

# CS107 Lecture 4

## Bits and Bytes; Bitwise Operators

reading:

*Bryant & O'Hallaron, Ch. 2.1*

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Based on slides created by Cynthia Lee, Chris Gregg, Jerry Cain, Lisa Yan and others.

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# Updates / Announcements

- Labs start this week!
  - Learning goals: practice with bit ops, exploring binary representations, GDB practice!
- Assign1 released later today
  - Post-assignment goals: familiarity with bitwise manipulation, working around limitations of computer arithmetic, understanding real-world software impacts of integer representations, practicing debugging strategies with GDB.
  - We are eager to help in helper hours! Focusing on helping you get unstuck to continue making progress, and helping you drive your own debugging. Building familiarity with debugging will pay off tremendously throughout the quarter!

# CS107 Topic 1

## How can a computer represent integer numbers?

Why is answering this question important?

- Helps us understand the limitations of computer arithmetic (last time)
- Shows us how to more efficiently perform arithmetic (today)
- Shows us how we can encode data more compactly and efficiently (today)

**assign1:** implement 3 programs that manipulate binary representations to (1) work around the limitations of arithmetic with addition, (2) simulate an evolving colony of cells, and (3) print Unicode text to the terminal.

# Today

Today, we'll learn about a new set of operators to manipulate bits. For example:

```
int x = 2;
```

```
// NEW: shift all bits X places to the left or right
```

```
x = x << 1; // now x is 4!
```

```
// NEW: check if the least significant bit is a 0
```

```
if (x & 1 == 0) {...
```

This is useful because we can perform some arithmetic more efficiently, and also store data more compactly in individual bits.

# Learning Goals

- Learn about the bitwise C operators and how to use them to manipulate bits
- Understand when to use one bitwise operator vs. another in your program
- Get practice with writing programs that manipulate binary representations

# Lecture Plan

- **Recap and continuing:** Integer Representations
- Bitwise Operators
- Bitmasks

# Lecture Plan

- **Recap and continuing: Integer Representations**
- Bitwise Operators
- Bitmasks

# Bits and Bytes So Far

All data, including integer variables, are ultimately stored in memory in binary:

```
int x = 5;    // really 0b0...0101 in memory!
```

- Unsigned numbers store the direct binary representation of its value
- Signed numbers use **two's complement** to store its positive/negative/0 value
- Overflow occurs when we exceed the the minimum or maximum value of the bit representation – it can cause some funky bugs!
  - If an operation needs more bits to represent the result than we have, upper bits are lost

# Base 10 vs. Binary vs. Hex

- Let's take a byte (8 bits):

165

Base-10: Human-readable,  
but cannot easily interpret on/off bits

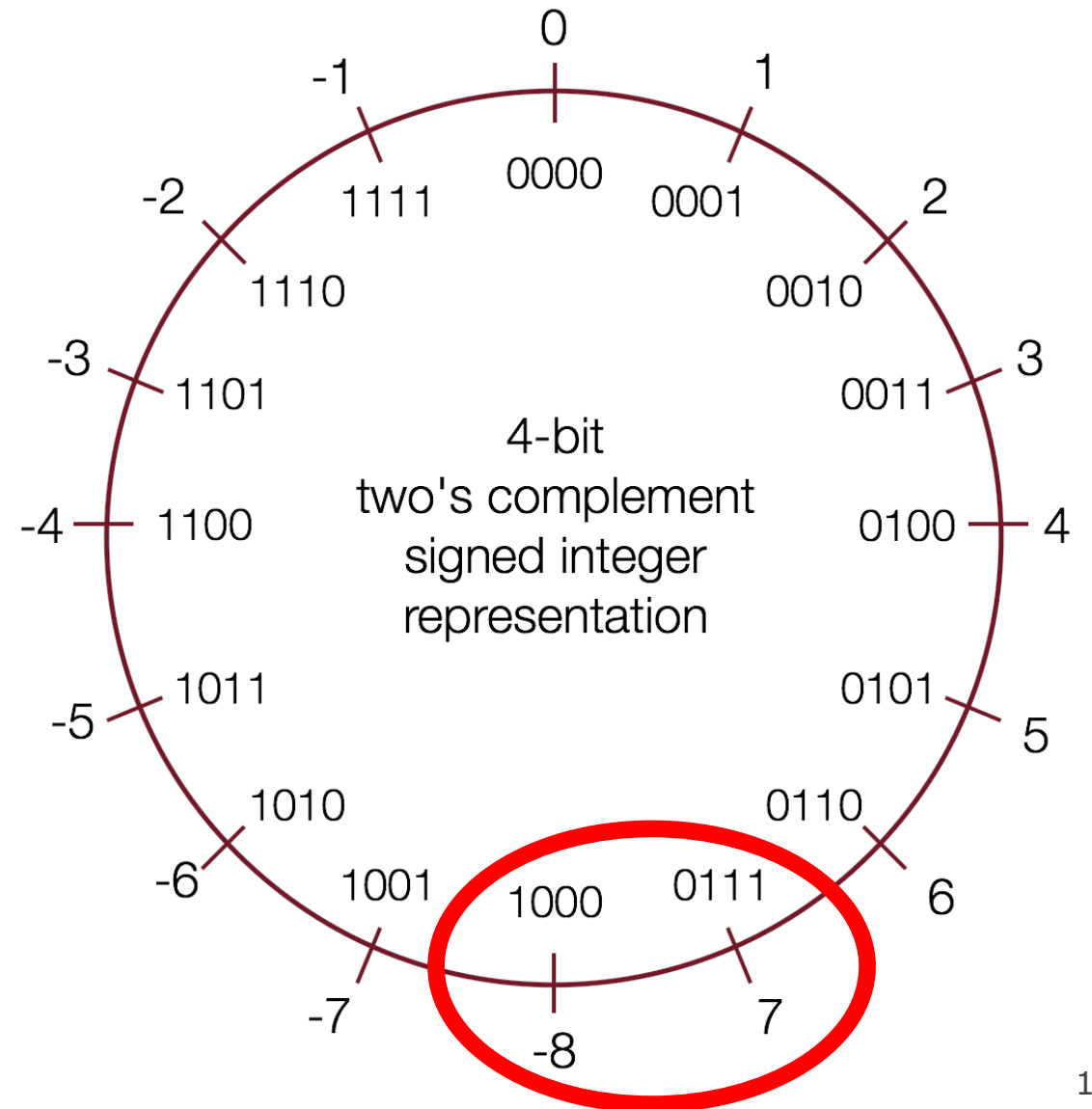
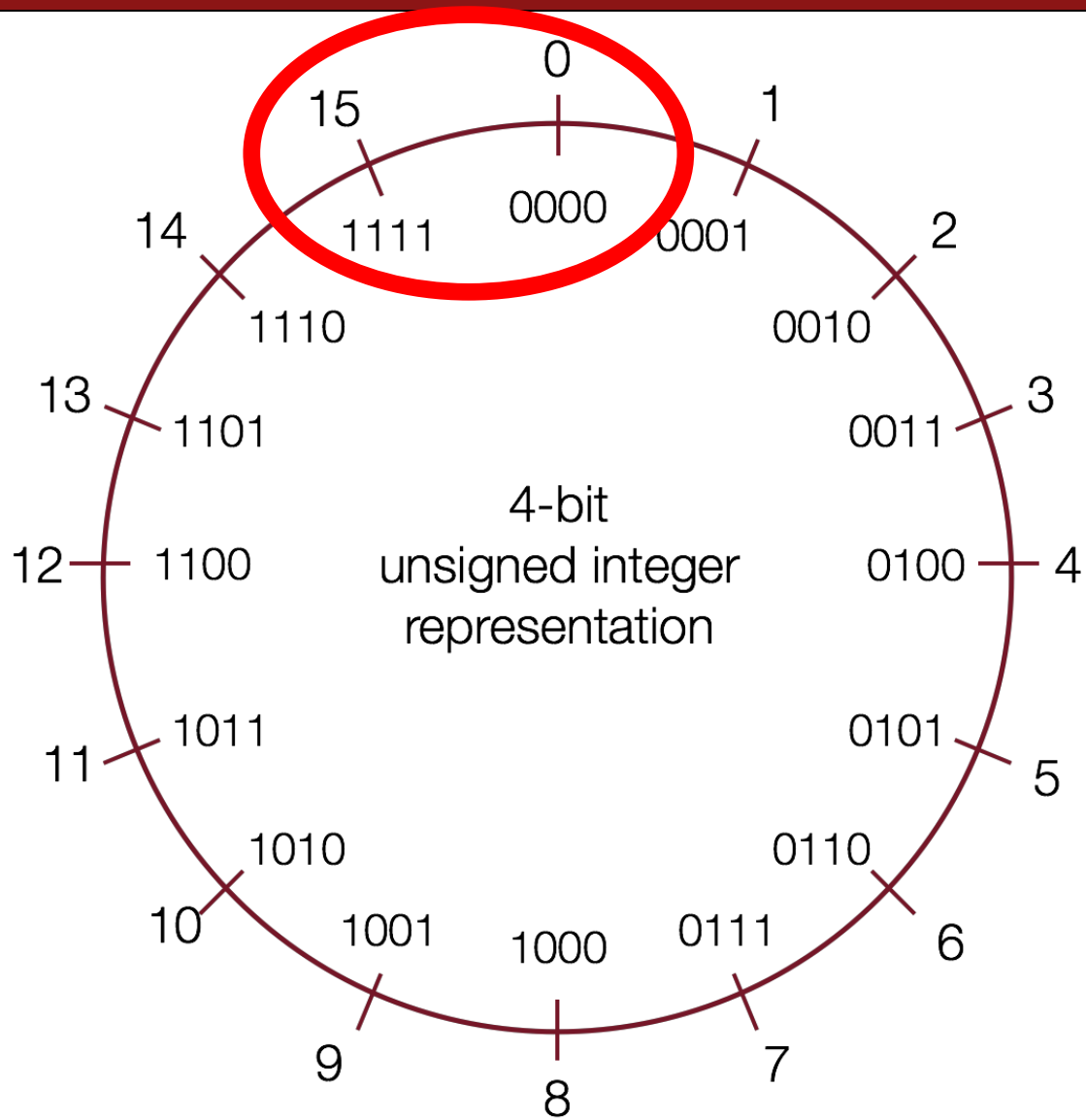
0b10100101

Base-2: Yes, computers use this,  
but not human-readable

0xa5

Base-16: Easy to convert to Base-2,  
More “portable” as a human-readable format  
(fun fact: a half-byte is called a nibble or nybble)

# Overflow



# Min and Max Integer Values

In C, there are various constants that represent these minimum and maximum values: `INT_MIN`, `INT_MAX`, `UINT_MAX`, `LONG_MIN`, `LONG_MAX`, `ULONG_MAX`, ...

# Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;  
short sx = x; // -12345!
```

```
x = 0000 0000 0000 0000 1100 1111 1100 0111  
sx =                1100 1111 1100 0111
```

# Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;  
short sx = x; // still -3
```

```
x = 1111 1111 1111 1111 1111 1111 1111 1101  
sx =                1111 1111 1111 1101
```

# Expanding Bit Representations

Sometimes, we want to carry over a value to a larger variable (e.g. make an **int** and set it equal to a **short**).

- For **unsigned** values, C adds *leading zeros* to the representation (“zero extension”)
- For **signed** values, C *repeats the sign of the value* for new digits (“sign extension”)

# Expanding Bit Representation

If we want to **expand** the bit size of an **unsigned** number, *C adds leading zeros*.

```
unsigned short s = 4;  
unsigned int i = s; // still 4
```

```
s =           0000 0000 0000 0100  
i = 0000 0000 0000 0000 0000 0000 0000 0100
```

# Expanding Bit Representation

If we want to **expand** the bit size of an **signed** number, *C repeats the sign bit*.

```
short s = 4;  
int i = s; // still 4
```

```
s =           0000 0000 0000 0100  
i = 0000 0000 0000 0000 0000 0000 0000 0100
```

# Expanding Bit Representation

If we want to **expand** the bit size of an **signed** number, *C repeats the sign bit*.

```
short s = -4;  
int i = s; // still -4
```

```
s =           1111 1111 1111 1100  
i = 1111 1111 1111 1111 1111 1111 1111 1100
```

# Casting

You can cast something to another type (treat as other type temporarily) by putting that type in parentheses in front of the value:

```
short s = -12345;  
...(unsigned short)s...
```

Casting between variable types can cause tricky issues; **the bits remain the same but are interpreted differently.**

Here, `s` is -12345, but casted it is 53191! (**1100 1111 1100 0111** in binary)

# Casting

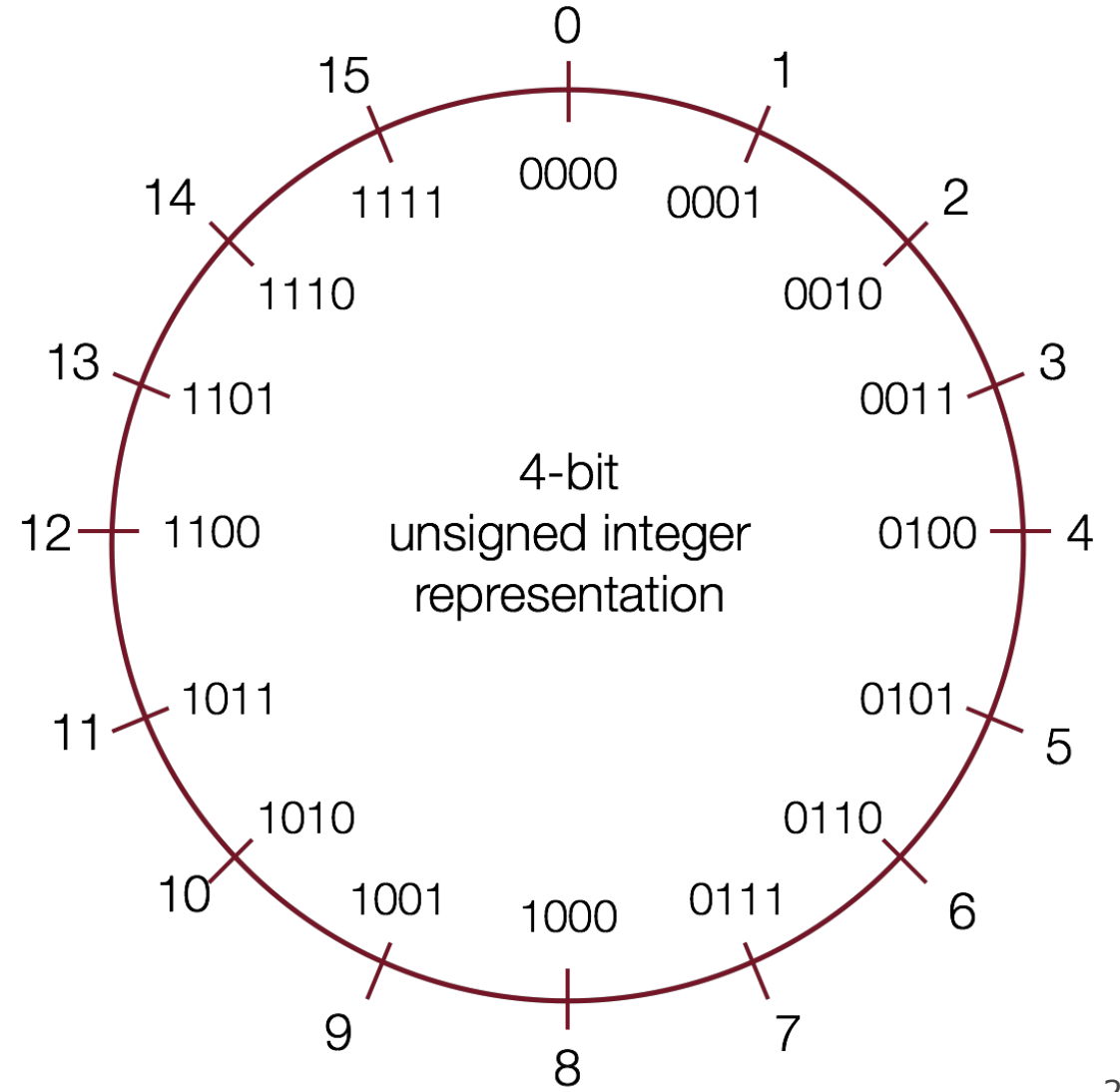
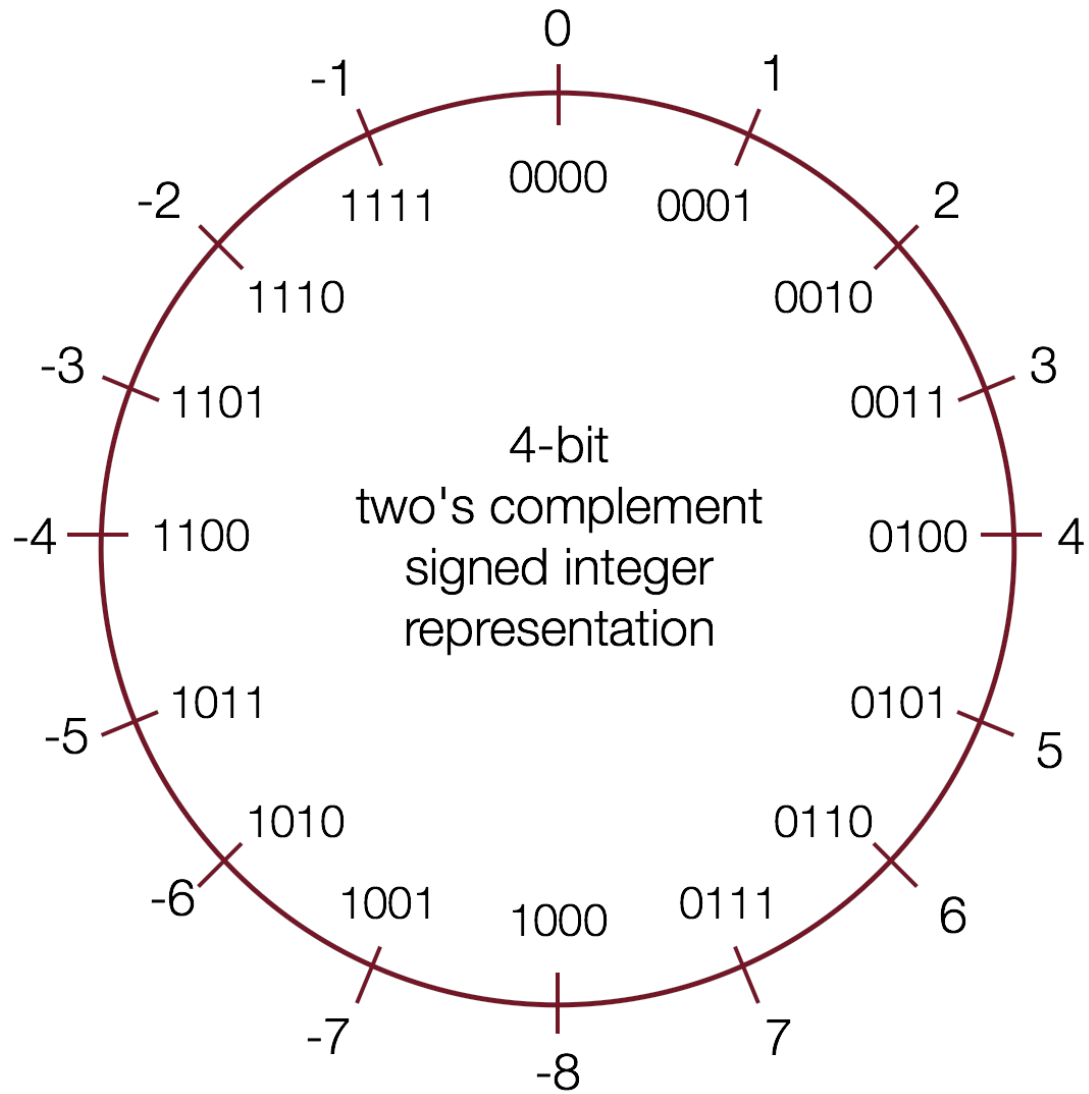
You can store the result as well:

```
short s = -12345;  
unsigned short us = (unsigned short)s; // 53191!
```

You can also use the **U** suffix after a number literal to treat it as unsigned:

**-12345U**

# Casting



# Comparisons Between Different Types

**Be careful** when comparing signed and unsigned integers. **C will implicitly cast** the signed argument to unsigned, and then performs the operation assuming both numbers are non-negative.

```
int x = -1;    // 1111 1111 1111 1111 1111 1111 1111 1111
unsigned int y = 0;
if (x < y) { ... // will be false!!
```

Note: when doing <, >, <=, >= comparison between different size types, it will *promote to the larger type*.

# Lecture Plan

- Recap and continuing: Integer Representations
- **Bitwise Operators**
- Bitmasks

# Bitwise Operators

- You're already familiar with many operators in C:
  - **Arithmetic operators:** +, -, \*, /, %
  - **Comparison operators:** ==, !=, <, >, <=, >=
  - **Logical Operators:** &&, ||, !
- Today, we're introducing a new category of operators: **bitwise operators:**
  - &, |, ~, ^, <<, >>

# And (&)

AND is a binary operator. The AND of 2 bits is 1 if both bits are 1, and 0 otherwise.

**output = a & b;**

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

& with 1 to let a bit through, & with 0 to zero out a bit

# Or (|)

OR is a binary operator. The OR of 2 bits is 1 if either (or both) bits is 1.

$$\text{output} = a \mid b;$$

a	b	output
0	0	0
0	1	1
1	0	1
1	1	1

| with 1 to turn on a bit, | with 0 to let a bit go through

# Not ( $\sim$ )

NOT is a unary operator. The NOT of a bit is 1 if the bit is 0, or 0 otherwise.

**output =  $\sim$ a;**

a	output
0	1
1	0

# Exclusive Or (^)

Exclusive Or (XOR) is a binary operator. The XOR of 2 bits is 1 if *exactly* one of the bits is 1, or 0 otherwise.

$$\text{output} = a \wedge b;$$

a	b	output
0	0	0
0	1	1
1	0	1
1	1	0

$\wedge$  with 1 to flip a bit,  $\wedge$  with 0 to let a bit go through

# Operators on Multiple Bits

When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
0110
& 1100
----
0100
```

OR

```
0110
| 1100
----
1110
```

XOR

```
0110
^ 1100
----
1010
```

NOT

```
~ 1100
----
0011
```

# Bit Operators

```
int x = 6; // 0000 ... 0110
```

```
int y = 5; // 0000 ... 0101
```

```
// 4
```

```
int anded = x & y; // 0000 ... 0100
```

```
// 7
```

```
int ored = x | y; // 0000 ... 0111
```

```
// -7
```

```
int notX = ~x; // 1111 ... 1111 1111 1001
```

```
int xored = x ^ y; // what would this give us?
```

If  $x = 6$  (0110) and  $y = 5$  (0101), what would  $x \wedge y$  be?

0

1

2

3

4

If  $x = 6$  (0110) and  $y = 5$  (0101), what would  $x \wedge y$  be?

0  0%

1  0%

2  0%

3  0%

4  0%

If  $x = 6$  (0110) and  $y = 5$  (0101), what would  $x \wedge y$  be?

0  0%

1  0%

2  0%

3  0%

4  0%

# Operators on Multiple Bits

When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND	OR	XOR	NOT
0110	0110	0110	
& 1100	1100	^ 1100	~ 1100
----	----	----	----
0100	1110	1010	0011

**Note:** these are different from the logical operators AND (&&), OR (||) and NOT (!).

# Bit Operators

```
int x = 4; // 0000 ... 0100
```

```
int y = 5; // 0000 ... 0101
```

```
// This is checking if x and y are both nonzero
```

```
if (x && y) { ...
```

```
// This is checking if the result of x & y is nonzero
```

```
if (x & y) { ...
```

# Lecture Plan

- Recap and continuing: Integer Representations
- Bitwise Operators
- **Bitmasks**

# Bitmasks

We will frequently want to manipulate or isolate out specific bits in a larger collection of bits.

**Motivating Example:** Bit vectors

# Bit Vectors and Sets

Instead of using arrays of e.g., Booleans in our programs, sometimes it's beneficial to store that information in bits instead – more compact.

- **Example:** we can represent current courses taken using a **char** and manipulate its contents using bit operators.

0	0	1	0	0	0	1	1
CS161	CS109	CS103	CS111	CS107	CS107E	CS106B	CS106A

# Bit Masking

```
#define CS106A 0x1    /* 0000 0001 */
#define CS106B 0x2    /* 0000 0010 */
#define CS107E 0x4    /* 0000 0100 */
#define CS107  0x8    /* 0000 1000 */
#define CS111  0x10   /* 0001 0000 */
#define CS103  0x20   /* 0010 0000 */
#define CS109  0x40   /* 0100 0000 */
#define CS161  0x80   /* 1000 0000 */
```

```
// Bit pattern: 0000 1011
```

```
unsigned char myClasses = CS106A | CS106B | CS107;
```

# Bit Vectors and Sets

0	0	1	0	0	0	1	1
CS167	CS109	CS103	CS111	CS107	CS107E	CS106B	CS106A

- How do we find the union of two sets of courses taken? Use OR:

```
    00100011
  | 01100001
  -----
    01100011
```

# Bit Masking

```
#define CS106A 0x1      /* 0000 0001 */
#define CS106B 0x2      /* 0000 0010 */
#define CS107E 0x4      /* 0000 0100 */
#define CS107  0x8      /* 0000 1000 */
#define CS111  0x10     /* 0001 0000 */
#define CS103  0x20     /* 0010 0000 */
#define CS109  0x40     /* 0100 0000 */
#define CS161  0x80     /* 1000 0000 */
```

```
unsigned char myClasses = CS106A | CS106B | CS107;
unsigned char otherClasses = CS106A | CS106B | CS103;
```

```
// 0010 1011
```

```
unsigned char either = myClasses | otherClasses;
```

# Bit Vectors and Sets

0	0	1	0	0	0	1	1
CS167	CS109	CS103	CS111	CS107	CS107E	CS106B	CS106A

- How do we find the intersection of two sets of courses taken? Use AND:

```
    00100011
&   01100001
-----
    00100001
```

# Bit Masking

**Example:** how do we update our bit vector to indicate we've taken CS107?

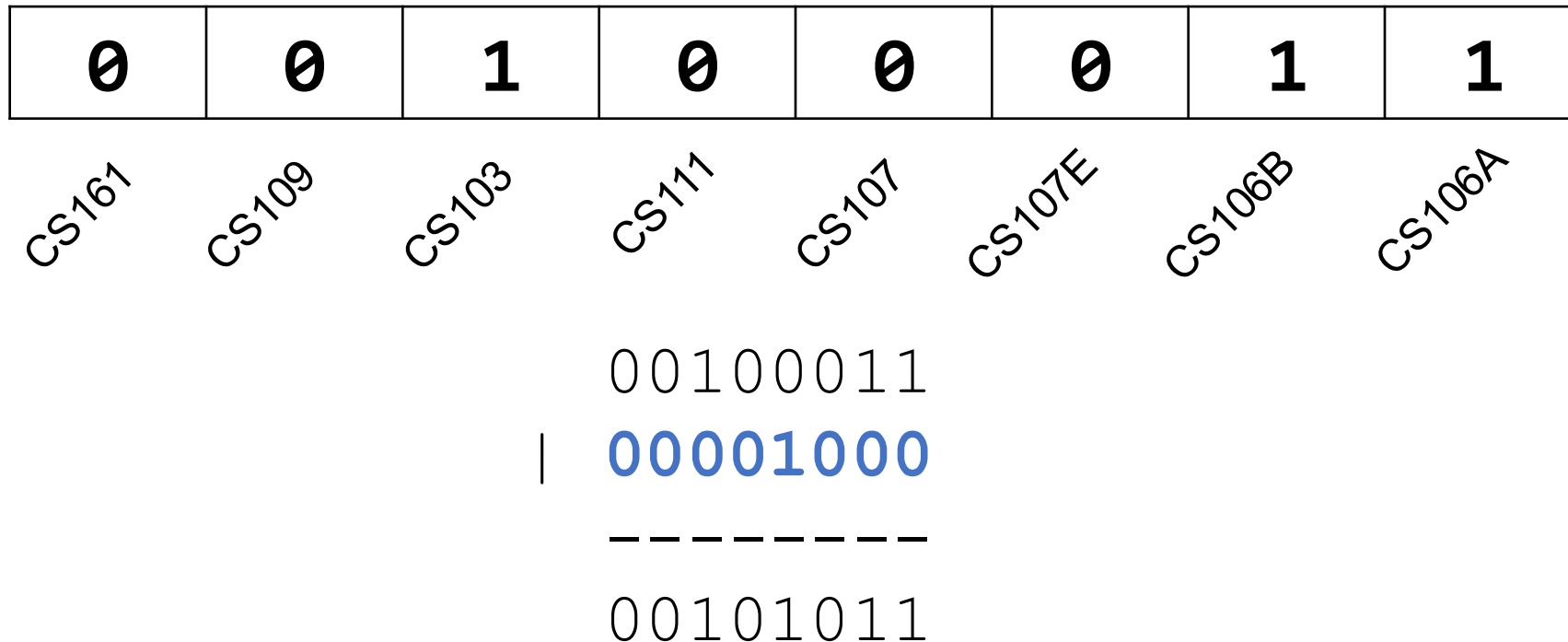
<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>
CS167	CS109	CS103	CS111	CS107	CS107E	CS106B	CS106A

```
00100011
| 00001000
-----
00101011
```

# Bit Masking

**Example:** how do we update our bit vector to indicate we've taken CS107?

A bitmask is a constructed bit pattern that we can use, along with bit operators, to manipulate a value.



# Bit Masking

```
#define CS106A 0x1      /* 0000 0001 */
#define CS106B 0x2      /* 0000 0010 */
#define CS107E 0x4      /* 0000 0100 */
#define CS107  0x8      /* 0000 1000 */
#define CS111  0x10     /* 0001 0000 */
#define CS103  0x20     /* 0010 0000 */
#define CS109  0x40     /* 0100 0000 */
#define CS161  0x80     /* 1000 0000 */

char myClasses = ...;
myClasses = myClasses | CS107;    // Add CS107
```

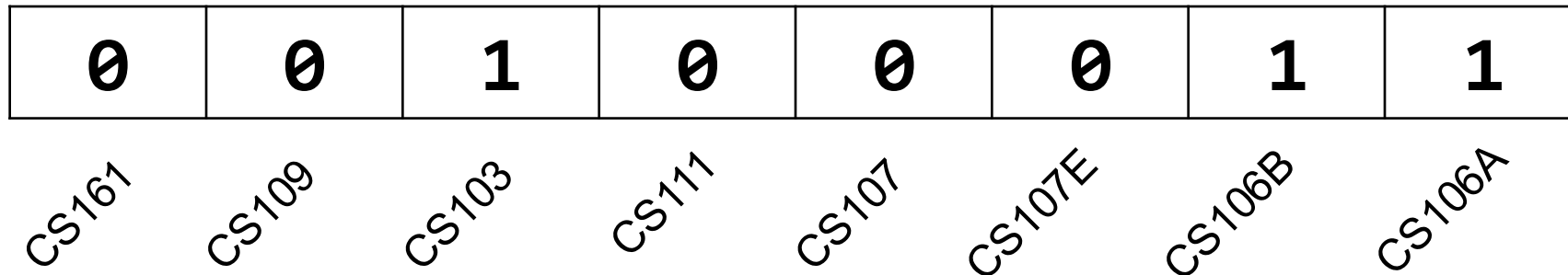
# Bit Masking

```
#define CS106A 0x1      /* 0000 0001 */
#define CS106B 0x2      /* 0000 0010 */
#define CS107E 0x4      /* 0000 0100 */
#define CS107  0x8      /* 0000 1000 */
#define CS111  0x10     /* 0001 0000 */
#define CS103  0x20     /* 0010 0000 */
#define CS109  0x40     /* 0100 0000 */
#define CS161  0x80     /* 1000 0000 */
```

```
char myClasses = ...;
myClasses |= CS107;    // Add CS107
```

# Bit Masking

**Example:** how do we update our bit vector to indicate we've *not* taken CS103?



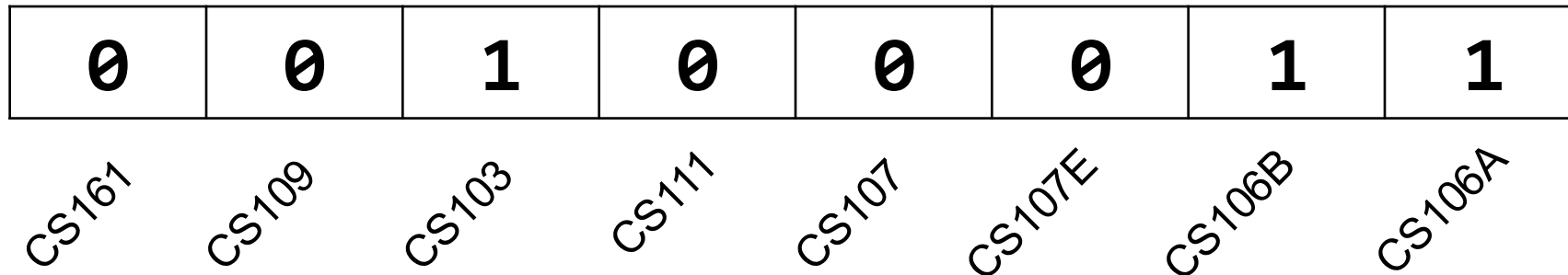
1. What operator is good for turning off certain bits?
2. What mask do we want?
3. How do we create that mask?

```
00100011
& 11011111
-----
00000011
```

```
char myClasses = ...;
myClasses = myClasses & ~CS103; // Remove CS103
```

# Bit Masking

**Example:** how do we update our bit vector to indicate we've *not* taken CS103?

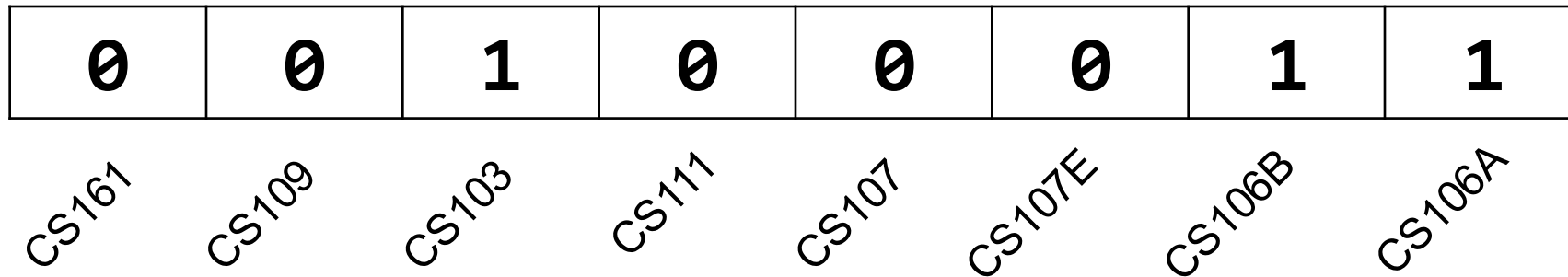


```
00100011
& 11011111
-----
00000011
```

```
char myClasses = ...;
myClasses &= ~CS103; // Remove CS103
```

# Bit Masking

- **Example:** how do we check if we've taken CS106B?

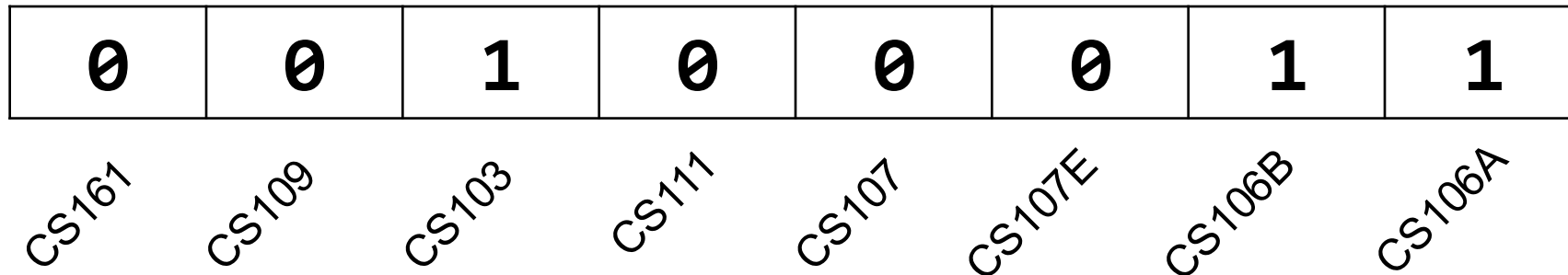


```
00100011
& 00000010
-----
00000010
```

```
char myClasses = ...;
if (myClasses & CS106B) {...
    // taken CS106B!
```

# Bit Masking

- **Example:** how do we check if we've *not* taken CS107?



```
00100011
& 00001000
-----
00000000
```

```
char myClasses = ...;
if (!(myClasses & CS107)) {...
    // not taken CS107!
```

# Bitwise Operator Tricks

- $|$  with 1 is useful for turning select bits on
- $\&$  with 0 is useful for turning select bits off
- $|$  is useful for taking the union of bits
- $\&$  is useful for taking the intersection of bits
- $\wedge$  is useful for flipping select bits
- $\sim$  is useful for flipping all bits

## 3 Step Process:

1. What operator is good for the task we are trying to do?
2. What mask do we need for this operation?
3. How do we create that mask?

# Recap

- **Recap and continuing:** Integer Representations
- Bitwise Operators
- Bitmasks

**Lecture 4 takeaways:** We can use bit operators like  $\&$ ,  $|$ ,  $\sim$ , etc. to manipulate the binary representation of values. A number is a bit pattern that can be manipulated arithmetically or bitwise at your convenience!

# Extra Practice

# Hexadecimal and Truncation

For each initialization of x, what will be printed?

i. `x = 130; // 0x82`

ii. `x = -132; // 0xff7c`

iii. `x = 25; // 0x19`

```
short x = _____;  
char cx = x;  
printf("%d", cx);
```



# Hexadecimal and Truncation

For each initialization of x, what will be printed?

**-126** i. `x = 130; // 0x82`

**124** ii. `x = -132; // 0xff7c`

**25** iii. `x = 25; // 0x19`

```
short x = _____;  
char cx = x;  
printf("%d", cx);
```

# Limits and Comparisons

2. Will the following char comparisons evaluate to true or false?

i. `-7 < 4`      **true**

iii. `(char) 130 > 4`      **false**

ii. `-7 < 4U`      **false**

iv. `(char) -132 > 2`      **true**

By default, numeric constants in C are signed ints, unless they are suffixed with u (unsigned) or L (long).

# Bitwise Warmup

How can we use bitmasks + bitwise operators to...

0b00001101

1. ...turn **on** a particular set of bits?

0b00001101

---

0b00001111

2. ...turn **off** a particular set of bits?

0b00001101

---

0b00001001

3. ...**flip** a particular set of bits?

0b00001101

---

0b00001011



# Bitwise Warmup

How can we use bitmasks + bitwise operators to...

0b00001101

1. ...turn **on** a particular set of bits? **OR**

```
0b00001101
0b00000010 |
-----
0b00001111
```

2. ...turn **off** a particular set of bits? **AND**

```
0b00001101
0b11111011 &
-----
0b00001001
```

3. ...**flip** a particular set of bits? **XOR**

```
0b00001101
0b00000110 ^
-----
0b00001011
```