Programming Abstractions

CS106B

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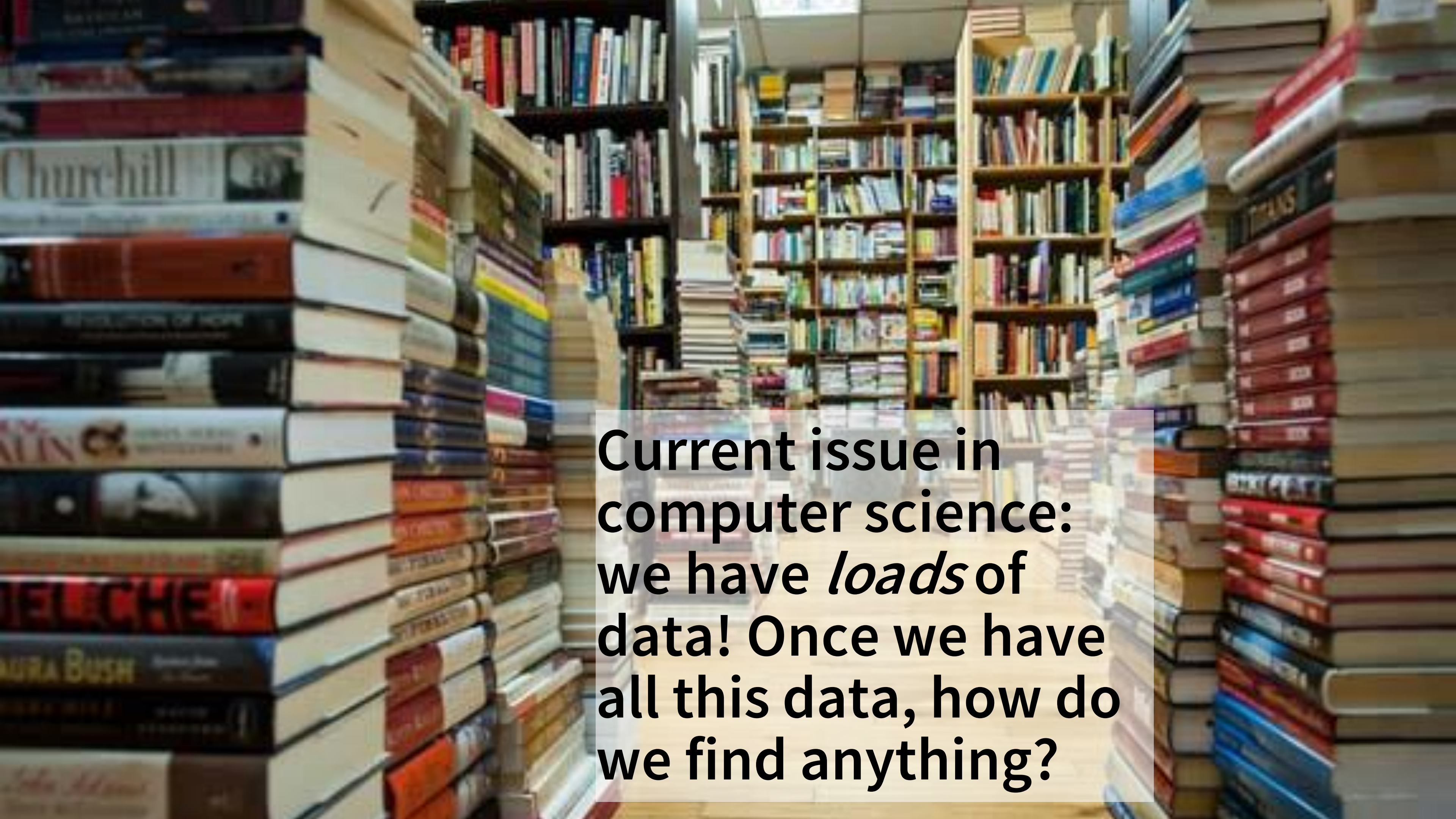
Today's Topics

- Recursion!
  - Algorithm performance analysis with Big-O
- Next time:
  - More recursion! It's Recursion Week!
  - Like Shark Week, but more nerdy

- **For important announcements, be sure to see the weekly announcements post on the Ed Q&A board! https://edstem.org**
- **Also on Ed: live lecture Q&A with Chris & Jonathan**

Binary Search

AN ELEGANT SOLUTION TO THE PROBLEM OF TOO MUCH DATA



Current issue in computer science: we have *loads* of data! Once we have all this data, how do we find anything?

The context:

- You have a **collection of numbers**
  - Say product IDs for items in stock in a store
  - We're going to store our collection of numbers in a Vector
  - We're going to keep them *in sorted order*

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- It's important to be able to **find out whether you have a particular number** in your collection or not
  - A customer asks, "Do you have item 8 in stock?" (Yes.)
  - A customer asks, "Do you have item 55 in stock?" (No.)

- **Key question: How long does this take?**

Does this list of numbers contain X?

Context: we have a collection of numbers in a Vector, in sorted order.

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- **Basic approach: Start at the front and proceed forward until you find:**
  - X (answer Yes)
  - A number greater than X (answer No)
  - End of the list (answer No)

- Key observation: each time you compare against the contents of a cell of the Vector and it's not X, you rule out *1* of the N cells in the Vector

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- **Efficiency Hack: Jump to the middle of the Vector and look there to find:**
  - X (answer Yes)
  - A number greater than X (rule out entire second half of Vector)

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

  - A number less than X (rule out entire first half of Vector)

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- Key observation: with *one* comparison, you ruled out *N/2* of the N cells in the Vector!

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- **Efficiency Hack: Jump to the middle of the Vector and look there to find:**
  - X (answer Yes)
  - A number greater than X (rule out entire second half of Ve

0	1	2	3	4	5	6	7	8	9	1
2	7	8	13	25	29	33	51	89	90	9

  - A number less than X (rule out entire first half of Vector)

0	1	2	3	4	5	6	7	8	9	1
2	7	8	13	25	29	33	51	89	90	95

Now we could do our Basic Approach, but in **half the time.** Thanks, Efficiency Hack!!

- Key observation: with *one* comparison, you ruled out *N/2* of the N
the Vector!

…but I have an even better idea…

Does this list of numbers contain X?

Context: we have a collection of numbers in a Vector, in sorted order.

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- **Extreme Efficiency Hack: Keep jumping to the middle!**
  - Let's say our first jump to the middle found a number less than X, so we ruled out the whole first half:

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

  - Now jump to the middle of the remaining second half:

0	1	2	3	4	5	6	7	8	9	10
2	7	8	13	25	29	33	51	89	90	95

- Key observation: we do one piece of work, then delegate the rest. **Recursion!!**

Basic Recursive Function Design Pattern

Always two parts:

Base case:

- This problem is so tiny, it's hardly a problem anymore! Just give answer.

Recursive case:

- This problem is still a bit large, let's (1) bite off just one piece, and (2) delegate the remaining work to recursion.

Binary Search pseudocode

- We'll write the real C++ code together on Friday, but here's the outline/pseudocode of how it works:

```
bool binarySearch(Vector<int>& data, int key)
{
    if (data.size() == 0) {
        return false;
    }
    if (key == data[midpoint]) {
        return true;
    } else if (key < data[midpoint]) {
        return binarySearch(data[first half only], key);
    } else {
        return binarySearch(data[second half only], key);
    }
}
```

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        return binarySearch(data[second half only], key);
    }
}
```

Base case: we shrank the search problem so tiny it no longer exists!

Recursive case:

Do one piece of work (comparison)

Delegate the rest of the work

Big-O Performance Analysis

A WAY TO COMPARE THE NUMBER OF STEPS TO RUN THESE FUNCTIONS

Big-O analysis in computer science

Vector

web.stanford.edu/dept/cs_edu/resources/cslib_docs/Vector.html

The Stanford `libcs106` library, Fall Quarter 2022

`#include "vector.h"`

`class Vector<ValueType>`

This class stores an ordered list of values similar to an array. It supports traditional array selection using square brackets, as well as inserting and removing elements. Operations that access elements by index run in O(1) time. Operations, such as insert and remove, that must rearrange elements run in O(N) time.

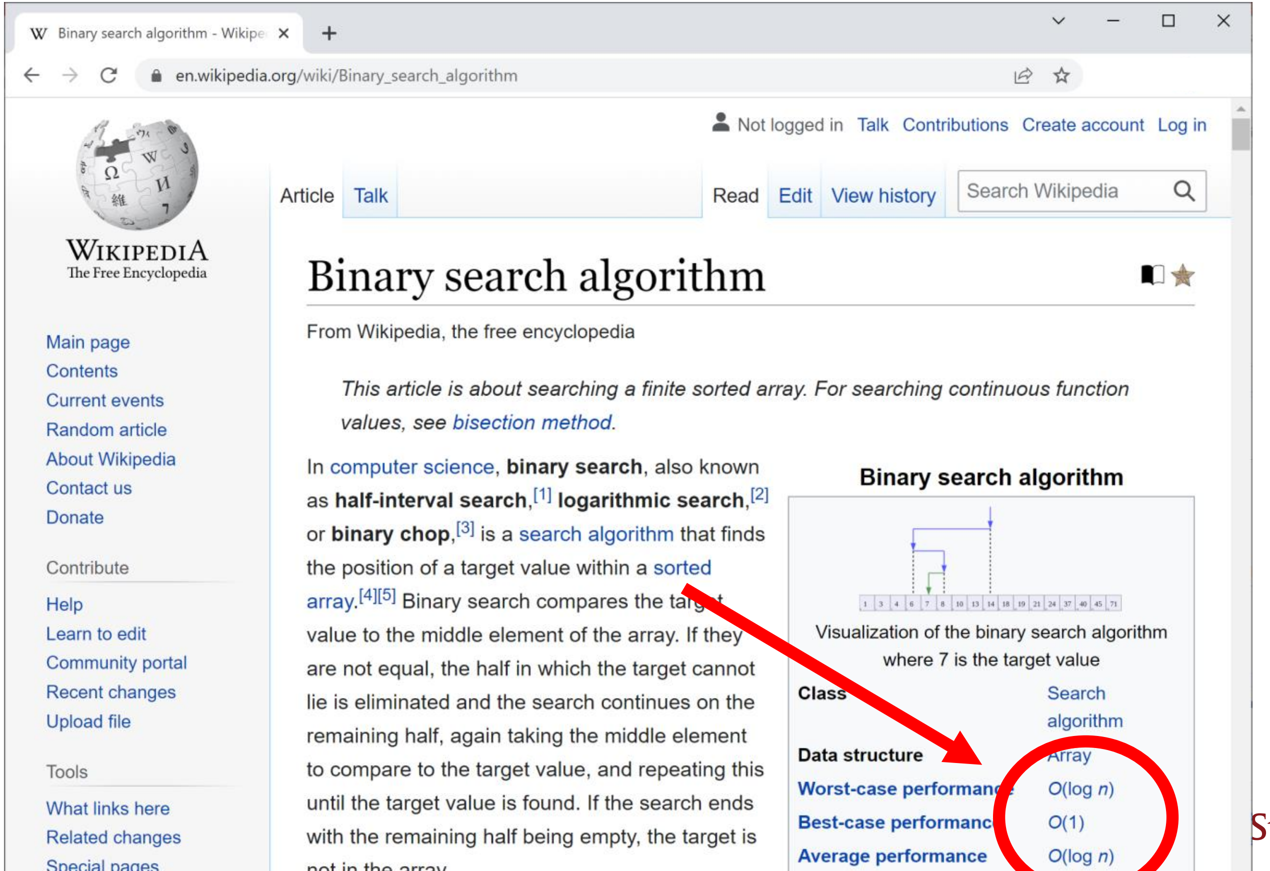
Constructor

`Vector()`	O(1)	Initializes a new empty vector.
`Vector(n, value)`	O(N)	Initializes a new vector storing *n* copies of the given value.

Methods

`add(value)`	O(1)	Adds a new value to the end of this vector.
`clear()`	O(1)	Removes all elements from this vector.
`equals(vec)`	O(N)	Returns **true** if the two vectors contain the same elements in the same order.
`get(index)`	O(1)	Returns the element at the specified index in this vector.

Big-O analysis in computer science



Formal definition of big-O

We say a function $f(n)$ is "big-O" of another function $g(n)$
(written $f(n) = O(g(n))$)

if and only if

there exist positive constants c and n_0 such that

$$f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0.$$

c says "we don't care about constant coefficients"

n_0 says "we only care about performance on big data sets"

Before we start, let's get introduced

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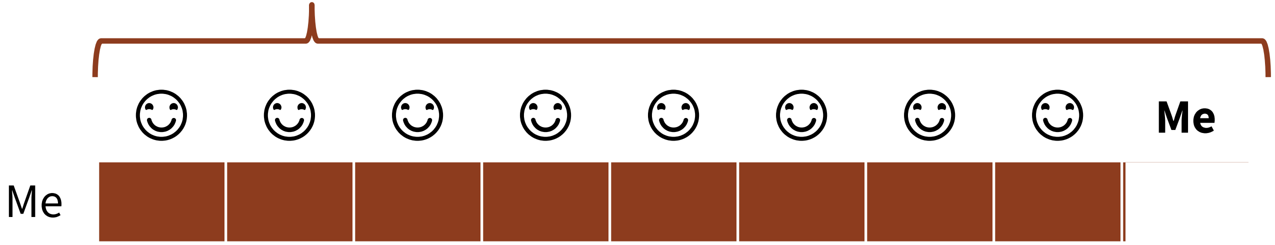
Lets say I want to meet each of you today with a handshake and *you* tell *me* your name…

How many introductions need to happen?

There are **N** people in the room including me



But do I need to shake hands with myself, or tell myself my name?

N-1 introductions

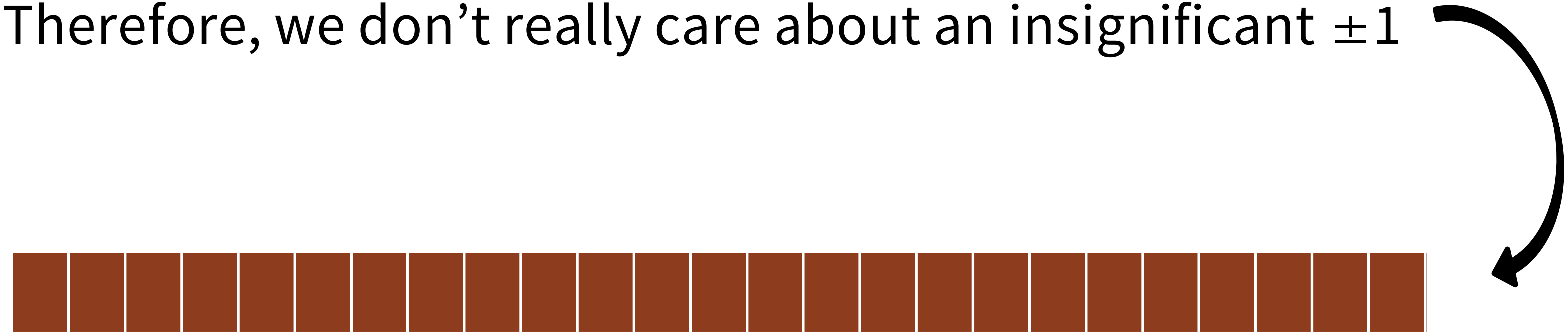
Stanford University

Putting this in Big-O terms

Big-O is a way of categorizing amount of work to be done in general terms, with a focus on:

- ***Rate of growth*** as a function of the problem size N
- What that rate looks like ***on the horizon*** (i.e., for large N)

Therefore, we don't really care about an insignificant ±1



Stanford University

Putting this in Big-O terms

For the first handshake problem, the rate N is important and the -1 constant is not, so **N – 1** introductions becomes:

$$O(N-1) \rightarrow O(N)$$

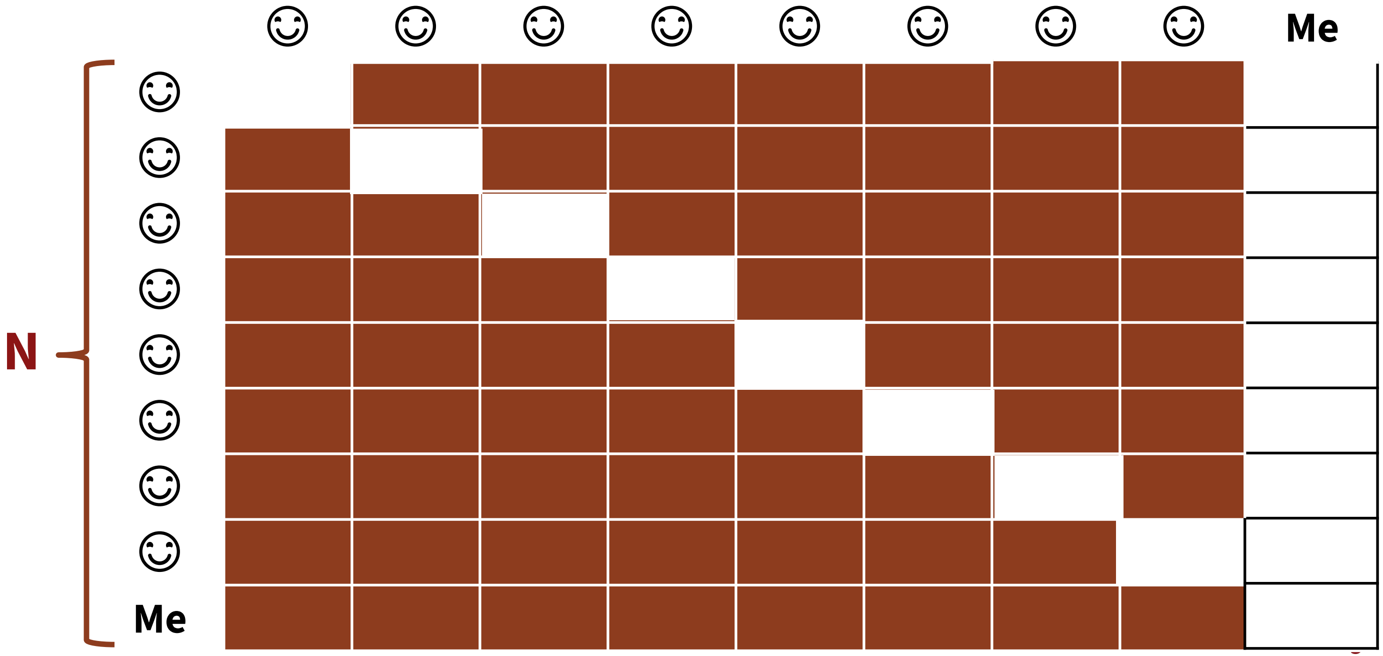
Similarly, if we said that each introduction **takes 3 seconds**, the amount of time is **3(N – 1) = 3N – 3**, but we disregard the constant 3s:

$$O(3N - 3) \rightarrow O(N)$$

Before we start, let’s get introduced

What if I not only want you to be introduced to me, but to each other? (Note: I don’t need to tell anyone my name, and nobody needs to tell themselves their own name, so subtract those out.)

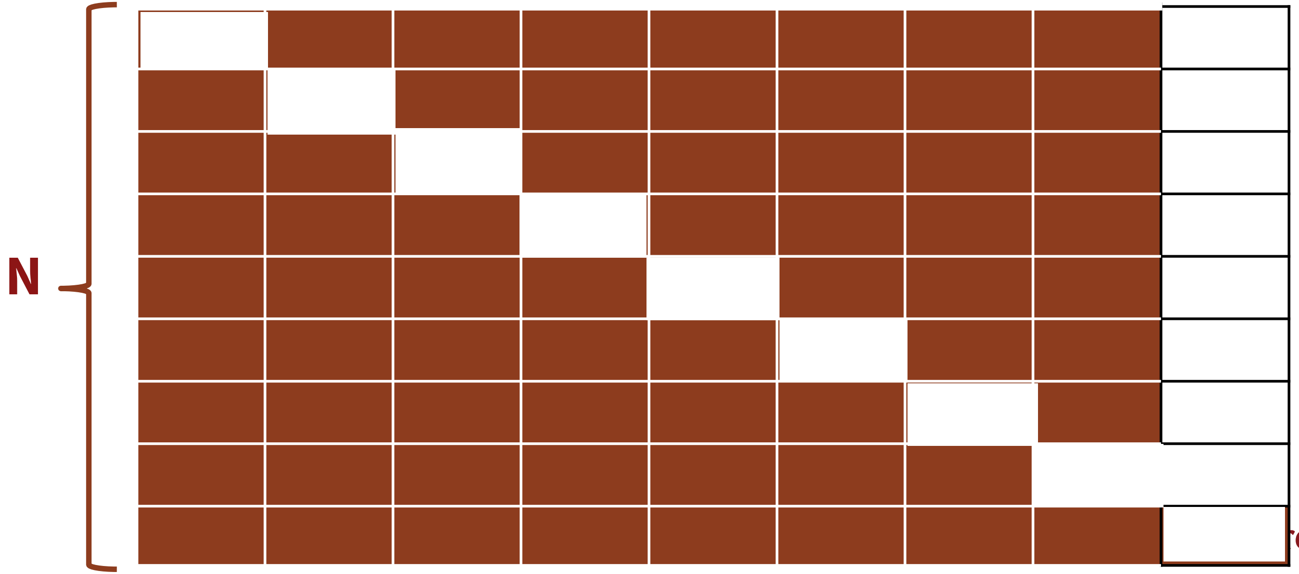
Now how many introductions? **N^2**



Before we start, let's get introduced

What if I not only want you to be introduced to me, but to each other? (Note: I don't need to tell anyone my name, and nobody needs to tell themselves their own name, so subtract those out.)

Now how many introductions? $(\mathbf{N - 1})^2 = \mathbf{N}^2 - \mathbf{2N + 1}$



Putting this in Big-O terms

For the second handshake problem, the introductions was **N^2 - N**:

$$O(\quad N^2 - 2N + 1) \rightarrow O(N^2)$$

But wait, didn't we just say that a term of +/- N *was* important?

For Big-O, we only care about the **largest term** of the polynomial

Big-O and Binary Search

SPOILER: FAST!!

Binary search

2	7	8	13	25	29	33	51	89	90	95

Jump right to the middle of the region to search, then repeat this process of roughly cutting the array in half again and again until we either find the item or (worst case) cut it down to nothing.

Worst case cost is number of times we can divide length in half:

$$O(\log_2 N)$$

Putting it all together

Binary search

Handshake #1

Handshake #2

MANY important optimization and other problems

$\log_2 n$	n	$n \log_2 n$	n^2	2^n
2	**4**	8	16	
3	**8**	24	64	256
4	**16**	64	256	65,536
5	**32**	160	1,024	4,294,967,296
6	**64**			
7	**128**			
8	**256**			
9	**512**			
10	**1,024**			
30	**2,700,000,000**			

Naïve Recursive Fibonacci ($O(1.6^n)$)

$\log_2 n$	n	$n \log_2 n$	n^2	2^n
2	**4**	8	16	16
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2.4s

Easy!

Traveling Salesperson Problem:
We have a bunch of cities to visit. In what order should we visit them to minimize total travel distance?

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We have a bunch of cities to visit. In what order should we visit them to minimize total travel distance?

Exhaustively try all orderings: **$O(n!)$**

Use current best known algorithm: **$O(n^2 2^n)$**

Maybe we could invent an algorithm that fits in our rightmost column: **$O(2^n)$**

So let's say we come up with a way to solve Traveling Salesperson Problem in $O(2^n)$.

It would take 4 days to solve Traveling Salesperson Problem on 50 state capitals.

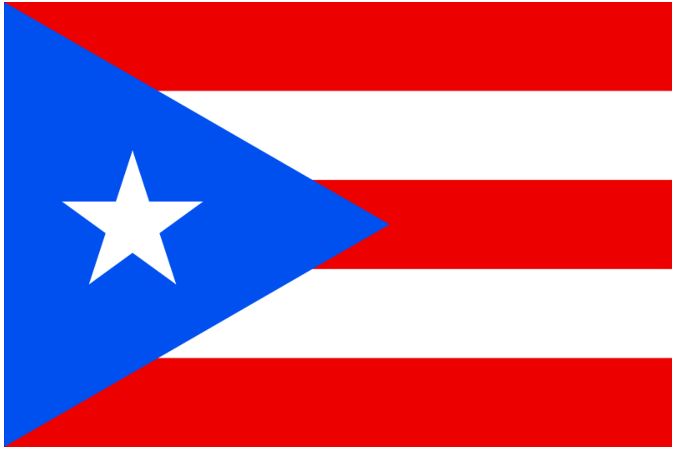
Two *tiny* little updates

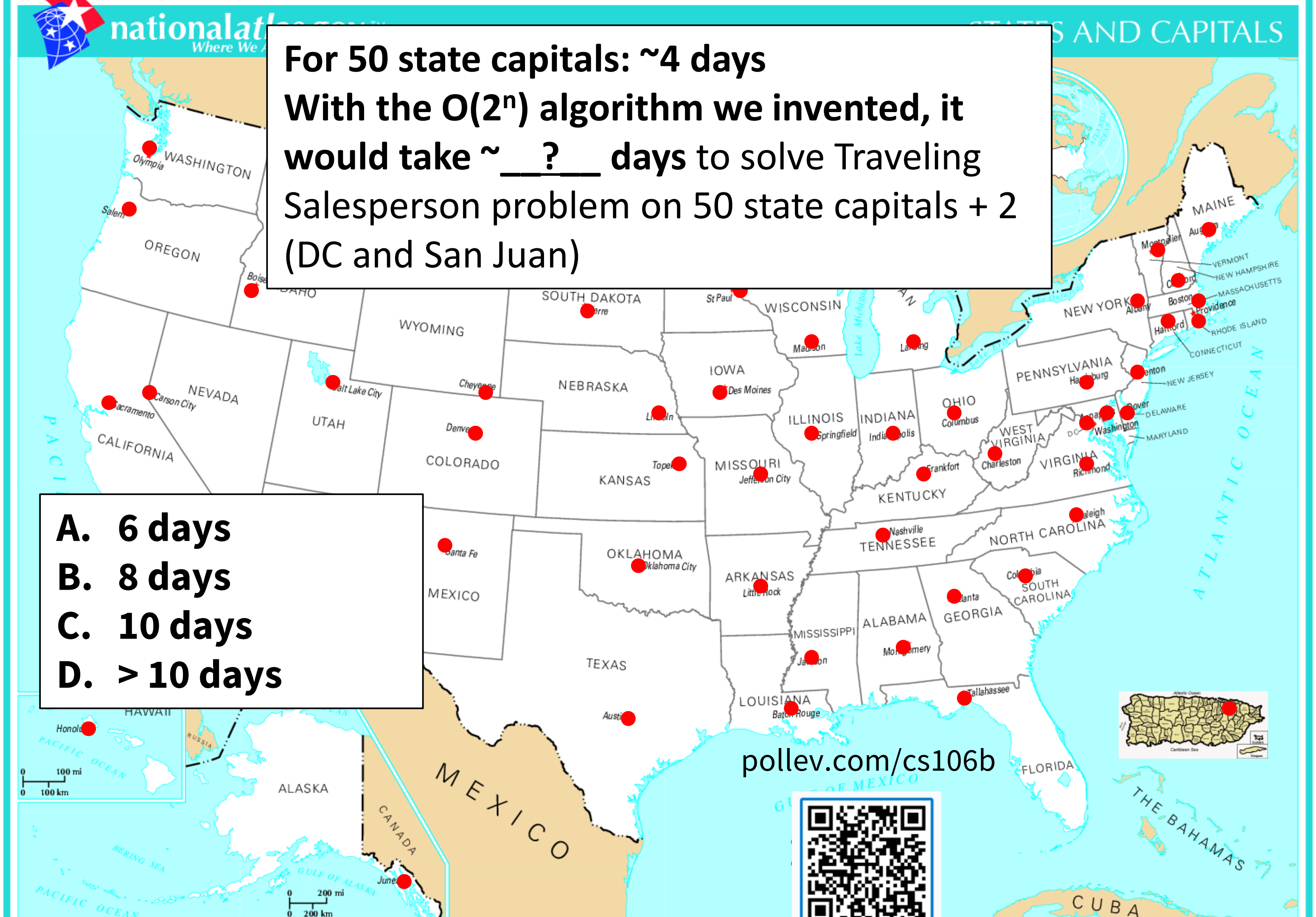
Imagine we approve statehood for US territory Puerto Rico

- Add San Juan, the capital city

Also add Washington, DC

Now <u>52</u> capital cities instead of <u>50</u>





For 50 state capitals: ~4 days

With the $O(2^n)$ algorithm we invented, it would take ~__?__ days to solve Traveling Salesperson problem on 50 state capitals + 2 (DC and San Juan)

- **A. 6 days**
- **B. 8 days**
- **C. 10 days**
- **D. > 10 days**

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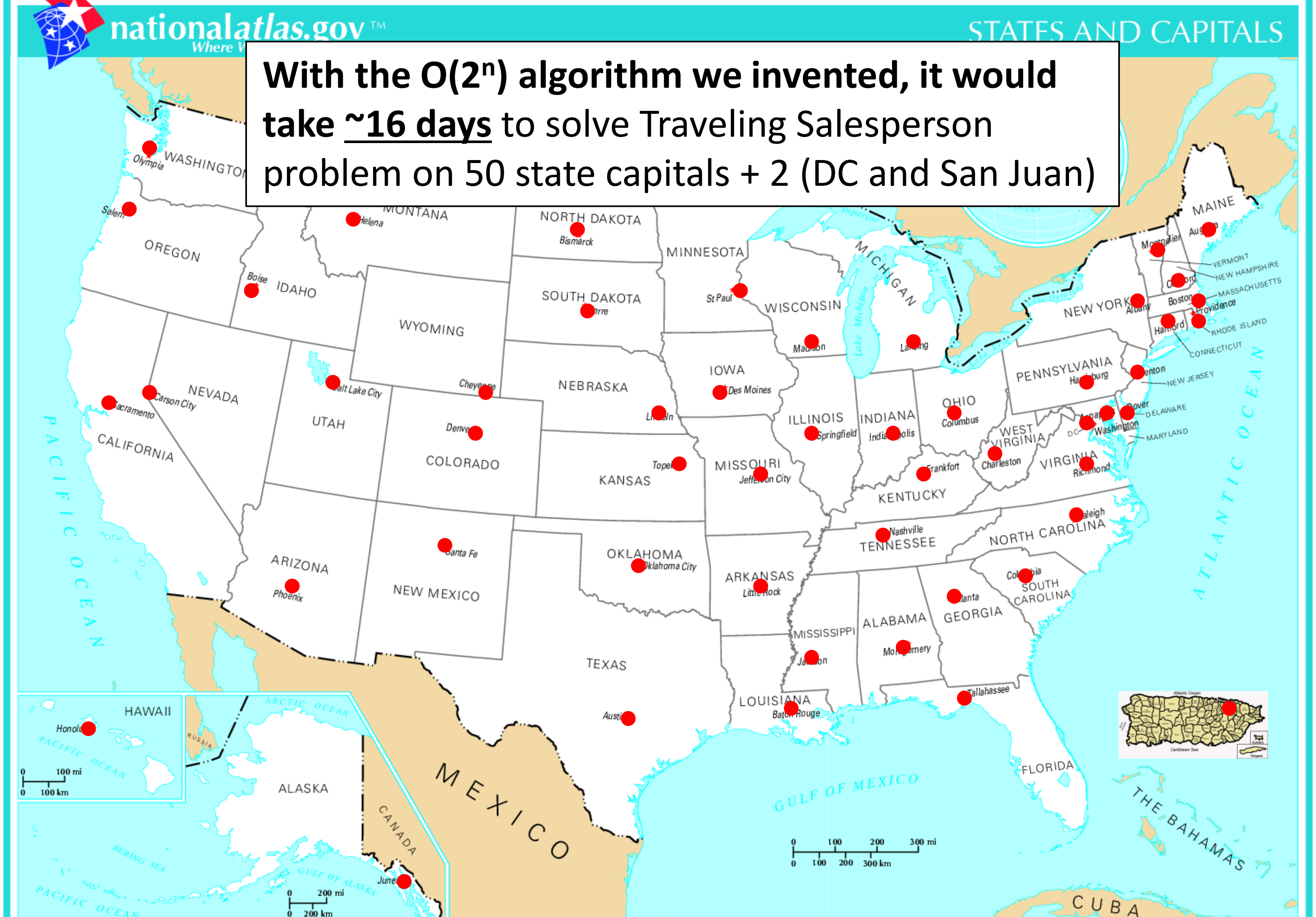
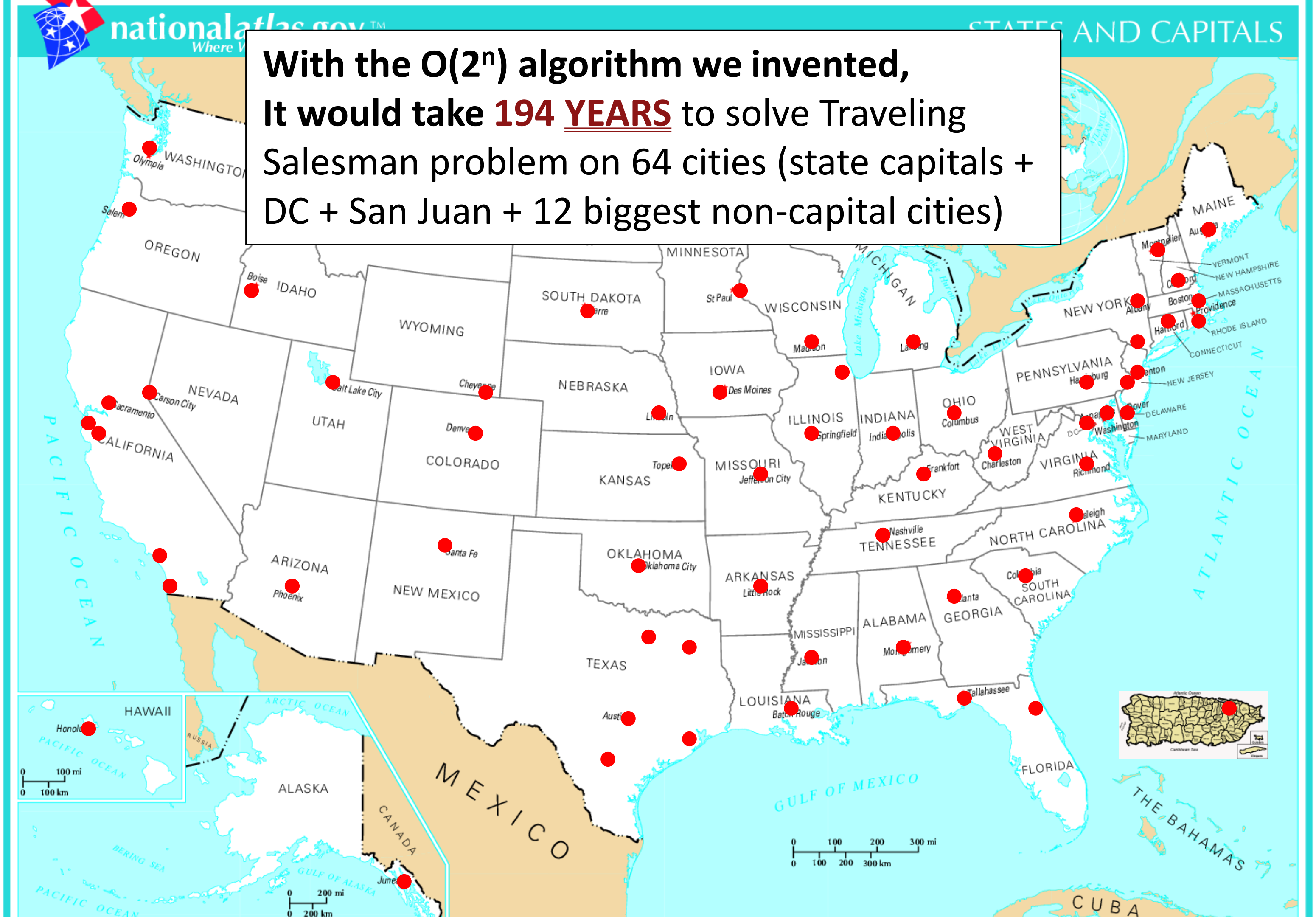
With the $O(2^n)$ algorithm we invented, it would take <u>~16 days</u> to solve Traveling Salesperson problem on 50 state capitals + 2 (DC and San Juan)



Sacramento is not exactly the most interesting or important city in California (sorry, Sacramento).

What if we add the 12 biggest non-capital cities in the United States to our map?

With the $O(2^n)$ algorithm we invented,
It would take 194 YEARS to solve Traveling Salesman problem on 64 cities (state capitals + DC + San Juan + 12 biggest non-capital cities)



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194 **YEARS**

$\log_2 n$	n	$n \log_2 n$	n^2	2^n
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3.59E+21 **YEARS**

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3,590,000,000,000,000,000,000
YEARS

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For comparison: there are about 10E+80 atoms in the universe. No big deal.

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1.42E+137 **YEARS**

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10	**1,024**	10,240 (.000003s)	1,048,576 (.0003s)	1.80×10^{308}
30	**2,700,000,000**	84,591,843,105 (28s)	7,290,000,000,000,000,000 (77 years)	LOL

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2^n is clearly infeasible, but **look at $\log_2 n$—only a tiny fraction of a second!**

In Conclusion

- **NOT worth doing:** Optimization of your code that **just trims** a bit
  - Like that +/-1 handshake—we don't need to worry ourselves about it!
  - Just write clean, easy-to-read code!!!!!
- **COULD BE worth doing:** Optimization of your code that **changes Big-O**
  - **If** performance of a particular function is important, focus on this!
  - *(but if performance of the function is not very important, for example it will only run on small inputs, just focus on writing clean, easy-to-read code!!)*

- (Also remember that efficiency is not necessarily a virtue—first and foremost focus on correctness, both technical correctness and ethical/moral correctness)