CS107 Spring 2019, Lecture 2 Bits and Bytes; Integer Representations

reading:

Bryant & O'Hallaron, Ch. 2.2-2.3

CS107 Topic 1: How can a computer represent integer numbers?

Demo: Unexpected Behavior

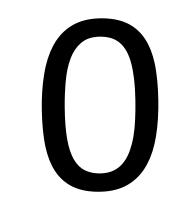


Plan For Today

- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Break: Announcements
- Signed Integers
- Casting and Combining Types

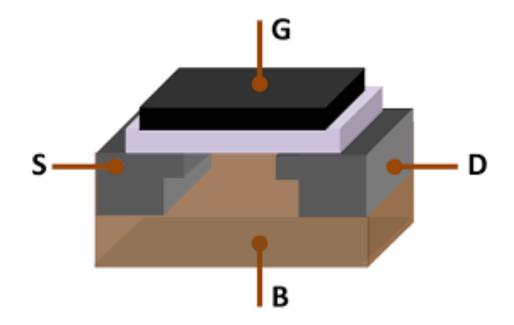
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Bits

• Computers are built around the idea of two states: "on" and "off". Transistors represent this in hardware, and bits represent this in software!

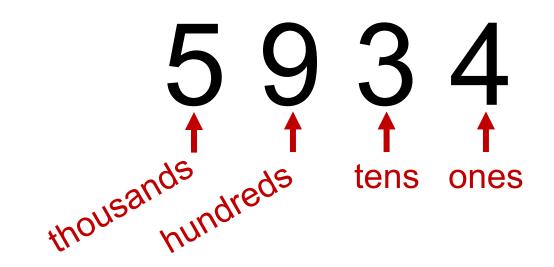


One Bit At A Time

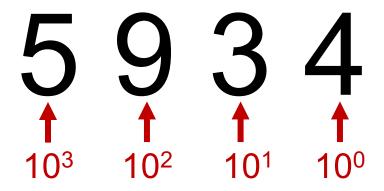
- We can combine bits, like with base-10 numbers, to represent more data.
 bits = 1 byte.
- Computer memory is just a large array of bytes! It is byte-addressable; you can't address (store location of) a bit; only a byte.
- Computers still fundamentally operate on bits; we have just gotten more creative about how to represent different data as bits!
 - Images
 - Audio
 - Video
 - Text
 - And more...

5934

Digits 0-9 (0 to base-1)



$$= 5*1000 + 9*100 + 3*10 + 4*1$$



5 9 3 4 3 2 1 0

10^X:

2X:

Digits 0-1 (0 to base-1)



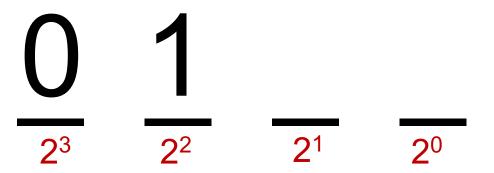
$$= 1*8 + 0*4 + 1*2 + 1*1 = 11_{10}$$



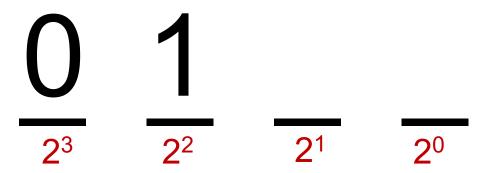
$$= 1*8 + 0*4 + 1*2 + 1*1 = 11_{10}$$

- Strategy:
 - What is the largest power of 2 ≤ 6?

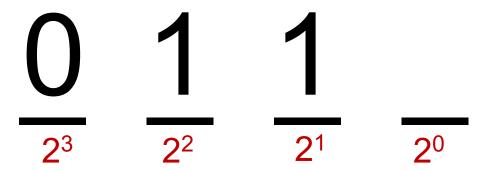
- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$



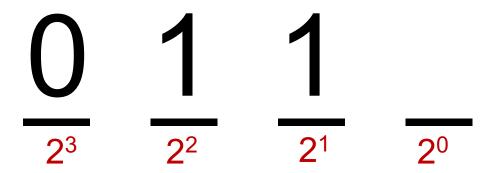
- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$
 - Now, what is the largest power of $2 \le 6 2^2$?



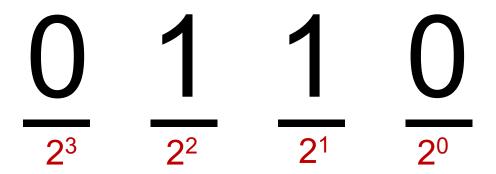
- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$
 - Now, what is the largest power of $2 \le 6 2^2$? **2**¹=**2**



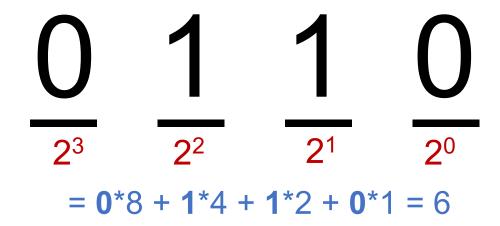
- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$
 - Now, what is the largest power of $2 \le 6 2^2$? $2^1 = 2$
 - $6-2^2-2^1=0!$



- Strategy:
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Practice: Base 2 to Base 10

What is the base-2 value 1010 in base-10?

```
a) 20 (text code: 641180)
```

```
b) 101 (text code: 642224)
```

```
c) 10 (text code: 642225)
```

d) 5 (text code: 642226)

e) Other (text code: 642227)

Respond at pollev.com/nicktroccoli901 or text a code above to 22333.

To show this poll

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Practice: Base 10 to Base 2

What is the base-10 value 14 in base 2?

```
a) 1111 (text code: 642232)
```

```
b) 1110 (text code: 642233)
```

c) **1010** (text code: 642235)

d) Other (text code: 642236)

Respond at pollev.com/nicktroccoli901 or text a code above to 22333.

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• What is the minimum and maximum base-10 value a single byte (8 bits) can store?

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What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = ?



2^x:

What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = ?

• Strategy 1: $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$

2^x:

What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = 255

- Strategy 1: $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$
- Strategy 2: $2^8 1 = 255$

2^x:

Multiplying by Base

$$1453 \times 10 = 1453$$
0
 $1101_2 \times 2 = 1101$ 0

Key Idea: inserting 0 at the end multiplies by the base!

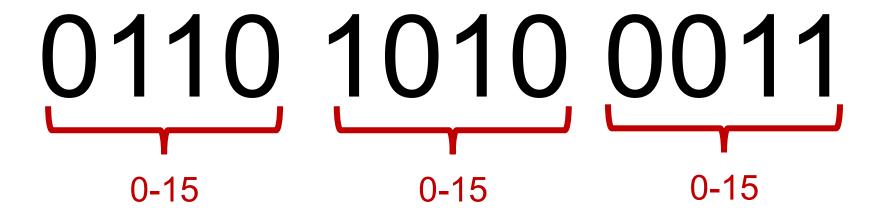
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- When working with bits, oftentimes we have large numbers with 32 or 64 bits.
- Instead, we'll represent bits in base-16 instead; this is called hexadecimal.

0110 1010 0011

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- When working with bits, oftentimes we have large numbers with 32 or 64 bits.
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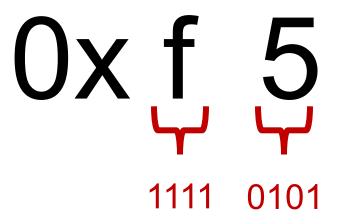
Each is a base-16 digit!

• Hexadecimal is *base-16*, so we need digits for 1-15. How do we do this?

0 1 2 3 4 5 6 7 8 9 a b c d e f

Hex digit	0	1	2	3	4	5	6	7
Decimal value	0	1	2	3	4	5	6	7
Binary value	0000	0001	0010	0011	0100	0101	0110	0111
Hex digit	8	9	A	В	С	D	E	F
Decimal value	8	9	10	11	12	13	14	15
Binary value	1000	1001	1010	1011	1100	1101	1110	1111

- We distinguish hexadecimal numbers by prefixing them with **0x**, and binary numbers with **0b**.
- E.g. **0xf5** is **0b11110101**



What is **0x173A** in binary?

What is **0x173A** in binary?

Hexadecimal	1	7	3	A
Binary	0001	0111	0011	1010

What is **0b11110010110110110011** in hexadecimal? (*Hint: start from the right*)

What is **0b11110010110110110011** in hexadecimal? (*Hint: start from the right*)

Binary	11	1100	1010	1101	1011	0011
Hexadecimal	3	\mathbf{C}	A	D	В	3

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Number Representations

- Unsigned Integers: positive and 0 integers. (e.g. 0, 1, 2, ... 99999...
- **Signed Integers:** negative, positive and 0 integers. (e.g. ...-2, -1, 0, 1,... 9999...)

• Floating Point Numbers: real numbers. (e,g. 0.1, -12.2, 1.5x10¹²)

Number Representations

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→ Stay tuned until week 5!

32-Bit and 64-Bit



- In the early 2000's, most computers were **32-bit**. This means that pointers in programs were **32 bits**.
- 32-bit pointers could store a memory address from 0 to 2³²-1, for a total of **2³²** bytes of addressable memory. This equals **4 Gigabytes**, meaning that 32-bit computers could have at most **4GB** of memory (RAM)!
- Because of this, computers transitioned to **64-bit**. This means that pointers in programs were **64 bits**.
- 64-bit pointers could store a memory address from 0 to 2⁶⁴-1, for a total of **2**⁶⁴ bytes of addressable memory. This equals **16 Exabytes**, meaning that 64-bit computers could have at most **1024*1024*1024 GB** of memory (RAM)!

Number Representations

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
$int32_t$	$uint32_t$	4	4
$int64_t$	$\mathrm{uint}64_\mathrm{t}$	8	8
char *		4	8
float		4	4
double		8	8

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Myth

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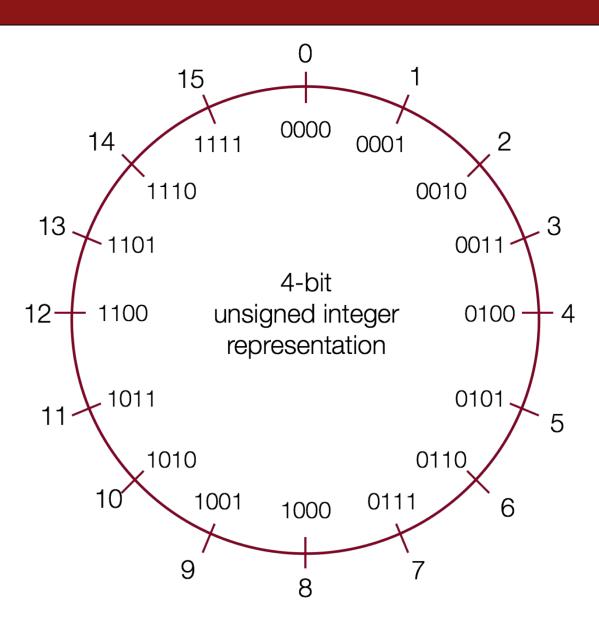
Unsigned Integers

- An **unsigned** integer is 0 or a positive integer (no negatives).
- We have already discussed converting between decimal and binary, which is a nice 1:1 relationship. Examples:

```
0b0001 = 1
0b0101 = 5
0b1011 = 11
0b1111 = 15
```

• The range of an unsigned number is $0 \rightarrow 2^w - 1$, where w is the number of bits. E.g. a 32-bit integer can represent 0 to $2^{32} - 1$ (4,294,967,295).

Unsigned Integers



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Announcements

- Sign up for Piazza on the Help page if you haven't already!
- Lab signups opened earlier this week, start next week.
 - Labs posted on the course website at the start of each week
- Office Hours started earlier this week
 - You must fill out signup questions completely when signing up
- Please send course staff OAE letters for accommodations!

Let's Take A Break

To ponder during the break:

A **signed** integer is a negative, 0, or positive integer. How can we represent both negative and positive numbers in binary?

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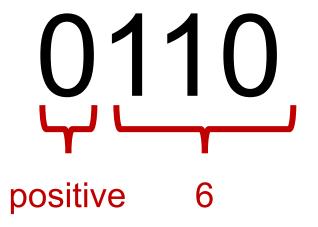
Signed Integers

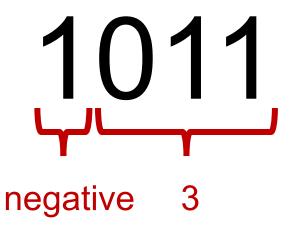
- A **signed** integer is a negative integer, 0, or a positive integer.
- *Problem:* How can we represent negative *and* positive numbers in binary?

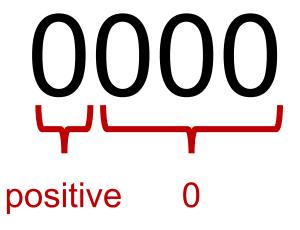
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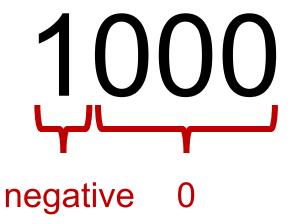
- A **signed** integer is a negative integer, 0, or a positive integer.
- *Problem:* How can we represent negative *and* positive numbers in binary?

Idea: let's reserve the *most* significant bit to store the sign.









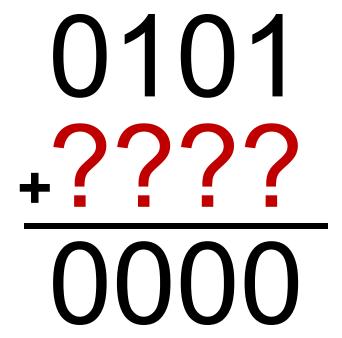


```
1\ 000 = -0 0\ 0000 = 0
1\ 001 = -1 0\ 001 = 1
1\ 010 = -2 0\ 010 = 2
1\ 011 = -3 0\ 011 = 3
1\ 100 = -4 0\ 100 = 4
1\ 101 = -5 0\ 101 = 5
1\ 110 = -6 0\ 110 = 6
1\ 111 = -7 0\ 111 = 7
```

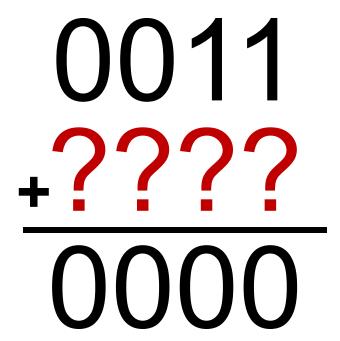
We've only represented 15 of our 16 available numbers!

- **Pro:** easy to represent, and easy to convert to/from decimal.
- Con: +-0 is not intuitive
- Con: we lose a bit that could be used to store more numbers
- Con: arithmetic is tricky: we need to find the sign, then maybe subtract (borrow and carry, etc.), then maybe change the sign...this might get ugly!

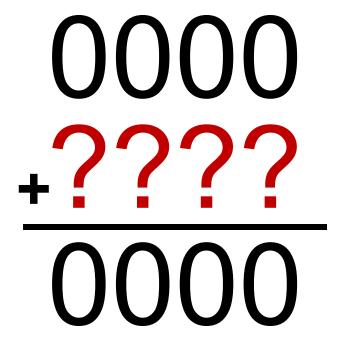
Can we do better?

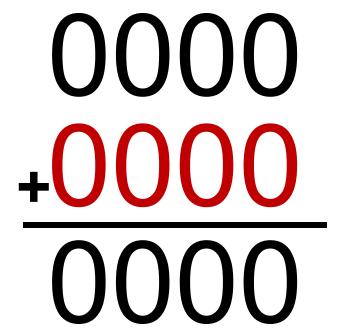


$$0101 \\ +1011 \\ \hline 0000$$



$$0011 \\ +1101 \\ \hline 0000$$

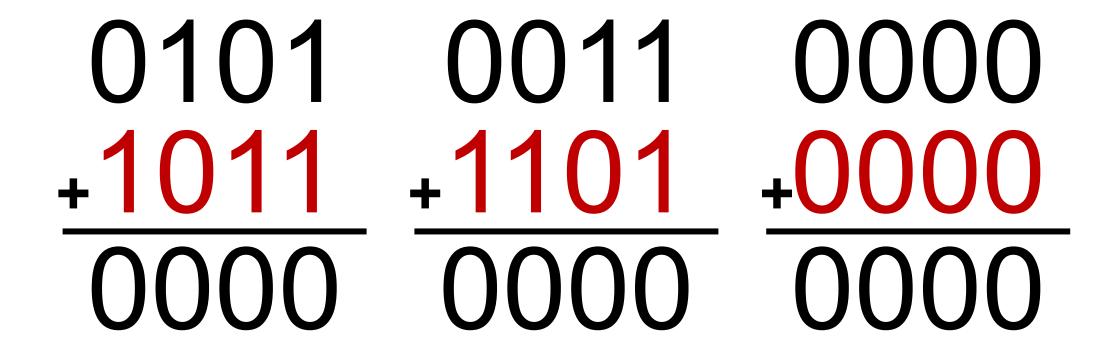




Decimal	Positive	Negative
0	0000	0000
1	0001	1111
2	0010	1110
3	0011	1101
4	0100	1100
5	0101	1011
6	0110	1010
7	0111	1001

Decimal	Positive	Negative
8	1000	1000
9	1001 (same as -7!)	NA
10	1010 (same as -6!)	NA
11	1011 (same as -5!)	NA
12	1100 (same as -4!)	NA
13	1101 (same as -3!)	NA
14	1110 (same as -2!)	NA
15	1111 (same as -1!)	NA

There Seems Like a Pattern Here...



The negative number is the positive number inverted, plus one!

There Seems Like a Pattern Here...

A binary number plus its inverse is all 1s.

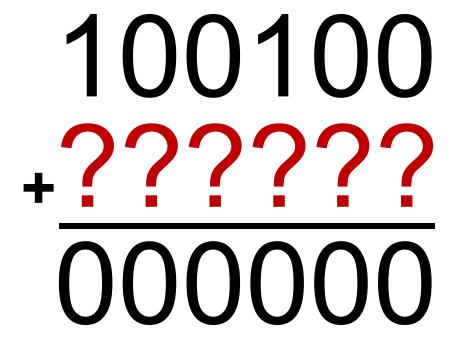
Add 1 to this to carry over all 1s and get 0!

0101
+1010
1111

1111 +001 0000

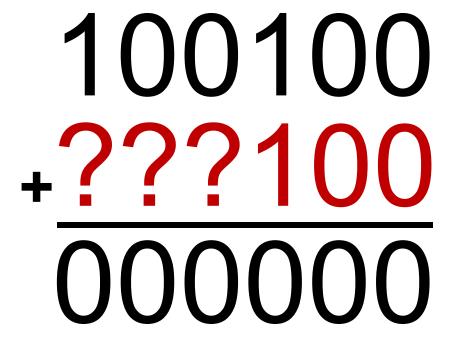
Another Trick

• To find the negative equivalent of a number, work right-to-left and write down all digits through when you reach a 1. Then, invert the rest of the digits.



Another Trick

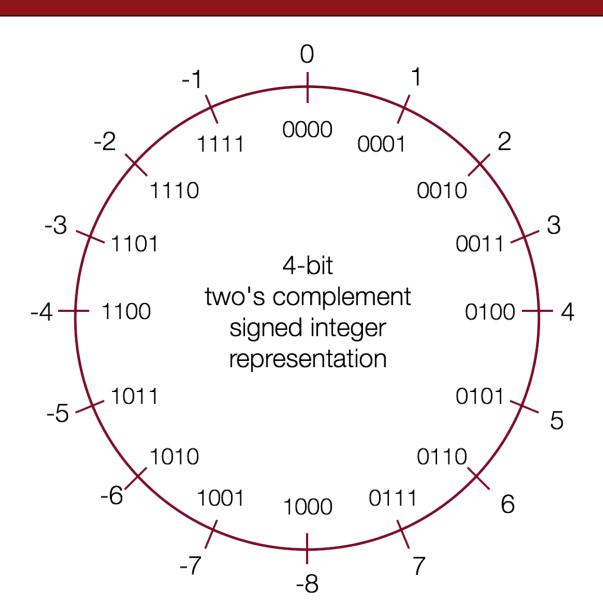
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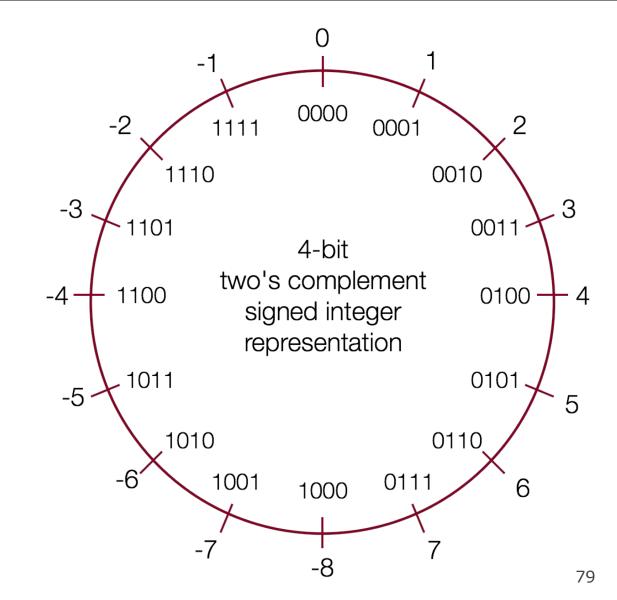
Another Trick

• To find the negative equivalent of a number, work right-to-left and write down all digits through when you reach a 1. Then, invert the rest of the digits.

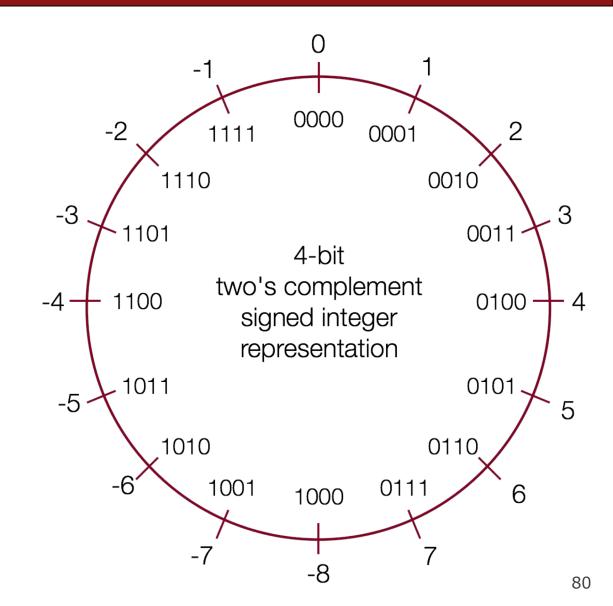
 $100100 \\ + 01100 \\ \hline 00000$



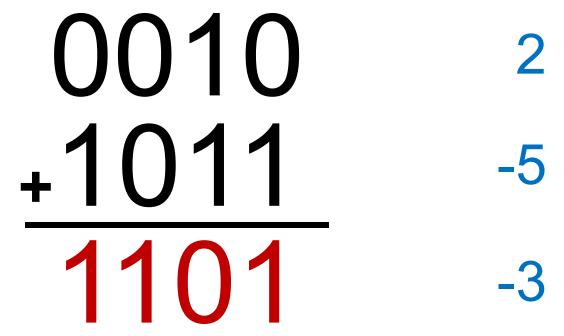
- In two's complement, we represent a positive number as itself, and its negative equivalent as the two's complement of itself.
- The **two's complement** of a number is the binary digits inverted, plus 1.
- This works to convert from positive to negative, and back from negative to positive!



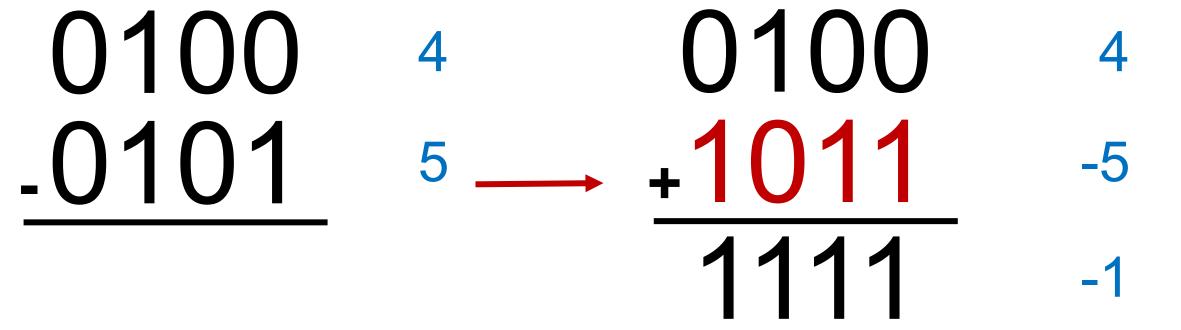
- **Con:** more difficult to represent, and difficult to convert to/from decimal and between positive and negative.
- Pro: only 1 representation for 0!
- **Pro:** all bits are used to represent as many numbers as possible
- **Pro:** it turns out that the most significant bit *still indicates the sign* of a number.
- Pro: arithmetic is easy: we just add!



• Adding two numbers is just...adding! There is no special case needed for negatives. E.g. what is 2 + -5?



• Subtracting two numbers is just performing the two's complement on one of them and then adding. E.g. 4-5=-1.



• While you don't need to worry about multiplication, it turns out that with two's complement, multiplying two numbers is just multiplying, and discarding overflow digits! E.g. $-2 \times -3 = 6$.

```
1110 (-2)
    x1101 (-3)
      1110
   0000
 1110
+1110
<del>1011</del>0110
```

Practice: Two's Complement

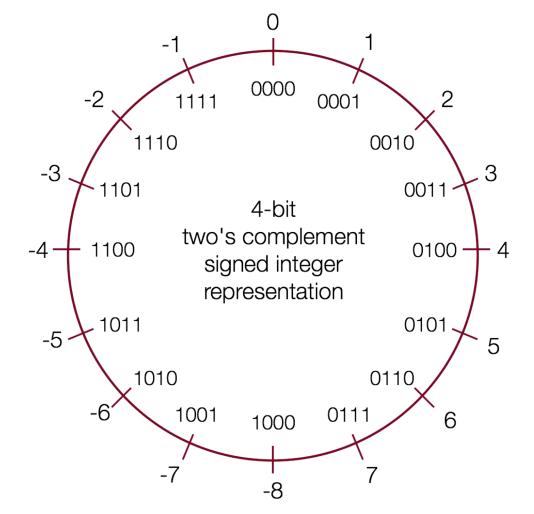
What are the negative or positive equivalents of the numbers below?

- a) -4 (1100)
- b) 7 (0111)
- c) 3 (0011)
- d) -8 (1000)

Practice: Two's Complement

What are the negative or positive equivalents of the numbers below?

- a) -4 (1100)
- b) 7 (0111)
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Overflow and Underflow

• If you exceed the **maximum** value of your bit representation, you wrap around or overflow back to the **smallest** bit representation.

$$0b1111 + 0b1 = 0b0000$$

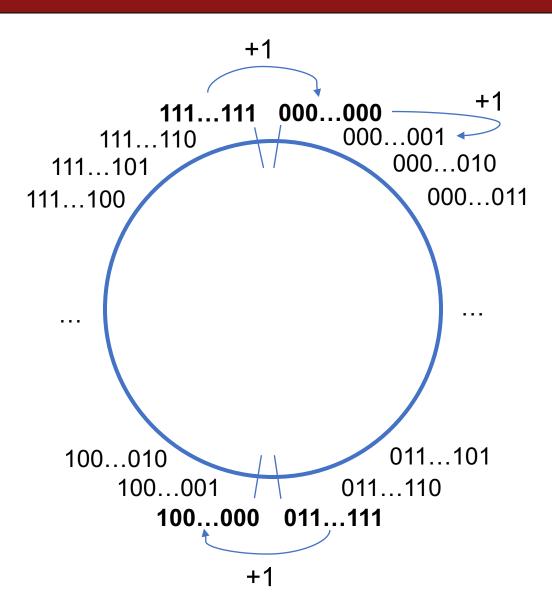
• If you go below the **minimum** value of your bit representation, you wrap around or underflow back to the **largest** bit representation.

$$0b0000 - 0b1 = 0b1111$$

Min and Max Integer Values

Туре	Width (bytes)	Width (bits)	Min in hex (name)	Max in hex (name)
char	1	8	80 (CHAR_MIN)	7F (CHAR_MAX)
unsigned char	1	8	0	FF (UCHAR_MAX)
short	2	16	8000 (SHRT_MIN)	7FFF (SHRT_MAX)
unsigned short	2	16	0	FFFF (USHRT_MAX)
int	4	32	8000000 (INT_MIN)	7FFFFFF (INT_MAX)
unsigned int	4	32	0	FFFFFFF (UINT_MAX)
long	8	64	80000000000000 (LONG_MIN)	7FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
unsigned long	8	64	0	FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

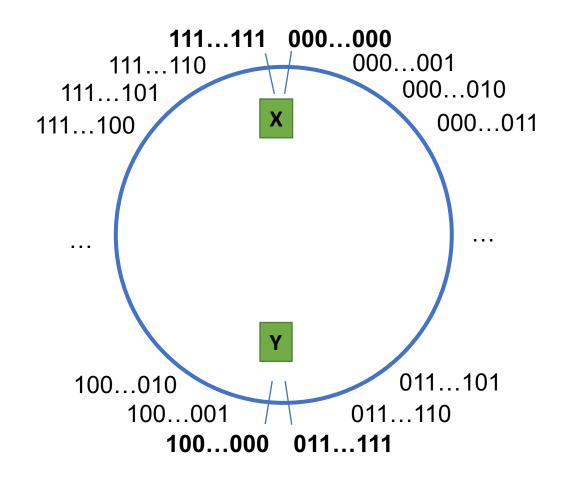
Overflow and Underflow



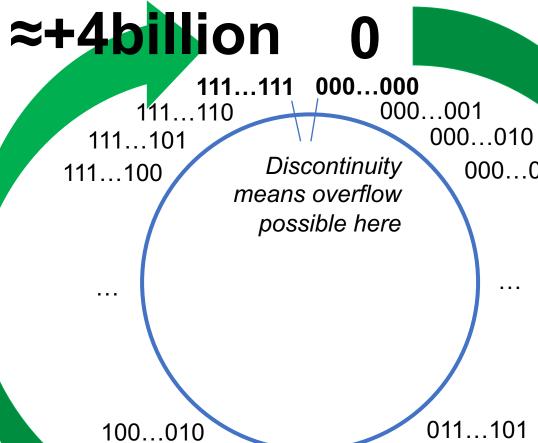
Overflow

At which points can overflow occur for signed and unsigned int? (assume binary values shown are all 32 bits)

- A. Signed and unsigned can both overflow at points X and Y
- B. Signed can overflow only at X, unsigned only at Y
- C. Signed can overflow only at Y, unsigned only at X
- D. Signed can overflow at X and Y, unsigned only at X
- E. Other



Unsigned Integers



100...001

100...000

positive numbers

More increasing

Increasing positive numbers

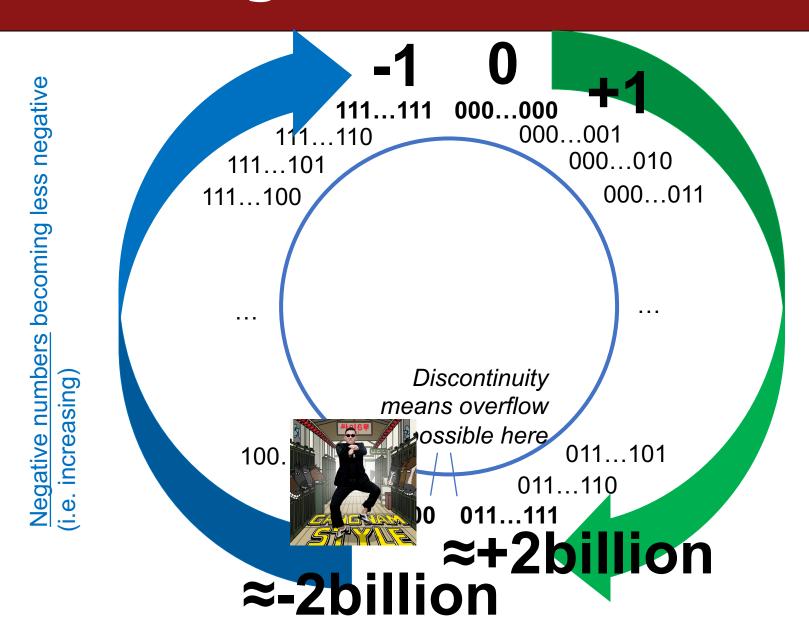
000...011

. . .

011...110

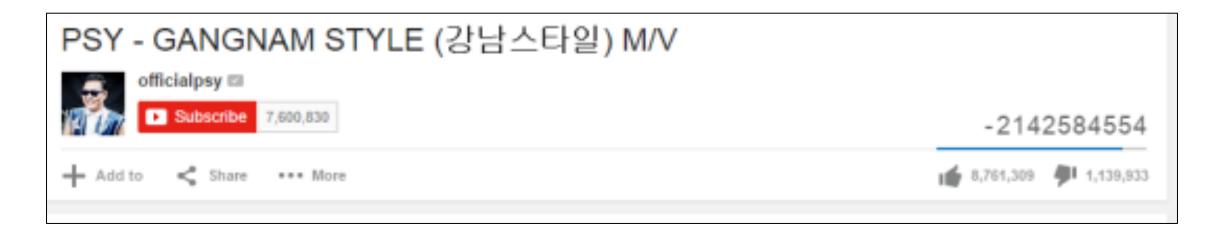
011...111

Signed Numbers



Increasing positive numbers

Overflow In Practice: PSY



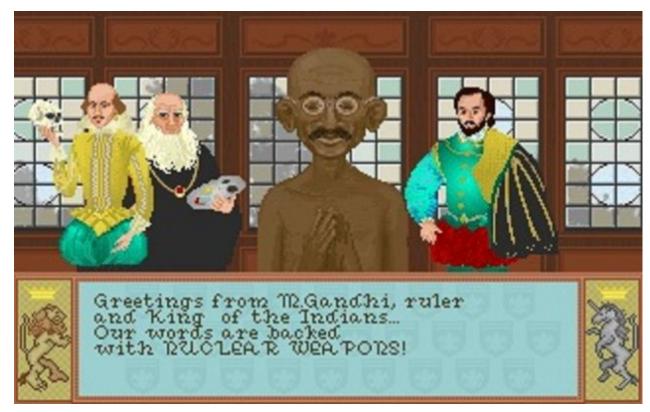
YouTube: "We never thought a video would be watched in numbers greater than a 32-bit integer (=2,147,483,647 views), but that was before we met PSY. "Gangnam Style" has been viewed so many times we had to upgrade to a 64-bit integer (9,223,372,036,854,775,808)!"

Overflow In Practice: Timestamps

- Many systems store timestamps as the number of seconds since Jan. 1, 1970 in a signed 32-bit integer.
- **Problem:** the latest timestamp that can be represented this way is 3:14:07 UTC on Jan. 13 2038!

Underflow In Practice: Gandhi

- In the game "Civilization", each civilization leader had an "aggression" rating. Gandhi was meant to be peaceful, and had a score of 1.
- If you adopted "democracy", all players' aggression reduced by 2. Gandhi's went from 1 to **255**!
- Gandhi then became a big fan of nuclear weapons.



https://kotaku.com/why-gandhi-is-such-an-asshole-in-civilization-1653818245

Recap

- Bits and Bytes
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Next time: How can we manipulate individual bits and bytes?

Extra Slides

printf and Integers

- There are 3 placeholders for 32-bit integers that we can use:
 - %d: signed 32-bit int
 - %u: unsigned 32-bit int
 - %x: hex 32-bit int
- As long as the value is a 32-bit type, printf will treat it according to the placeholder!

Casting

What happens at the byte level when we cast between variable types? The
bytes remain the same! This means they may be interpreted differently
depending on the type.

```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);

This prints out: "v = -12345, uv = 4294954951". Why?
```

Casting

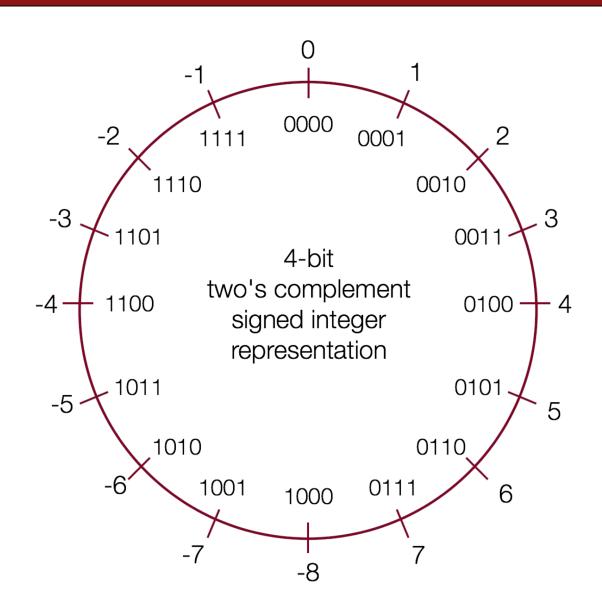
What happens at the byte level when we cast between variable types? The
bytes remain the same! This means they may be interpreted differently
depending on the type.

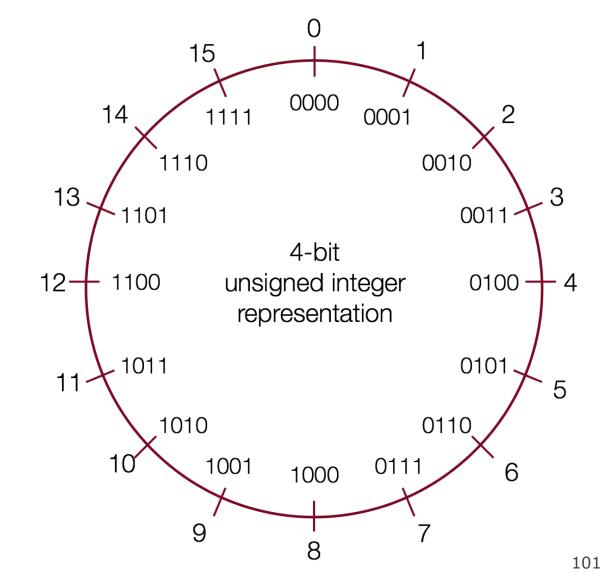
```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);
```

The bit representation for -12345 is **0b11000000111001**.

If we treat this binary representation as a positive number, it's huge!

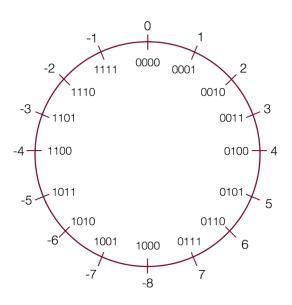
Casting





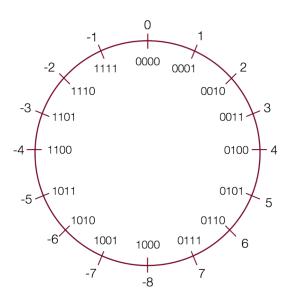
• Be careful when comparing signed and unsigned integers. C will implicitly cast the signed argument to unsigned, and then performs the operation assuming both numbers are non-negative.

Expression	Туре	Evaluation	Correct?
0 == 0U			
-1 < 0			
-1 < 0U			
2147483647 > -			
2147483647 - 1			
2147483647U > -			
2147483647 - 1			
2147483647 >			
(int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



• Be careful when comparing signed and unsigned integers. C will implicitly cast the signed argument to unsigned, and then performs the operation assuming both numbers are non-negative.

Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
2147483647 > - 2147483647 - 1	Signed	1	yes
2147483647U > - 2147483647 - 1	Unsigned	0	No!
2147483647 > (int)2147483648U	Signed	1	No!
-1 > -2	Signed	1	yes
(unsigned)-1 > -2	Unsigned	1	yes



Which many of the following statements are true? (assume that variables are set to values that place them in the spots shown)

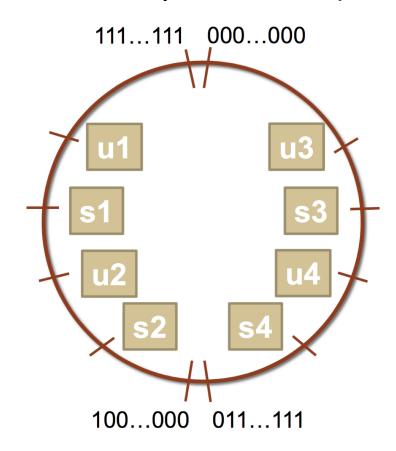
```
s3 > u3
u2 > u4
```

s2 > s4

s1 > s2

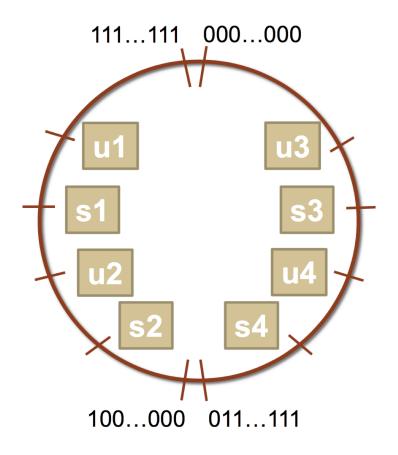
u1 > u2

s1 > u3



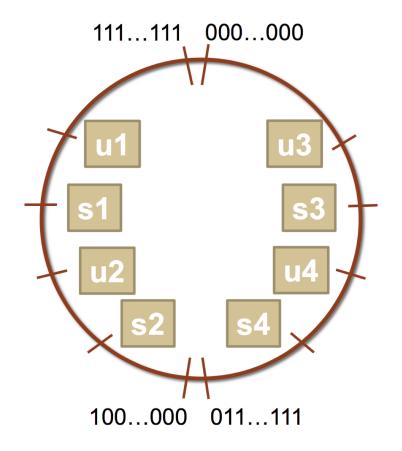
Which many of the following statements are true? (assume that variables are set to values that place them in the spots shown)

```
s3 > u3
u2 > u4
s2 > s4
s1 > s2
u1 > u2
s1 > u3
```



Which many of the following statements are true? (assume that variables are set to values that place them in the spots shown)

```
s3 > u3 - true
u2 > u4 - true
s2 > s4 - false
s1 > s2 - true
u1 > u2 - true
s1 > u3 - true
```



Expanding Bit Representations

- Sometimes, we want to convert between two integers of different sizes (e.g. short to int, or int to long).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a smaller data type to a bigger data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For signed values, we can repeat the sign of the value for new digits ("sign extension"
- Note: when doing <, >, <=, >= comparison between different size types, it will promote to the larger type.

Expanding Bit Representation

Expanding Bit Representation

```
short s = 4;
// short is a 16-bit format, so
                              s = 0000 \ 0000 \ 0000 \ 0100b
int i = s;
- or -
short s = -4;
                              s = 1111 \ 1111 \ 1111 \ 1100b
// short is a 16-bit format, so
int i = s;
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

```
0000 0000 0000 0000 1100 1111 1100 0111
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1100 1111 1100 0111
```

This is -12345! And when we cast sx back an int, we sign-extend the number.

```
1111 1111 1111 1111 1100 1111 1100 0111 // still -12345
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

```
1111 1111 1111 1111 1111 1111 1111 1101
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1111 1111 1111 1101
```

This is -3! If the number does fit, it will convert fine. y looks like this:

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

0000 0000 0000 0001 1111 0100 0000 0000

When we cast x to a short, it only has 16-bits, and C truncates the number:

1111 0100 0000 0000

This is 62464! Unsigned numbers can lose info too. Here is what y looks like:

0000 0000 0000 1111 0100 0000 0000 // still 62464

The size of Operator

• **sizeof** takes a variable type as a parameter and returns the number of bytes

that type uses.

```
printf("sizeof(char): %d\n", (int) sizeof(char));
printf("sizeof(short): %d\n", (int) sizeof(short));
printf("sizeof(int): %d\n", (int) sizeof(int));
printf("sizeof(unsigned int): %d\n", (int) sizeof(unsigned int));
printf("sizeof(long): %d\n", (int) sizeof(long));
printf("sizeof(long long): %d\n", (int) sizeof(long long));
printf("sizeof(size_t): %d\n", (int) sizeof(size_t));
printf("sizeof(void *): %d\n", (int) sizeof(void *));
```

```
$ ./sizeof
sizeof(char): 1
sizeof(short): 2
sizeof(int): 4
sizeof(unsigned int): 4
sizeof(long): 8
sizeof(long long): 8
sizeof(size_t): 8
sizeof(void *): 8
```

Туре	Width in bytes	Width in bits
char	1	8
short	2	16
int	4	32
long	8	64
void *	8	64