# CS107 Lecture 5 Bitwise Operators, Continued 

reading:
Bryant \& O'Hallaron, Ch. 2.1

## CS107 Topic 1

## How can a computer represent integer numbers?

Why is answering this question important?

- Helps us understand the limitations of computer arithmetic (last week)
- Shows us how to more efficiently perform arithmetic (today)
- Shows us how we can encode data more compactly and efficiently (last time)
assign1: implement 3 programs that manipulate binary representations to (1) work around the limitations of arithmetic with addition, (2) simulate a chamber of gas particles, and (3) print Unicode text to the terminal.


## Learning Goals

- Learn about the bit shift operators
- Understand when to use one bitwise operator vs. another in your program
- Get practice with writing programs that manipulate binary representations


## Lecture Plan

- Recap: Bit Operators so far
- Bit Shift Operators
- Example: Powers of 2
- Demo: GDB


## Lecture Plan

- Recap: Bit Operators so far
- Bit Shift Operators
- Example: Powers of 2
- Demo: GDB


## Bits and Bytes So Far

> // 1. Data is really stored in binary
> int $x=5 ; ~ / / ~ r e a l l y ~ 0 b 00 . . .0101 ~ i n ~ m e m o r y!~$
// 2. We know what that binary representation is for integers int $y=-5$; // two’s complement: 0b111...11011
// 3. We can use/manipulate a binary representation with bit ops $x$ |= 0x2; // turn on the 2nd bit from the right: 0b00...0111
// 4. A variable and its binary representation are // one and the same printf("\%d\n", x); // prints 7!

## Bitwise OR (I)

| with 1 is useful for turning select bits on.
int $x=5 ; / / 0 b 101$
// Turn on the $2^{\text {nd }}$ bit from the right x |= 0x2; // 0b111
| is useful for taking the union of bits.
int $x=5 ; \quad / / 0 b 00101$
int $y=26 ; \quad / / 0 b 11010$
int $z=x$ | y; // 0b11111
printf("\%d\n", z); // 31

## Bitwise AND (\&)

\& with 0 is useful for turning select bits off.
int $x$ = 5; // 0b101
// Turn off the 3rd bit from the right $x \quad \&=-5 ; / /-5$ is 0b111... 1011
\& is useful for taking the intersection of bits.
int $x=21 ; \quad / / 0 b 10101$
int $y=27 ; \quad / / 0 b 11011$
int $z=x \& y ; / / 0 b 10001$
printf("\%d\n", z); // 17

## Bitwise XOR (^)

^ with 1 is useful for flipping select bits.
int $x=5 ; / / 0 b 101$
// Flip the $2^{\text {nd }}$ bit from the right $x^{\wedge}=0 x 2 ; ~ / / ~ 0 b 111$

## Bitwise NOT (~)

## ~ is useful for flipping all bits.

int x = 5; // 0b101
// Flip all bits
x = ~x; // 0b11111...1010, which is -6
// Take two's complement (same as negating) int $y=\sim x+1 ; ~ / / ~ s a m e ~ a s ~-x ~$

## Bit Vectors and Sets

Instead of using arrays of e.g., Booleans in our programs, sometimes it's beneficial to store that information in bits instead - more compact.

- Example: we can represent current courses taken using a char and manipulate its contents using bit operators.

| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |

## Bit Vectors and Sets

| \#define CS106A 0x1 | /* 00000001 |
| :---: | :---: |
| \#define CS106B 0x2 | /* 00000010 |
| \#define CS107E 0x4 | /* 00000100 |
| \#define CS107 0x8 | /* 00001000 |
| \#define CS111 0x10 | /* 00010000 |
| \#define CS103 0x20 | /* 00100000 |
| \#define CS109 0x40 | /* 01000000 |
| \#define CS161 0x80 | /* 10000000 |
| char myClasses = ...; |  |
| myClasses \|= CS107; // Add CS107 |  |
| if (myClasses \& CS1 | ) \{ |
| // taken CS106B |  |

## Practice: Bit Masking

Practice: write an expression that, given a 32-bit integer j, flips ("complements") the least-significant byte, and preserves all other bytes.

1. What operator is good for flipping certain bits?
2. What mask do we want?
3. How do we create that mask?
j ^ 0xff

## Lecture Plan

## - Recap: Bit Operators so far

- Bit Shift Operators
- Example: Powers of 2
- Demo: GDB


## Left Shift (<<)

The LEFT SHIFT operator shifts a bit pattern a certain number of positions to the left. New lower order bits are filled in with 0 s , and bits shifted off the end are lost.

```
x << k; // evaluates to x shifted to the left by k bits
x <<= k; // shifts x to the left by k bits
```

8-bit examples:

```
0 0 1 1 0 1 1 1 ~ \ll ~ 2 ~ r e s u l t s ~ i n ~ 1 1 0 1 1 1 0 0 ~
01100011 << 4 results in 00110000
10010101 << 4 results in 01010000
```


## Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

$$
\begin{array}{ll}
\mathbf{x} \gg \mathbf{k} ; & / / \text { evaluates to } x \text { shifted to the right by } k \text { bits } \\
\mathbf{x} \gg=\mathbf{k} ; & / / \text { shifts } x \text { to the right by } k \text { bits }
\end{array}
$$

Question: how does it fill in the new higher-order bits?

## Right Shift (>>)

There are two kinds of right shifts, depending on the value and type you are shifting:

- Unsigned numbers are right-shifted by filling new high-order bits with Os ("logical right shift").
- Signed numbers are right-shifted by filling new high-order bits with the most significant bit ("arithmetic right shift").

This way, the sign of the number (if applicable) is preserved!

## Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

$$
\begin{aligned}
& \text { x >> k; // evaluates to } x \text { shifted to the right by k bit } \\
& \mathbf{x} \boldsymbol{\gg =} \mathbf{k ;} \text { // shifts } x \text { to the right by k bits }
\end{aligned}
$$

unsigned short x = 2; // 0000000000000010
x >>= 1; // 0000000000000001 printf("\%u\n", x); // 1

## Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

$$
\begin{aligned}
& \text { x >> k; // evaluates to } x \text { shifted to the right by k bit } \\
& \mathbf{x} \boldsymbol{\gg =} \mathbf{k ;} \text { // shifts } x \text { to the right by k bits }
\end{aligned}
$$

```
short x = 2; // 0000 0000 0000 0010
x >>= 1; // 0000 0000 0000 0001
printf("%d\n", x); // 1
```


## Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

$$
\begin{aligned}
& \text { x >> k; // evaluates to } x \text { shifted to the right by k bit } \\
& \mathbf{x} \gg=\text { k; // shifts } x \text { to the right by k bits }
\end{aligned}
$$

```
short x = -2; // 1111 1111 1111 1110
x >>= 1; // 1111 1111 1111 1111
printf("%d\n", x); // -1
```


## Shifting and Masking

Suppose we have a 32-bit number. int $x=0 b 1010010 ;$
How can we use bit operators to design a mask that turns on the i-th bit of a number for any $i(0,1,2, \ldots, 31)$ ?

1. What operator is good for turning on certain bits?
2. What mask do we want?
3. How do we create that mask?

Respond on PollEv for \#3: pollev.com/cs107 or text CS107 to 22333 once to join.


What mask would help us turn on the i-th bit of a number?

Nobody has responded yet.
Hang tight! Responses are coming in.

## Shifting and Masking

Suppose we have a 32-bit number.
int x = 0b1010010;

How can we use bit operators to design a mask that turns on the i-th bit of a number for any $i(0,1,2, \ldots, 31)$ ?

1. What operator is good for turning on certain bits?
2. What mask do we want?
3. How do we create that mask?
$x \mid(1 \ll i)$

## Shifting and Masking

Suppose we have a 32-bit number.
int x = 0b1010010;

How can we use bit operators to design a mask that turns on the i-th bit of a number for any i ( $0,1,2, \ldots, 31$ )?
$x \mid(1 \ll i)$

What if x is a 64 -bit number (e.g. long) and i could be $0-63$ ? It turns out there's a problem with this expression...

## Bit Operator Pitfalls

- The default type of a number literal in your code is an int.
- Let's say you want a long with the index-32 bit as 1 :
long num = 1 << 32;
- This doesn't work! 1 is by default an int, and you can't shift an int by 32 because it only has 32 bits. You must specify that you want 1 to be a long.
long num $=1 \underline{L}$ << 32;


## Shifting and Masking

Suppose we have a 64-bit number.
long x = 0b1010010;

How can we use bit operators to design a mask that turns on the i-th bit of a number for any $i(0,1,2, \ldots, 63)$ ?
$x \mid(1 L \ll i)$

## Number Literal Suffixes

$\mathbf{U}$ makes a literal unsigned, and $\mathbf{L}$ makes a literal a long.

```
int w = -5 >> 1; // 0b1111...1101, -5
int x = -5U >> 1; // 0b0111...1101, 2147483645
int y = 1 << 32; // 0! (technically undefined)
int z = 1L << 32; // 4294967296
```


## $1 L \ll i$

## What does $\mathbf{1 L}$ << i represent numerically?

A power of 2! Specifically, $2^{i}$.

## Lecture Plan

## - Recap: Bit Operators so far

## - Bit Shift Operators

- Example: Powers of 2


## - Demo: GDB

## Powers of 2

## Challenge: without using loops or math library functions, how could we detect whether a number is a power of 2 ?

What is true about a power of 2 but not other numbers?

## Powers of 2

Key idea: A power of 2 minus 1 will have all bits below the original bit be 1 , and everything else be 0. E.g.

Ob10000-1 = $0 b 01111$
0b100-1 = 0b011

Not true for other non-power-of-2 numbers:
Ob10010-1 = 0b10001

Cool idea: no bits overlap between a power of 2 and a power of 2 minus 1 . How is this handy?

## Demo: Powers of 2



## Lecture Plan

## - Recap: Bit Operators so far

## - Bit Shift Operators

## - Examnle: Powers of 2

- Demo: GDB


## Introducing GDB

Is there a way to step through the execution of a program and print out its
values as it's running? E.g., to view binary representations? Yes!

## The GDB Debugger

GDB is a command-line debugger, a text-based debugger with similar functionality to other debuggers you may have used, such as in Qt Creator

- It lets you put breakpoints at specific places in your program to pause there
- It lets you step through execution line by line
- It lets you print out values of variables in various ways (including binary)
- It lets you track down where your program crashed
- And much, much more!

GDB is essential to your success in CS107 this quarter! We'll be building our familiarity with GDB over the course of the quarter.
GDB Guide: cs107.stanford.edu/resources/gdb.html

## gdb on a program

-gdb myprogram
-b
run gdb on executable Set breakpoint on a function (e.g., b main) or line (b 42)
Run with provided args control forward execution (next, step into, continue) print variable ( $p$ varname) or evaluated expression (p 3L << 10)

- p/t, p/x binary and hex formats.
- p/d, p/u, p/c
- info

Important: gdb does not run the current line until you hit "next"

## Demo: Bitmasks and GDB



## gdb: highly recommended

At this point, setting breakpoints/stepping in gdb may seem like overkill for what could otherwise be achieved by copious printf statements.
However, gdb is incredibly useful for assign1 (and all assignments):

- A fast "C interpreter": $\mathrm{p}+$ <expression>
- Sandbox/try out ideas around bitshift operators, signed/unsigned types, etc.
- Can print values out in binary!
- Once you're happy, then make changes to your C file
- Tip: Open two terminal windows and SSH into myth in both
- Keep one for emacs, the other for gdb/command-line
- Easily reference C file line numbers and variables while accessing gdb
- Tip: Every time you update your C file, make and then rerun gdb.

Gdb takes practice! But the payoff is tremendous! $;$

## Recap

- Recap: Bit Operators so far
- Bit Operators + GDB Demo: Courses
- Demo 2: Practice and Powers of 2
- Bit Shift Operators

Lecture 5 takeaways: We can use bit operators like \&, |, ~, <<, etc. to manipulate the binary representation of values. A number is a bit pattern that can be manipulated arithmetically or bitwise at your convenience!

Next time: How can a computer represent and manipulate more complex data like text?

## Extra Practice

## Shift Operation Pitfalls

1. Technically, the C standard does not precisely define whether a right shift for signed integers is logical or arithmetic. However, almost all compilers/machines use arithmetic, and you can most likely assume this.
2. Operator precedence can be tricky! For example:
$1 \ll 2+3 \ll 4$ means $1 \ll(2+3) \ll 4$ because addition and subtraction have higher precedence than shifts! Always use parentheses to be sure:
```
(1<<2) + (3<<4)
```


## Color Wheel

- Another application for storing data efficiently in binary is representing colors.
- A color representation commonly consists of opacity (how transparent or opaque it is), and how much red/green/blue is in the color.
- Key idea: we can encode each of these in 1 byte, in a value from 0-255! Thus, an entire color can be represented in one 4-byte integer.
$0 x 42530144$


## Demo: Color Wheel



## Bit Masking

Bit masking is also useful for integer representations as well. For instance, we might want to check the value of the most-significant bit, or just one of the middle bytes.

- Example: If I have a 32-bit integer $\mathbf{j}$, what operation should I perform if I want to get just the lowest byte in $\mathbf{j}$ ?

```
int j = ...;
int k = j & 0xff; // mask to get just lowest byte
```


## Practice: Bit Masking

- Practice 1: write an expression that, given a 32-bit integer j , sets its leastsignificant byte to all 1 s , but preserves all other bytes.
- Practice 2: write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.


## Practice: Bit Masking

- Practice 1: write an expression that, given a 32-bit integer j, sets its leastsignificant byte to all 1s, but preserves all other bytes.
j | 0xff
- Practice 2: write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.
j ^ ~0xff


## More Exercises

Suppose we have a 64-bit number. long x = 0b1010010;
How can we use bit operators, and the constant 1L or $-1 L$ to...

- ...design a mask that zeros out (i.e., turns off) the bottom i bits (and keeps the rest of the bits the same)?


## More Exercises

Suppose we have a 64-bit number. long x = 0b1010010;
How can we use bit operators, and the constant 1L or $-1 L$ to...

- ...design a mask that zeros out (i.e., turns off) the bottom i bits (and keeps the rest of the bits the same)?
$x \&(-1 L \ll i)$

