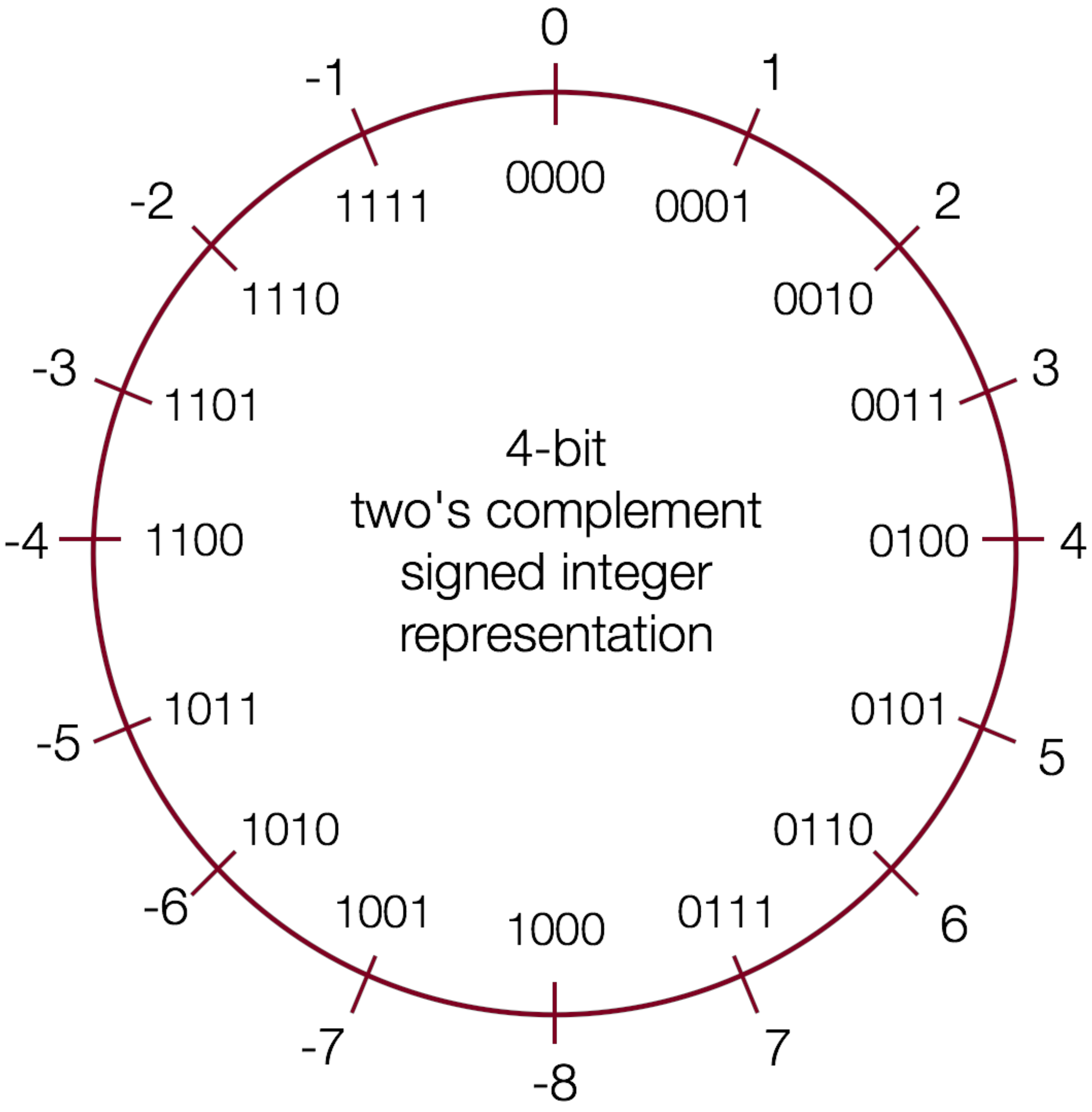
CS 107
Lecture 2: Integer Representations

Friday, January 7, 2022

Computer Systems
Winter 2022
Stanford University
Computer Science Department

Reading: Reader: Bits and Bytes, Textbook: Chapter 2.2

Lecturer: Chris Gregg



Today's Topics

- Logistics
  - Assign0 — Due Monday
  - Labs start Tuesday
  - Office hours in full coverage
- Reading: Reader: Bits and Bytes, Textbook: Chapter 2.2 (very mathy…)
- Integer Representations
  - Unsigned numbers
  - Signed numbers
    - two's complement
  - Signed vs Unsigned numbers
  - Casting in C
  - Signed and unsigned comparisons
  - The `sizeof` operator
  - Min and Max integer values
  - Truncating integers
  - two's complement overflow

Information Storage

In C, everything can be thought of as a block of 8 bits

Information Storage

In C, everything can be thought of as a block of 8 bits called a "byte"

Information Storage

We will discuss manipulating bytes on a bit-by-bit level, but we won't be able to consider an individual bit on its own.

In a computer, the memory system is simply a large array of bytes (sound familiar, from CS106B?)

values (chars):	7	2	8	3	14	99	-6	3	45	11
address (decimal):	200	201	202	203	204	205	206	207	208	209
address (hex):	0xc8	0xc9	0xca	0xcb	0xcc	0xcd	0xce	0xcf	0xd0	0xd1

Each address (a pointer!) represents the next byte in memory.

E.g., address 0 is a byte, then address 1 is the next full byte, etc.

Again: you can't address a bit. You must address at the byte level.

Byte Range

Because a byte is made up of 8 bits, we can represent the range of a byte as follows:

`00000000` to `11111111`

This range is 0 to 255 in decimal.

But, neither binary nor decimal is particularly convenient to write out bytes (binary is too long, and decimal isn't numerically friendly for byte representation)

So, we use "hexadecimal," (base 16).

Hexadecimal

Hexadecimal has 16 digits, so we augment our normal 0-9 digits with six more digits: A, B, C, D, E, and F.

Figure 2.2 in the textbook shows the hex digits and their binary and decimal values:

Hex digit	0	1	2	3	4	5	6	7
Decimal value	0	1	2	3	4	5	6	7
Binary value	0000	0001	0010	0011	0100	0101	0110	0111

Hex digit	8	9	A	B	C	D	E	F
Decimal value	8	9	10	11	12	13	14	15
Binary value	1000	1001	1010	1011	1100	1101	1110	1111

Hexadecimal

- In C, we write a hexadecimal with a starting `0x`. So, you will see numbers such as `0xfa1d37b`, which means that it is a hex number.
- You should memorize the binary representations for each hex digit. One trick is to memorize A (`1010`), C (`1100`), and F (`1111`), and the others are easy to figure out.
- Let's practice some hex to binary and binary to hex conversions:

Convert: `0x173A4C` to binary.

Hexadecimal	1	7	3	A	4	C
Binary	0001	0111	0011	1010	0100	1100

`0x173A4C` is binary `0b000101110011101001001100`

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Hexadecimal

Convert: `0b1111001010110110110011` to hexadecimal.

Binary	11	1100	1010	1101	1011	0011
Hexadecimal	3	C	A	D	B	3

(start from the **right**)

`0b1111001010110110110011`
is hexadecimal `3CADB3`

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Decimal to Hexadecimal

To convert from decimal to hexadecimal, you need to repeatedly divide the number in question by 16, and the remainders make up the digits of the hex number:

314156 decimal:

```
314,156 / 16 = 19,634 with 12 remainder: C
19,634 / 16  = 1,227 with 2 remainder:   2
1,227 / 16   = 76 with 11 remainder:     B
76 / 16 = 4 with 12 remainder:           C
4 / 16 = 0 with 4 remainder:             4

Reading from bottom up: 0x4CB2C
```

Hexidecimal to Decimal

To convert from hexadecimal to decimal, multiply each of the hexadecimal digits by the appropriate power of 16:

```
0x7AF:

7 * 16^2 + 10 * 16 + 15
= 7 * 256 + 160 + 15
= 1792 + 160 + 15 = 1967
```

Let the computer do it!

Honestly, hex to decimal and vice versa are easy to let the computer handle. You can either use a search engine (Google does this automatically), or you can use a python one-liner:

4. cgregg@myth10: ~ (ssh)

```
cgregg@myth10:~$ python -c "print(hex(314156))"
0x4cb2c
cgregg@myth10:~$ python -c "print(0x7af)"
1967
cgregg@myth10:~$
```

Let the computer do it!

You can also use Python to convert to and from binary:

```
4. cgregg@myth10: ~ (ssh)
cgregg@myth10:~$ python -c "print(bin(0x173A4C))"
0b101110011101001001100
cgregg@myth10:~$ python -c "print(hex(0b1111001010110110110011))"
0x3cadb3
cgregg@myth10:~$
```

(but you should memorize this as it is easy and you will use it frequently)

Integer Representations

Integer Representations

The C language has two different ways to represent numbers, unsigned and signed:

unsigned: can only represent non-negative numbers

signed: can represent negative, zero, and positive numbers

We are going to talk about these representations, and also about what happens when we expand or shrink an encoded integer to fit into a different type (e.g., `int` to `long`)

Unsigned Integers

For positive (unsigned) integers, there is a 1-to-1 relationship between the decimal representation of a number and its binary representation. If you have a 4-bit number, there are 16 possible combinations, and the unsigned numbers go from 0 to 15:

```
0b0000 = 0     0b0001 = 1     0b0010 = 2     0b0011 = 3
0b0100 = 4     0b0101 = 5     0b0110 = 6     0b0111 = 7
0b1000 = 8     0b1001 = 9     0b1010 = 10    0b1011 = 11
0b1100 = 12    0b1101 = 13    0b1110 = 14    0b1111 = 15
```

The range of an unsigned number is $0 \rightarrow 2^w - 1$, where w is the number of bits in our integer. For example, a 32-bit `int` can represent numbers from 0 to $2^{32} - 1$, or 0 to 4,294,967,295.

Signed Integers: How do we represent them?

What if we want to represent negative numbers? We have choices!

One way we could encode a negative number is simply to designate some bit as a "sign" bit, and then interpret the rest of the number as a regular binary number and then apply the sign. For instance, for a four-bit number:

0 001 = 1	1 001 = -1
0 010 = 2	1 010 = -2
0 011 = 3	1 011 = -3
0 100 = 4	1 100 = -4
0 101 = 5	1 101 = -5
0 110 = 6	1 110 = -6
0 111 = 7	1 111 = -7

This might be okay...but we've only represented 14 of our 16 available numbers...

Signed Integers: How do we represent them?

0 001 = 1
0 010 = 2
0 011 = 3
0 100 = 4
0 101 = 5
0 110 = 6
0 111 = 7

1 001 = -1
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What about 0 000 and 1 000? What should they represent?

Well...this is a bit tricky!

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Well...this is a bit tricky!

Let's look at the bit patterns: 0 000 1 000

Should we make the 0 000 just represent decimal 0? What about 1 000? We could make it 0 as well, or maybe -8, or maybe even 8, but none of the choices are nice.

Signed Integers: How do we represent them?

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Should we make the 0 000 just represent decimal 0? What about 1 000? We could make it 0 as well, or maybe -8, or maybe even 8, but none of the choices are nice.

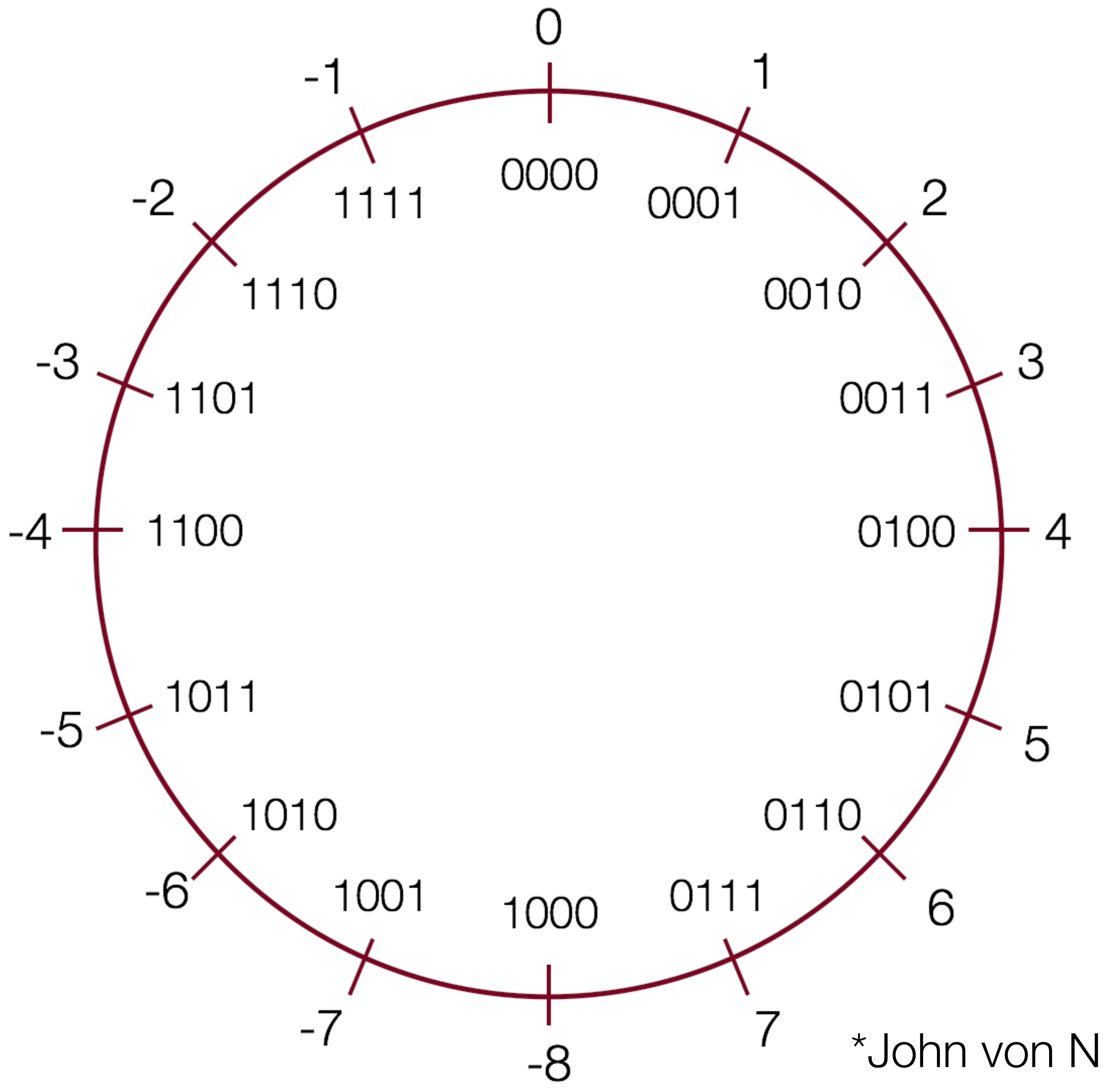
Fine. Let's just make 0 000 to be equal to decimal 0. How does arithmetic work? Well…to add two numbers, you need to know the sign, then you might have to subtract (borrow and carry, etc.), and the sign might change…this is going to get ugly!

Signed Integers: How do we represent them?

There is a better way!

Signed Integers: How do we represent them?

Behold: the "two's complement" circle:



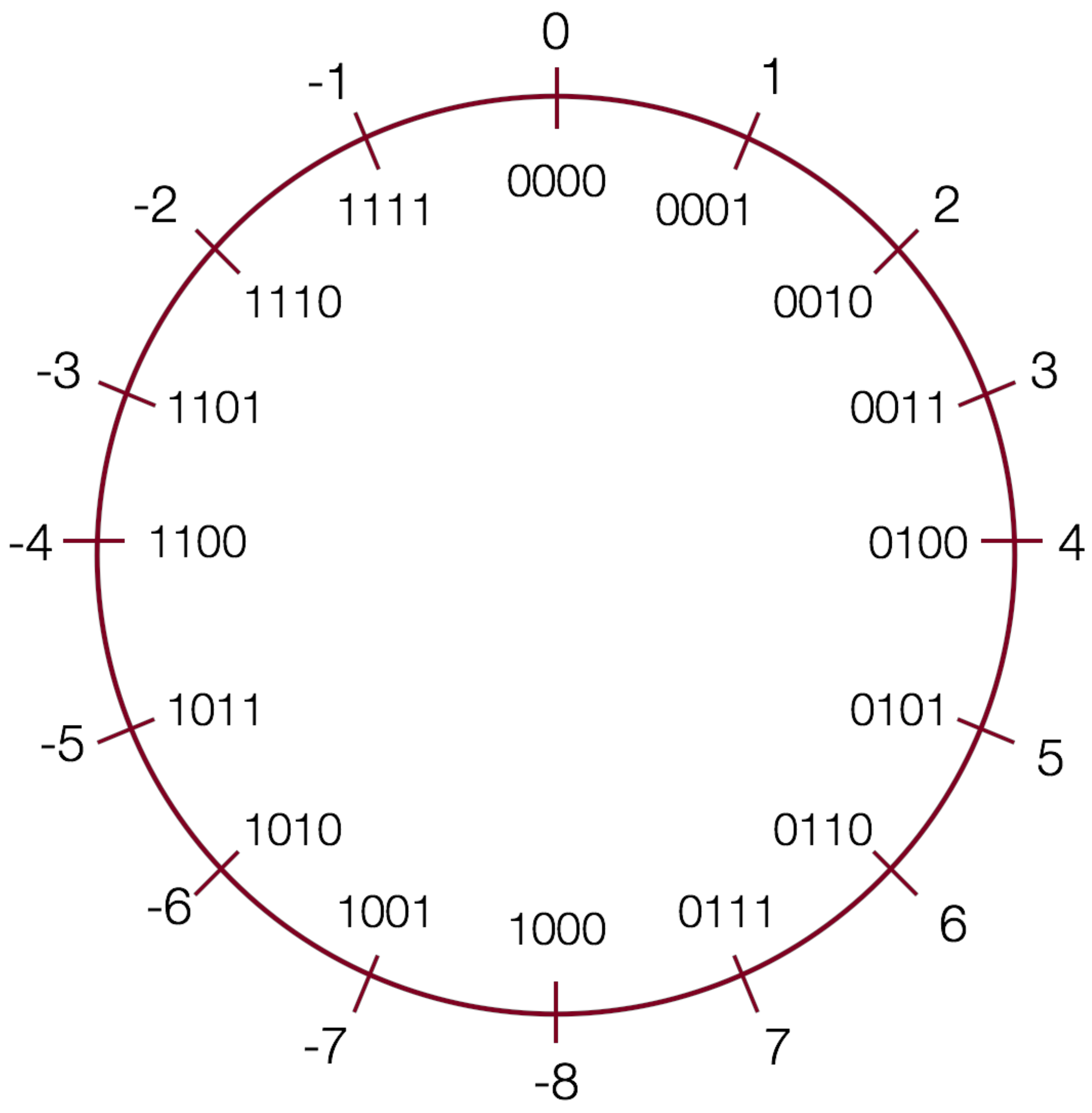
In the early days of computing*, two's complement was determined to be an excellent way to store binary numbers.

In two's complement notation, positive numbers are represented as themselves (phew), and negative numbers are represented as the two's complement of themselves (definition to follow).

This leads to some amazing arithmetic properties!

*John von Neumann suggested it in 1945, for the EDVAC computer.

Two's Complement



A two's-complement number system encodes positive and negative numbers in a binary number representation. The weight of each bit is a power of two, except for the most significant bit, whose weight is the negative of the corresponding power of two.

Definition: For vector $\vec{x} = [x_{w-1}, x_{w-2}, \ldots, x_0]$ of an w-bit integer $x_{w-1}x_{w-2}\ldots x_0$ is given by the following formula:

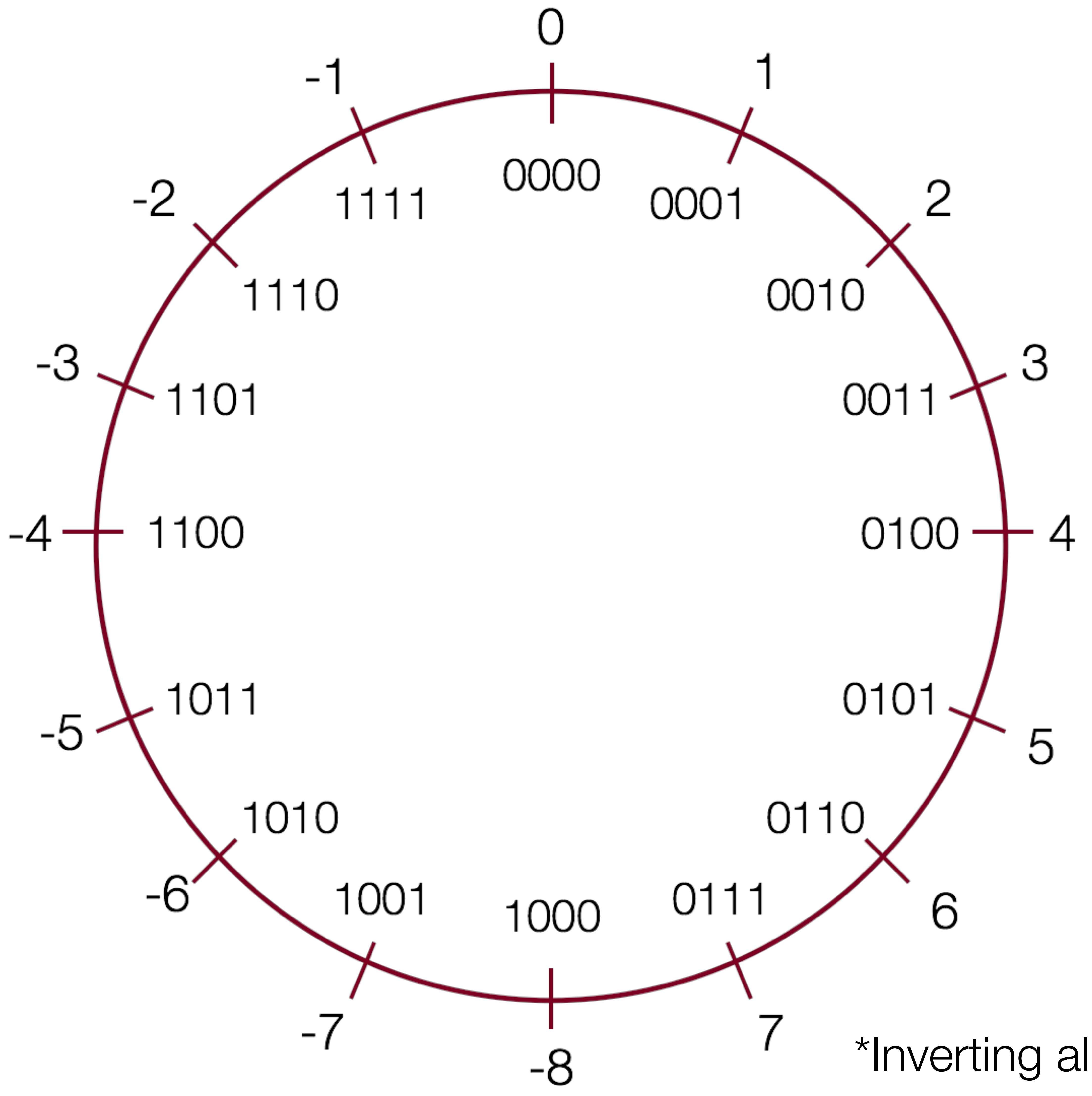
$$B2T_w(\vec{x}) = -x_{w-1}2^{w-1} + \sum_{i=0}^{w-2} x_i 2^i .$$

$B2T_w$ means "Binary to Two's complement function"

In practice, a negative number in two's complement is obtained by inverting all the bits of its positive counterpart*, and then adding 1.

*Inverting all the bits of a number is its "one's complement"

Two's Complement



In practice, a negative number in two's complement is obtained by inverting all the bits of its positive counterpart*, and then adding 1, or: `x = ~x + 1`

Example: The number 2 is represented as normal in binary: 0010

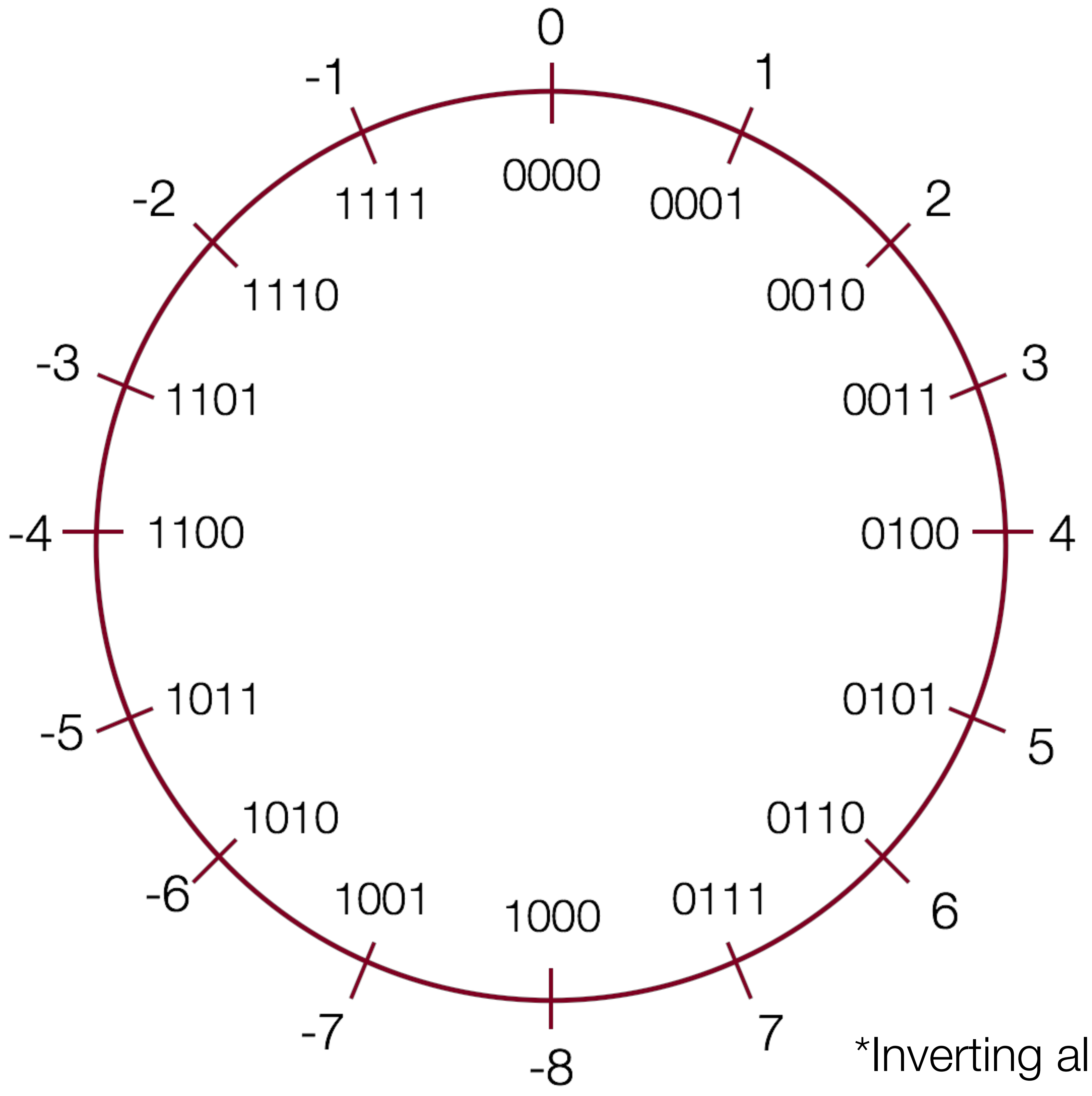
-2 is represented by inverting the bits, and adding 1:

0010 ☞ 1101

```
 1101
+   1
 1110
```

*Inverting all the bits of a number is its "one's complement"

Two's Complement



Trick: to convert a positive number to its negative in two's complement, start from the right of the number, and write down all the digits until you get to a 1. Then invert the rest of the digits:

Example: The number 2 is represented as normal in binary: 0010

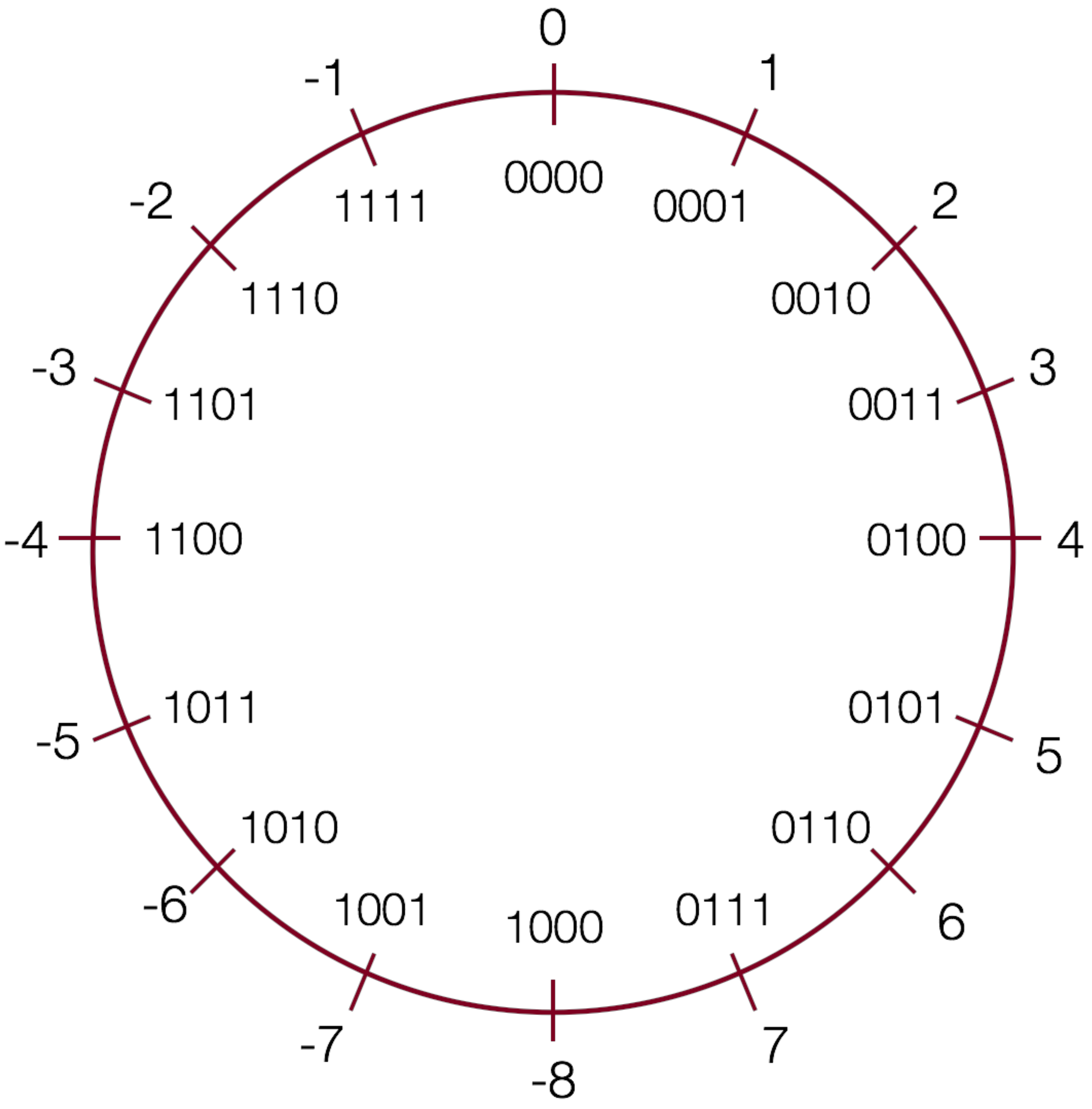
Going from the right, write down numbers until you get to a 1:

10

Then invert the rest of the digits:
1110

*Inverting all the bits of a number is its "one's complement"

Two's Complement



To convert a negative number to a positive number, perform the same steps!

Example: The number -5 is represented in two's complements as: 1011

5 is represented by inverting the bits, and adding 1:

1011 ☞ 0100

$$\begin{array}{r} 0100 \\ +\quad 1 \\ \hline 0101 \end{array}$$

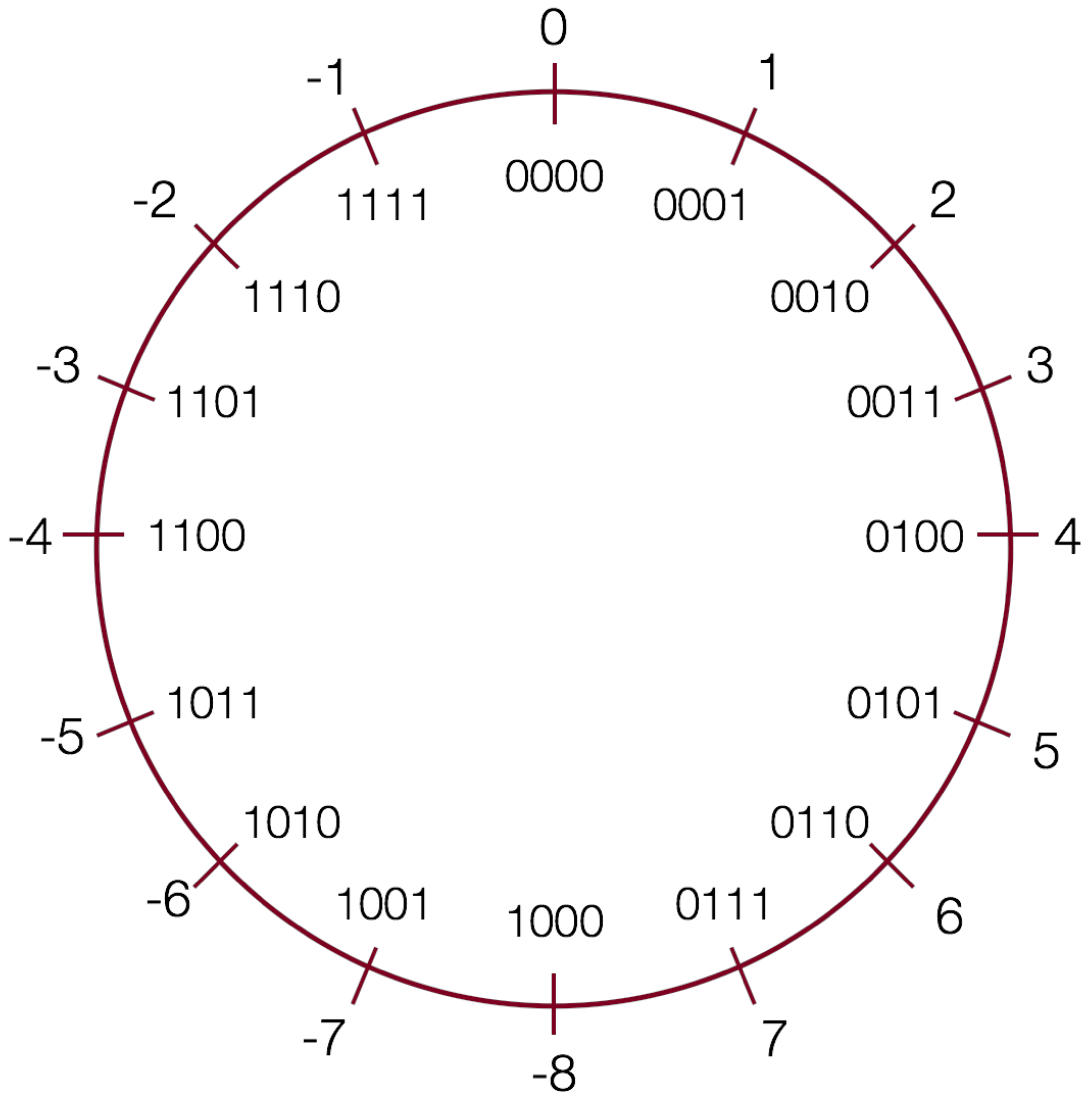
Shortcut: start from the right, and write down numbers until you get to a 1:

1

Now invert all the rest of the digits:

0101

Two's Complement: Neat Properties

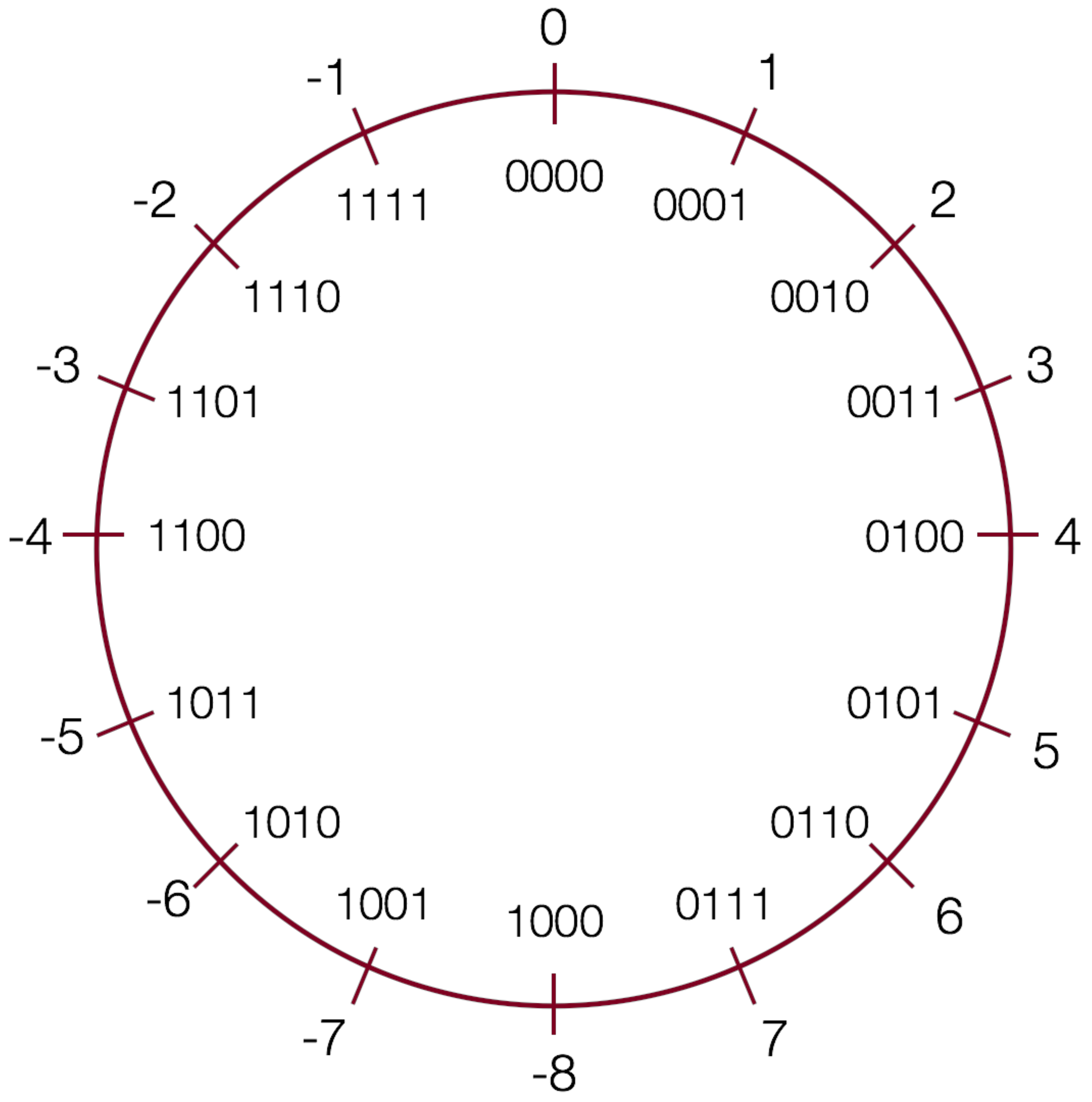


There are a number of useful properties associated with two's complement numbers:

1. There is only one zero (yay!)
2. The highest order bit (left-most) is 1 for negative, 0 for positive (so it is easy to tell if a number is negative)
3. Adding two numbers is just…adding! Example:
   2 + -5 = -3

 0010 ☞ 2
<u>+1011</u> ☞ -5
 1101 ☞ -3 decimal (wow!)

Two's Complement: Neat Properties



More useful properties:

4. Subtracting two numbers is simply performing the two's complement on one of them and then adding.
   Example:
   4 - 5 = -1

0100 ☞ 4, 0101 ☞ 5

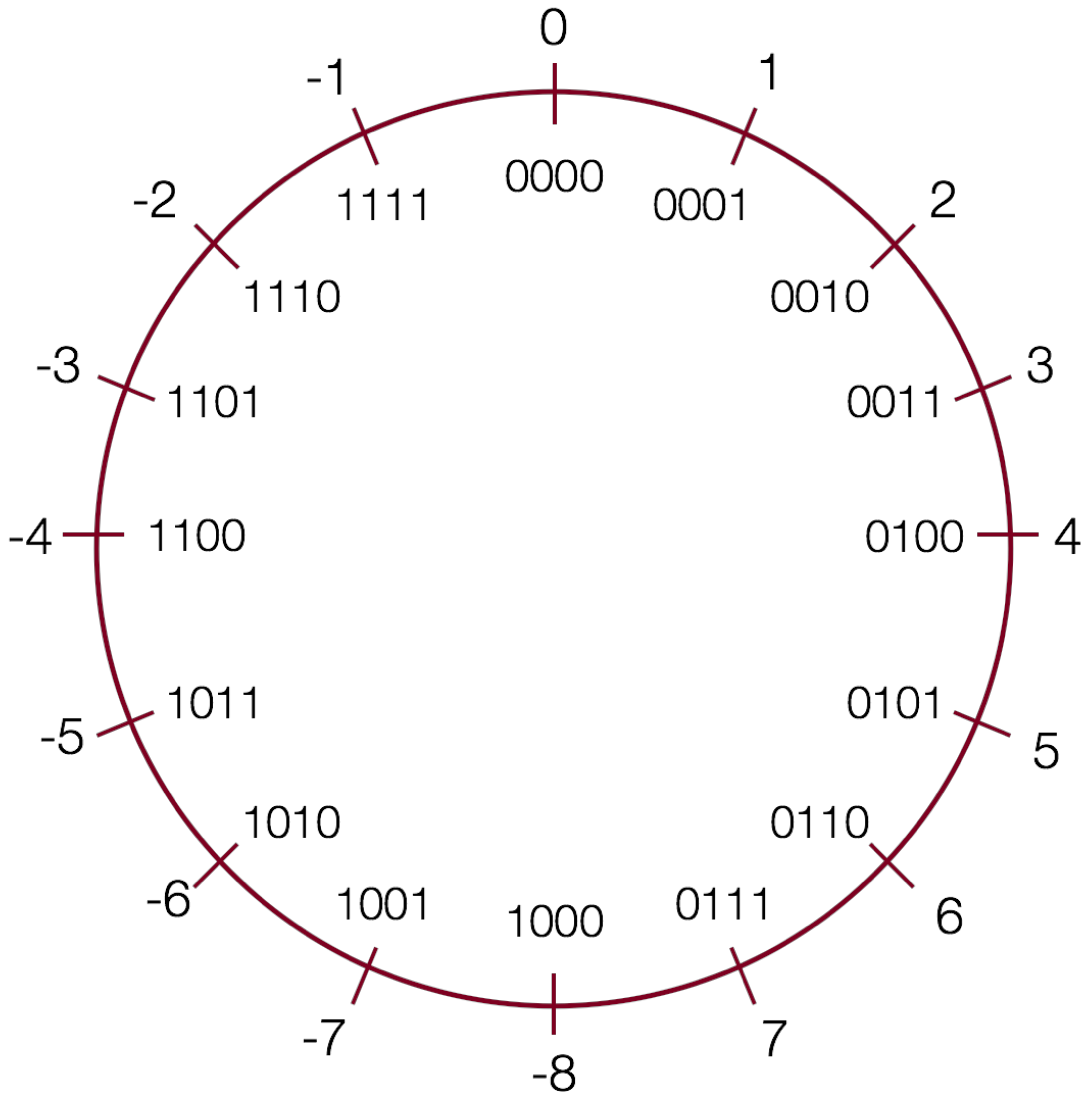
Find the two's complement of 5: 1011
add:

 0100 ☞ 4

+1011 ☞ -5

 1111 ☞ -1 decimal

Two's Complement: Neat Properties



More useful properties:

5. Multiplication of two's complement works just by multiplying (throw away overflow digits).

Example: -2 * -3 = 6

1110 ☞ -2

x1101 ☞ -3

1110

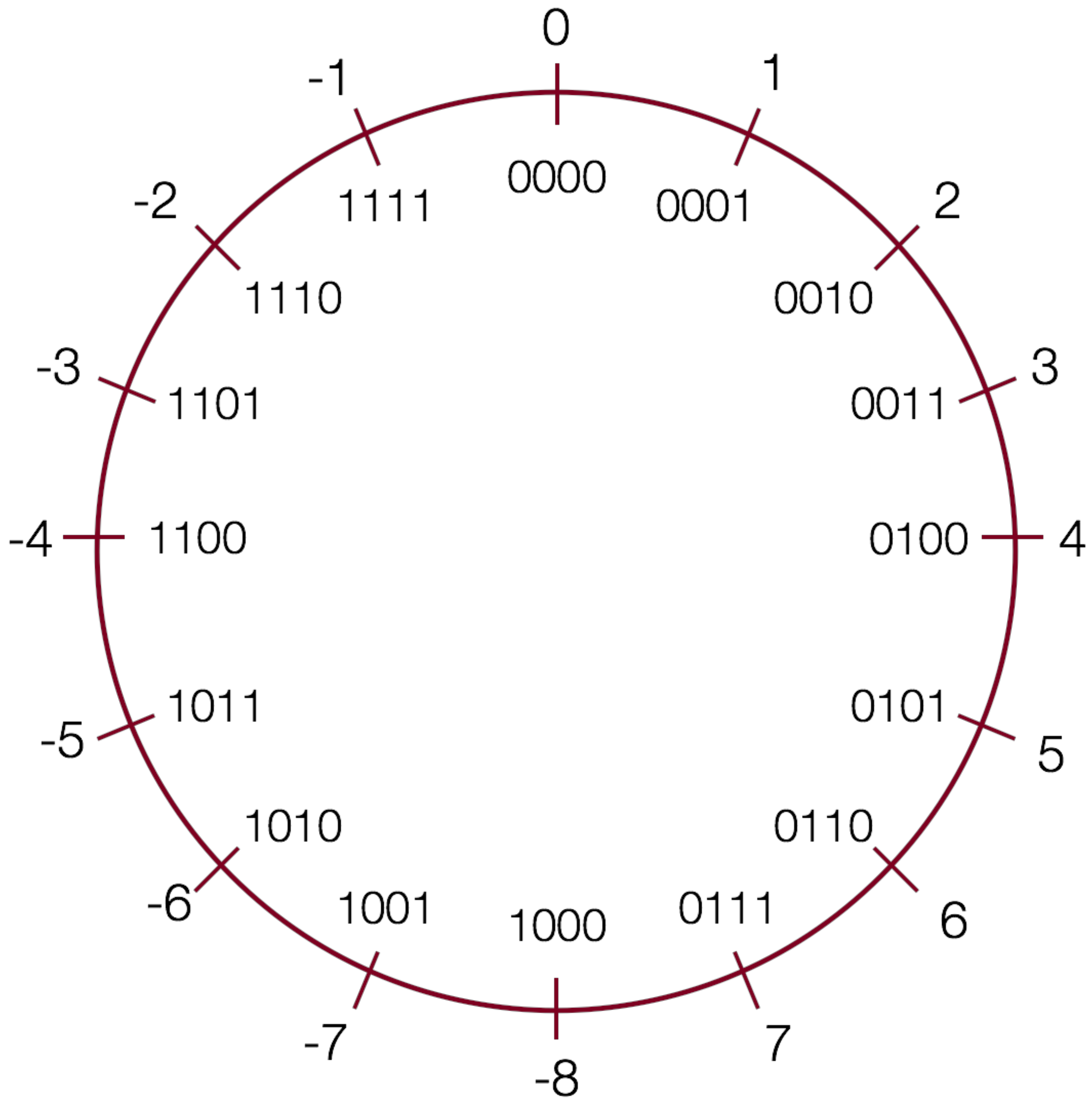
0000

1110

+1110

~~1011~~**0110** ☞ 6

Two's Complement: Powers of two remain!



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$$B2T_w(\vec{x}) = -x_{w-1}2^{w-1} + \sum_{i=0}^{w-2} x_i 2^i.$$

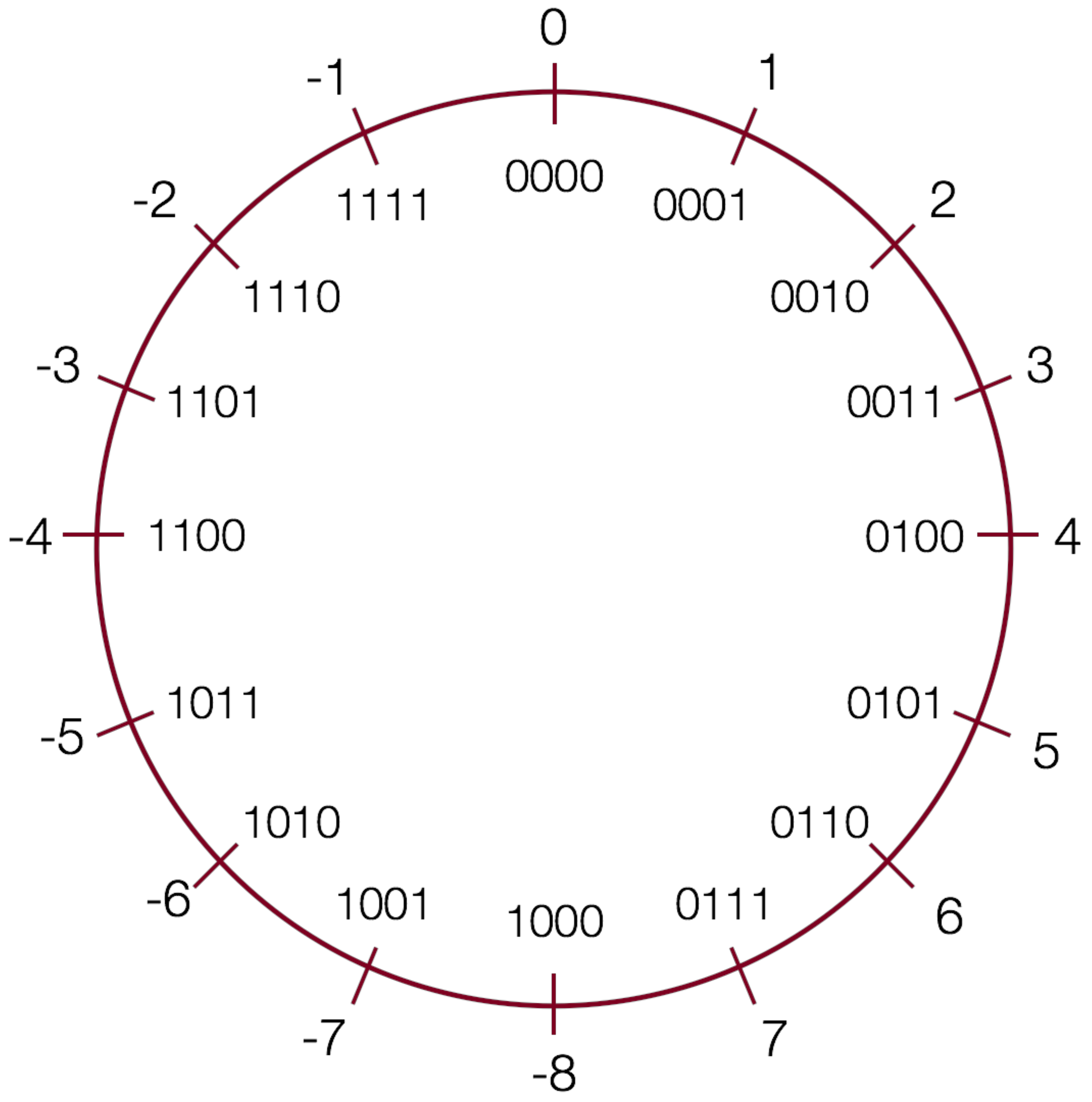
From the definition of a two's complement number, we can see that we are still dealing with bits being equal to their powers-of-two place: there isn't anything magical about the placement of the bits:

-5 = 1 0 1 1

(1 * -2³) + (0 * 2²) + (1 * 2¹) + (1 * 2⁰)

$-5 = (1 * -2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0)$

Practice



Convert the following 4-bit numbers from positive to negative, or from negative to positive using two's complement notation:

a. -4 (1100) ☞

b. 7 (0111) ☞

c. 3 (0011) ☞

d. -8 (1000) ☞

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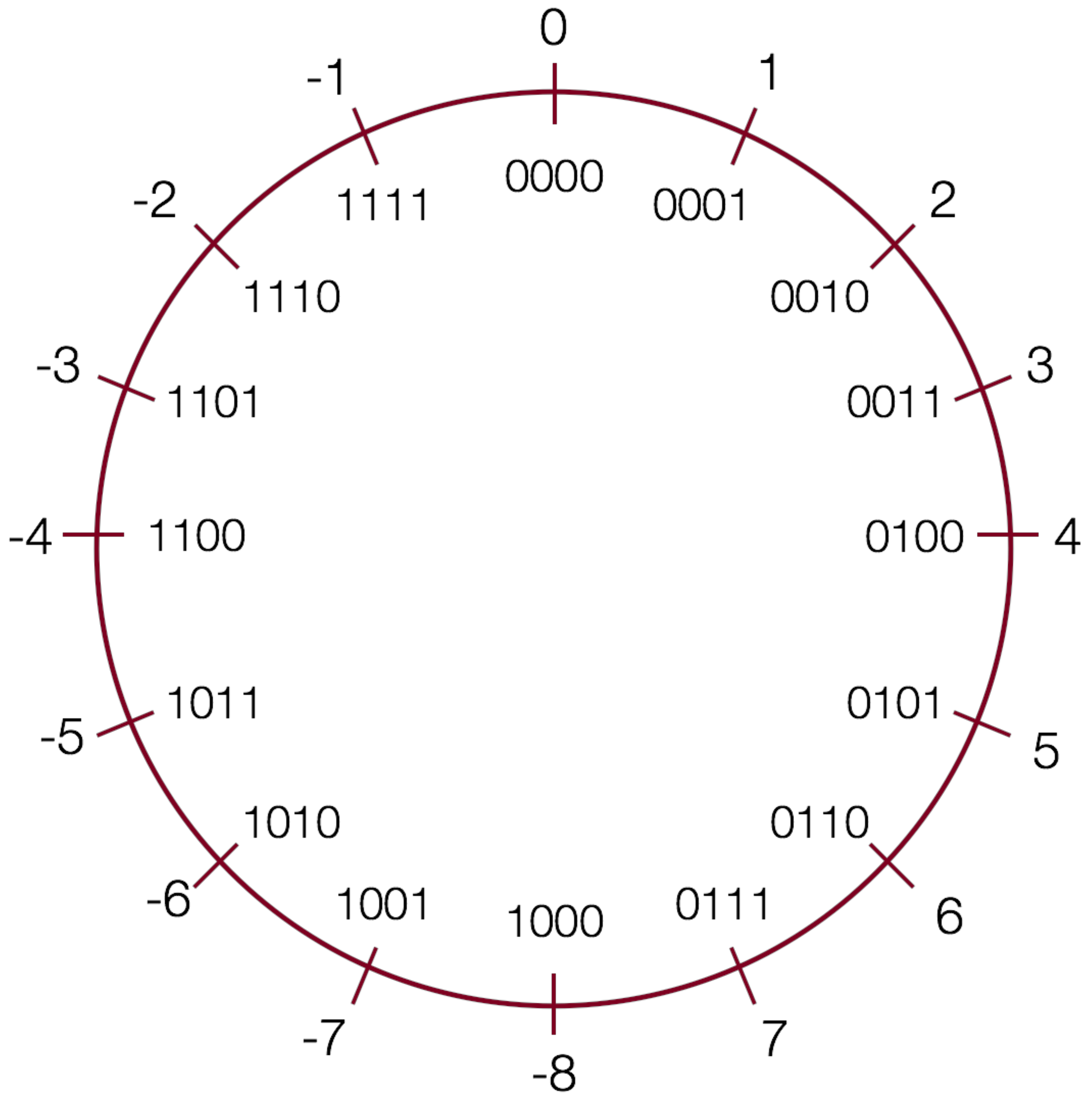

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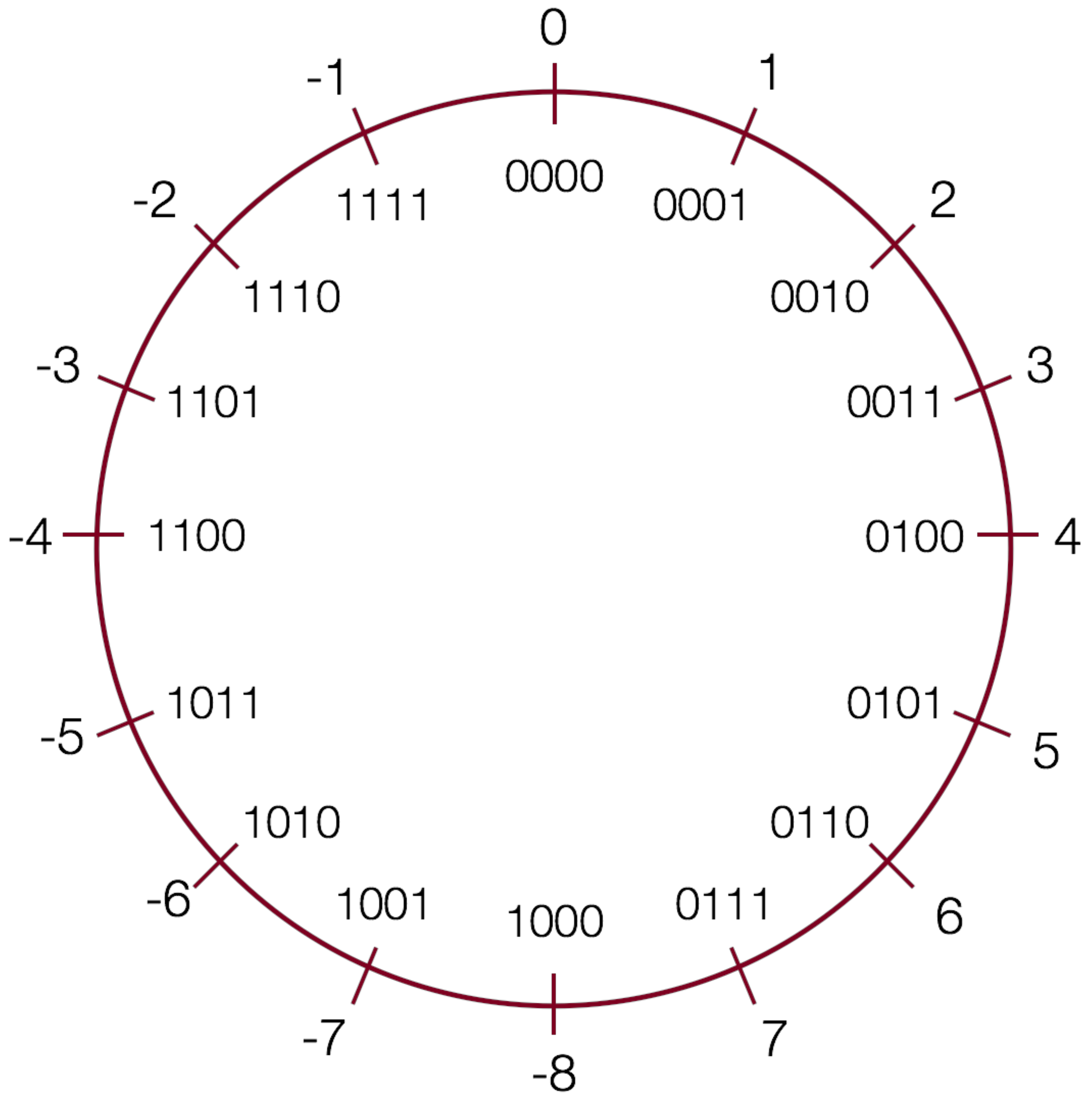
a. -4 (1100) ☞ 0100

b. 7 (0111) ☞ 1001

c. 3 (0011) ☞ 1101

d. -8 (1000) ☞ 1000 (?! If you look at the chart, +8 cannot be represented in two's complement with 4 bits!)

Practice



Convert the following 8-bit numbers from positive to negative, or from negative to positive using two's complement notation:

a. -4 (11111100) ☞ 00000100

b. 27 (00011011) ☞ 11100101

c. -127 (10000001) ☞ 01111111

d. 1 (00000001) ☞ 11111111

Casting Between Signed and Unsigned

Converting between two numbers in C can happen explicitly (using a parenthesized cast), or implicitly (without a cast):

explicit

```
int tx, ty;
unsigned ux, uy;
…
tx = (int) ux;
uy = (unsigned) ty;
```

implicit

```
int tx, ty;
unsigned ux, uy;
…
tx = ux; // cast to signed
uy = ty; // cast to unsigned
```

When casting: **the underlying bits do not change**, so there isn't any conversion going on, except that the variable is treated as the type that it is. You cannot convert a signed number to its unsigned counterpart using a cast!

Casting Between Signed and Unsigned

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```c
// test_cast.c
#include<stdio.h>
#include<stdlib.h>

int main() {
    int v = -12345;
    unsigned int uv = (unsigned int) v;

    printf("v = %d, uv = %u\n",v,uv);

    return 0;
}
```

```
$ ./test_cast
v = -12345, uv = 4294954951
```

Casting Between Signed and Unsigned

`printf` has three 32-bit integer representations:

`%d` : signed 32-bit int
`%u` : unsigned 32-bit int
`%x` : hex 32-bit int

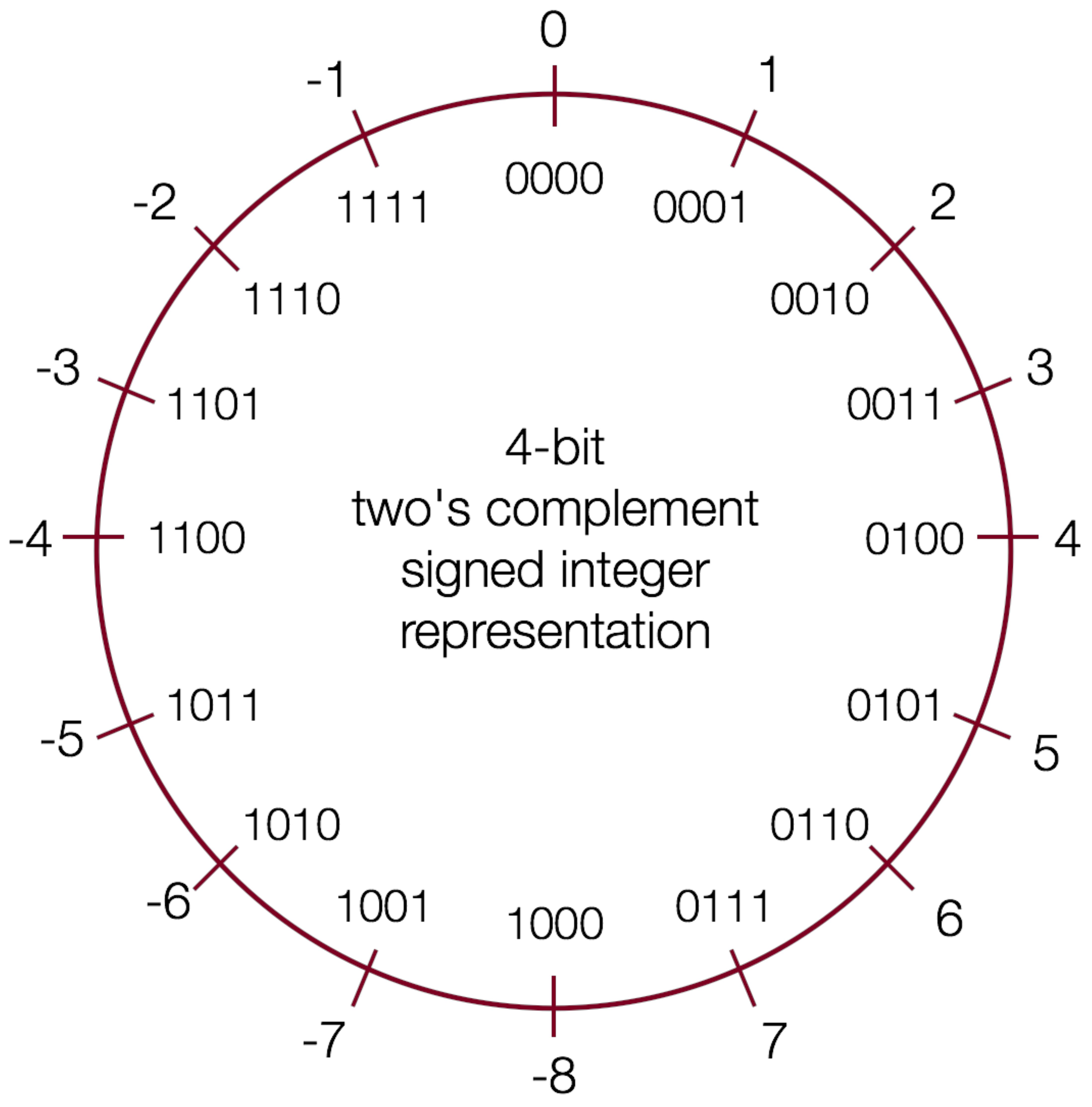
As long as the value is a 32-bit type, printf will treat it according to the formatter it is applying:

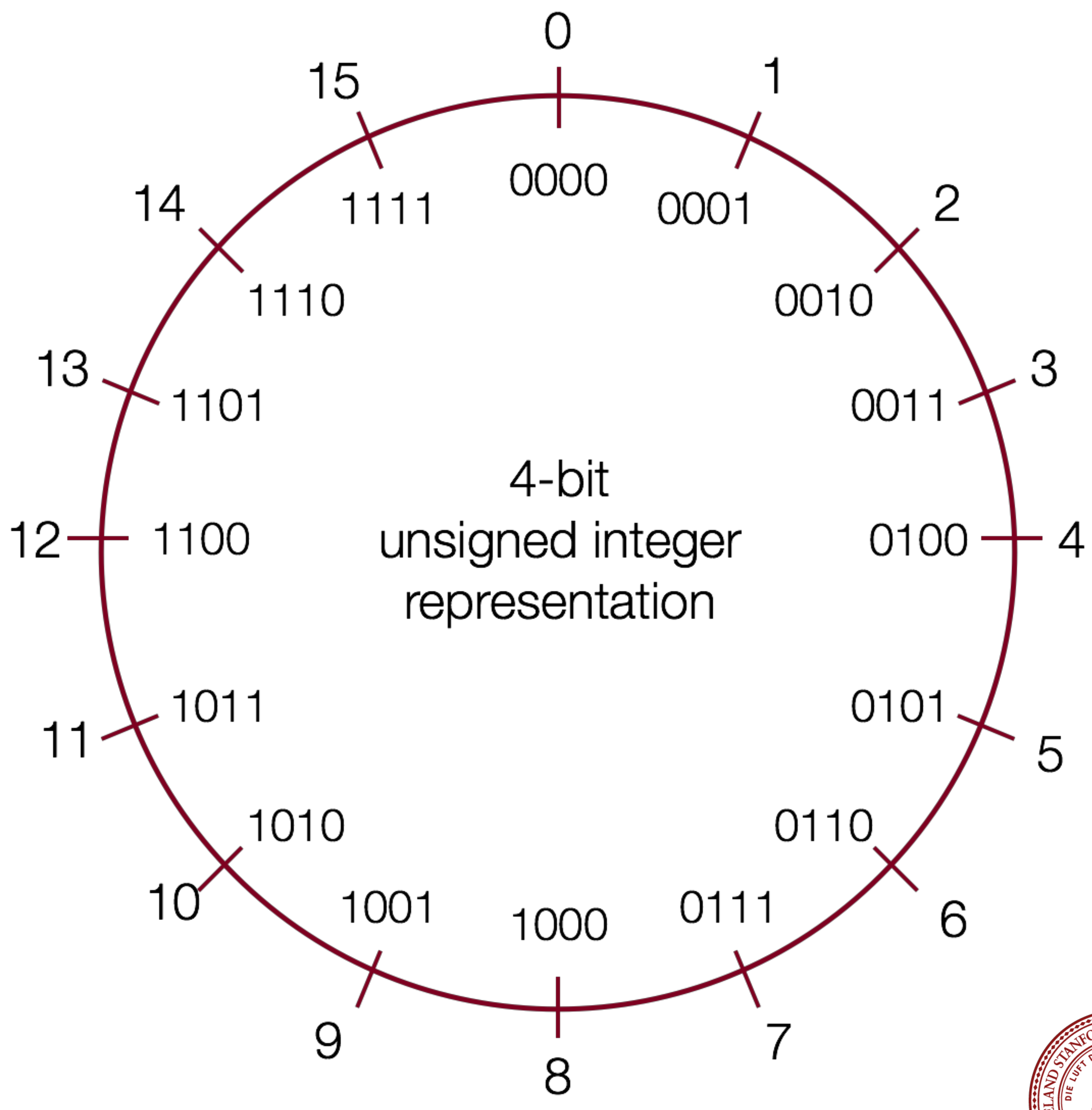
```
int x = -1;
unsigned u = 3000000000; // 3 billion

printf("x = %u = %d\n", x, x);
printf("u = %u = %d\n", u, u);
```

```
$ ./test_printf
x = 4294967295 = -1
u = 3000000000 = -1294967296
```

Signed vs Unsigned Number Wheels



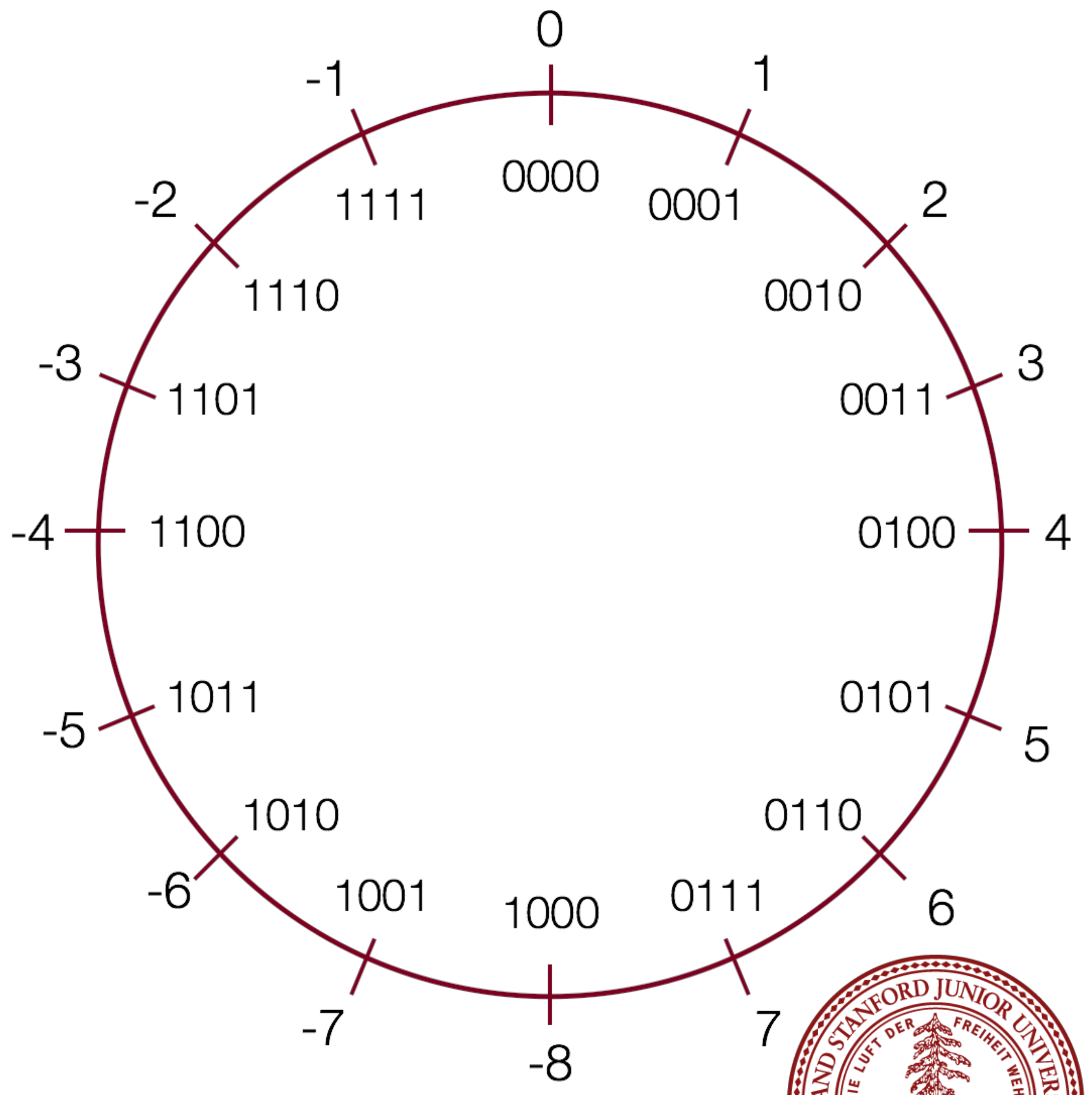


Comparison between signed and unsigned integers

When a C expression has combinations of signed and unsigned variables, you need to be careful!

If an operation is performed that has both a signed and an unsigned value, **C implicitly casts the signed argument to unsigned** and performs the operation assuming both numbers are non-negative. Let's take a look…

Expression	Type	Evaluation
`0 == 0U`		
`-1 < 0`		
`-1 < 0U`		
`2147483647 > -2147483647 - 1`		
`2147483647U > -2147483647 - 1`		
`2147483647 > (int)2147483648U`		
`-1 > -2`		
`(unsigned)-1 > -2`		

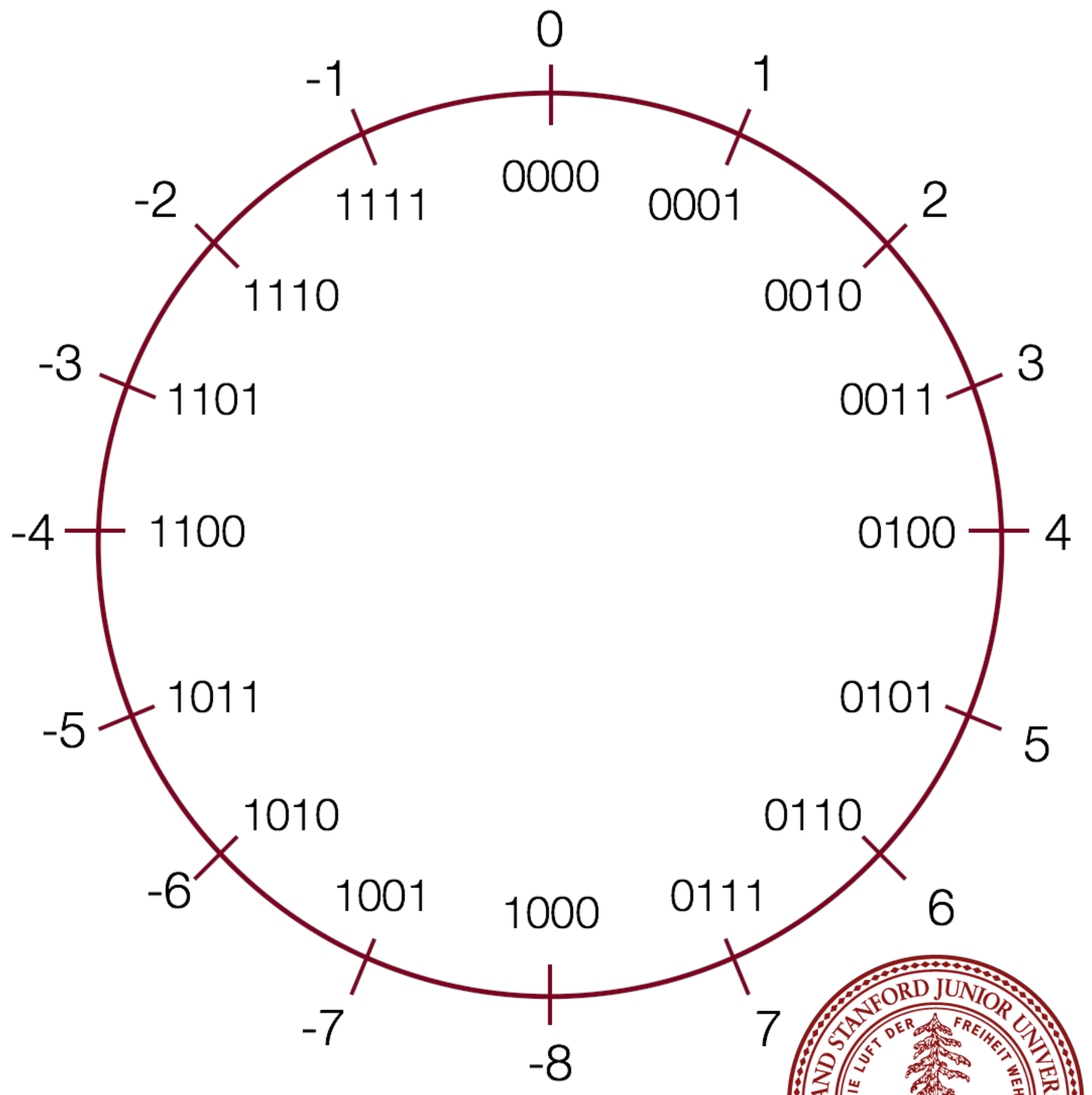


Comparison between signed and unsigned integers

When a C expression has combinations of signed and unsigned variables, you need to be careful!

If an operation is performed that has both a signed and an unsigned value, **C implicitly casts the signed argument to unsigned** and performs the operation assuming both numbers are non-negative. Let's take a look…

Expression	Type	Evaluation
`0 == 0U`	Unsigned	1
`-1 < 0`	Signed	1
`-1 < 0U`	Unsigned	0
`2147483647 > -2147483647 - 1`	Signed	1
`2147483647U > -2147483647 - 1`	Unsigned	0
`2147483647 > (int)2147483648U`	Signed	1
`-1 > -2`	Signed	1
`(unsigned)-1 > -2`	Unsigned	1



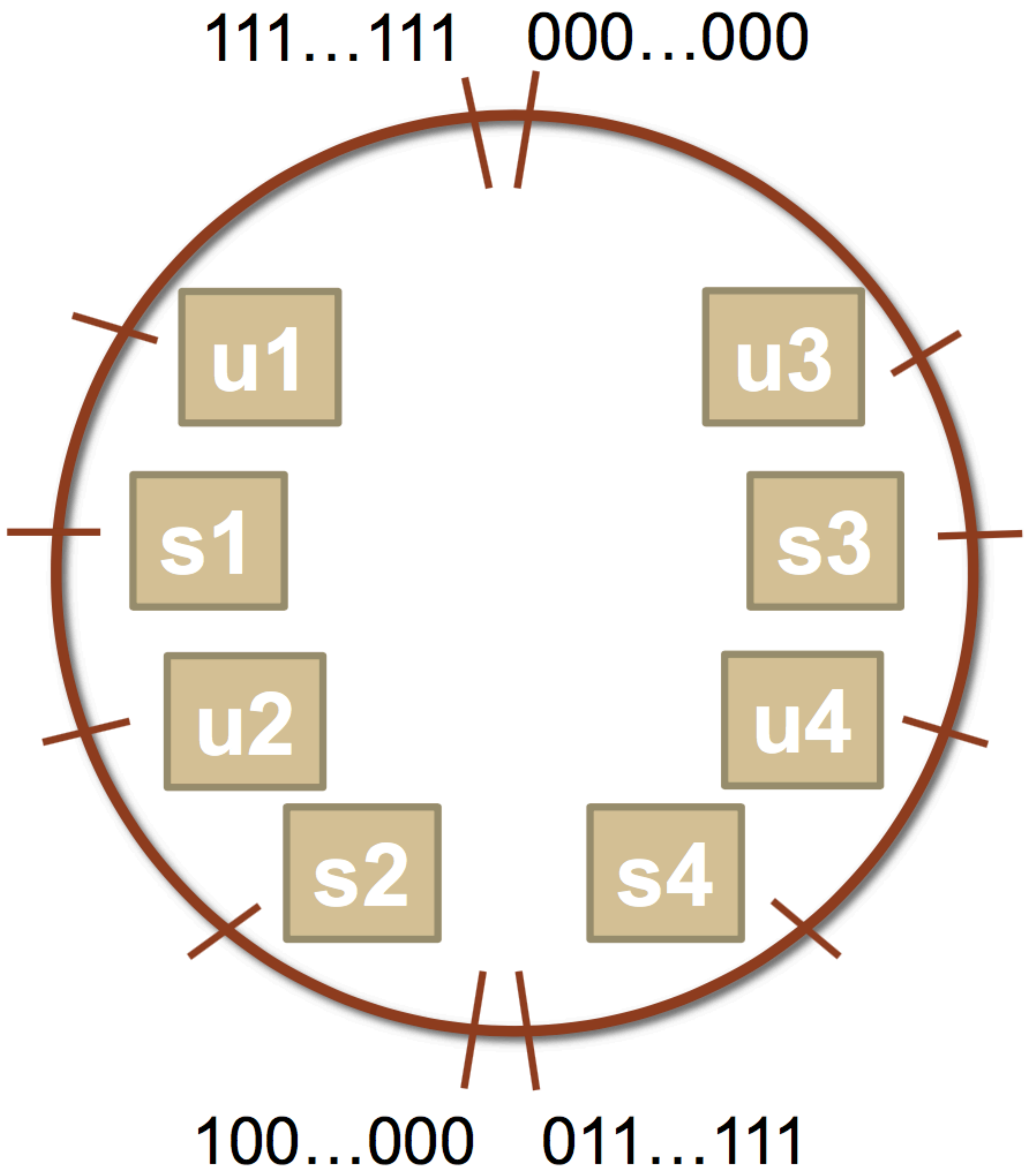
Comparison between signed and unsigned integers

Let's try some more…a bit more abstractly.

```
int s1, s2, s3, s4;
unsigned int u1, u2, u3, u4;
```

Which many of the following statements are true? *(assume that variables are set to values that place them in the spots shown)*

```
s3 > u3
u2 > u4
s2 > s4
s1 > s2
u1 > u2
s1 > u3
```



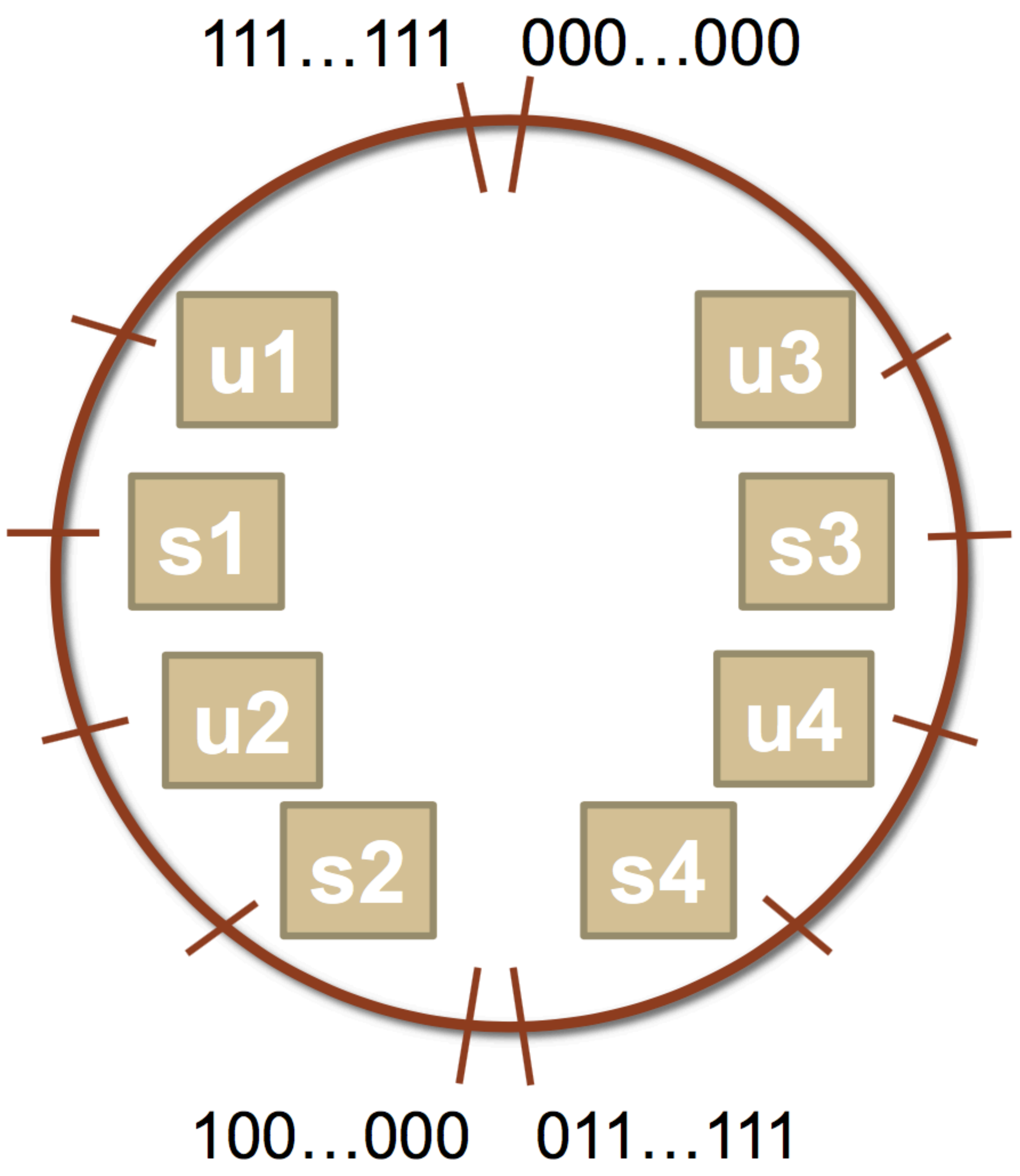
Comparison between signed and unsigned integers

Let's try some more…a bit more abstractly.

```
int s1, s2, s3, s4;
unsigned int u1, u2, u3, u4;
```

Which many of the following statements are true? *(assume that variables are set to values that place them in the spots shown)*

```
s3 > u3 : true
u2 > u4 : true
s2 > s4 : false
s1 > s2 : true
u1 > u2 : true
s1 > u3 : true
```



The `sizeof` Operator

As we have seen, integer types are limited by the number of bits they hold. On the 64-bit myth machines, we can use the **`sizeof`** operator to find how many bytes each type uses:

```
int main() {
    printf("sizeof(char): %d\n", (int) sizeof(char));
    printf("sizeof(short): %d\n", (int) sizeof(short));
    printf("sizeof(int): %d\n", (int) sizeof(int));
    printf("sizeof(unsigned int): %d\n", (int) sizeof(unsigned int));
    printf("sizeof(long): %d\n", (int) sizeof(long));
    printf("sizeof(long long): %d\n", (int) sizeof(long long));
    printf("sizeof(size_t): %d\n", (int) sizeof(size_t));
    printf("sizeof(void *): %d\n", (int) sizeof(void *));
    return 0;
}
```

```
$ ./sizeof
sizeof(char): 1
sizeof(short): 2
sizeof(int): 4
sizeof(unsigned int): 4
sizeof(long): 8
sizeof(long long): 8
sizeof(size_t): 8
sizeof(void *): 8
```

Type	Width in bytes	Width in bits
char	1	8
short	2	16
int	4	32
long	8	64
void *	8	64

MIN and MAX values for integers

Because we now know how bit patterns for integers works, we can figure out the maximum and minimum values, designated by **`INT_MAX`**, **`UINT_MAX`**, **`INT_MIN`**, (etc.), which are defined in **`limits.h`**

Type	**Width (bytes)**	**Width (bits)**	**Min in hex (name)**	**Max in hex (name)**
char	1	8	80 (CHAR_MIN)	7F (CHAR_MAX)
unsigned char	1	8	0	FF (UCHAR_MAX)
short	2	16	8000 (SHRT_MIN)	7FFF (SHRT_MAX)
unsigned short	2	16	0	FFFF (USHRT_MAX)
int	4	32	80000000 (INT_MIN)	7FFFFFFF (INT_MAX)
unsigned int	4	32	0	FFFFFFFF (UINT_MAX)
long	8	64	8000000000000000 (LONG_MIN)	7FFFFFFFFFFFFFFF (LONG_MAX)
unsigned long	8	64	0	FFFFFFFFFFFFFFFF (ULONG_MAX)

Expanding the bit representation of a number

Sometimes we want to convert between two integers having different sizes. E.g., a `short` to an `int`, or an `int` to a `long`.

We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a smaller data type to a bigger data type.

This is easy for unsigned values: simply add leading zeros to the representation (called "zero extension").

```
unsigned short s = 4;
// short is a 16-bit format, so                   s = 0000 0000 0000 0100b

unsigned int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

Expanding the bit representation of a number

For signed values, we want the number to remain the same, just with more bits. In this case, we perform a "sign extension" by repeating the sign of the value for the new digits. E.g.,

```
short s = 4;
// short is a 16-bit format, so                     s = 0000 0000 0000 0100b

int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

— or —

```
short s = -4;
// short is a 16-bit format, so                     s = 1111 1111 1111 1100b

int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 1111 1100b
```

Sign-extension Example

```
// show_bytes() defined on pg. 45, Bryant and O'Halloran
int main() {
    short sx = -12345;          // -12345
    unsigned short usx = sx; // 53191
    int x = sx;                 // -12345
    unsigned ux = usx;          // 53191

    printf("sx = %d:\t", sx);
    show_bytes((byte_pointer) &sx, sizeof(short));
    printf("usx = %u:\t", usx);
    show_bytes((byte_pointer) &usx, sizeof(unsigned short));
    printf("x  = %d:\t", x);
    show_bytes((byte_pointer) &x, sizeof(int));
    printf("ux = %u:\t", ux);
    show_bytes((byte_pointer) &ux, sizeof(unsigned));

    return 0;
}
```

```
$ ./sign_extension
sx = -12345:   c7 cf
usx = 53191:   c7 cf
x   = -12345:  c7 cf ff ff
ux = 53191:    c7 cf 00 00
```

(careful: this was printed on the little-endian myth machines!)

Back to right shift: arithmetic -vs- logical

The right-shift (>>) operator behaves differently for unsigned and signed numbers:

- **Unsigned** numbers are **logically**-right shifted (by shifting in 0s, always)
- **Signed** numbers are **arithmetically**-right shifted (by shifting in the sign bit)

```
$ ./right_shift
a = 1048576:        00 00 10 00
a >> 8 = 4096:      00 10 00 00
b = -1048576:       00 00 f0 ff
b >> 8 = -4096:     00 f0 ff ff
```

(run on a little-endian machine)

```
// show_bytes() defined on pg. 45, Bryant and O'Halloran
int main() {
    int a = 1048576;
    int a_rs8 = a >> 8;

    int b = -1048576;
    int b_rs8 = b >> 8;

    printf("a = %d:\t", a);
    show_bytes((byte_pointer) &a, sizeof(int));

    printf("a >> 8 = %d:\t", a_rs8);
    show_bytes((byte_pointer) &a_rs8, sizeof(int));

    printf("b = %d:\t", b);
    show_bytes((byte_pointer) &b, sizeof(int));

    printf("b >> 8 = %d:\t", b_rs8);
    show_bytes((byte_pointer) &b_rs8, sizeof(int));
    return 0;
}
```

Truncating Numbers: Signed

What if we want to reduce the number of bits that a number holds? E.g.

```
int x = 53191;
short sx = (short) x;
int y = sx;
```

What happens here? Let's look at the bits in `x` (a 32-bit `int`), `53191`:

`0000 0000 0000 0000 1100 1111 1100 0111`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1100 1111 1100 0111`

What is this number in decimal? Well, it must be negative (b/c of the initial 1), and it is `-12345`.

Truncating Numbers: Signed

What if we want to reduce the number of bits that a number holds? E.g.

```
int x = 53191;           // 53191
short sx = (short) x;  // -12345
int y = sx;
```

This is a form of *overflow*! We have altered the value of the number. Be careful!

We don't have enough bits to store the int in the short for the value we have in the `int`, so the strange values occur.

What is y above? We are converting a short to an int, so we sign-extend, and we get -12345!

`1100 1111 1100 0111` becomes

`1111 1111 1111 1111 1100 1111 1100 0111`

Play around here: http://www.convertforfree.com/twos-complement-calculator/

Truncating Numbers: Signed

If the number does fit into the smaller representation in the current form, it will convert just fine.

```
int x = -3;           // -3
short sx = (short) -3; // -3
int y = sx; // -3
```

x: `1111 1111 1111 1111 1111 1111 1111 1101` becomes

sx: `1111 1111 1111 1101`

Play around here: http://www.convertforfree.com/twos-complement-calculator/

Truncating Numbers: Unsigned

We can also lose information with unsigned numbers:

```
unsigned int x = 128000;
unsigned short sx = (short) x;
unsigned int y = sx;
```

Bit representation for x = 128000 (32-bit unsigned int):

```
0000 0000 0000 0001 1111 0100 0000 0000
```

Truncated unsigned short sx:

```
                    1111 0100 0000 0000
```

which equals 62464 decimal.

Converting back to an unsigned int, y = 62464

Overflow in Unsigned Addition

When integer operations overflow in C, the runtime does not produce an error:

```
#include<stdio.h>
#include<stdlib.h>
#include<limits.h> // for UINT_MAX

int main() {
    unsigned int a = UINT_MAX;
    unsigned int b = 1;
    unsigned int c = a + b;

    printf("a = %u\n",a);
    printf("b = %u\n",b);
    printf("a + b = %u\n",c);

    return 0;
}
```

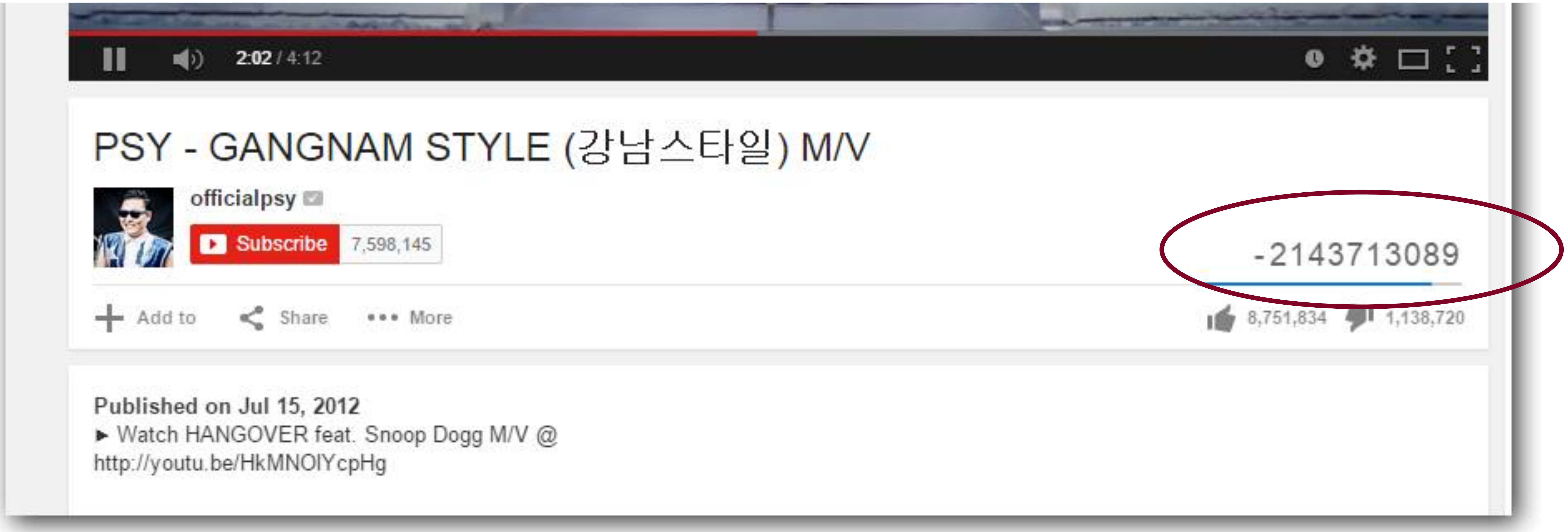
```
$ ./unsigned_overflow
a = 4294967295
b = 1
a + b = 0
```

Technically, unsigned integers in C don't overflow, they just wrap. You need to be aware of the size of your numbers. Here is one way to test if an addition will fail:

```
// for addition
#include <limits.h>
unsigned int a = <something>;
unsigned int x = <something>;
if (a > UINT_MAX - x) /* `a + x` would overflow */;
```

Overflow in Signed Addition

Signed overflow wraps around to the negative numbers:



YouTube fell into this trap — their view counter was a signed, 32-bit int. They fixed it after it was noticed, but for a while, the view count for Gangnam Style (the first video with over `INT_MAX` number of views) was negative.

Overflow in Signed Addition

In the news on January 5, 2022 (!):

ars TECHNICA — BIZ & IT | TECH | SCIENCE | POLICY | CARS | GAMING & CUL

GOOD THING ANDROID IS GREAT AT ROLLING OUT UPDATES —

Google fixes nightmare Android bug that stopped user from calling 911

An integer overflow/underflow crash lets misbehaving apps lock users out of 911.

RON AMADEO - 1/5/2022, 3:09 PM

https://arstechnica.com/gadgets/2022/01/google-fixes-nightmare-android-bug-that-stopped-user-from-calling-911/

Overflow in Signed Addition

Signed overflow wraps around to the negative numbers.

```
#include<stdio.h>
#include<stdlib.h>
#include<limits.h> // for INT_MAX

int main() {
    int a = INT_MAX;
    int b = 1;
    int c = a + b;

    printf("a = %d\n",a);
    printf("b = %d\n",b);
    printf("a + b = %d\n",c);

    return 0;
}
```

```
$ ./signed_overflow
a = 2147483647
b = 1
a + b = -2147483648
```

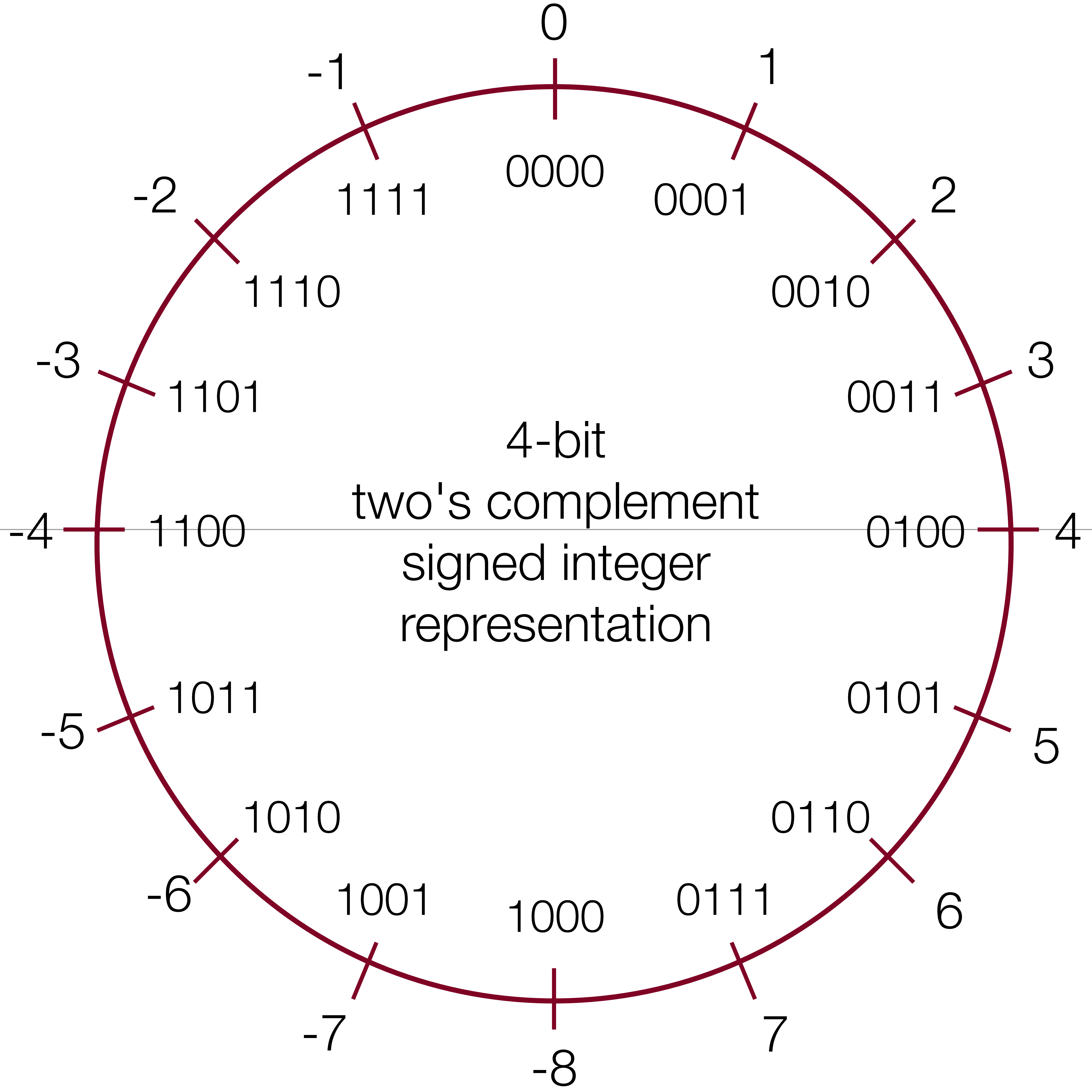
Technically, signed integers in C *produce undefined behavior* when they overflow. On two's complement machines (virtually all machines these days), it does overflow predictably. You can test to see if your addition will be correct:

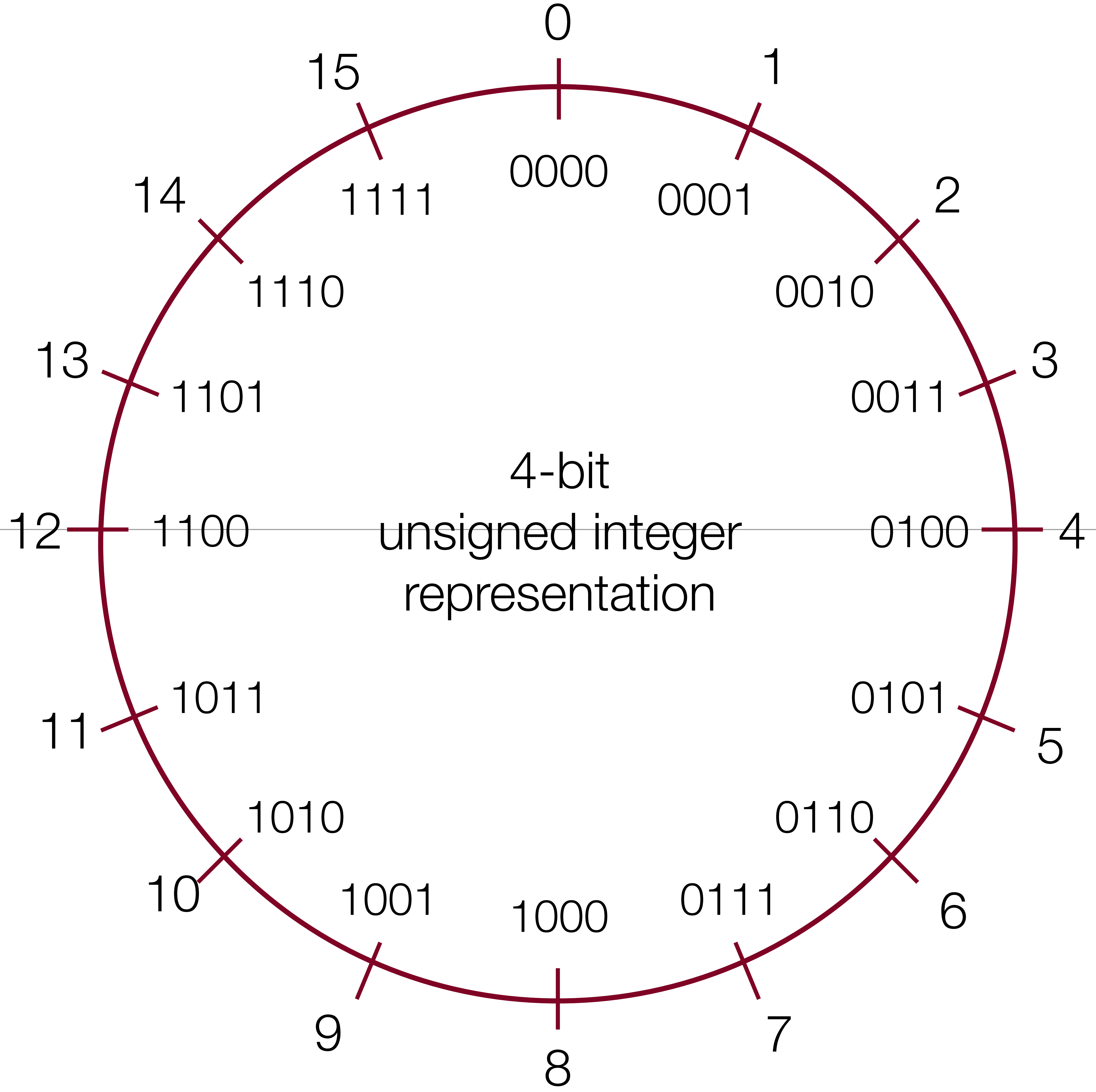
```
// for addition
#include <limits.h>
int a = <something>;
int x = <something>;
if ((x > 0) && (a > INT_MAX - x)) /* `a + x` would overflow */;
if ((x < 0) && (a < INT_MIN - x)) /* `a + x` would underflow */;
```

References and Advanced Reading

- **References:**
  - Two's complement calculator: http://www.convertforfree.com/twos-complement-calculator/
  - Wikipedia on Two's complement: https://en.wikipedia.org/wiki/Two%27s_complement
  - The `sizeof` operator: http://www.geeksforgeeks.org/sizeof-operator-c/

- **Advanced Reading:**
  - Signed overflow: https://stackoverflow.com/questions/16056758/c-c-unsigned-integer-overflow
  - Integer overflow in C: https://www.gnu.org/software/autoconf/manual/autoconf-2.62/html_node/Integer-Overflow.html
  - https://stackoverflow.com/questions/34885966/when-an-int-is-cast-to-a-short-and-truncated-how-is-the-new-value-determined


4-bit
two's complement
signed integer
representation
0
0000
1
0001
2
0010
3
0011
4
0100
5
0101
6
0110
7
0111
-8
1000
-7
1001
-6
1010
-5
1011
-4
1100
-3
1101
-2
1110
-1
1111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0
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7
0111
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1000
9
1001
10
1010
11
1011
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1100
13
1101
14
1110
15
1111