CS107 Lecture 4

Bits and Bytes; Bitwise Operators

Reading: Bryant & O’Hallaron, Ch. 2.1

Ed Discussion: https://edstem.org/us/courses/65949/discussion/5374105

This document is copyright (C) Stanford Computer Science, licensed under Creative Commons Attribution 2.5 License. All rights reserved.
Based on slides created by Cynthia Lee, Chris Gregg, Nick Troccoli, Lisa Yan, Jerry Cain and others.

Casting

What happens at the byte level when we cast between variable types? **The bytes remain the same!** **This means they may be interpreted differently depending on the type.**

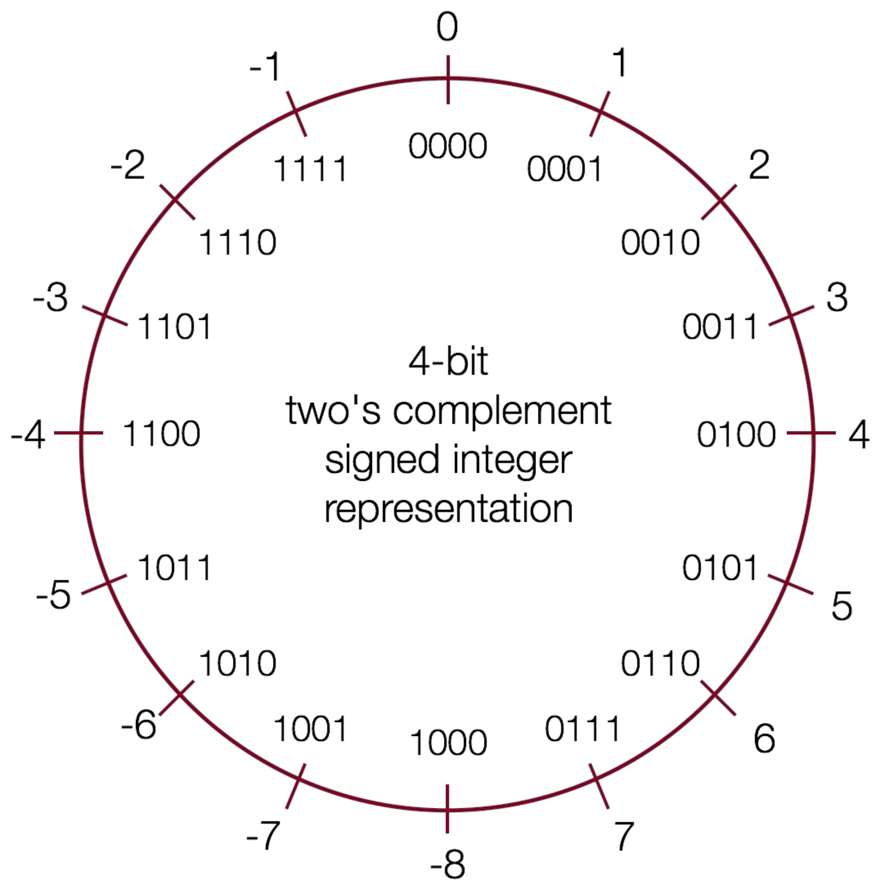
```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);
```

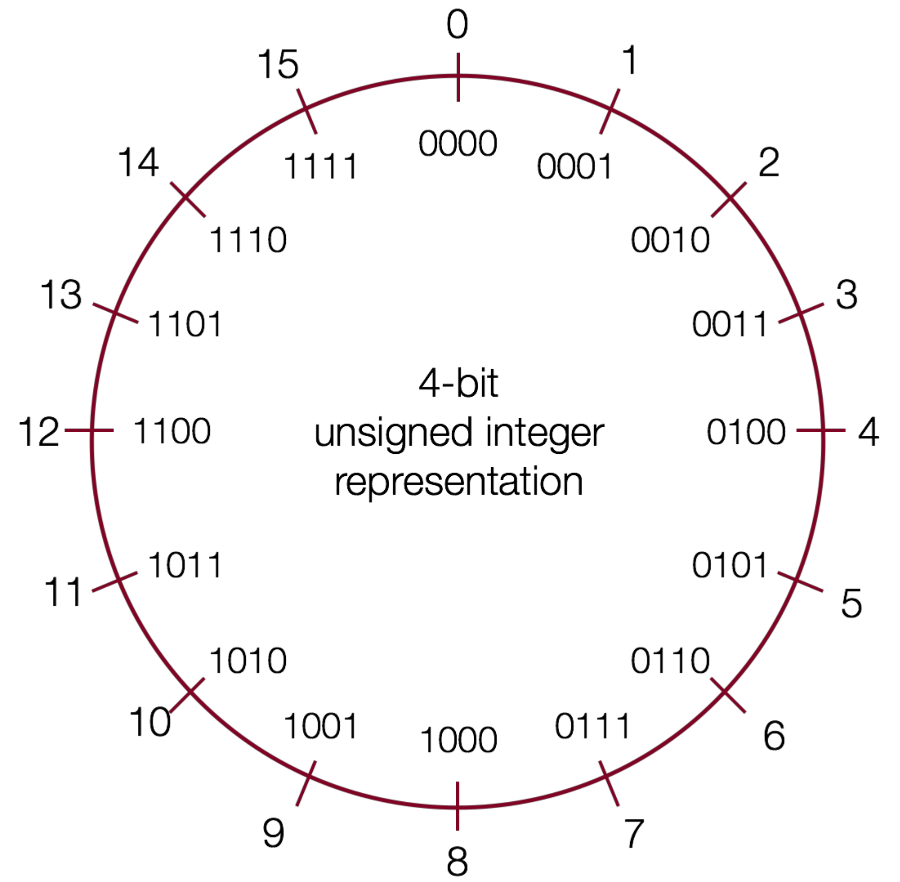
This prints out: `"v = -12345, uv = 4294954951"`. **Why?**

The bit representation for -12345 is
0b**11111111111111111100111111000111.**

If we treat this as an unsigned, inherently positive number, it's *huge*!

Casting





Casting

You can cast something to another type by putting that type in parentheses in front of the value:

```
int v = -12345;
...(unsigned int)v...
```

You can also use the **U** suffix after a number literal to treat it as unsigned:

```
-12345U
```

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

`Expression`	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

`Expression`	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

`Expression`	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

`Expression`	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

`Expression`	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!
`2147483647 > -2147483648`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!
`2147483647 > -2147483648`	Signed	true	yes

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!
`2147483647 > -2147483648`	Signed	true	yes
`2147483647U > -2147483648`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
0 == 0U	Unsigned	true	yes
-1 < 0	Signed	true	yes
-1 < 0U	Unsigned	false	No!
2147483647 > -2147483648	Signed	true	yes
2147483647U > -2147483648	Unsigned	false	No!

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!
`2147483647 > -2147483648`	Signed	true	yes
`2147483647U > -2147483648`	Unsigned	false	No!
`-1 > -2`			
`(unsigned)-1 > -2`			

Comparisons Between Different Types

Be careful when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned and then evaluate the expression assuming both numbers are unsigned and nonnegative.

Expression	Comparison Type?	Evaluates To?	Mathematically correct?
`0 == 0U`	Unsigned	true	yes
`-1 < 0`	Signed	true	yes
`-1 < 0U`	Unsigned	false	No!
`2147483647 > -2147483648`	Signed	true	yes
`2147483647U > -2147483648`	Unsigned	false	No!
`-1 > -2`	Signed	true	yes
`(unsigned)-1 > -2`	Unsigned	true	yes

Expanding Bit Representations

- Sometimes, we need to convert between two integers of different sizes (e.g. **`short`** to **`int`**, or **`int`** to **`long`**).
- We might not be able to convert from a bigger data type to a smaller data type and retain all information, but we should always be able to convert from a **smaller** data type to a **larger** data type.
- For **unsigned** values, we can prepend *leading zeros* to the representation ("zero extension")
- For **signed** values, we can *repeat the sign of the value* for new digits ("sign extension")
- Note: when doing <, >, <=, >= comparison between different size types, it will *promote the smaller type to the larger one*.

Expanding Bit Representation

```
unsigned short s = 4;
// short is a 16-bit format, so                    s = 0000 0000 0000 0100b

unsigned int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

Expanding Bit Representation

```
short s = 4;
// short is a 16-bit format, so                     s = 0000 0000 0000 0100b

int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

— or —

```
short s = -4;
// short is a 16-bit format, so                     s = 1111 1111 1111 1100b

int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 1111 1100b
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

`0000 0000 0000 0000 1100 1111 1100 0111`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1100 1111 1100 0111`

This is -12345! And when we cast sx back an int, we sign-extend the number.

`1111 1111 1111 1111 1100 1111 1100 0111` // still -12345

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

1111 1111 1111 1111 1111 1111 1111 1101

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

1111 1111 1111 1101

This is -3! **If the number does fit, it will convert fine.** y looks like this:

1111 1111 1111 1111 1111 1111 1111 1101 // still -3

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

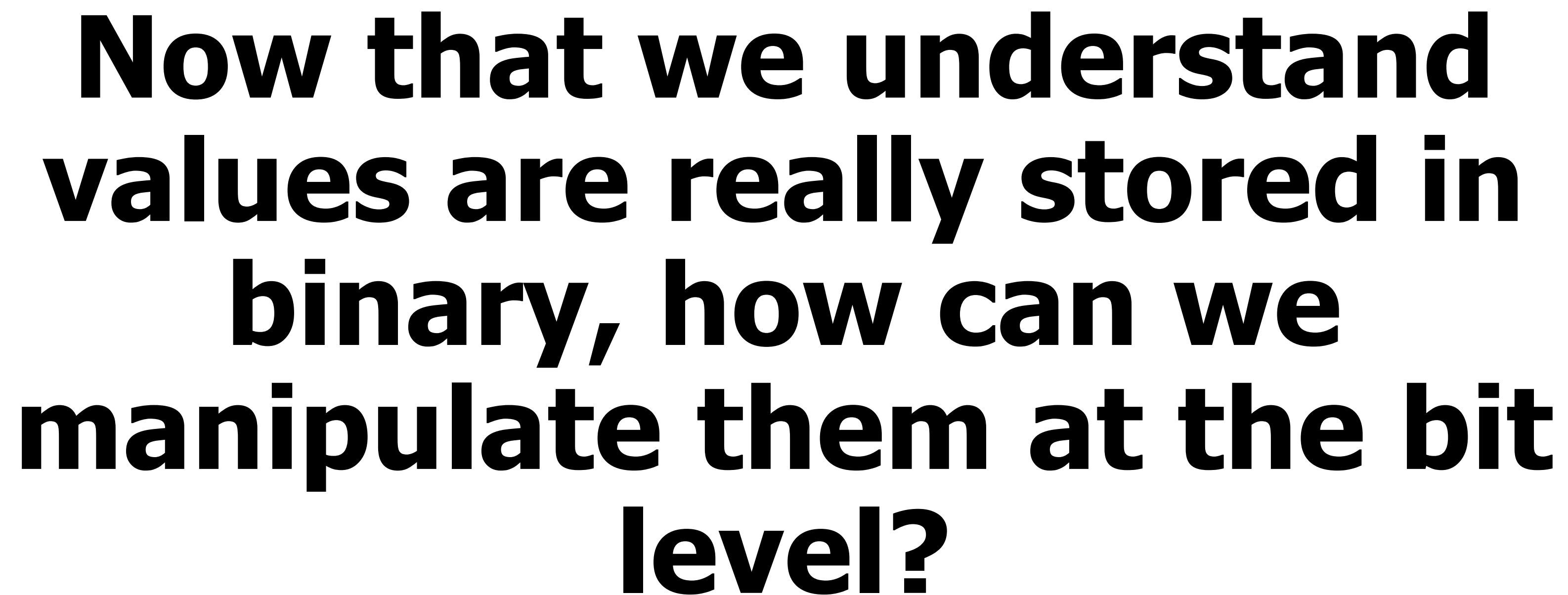
0000 0000 0000 0001 1111 0100 0000 0000

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

1111 0100 0000 0000

This is 62464! **Unsigned numbers can lose info too.** Here is what y looks like:

0000 0000 0000 0000 1111 0100 0000 0000 // still 62464


Now that we understand values are really stored in binary, how can we manipulate them at the bit level?

Bitwise Operators

- You’re already familiar with many operators in C:
  - **Arithmetic operators:** +, -, *, /, %
  - **Comparison operators:** ==, !=, <, >, <=, >=
  - **Logical Operators**: &&, ||, !
- Today and Wednesday, we’ll be discussing a new category of operators: **bitwise operators:**

&, |, ~, ^, <<, >>

And (&)

AND is a binary operator. The AND of 2 bits is 1 if both bits are 1, and 0 otherwise.

```
output = a & b;
```

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

& with 1 to let a bit through, & with 0 to zero out a bit

Or (|)

OR is a binary operator. The OR of 2 bits is 1 if either (or both) bits is 1.

```
output = a | b;
```

a	b	output
0	0	0
0	1	1
1	0	1
1	1	1

| with 1 to turn on a bit, | with 0 to let a bit go through

Not (~)

NOT is a unary operator. The NOT of a bit is 1 if the bit is 0, or 1 otherwise.

```
output = ~a;
```

a	output
0	1
1	0

Exclusive Or (^)

Exclusive Or (XOR) is a binary operator. The XOR of 2 bits is 1 if *exactly* one of the bits is 1, or 0 otherwise.

```
output = a ^ b;
```

a	b	output
0	0	0
0	1	1
1	0	1
1	1	0

^ with 1 to flip a bit, ^ with 0 to let a bit go through unmodified

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
  0110
& 1100
  ----
  0100
```

OR

```
  0110
| 1100
  ----
  1110
```

XOR

```
  0110
^ 1100
  ----
  1010
```

NOT

```
~ 1100
  ----
  0011
```

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND
```
  0110
& 1100
  ----
  0100
```

OR
```
  0110
| 1100
  ----
  1110
```

XOR
```
  0110
^ 1100
  ----
  1010
```

NOT
```
~ 1100
  ----
  0011
```

Note: these are different from the logical operators AND (&&), OR (||) and NOT (!).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND	OR	XOR	NOT
0110 & 1100 ---- 0100	0110 | 1100 ---- 1110	0110 ^ 1100 ---- 1010	~ 1100 ---- 0011

This is different from logical AND (&&). The logical AND returns true if both are nonzero, or false otherwise. With &&, this would be 6 && 12, which would evaluate to **true** (1).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
  0110
& 1100
  ----
  0100
```

OR

```
  0110
| 1100
  ----
  1110
```

XOR

```
  0110
^ 1100
  ----
  1010
```

NOT

```
~ 1100
  ----
  0011
```

This is different from logical OR (||). The logical OR returns true if either are nonzero, or false otherwise. With ||, this would be 6 || 12, which would evaluate to **true** (1).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
  0110
& 1100
  ----
  0100
```

OR

```
  0110
| 1100
  ----
  1110
```

XOR

```
  0110
^ 1100
  ----
  1010
```

NOT

```
~ 1100
  ----
  0011
```

This is different from logical NOT (!). The logical NOT returns true if this is zero, and false otherwise. With !, this would be !12, which would evaluate to **false** (0).

Bitmasks

We will frequently want to manipulate or otherwise isolate specific bits in a larger collection of them. A **bitmask** is a constructed bit pattern that we can use, along with standard bit operators like **&**, **|**, **^**, **~**, **<<**, and **>>**, to do this.

Motivating Example: Bit vectors

Aside: C++ relies on bit vectors to efficiently implement **vector<bool>**.

Bit Vectors and Sets

Instead of using arrays of Booleans, one can more compactly store Boolean information in bits instead.

- **Example:** we can represent current courses taken using a **`char`** and manipulate its contents using bit operators.

0	0	1	0	0	0	1	1
CS161	CS109	CS103	CS111	CS107	CS106AX	CS106B	CS106A

Bit Vectors and Sets

0	0	1	0	0	0	1	1
CS161	CS109	CS103	CS111	CS107	CS106AX	CS106B	CS106A

- How do we find the union of two sets of courses taken? Use OR:

```
  00100011
| 01100001
  --------
  01100011
```

Bit Vectors and Sets

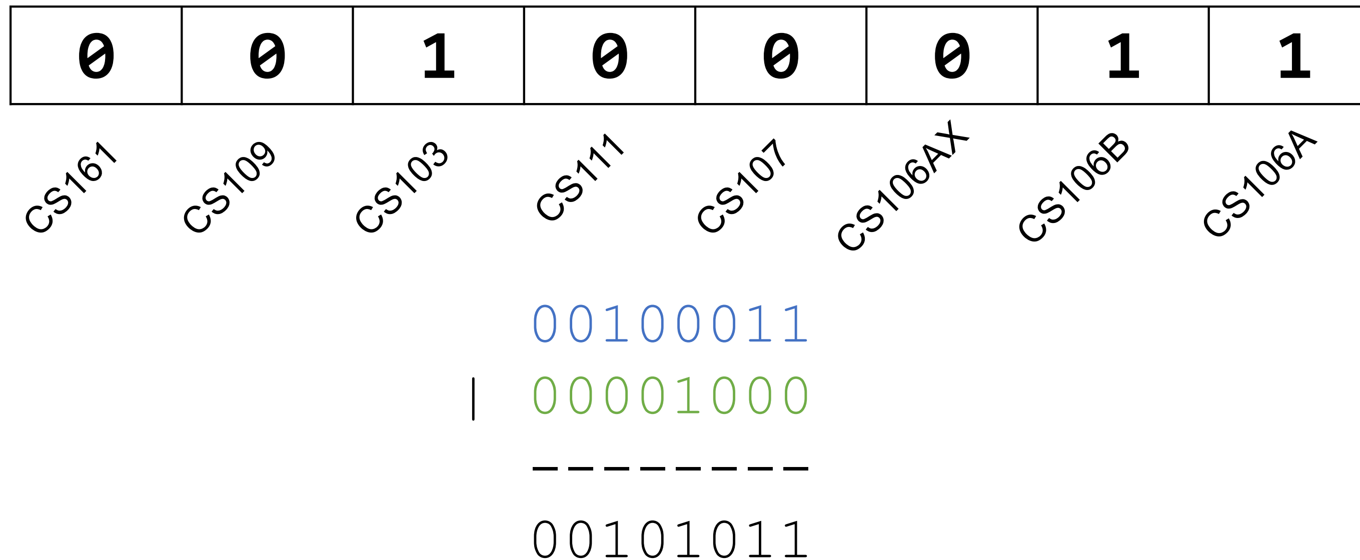
0	0	1	0	0	0	1	1
CS161	CS109	CS103	CS111	CS107	CS106AX	CS106B	CS106A

- How do we find the intersection of two sets of courses taken? Use AND:

```
   00100011
&  01100001
   --------
   00100001
```

Bit Masking

Example: how do we update our bit vector to indicate we've taken CS107?



Bit Masking

```
#define CS106A  0x1    /* 0000 0001 */
#define CS106B  0x2    /* 0000 0010, or 0x1 << 1 */
#define CS106AX 0x4    /* 0000 0100, or 0x1 << 2 */
#define CS107   0x8    /* 0000 1000, or 0x1 << 3 */
#define CS111   0x10   /* 0001 0000, or 0x1 << 4 */
#define CS103   0x20   /* 0010 0000, or 0x1 << 5 */
#define CS109   0x40   /* 0100 0000, or 0x1 << 6 */
#define CS161   0x80   /* 1000 0000, or 0x1 << 7 */

char myClasses = ...;
myClasses = myClasses | CS107;   // include CS107!
```

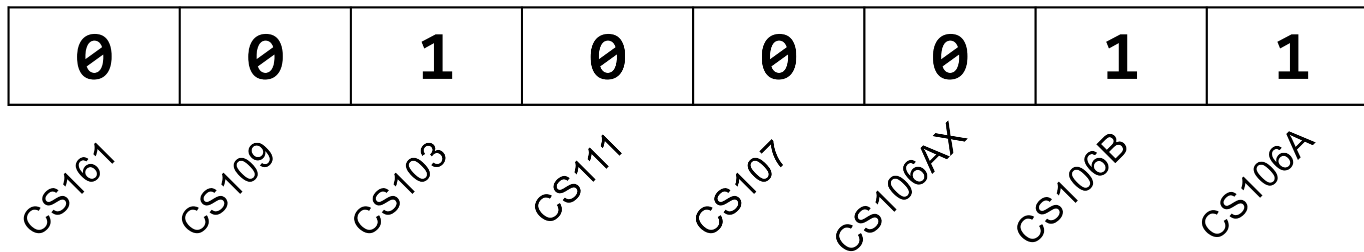
Bit Masking

```
#define CS106A  0x1    /* 0000 0001 */
#define CS106B  0x2    /* 0000 0010, or 0x1 << 1 */
#define CS106AX 0x4    /* 0000 0100, or 0x1 << 2 */
#define CS107   0x8    /* 0000 1000, or 0x1 << 3 */
#define CS111   0x10   /* 0001 0000, or 0x1 << 4 */
#define CS103   0x20   /* 0010 0000, or 0x1 << 5 */
#define CS109   0x40   /* 0100 0000, or 0x1 << 6 */
#define CS161   0x80   /* 1000 0000, or 0x1 << 7 */

char myClasses = ...;
myClasses |= CS107;   // include CS107!
```

Bit Masking

- **Example:** how do we update our bit vector to indicate we've *dropped* CS103?

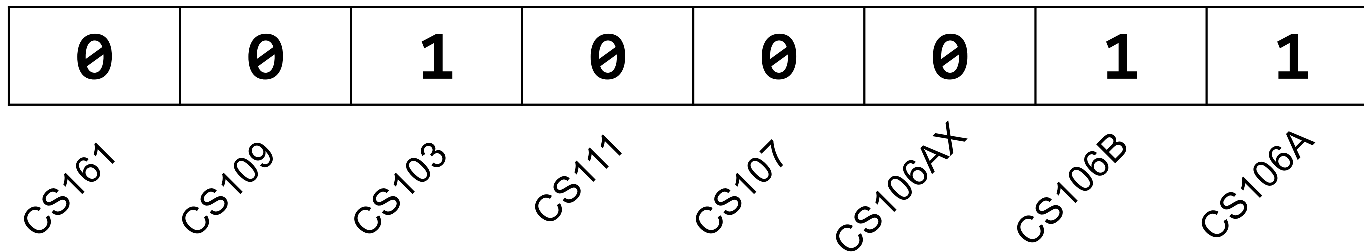



```
  00100011
& 11011111
  --------
  00000011
```

```
char myClasses = ...;
myClasses &= ~CS103;    // Drop CS103
```

Bit Masking

- **Example:** how do we check if we've taken CS106B?




```
  00100011
& 00000010
  --------
  00000010
```

```
char myClasses = ...;
if (myClasses & CS106B) {...
    // taken CS106B!
```

Bitwise Operator Tricks

- | with 1 is useful for turning select bits on
- & with 0 is useful for turning select bits off
- | is useful for taking the union of bits
- & is useful for taking the intersection of bits
- ^ is useful for flipping isolated bits
- ~ is useful for flipping all bits