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CS109

Section #5
Oct 31, 2018

Section #5: Code Analysis Solution

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1. Binary Tree:

Let X_1 and X_2 be number of nodes the left and right calls to `randomTree`.
$E[X_1] = E[X_2] = E[X]$.

$$\begin{aligned} E[X] &= p \cdot E[X \mid \texttt{if}] + (1-p)E[X \mid \texttt{else}] \\ &= p \cdot E[1 + X_1 + X_2] + (1-p) \cdot 0 \\ &= p \cdot (1 + E[X] + E[X]) \\ &= p + 2pE[X] \\ (1-2p)E[X] &= p \\ E[X] &= \frac{p}{1-2p} \end{aligned}$$

2. Girl Scout Cookies

a. $X = Noa + Chaya \sim N(\mu + \mu = 2\mu, \sigma^2 + \sigma^2 = 2\sigma^2)$

b. $Y = 2 \cdot Maria \sim N(\mu \cdot 2 = 2\mu, (2\sigma)^2 = 4\sigma^2)$

3. Timing Attack

a. Let Y be the amount of time to execute k lines. $Y = \sum_{i=1}^{k} X_i$ where X_i is the amount of time to execute line i. $X_i \sim N(\mu = 5, \sigma^2 = 0.5)$.
Since Y is the sum of independent normals:

$$\begin{aligned} Y &\sim N(\mu = \sum_{i=1}^{k} 5, \sigma^2 = \sum_{i=1}^{k} 0.5) \\ &\sim N(\mu = 5k, \sigma^2 = 0.5k) \end{aligned}$$

b. From last problem:
Time to run 6 lines of code $A \sim N(\mu = 30, \sigma^2 = 3)$
Time to run 4 lines of code $B \sim N(\mu = 20, \sigma^2 = 2)$

$$\begin{aligned} -B &\sim N(\mu = -20, \sigma^2 = 2) \\ A - B &\sim N(\mu = 10, \sigma^2 = 5) \\ P(A > B) &= P(A - B > 0) \\ &= 1 - F_{A-B}(0) \\ &= 1 - \Phi\Big(\frac{0 - 10}{\sqrt{5}}\Big) \\ &\approx 1.0 \end{aligned}$$

c. Let M be the event that the first letter matched.

$$\begin{aligned}
\frac{P(M^C|T=27)}{P(M|T=27)} &= \frac{f(T=27|M^C)P(M^C)}{f(T=27|M)P(M)} \\
&= \frac{f(T=27|M^C)\frac{25}{26}}{f(T=27|M)\frac{1}{26}} \\
&= \frac{25}{26} \cdot \frac{f(T=27|M^C)}{f(T=27|M)} \\
&= \frac{25}{26} \cdot \frac{\frac{1}{\sqrt{6\pi}} e^{-\frac{(27-30)^2}{6}}}{\frac{1}{\sqrt{8\pi}} e^{-\frac{(27-40)^2}{8}}} \\
&= \frac{25}{26} \cdot \frac{\sqrt{8}}{\sqrt{6}} \cdot \frac{e^{-\frac{9}{6}}}{e^{-\frac{169}{8}}} \\
&\approx 370 \text{ million}
\end{aligned}$$

d. $7 \cdot 26 + 7 = 189$