Will Monroe
CS 109

July 19, 2017

Practice Midterm Examination

Problems by Mehran Sahami and Chris Piech

This is a closed book, closed calculator, closed computer exam. You are, however, allowed to use one 8½" × 11" page (front and back; or two pages, one side only) of notes in the exam. The last page of the exam is a Standard Normal Table, in case you need it.

You have 2 hours (120 minutes) to take the exam. The exam is 120 points, meant to roughly correspond to one point per minute of the exam. You may want to use the point allocation for each problem as an indicator for pacing yourself on the exam.

In the event of an incorrect answer, any explanation you provide of how you obtained your answer can potentially allow us to give you partial credit for a problem. For example, describe the distributions and parameter values you used, where appropriate. It is fine for your answers to include summations (but not integrals), products, factorials, exponentials, and combinations, unless the question specifically asks for a numeric quantity or closed form. Where numeric answers are required, the use of fractions is fine.

THE STANFORD UNIVERSITY HONOR CODE

A. The Honor Code is an undertaking of the students, individually and collectively:

   (1) that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;

   (2) that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

B. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid as far as practicable, academic procedures that create temptations to violate the Honor Code.

C. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to create optimal conditions for honorable academic work.

I acknowledge and accept the letter and spirit of the Honor Code:

Signature: _____

Name (print): _____

SUNetID: _____

Email (preferably as used on Gradescope): _____

1. [15 points] Will comes to class with 12 drinks in his cooler as follows:
   - 5 CapriSuns (Drink type A)
   - 4 Cokes (Drink type B)
   - 3 melted Otter Pops (Drink type C)

   Note that all drinks of the same type are *indistinguishable*.

   a. In how many distinct ways can Will distribute the drinks to 12 students, where each student only gets one drink?

   b. Say that out of the 12 students, there is a particular pair of students (call them Larry and Sergey) who are only happy if they both receive the same type of drink. How many distinct ways can drinks be distributed to the 12 students (where each student only gets one drink) such that Larry and Sergey are happy?

   c. Now say that Will again starts with the 12 drinks described above, but there are only 10 students in the class. He distributes drinks to all 10 students (where each student only gets one drink, so 2 drinks remain in Will's bag). In how many distinct ways can drinks be distributed under these conditions?

2. [20 points] You arrive at a party and see that there are two pizzas (one cheese and one pepperoni) that each start with 12 slices. Slices of pizza are eaten sequentially, where it is equally likely that each slice eaten is either cheese or pepperoni (assuming that slices of both types remain). What is the probability that *at least* 3 slices of the pepperoni pizza remain at the time that the last slice of the cheese pizza is eaten?

   (Recall that it is fine for your answers below to include summations, products, factorials, exponentials, or combinations.)

3. [20 points] Say you have 500 songs on your MP3 player. You put it in "random" mode, causing songs to be randomly selected (with replacement) with equal probability. While doing your CS109 problem set, you listen to 200 songs.

   (Recall that it is fine for your answers below to include summations, products, factorials, exponentials, or combinations. You can also define intermediate variables in your answer as long as you clearly state how to compute their values.)

   a. Say we now randomly (with equal probability) select a song from your MP3 player. What is the *exact* probability you heard that particular song *more than* 4 times while doing your CS109 problem set?

   b. What is the (approximate) probability that there are exactly 3 songs that you heard *more than* 4 times each?

4. [20 points] Four 6-sided dice are rolled. The dice are fair, so each one has equal probability of producing a value in $\{1, 2, 3, 4, 5, 6\}$. Let X = the *minimum* of the four values rolled. (It is fine if more than one of the dice has the minimal value.)

   (Note: You can define intermediate variables in your answers as long as you clearly state how to compute their values.)

   a. What is $P(X \geq k)$ as a function of k?

   b. What is E[X]?

   c. Let T = the sum of the values rolled on the four dice. Let S = the sum of the largest *three* values on the four dice. In other words, $S = T - X$. What is $E[S]$?

5. [20 points] Spotify notices that users do not listen to songs with equal probability. Instead, the probability that a random play (an instance of a user listening to a song) is of the i-th most popular song is distributed as a Zipf random variable. As we (briefly) covered in class, the Zipf random variable is discrete and has PMF:

$$P(X = i) = \frac{\frac{1}{i}}{\sum_{n=1}^{N} \frac{1}{n}}$$

It is parameterized by N, the total number of songs. Spotify has $N =$ 30 million ($3 \cdot 10^7$) songs.

It may be useful in this problem to know that $\sum_{n=1}^{3 \cdot 10^7} \frac{1}{n} \approx 17.8$.

a. What is the probability that a random play is of the *10th most popular* song?

b. If there are 1 billion plays on a given day, what is the probability that the most popular song is listened to more than 100 million times that day? Use an approximation. Remember you can include summations in your answers and use variables to represent intermediate values, as long as you define clearly how to compute each one.

6. [25 points] Say that two different manufacturers (call them A and B) are equally likely to produce screens for laptops. The lifetimes for the screens (measured in hundreds of hours) manufactured by each company are *independently* distributed as follows:

   - Manufacturer A: lifetime of screens are normally distributed: $N(20, 4)$
   - Manufacturer B: lifetime of screens are exponentially distributed: $\text{Exp}(1/20)$

   Say we bought a laptop, have used it for 18 hundred hours so far, and the screen is still working at this point in time.

   a. At this point in time, what is the probability that manufacturer A produced the screen?

   b. At this point in time, what is the probability that manufacturer B produced the screen?

Standard Normal Table

An entry in the table is the area under the curve to the left of z, $P(Z \leq z) = \Phi(z)$.

Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998