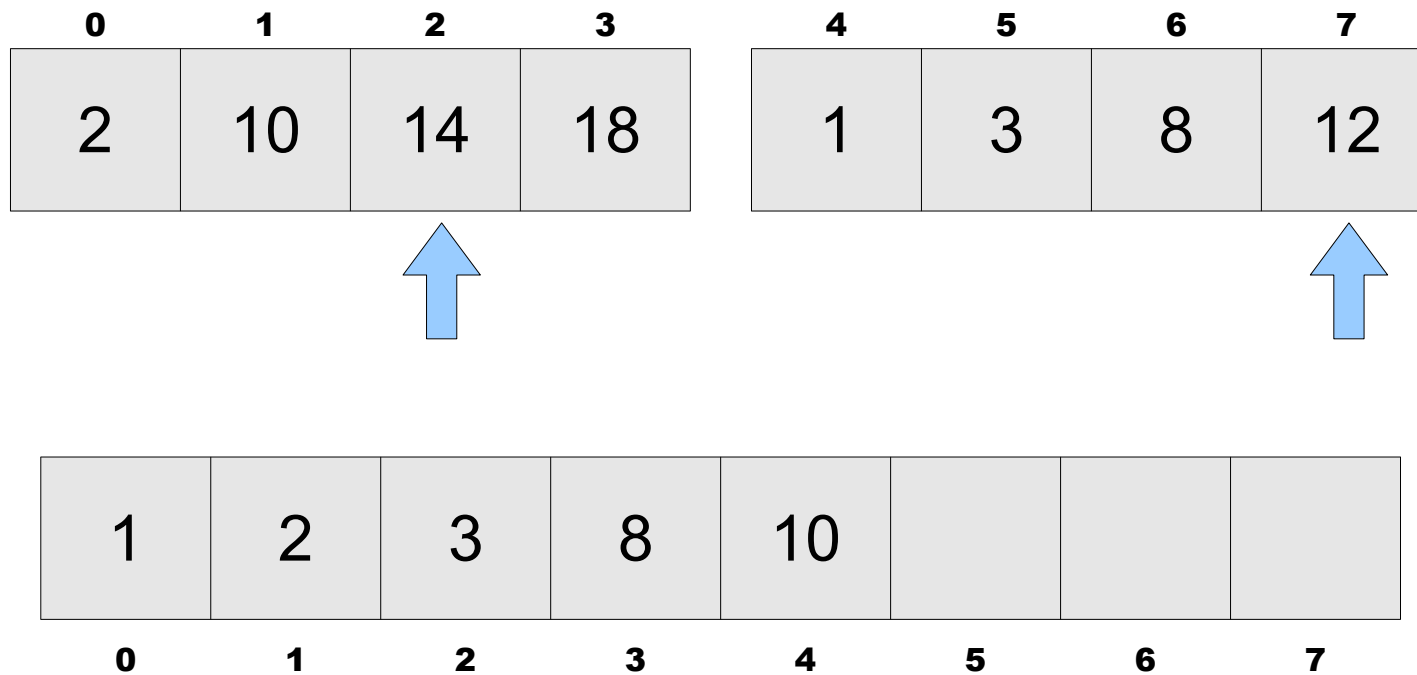
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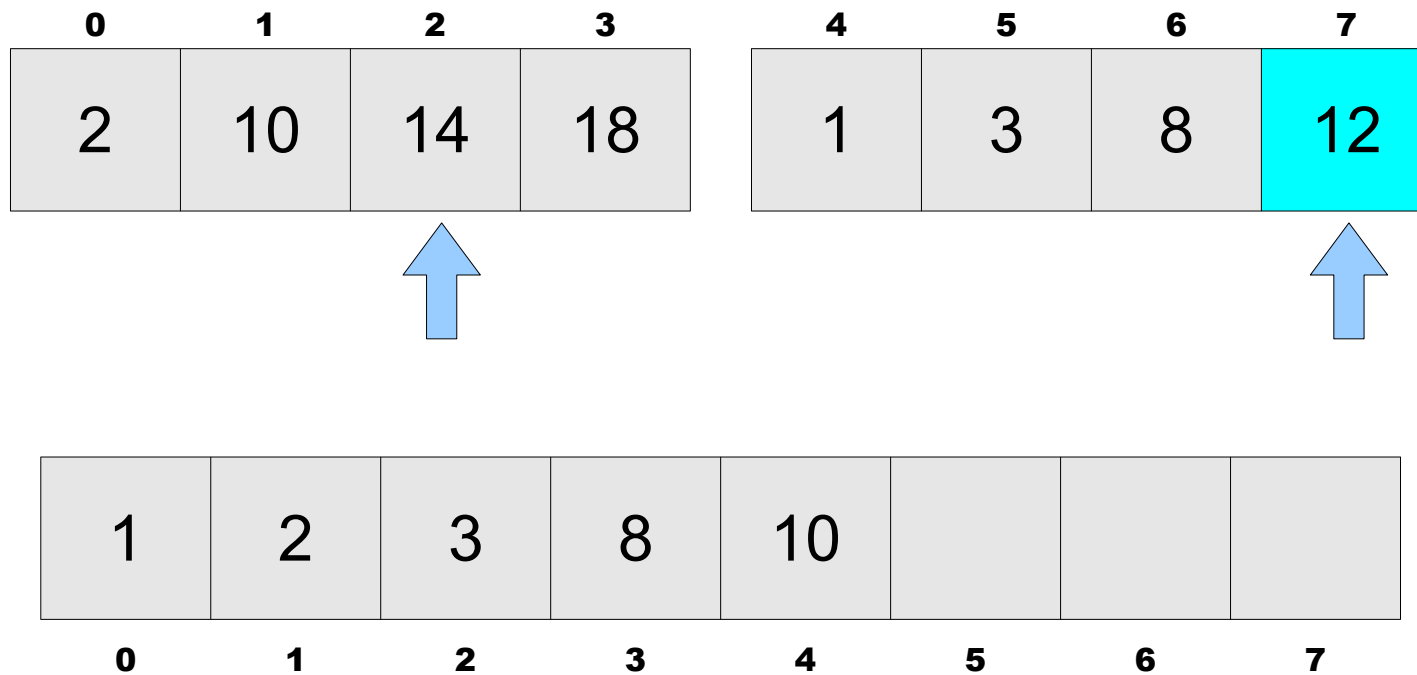
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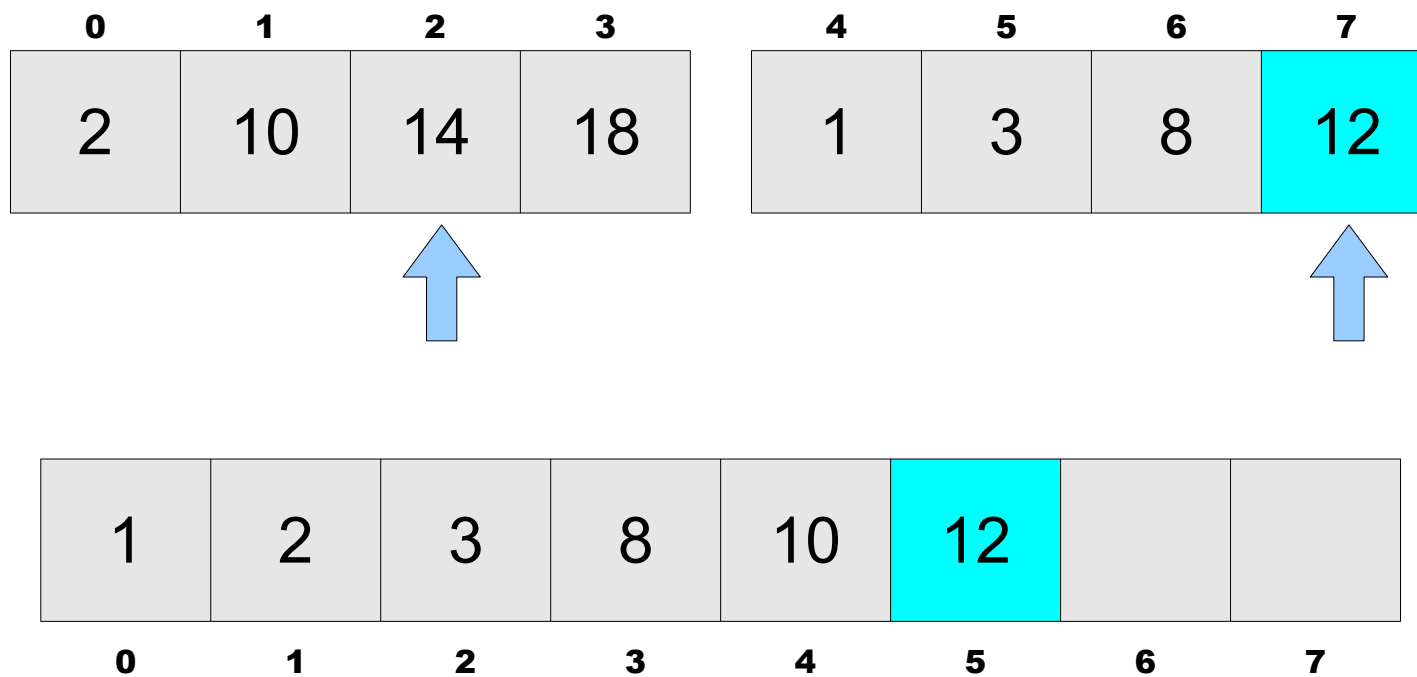
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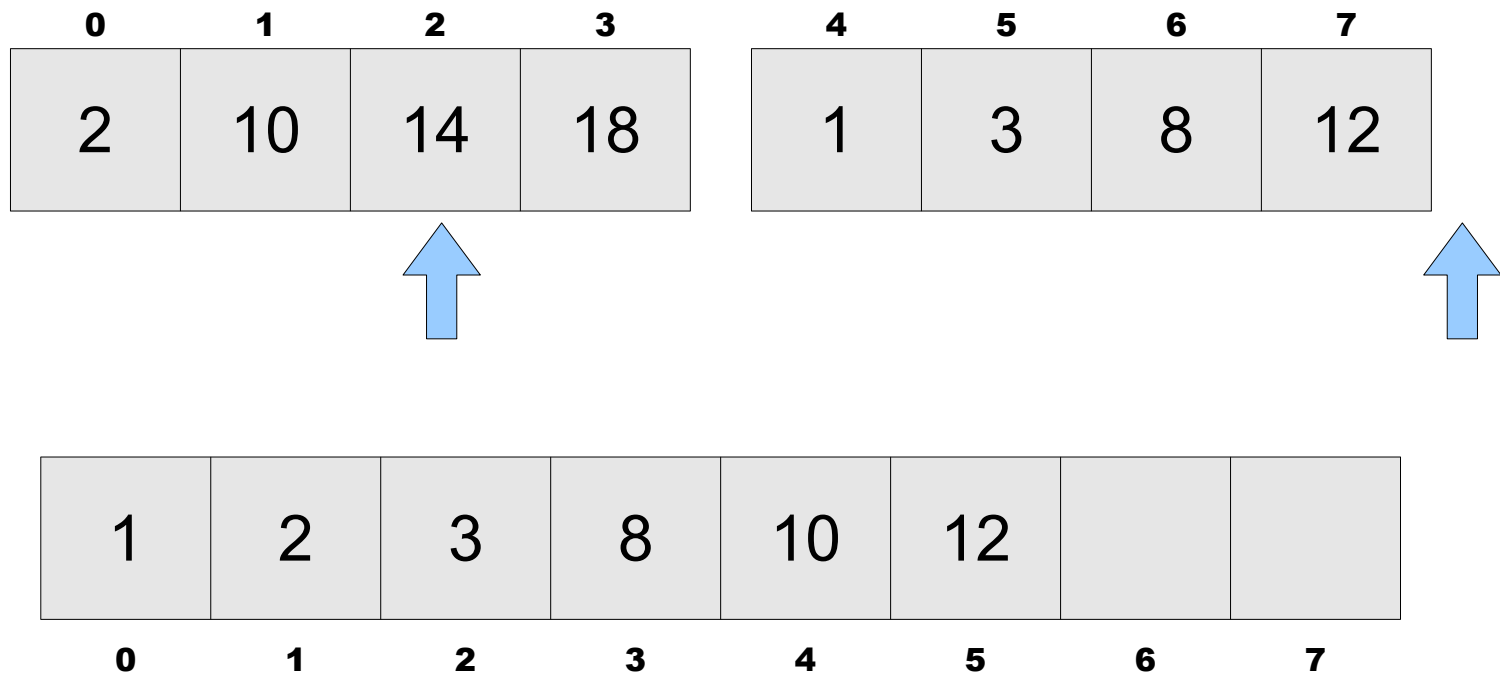
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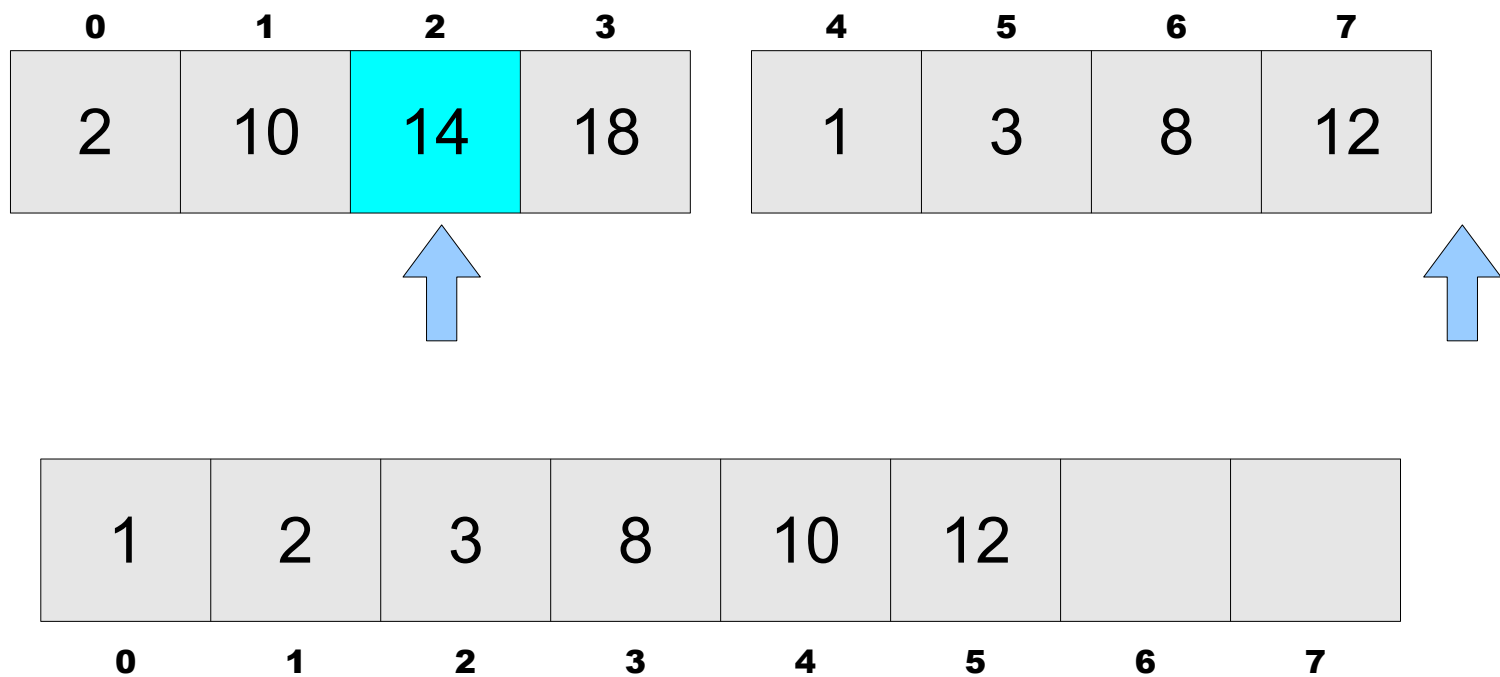
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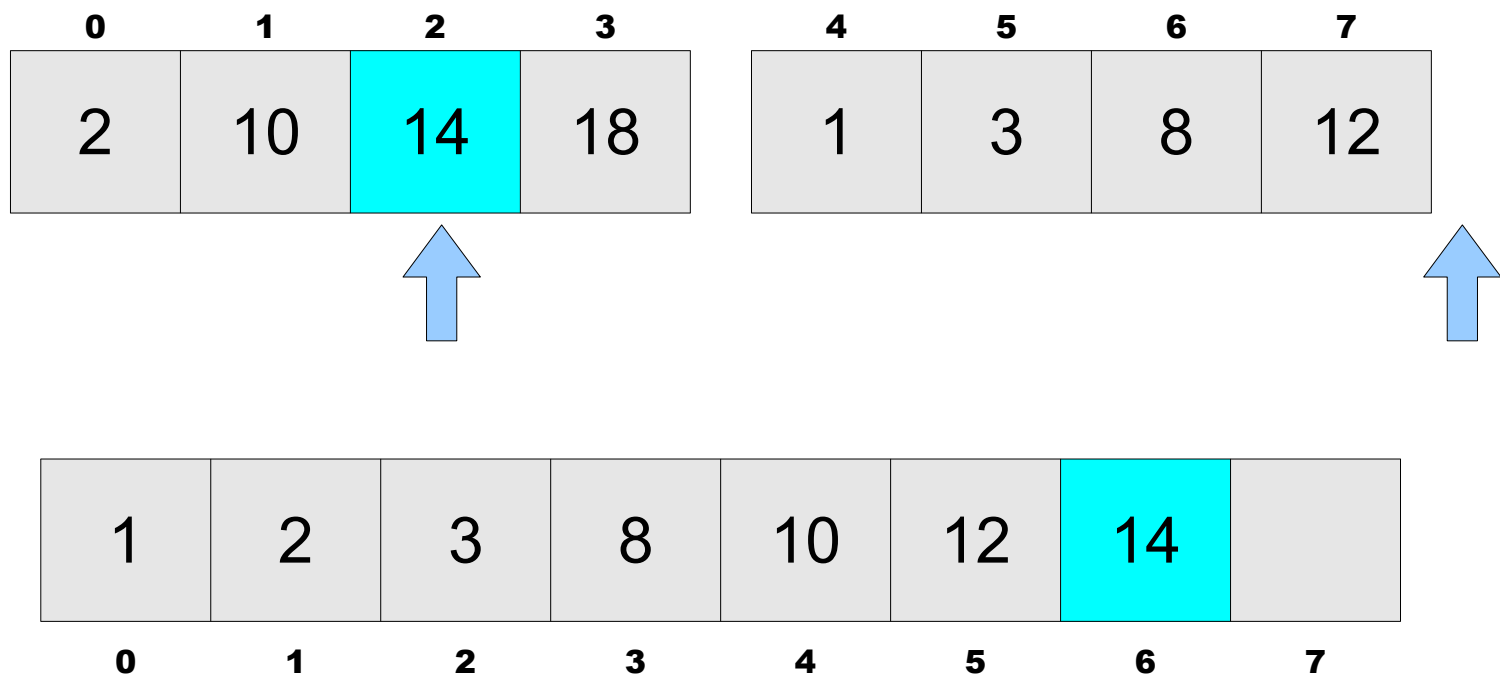
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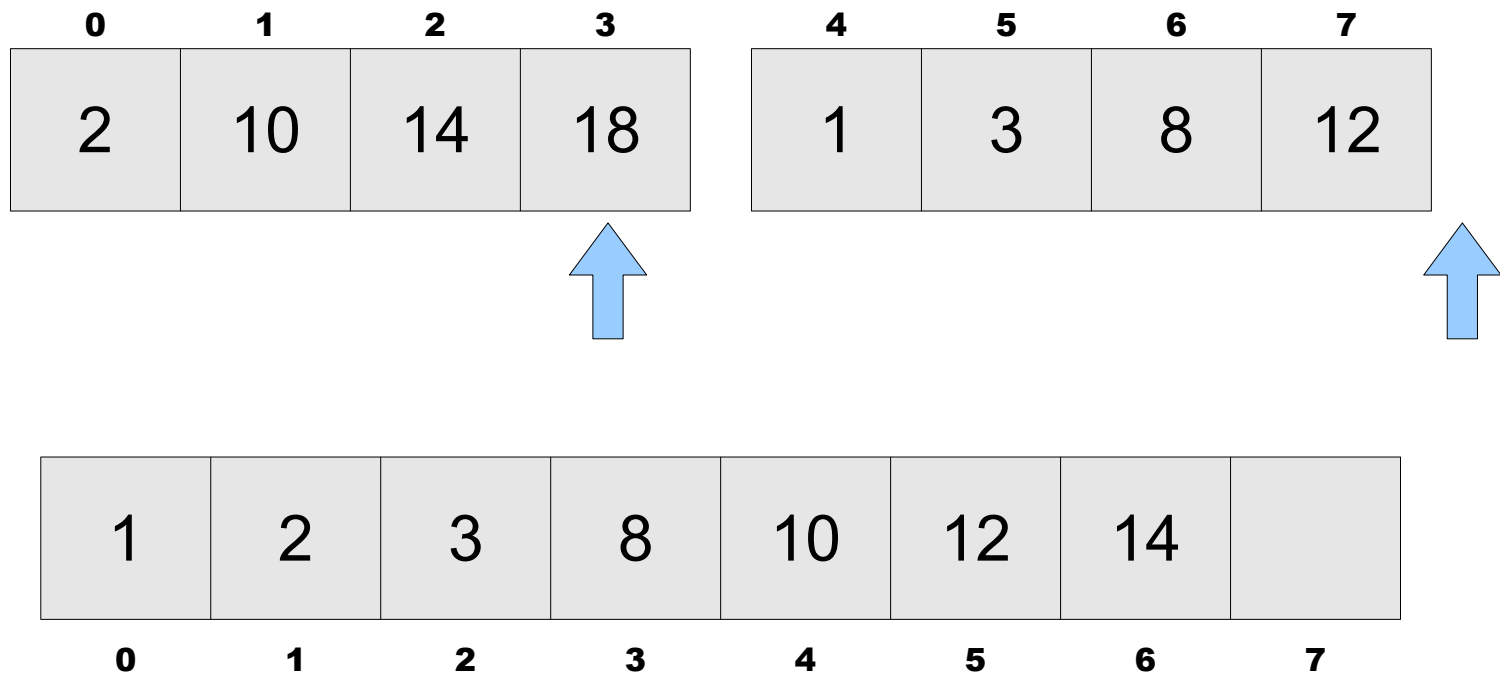
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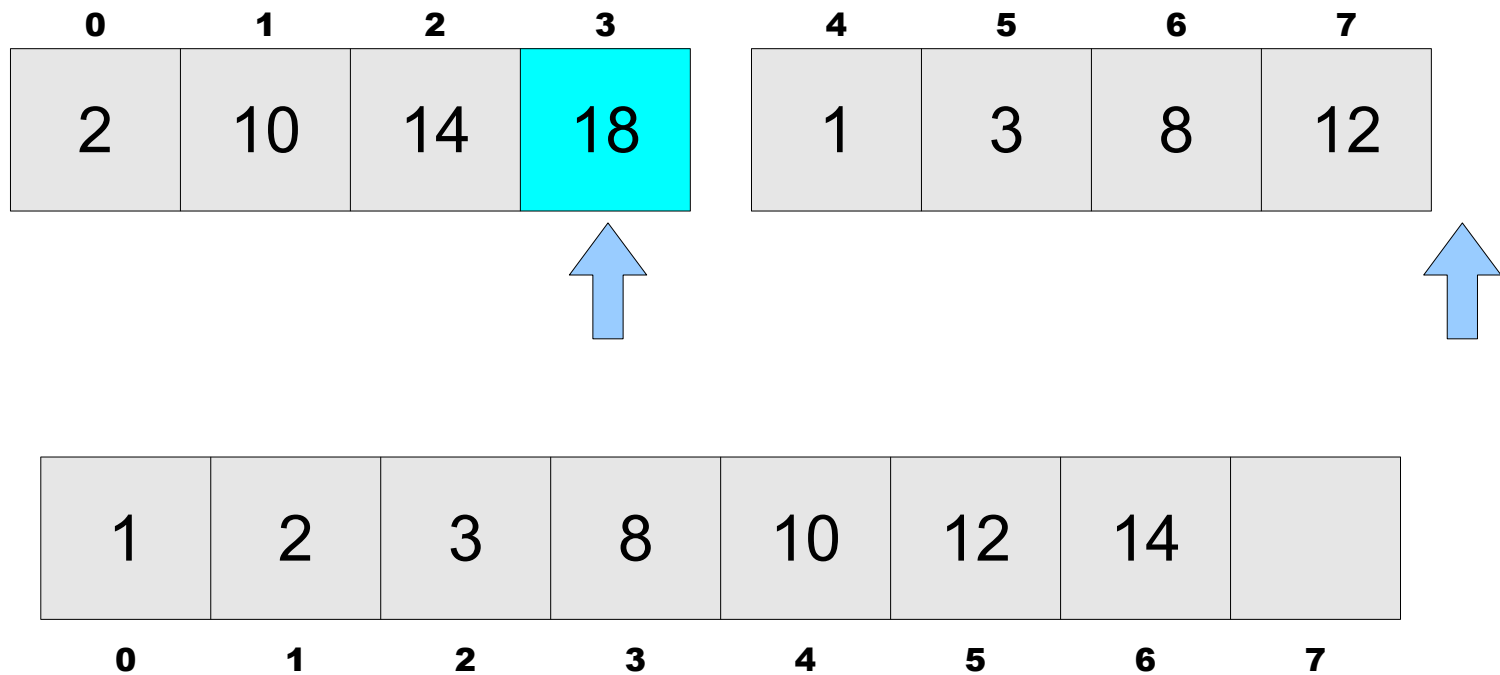
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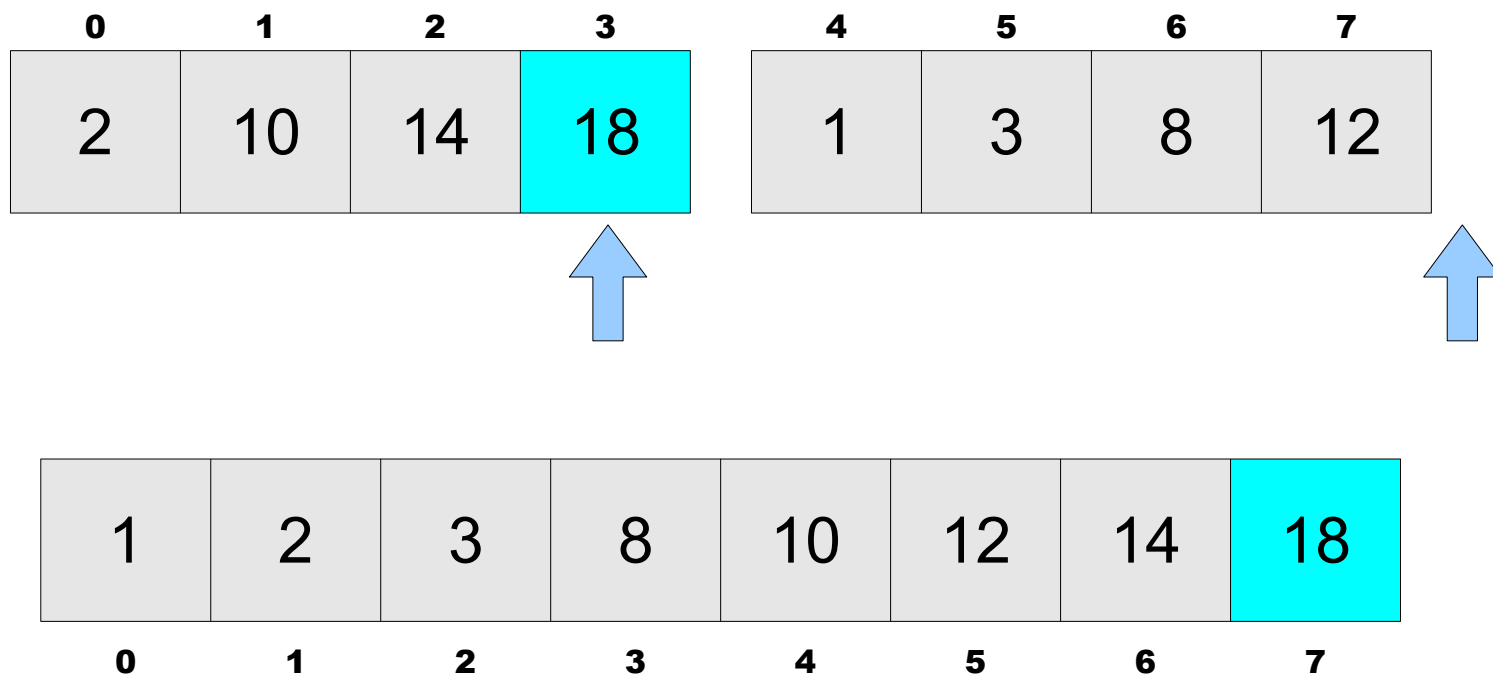
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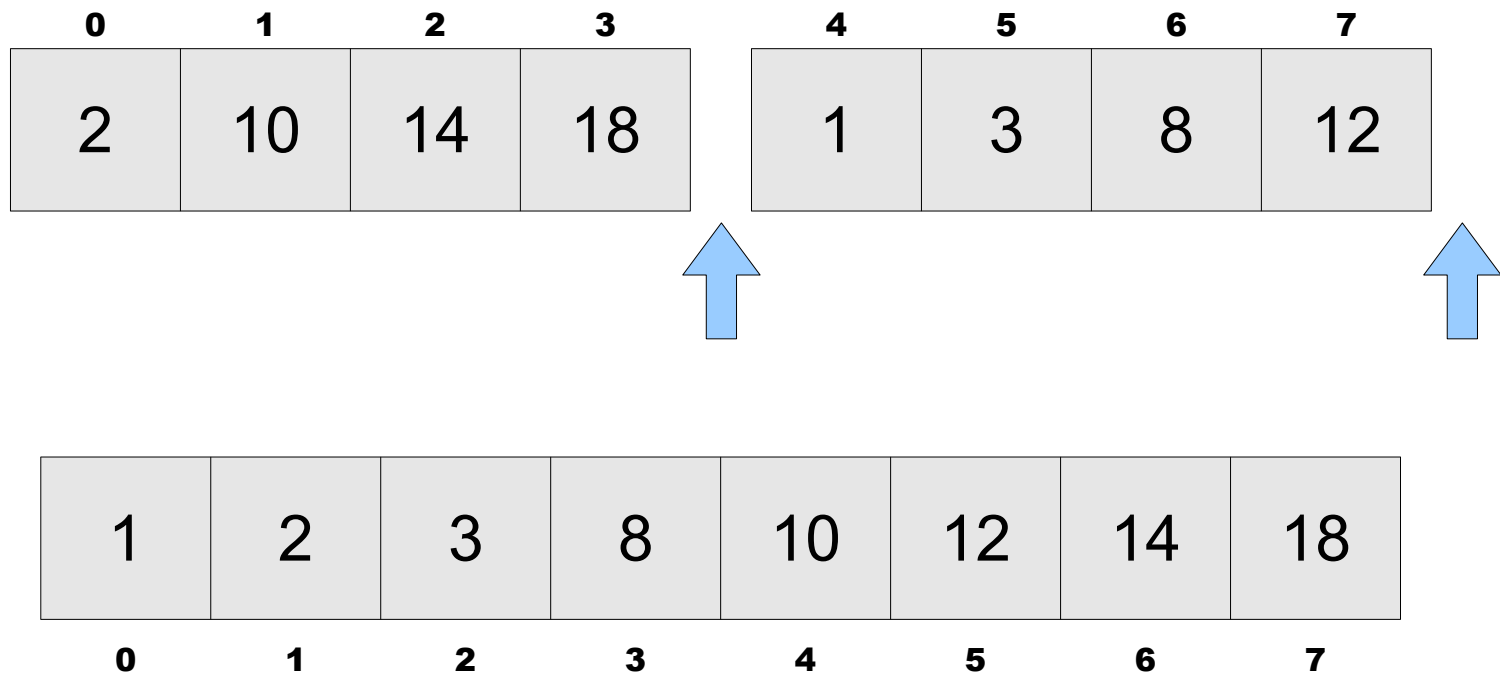
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TADA

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Merge Sort

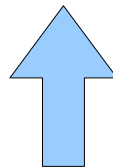
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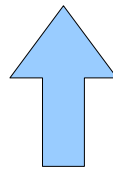


MID

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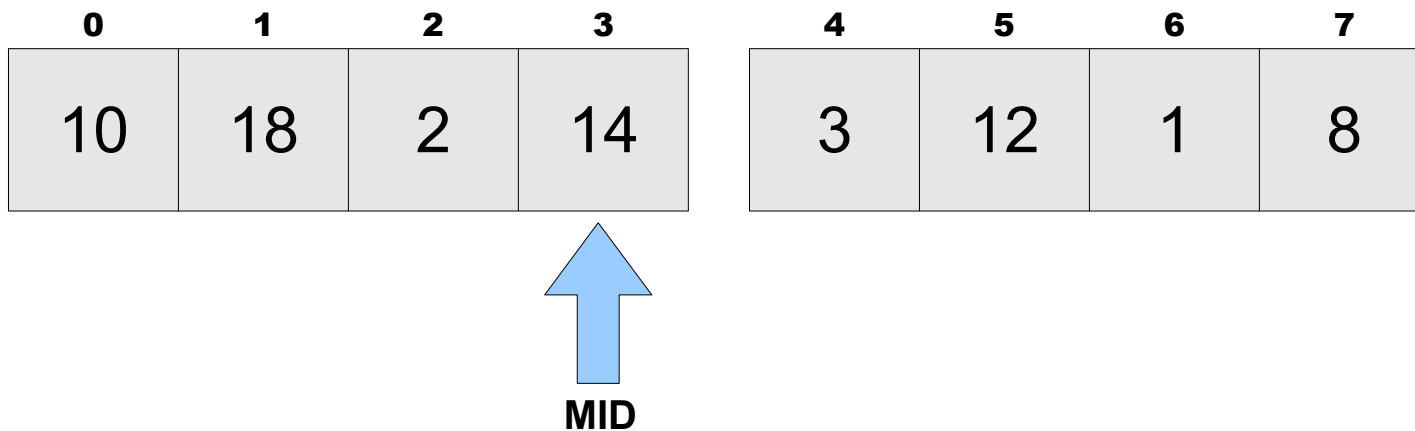


MID

Recall: $\text{mid} = \text{lo} + (\text{hi} - \text{lo}) / 2;$

Merge Sort

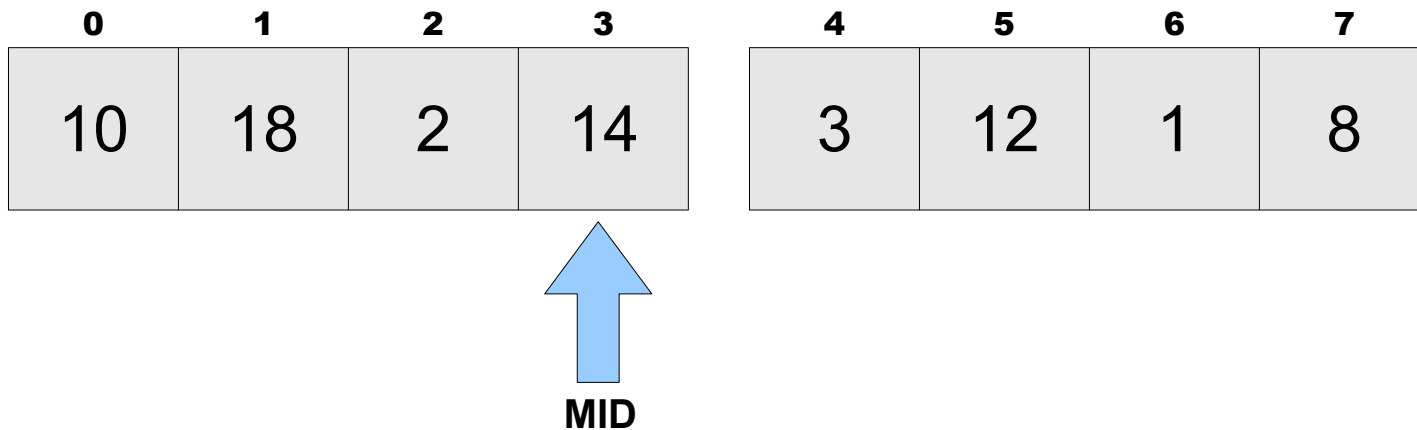
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Recall: $\text{mid} = \text{lo} + (\text{hi} - \text{lo}) / 2;$

mergeSort(v, lo, hi) →

- mergeSort(v, lo, mid)**
- mergeSort(v, mid + 1, hi)**

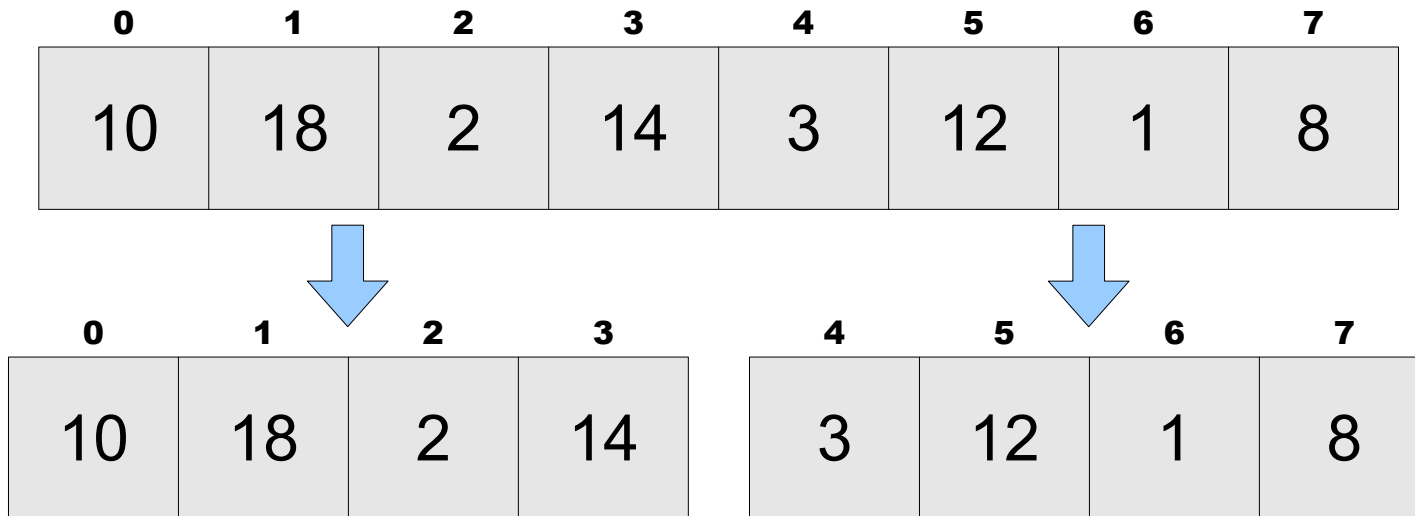
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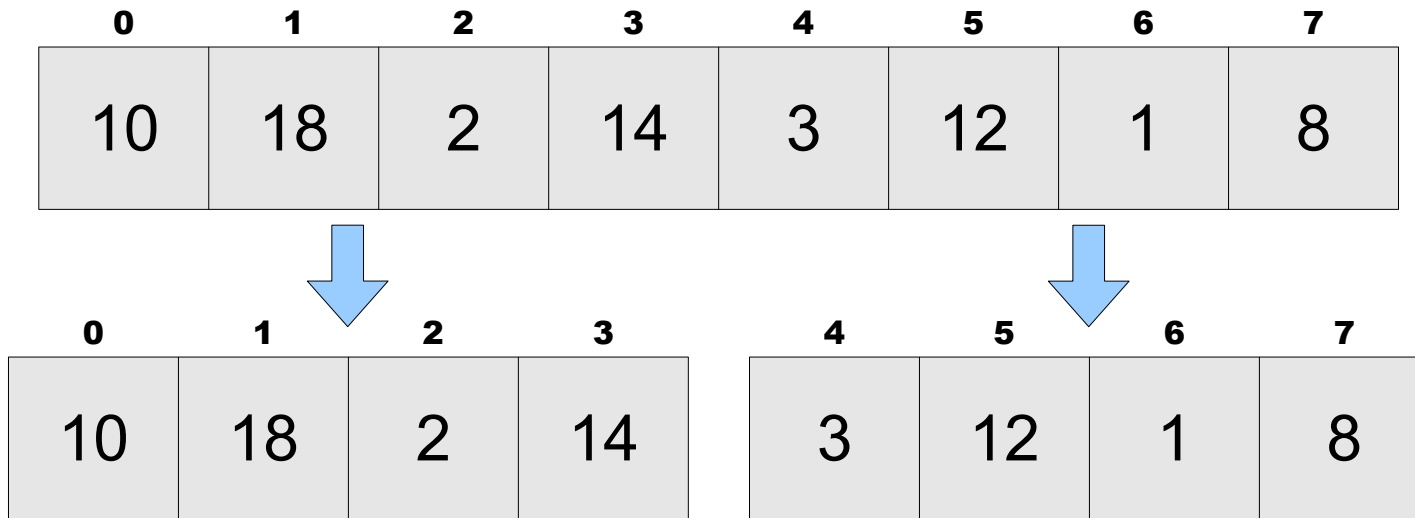
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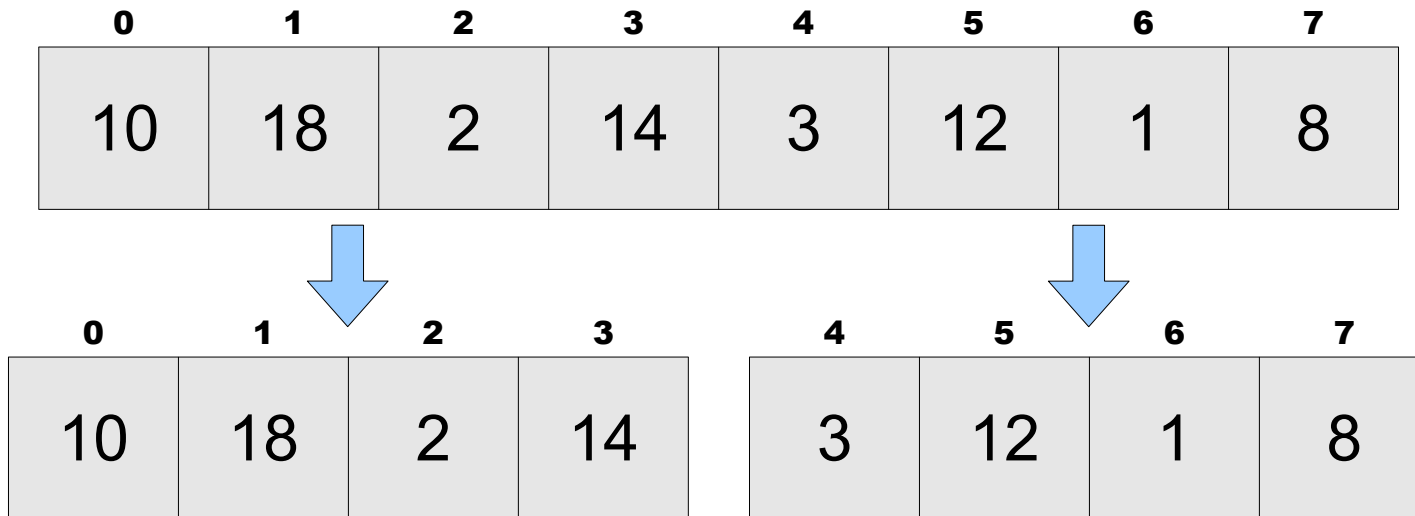


mergeSort(v, 0, 7) →

- mergeSort(v, 0, 3)**
- mergeSort(v, 4, 7)**

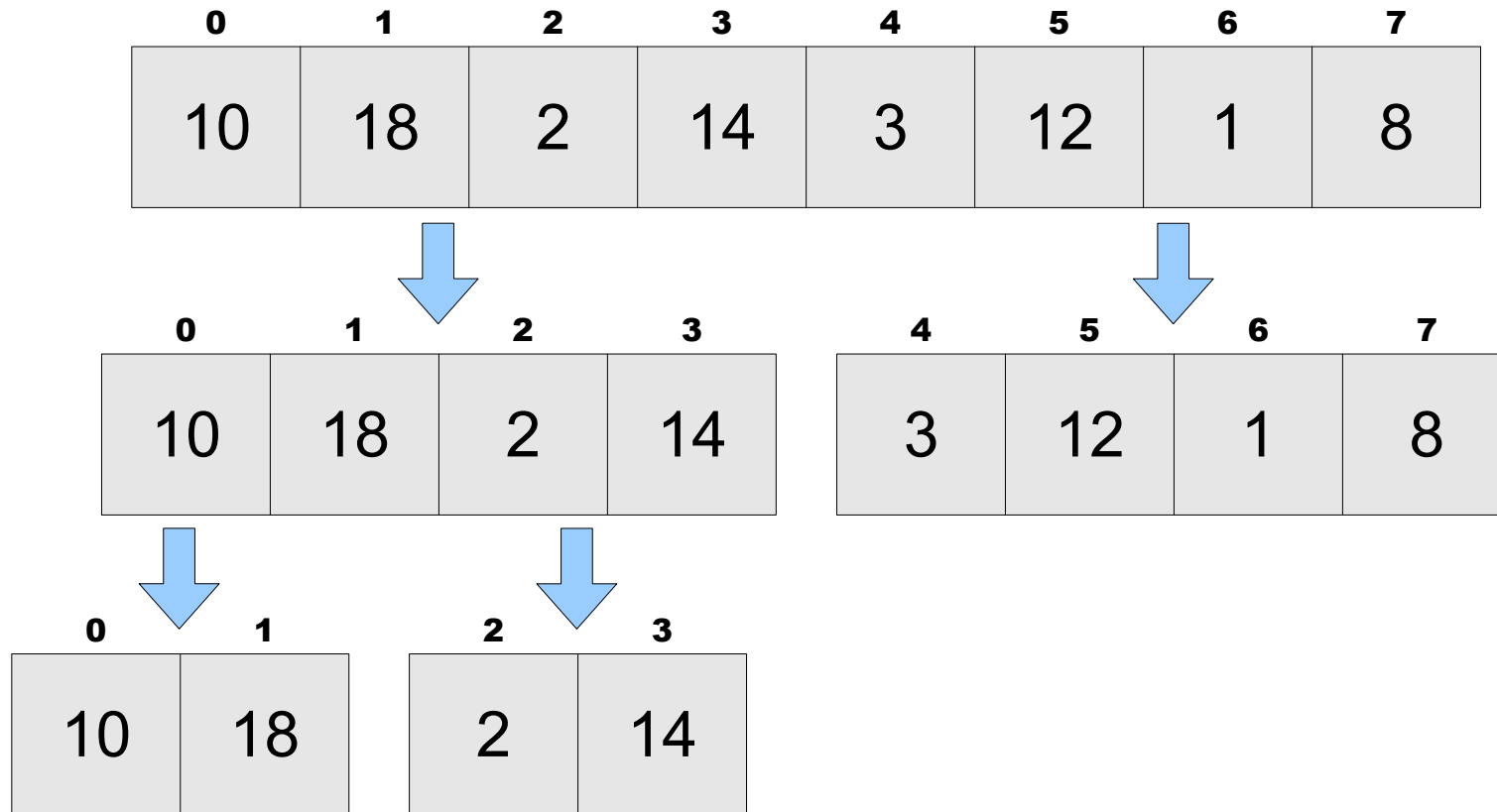
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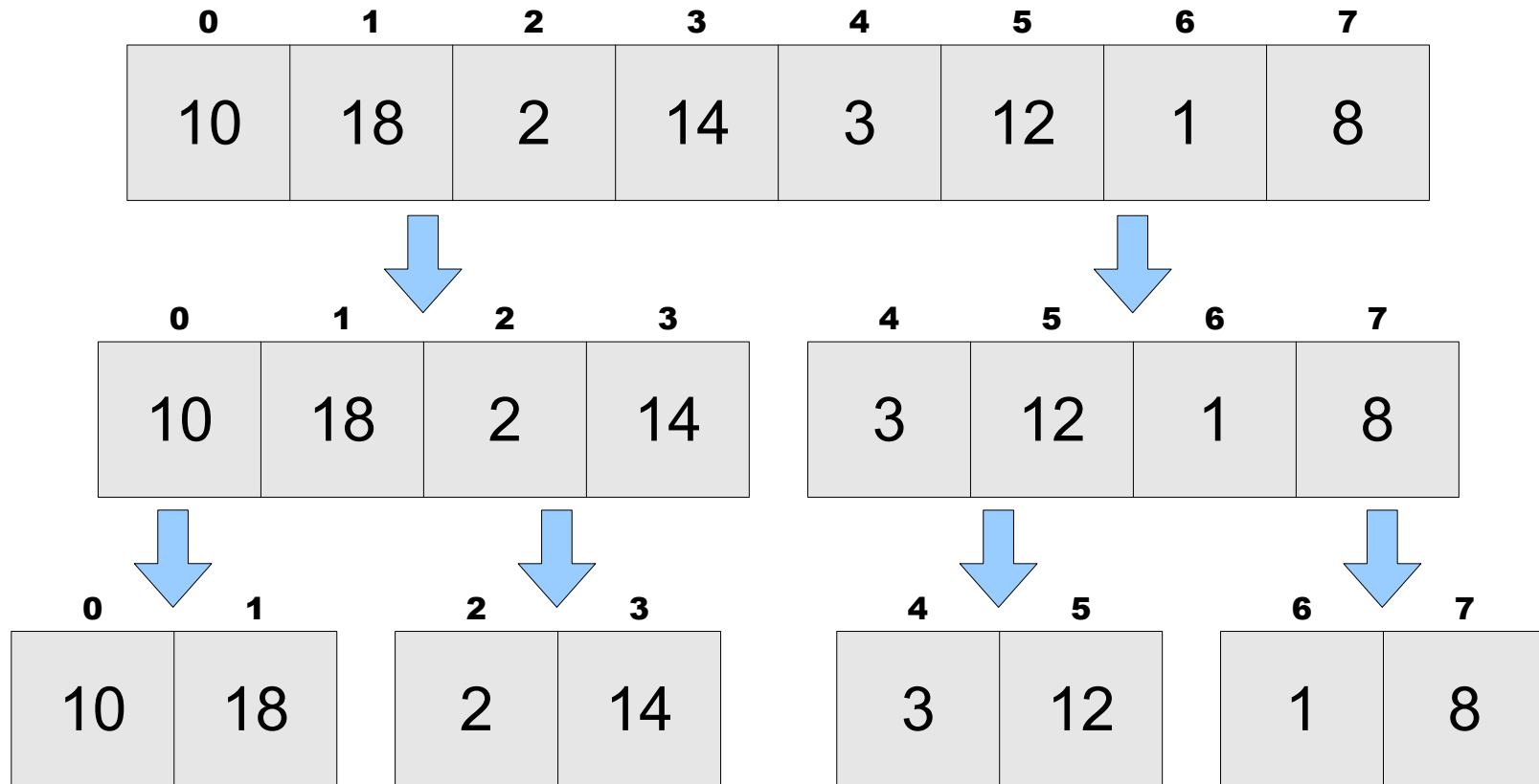
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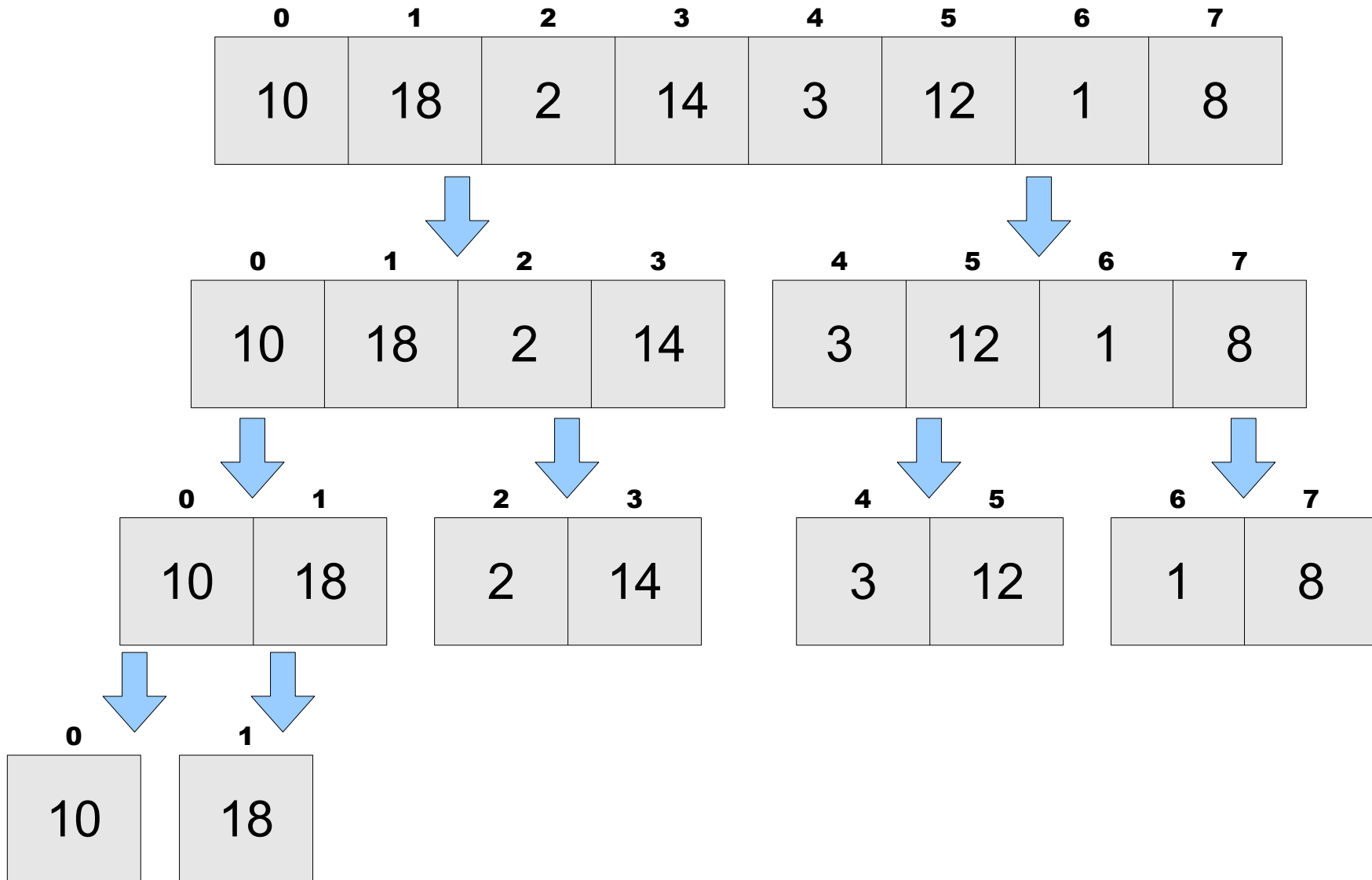
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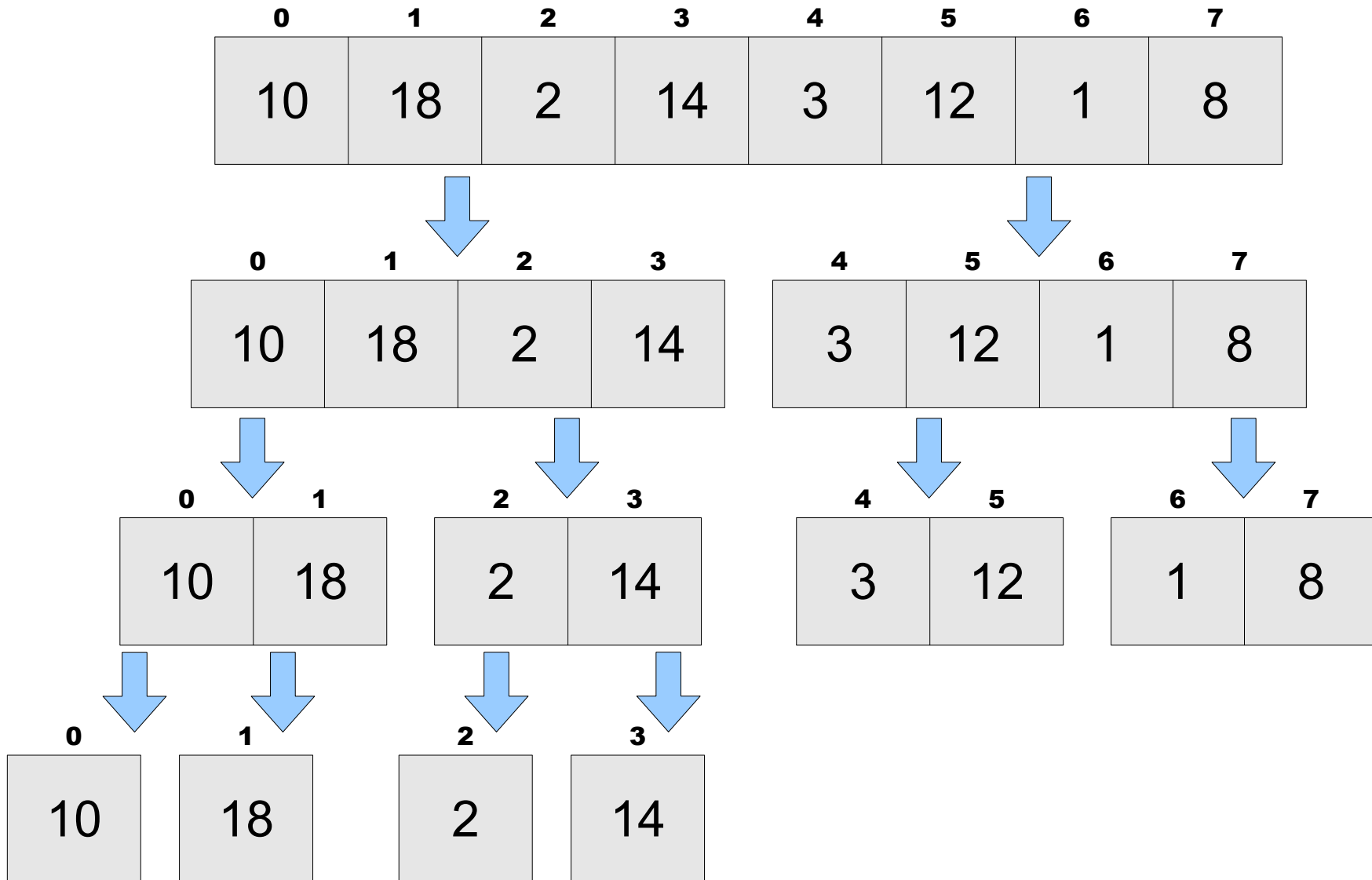
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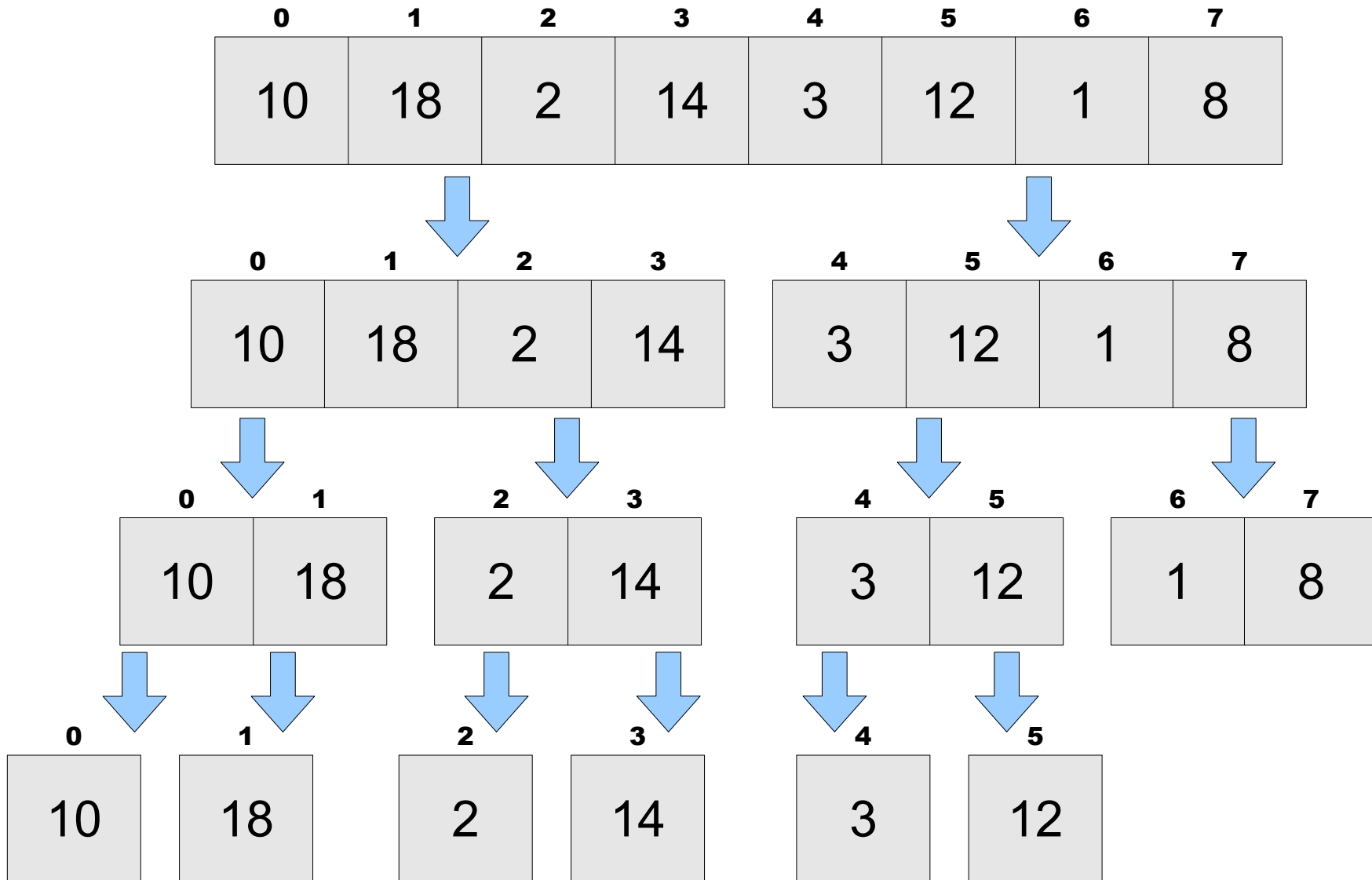
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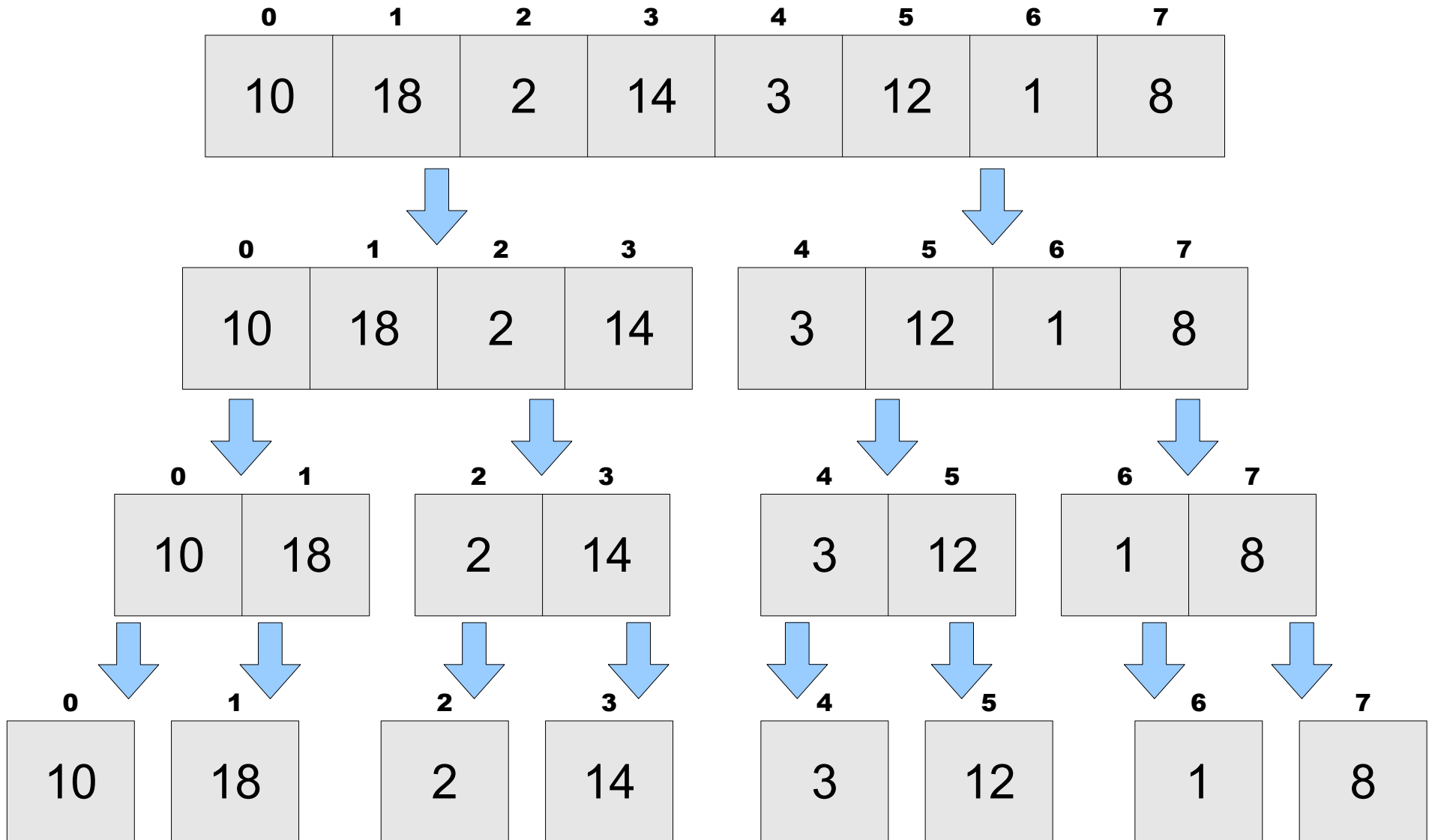
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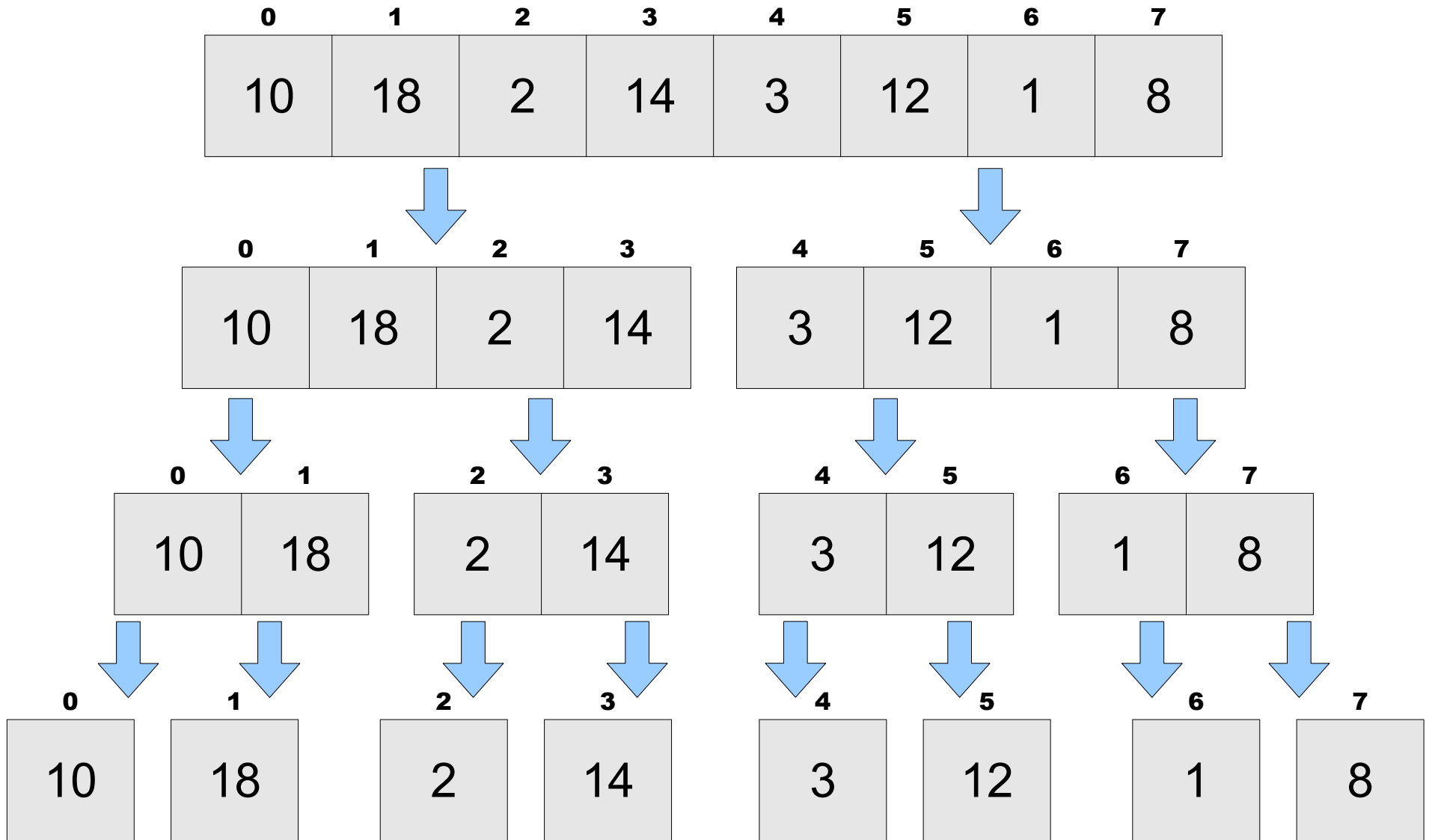
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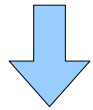
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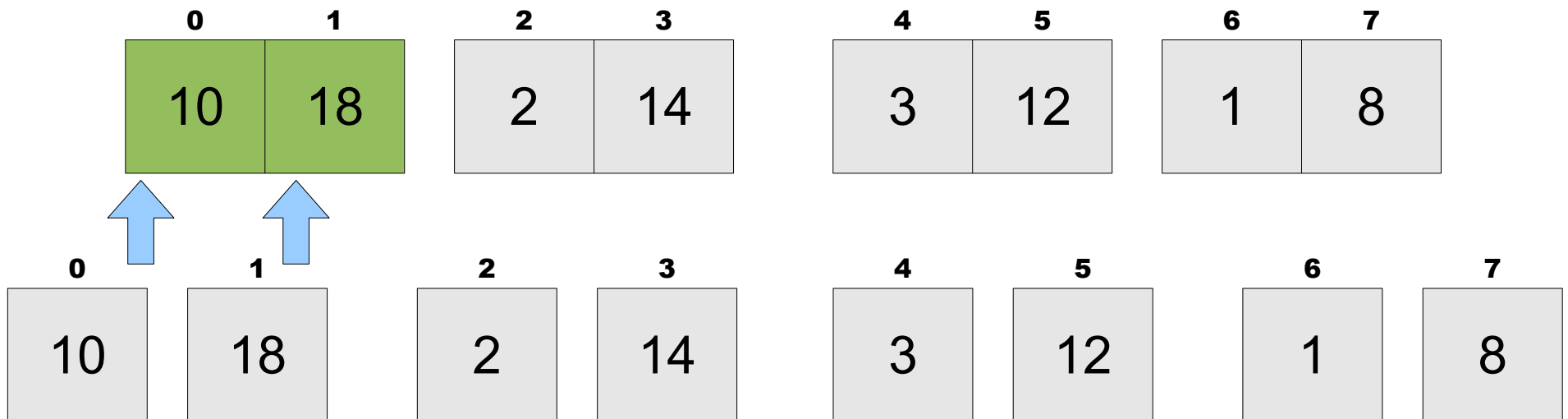
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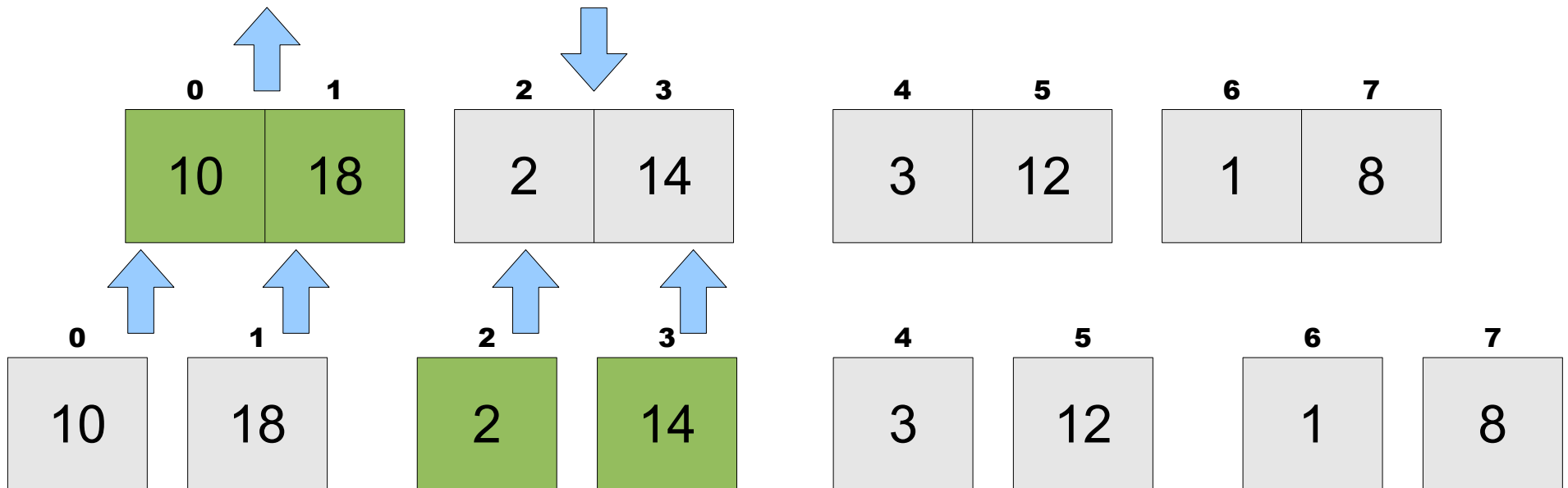
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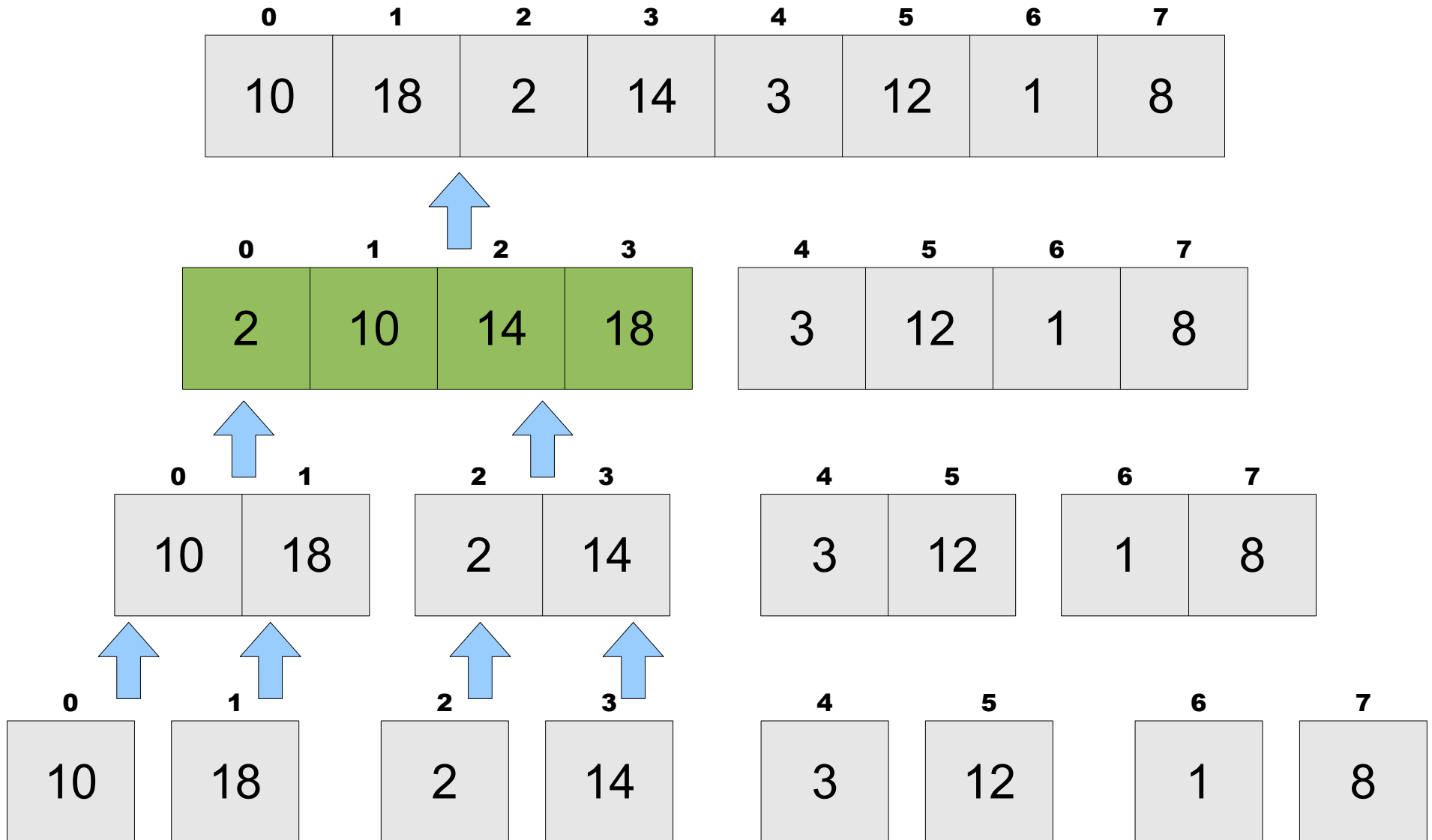
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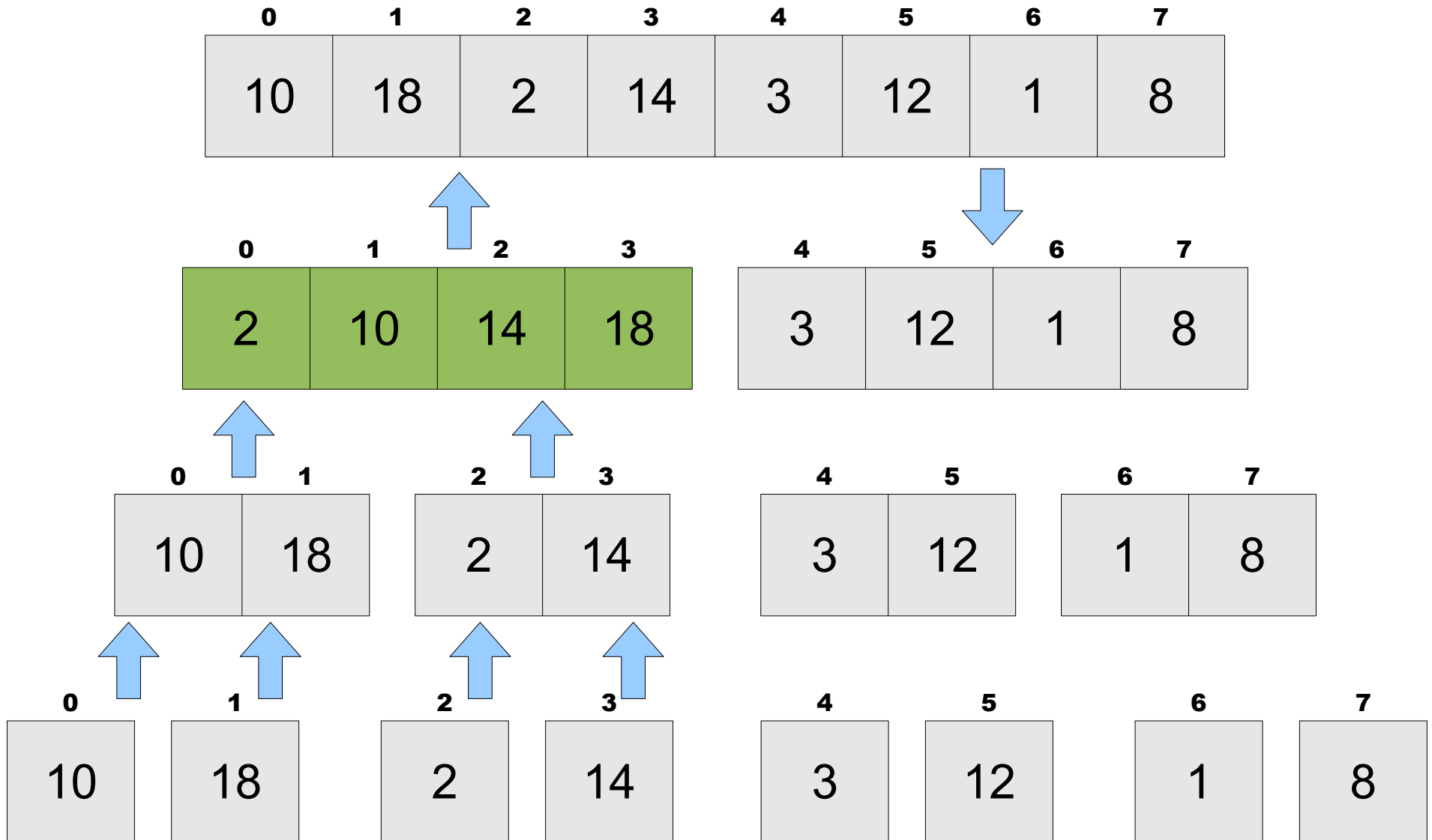
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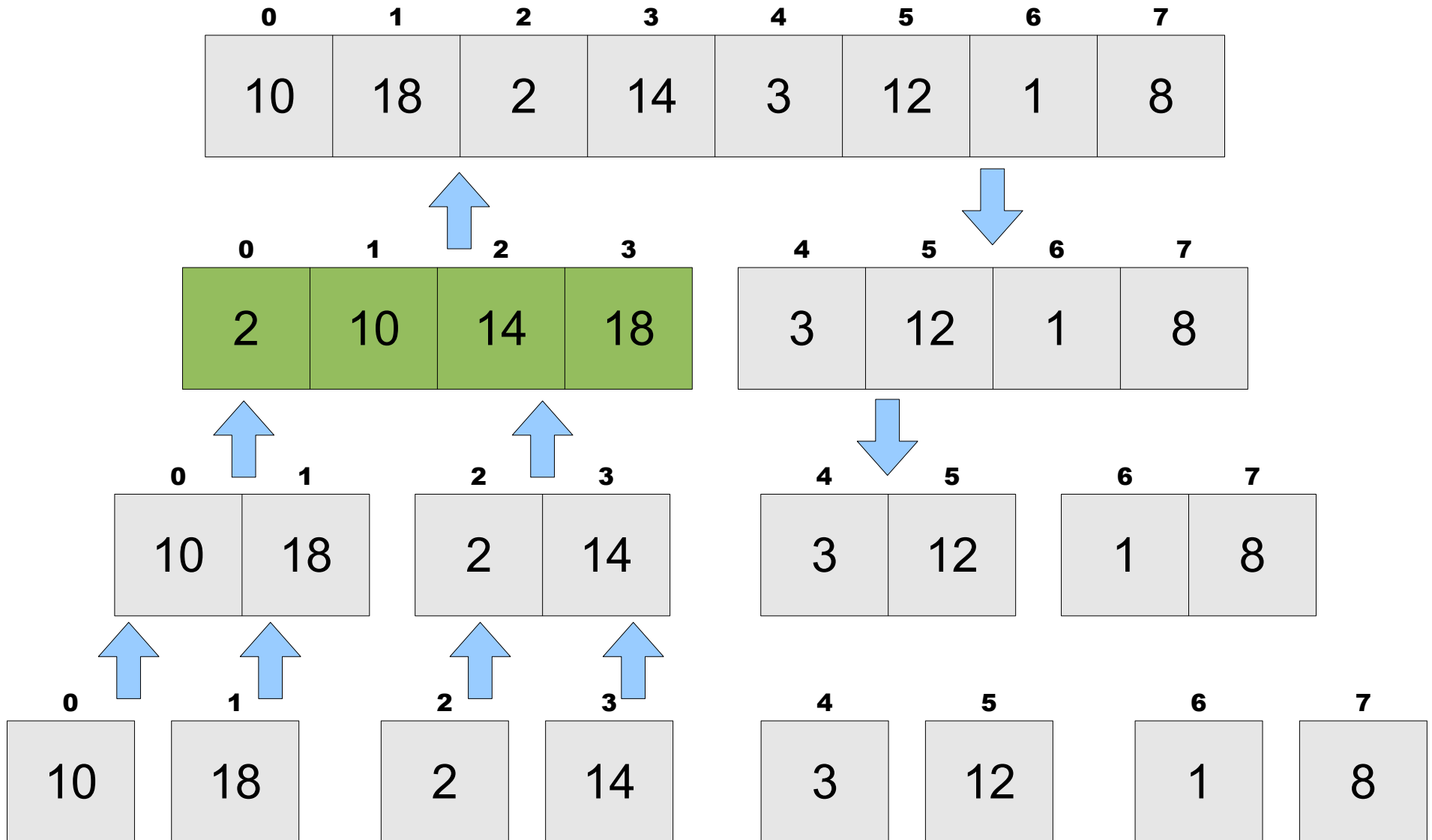
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Merge Sort

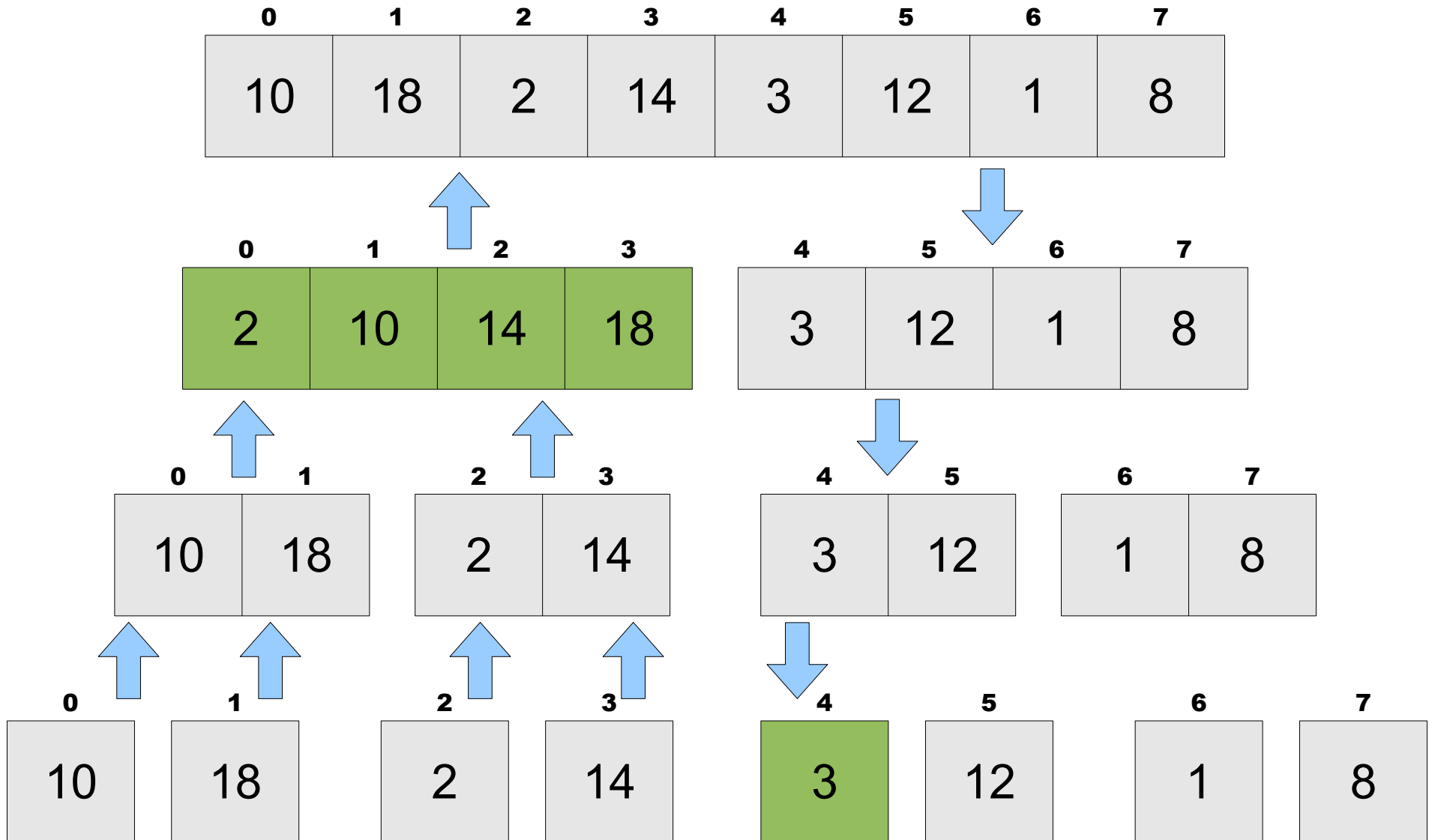
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

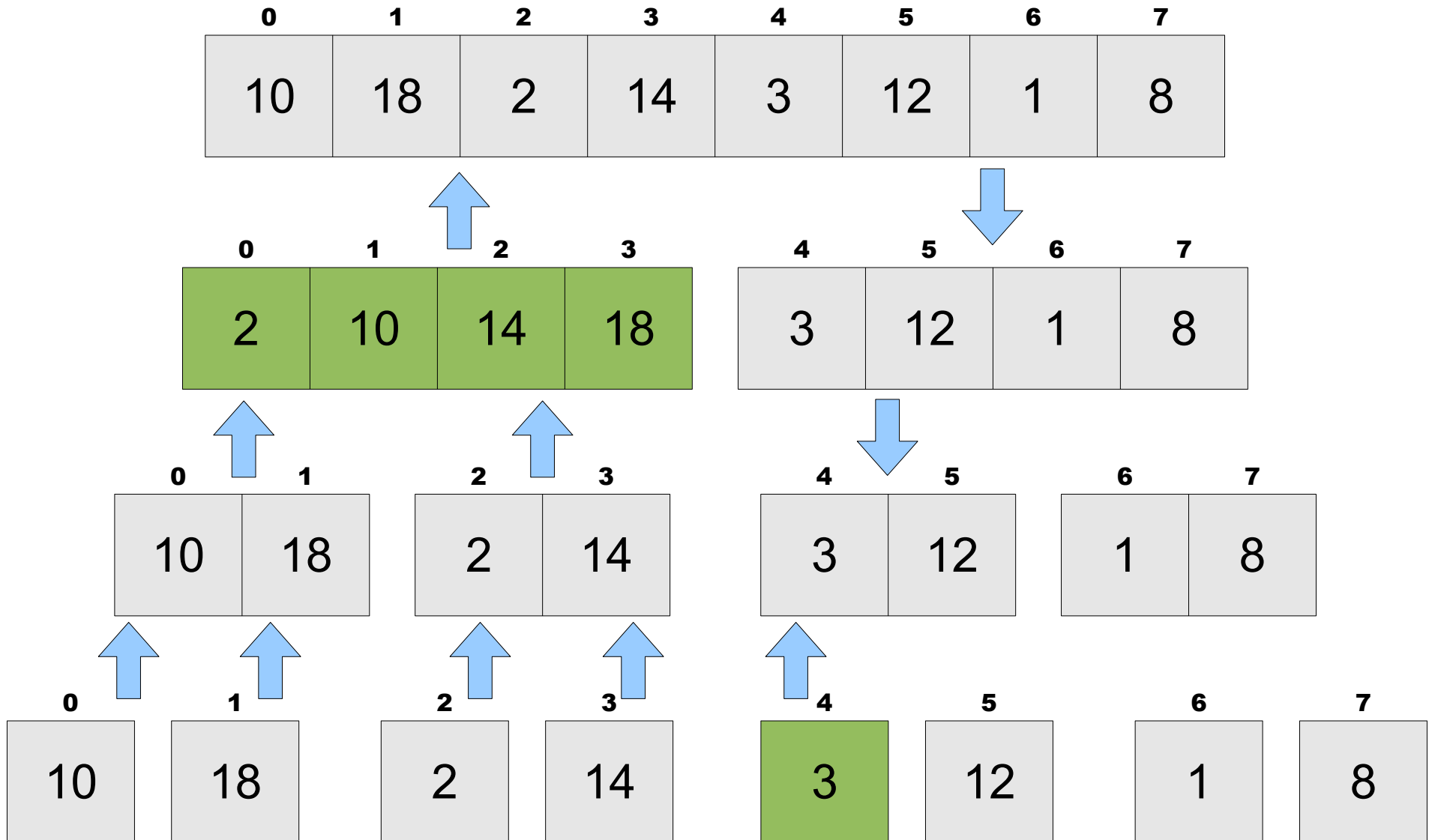
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

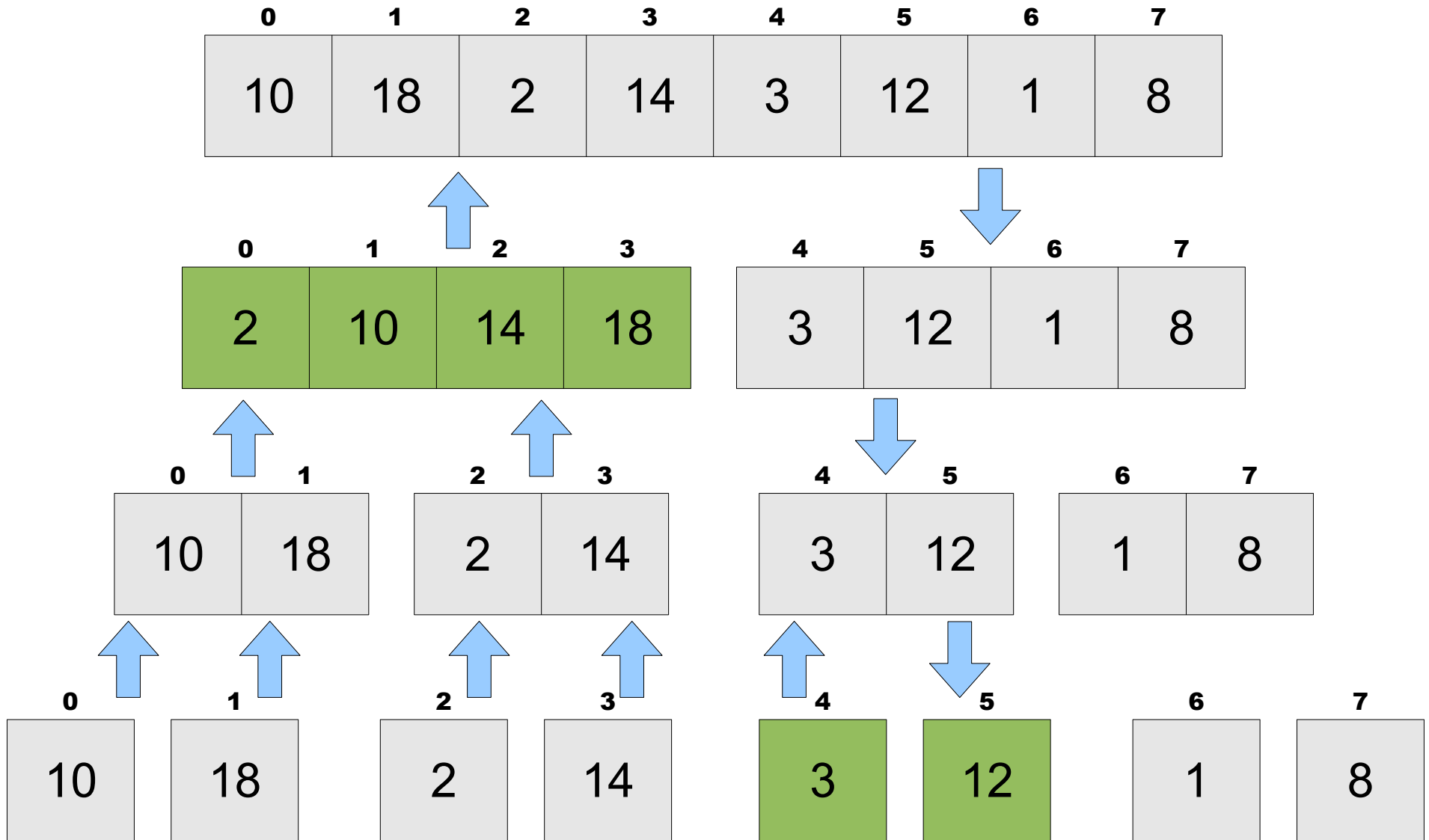
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

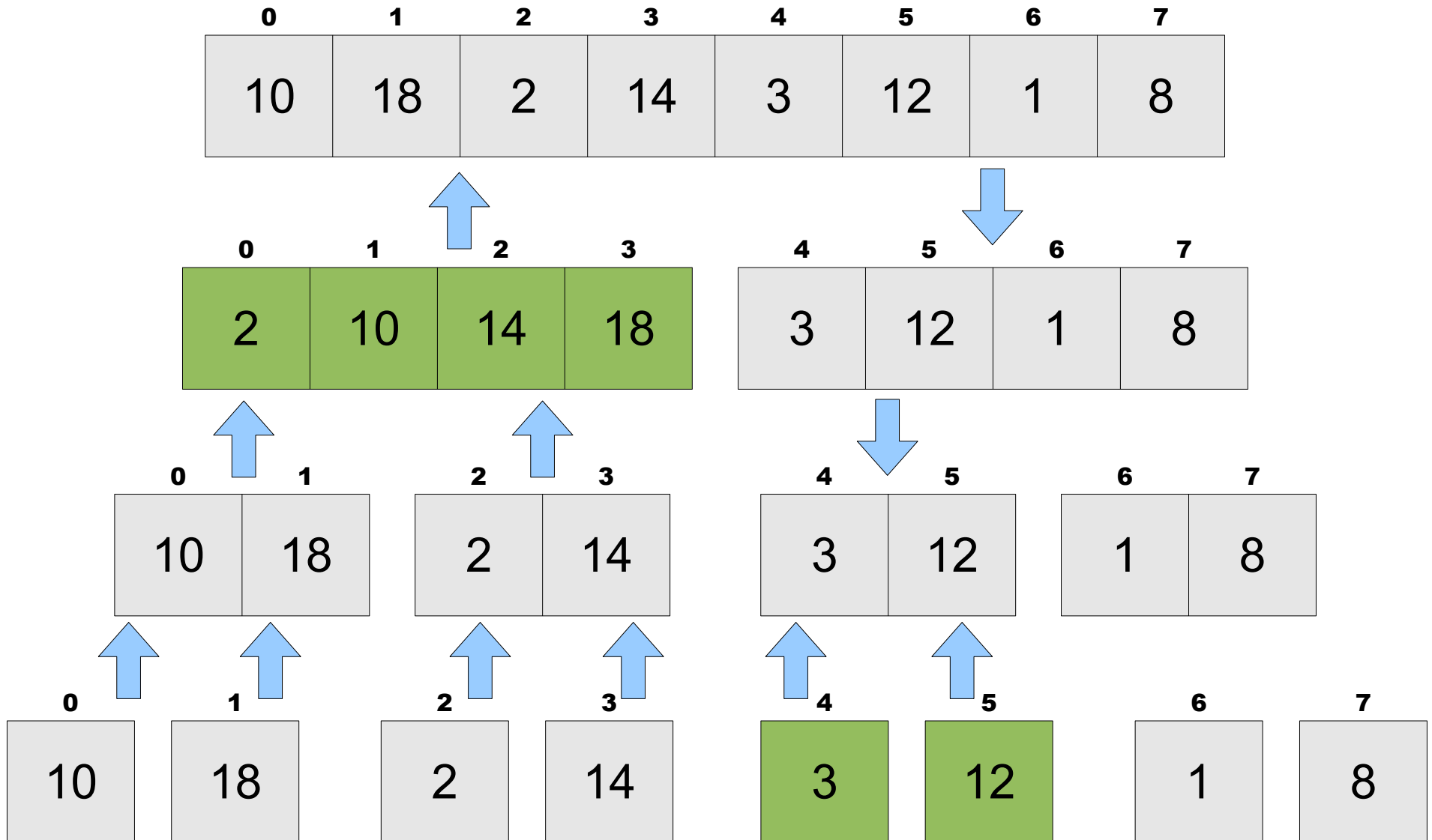
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

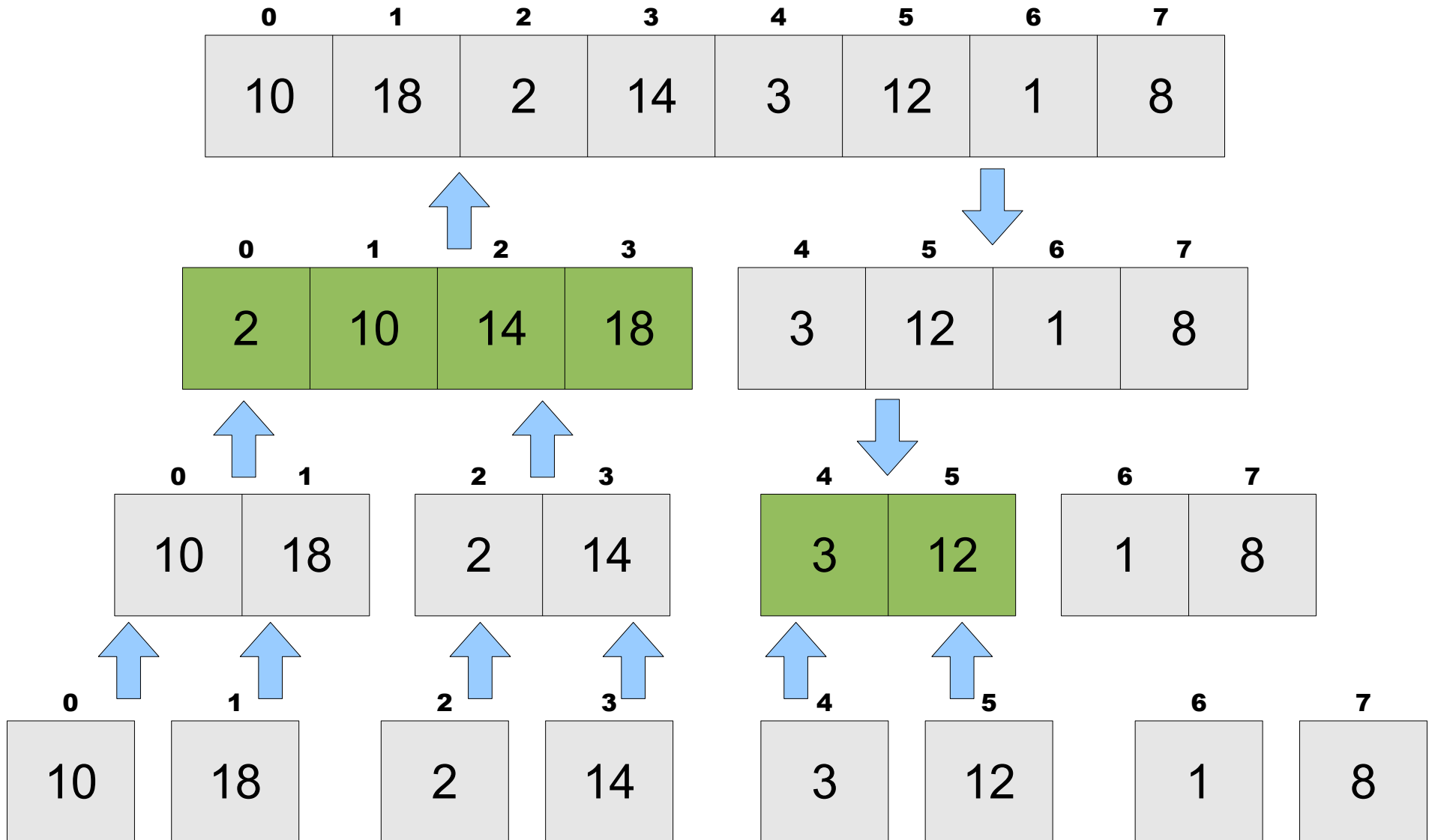
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

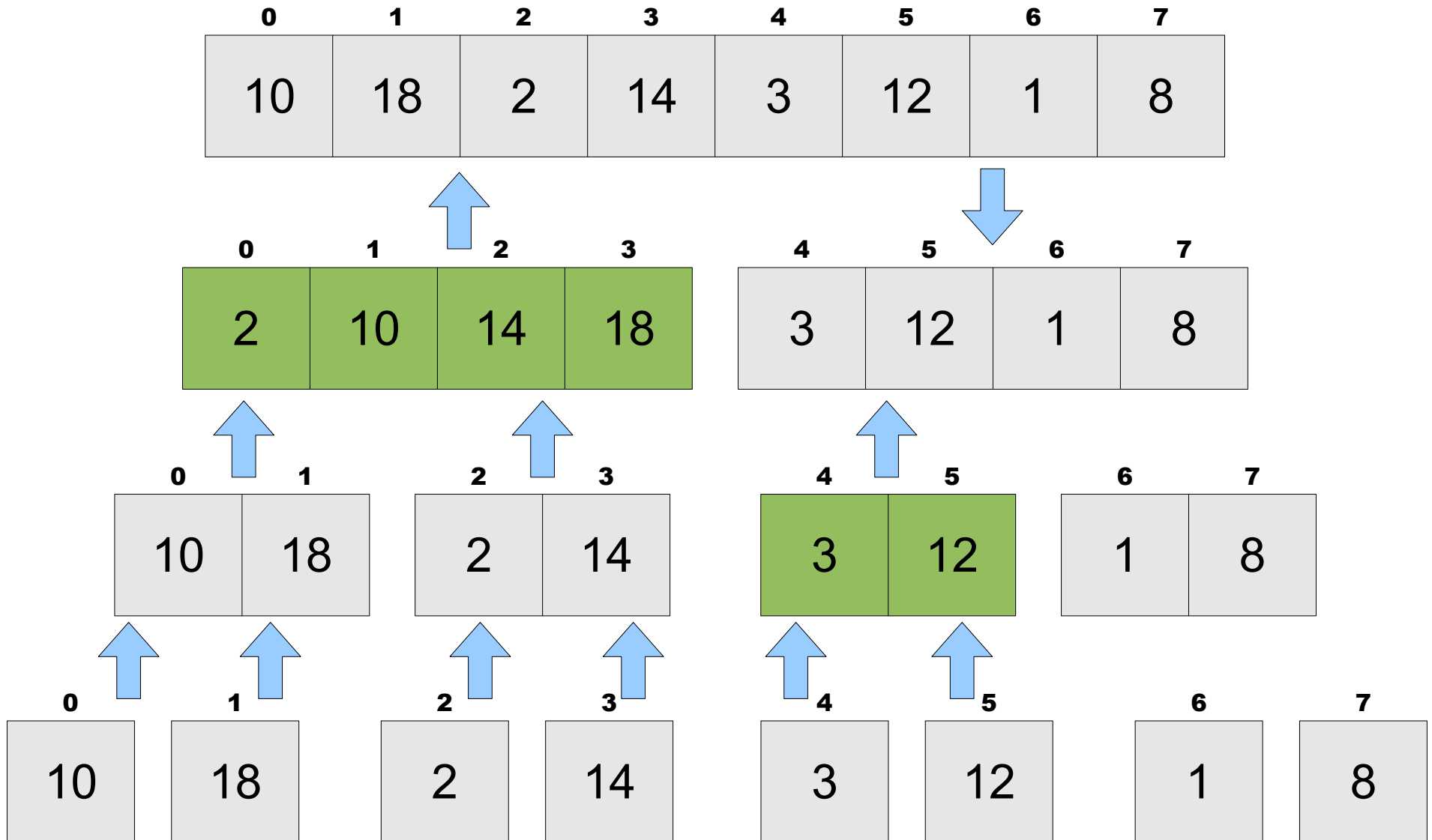
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

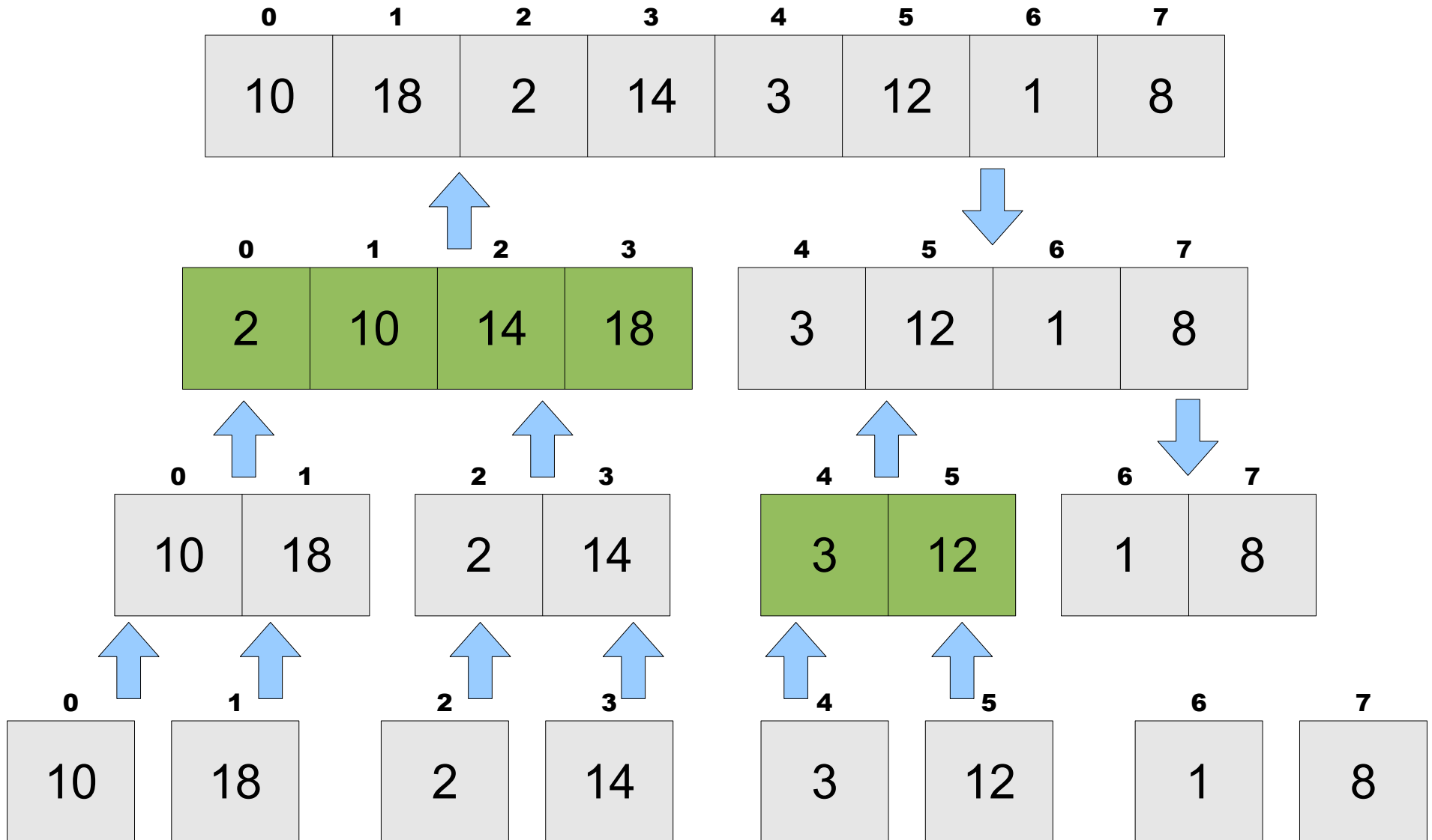
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

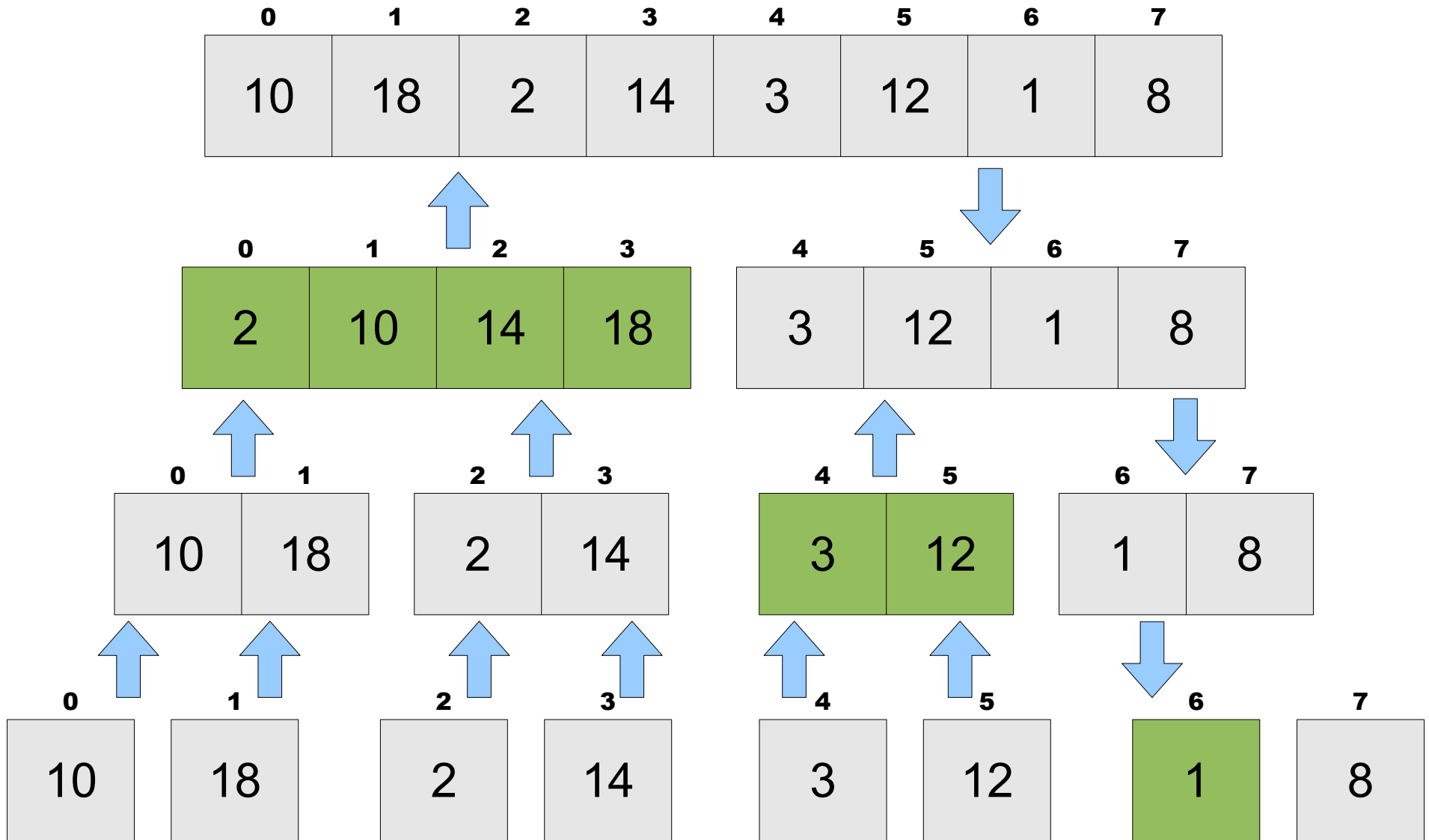
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

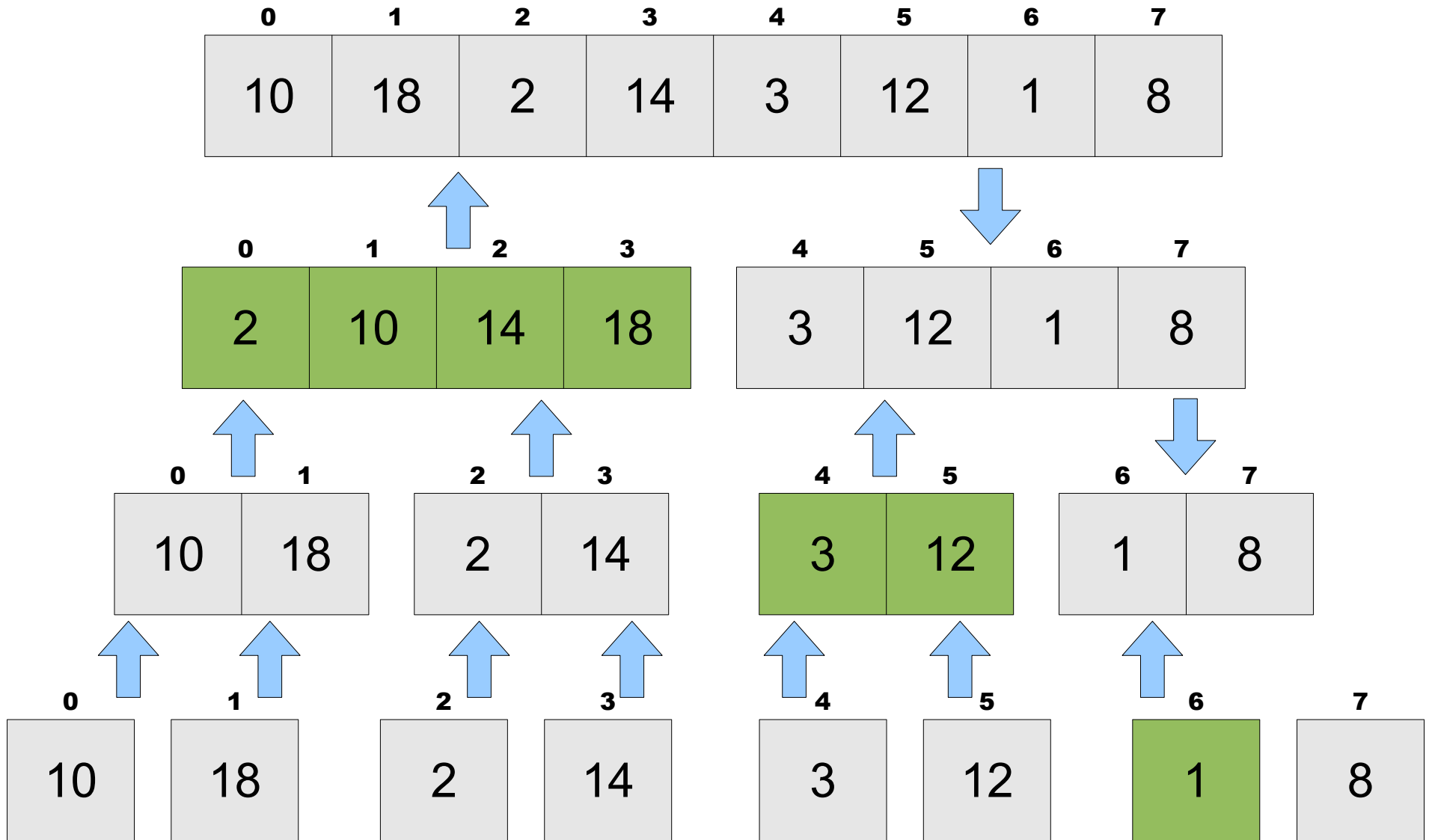
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

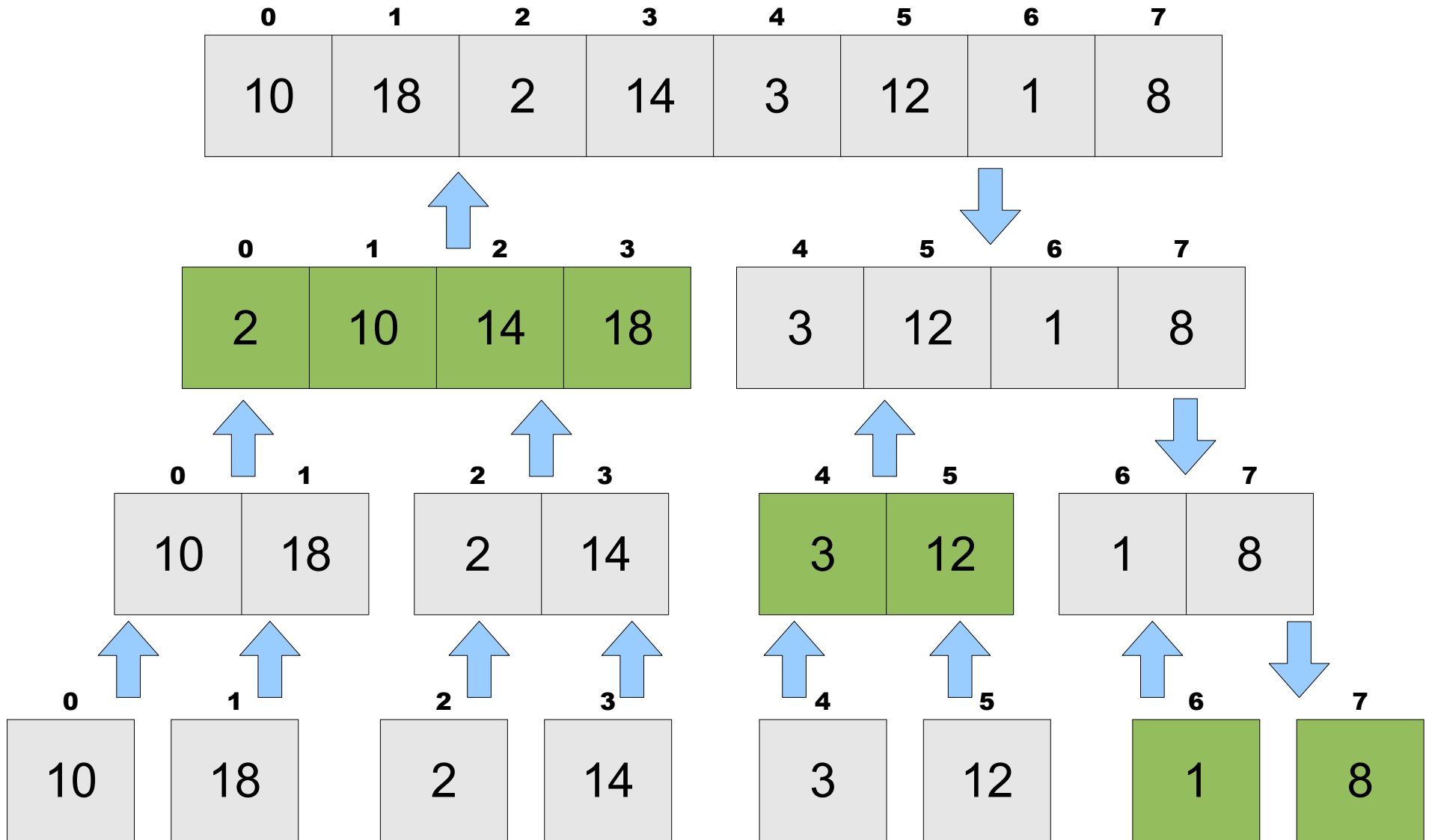
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

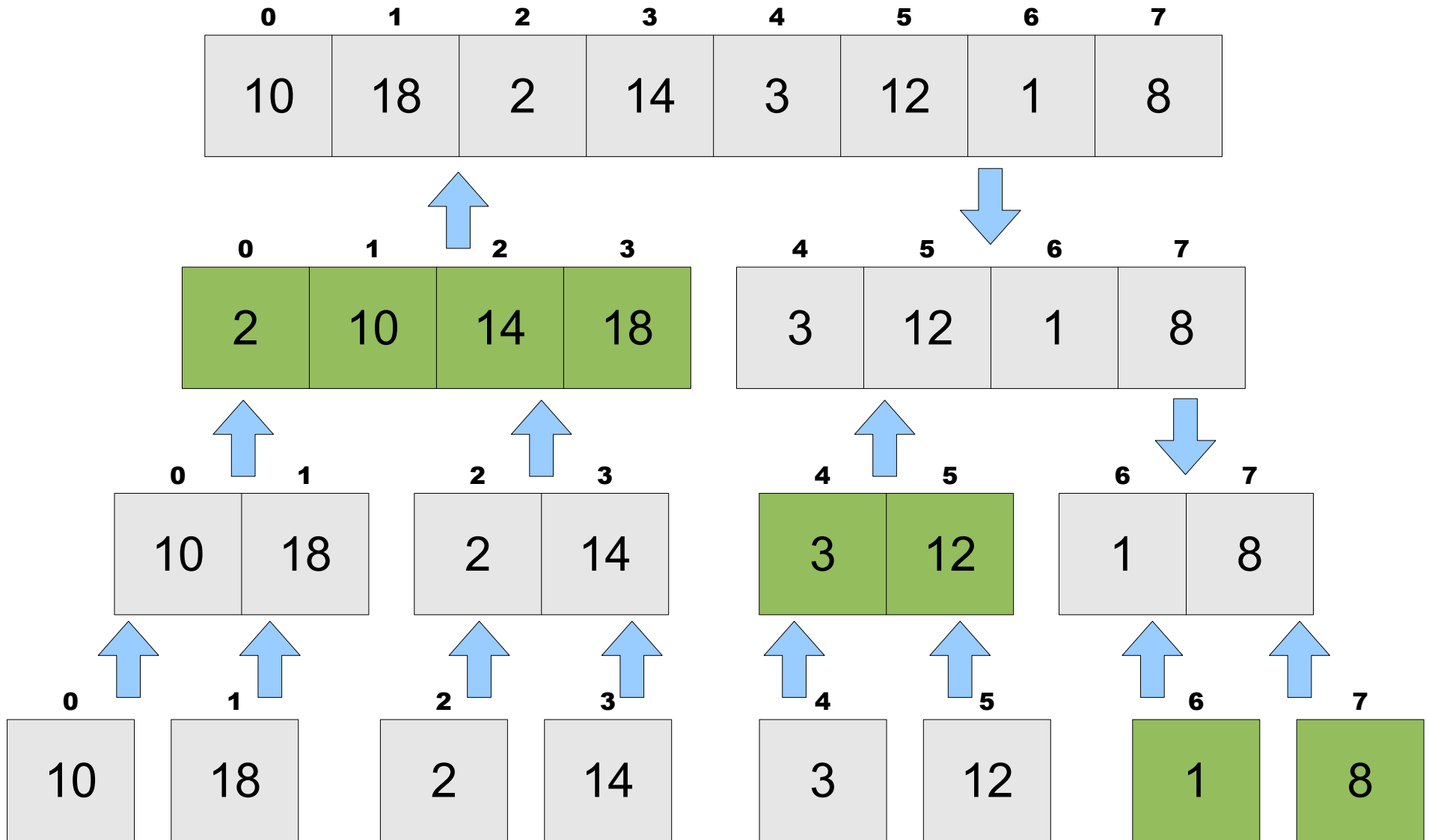
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

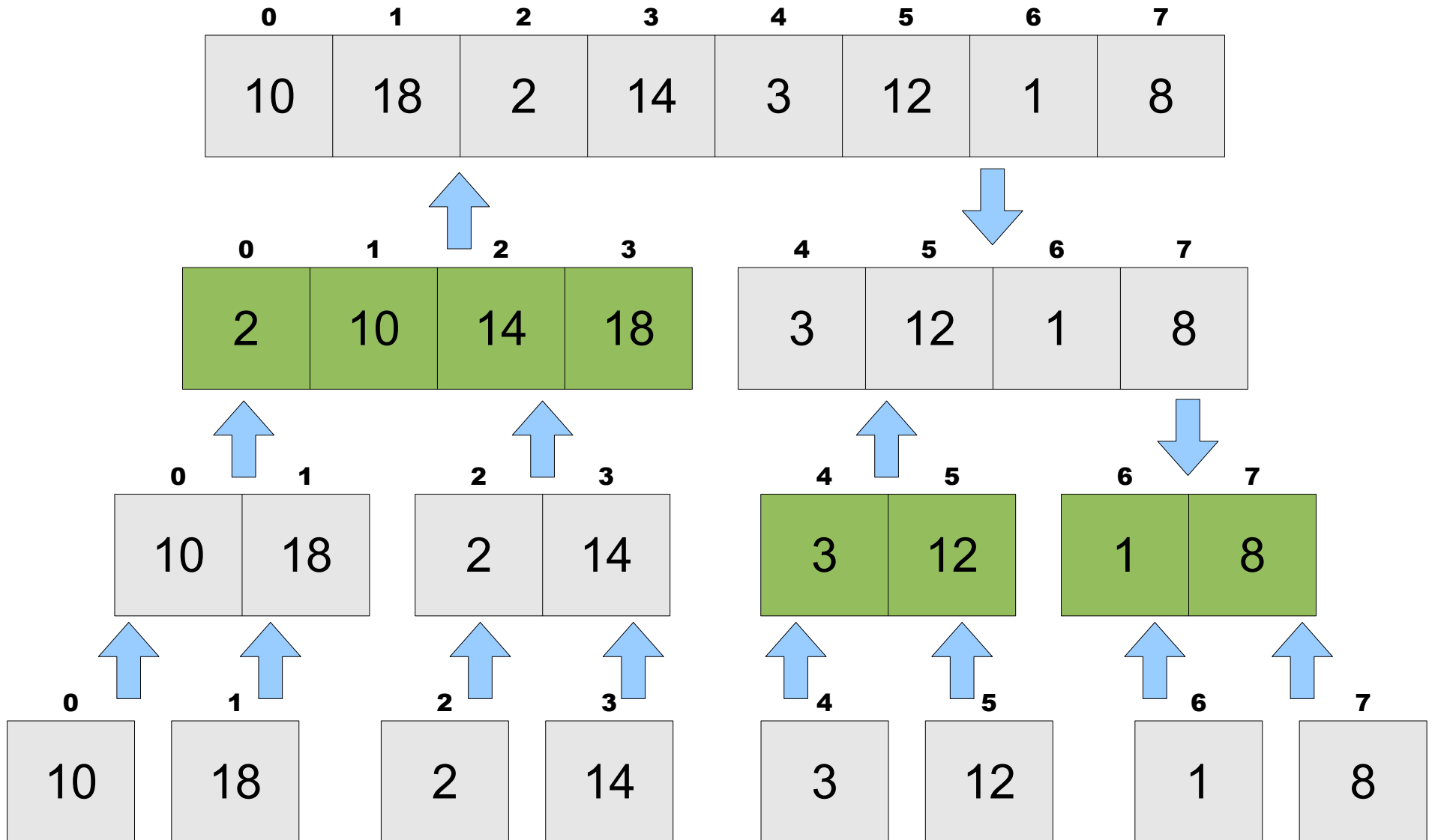
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

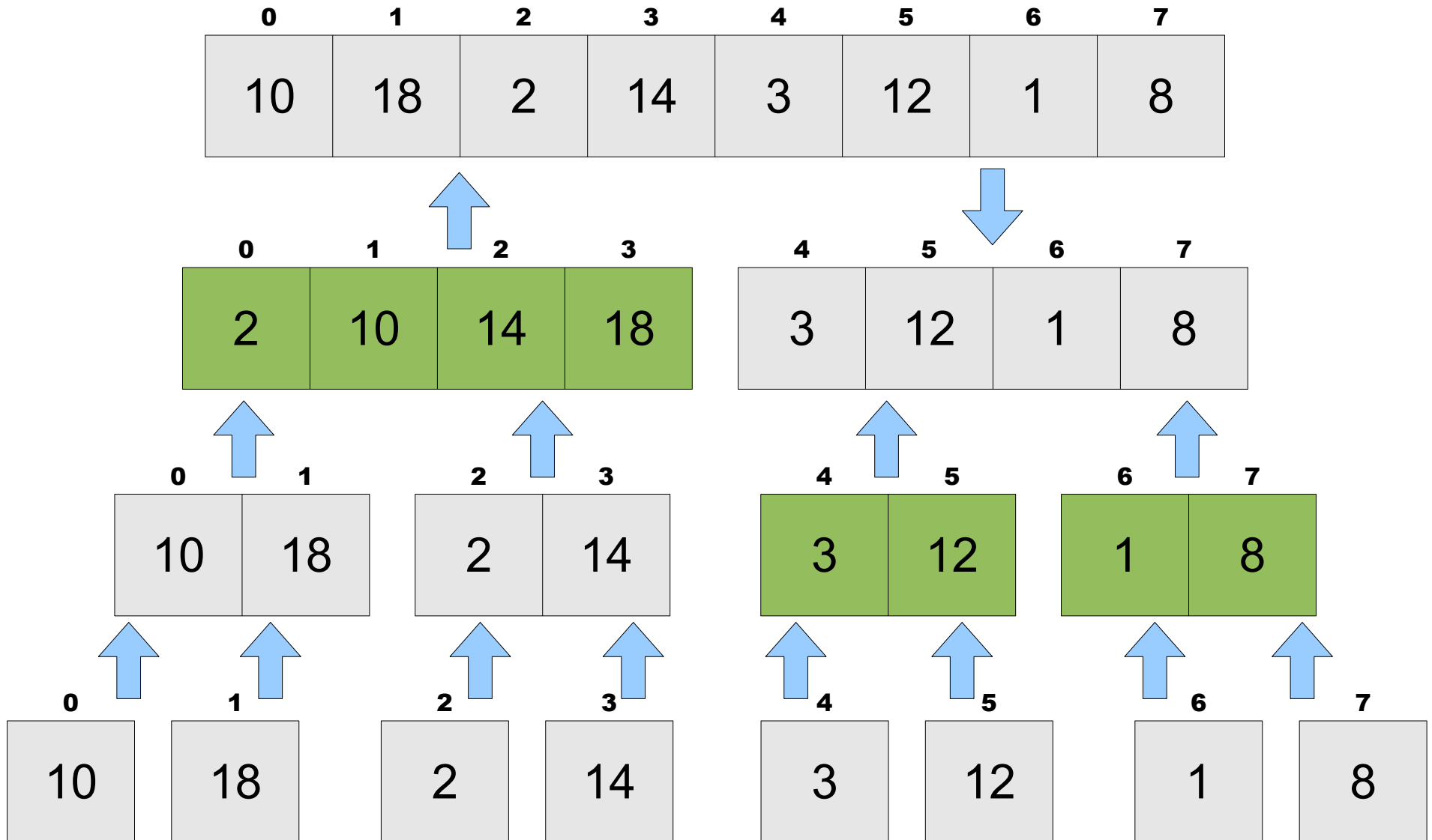
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

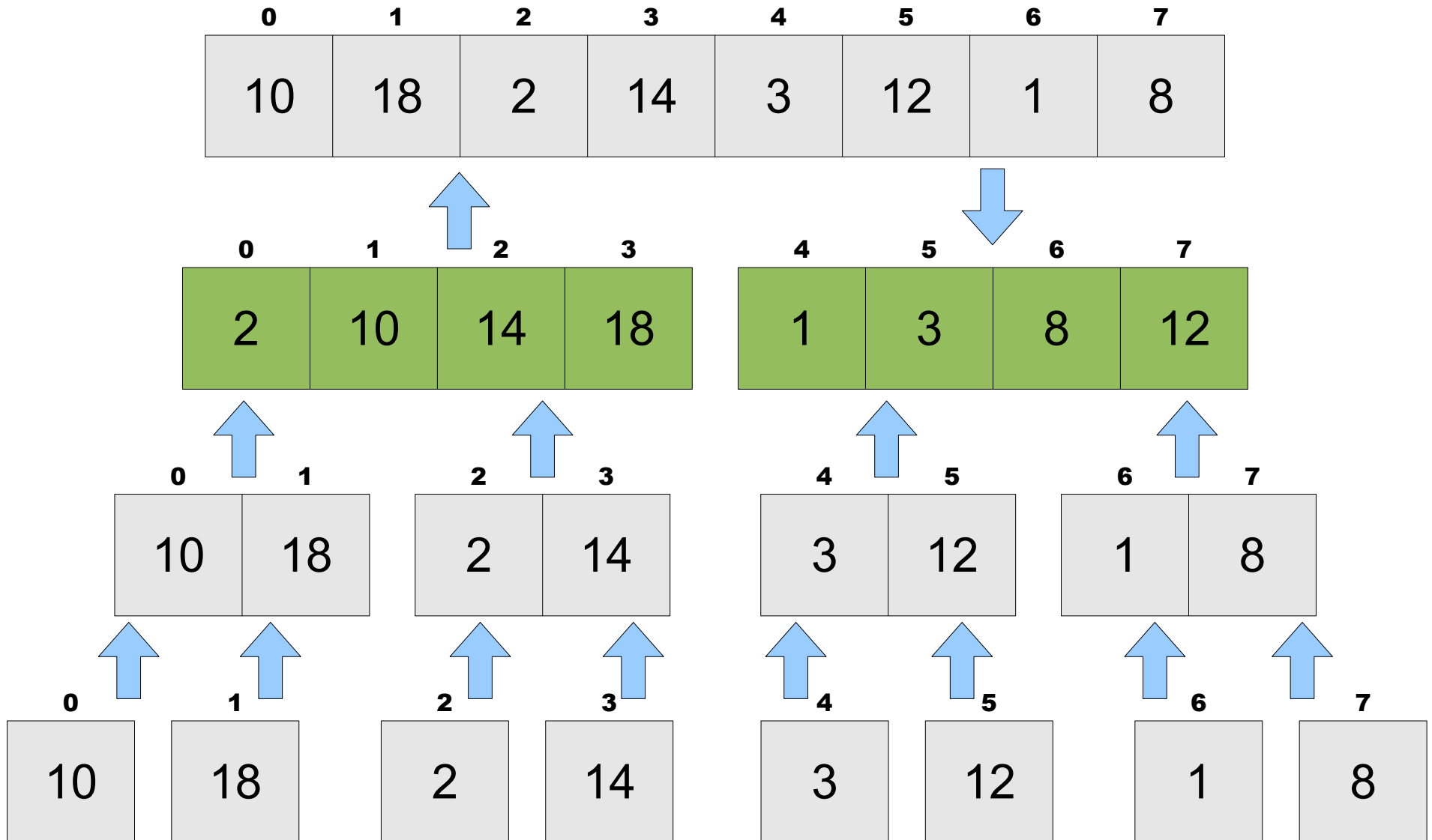
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

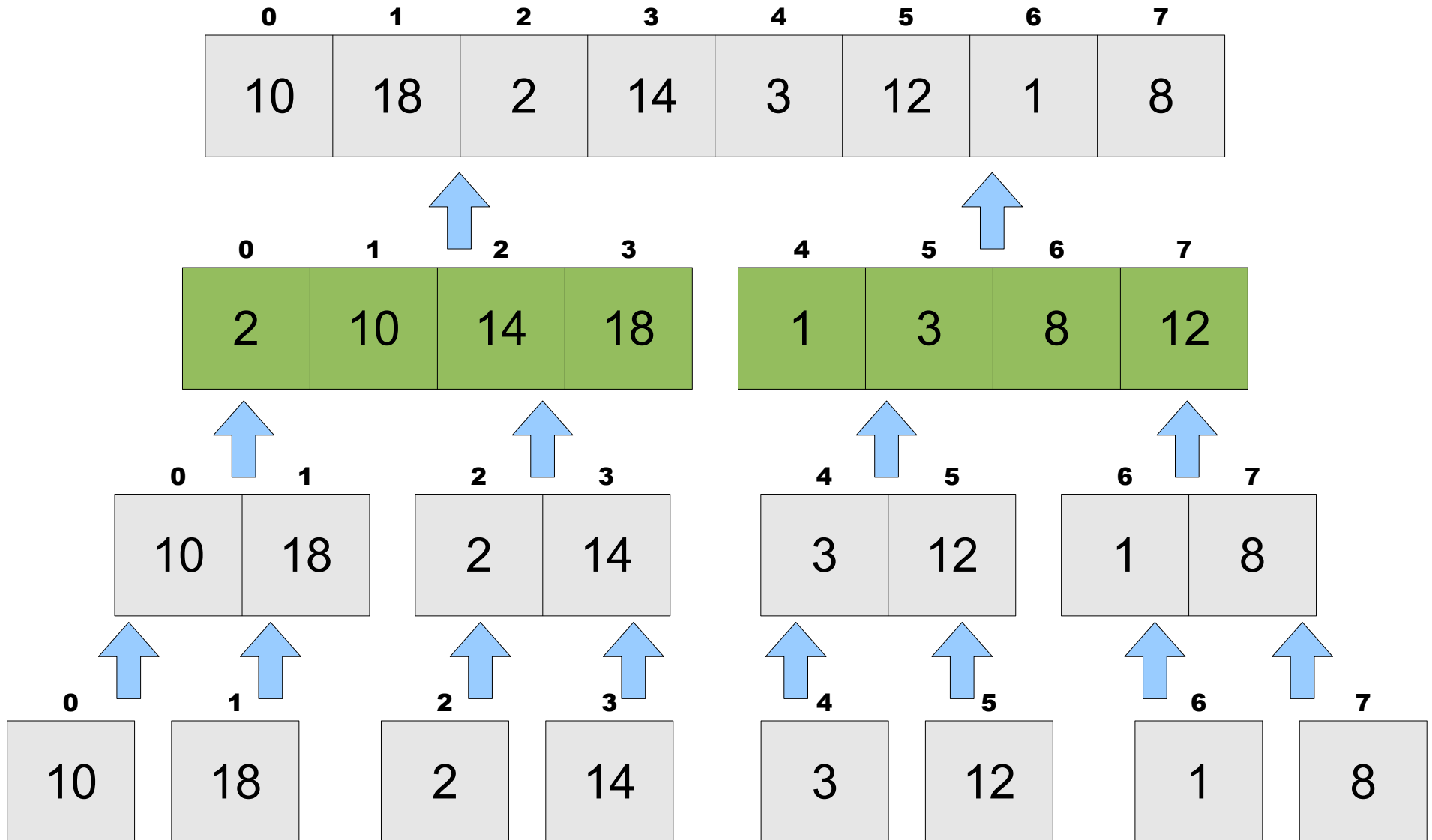
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

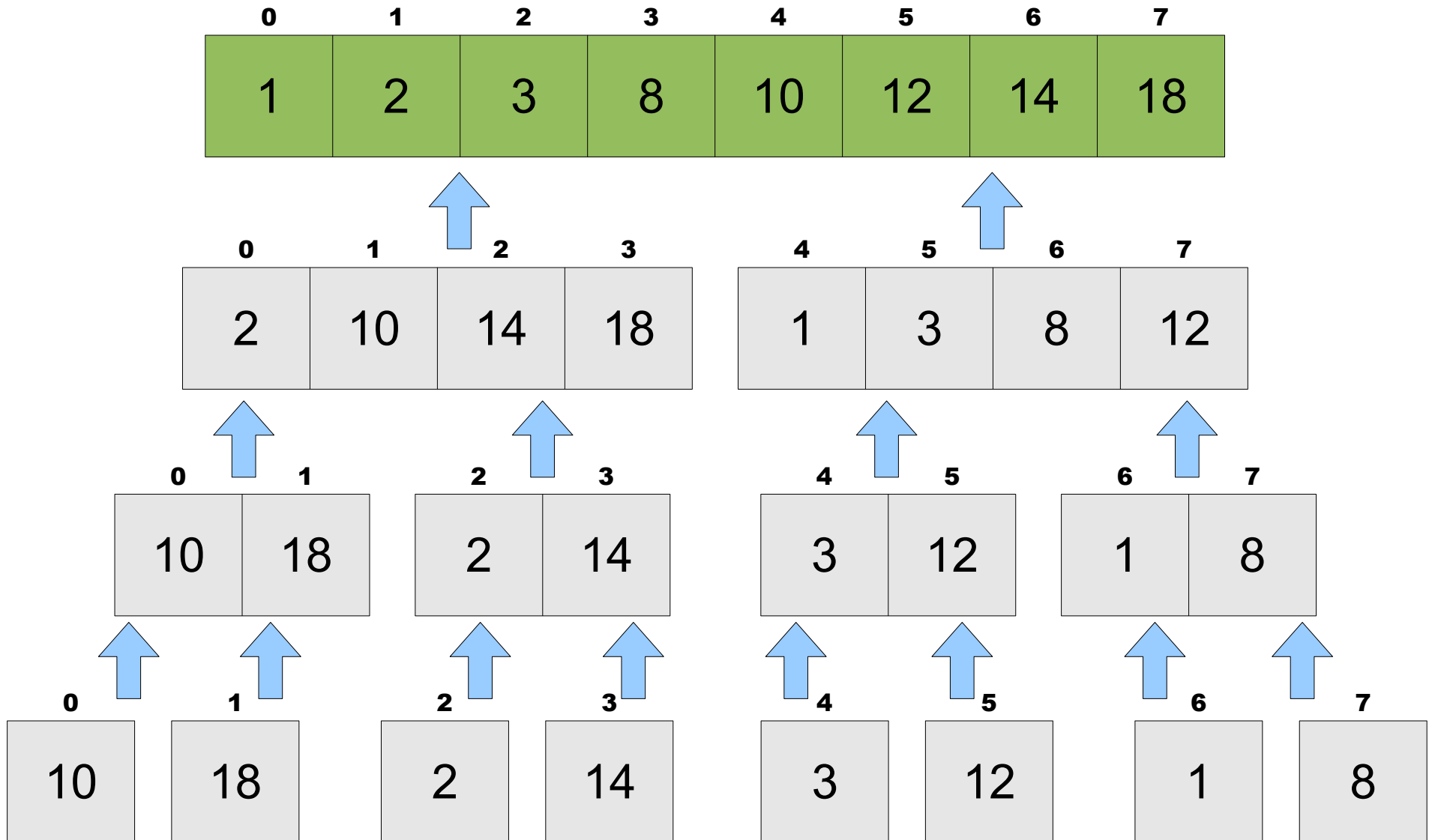
(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

Merge Sort

(break down the vector, then merge the pieces back together)

0	1	2	3	4	5	6	7
1	2	3	8	10	12	14	18

*** TADA! ***

Merge Sort

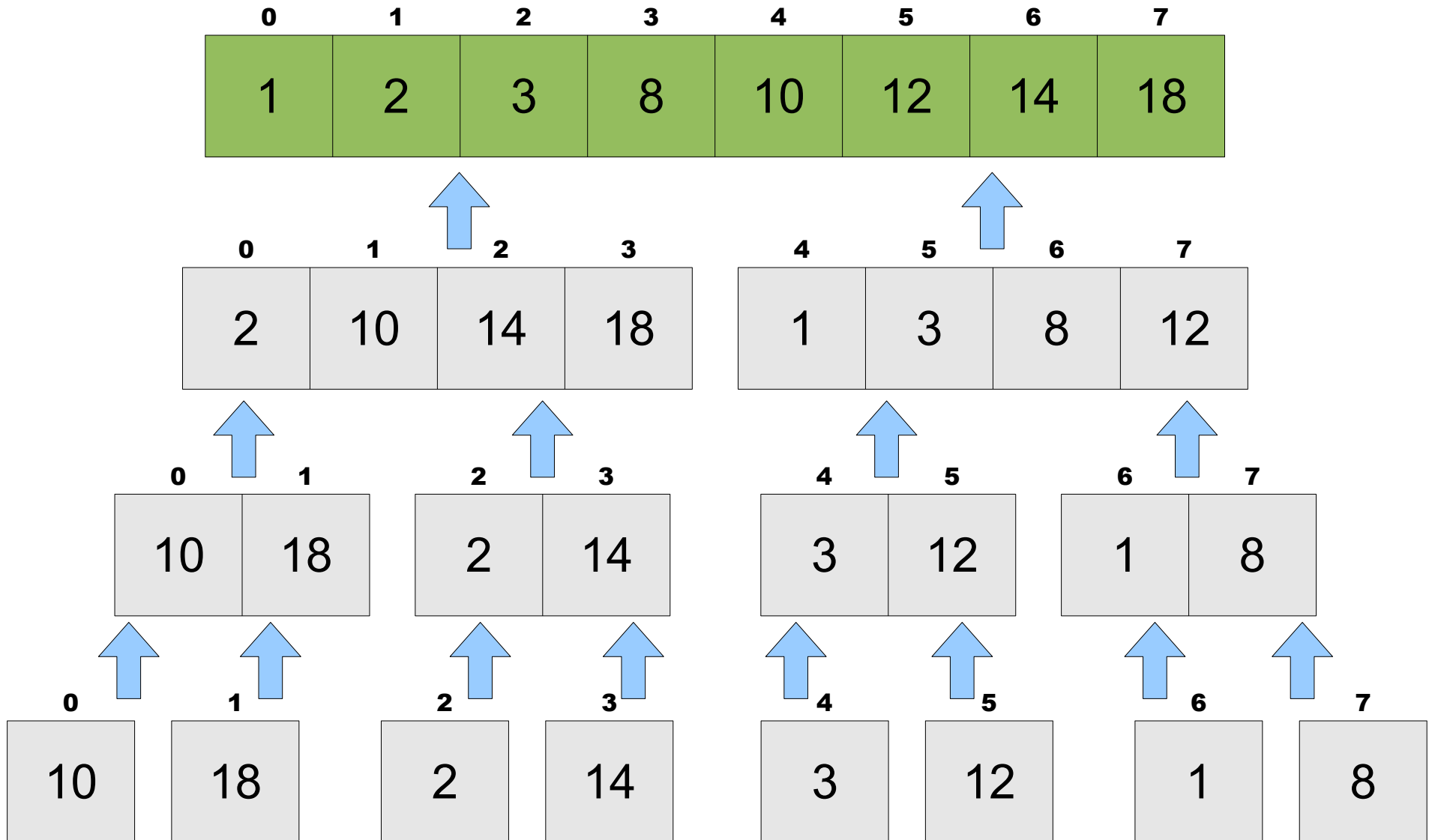
(break down the vector, then merge the pieces back together)

0	1	2	3	4	5	6	7
1	2	3	8	10	12	14	18

1. What's the **worst-case** Big-Oh runtime?
2. What's the **best-case** Big-Oh runtime?
3. Trace the results of **each recursive sub-call** to Merge Sort.

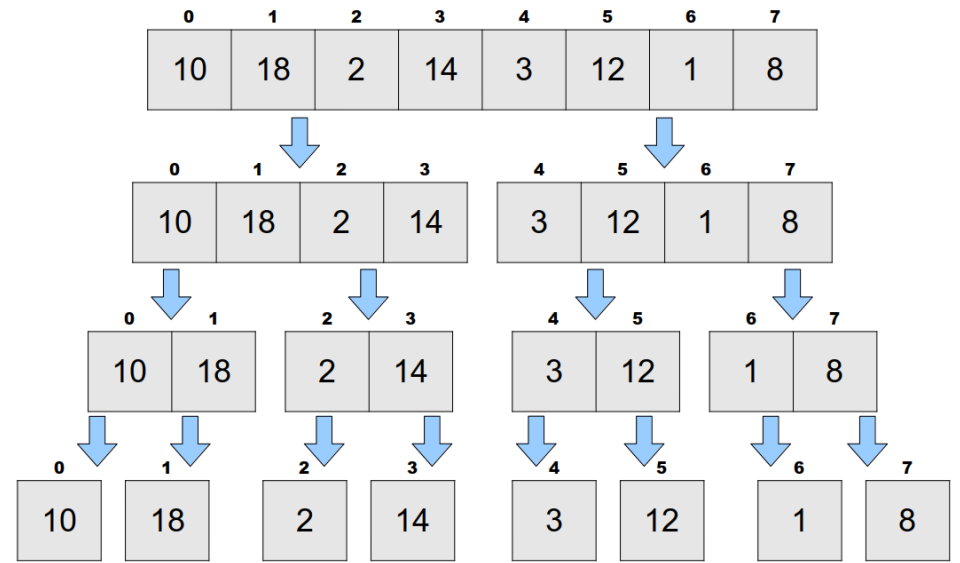
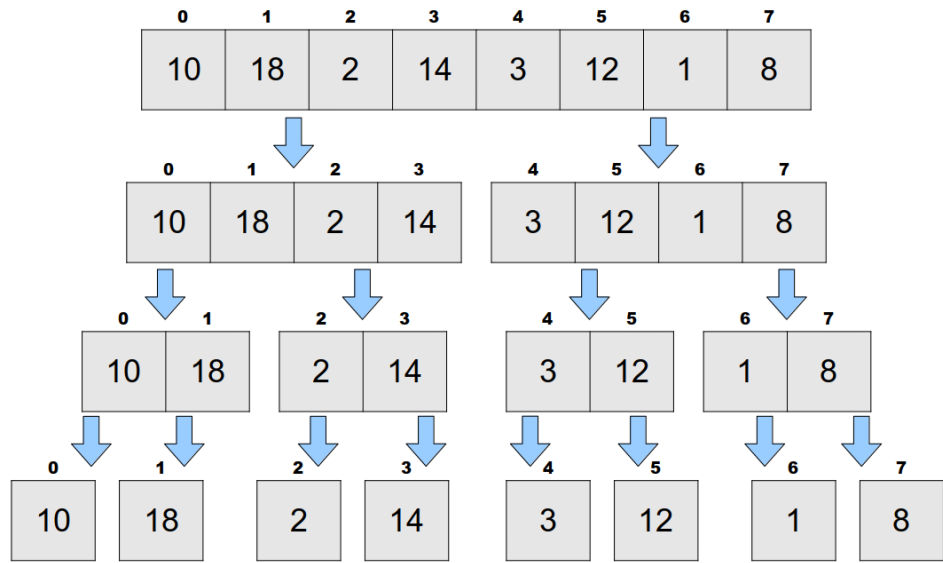
Merge Sort

(break down the vector, then merge the pieces back together)



We've reached our **base cases**. A vector with one element is **sorted**.

10	18	2	14	3	12	1	8	10	18	2	14	3	12	1	8
----	----	---	----	---	----	---	---	----	----	---	----	---	----	---	---



Merge Sort

(break down the vector, then merge the pieces back together)

0	1	2	3	4	5	6	7
1	2	3	8	10	12	14	18

Let's look at some code!