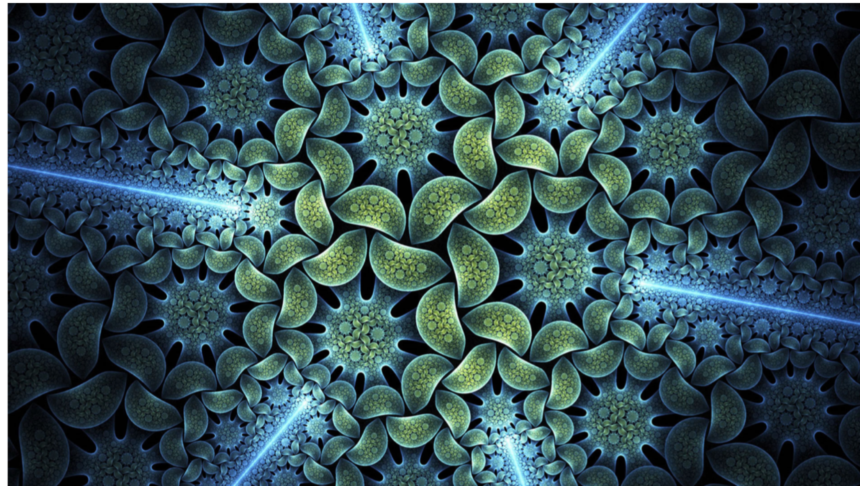
CS 106X, Lecture 9
Fractals

reading:

Programming Abstractions in C++, Chapter 8.4



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Based on slides created by Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, Marty Stepp, Ashley Taylor and others.

Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

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Announcements

- HW3 – Recursion – going out at 3PM today
  - Fractals
  - Grammar Generator
  - Human Pyramid

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Recursion & Big-O

```
void reverseLines(ifstream& input) {
    string line;
    if (getline(input, line)) {
        reverseLines(input);
        cout << line << endl;
    }
}
```

- What is the Big-O of the above function?
- (What is N?)

How many times is this function called in total? x **What is the runtime of each individual function call?**

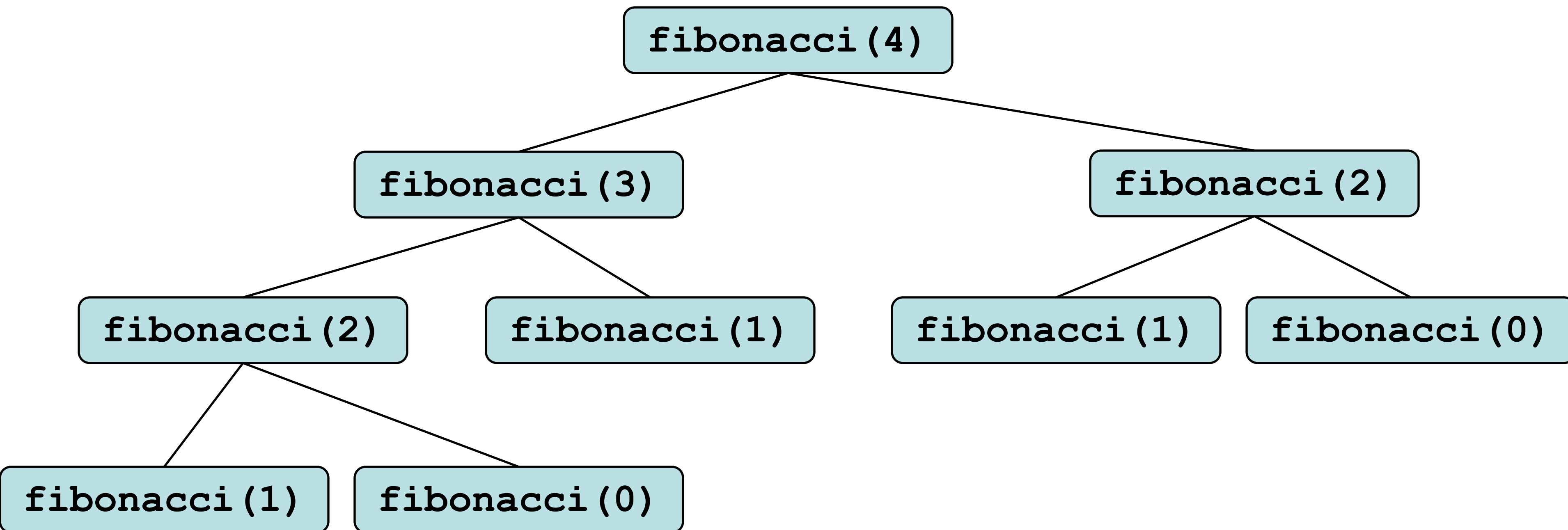
Recursion & Big-O

- The runtime of a recursive function is the number of function calls times the work done in each function call.
- The number of calls for a branching recursive function is usually

$$O(b^d)$$

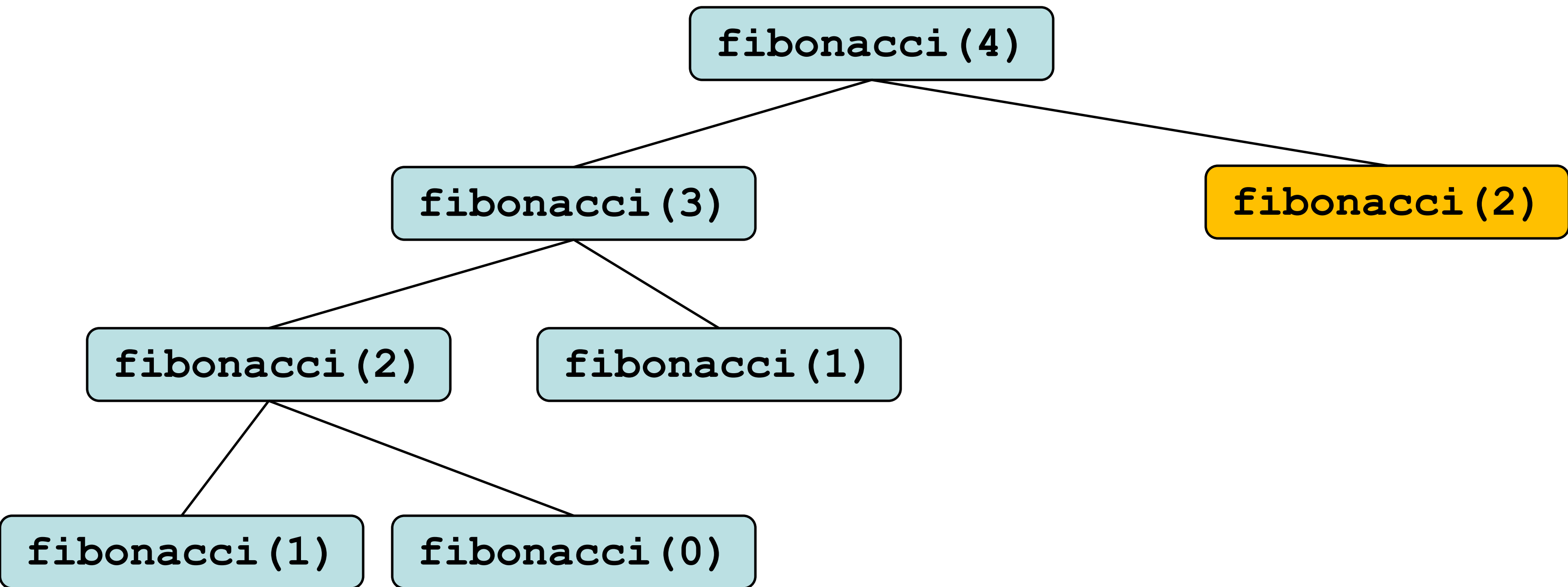
  where
  - **b** is the worst-case branching factor (# recursive calls per function execution)
  - **d** is the worst-case depth of the recursion (the longest path from the top of the recursive call tree to a base case).

Fibonacci: Big-O



- Each recursive call makes 2 *additional* recursive calls.
- The worst-case depth of the recursion is the index of the Fibonacci number we are trying to calculate (N).
- Therefore, the number of total calls is $O(2^N)$.
- Each individual function call does O(1) work. Therefore, the total runtime is $O(2^N) * O(1) = O(2^N)$.

Recursive Tree



Is there a way to remember what we already computed?

Memoized Fibonacci

```
// Returns the nth Fibonacci number (no error handling).
// This version uses memoization.
int fibonacci(int i, Map<int, int>& cache) {
    if (i < 2) {
        return i;
    } else if (cache.containsKey(i)) {
        return cache[i];
    } else {
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);
        cache[i] = result;
        return result;
    }
}
```

Wrapper Functions

```
Map<int, int> cache;
int sixthFibonacci = fibonacci(5, cache);  // 5
```

- The above function signature isn’t ideal; it requires the client to know to pass in an (empty) map.
- In general, the parameters we need for our recursion will not always match those the client will want to pass.
- Is there a way we can remove that requirement, while still memoizing?
- **YES!** A “wrapper” function is a function that “wraps” around the first call to a recursive function to abstract away any additional parameters needed to perform the recursion.

That's a Wrap(per)!

```
// "Wrapper" function that returns the nth Fibonacci number.
// This version calls the recursive version with an empty cache.
int fibonacci(int i) {
    Map<int, int> cache;
    return fibonacci(i, cache);
}

// Recursive function that returns the nth Fibonacci number.
// This version uses memoization.
int fibonacci(int i, Map<int, int>& cache) {
    if (i < 0) {
        throw "Illegal negative index";
    } else if (i < 2) {
        return i;
    } else if (cache.containsKey(i)) {
        return cache[i];
    } else {
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);
        cache[i] = result;
        return result;
    }
}
```

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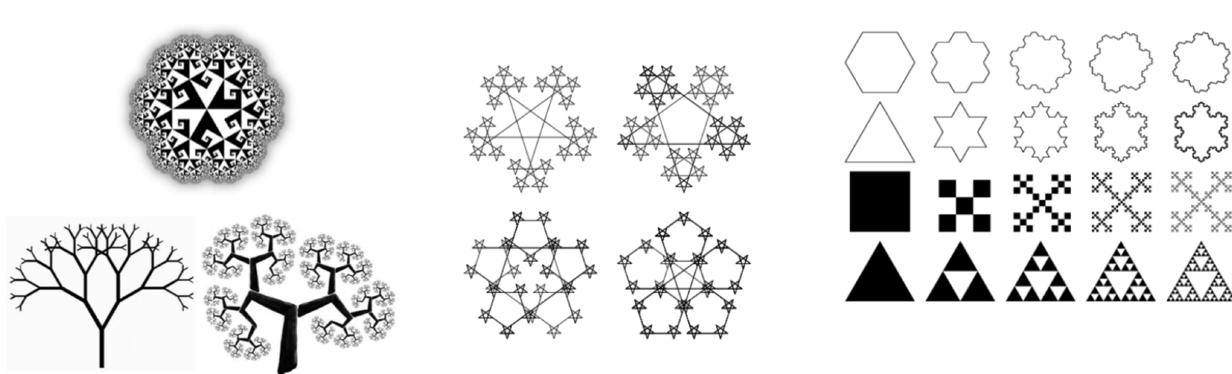
Fractals

A **fractal** is a recurring graphical pattern. Smaller instances of the same shape or pattern occur within the pattern itself.



Fractals in Nature

Many natural phenomena generate fractal patterns:

1. earthquake fault lines
2. animal color patterns
3. clouds
4. mountain ranges
5. snowflakes
6. crystals
7. DNA
8. ...

Cantor Fractal

Cantor Fractal

Parts of a cantor set image ... are Cantor set images

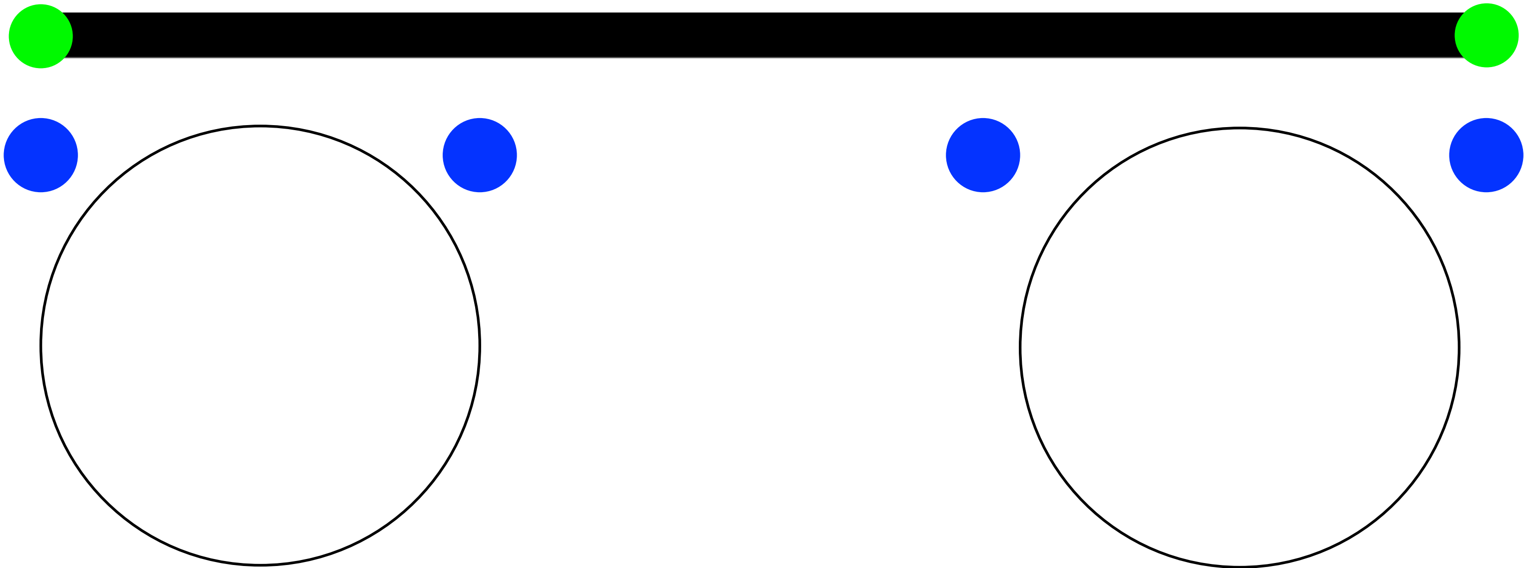
Cantor Fractal

Another cantor set **Another cantor set**

Level 1 Cantor Fractal

Level n Cantor

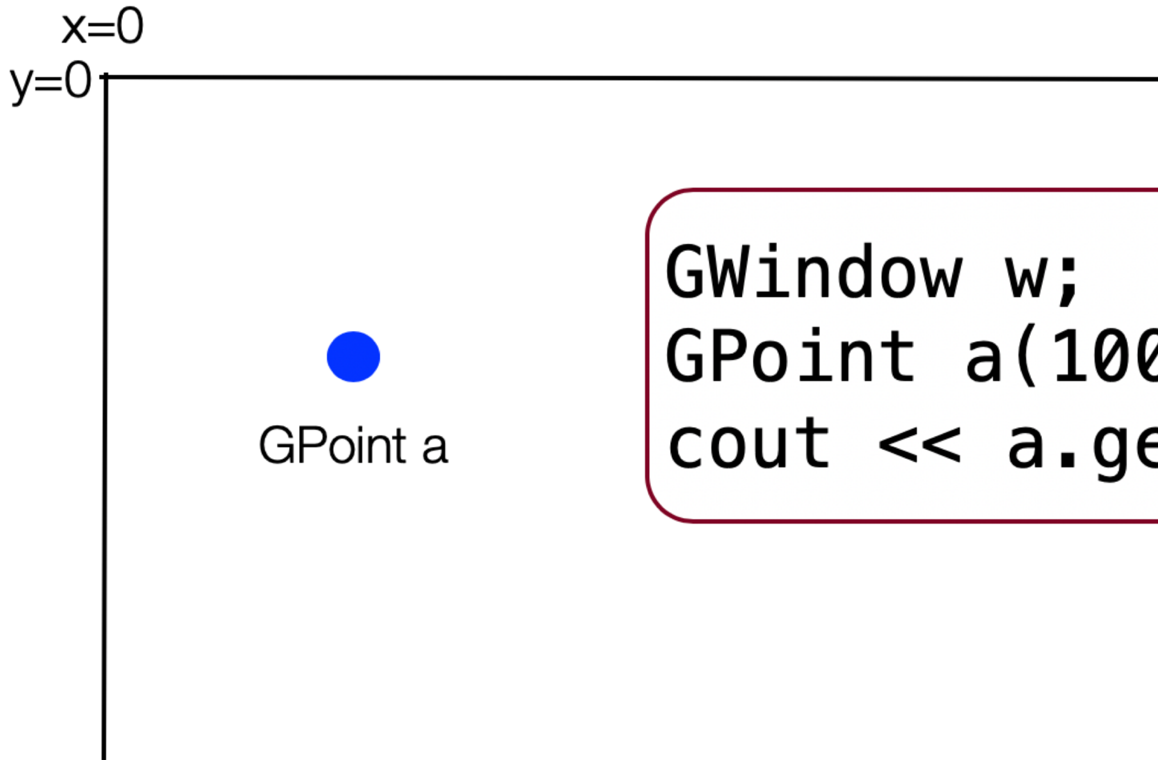
1. Draw a line from start to finish.



2. Draw a Cantor of size n-1

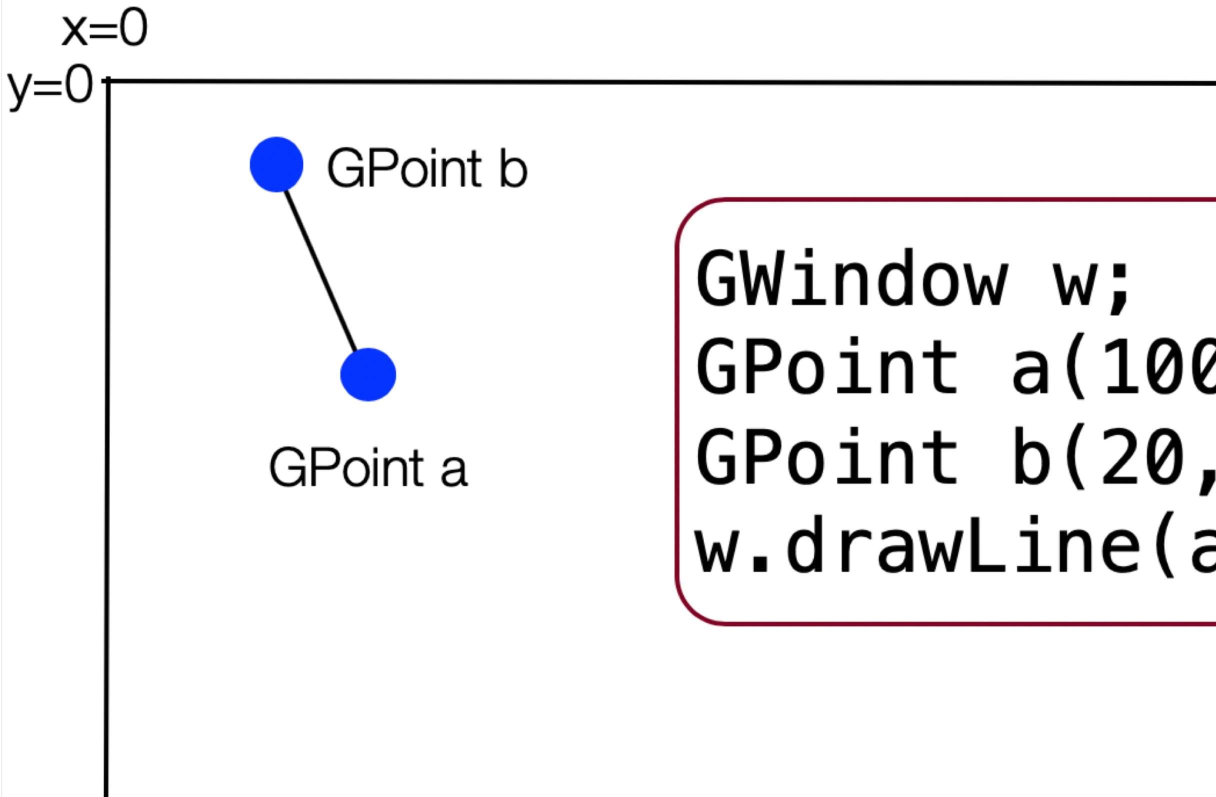
2. Draw a Cantor of size n-1

Stanford Graphics Libraries




```
GWindow w;
GPoint a(100, 100);
cout << a.getX() << endl;
```

Stanford Graphics Libraries




```
GWindow w;
GPoint a(100, 100);
GPoint b(20, 20);
w.drawLine(a, b);
```

Cantor Fractal

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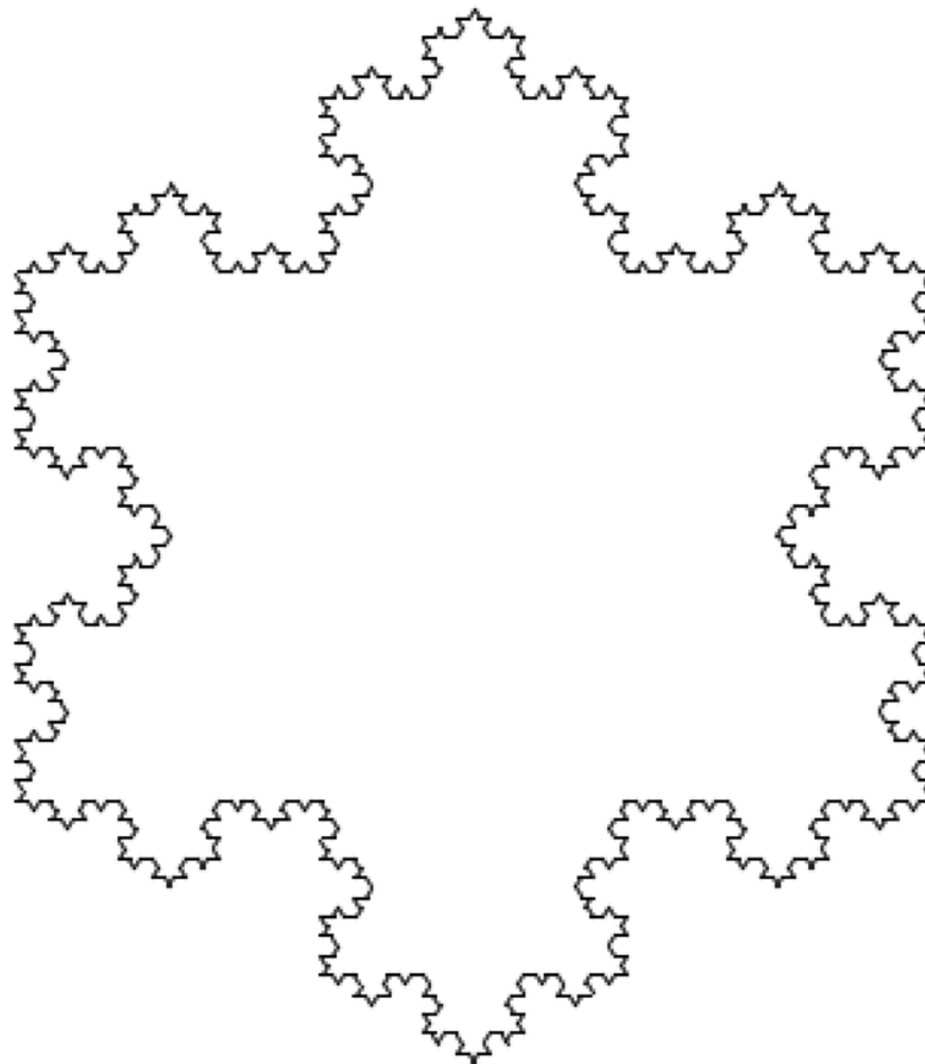
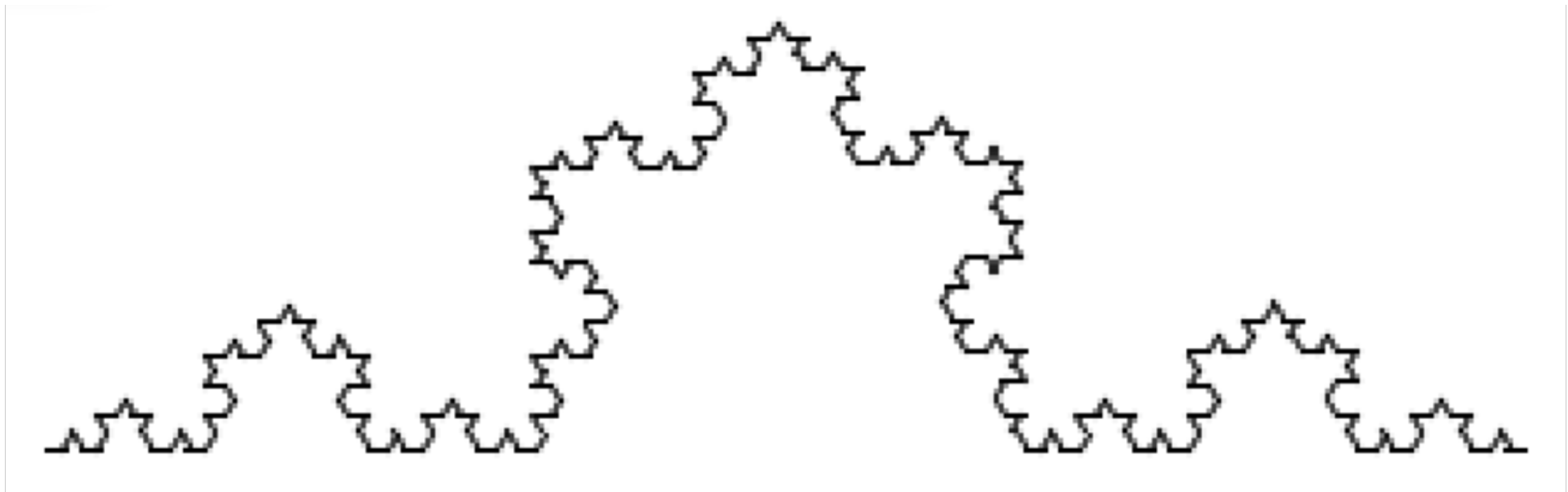
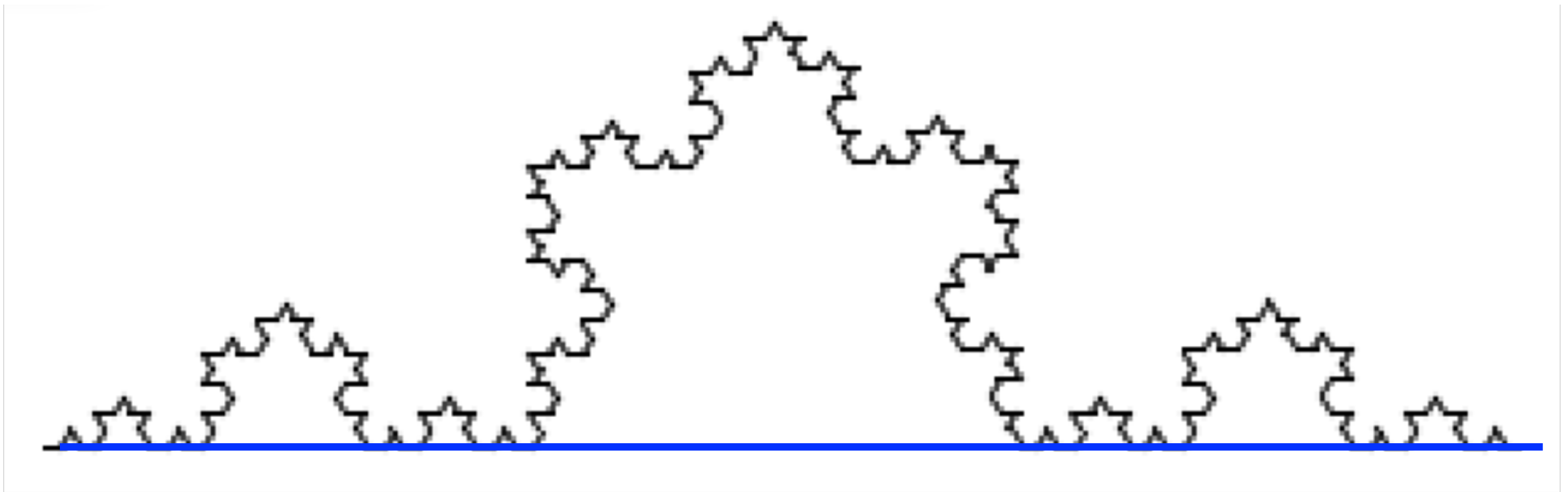
Snowflake



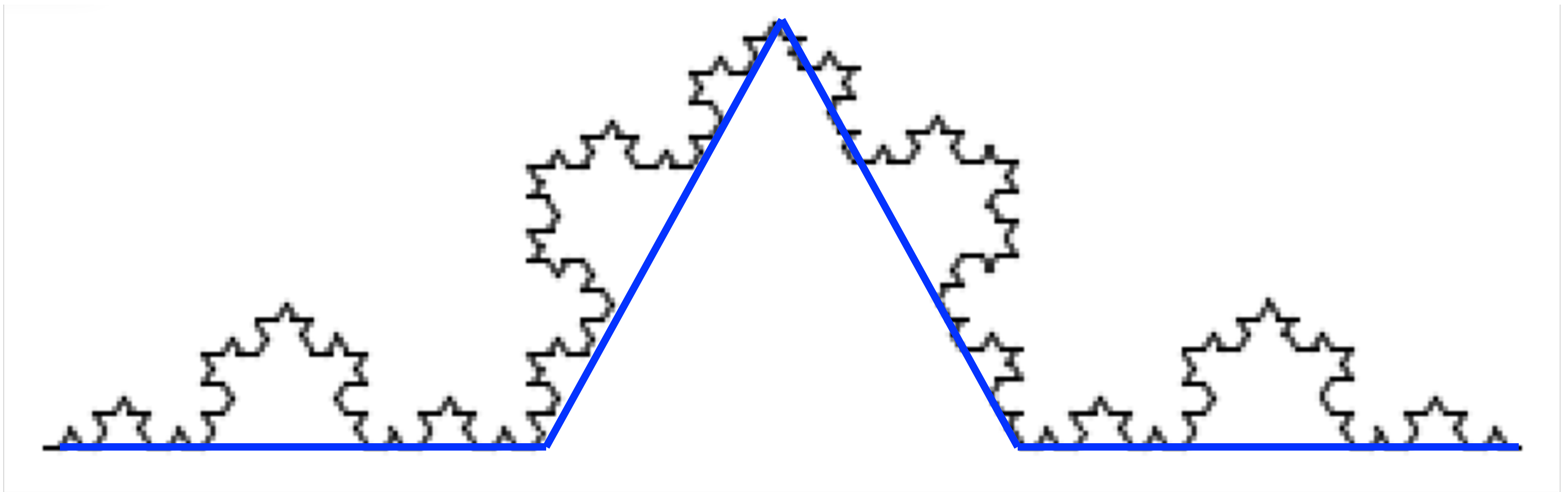
Snowflake



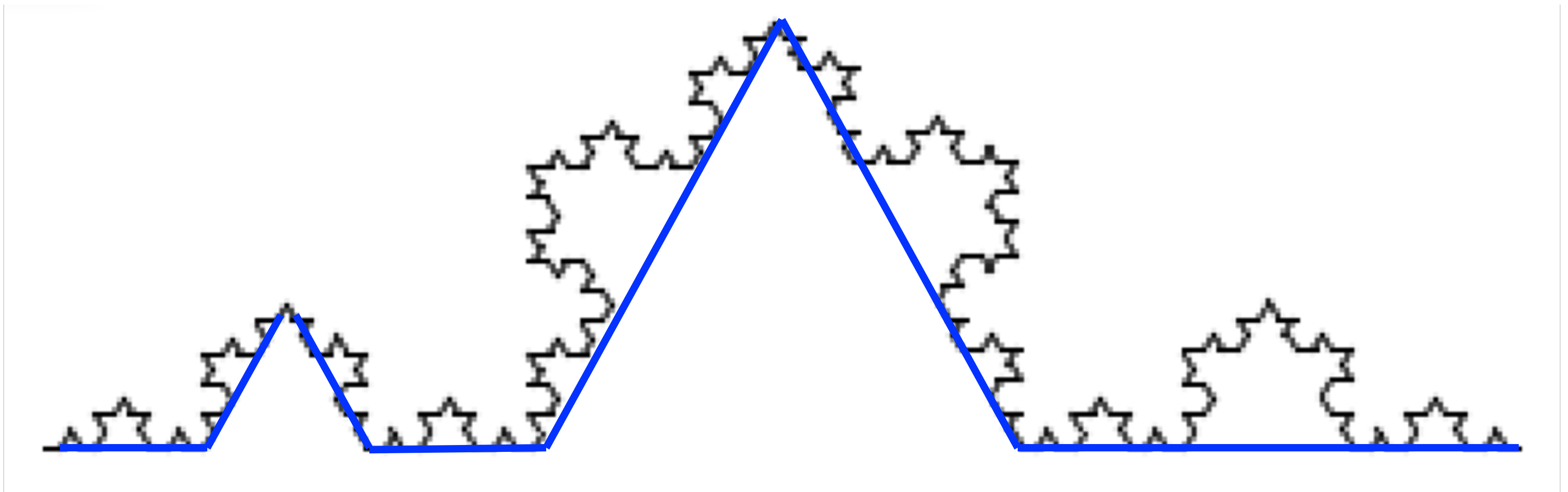
Snowflake



Snowflake

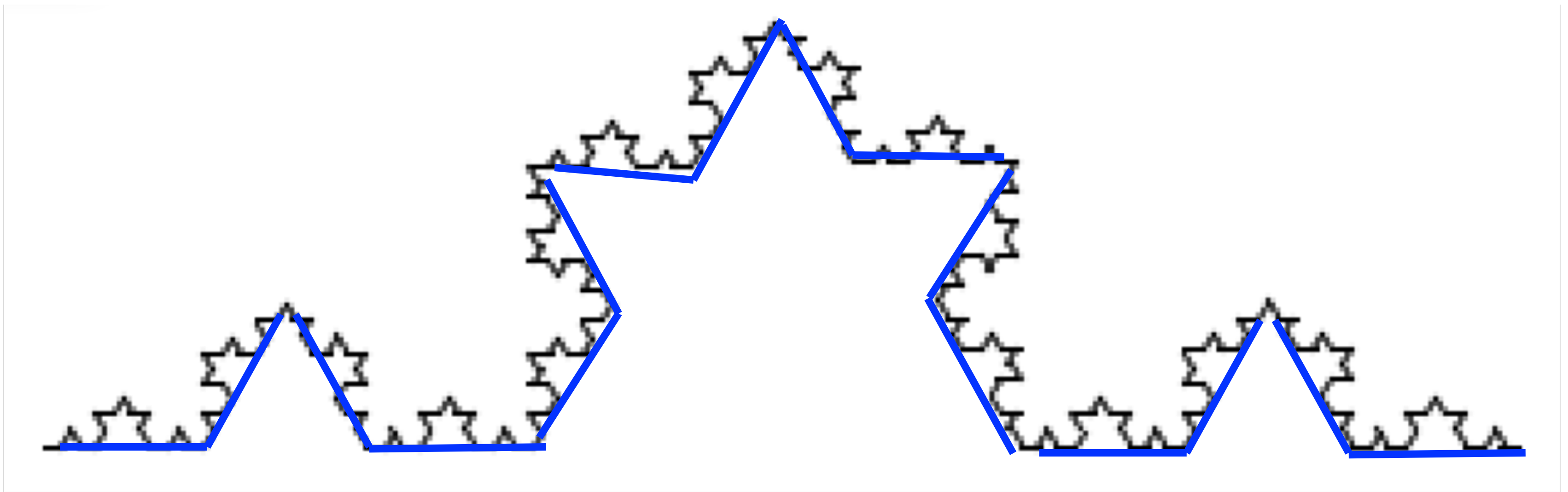


Snowflake



Snowflake

Snowflake



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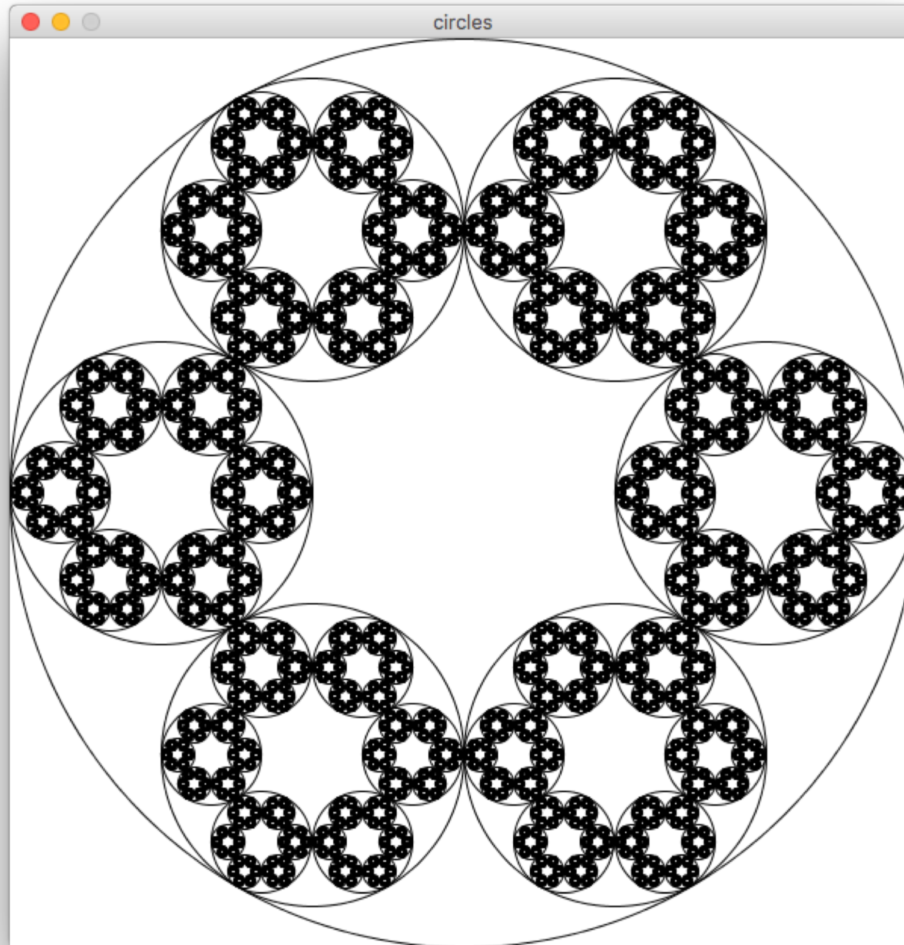
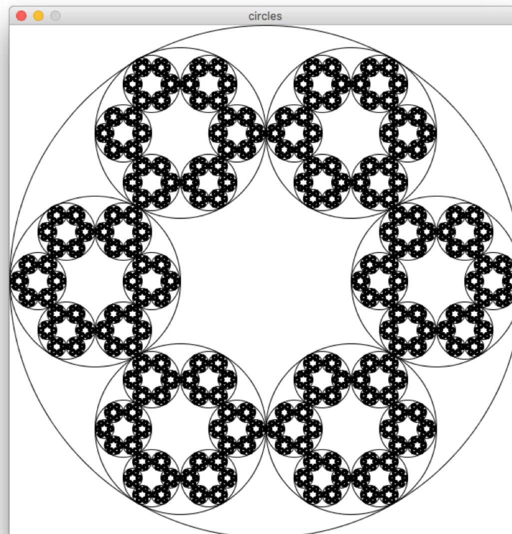
Emblem Fractal



Emblem Fractal

- We want to draw this figure at a given center and radius on-screen.
- An order-0 emblem is nothing
- An order-1 emblem is a circle of the specified size
- An order-n emblem is a circle of the specified size, containing 6 order n-1 emblems at increments of 60 degrees around the circle 2/3 away from the center, with 1/3 the radius.



Recap

- **Fractals**
  - Fractals are self-referential, and that makes for nice recursion problems!
  - Break the problem into a smaller, self-similar part, and don't forget your base case!

References and Advanced Reading

- **References:**
  - http://www.cs.utah.edu/~germain/PPS/Topics/recursion.html
  - **Why is iteration generally better than recursion?** http://stackoverflow.com/a/3093/561677

- **Advanced Reading:**
  - **Tail recursion:** http://stackoverflow.com/questions/33923/what-is-tail-recursion
  - **Interesting story on the history of recursion in programming languages:** http://goo.gl/P6Einb