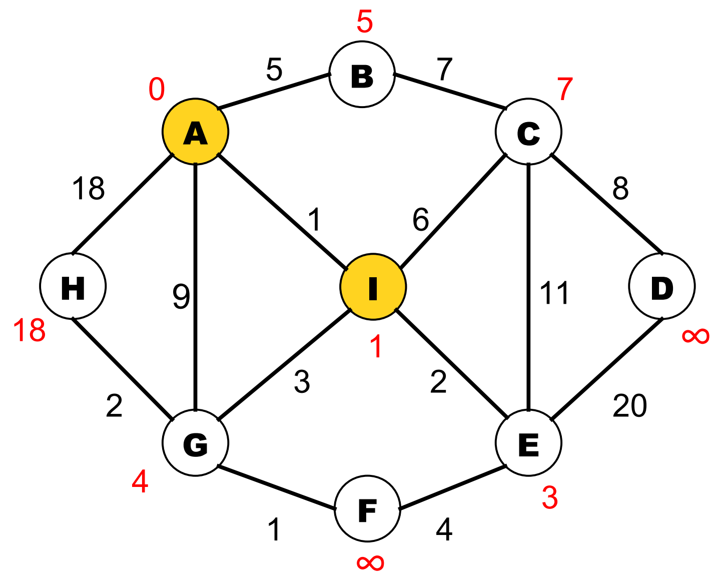
Dijkstra's Algorithm

An Illustration



Slides by **Sean Szumlanski** for **CS106B**, Programming Abstractions

Winter 2024

Dijkstra's Algorithm

Goal: Find the lowest-cost path from some start vertex (source) to every other vertex in the graph.

Assumptions: Edges all have non-negative weights.

Motivation: Sending a message to every other node in a network as fast as possible.

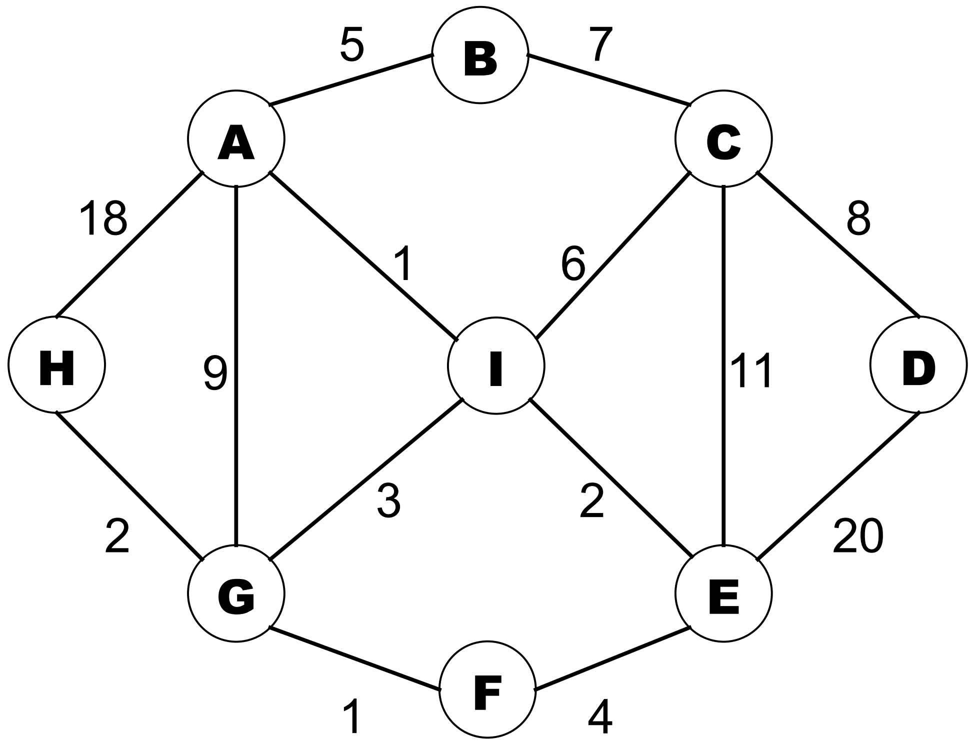
Shipping from a central distribution center, taking the shortest path to all destinations.

Modeling the spread of infectious diseases through social networks.

Evaluating degrees of separation between humans based on social network activity.

Dijkstra's Algorithm

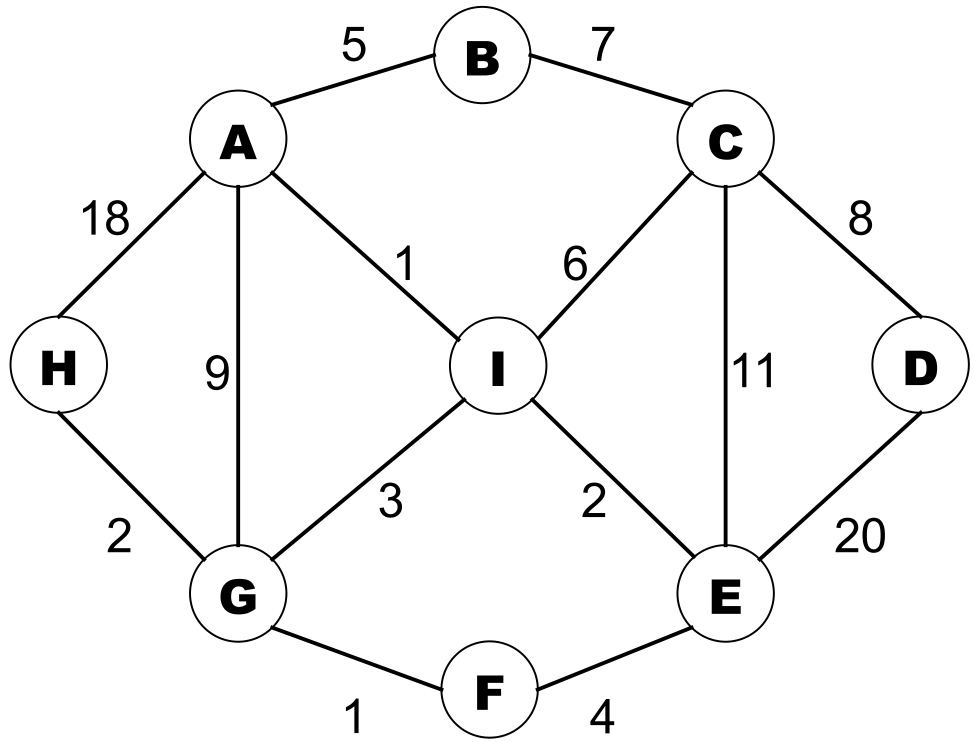
(calculating the cheapest path from a source vertex to all other vertices)



Let's trace through the algorithm to see how it works.

Dijkstra’s Algorithm

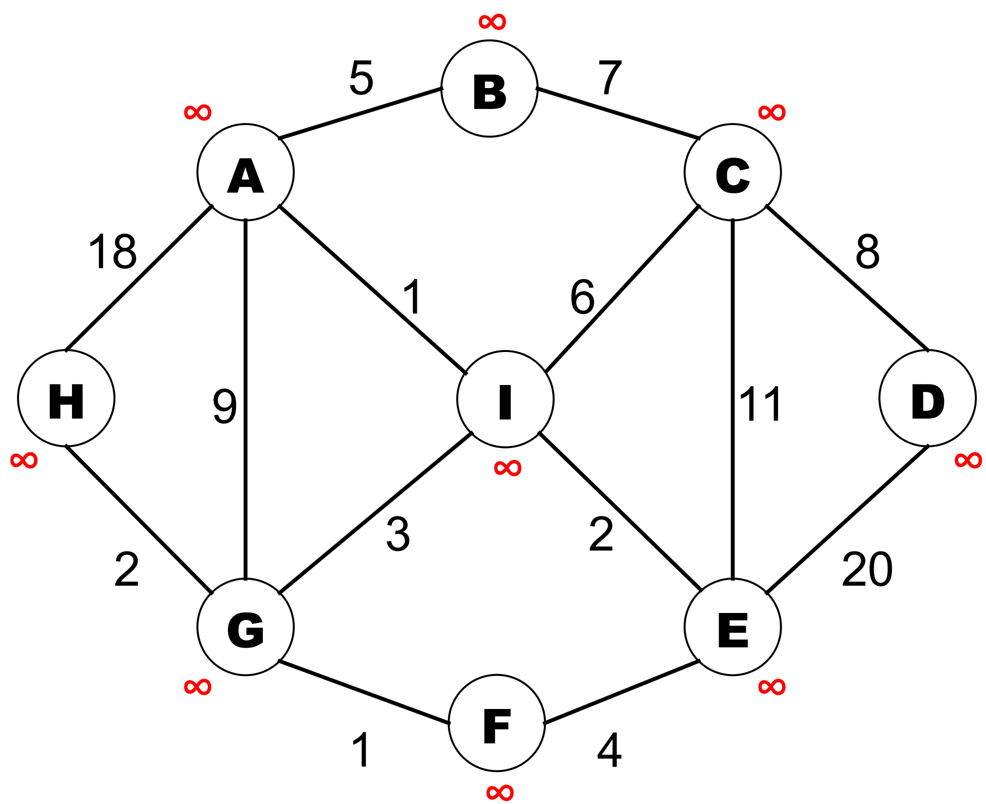
(calculating the cheapest path from a source vertex to all other vertices)



1: Initialize a value at each vertex to infinity (∞). Call these values **dist**[**i**].

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

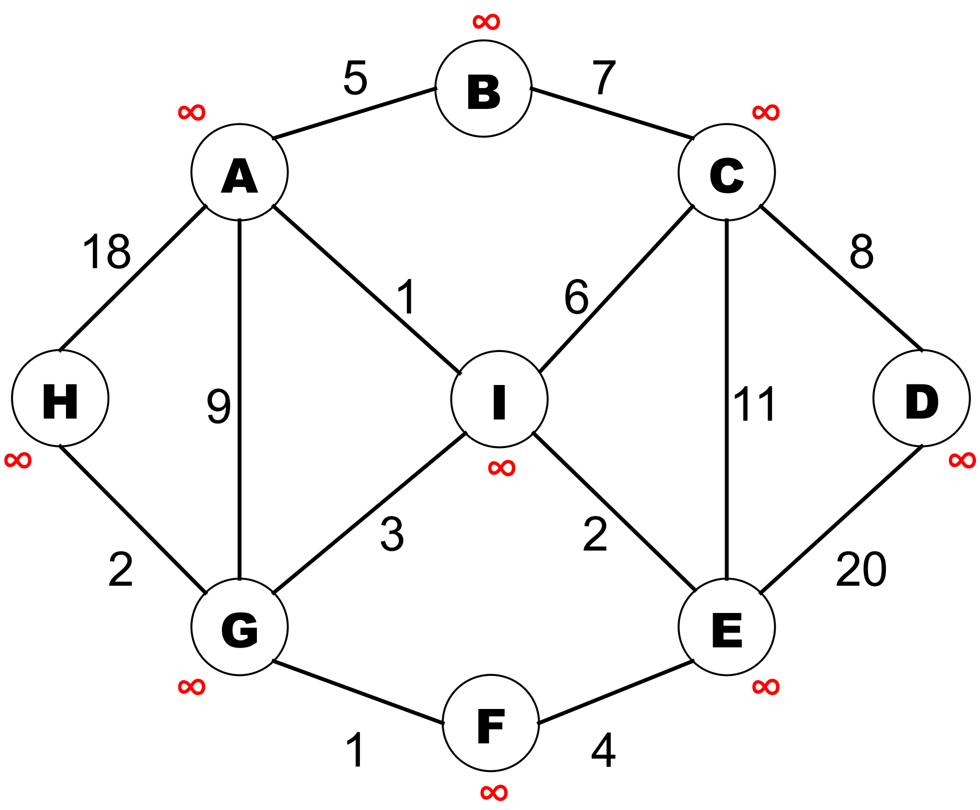


1: Initialize a value at each vertex to infinity (∞). Call these values **dist**[**i**].

Note: These ∞ values represent the cost of reaching each vertex from our source, using only intermediary vertices whose shortest paths we have already found.

Dijkstra's Algorithm

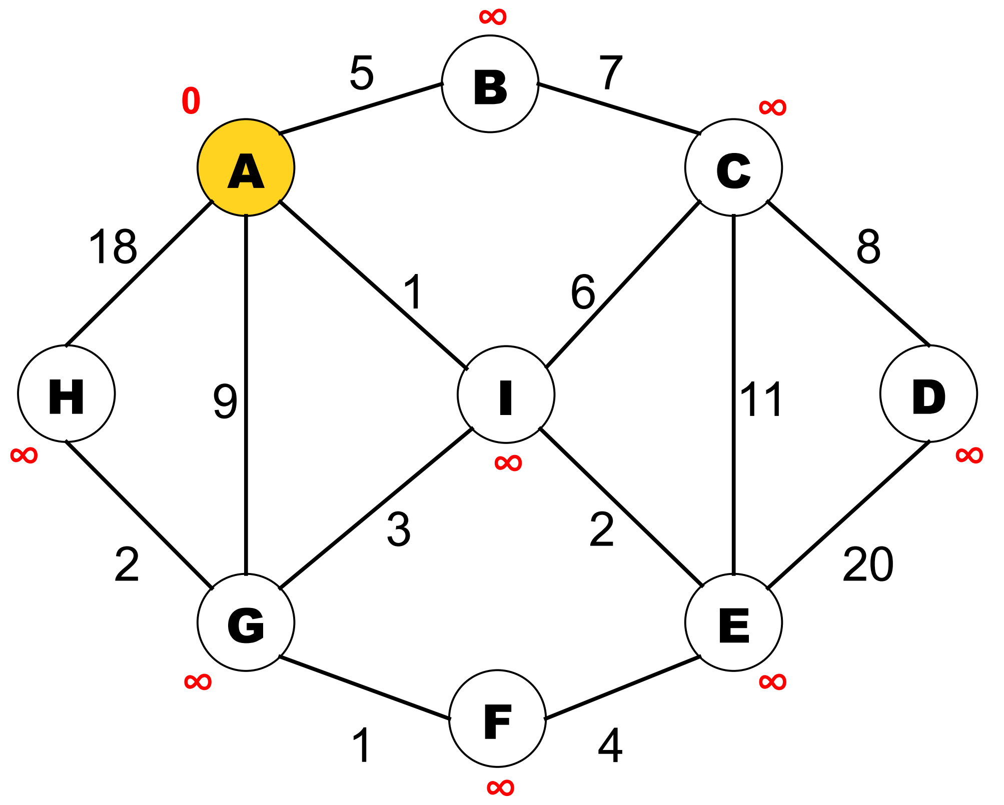
(calculating the cheapest path from a source vertex to all other vertices)



2: Initialize the value at our source vertex to zero and mark the source vertex as visited.

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

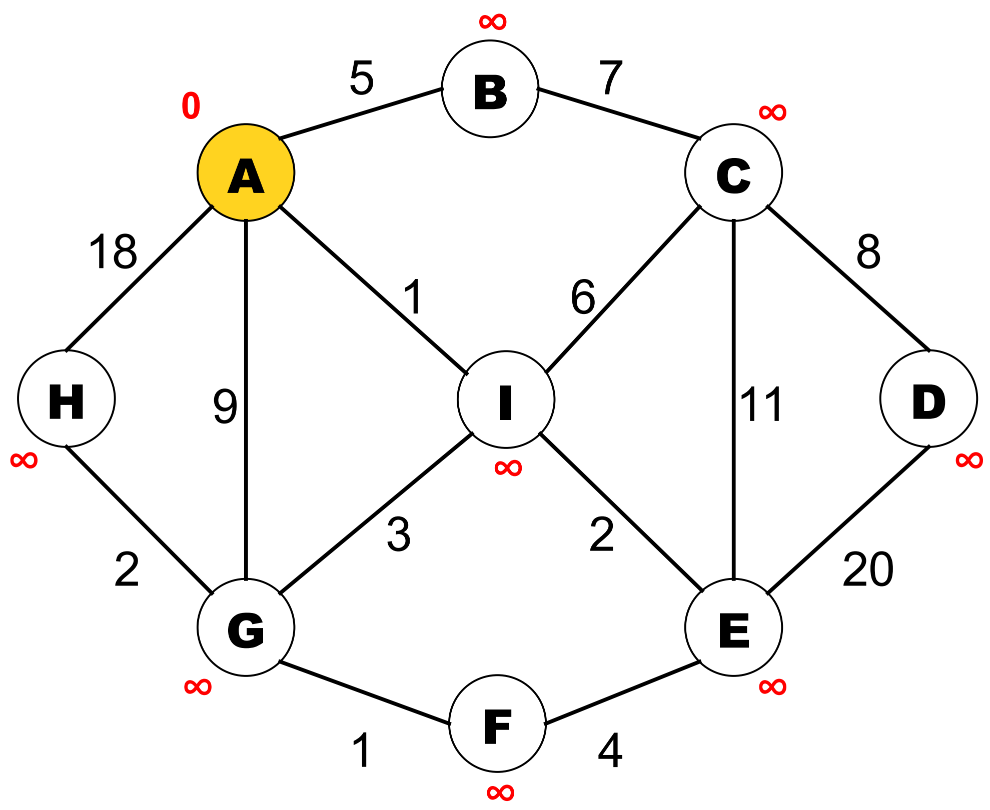


2: Initialize the value at our source vertex to zero and mark the source vertex as visited.

Note: Clearly we can get from vertex A to vertex A at a cost of zero...

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

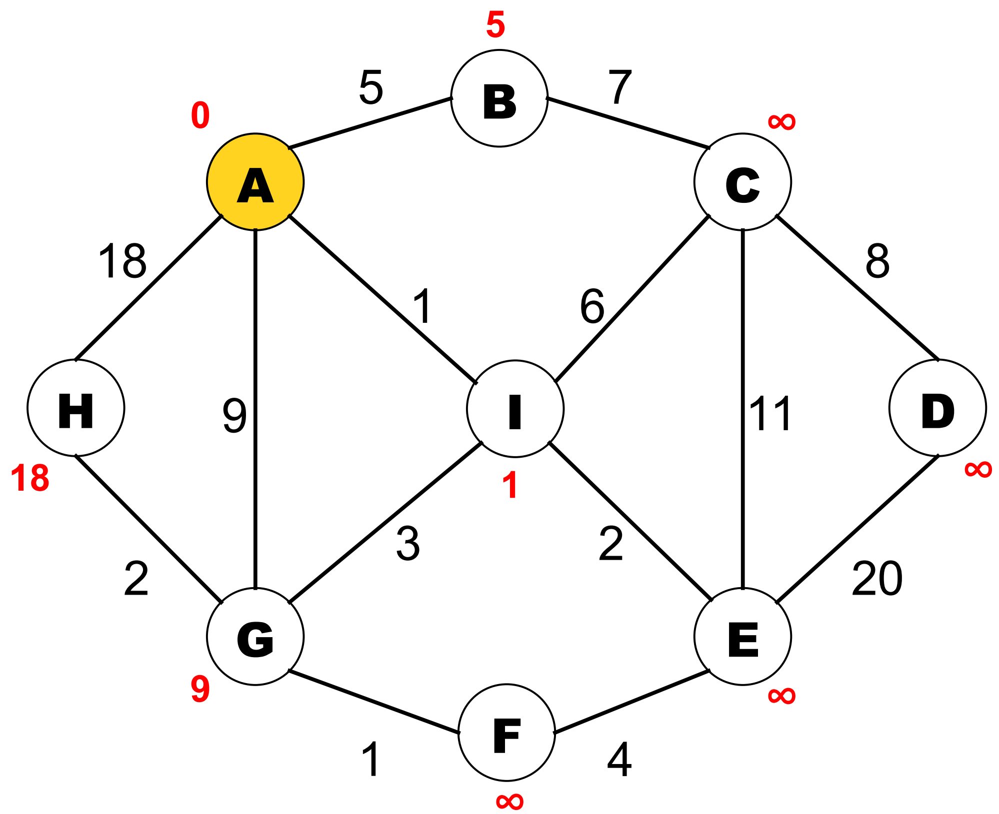


3: Update the cost of getting from the source vertex to every other vertex:

MIN{weight[source, i], dist[i]}

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



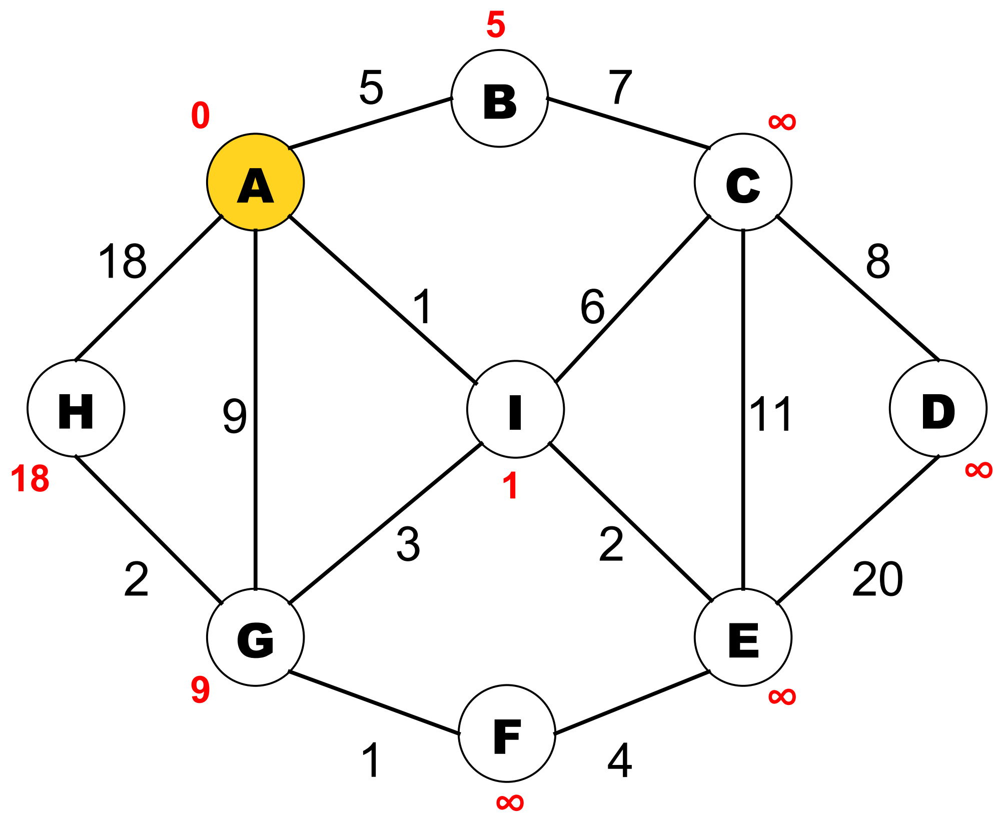
3: Update the cost of getting from the source vertex to every other vertex:

MIN{weight[source, i], dist[i]}

Note: These dist[i] values now represent the lowest-cost path from A using no intermediate vertices. (Unless we had negative edge weights...)

Dijkstra's Algorithm

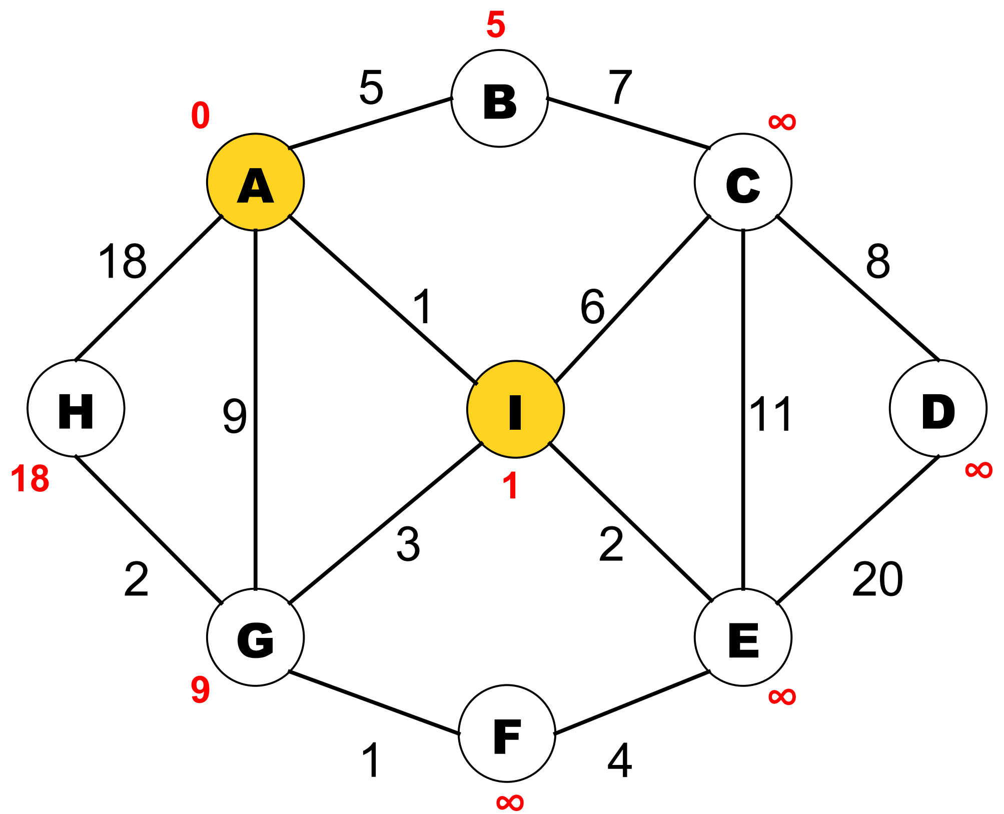
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

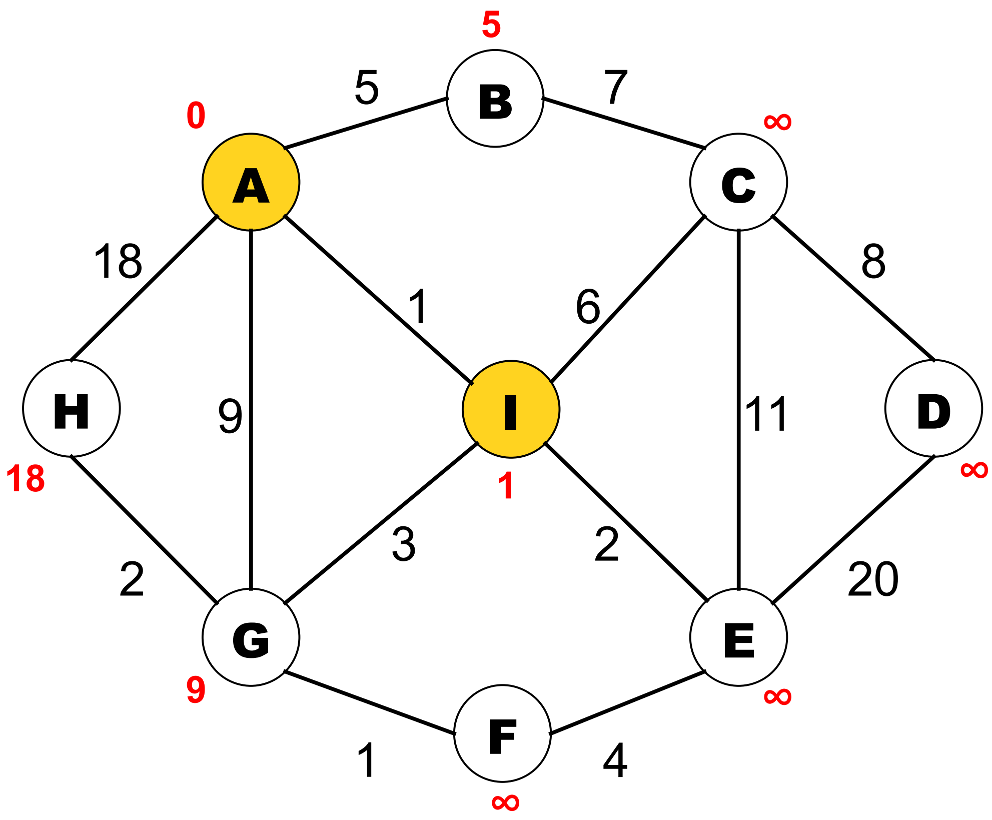


4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.

Note: Clearly we have found a shortest path from vertex A to vertex I, since any other path must go through edges of greater (or equal) weight.

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

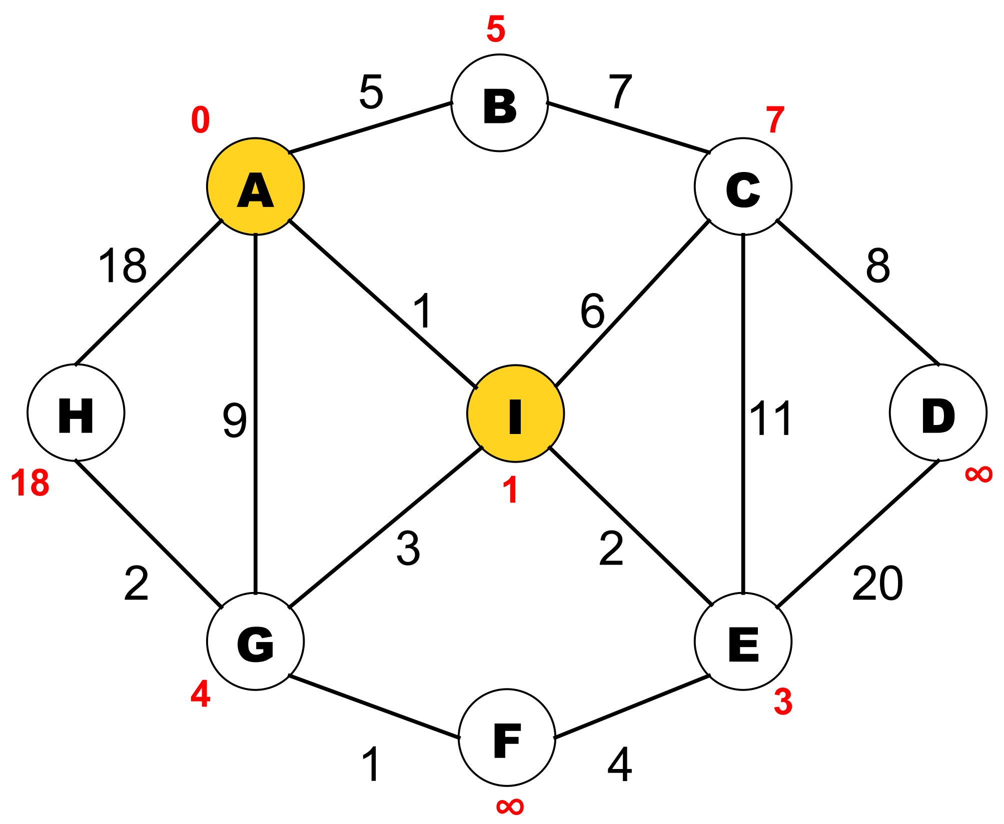


5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra’s Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



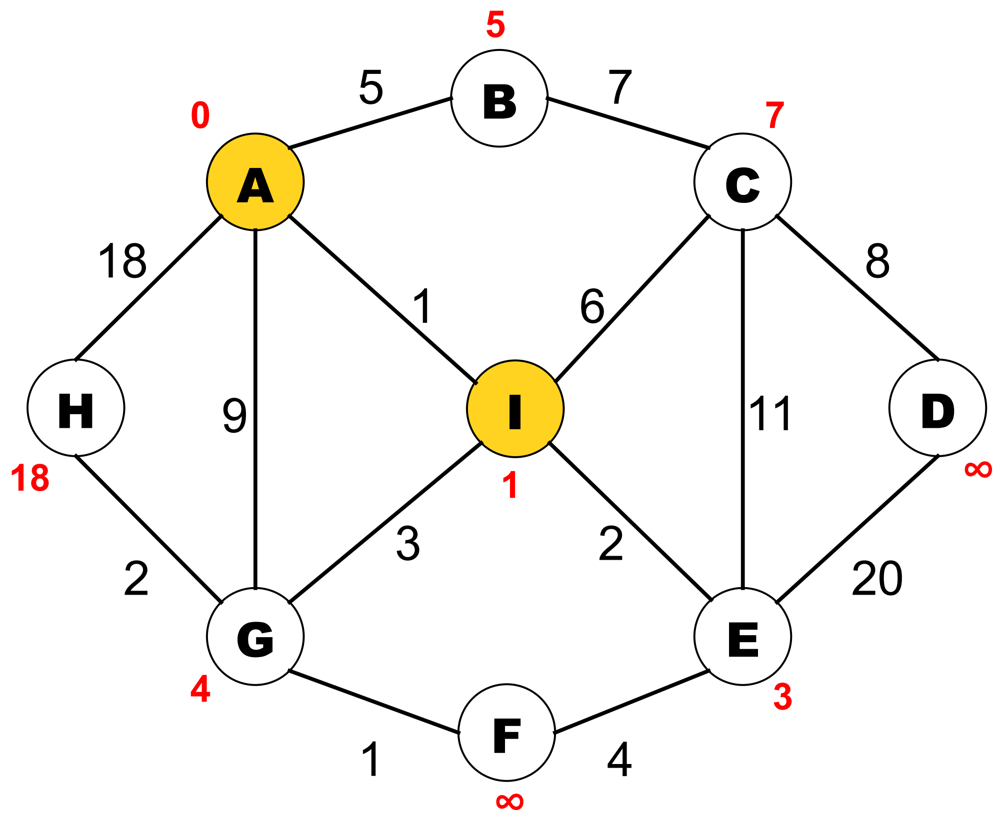
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertex I to be used as an intermediary vertex along the path.

Dijkstra’s Algorithm

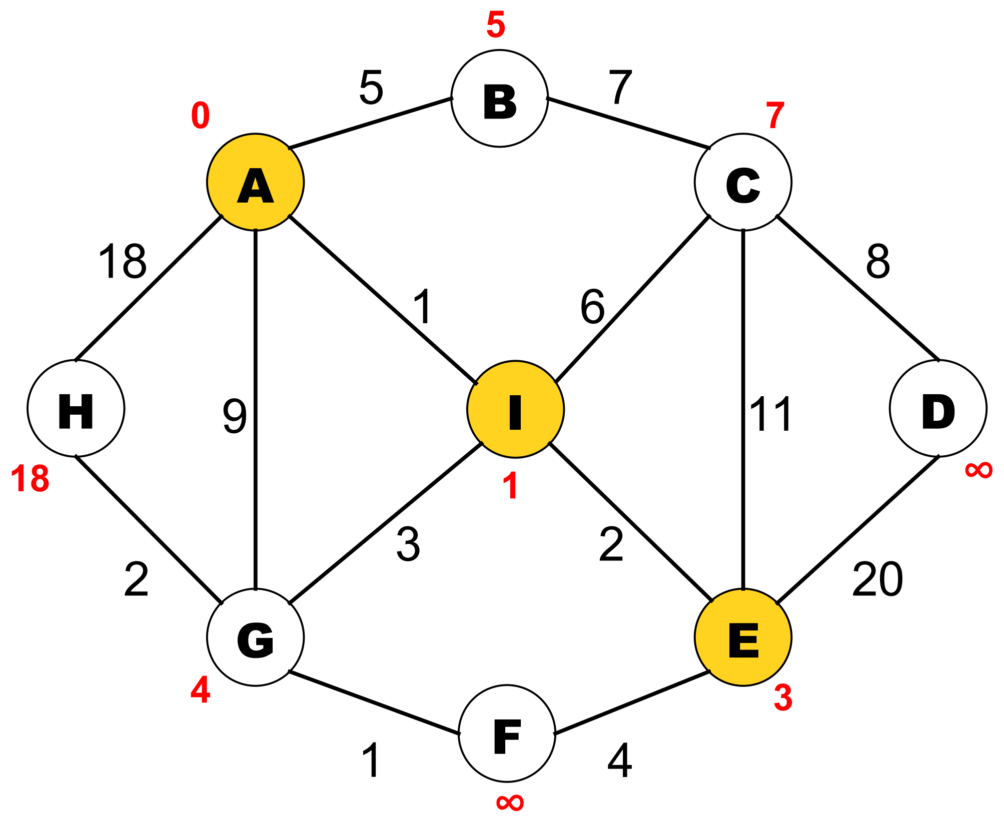
(calculating the cheapest path from a source vertex to all other vertices)



4: (again) Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.

Dijkstra’s Algorithm

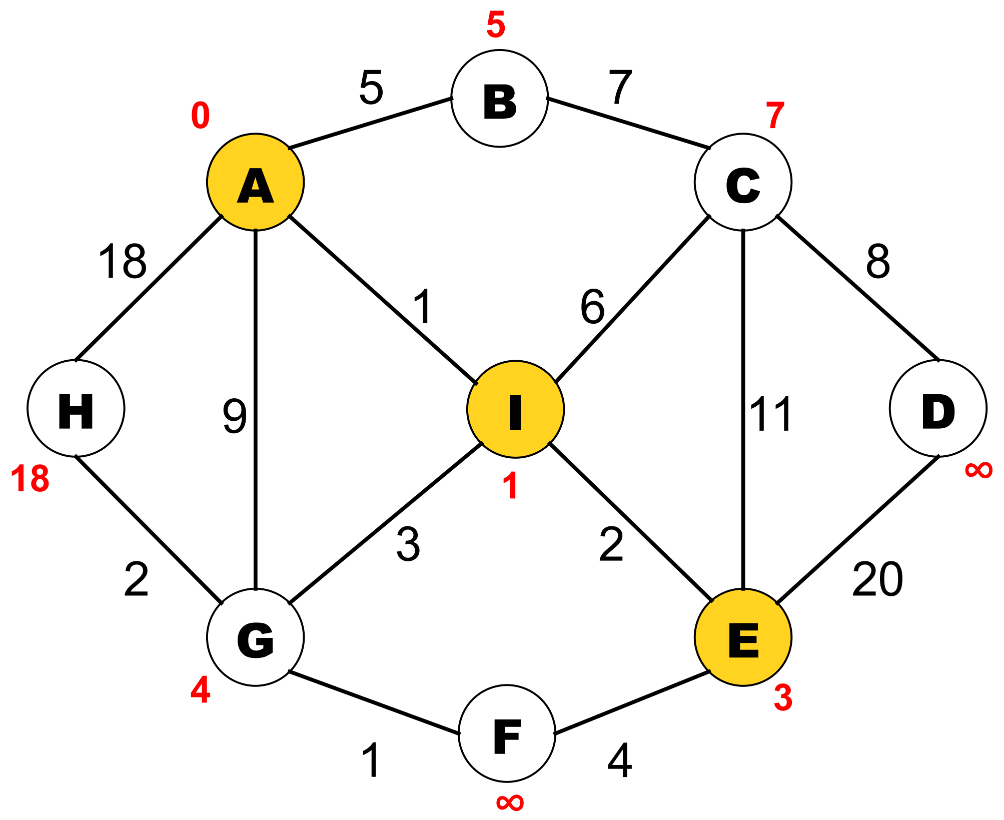
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

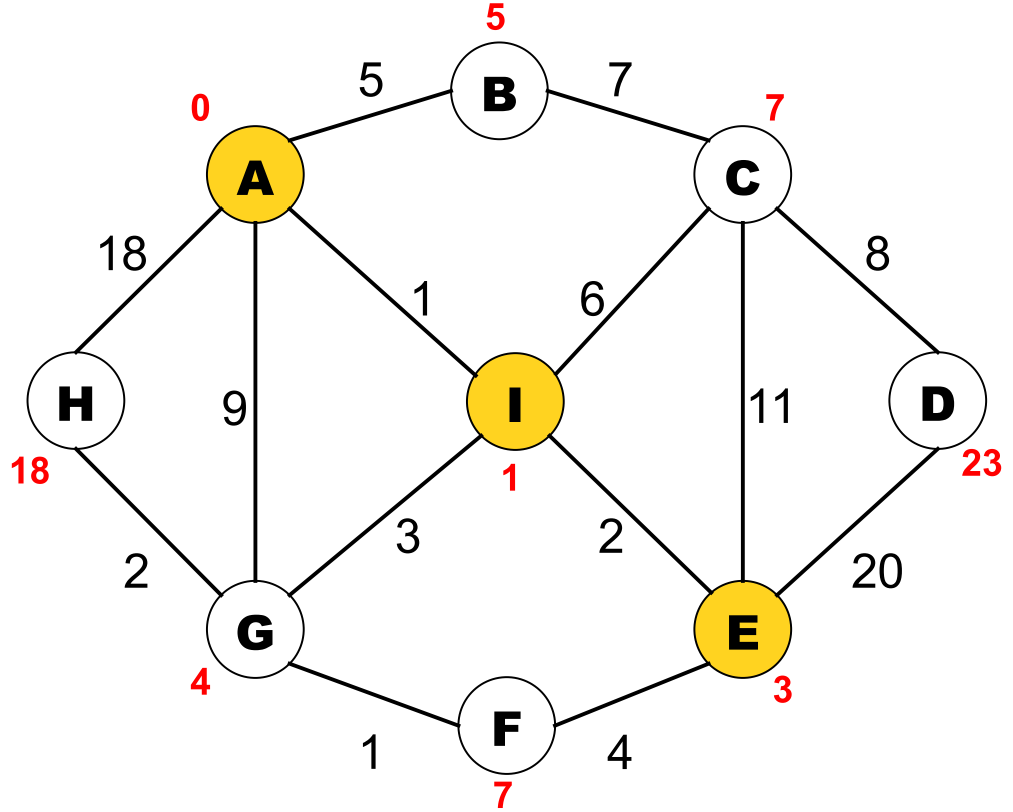


5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



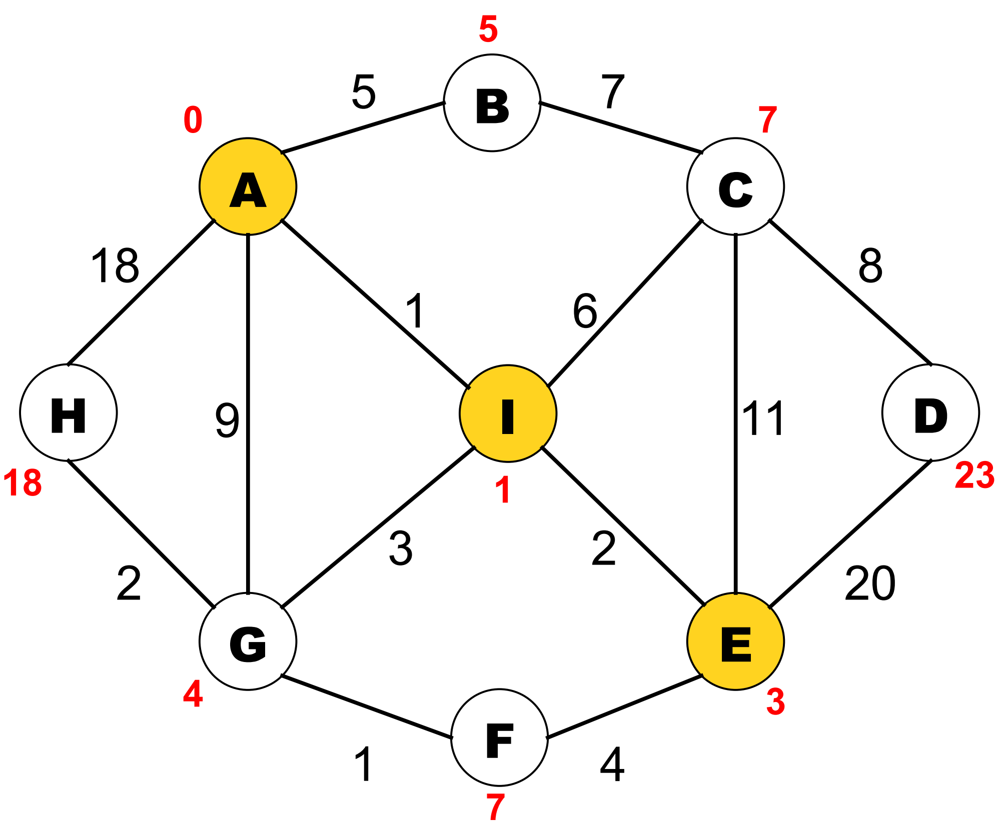
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I and E** to be used as intermediary vertices along the path.

Dijkstra’s Algorithm

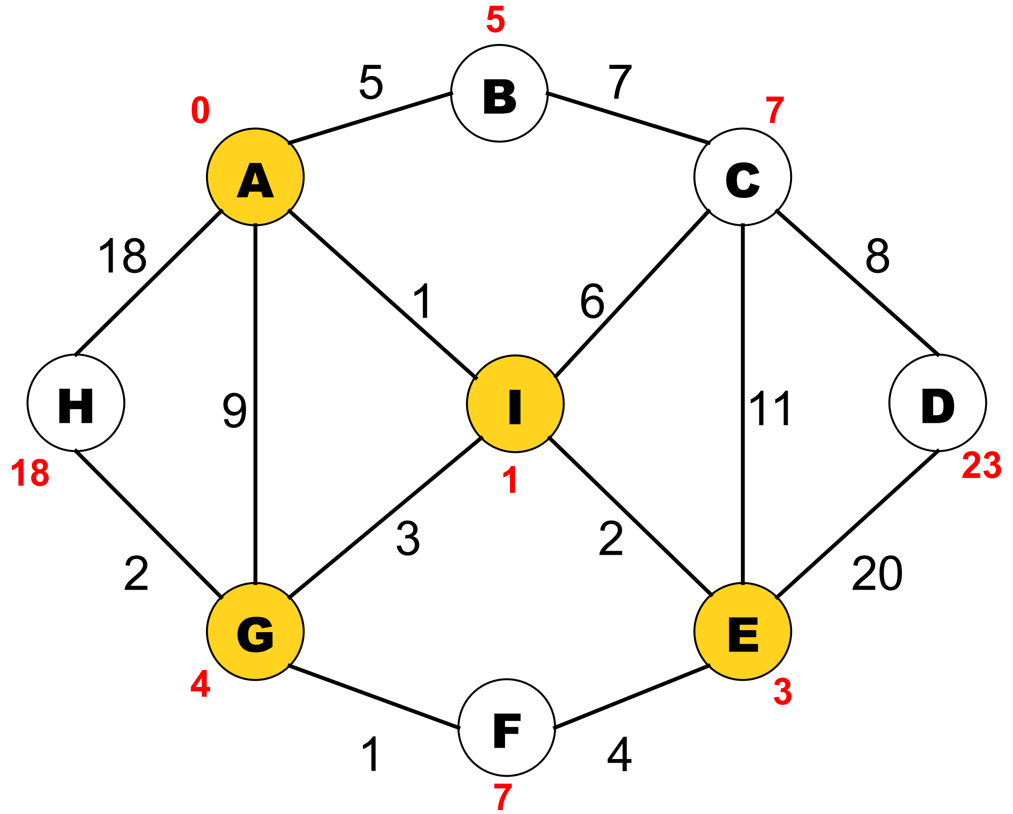
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

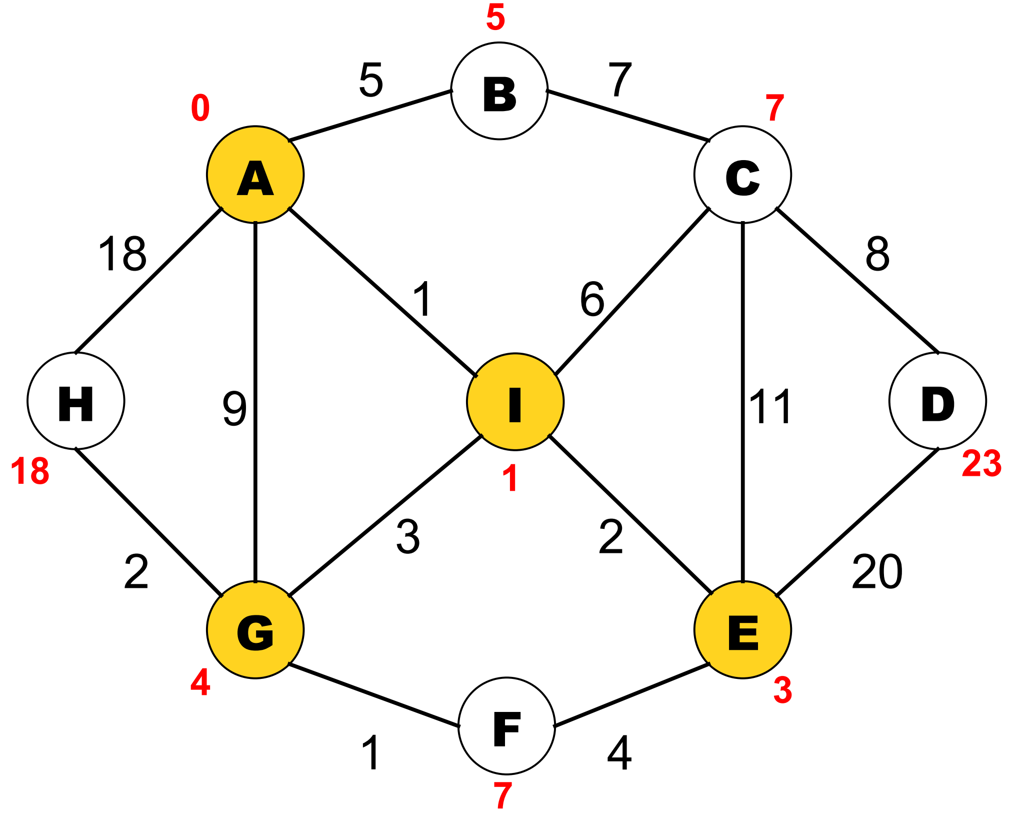
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra’s Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

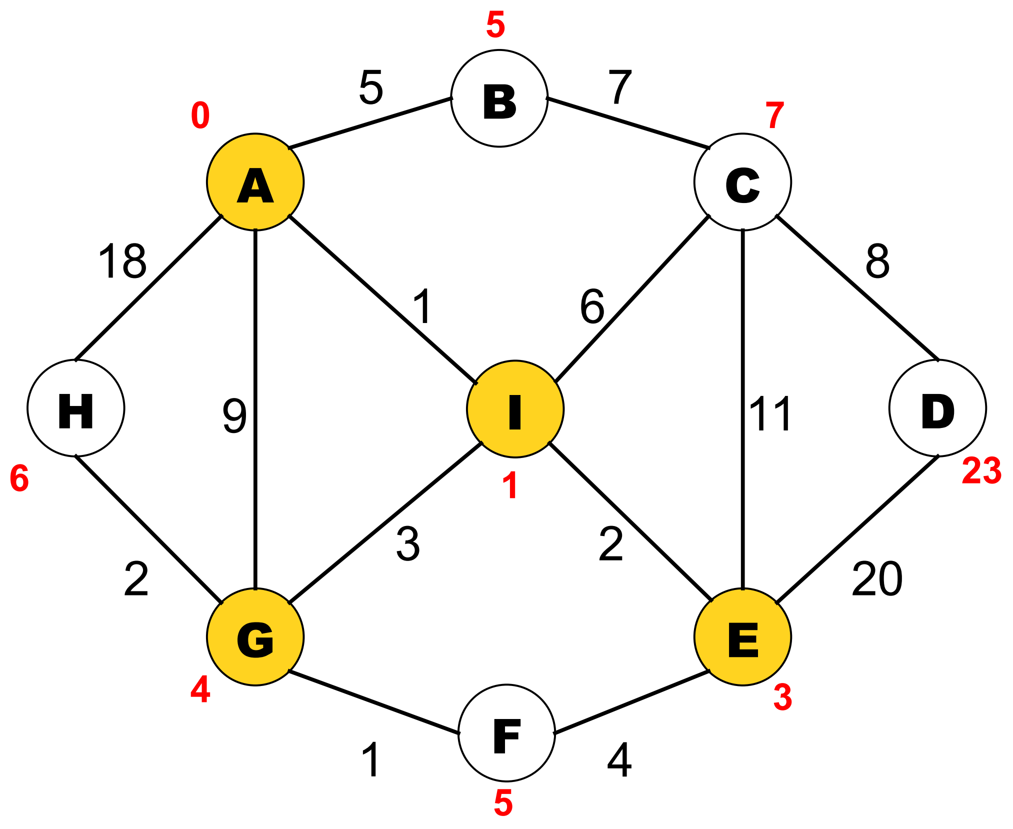


5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



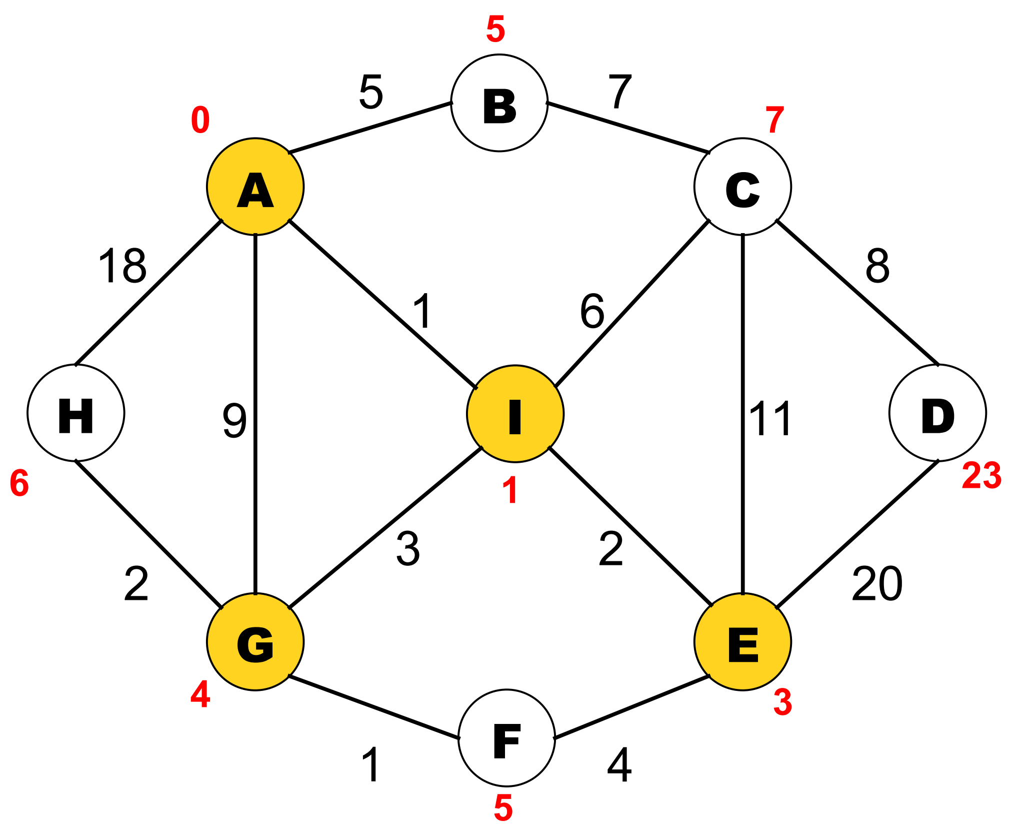
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I, E, and G** to be used as intermediary vertices along the path.

Dijkstra's Algorithm

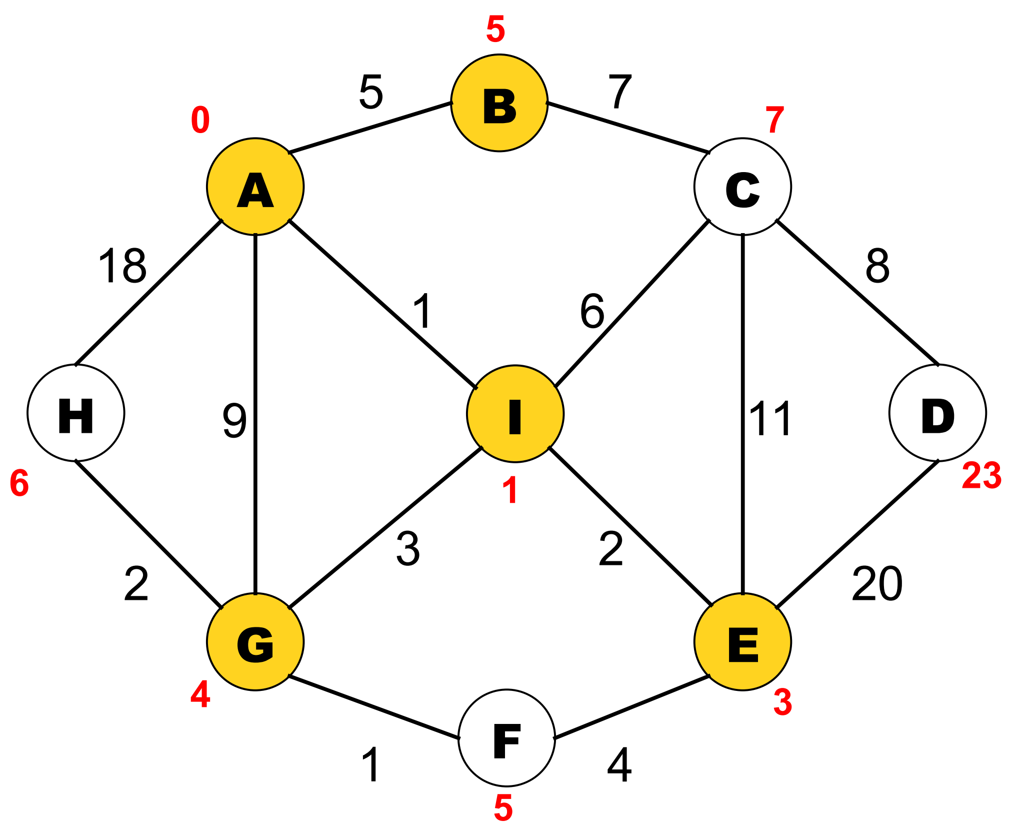
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

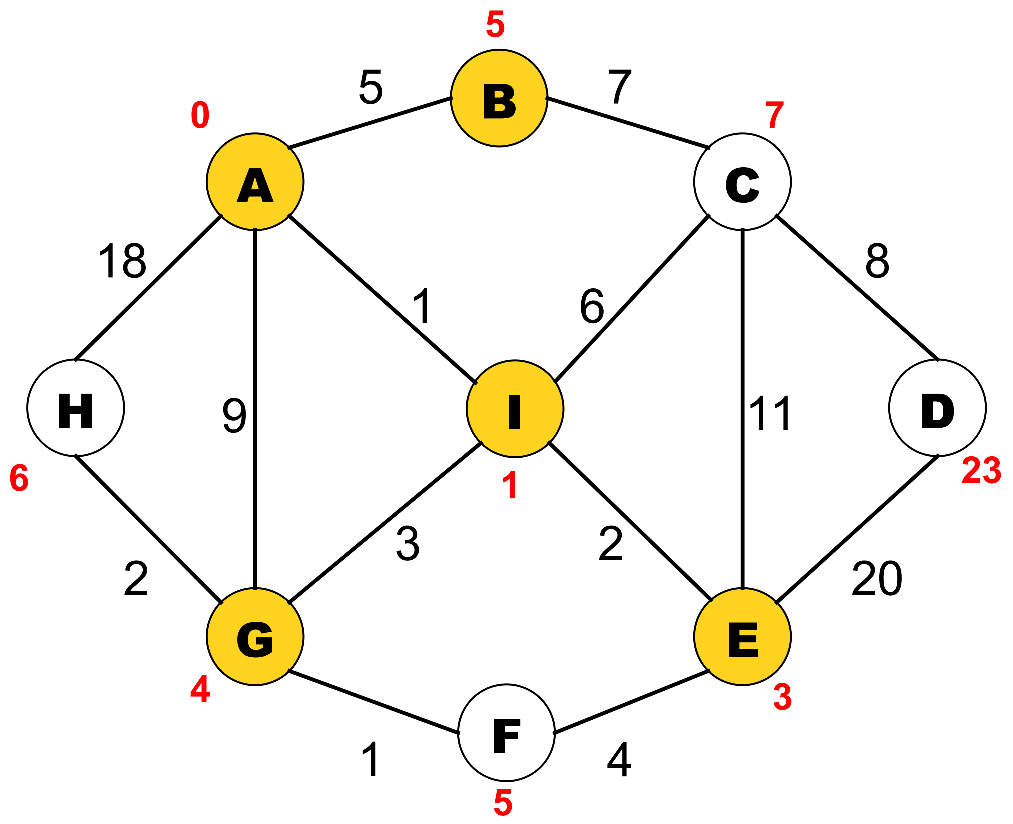
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra’s Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

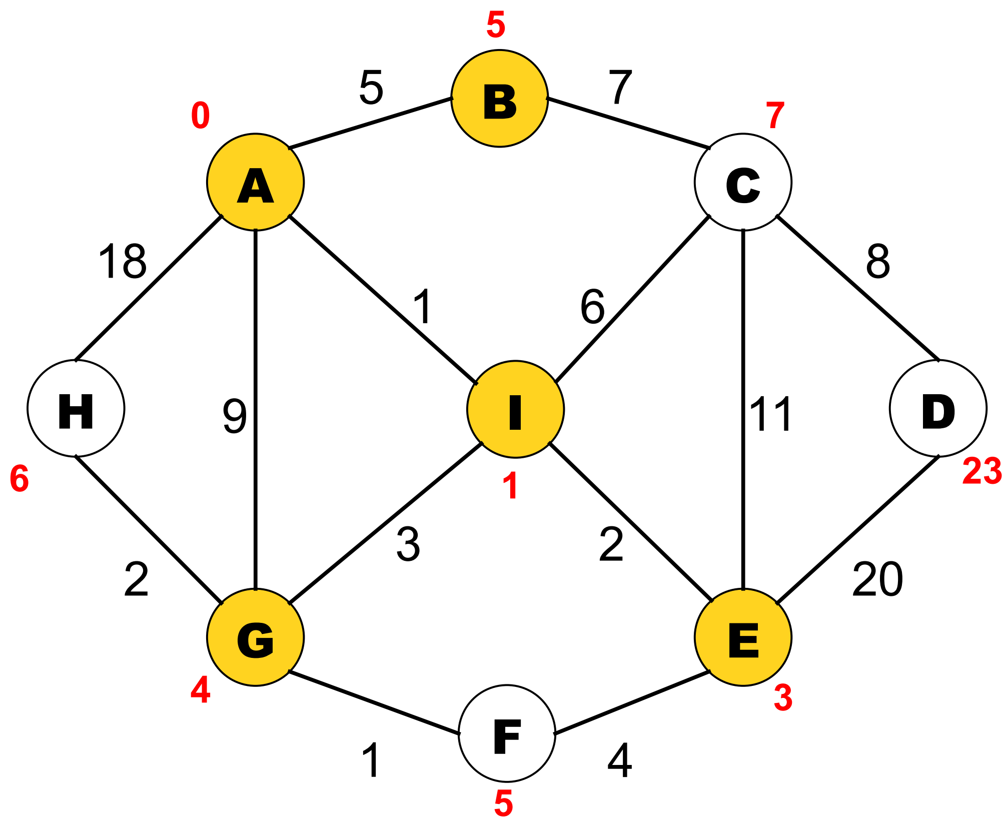


5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra’s Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



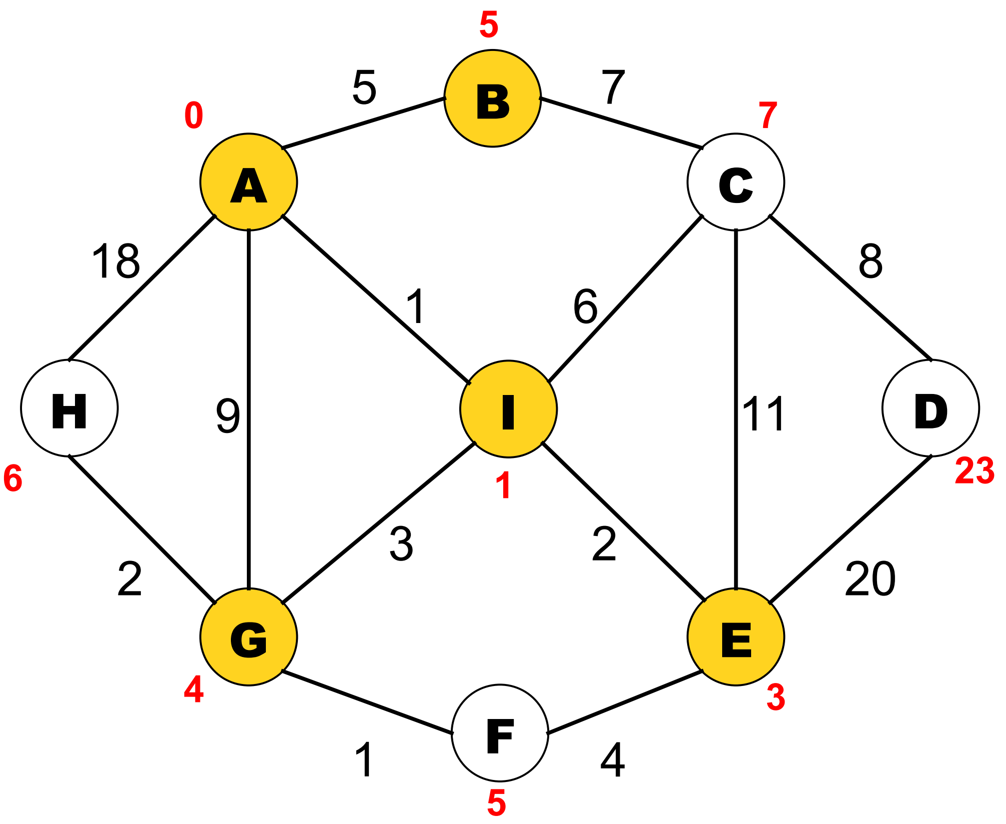
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I, E, G, and B** to be used as intermediary vertices along the path.

Dijkstra's Algorithm

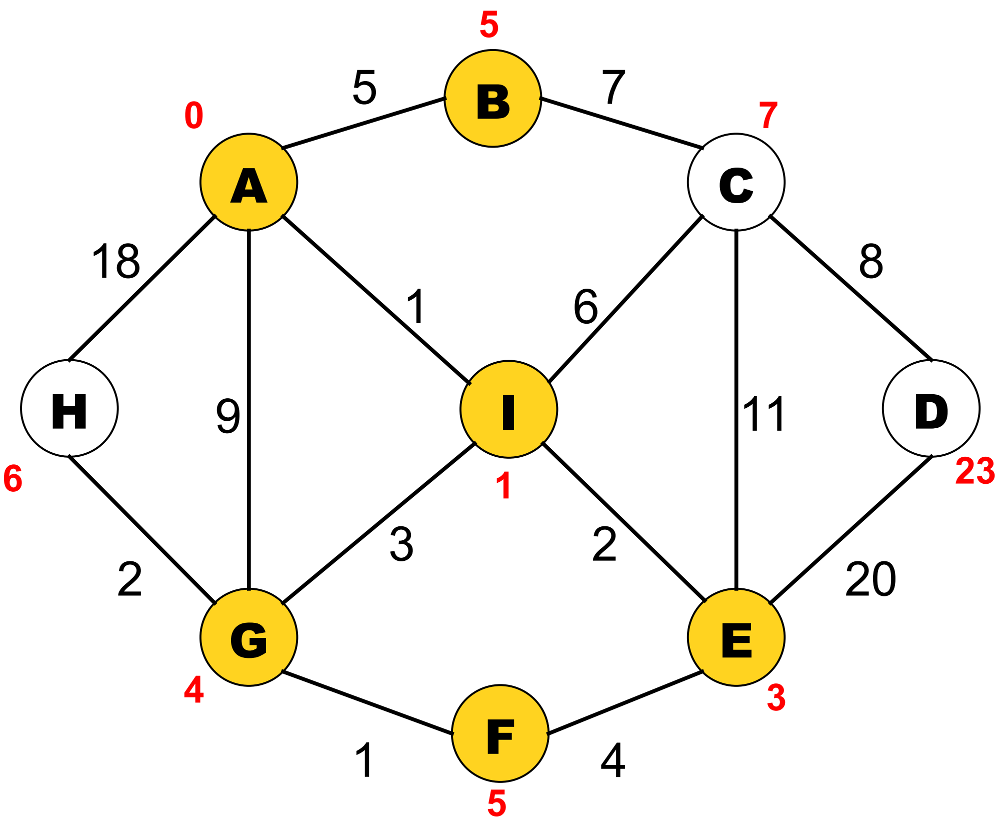
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

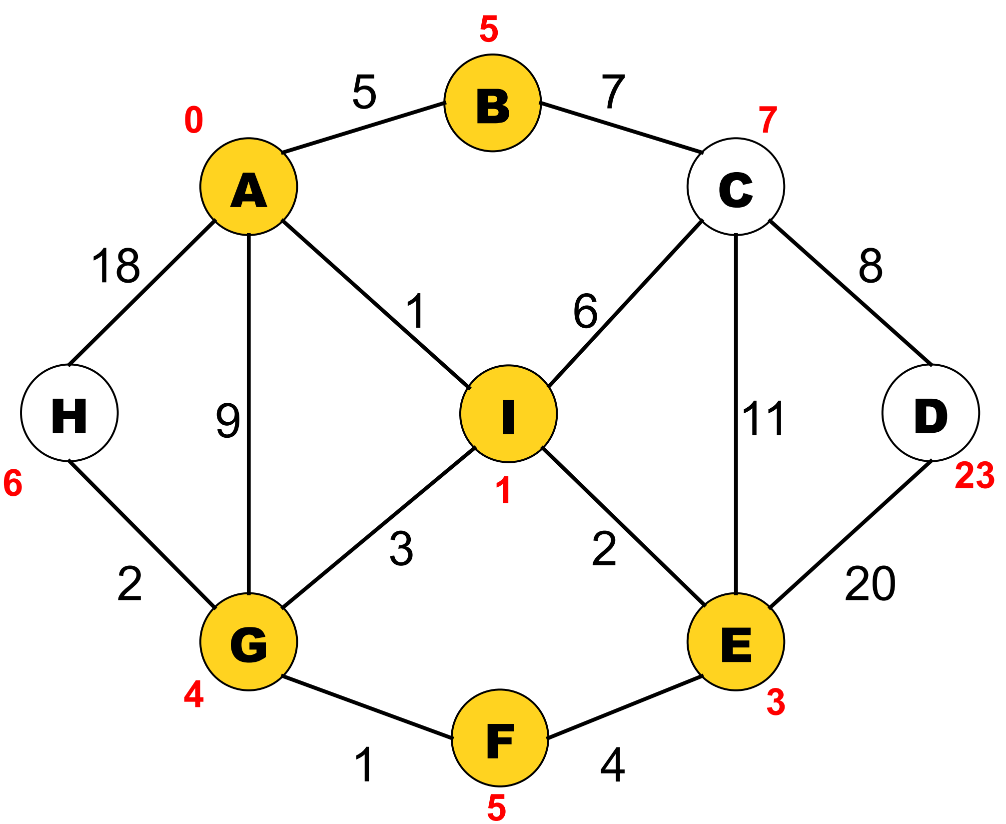
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

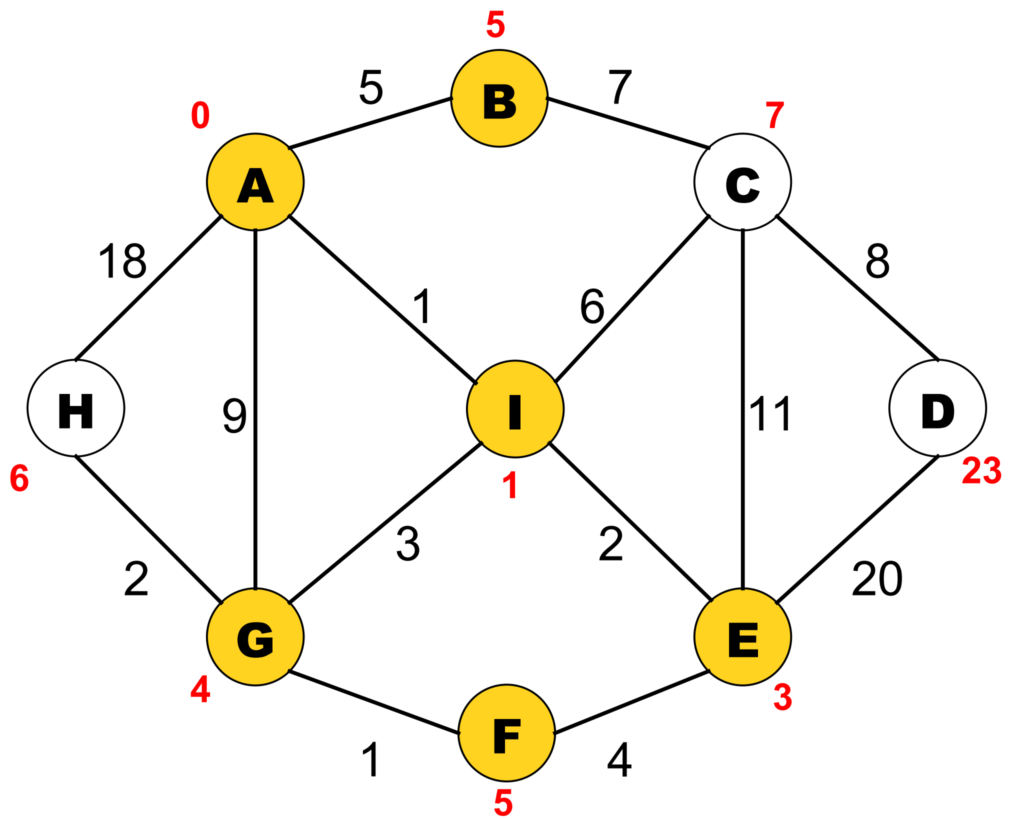


5: From that vertex, *i*, update the **dist[j]** values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



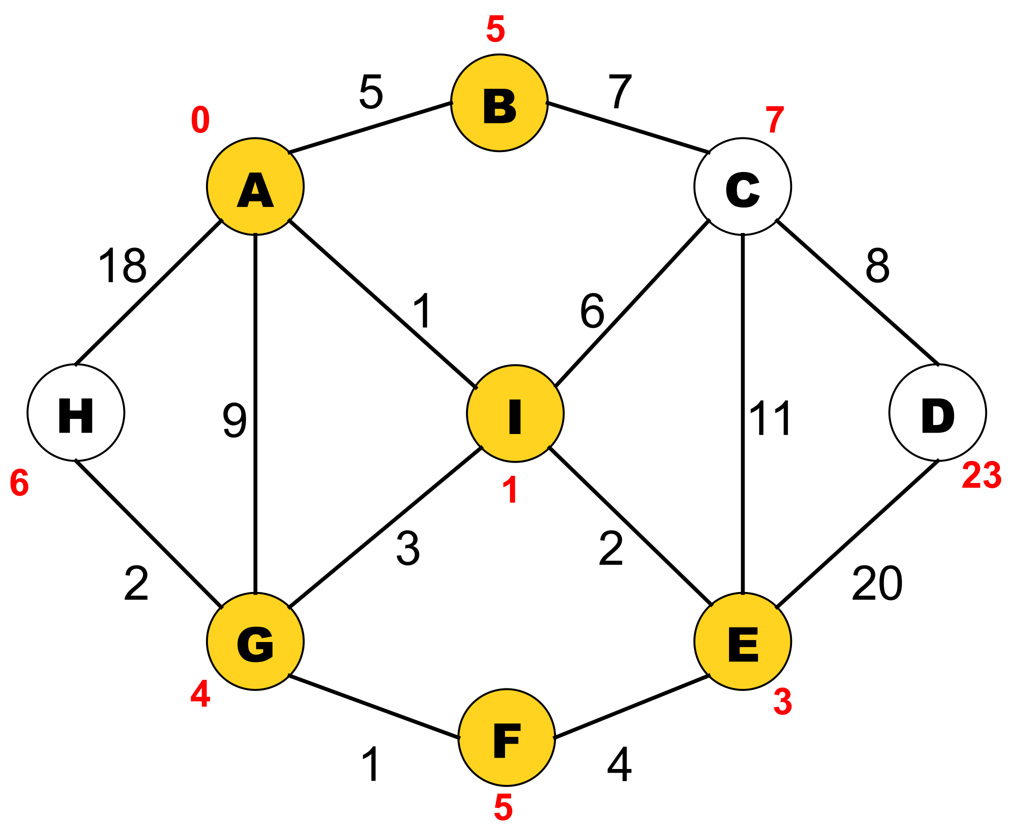
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I, E, G, B, and F** to be used as intermediary vertices along the path.

Dijkstra's Algorithm

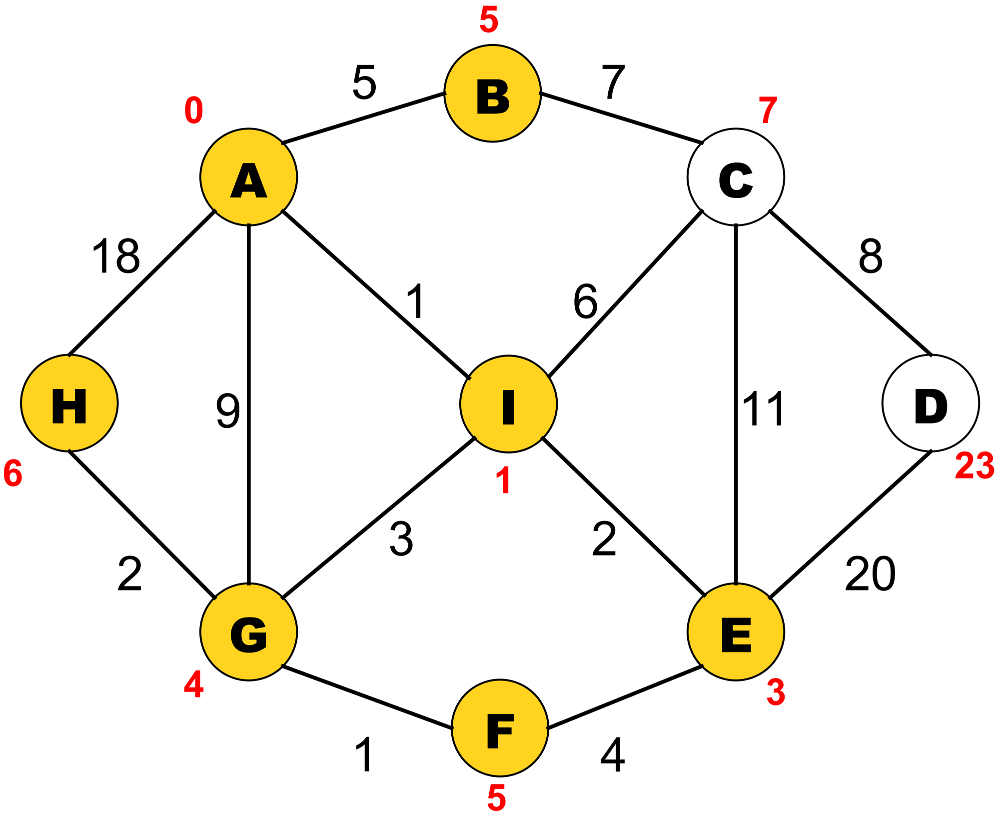
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra’s Algorithm

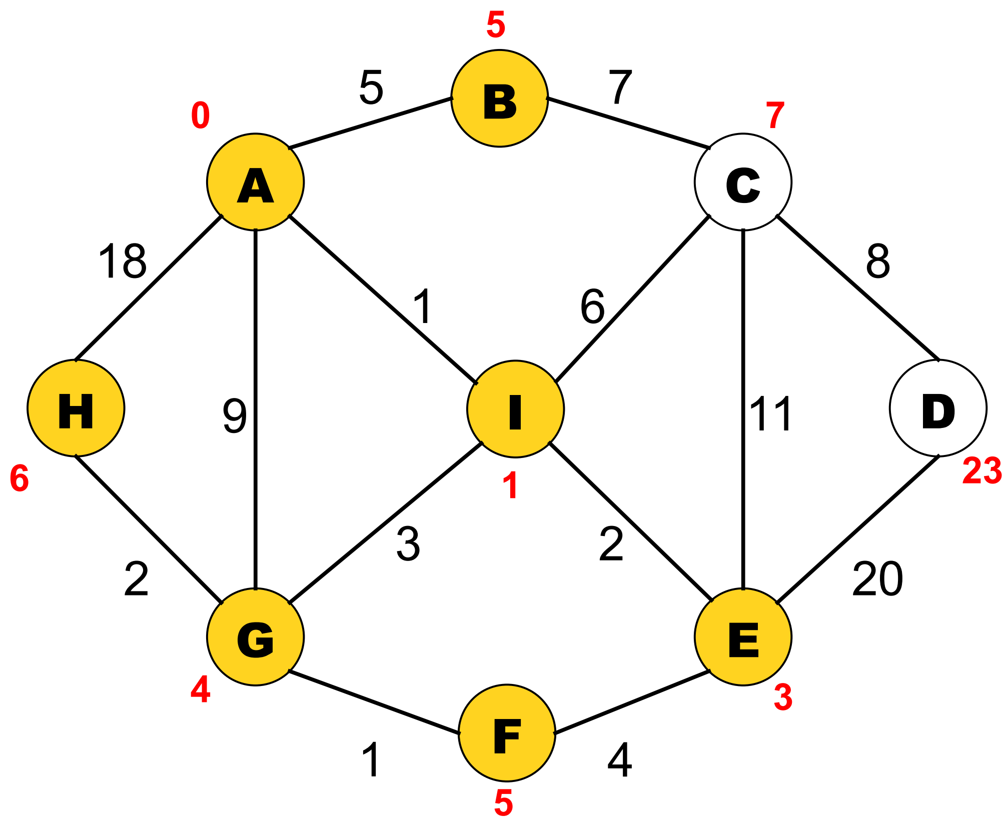
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



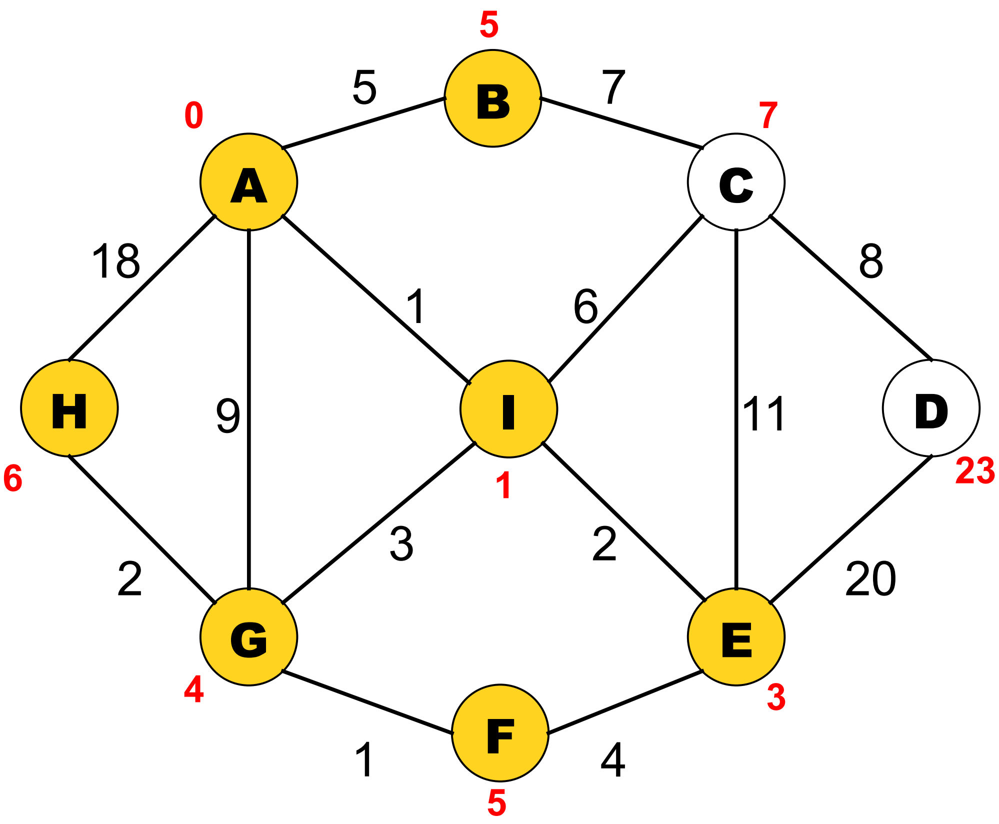
5: **(again)** From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I, E, G, B, F, and H** to be used as intermediary vertices along the path.

Dijkstra's Algorithm

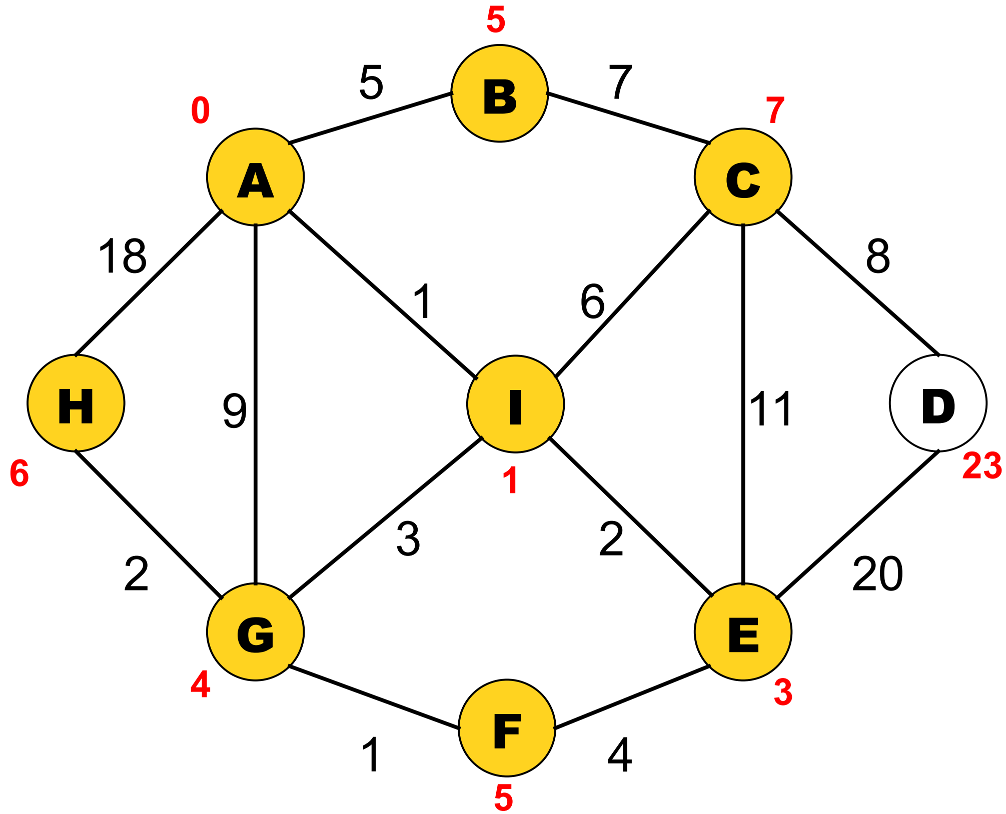
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

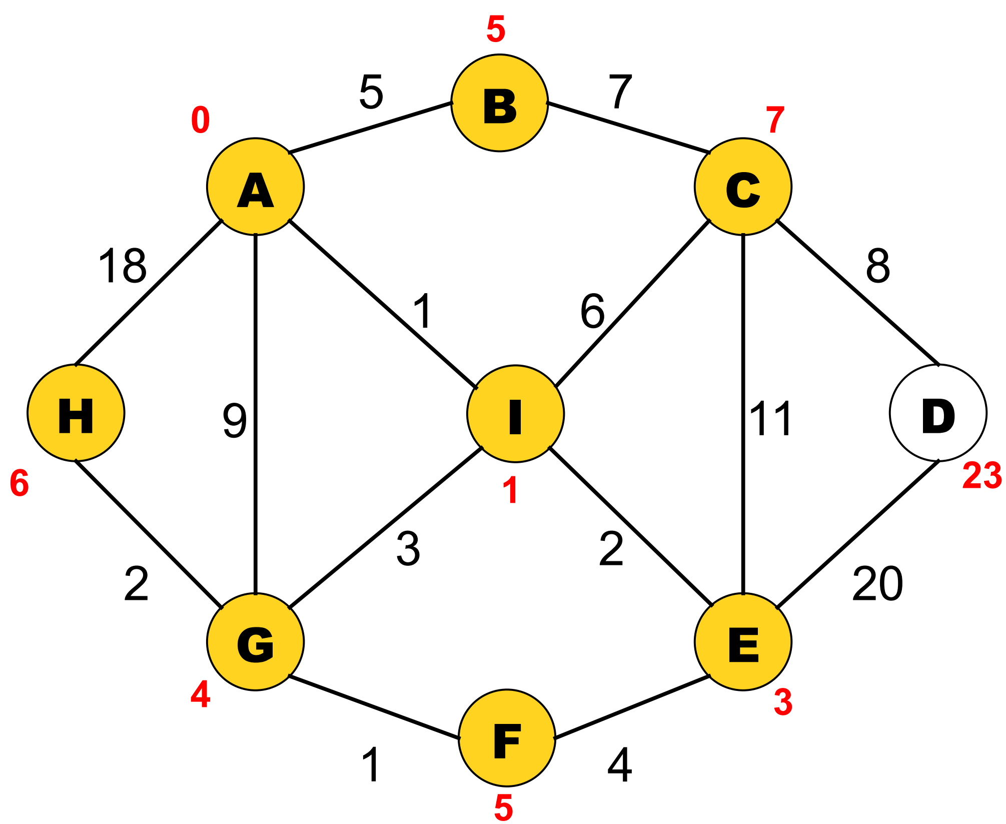
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

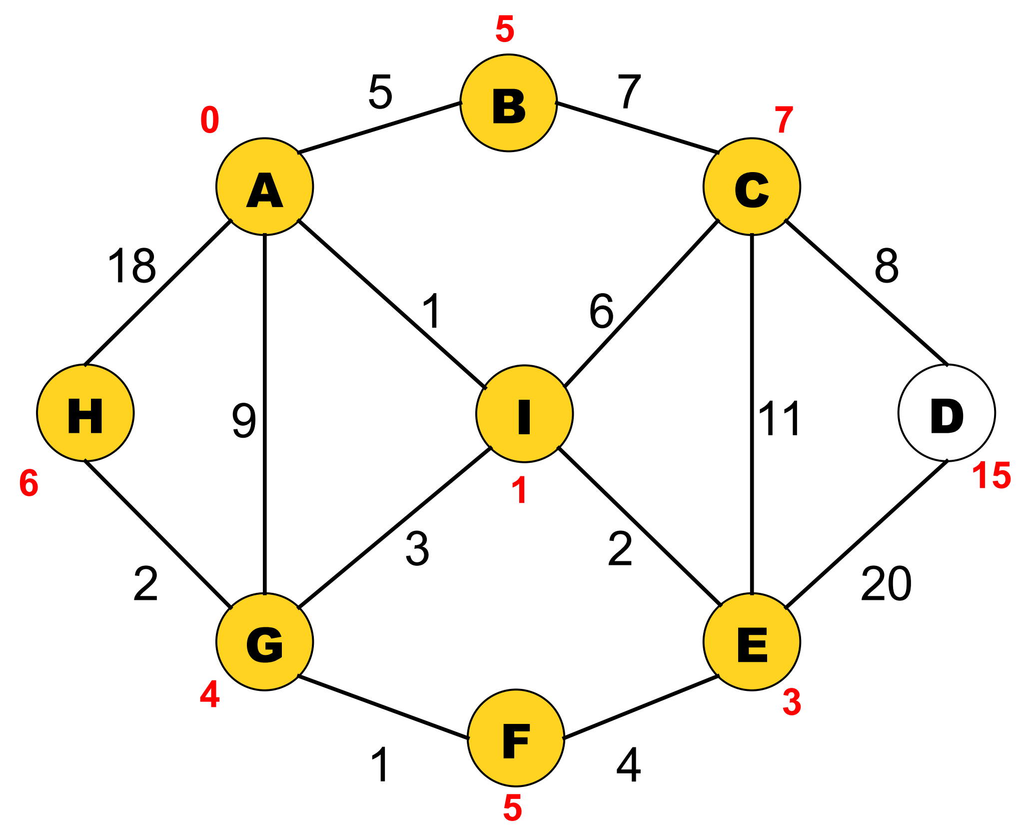


5: **(again)** From that vertex, *i*, update the **dist[j]** values for all adjacent vertices, *j*:

MIN{dist[i]+ weight[i, j], dist[j]}

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)



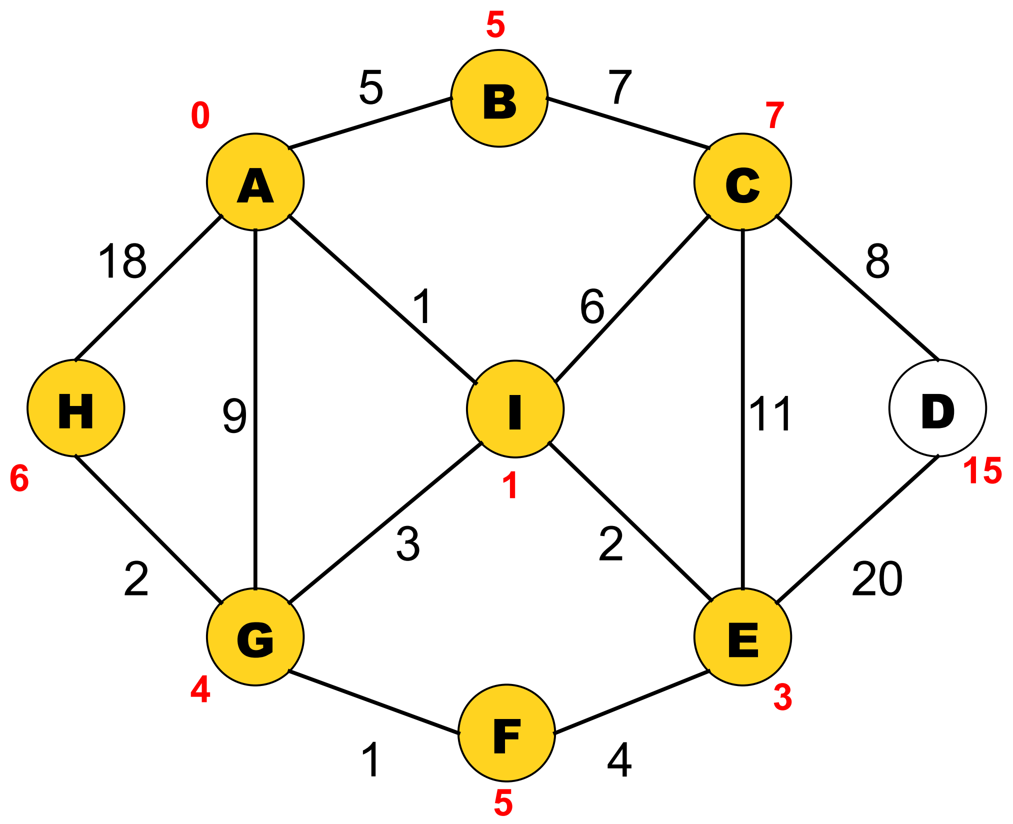
5: From that vertex, *i*, update the **dist**[**j**] values for all adjacent vertices, *j*:
(again)

MIN{dist[i]+ weight[i, j], dist[j]}

Note: The dist[i] values now indicate the lowest cost path from vertex A if we allow vertices **I, E, G, B, F, H, and C** to be used as intermediary vertices along the path.

Dijkstra's Algorithm

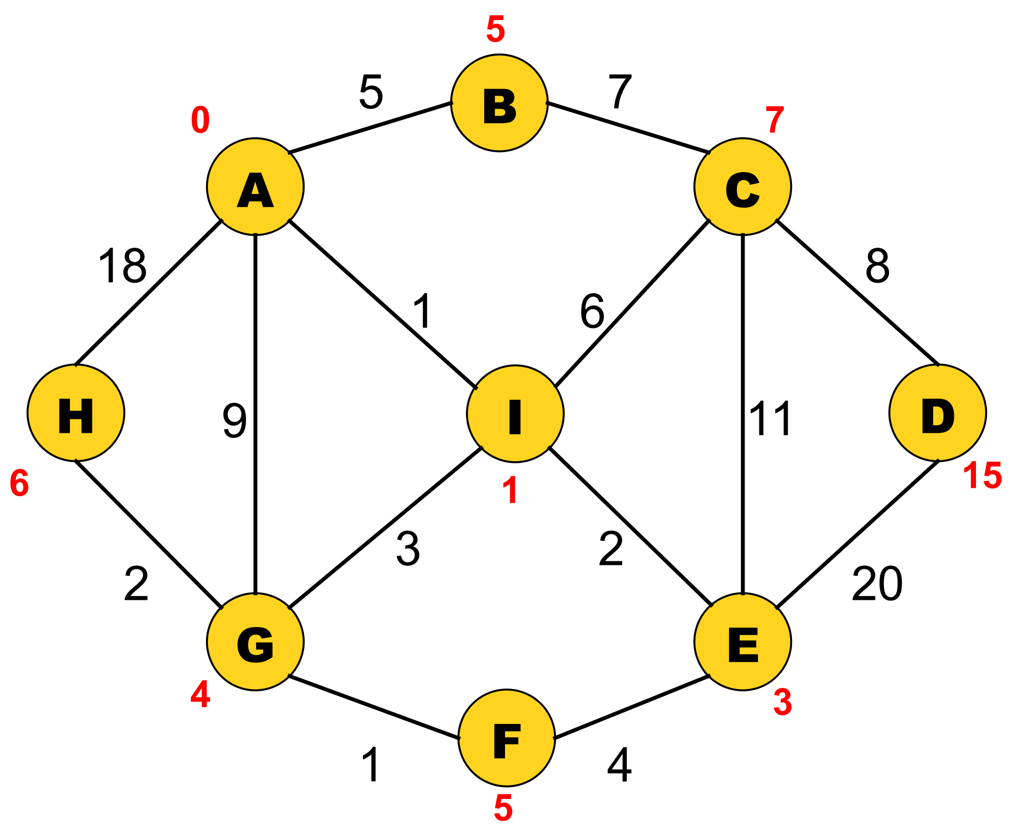
(calculating the cheapest path from a source vertex to all other vertices)



4: Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.
(again)

Dijkstra's Algorithm

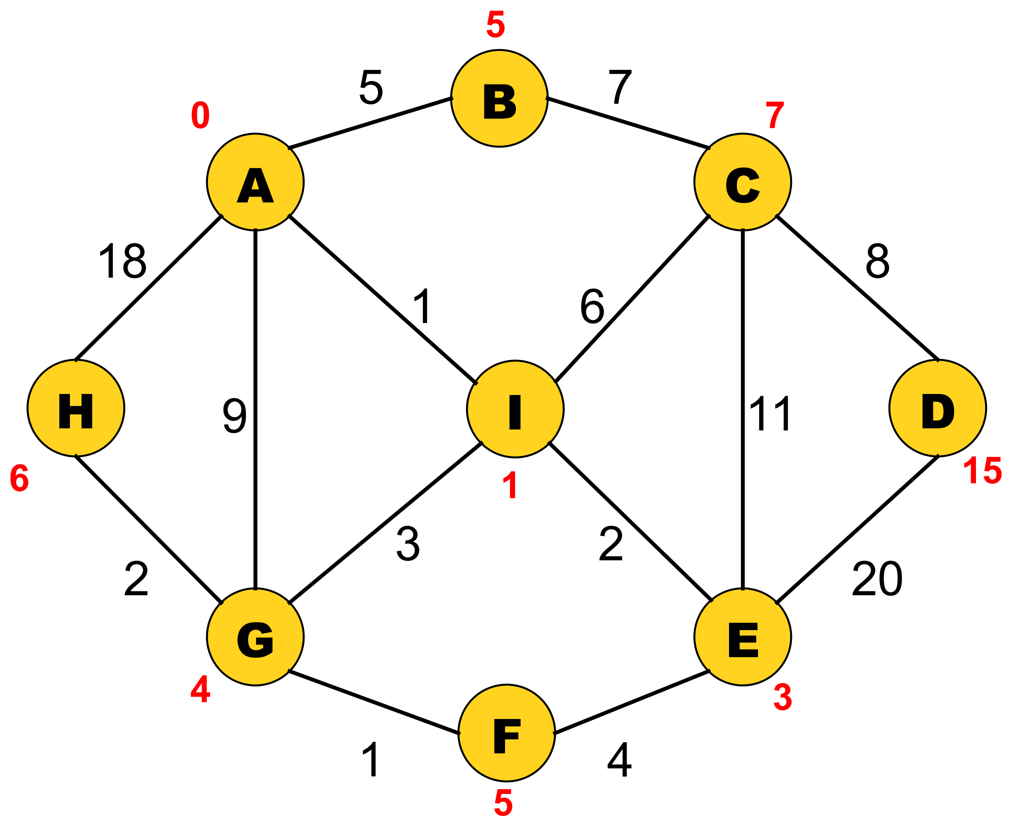
(calculating the cheapest path from a source vertex to all other vertices)



4: **(again)** Choose the unvisited vertex with the smallest **dist**[**i**] value and visit it.

Dijkstra's Algorithm

(calculating the cheapest path from a source vertex to all other vertices)

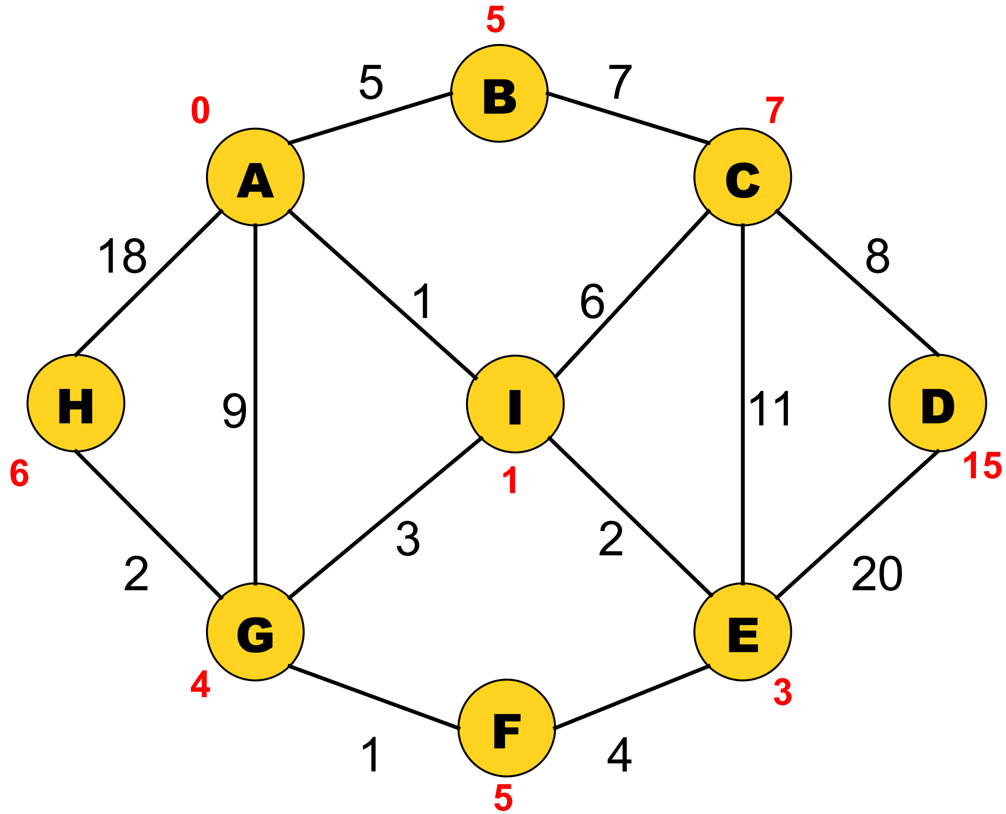


6: All vertices are visited, so terminate.

We now know the lowest cost to get from vertex A to each other vertex in the graph!

An idea for path recovery

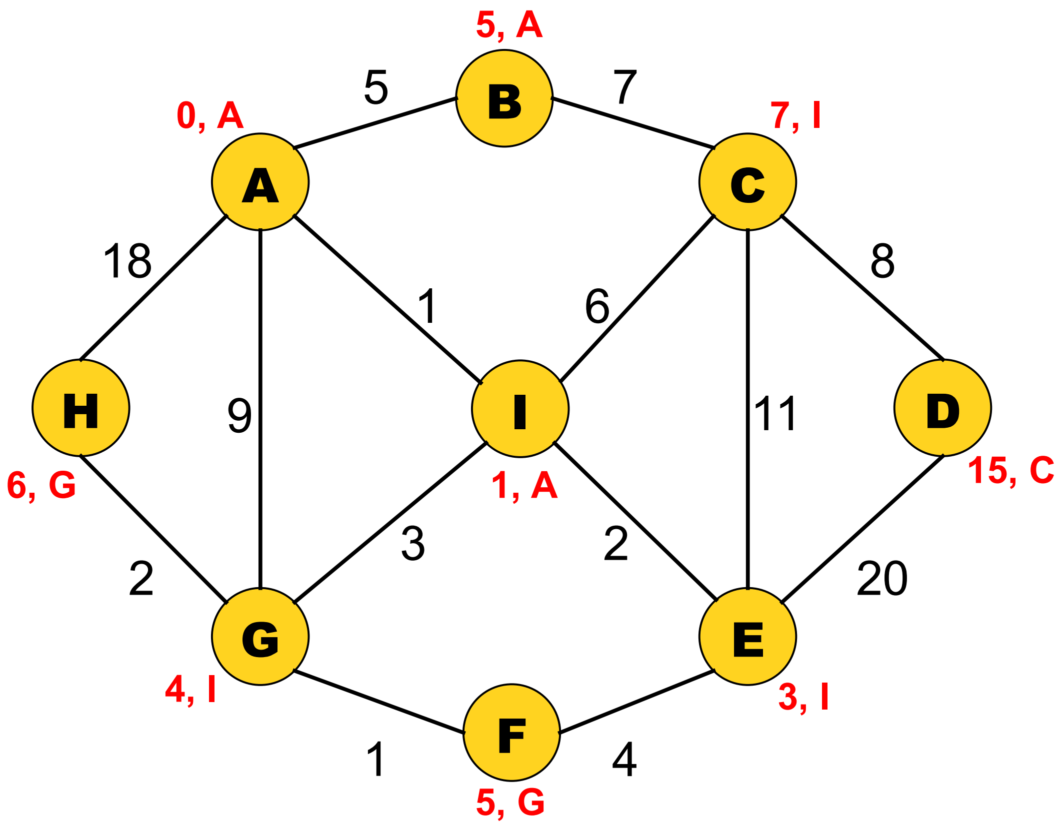
(with Dijkstra's Algorithm)



Idea: Keep track of the vertex we take every time we update a **dist**[**i**] value.

An idea for path recovery

(with Dijkstra's Algorithm)



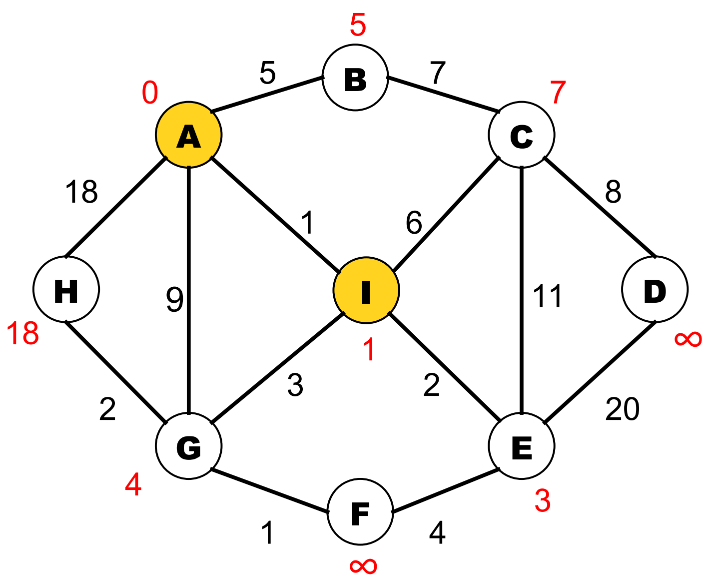
Idea: Keep track of the vertex we take every time we update a **dist**[**i**] value.

Now follow the vertices backward to the source to reconstruct the path.

For example, the path to D is **D** ← **C** ← **I** ← **A** (aka **A** → **I** → **C** → **D**)

The Algorithm

(with Dijkstra's Algorithm)



Initialize **dist**[**i**] to ∞.
Initialize **dist**[**source**] to 0.

while there are **unvisited vertices**:

  Find the unvisited vertex with minimum **dist**[**i**] value
  Visit that vertex.
  Update **dist**[**i**] for its unvisited neighbors.

We might want to halt if the minimum **dist**[**i**] value is ∞.

What is the runtime?