Programming Abstractions

CS106X

Cynthia Lee

Graphs Topics

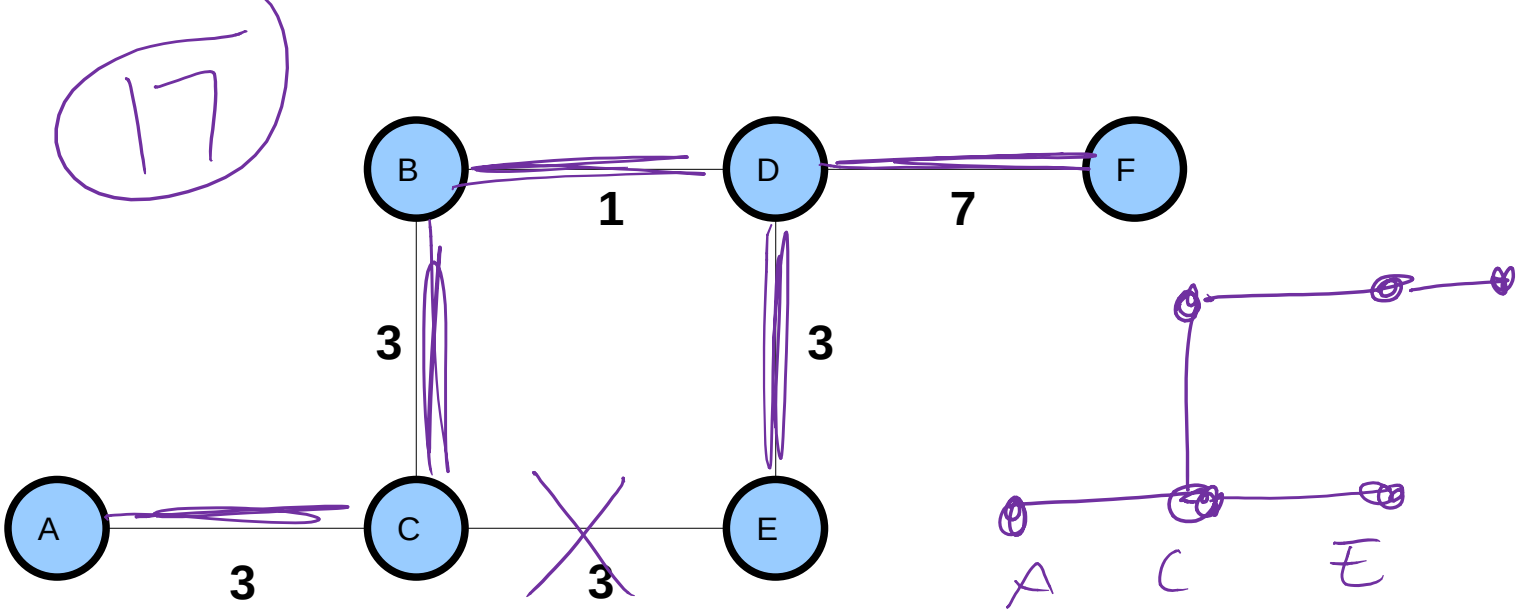
Graphs!

1. **Basics**
   - What are they? How do we represent them?
2. **Theorems**
   - What are some things we can prove about graphs?
3. **Breadth-first search on a graph**
   - Spoiler: just a very, very small change to tree version
4. **Dijkstra's shortest paths algorithm**
   - Spoiler: just a very, very small change to BFS
5. **A* shortest paths algorithm**
   - Spoiler: just a very, very small change to Dijkstra's
6. **Minimum Spanning Tree**
   - Kruskal's algorithm

A* Solving Super Mario (video)

https://youtu.be/DlkMs4ZHHr8

Minimum Spanning Tree



How many distinct minimum spanning trees are in this graph?

A. 0-1
B. 2-3
C. 4-5
D. 6-7
E. >7

Prim's Algorithm

Prim's algorithm

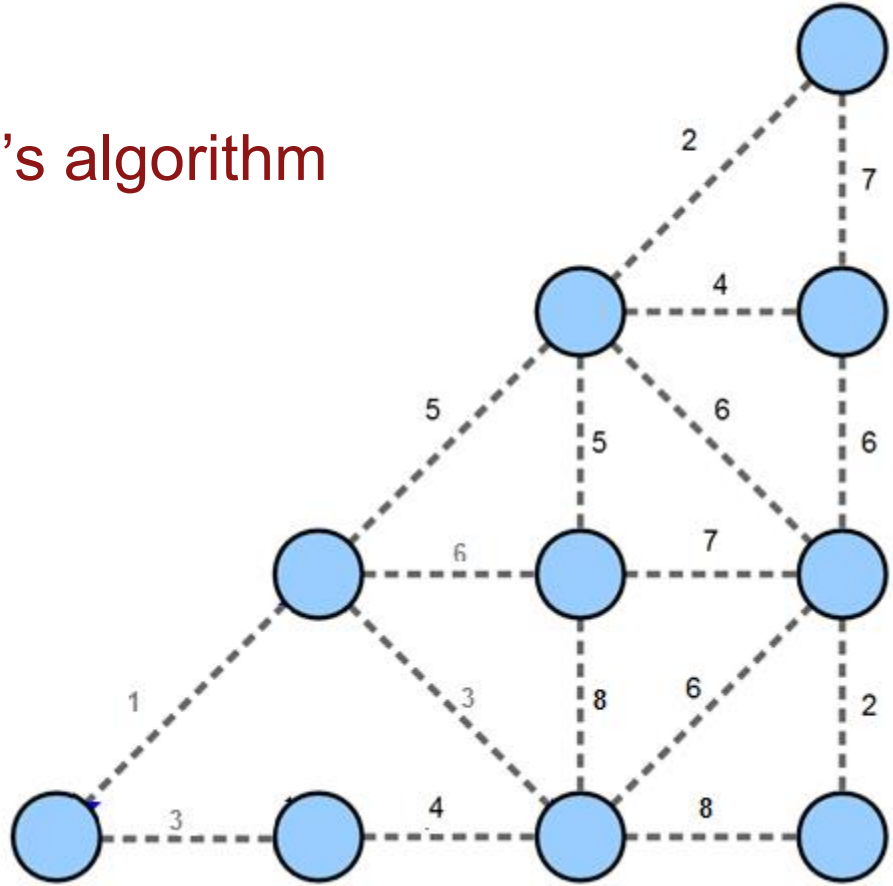
Arbitrarily choose start vertex

Add start vertex to MST

While vertices in MST < total vertices:

- Examine all edges that leave the current MST
- Choose the smallest one
- Add the end vertex of that edge to the MST

Prim's algorithm



Kruskal's Algorithm

Kruskal's algorithm

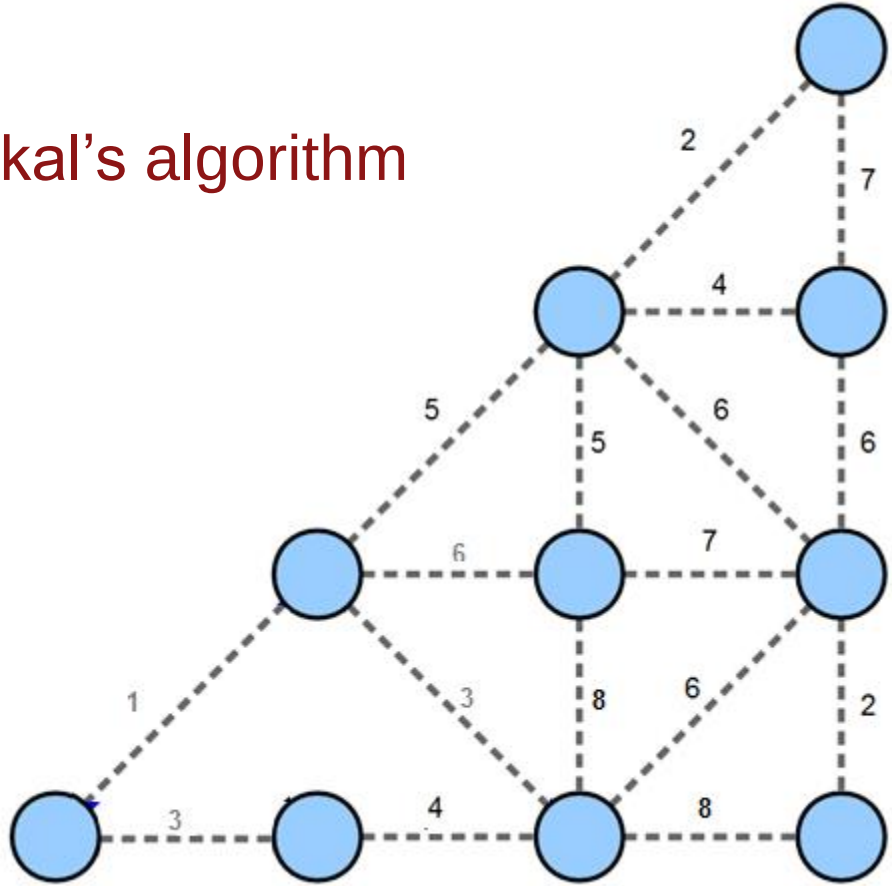
Remove all edges from graph

Place all edges in a PQ based on length/weight

While !PQ.isEmpty():

- Dequeue edge
- If the edge connects previous disconnected nodes or groups of nodes, keep the edge
- Otherwise discard the edge

Kruskal's algorithm



Kruskal's algorithm

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Efficiency of this step is key

Cluster management questions

The assignment handout asks you to consider questions such as:

- How will you keep track of which nodes are in each cluster?
- How will you determine which cluster a node belongs to?
- How will you merge together two clusters?

Cluster management strategies

[watch lecture for whiteboard hints]

The Good Will Hunting Problem

Video Clip

https://www.youtube.com/watch?v=N7b0cLn-wHU

“Draw all the homeomorphically irreducible trees with n=10.”

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In this case **“trees”** simply means **graphs with no cycles**

“with n = 10” (i.e., has **10 nodes**)

“homeomorphically irreducible”

- **No nodes of degree 2 allowed in your solutions**
  - For this problem, nodes of degree 2 are useless in terms of tree structure—they just act as a blip on an edge—and are therefore banned
- Have to be actually different
  - Ignore superficial changes in rotation or angles of drawing