

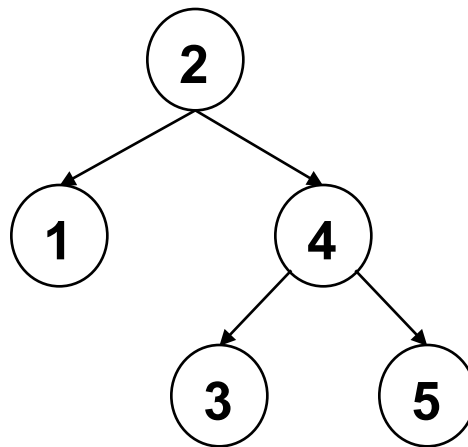
Combinatorics

The background of the slide is a photograph of a wooded area. In the foreground, there are several large, dark, geometric concrete sculptures. One is a truncated octahedron, another is a cube, and others are various polyhedrons. The ground is covered with fallen yellow and brown leaves. The background shows green trees and foliage.

CS 109
Lecture 2
March 30th, 2016

Binary Search Tree

- A **binary search tree (BST)**, is a binary tree where for *every* node n in the tree:
 - n 's value is **greater** than all the values in its **left** subtree.
 - n 's value is **less** than all the values in its **right** subtree.
 - both n 's left and right subtrees are binary search trees.



Binary Search Tree

- **Problem**: How many possible BSTs containing values 1, 2, and 3 have degenerate structure (i.e., each node in the BST has at most one child)?
- **Solution**: $3!$ ways to order 1, 2, and 3 for insertion

1, 2, 3

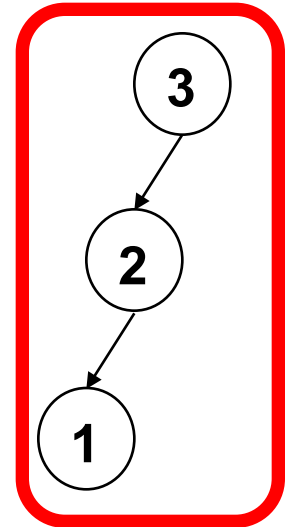
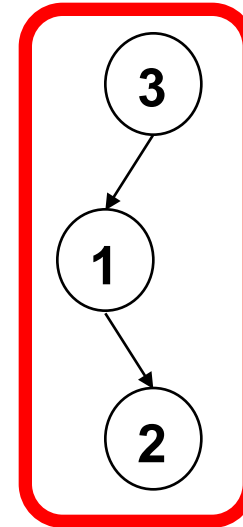
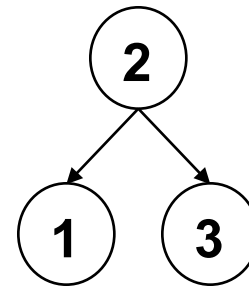
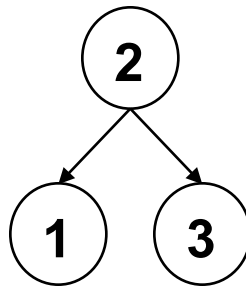
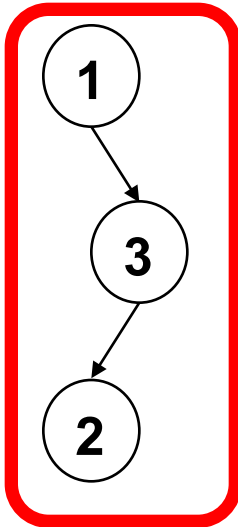
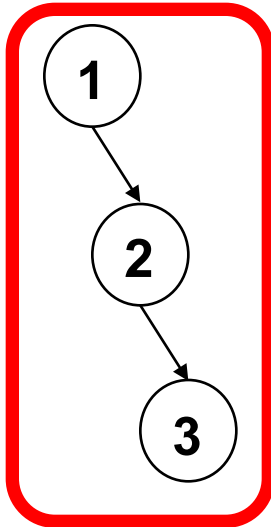
1, 3, 2

2, 1, 3

2, 3, 1

3, 1, 2

3, 2, 1



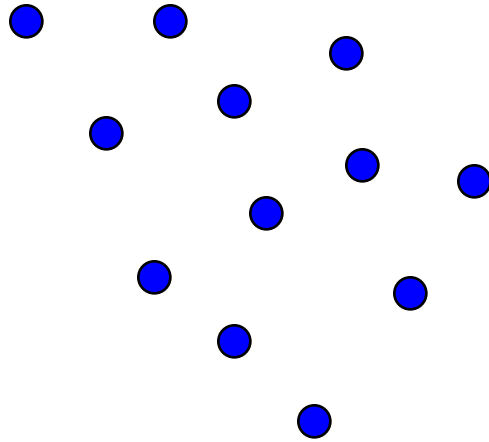
There are 4 degenerate BSTs possible.

Recursive definition of $\binom{n}{k}$

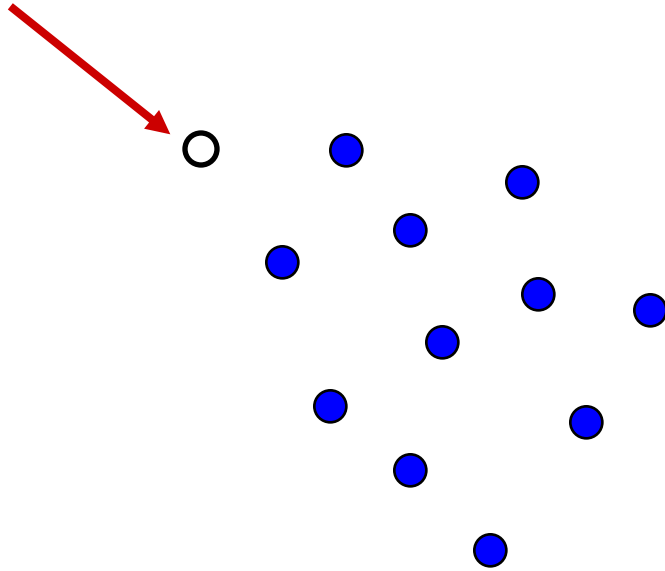
Let's write a function $C(n, k)$

The number of ways to select k objects from a set of n objects.

$C(n,k)$

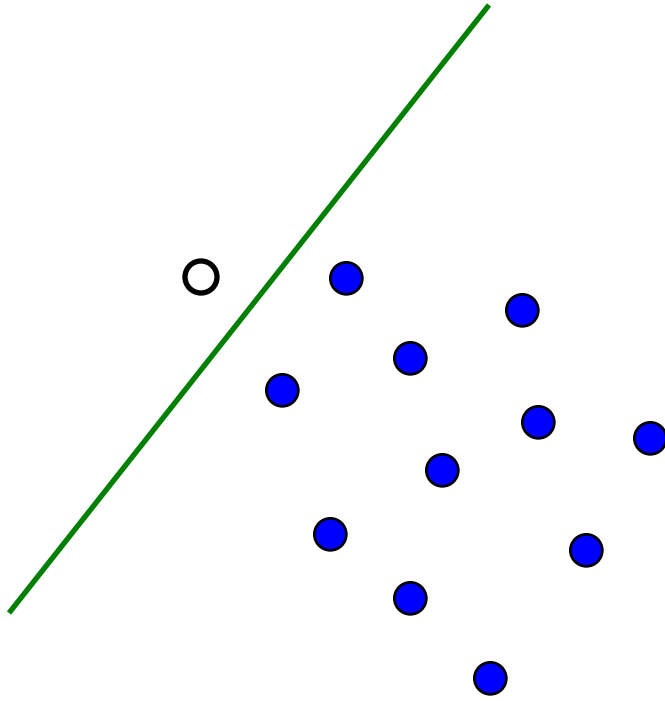


$$C(n,k)$$



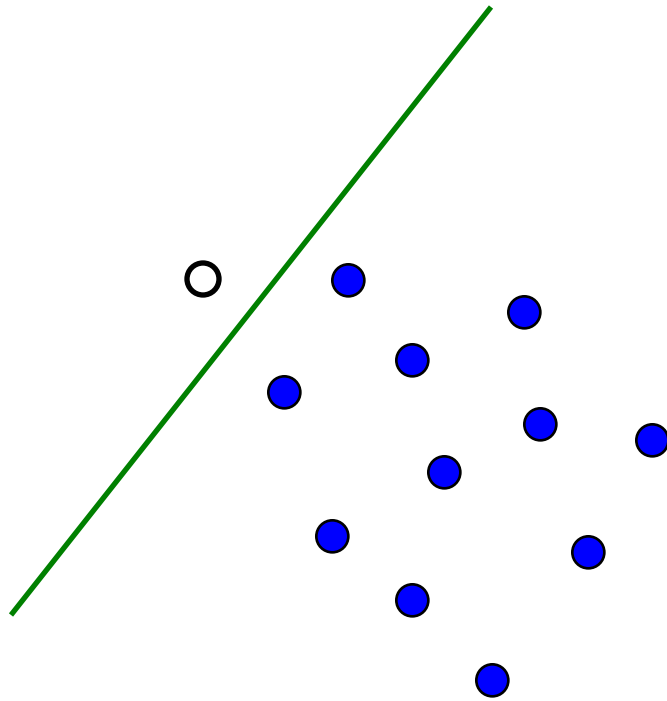
Select any one of the n points in the group

$C(n,k)$



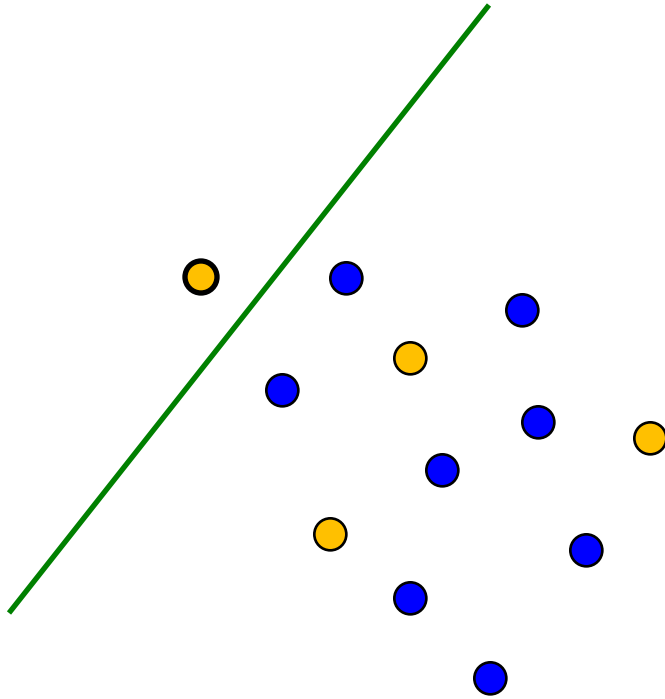
Separate this point from the rest

$C(n,4)$



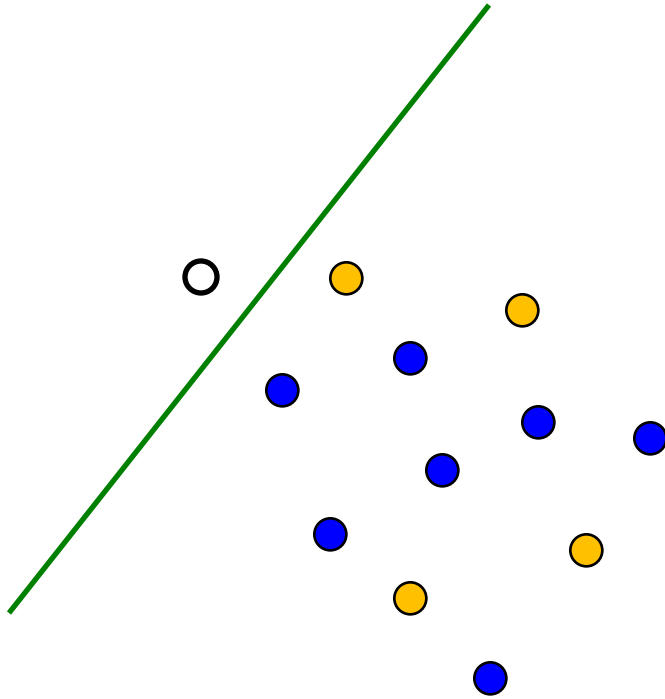
Let's consider specific problem $C(n, 4)$

$$C(n,4)$$



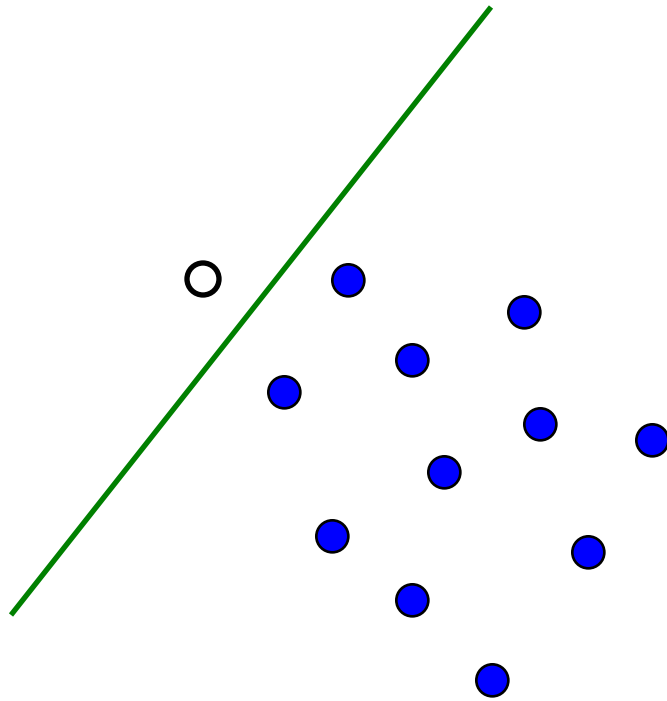
This point can be **included** in the 4 points we choose

$$C(n,4)$$



Or, it can be **excluded** from the 4 points we choose

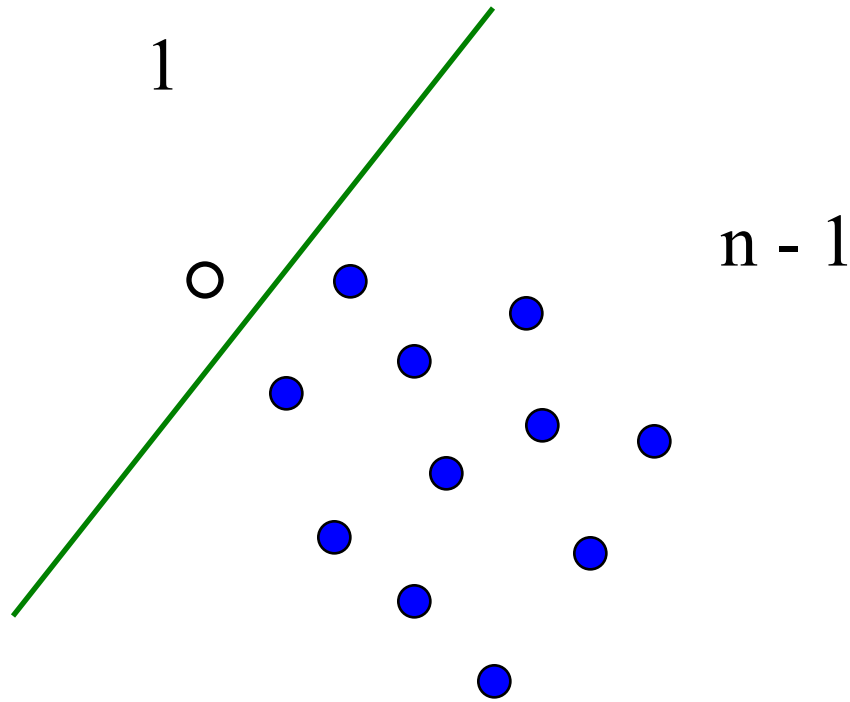
$C(n,k)$



Total number of solutions is

number of solutions including \circ
+
number of solutions not including \circ

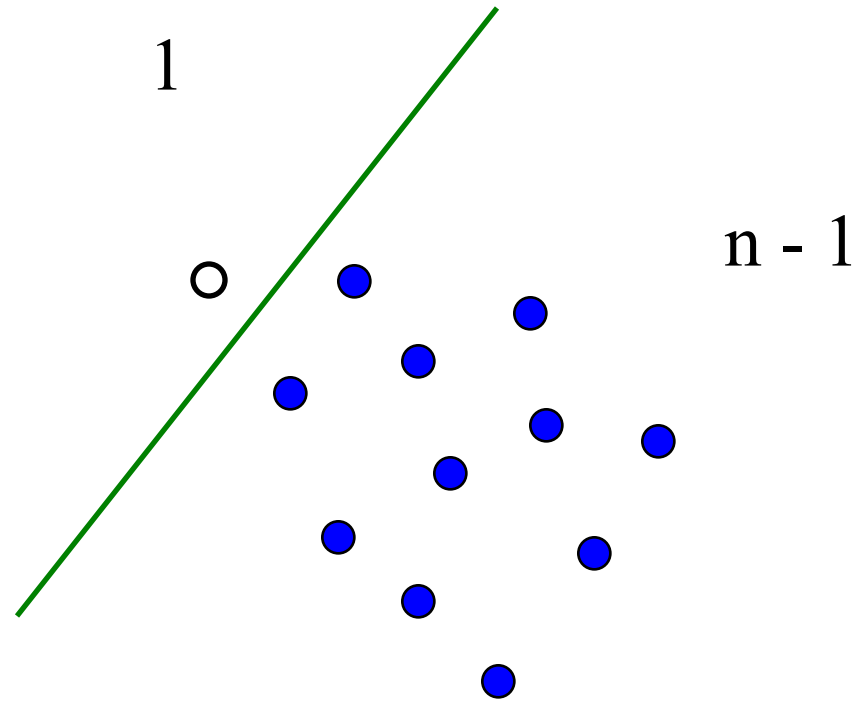
$C(n,k)$



Total number of solutions is

number of solutions including \circ
+
number of solutions not including \circ

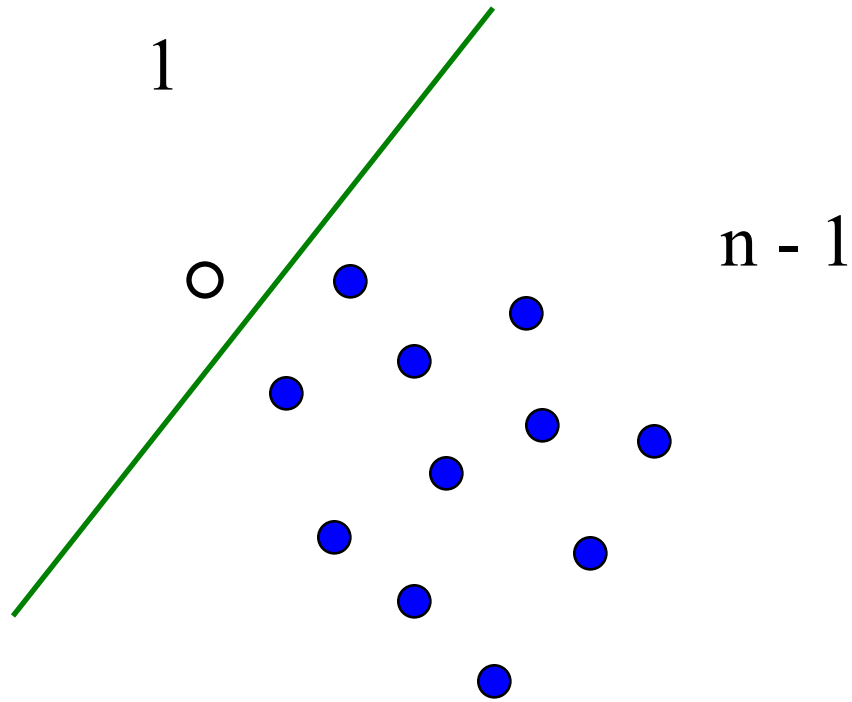
$C(n,k)$



number of solutions including \circ

$C(n-1, k-1)$

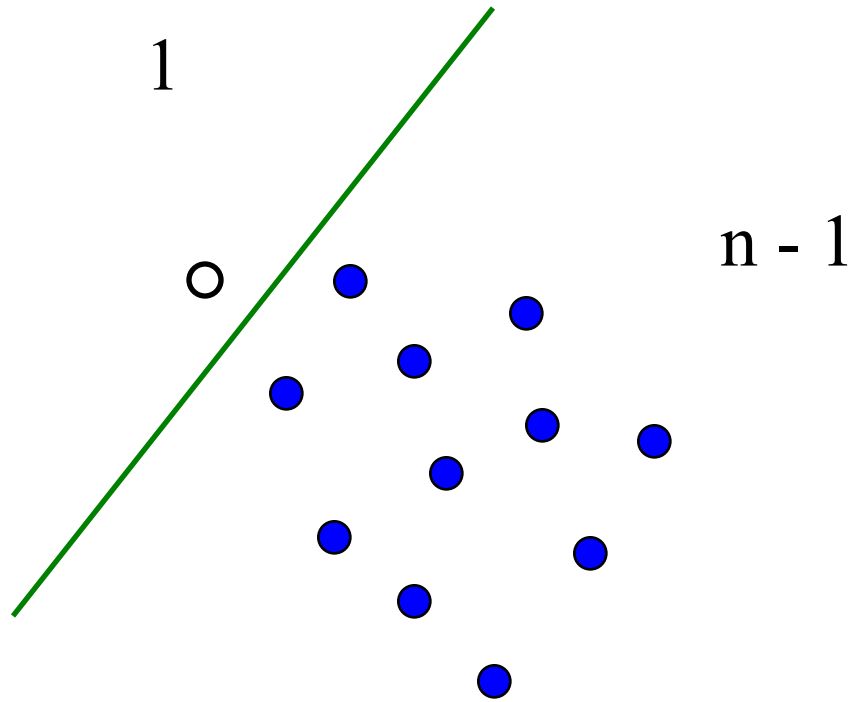
$C(n,k)$



number of solutions including \circ $C(n-1, k-1)$

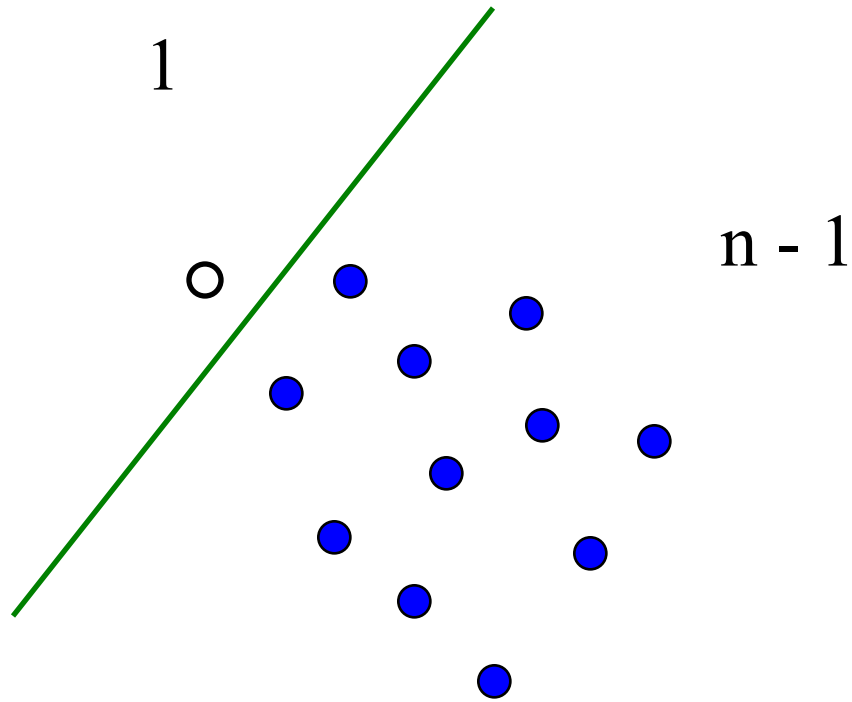
number of solutions not including \circ $C(n-1, k)$

$C(n,k)$



Total number of solutions is $C(n-1, k-1) + C(n-1, k)$

$C(n,k)$



```
int C(int n, int k)
{
    if (k == 0 || n == k) return (1);
    return (C(n-1, k-1) + C(n-1, k));
}
```