

# CS 109: Introduction to Probability for Computer Scientists

Will Monroe  
Summer 2017

with materials by  
Mehran Sahami  
and Chris Piech



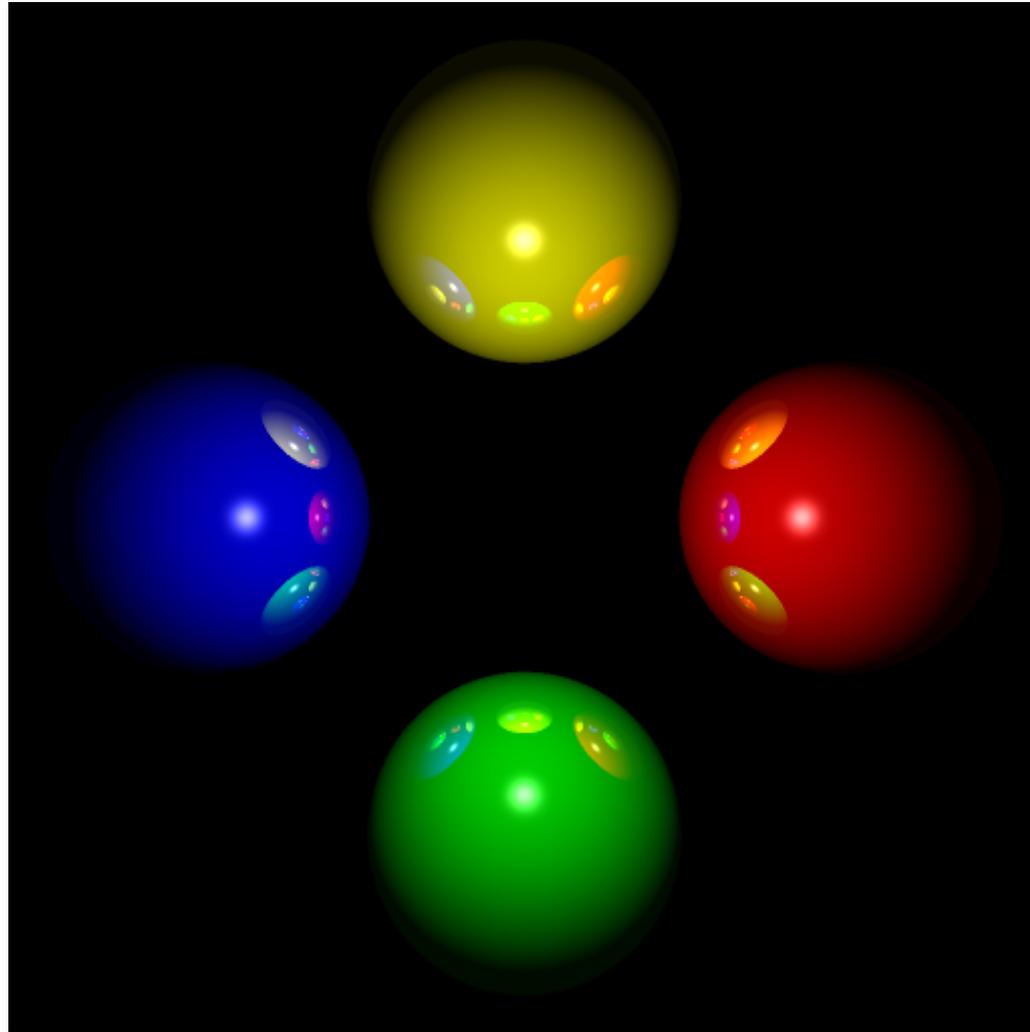
June 26, 2017

# Who am I?



Stanford undergrad

# Who am I?



Graphics



# Who am I?

Call me Will! 😄

or Mr. Monroe 👍

~~Professor Monroe~~

~~Dr. Monroe~~

~~Master Monroe?~~

# Who am I?



Reno, Nevada

# Traditional probability setups



Why probability with computers?

Theory

Artificial  
Intelligence

Biocomputation

Computer  
Engineering

Graphics

HCI

Information

Systems

Learning theory

Gene expression prediction

Caching, error correction

Monte Carlo rendering

Hypothesis testing

Language modeling

Randomized algorithms

Load balancing, queueing

# Quiz question

<https://b.socrative.com/login/student/>

Room: CS109SUMMER17

# Zika testing

<https://b.socrative.com/login/student/>

Room: CS109SUMMER17



0.08% of people have Zika

90% of people with Zika test positive

7% of people without Zika test positive

Someone tests positive. What's the probability they have Zika?

- A) <0.5%    B) 1%    C) 7%    D) 41%    E) 90%

# Zika testing

<https://b.socrative.com/login/student/>

Room: CS109SUMMER17



0.08% of people have Zika

90% of people with Zika test positive

7% of people without Zika test positive

Someone tests positive. What's the probability they have Zika?

Answer: B) 1%



~~Counting~~  
Combinatorial analysis

# An experiment



# General principle of counting

An experiment consisting of two or more **separate parts** has a number of outcomes equal to the **product** of the number of outcomes of each part.

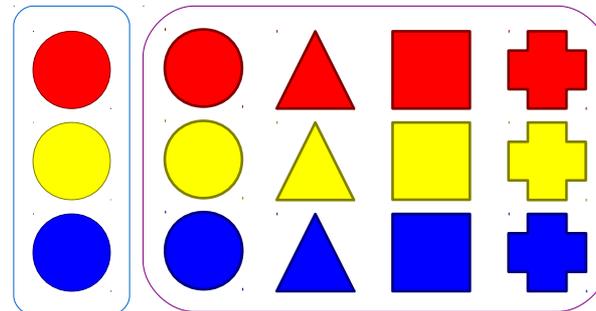


$$|A_1 \times A_2 \times \cdots \times A_n| = \prod_i |A_i|$$

shapes: 4



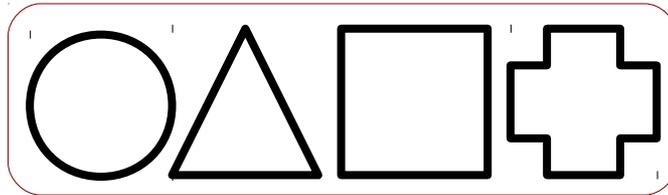
colors: 3



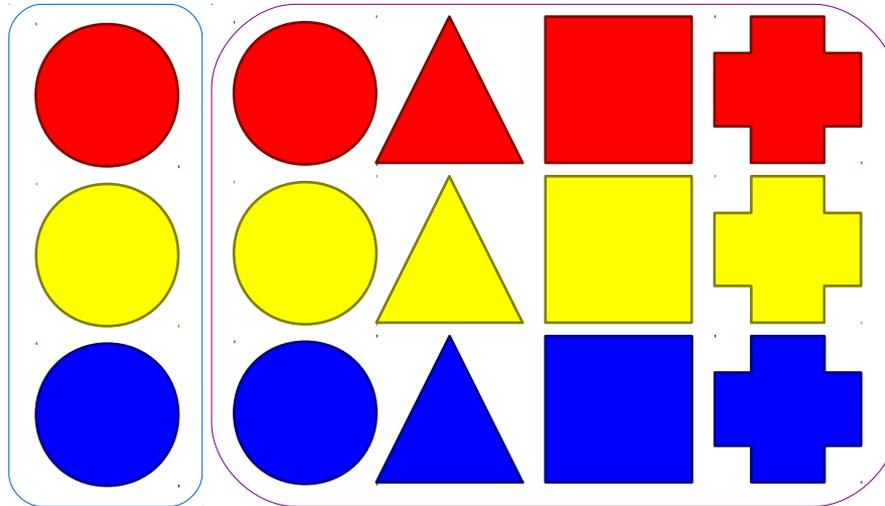
total:  
 $4 \cdot 3 = 12$

# General principle of counting

shapes: 4

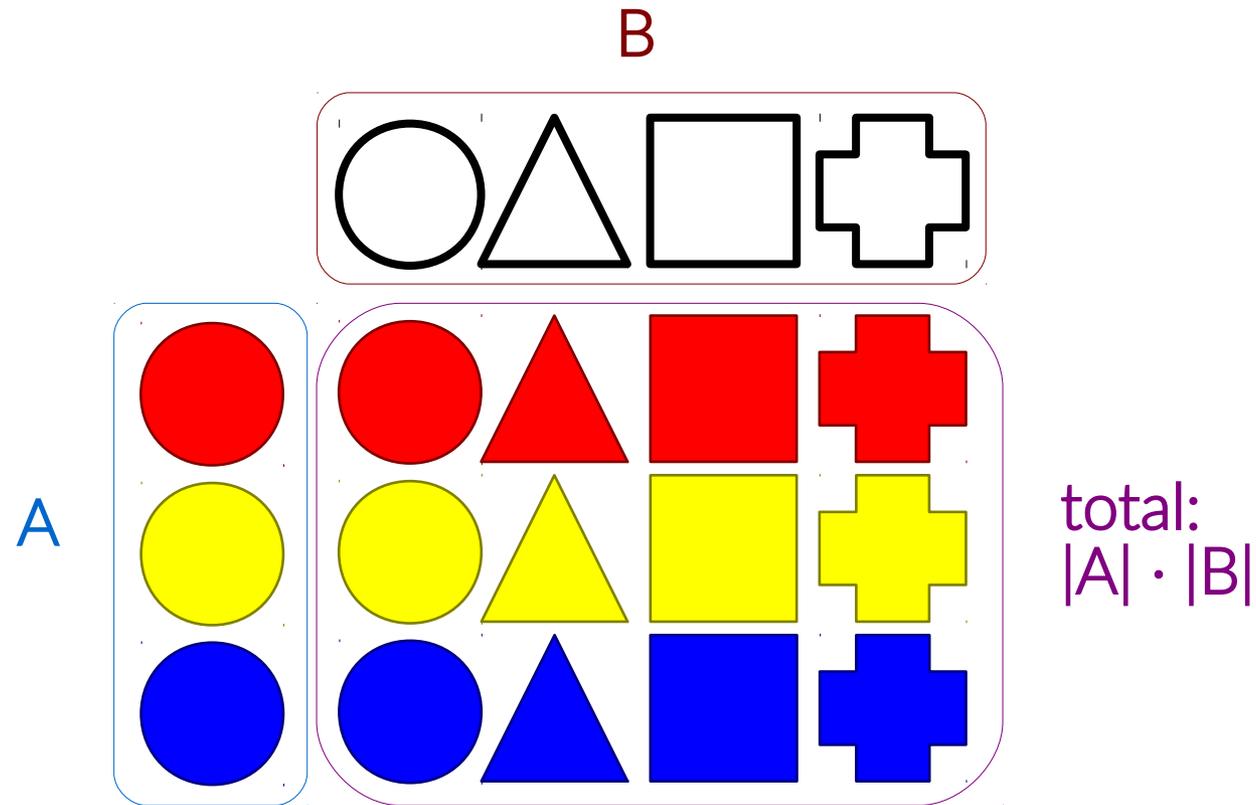


colors: 3



total:  
 $4 \cdot 3 = 12$

# General principle of counting



# Counting license plates



$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \\ = 17,576,000$$



$$10 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \\ = 175,760,000$$

(Approximate 2017 California population: 39,800,000)

# General principle of counting

An experiment consisting of two or more **separate parts** has a number of outcomes equal to the **product** of the number of outcomes of each part.

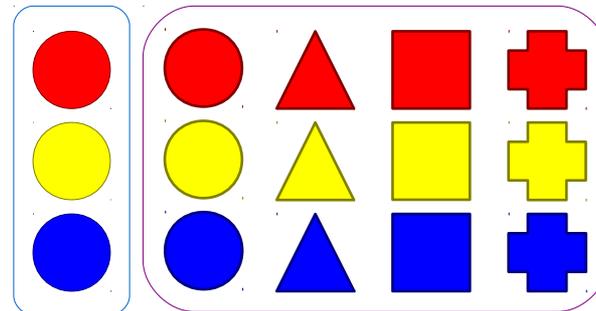


$$|A_1 \times A_2 \times \cdots \times A_n| = \prod_i |A_i|$$

shapes: 4



colors: 3



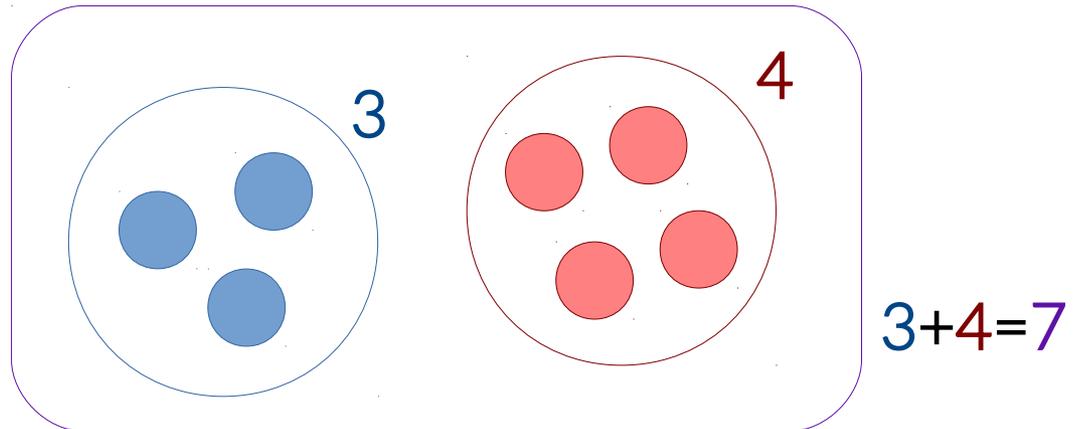
total:  
 $4 \cdot 3 = 12$

# Sum rule of counting

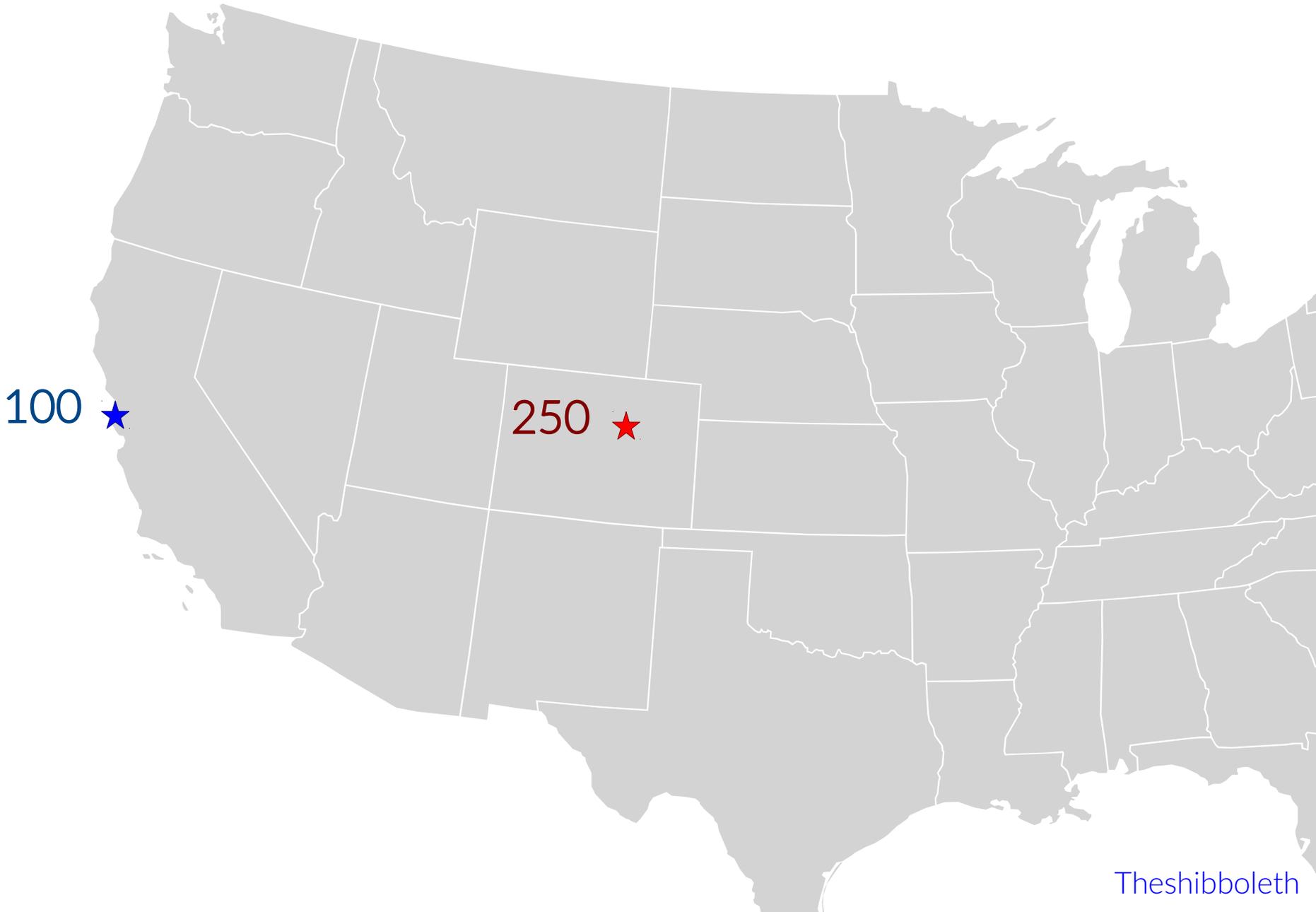
If two sets don't overlap, the **total number of elements** in the two sets is the **sum** of the number of elements of each set.



$$|A \cup B| = |A| + |B| \quad \text{if } A \cap B = \emptyset$$



# Datacenters

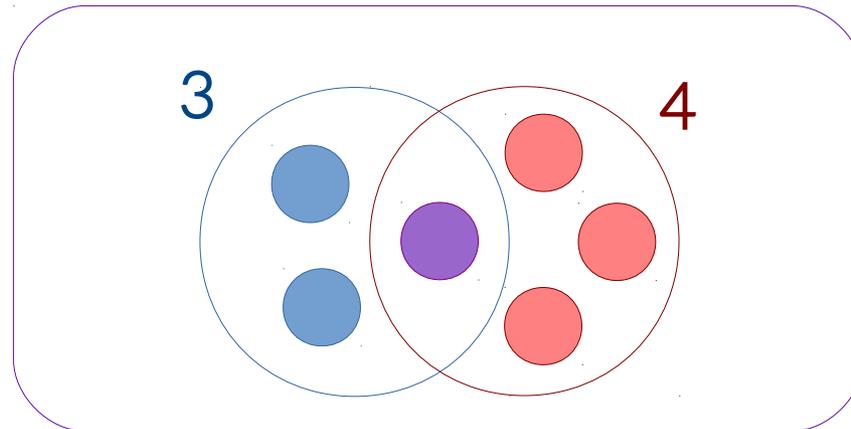


# Principle of Inclusion/Exclusion

The **total number of elements** in two sets is the sum of the number of elements of each set, **minus** the number of elements in both sets.

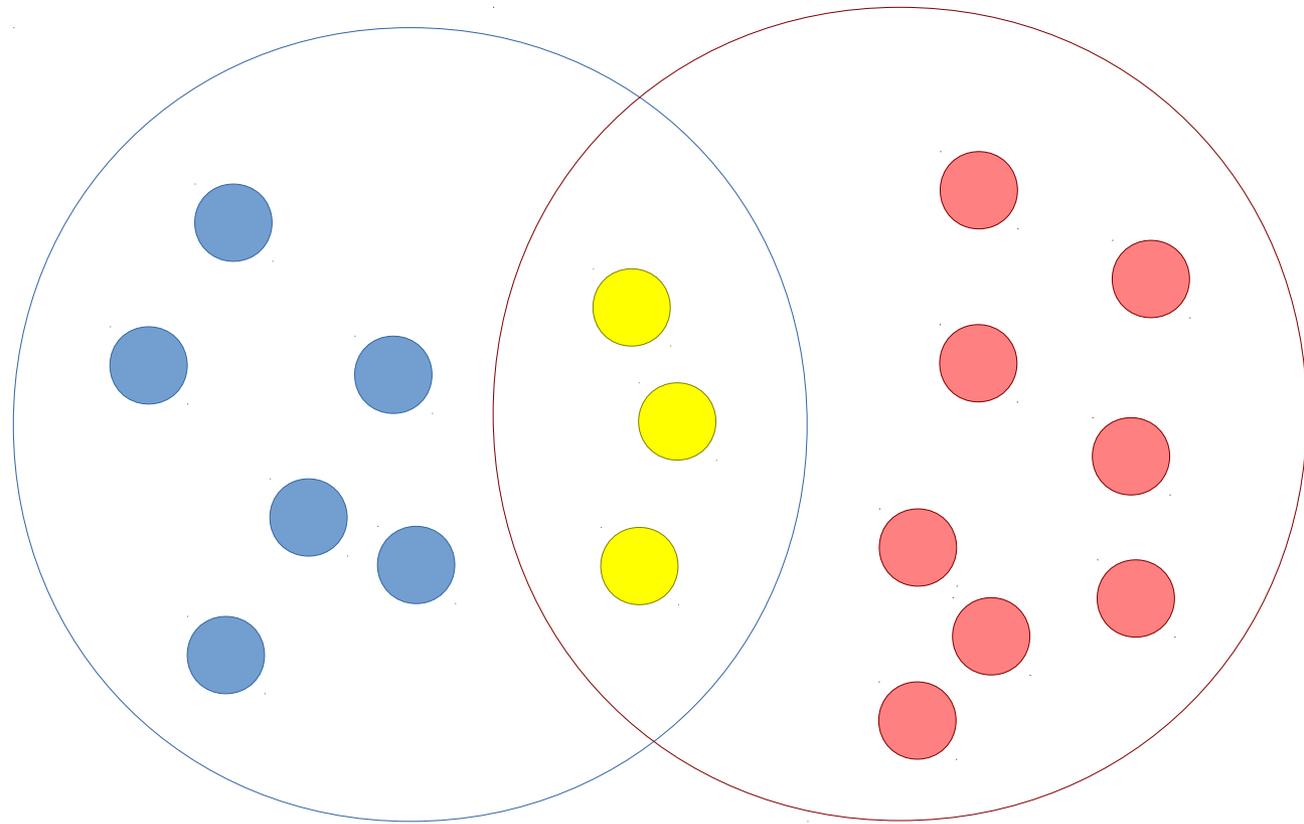
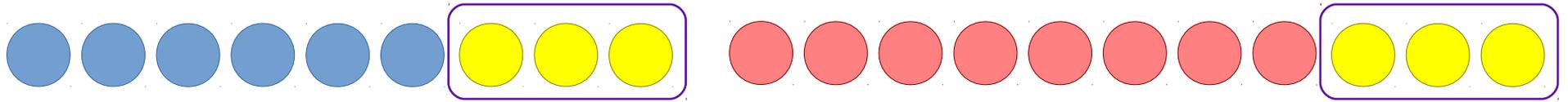


$$|A \cup B| = |A| + |B| - |A \cap B|$$



$$3 + 4 - 1 = 6$$

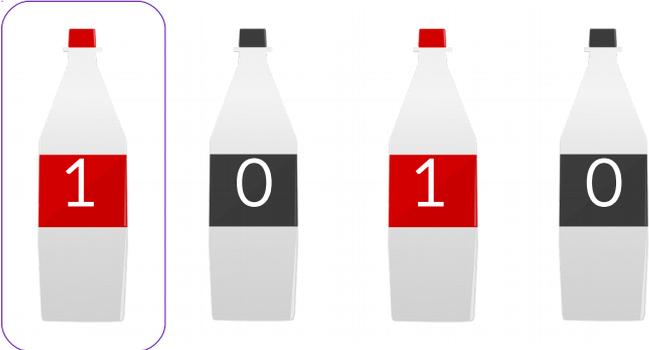
# Inclusion/Exclusion intuition



# Bit strings

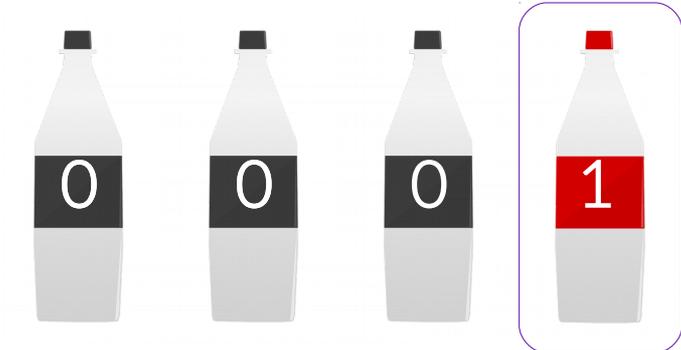
How many 4-bit strings are there that **start or end with 1**?

$A$



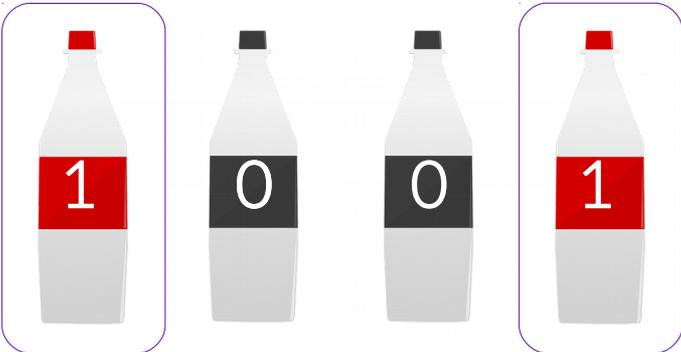
$|A| = 2^3$

$B$



$|B| = 2^3$

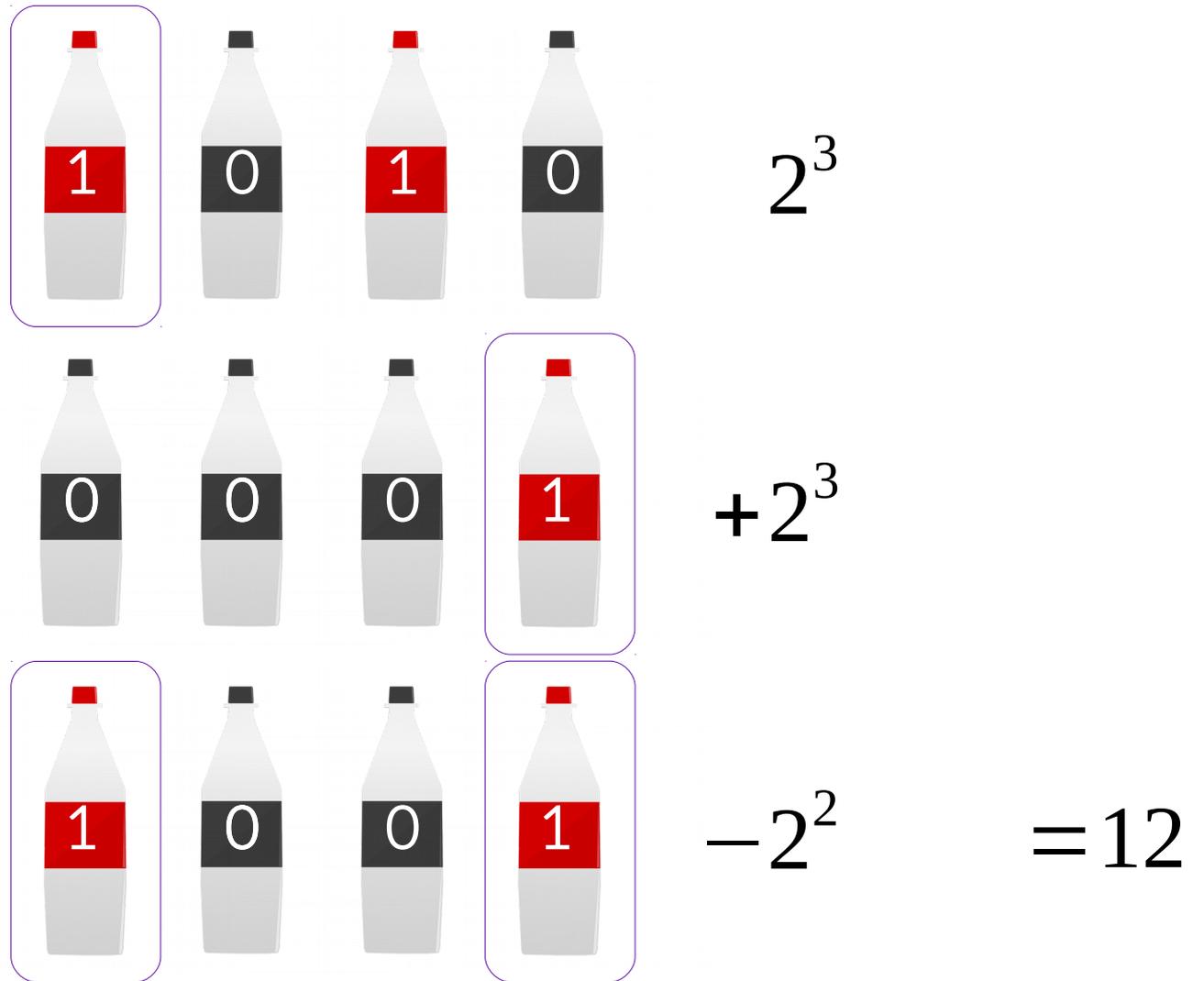
$A \cap B$



$|A \cap B| = 2^2$

# Bit strings

How many 4-bit strings are there that **start or end with 1**?

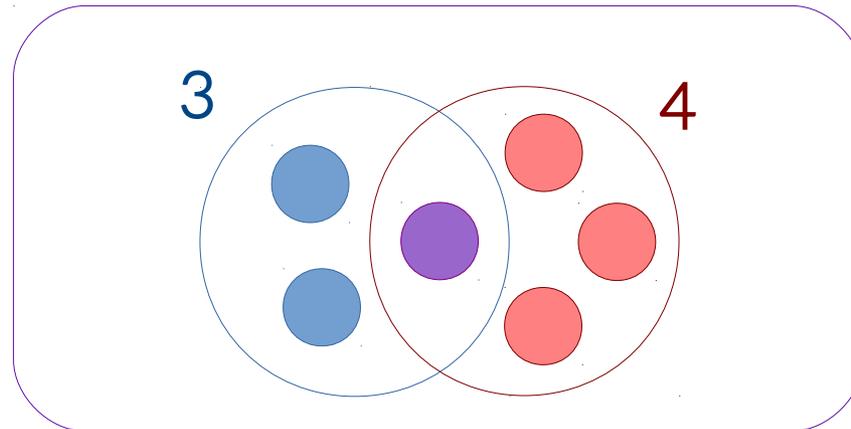


# Principle of Inclusion/Exclusion

The **total number of elements** in two sets is the sum of the number of elements of each set, **minus** the number of elements in both sets.



$$|A \cup B| = |A| + |B| - |A \cap B|$$



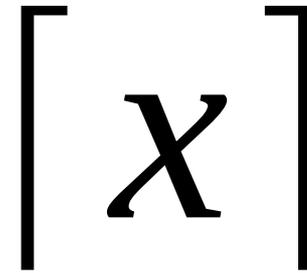
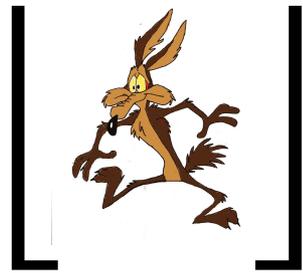
$$3+4-1 = 6$$

# Floor and ceiling

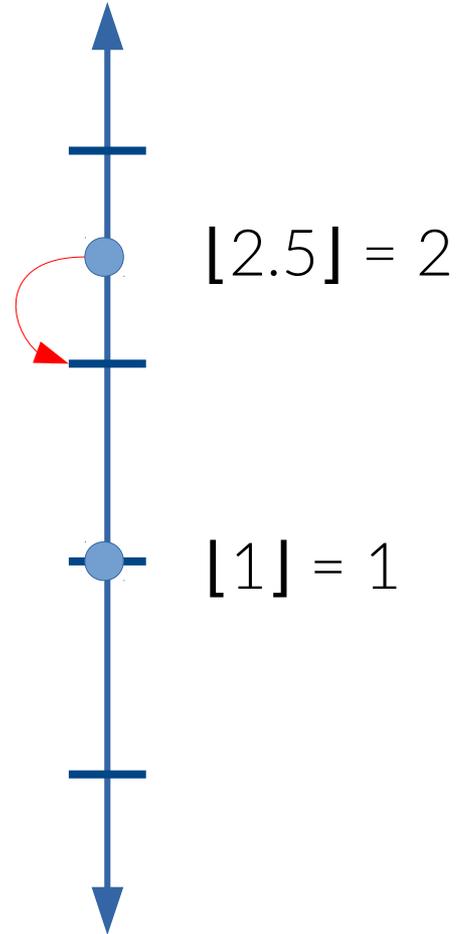
$\lfloor x \rfloor$

$\lceil x \rceil$

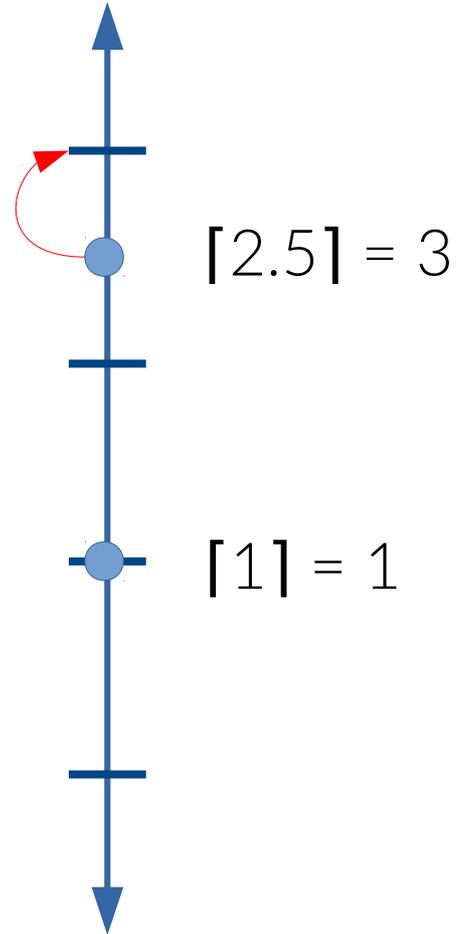
# Floor and ceiling



# Floor and ceiling

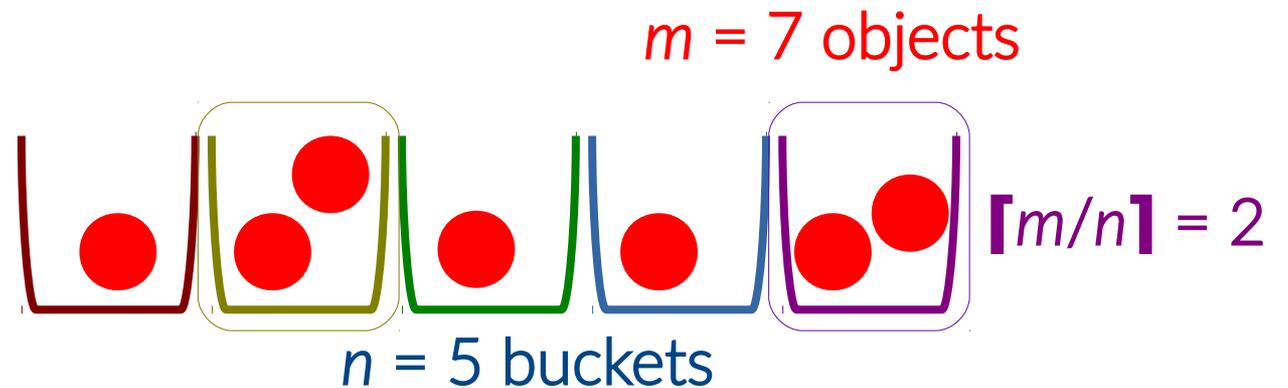


# Floor and ceiling



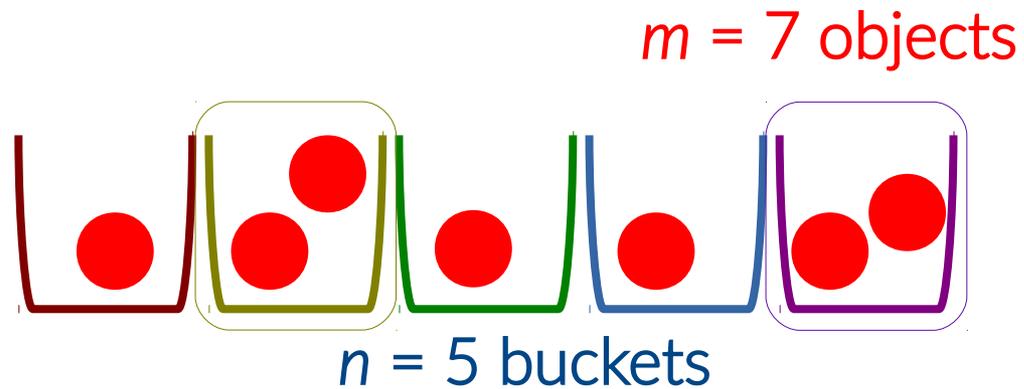
# General Pigeonhole Principle

If  $m$  objects are placed in  $n$  buckets, then **at least one bucket** must contain at least  $\lceil m/n \rceil$  objects.



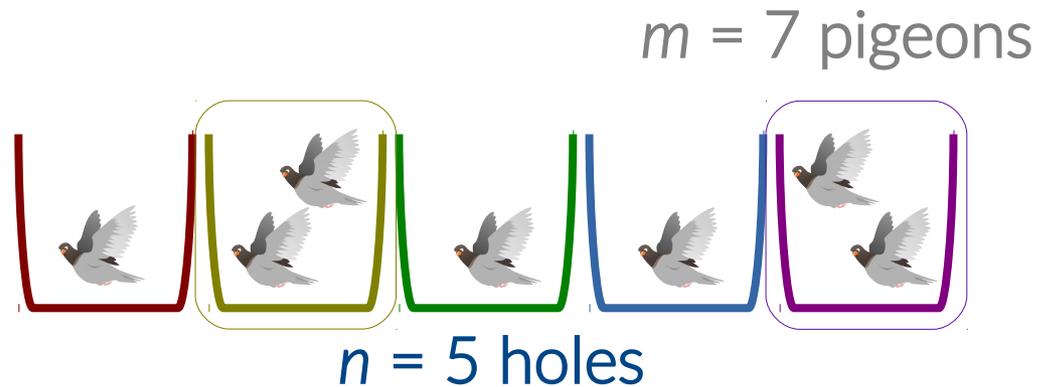
# Pigeonhole Principle intuition

More objects than buckets = Some objects have to share!



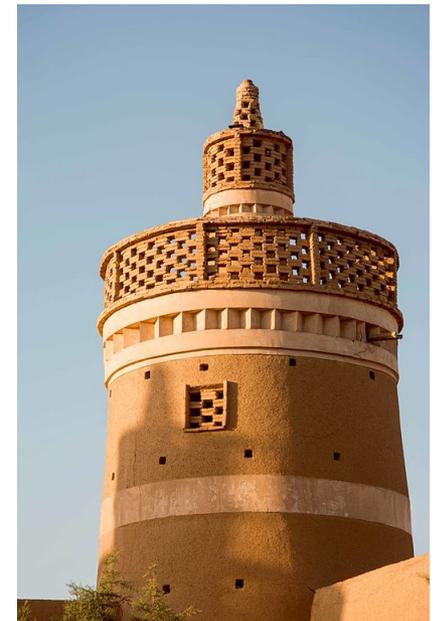
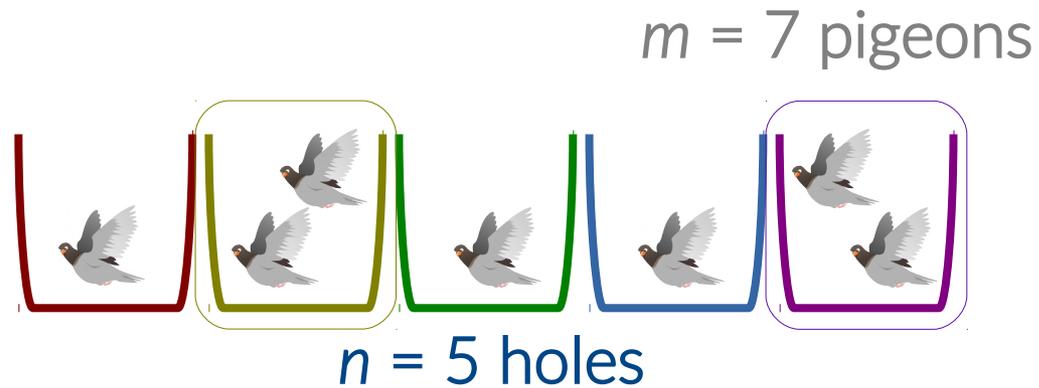
# Pigeonhole Principle intuition

More pigeons than holes = Some pigeons have to share!

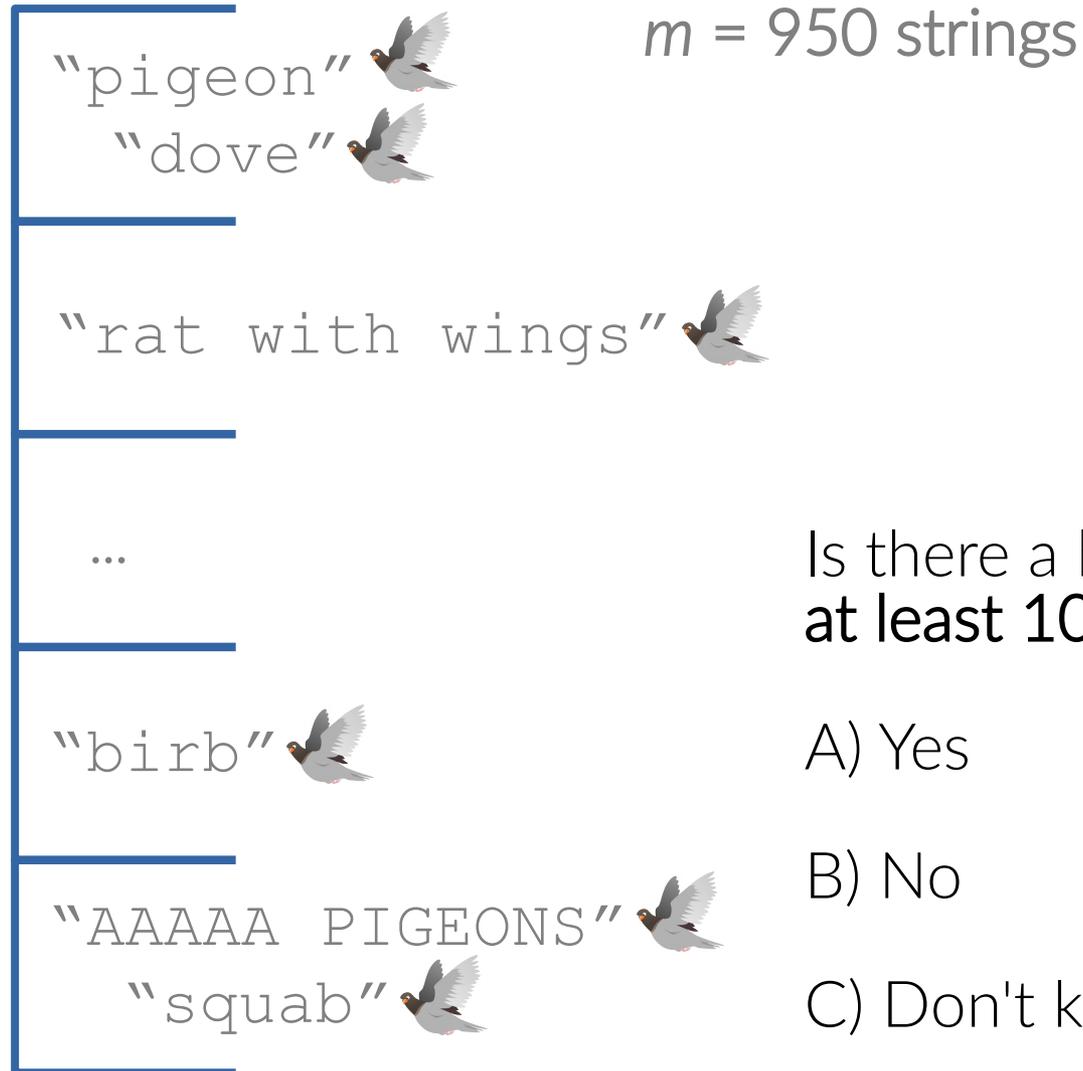


# Pigeonhole Principle intuition

More pigeons than holes = Some pigeons have to share!



# String hashing

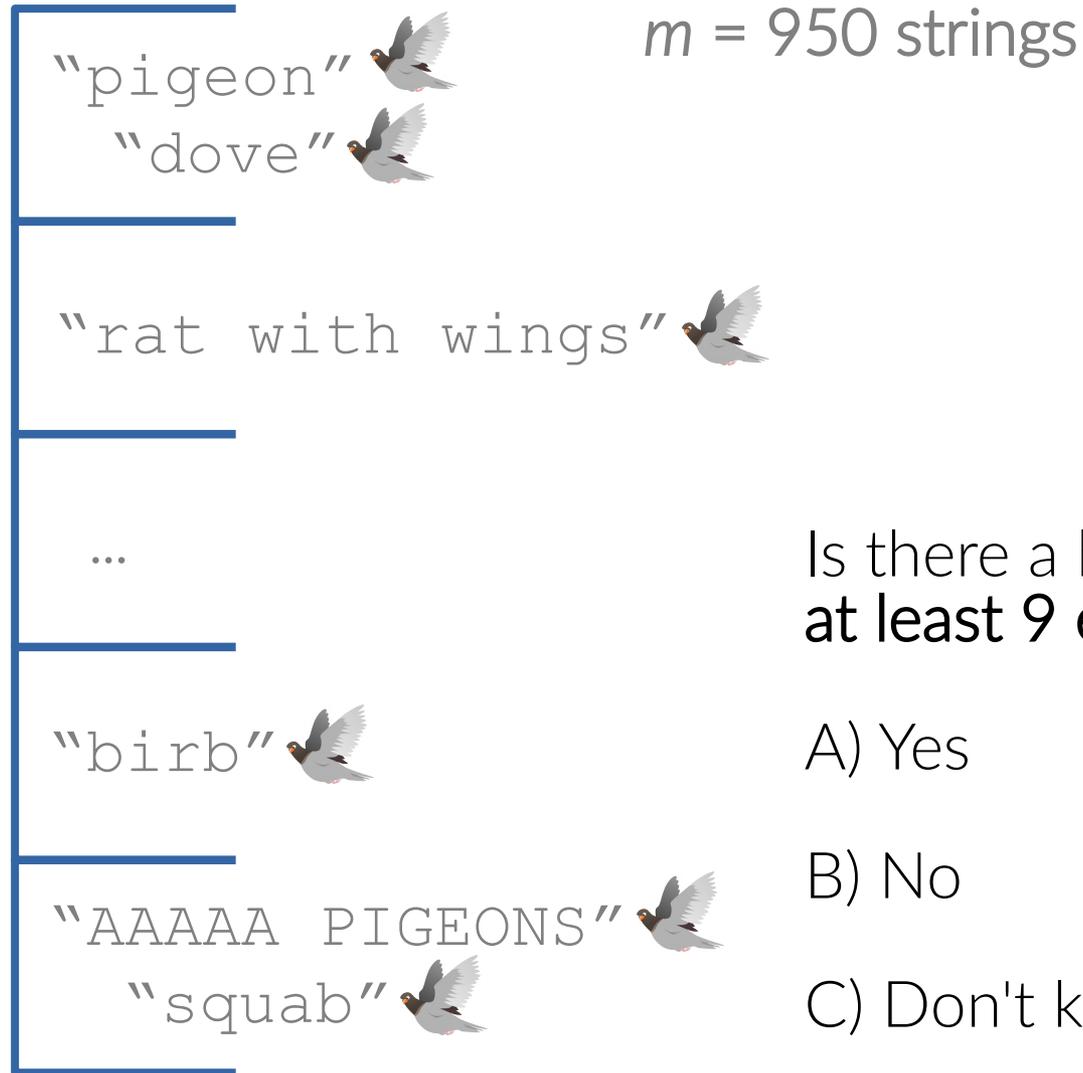


$n = 100$  buckets

Is there a bucket with at least 10 entries?

- A) Yes
- B) No
- C) Don't know

# String hashing



$n = 100$  buckets

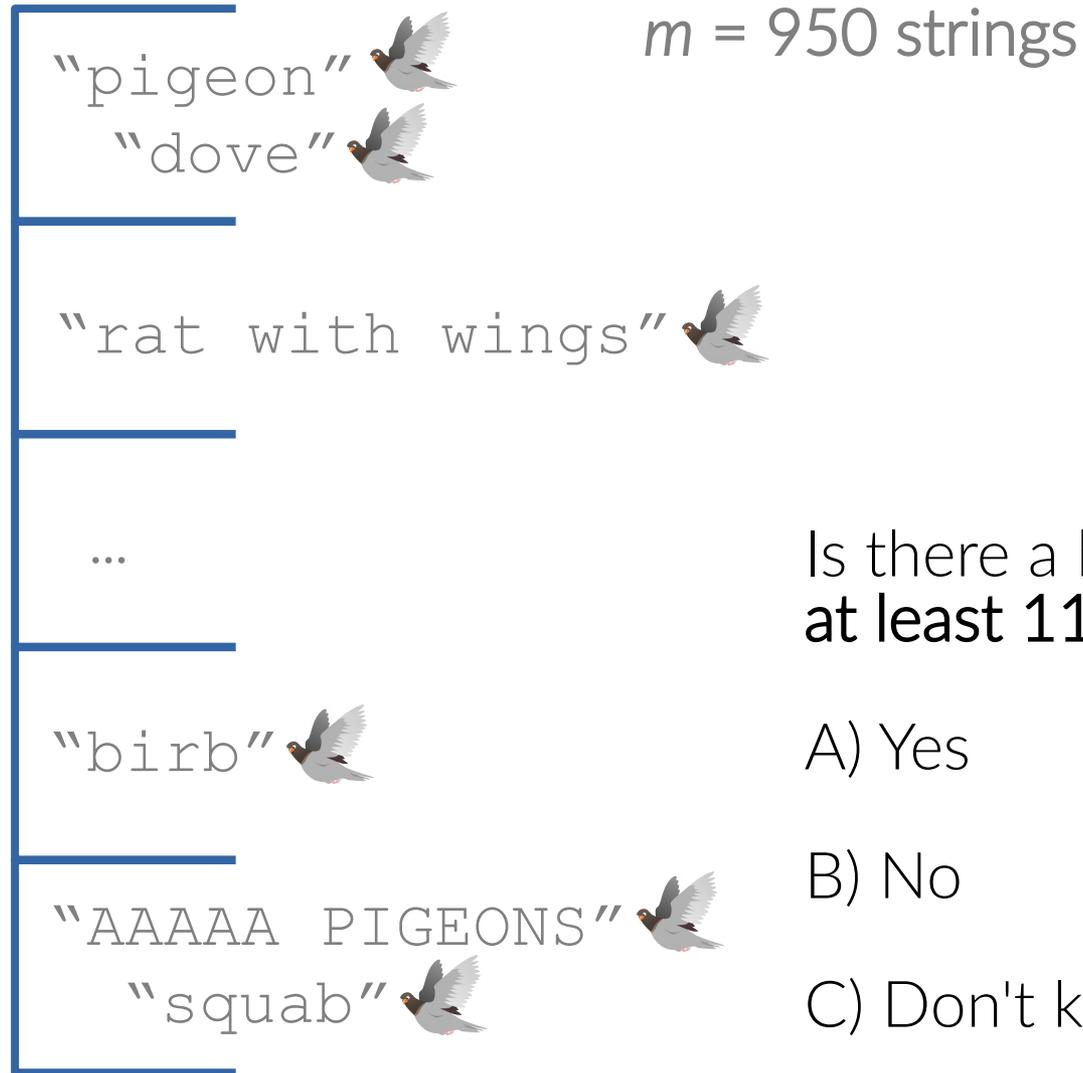
Is there a bucket with at least 9 entries?

A) Yes

B) No

C) Don't know

# String hashing

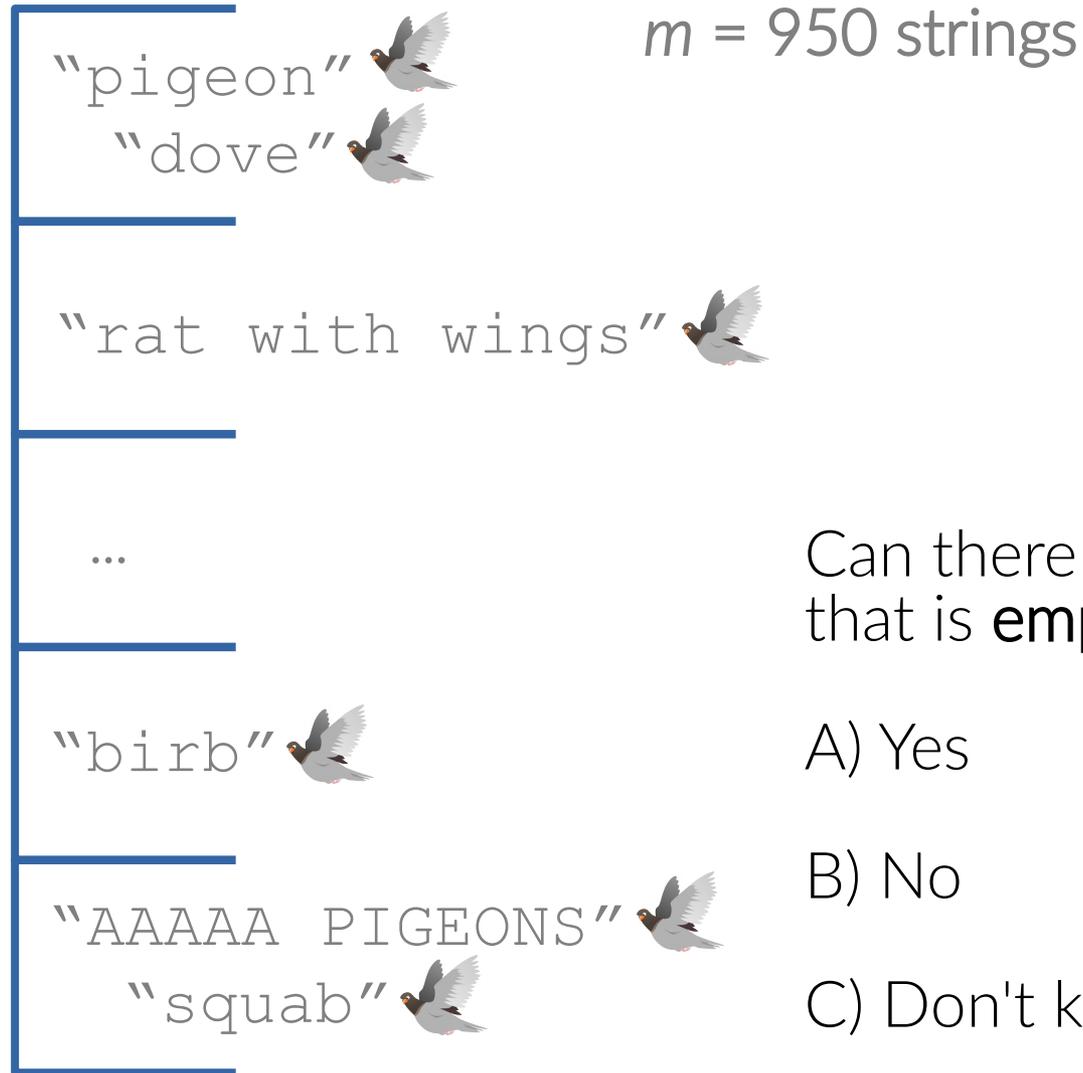


$n = 100$  buckets

Is there a bucket with at least **11** entries?

- A) Yes
- B) No
- C) Don't know

# String hashing



$n = 100$  buckets

Can there be a bucket that is **empty**?

A) Yes

B) No

C) Don't know

Break time!

A close-up photograph of a bicycle tire tread and spokes. The tire is black with a prominent tread pattern. The spokes are silver and arranged in a radial pattern. The background is blurred, showing more of the bicycle's frame and wheels. A red banner is overlaid at the bottom of the image.

# Course Logistics

# Course Website



<https://cs109.stanford.edu/>

# Teaching assistants



Michela Meister



Yuling Liu

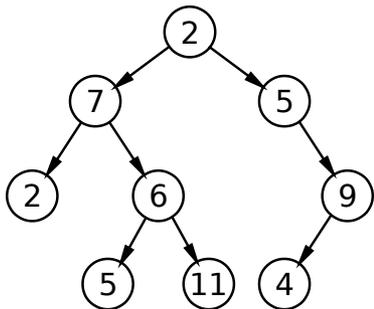


Brendan Corcoran

# Prereqs

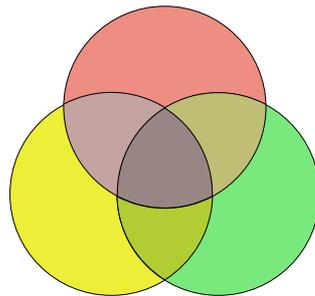
CS 106B/X

Programming  
Recursion  
Hash tables  
Binary trees



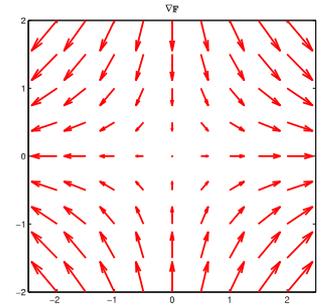
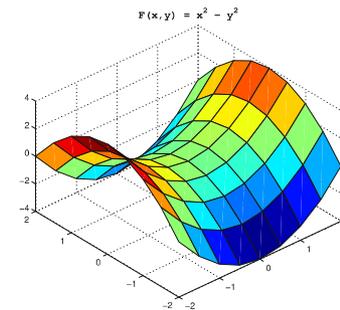
CS 103

Proofs (induction)  
Set theory

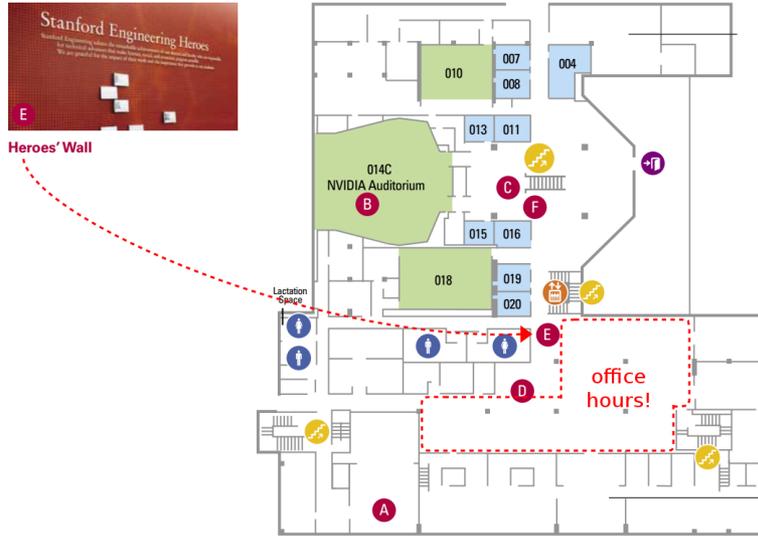


MATH 51/  
CME 100

Multivariate differentiation  
Multivariate integration  
Basic linear algebra  
(what's a vector?)



# Contacting us



Office hours

plazza

Forum

[cs109-sum1617-staff@lists.stanford.edu](mailto:cs109-sum1617-staff@lists.stanford.edu)

Email

# Units

Undergrads and  
SCPD students

5

Grad students

3, 4, 5

(same amount of work!  
~10 hours/week for  
assignments)

# Recorded lectures



<https://mvideox.stanford.edu/>

# Late days

[updated  
July 14]



free 24-hour “extensions”

# Grade components

45%	6 problem sets
20%	Midterm exam Tuesday, July 25, 7:00-9:00pm [email staff ASAP with conflicts]
30%	Final exam Saturday, August 19, 12:15-3:15pm [no alternate time!]
5%	Participation: Piazza, feedback

# Honor code

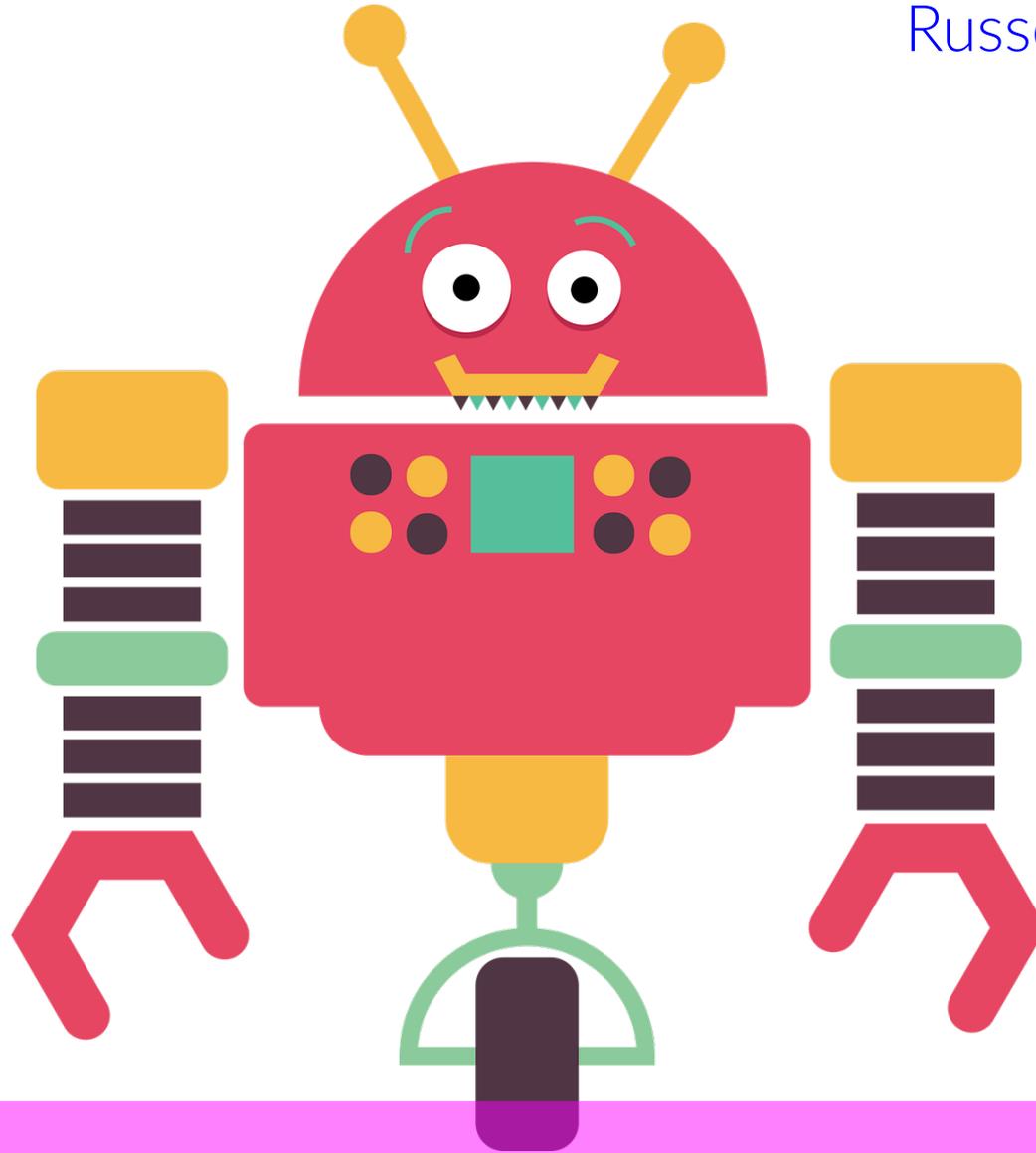
You **may**:

Talk with the course staff  
Talk with classmates  
(cite collaboration)  
Look up general material online

You **may not**:

Copy answers:  
from classmates  
from former students  
from the Internet  
Look at previous quarters' solutions

further reading:  
Russell & Norvig (2009)



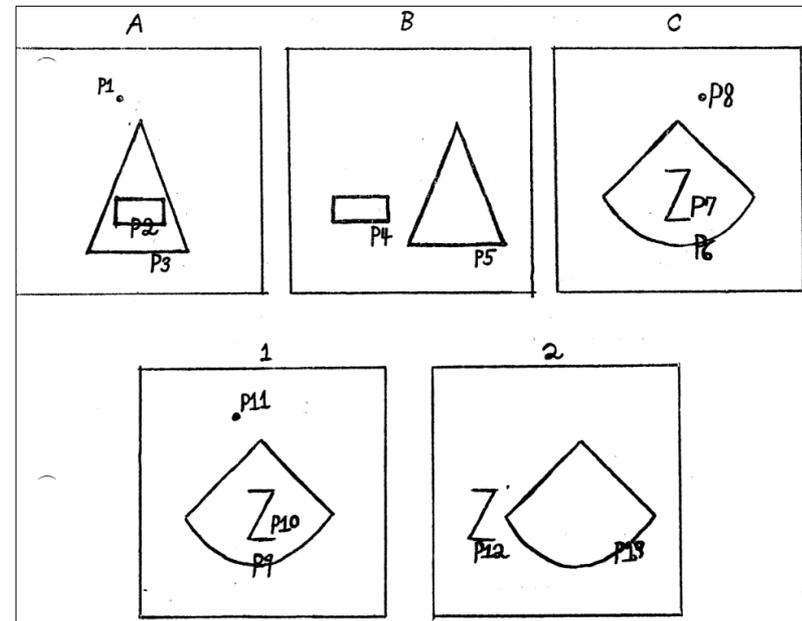
A very brief history of AI

# Early AI optimism

1952



1962



Evans (1962)

# Early AI optimism

"...in a visible future—the range of problems [AIs] can handle will be coextensive with the range to which the human mind has been applied."

–Herbert Simon, 1957  
(quoted in Russell & Norvig, 2003)

# Disillusionment (1974-1980)

“The spirit is willing but the flesh is weak”



(a plausible  
Russian  
translation)

“Спирт готов но мясо слабо.”



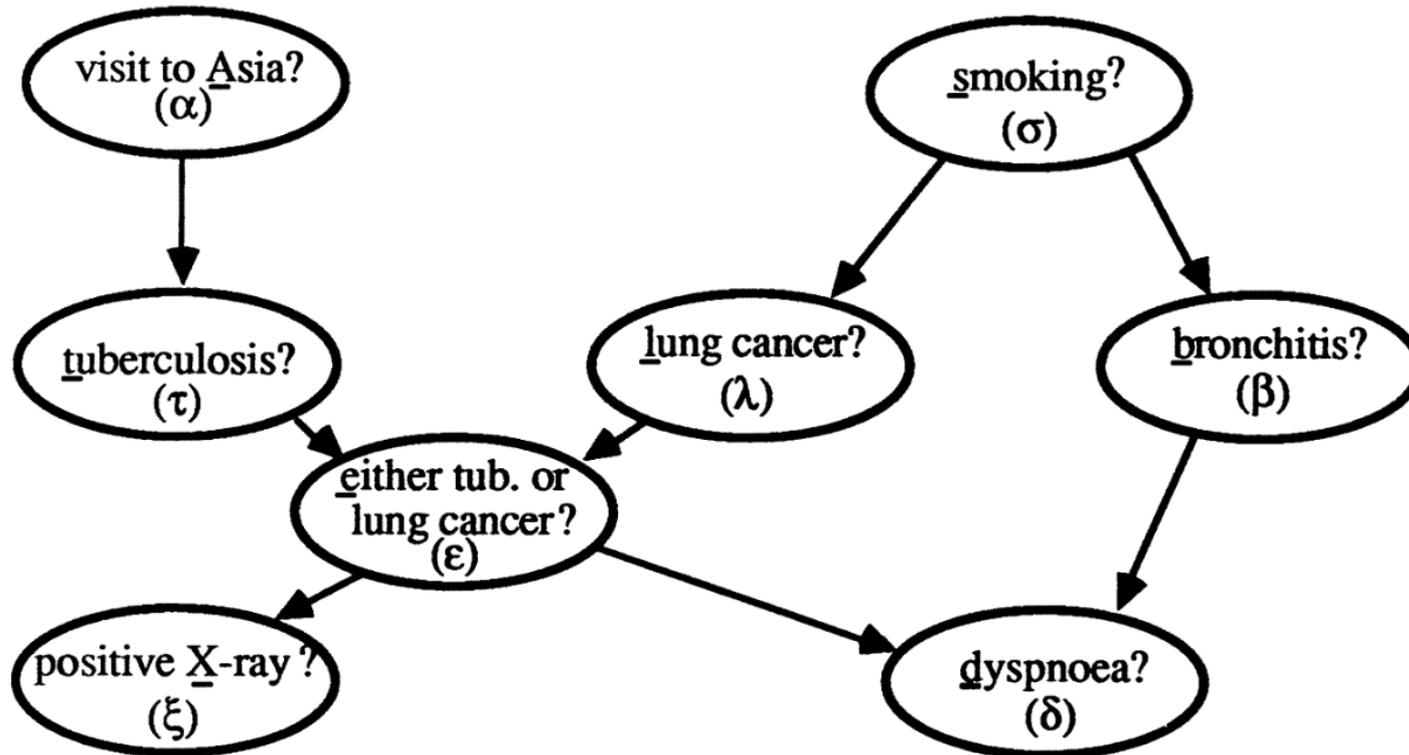
“The vodka is good but the meat is rotten”

**BRACE YOURSELVES**

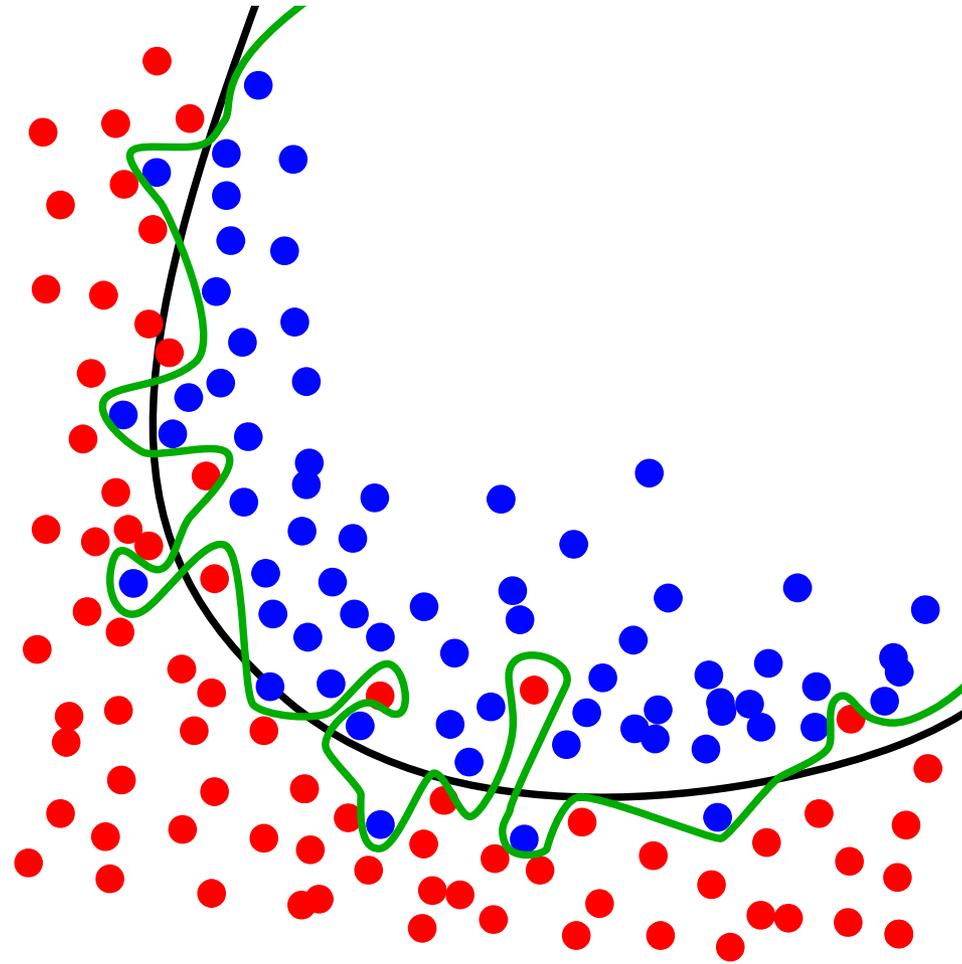


**WINTER IS COMING**

# Expert systems (1980s)

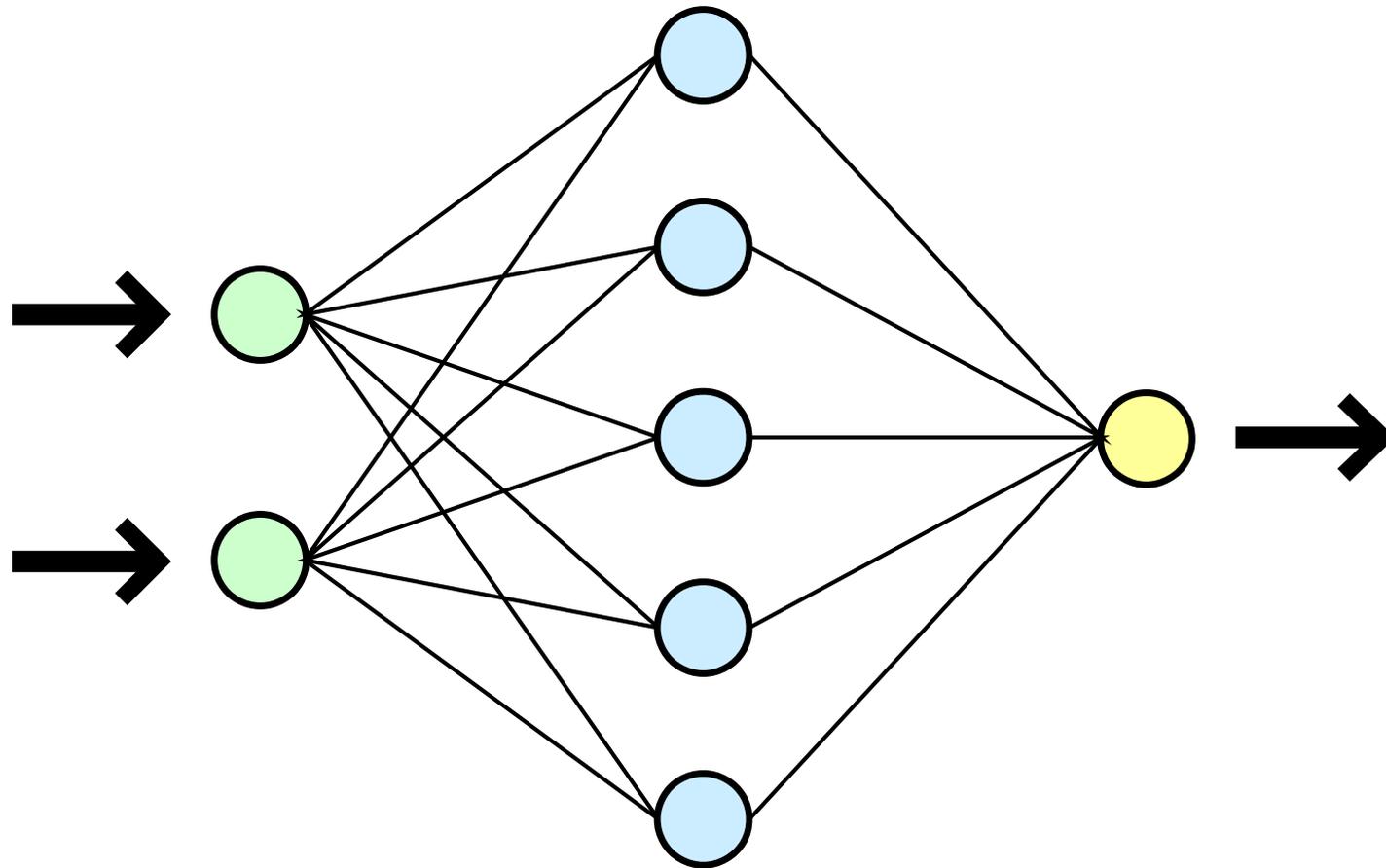


# The statistical revolution (1990s)



examples  $\rightarrow$  probability

# Neural nets return (2000s)



# Speech recognition



who is the current president of France?

# Image classification



## Easiest classes



## Hardest classes



# Solving speech recognition?

PROBABILITY

Cards ♡ ♦ ♠ ♣ 13 diff. cards  
4 each

red  $\frac{1}{2} \rightarrow 50\%$

diamond  $\frac{1}{4} \rightarrow 25\%$

#card:  
(including ace)

foods including ace other including a  
stick their nose in it or I don't worry

Stephen A. Smith

15:01 / 21:29

Teacher: ...includes— including ace.  
Student: 40.  
Teacher: (writing) —including—ace.  
Student: Isn't it 4 out of 40...?

# Solving object recognition?

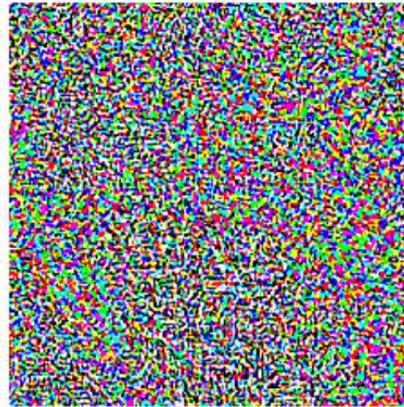


$\mathbf{x}$

“panda”

57.7% confidence

+ .007 ×



$\text{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y))$

“nematode”

8.2% confidence

=



$\mathbf{x} +$

$\epsilon \text{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y))$

“gibbon”

99.3 % confidence

# Solving question answering?

Q 🔒 how long has theresa may been pm ↻



how long has theresa may been pm × 🔍

[ALL](#) [NEWS](#) [VIDEOS](#) [IMAGES](#) [MAPS](#)

Theresa May / Height

1.72 trillion pm

