# CS107, Lecture 3 Bits and Bytes; Bitwise Operators

reading:

Bryant & O'Hallaron, Ch. 2.1

#### **Plan For Today**

- Recap: Integer Representations
- Truncating and Expanding
- Bitwise Operators and Masks
- **Demo 1:** Courses
- Break: Announcements
- **Demo 2:** Powers of 2
- Bit Shift Operators

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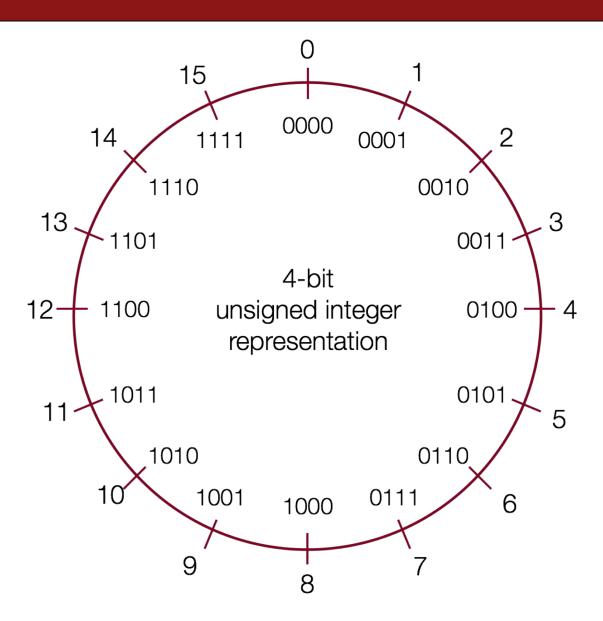
#### Base 2



#### Hexadecimal

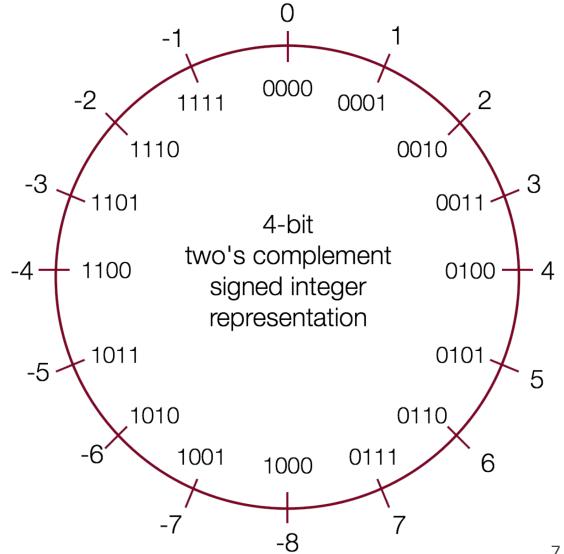
Hex digit	0	1	2	3	4	5	6	7
Decimal value	0	1	2	3	4	5	6	7
Binary value	0000	0001	0010	0011	0100	0101	0110	0111
Hex digit	8	9	A	В	С	D	E	F
Decimal value	8	9	10	11	12	13	14	15
Binary value	1000	1001	1010	1011	1100	1101	1110	1111

#### **Unsigned Integers**



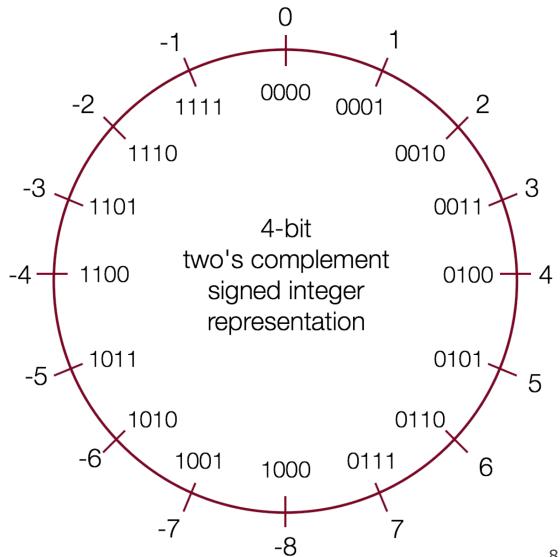
## Signed Integers: Two's Complement

- In two's complement, we represent a positive number as itself, and its negative equivalent as the two's complement of itself.
- The two's complement of a number is the binary digits inverted, plus 1.
- This works to convert from positive to negative, and back from negative to positive!



## Signed Integers: Two's Complement

- Con: more difficult to represent, and difficult to convert to/from decimal and between positive and negative.
- **Pro:** only 1 representation for 0!
- **Pro:** all bits are used to represent as many numbers as possible
- **Pro:** it turns out that the most significant bit still indicates the sign of a number.
- Pro: arithmetic is easy: we just add!



#### **Overflow and Underflow**

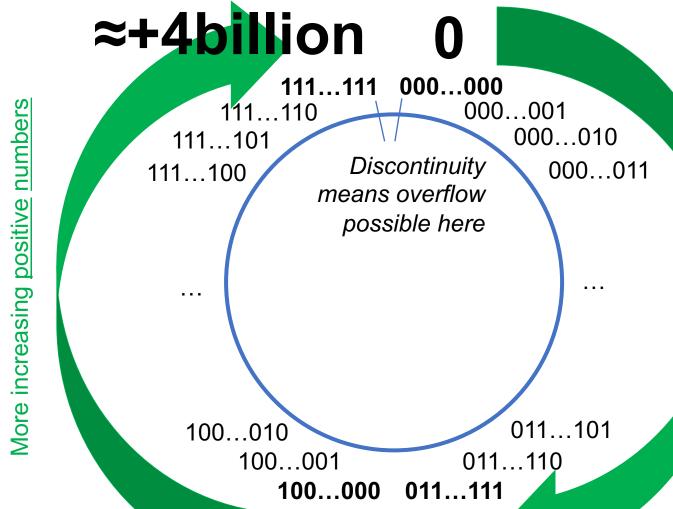
• If you exceed the **maximum** value of your bit representation, you wrap around or overflow back to the **smallest** bit representation.

```
0b1111 + 0b1 = 0b0000
```

• If you go below the **minimum** value of your bit representation, you wrap around or underflow back to the **largest** bit representation.

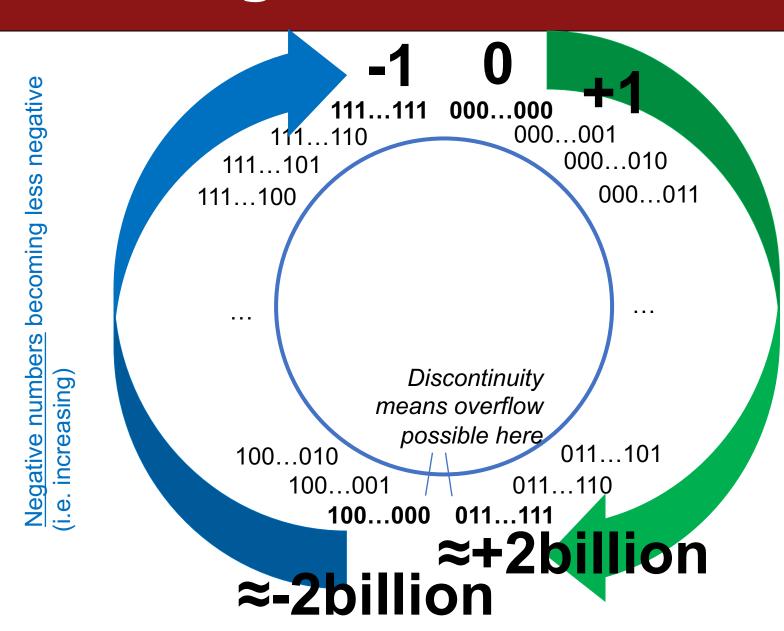
$$0b0000 - 0b1 = 0b1111$$

#### **Unsigned Integers**



Increasing positive numbers

#### **Signed Numbers**



Increasing positive numbers

#### Aside: ASCII

- ASCII is an encoding from common characters (letters, symbols, etc.) to bit representations (chars).
  - E.g. 'A' is 0x41
- Neat property: all uppercase letters, and all lowercase letters, are sequentially represented!
  - E.g. 'B' is 0x42

#### printf and Integers

- There are 3 placeholders for 32-bit integers that we can use:
  - %d: signed 32-bit int
  - %u: unsigned 32-bit int
  - %x: hex 32-bit int
- As long as the value is a 32-bit type, printf will treat it according to the placeholder!

## Casting

What happens at the byte level when we cast between variable types? The
bytes remain the same! This means they may be interpreted differently
depending on the type.

```
int v = -12345;
    unsigned int uv = v;
    printf("v = %d, uv = %u\n", v, uv);

This prints out: "v = -12345, uv = 4294954951". Why?
```

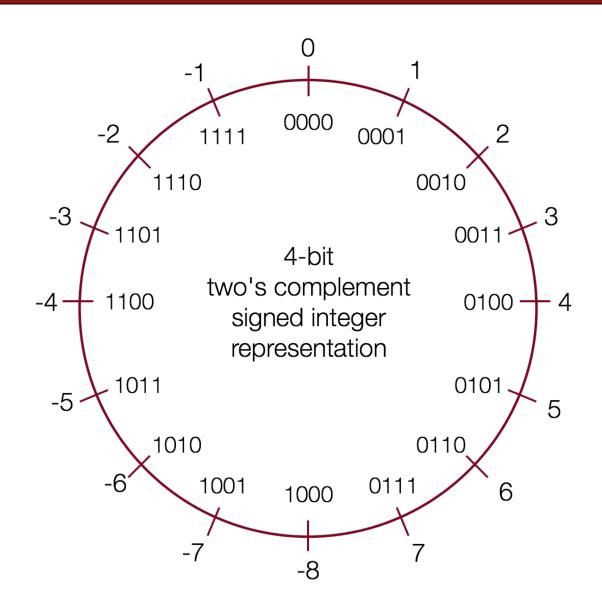
## Casting

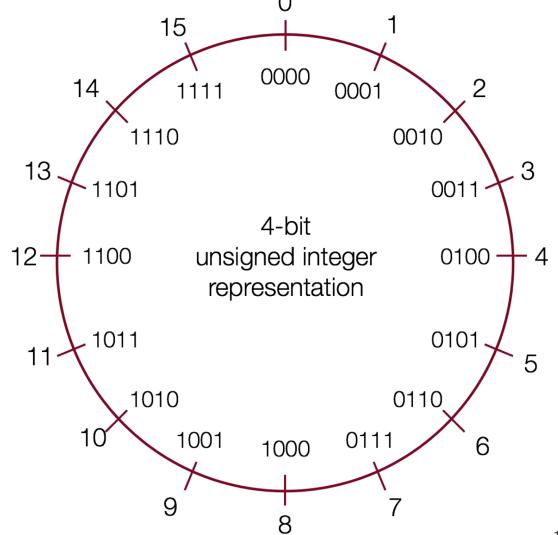
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int v = -12345;
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printf("v = %d, uv = %u\n", v, uv);
```

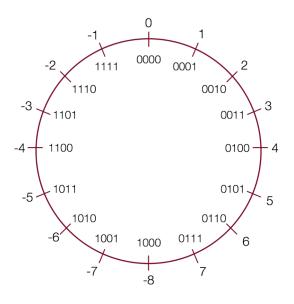
-12345 in binary is **1111 1111 1111 1111 1100 1111 1100 0111.** If we treat this binary representation as a positive number, it's *huge*!

#### Casting

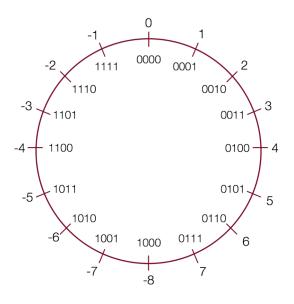




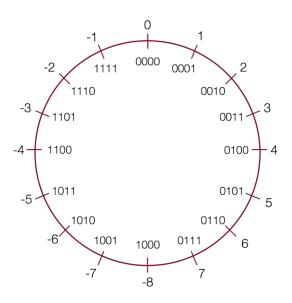
Expression	Туре	Evaluation	Correct?
0 == 0U			
-1 < 0			
-1 < 0U			
2147483647 > -			
2147483647 - 1			
2147483647U > -			
2147483647 - 1			
2147483647 >			
(int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



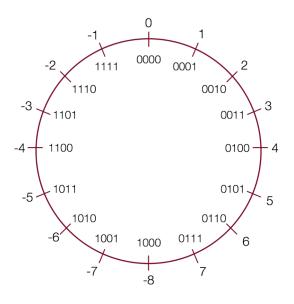
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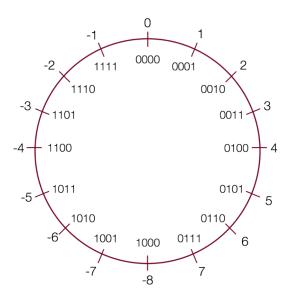
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-1 < 0	Signed	true	yes
-1 < 0U			
2147483647 > -			
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2147483647U > -			
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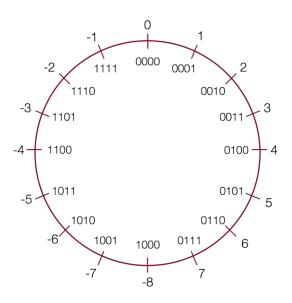
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-1 < 0	Signed	true	yes
-1 < OU	Unsigned	false	no!
2147483647 > -			
2147483647 - 1			
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2147483647 - 1			
2147483647 >			
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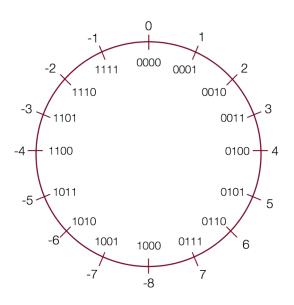
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2147483647U > - 2147483647 - 1			
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



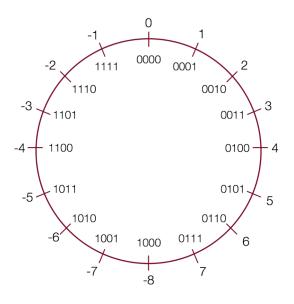
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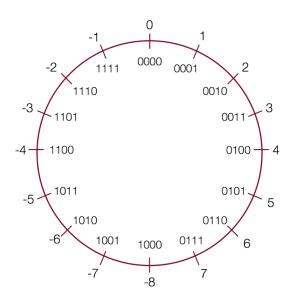
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-1 > -2	Signed	true	yes
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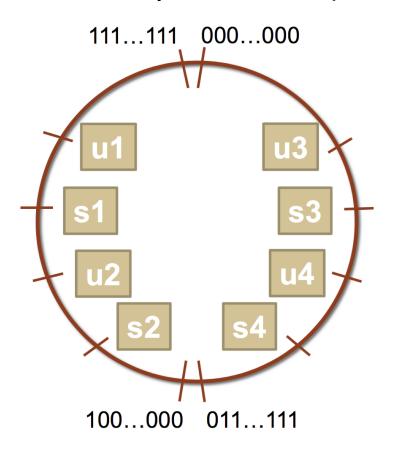
Which of the following statements are true? (assume that variables are set to values that place them in the spots shown)

1. 
$$s3 > u3$$

4. 
$$s1 > s2$$

5. 
$$u1 > u2$$

6. s1 > u3

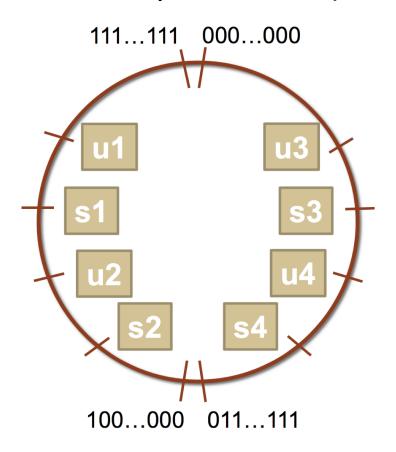


Which of the following statements are true? (assume that variables are set to values that place them in the spots shown)

1. 
$$s3 > u3 - true$$

4. 
$$s1 > s2$$

6. s1 > u3



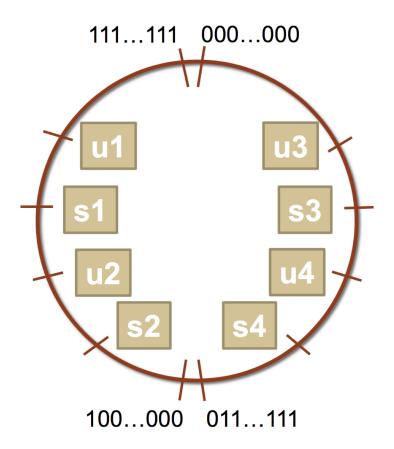
Which of the following statements are true? (assume that variables are set to values that place them in the spots shown)

```
1. s3 > u3 - true
```

2. 
$$u2 > u4 - true$$

4. 
$$s1 > s2$$

6. 
$$s1 > u3$$



Which of the following statements are true? (assume that variables are set to values that place them in the spots shown)

```
1. s3 > u3 - true
```

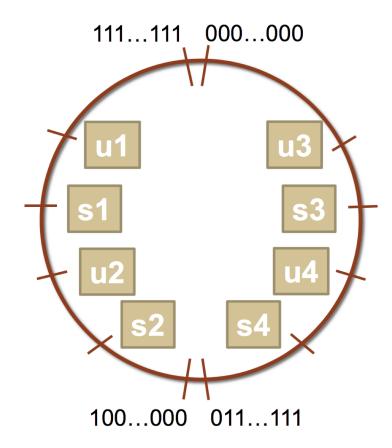
2. 
$$u2 > u4 - true$$

$$3. s2 > s4 - false$$

4. 
$$s1 > s2$$

5. 
$$u1 > u2$$

6. 
$$s1 > u3$$



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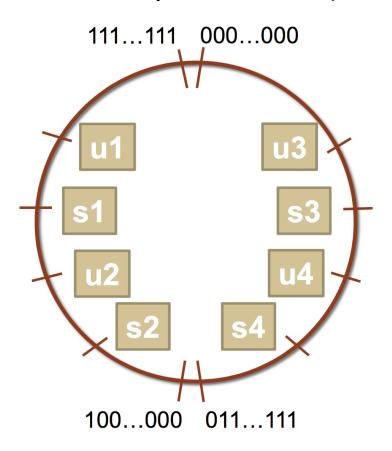
```
1. s3 > u3 - true
```

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$$s1 > s2$$
 - true

6. 
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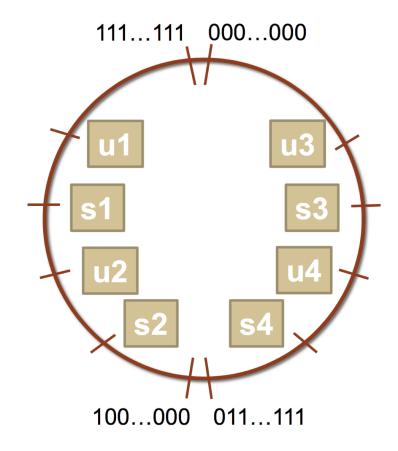
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```
    s3 > u3 - true
    u2 > u4 - true
    s2 > s4 - false
```

4. s1 > s2 - true

5. u1 > u2 - true

6. s1 > u3



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```

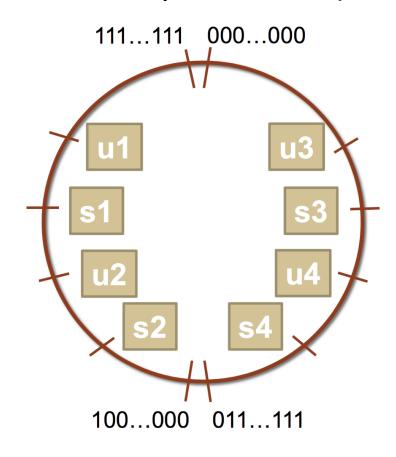
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$$3. s2 > s4 - false$$

4. 
$$s1 > s2$$
 - true

5. 
$$u1 > u2 - true$$

6. s1 > u3 - true



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#### **Expanding Bit Representations**

- Sometimes, we want to convert between two integers of different sizes (e.g. short to int, or int to long).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a **smaller** data type to a **bigger** data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For signed values, we can repeat the sign of the value for new digits ("sign extension"
- Note: when doing <, >, <=, >= comparison between different size types, it will promote to the larger type.

## **Expanding Bit Representation**

#### **Expanding Bit Representation**

```
unsigned short s = 32772;
// short is a 16-bit format, so
                               s = 1000 \ 0000 \ 0000 \ 0100b
unsigned int i = s;
- or -
short s = -4;
                               s = 1111 \ 1111 \ 1111 \ 1100b
// short is a 16-bit format, so
int i = s;
```

## **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

```
0000 0000 0000 0000 1100 1111 1100 0111
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1100 1111 1100 0111
```

This is -12345! And when we cast sx back an int, we sign-extend the number.

```
1111 1111 1111 1111 1100 1111 1100 0111 // still -12345
```

## **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

```
1111 1111 1111 1111 1111 1111 1111 1101
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1111 1111 1111 1101
```

This is -3! If the number does fit, it will convert fine. y looks like this:

## **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

0000 0000 0000 0001 1111 0100 0000 0000

When we cast x to a short, it only has 16-bits, and C truncates the number:

1111 0100 0000 0000

This is 62464! Unsigned numbers can lose info too. Here is what y looks like:

#### sizeof

sizeof takes a variable type as a parameter and returns its size in bytes.

```
sizeof(type)
```

#### For example:

```
sizeof(char) => 1
sizeof(short) => 2
sizeof(int) => 4
sizeof(unsigned int) => 4
sizeof(long) => 8
sizeof(char *) => 8
```

# Now that we understand binary representations, how can we manipulate them at the bit level?

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#### **Bitwise Operators**

- You're already familiar with many operators in C:
  - Arithmetic operators: +, -, \*, /, %
  - Comparison operators: ==, !=, <, >, <=, >=
  - Logical Operators: &&, ||,!
- Today, we're introducing a new category of operators: bitwise operators:
  - &, |, ~, ^, <<, >>

# And (&)

AND is a binary operator. The AND of 2 bits is 1 if both bits are 1, and 0 otherwise.

#### output = a & b;

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

# Or (|)

OR is a binary operator. The OR of 2 bits is 1 if either (or both) bits is 1.

outp	ut = a	b;
a	b	output
0	0	0
0	1	1
1	0	1
1	1	1

## **Not** (∼)

NOT is a unary operator. The NOT of a bit is 1 if the bit is 0, or 1 otherwise.

	◢ .~ .	4		
ou	Th		=	~a;
U	LP	u C		· u j

a	output
0	1
1	0

# Exclusive Or (^)

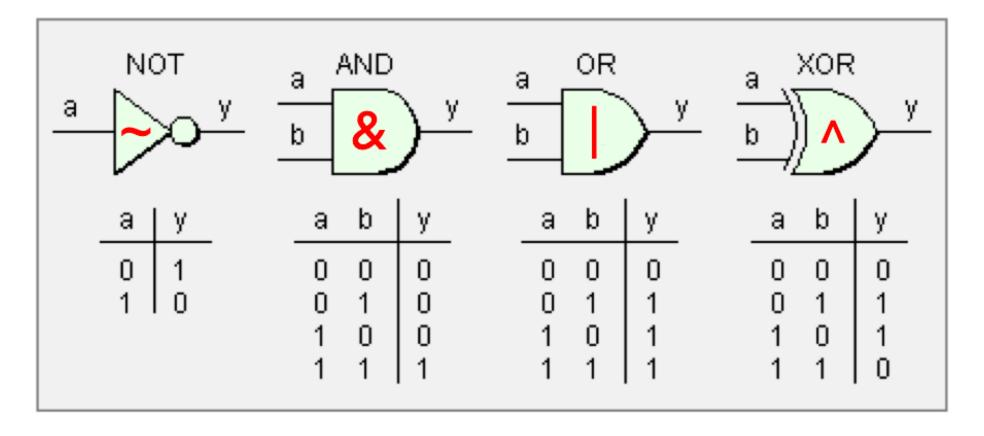
Exclusive Or (XOR) is a binary operator. The XOR of 2 bits is 1 if *exactly* one of the bits is 1, or 0 otherwise.

output	=	a	^	b;
--------	---	---	---	----

a	b	output
0	0	0
0	1	1
1	0	1
1	1	0

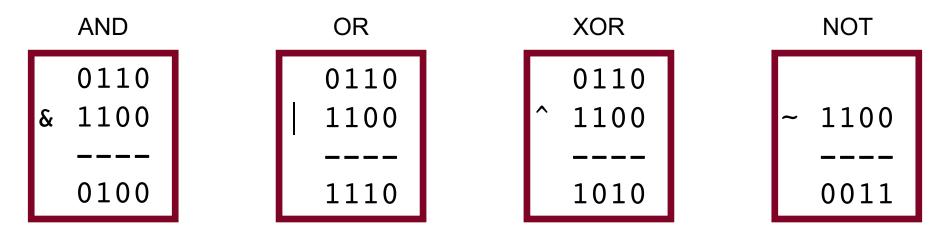
## An Aside: Boolean Algebra

 These operators are not unique to computers; they are part of a general area called **Boolean Algebra**. These are applicable in math, hardware, computers, and more!



#### **Operators on Multiple Bits**

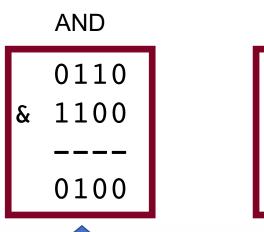
• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

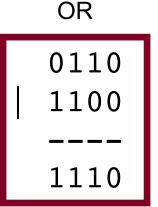


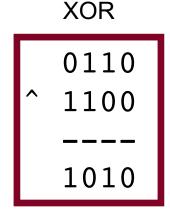
**Note:** these are different from the logical operators AND (&&), OR (||) and NOT (!).

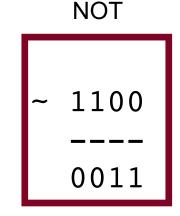
#### **Operators on Multiple Bits**

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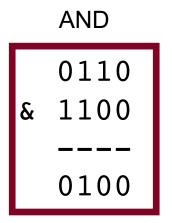


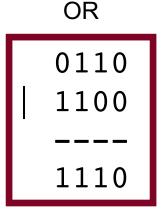


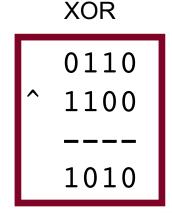
This is different from logical AND (&&). The logical AND returns true if both are nonzero, or false otherwise.

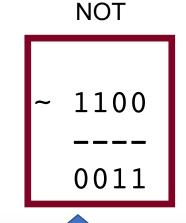
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• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:





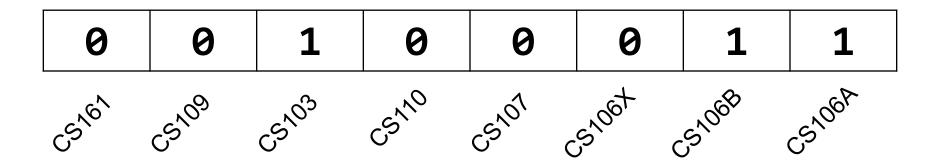




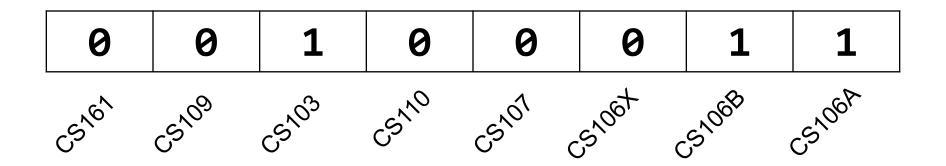
This is different from logical NOT (!). The logical NOT returns true if this is zero, and false otherwise.

#### **Bit Vectors and Sets**

- We can use bit vectors (ordered collections of bits) to represent finite sets, and perform functions such as union, intersection, and complement.
- Example: we can represent current courses taken using a char.

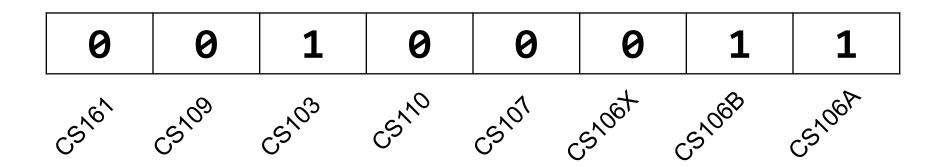


#### **Bit Vectors and Sets**



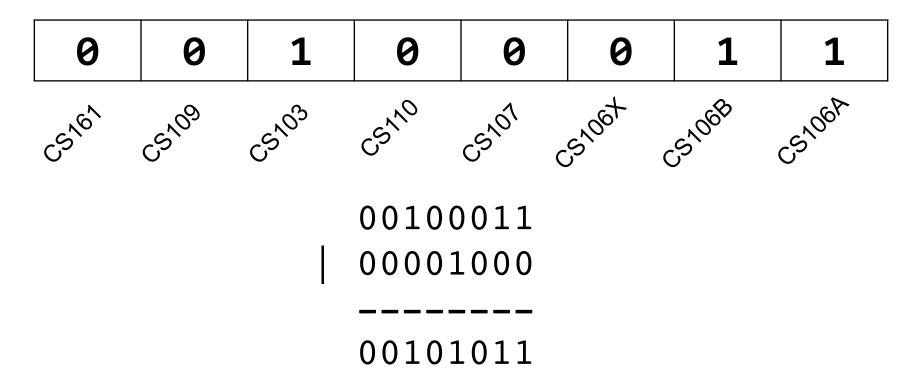
• How do we find the union of two sets of courses taken? Use OR:

#### **Bit Vectors and Sets**



• How do we find the intersection of two sets of courses taken? Use AND:

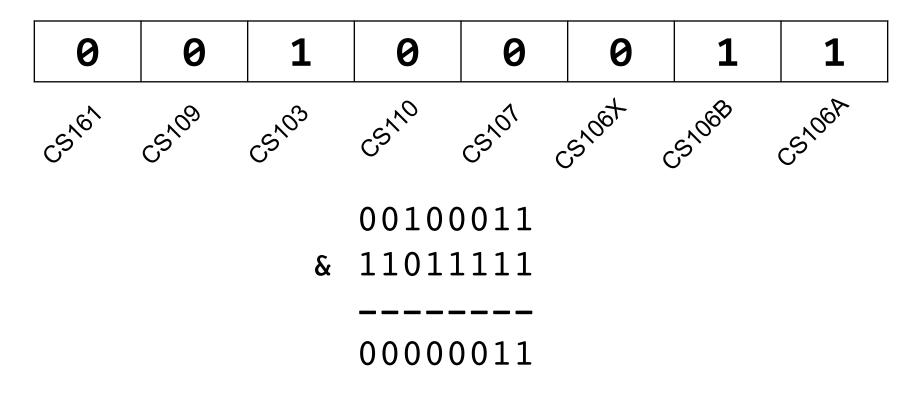
- We will frequently want to manipulate or isolate out specific bits in a larger collection of bits. A **bitmask** is a constructed bit pattern that we can use, along with bit operators, to do this.
- Example: how do we update our bit vector to indicate we've taken CS107?



```
#define CS106A 0x1
                 /* 0000 0001 */
                     /* 0000 0010 */
#define CS106B 0x2
#define CS106X 0x4
                 /* 0000 0100 */
#define CS107 0x8 /* 0000 1000 */
#define CS110 0x10
                  /* 0001 0000 */
#define CS103 0x20
                  /* 0010 0000 */
            0x40 /* 0100 0000 */
#define CS109
                     /* 1000 0000 */
#define CS161
             0x80
char myClasses = ...;
myClasses = myClasses | CS107; // Add CS107
```

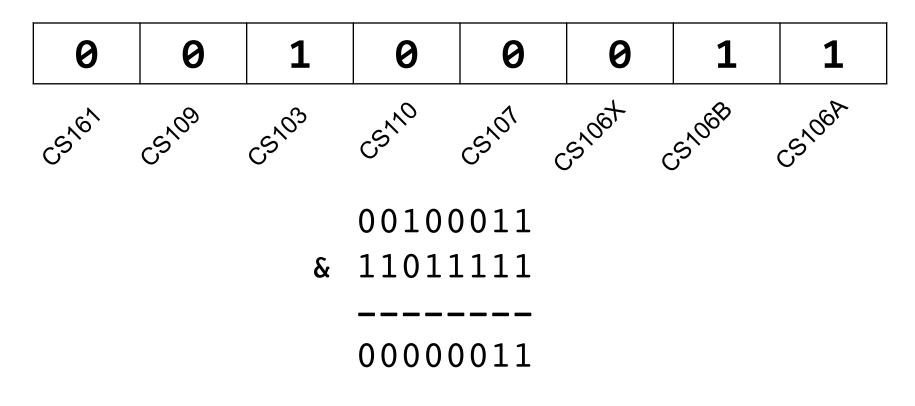
```
#define CS106A 0x1
                     /* 0000 0001 */
#define CS106B 0x2
                     /* 0000 0010 */
#define CS106X 0x4
                     /* 0000 0100 */
                     /* 0000 1000 */
#define CS107
             0x8
#define CS110 0x10
                     /* 0001 0000 */
                     /* 0010 0000 */
#define CS103
             0x20
#define CS109
                    /* 0100 0000 */
             0x40
#define CS161
             0x80
                     /* 1000 0000 */
char myClasses = ...;
myClasses |= CS107; // Add CS107
```

• Example: how do we update our bit vector to indicate we've not taken CS103?



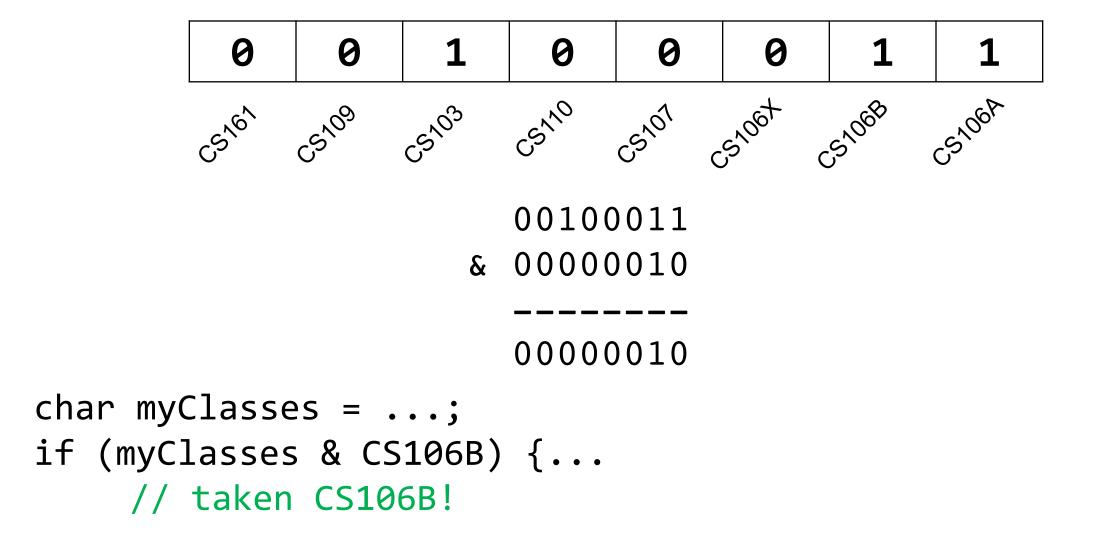
```
char myClasses = ...;
myClasses = myClasses & ~CS103; // Remove CS103
```

• Example: how do we update our bit vector to indicate we've not taken CS103?

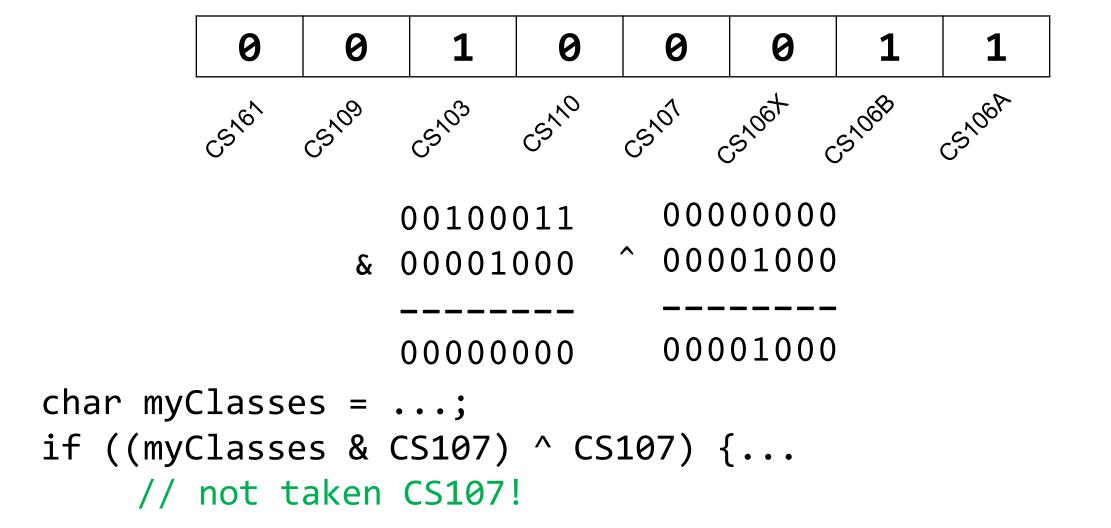


```
char myClasses = ...;
myClasses &= ~CS103; // Remove CS103
```

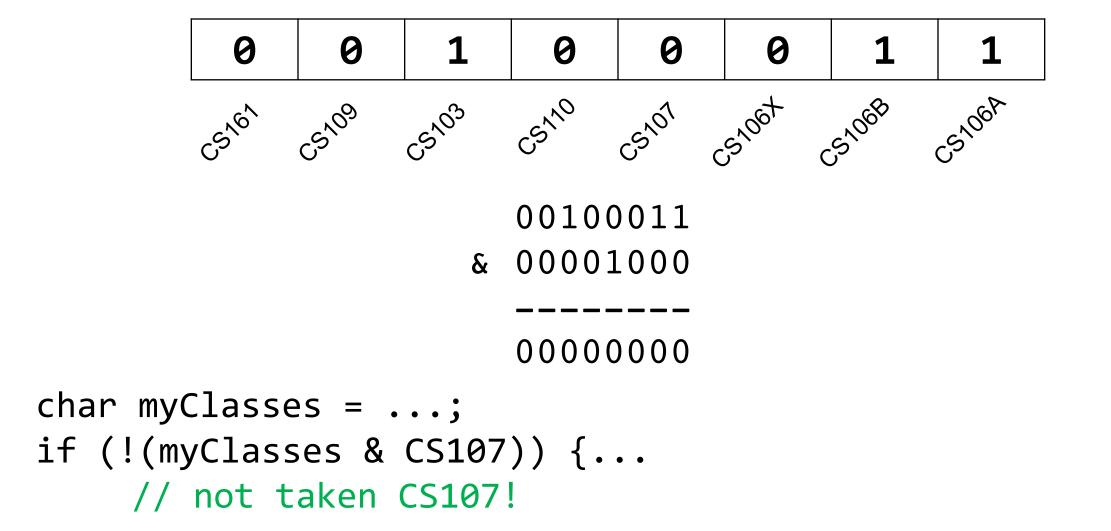
• Example: how do we check if we've taken CS106B?



• Example: how do we check if we've not taken CS107?



• Example: how do we check if we've not taken CS107?



# **Demo: Bitmasks and GDB**



• Bit masking is also useful for integer representations as well. For instance, we might want to check the value of the most-significant bit, or just one of the middle bytes.

• **Example:** If I have a 32-bit integer **j**, what operation should I perform if I want to get *just the lowest byte* in **j**?

```
int j = ...;
int k = j & 0xff;  // mask to get just lowest byte
```

## **Practice: Bit Masking**

• **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

• **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

## **Practice: Bit Masking**

• **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

• **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

#### **Plan For Today**

- **Recap**: Integer Representations
- Truncating and Expanding
- Bitwise Boolean Operators and Masks
- Demo 1: Courses
- Break: Announcements
- **Demo 2:** Powers of 2
- Bit Shift Operators

#### **Announcements**

- Please send us any OAE letters or athletics conflicts as soon as possible.
- Assignment 0 deadline tonight at 11:59PM PST
- Assignment 1 (Bit operations!) goes out tonight at Assignment 0 deadline
  - Saturated arithmetic
  - Cell Automata
  - Unicode and UTF-8
- Lab 1 this week!

#### Powers of 2

Without using loops, how can we detect if a binary number is a power of 2? What is special about its binary representation and how can we leverage that?

# Demo: Powers of 2



## **Plan For Today**

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# Left Shift (<<)

The LEFT SHIFT operator shifts a bit pattern a certain number of positions to the left. New lower order bits are filled in with 0s, and bits shifted off of the end are lost.

```
x << k; // shifts x to the left by k bits
```

#### 8-bit examples:

```
00110111 << 2 results in 11011100
01100011 << 4 results in 00110000
10010101 << 4 results in 01010000
```

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off of the end are lost.

```
x >> k; // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Idea: let's follow left-shift and fill with 0s.

```
short x = 2; // 0000 0000 0000 0010
x >> 1; // 0000 0000 0000 0001
printf("%d\n", x); // 1
```

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off of the end are lost.

```
x >> k; // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Idea: let's follow left-shift and fill with 0s.

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off of the end are lost.

```
x >> k; // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Problem: always filling with zeros means we may change the sign bit.

Solution: let's fill with the sign bit!

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off of the end are lost.

```
x >> k; // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Solution: let's fill with the sign bit!

```
short x = 2; // 0000 0000 0000 0010
x >> 1; // 0000 0000 0000 0001
printf("%d\n", x); // 1
```

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off of the end are lost.

```
x >> k; // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Solution: let's fill with the sign bit!

There are two kinds of right shifts, depending on the value and type you are shifting:

- Logical Right Shift: fill new high-order bits with 0s.
- Arithmetic Right Shift: fill new high-order bits with the most-significant bit.

Unsigned numbers are right-shifted using Logical Right Shift.

Signed numbers are right-shifted using Arithmetic Right Shift.

This way, the sign of the number (if applicable) is preserved!

#### **Shift Operation Pitfalls**

- 1. Technically, the C standard does not precisely define whether a right shift for signed integers is logical or arithmetic. However, almost all compilers/machines use arithmetic, and you can most likely assume this.
- 2. Operator precedence can be tricky! For example:

1<<2 + 3<<4 means 1 << (2+3) << 4 because addition and subtraction have higher precedence than shifts! Always use parentheses to be sure:

$$(1<<2) + (3<<4)$$

#### **Bit Operator Pitfalls**

- The default type of a number literal in your code is an int.
- Let's say you want a long with the index-32 bit as 1:

```
long num = 1 << 32;
```

• This doesn't work! 1 is by default an **int**, and you can't shift an int by 32 because it only has 32 bits. You must specify that you want 1 to be a **long**.

long num = 
$$1L << 32$$
;

#### Recap

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**Next time:** How can a computer represent and manipulate more complex data like text?

#### **Extra Slides**

# Demo: Absolute Value

