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CS109

Summer 2019
July 23rd

CS109 Midterm Exam

This is a closed calculator/computer exam. You are, however, allowed to use notes in the exam. The last page of the exam is a Standard Normal Table, in case you need it. You have 2 hours (120 minutes) to take the exam. The exam is 120 points, meant to roughly correspond to one point per minute of the exam. You may want to use the point allocation for each problem as an indicator for pacing yourself on the exam.

In the event of an incorrect answer, any explanation you provide of how you obtained your answer can potentially allow us to give you partial credit for a problem. For example, describe the distributions and parameter values you used, where appropriate. It is fine for your answers to include summations, products, factorials, exponents, integrals, and combinations, unless the question specifically asks for a numeric quantity or closed form. Where numeric answers are required, the use of fractions is fine.

I acknowledge and accept the letter and spirit of the honor code. I pledge to write more neatly than I have in my entire life:

Signature: _____

Family Name (print): _____

Given Name (print): _____

Email (preferably your gradescope email): _____

1 Year vs. Living Situation [15 points]

Below is a joint probability table that gives the relationship between student year and living situation for 109 students:

	Frosh	Sophomore	Junior	Senior	5+
One Room Double	0.3	0.12	0.04	0.01	0.0
Two Room Double	0.1	0.2	0.01	0.01	0.0
Single	0.0	0.05	0.1	0.01	a

a. (2 points) Either describe how to find or give an expression for the probability that a random 109 student is in their 5th+ year and is living in a single (denoted by "a" in the table).

b. (4 points) What is the probability that a random Junior in 109 is living in a single?

c. (4 points) If F is the event that a random student is a Frosh and S is the event that a random student is living in a Single, are F and S independent? Justify your answer.

d. (5 points) A student in a one room double shares their bedroom with one person, while students in two room doubles and singles share their bedrooms with no one. Give an expression for the expected number of people that a random student shares their bedroom with. You can refer to your answer to part a as "a."

2 Class Choice [25 points]

90 students are picking classes. There are 5 STEM classes, 10 humanities classes, and three writing classes available. Each student is required to take exactly two STEM classes, exactly two humanities classes, and exactly one writing class.

a. How many different ways are there for a single student to choose their classes if...

i. (3 points) there are no added constraints?

ii. (5 points) there are two STEM classes that conflict with one another (i.e. cannot be taken together) and one humanities class that conflicts with one writing class?

b. Let a “roster” refer to an assignment of the 90 (distinct) students to the three writing classes (where each student takes exactly one writing class). How many different rosters are possible if...

   i. (3 points) there are no added constraints?

   ii. (4 points) each class must have exactly 30 students in it?

   iii. (10 points) each class can have at most 40 students in it?

3 Infection Treatment [25 points]

Three drugs called A, B, and C are used to treat a particular infection. The effectiveness of each drug on a particular patient depends on whether or not that patient carries gene G, which 60% of people have. Specifically, if a patient carries gene G, then drugs A, B, and C have probabilities p_A, p_B, and p_C respectively of curing the patient. If a patient does not carry gene G, then the drugs have probabilities q_A, q_B, and q_C of curing the patient.

If a patient takes multiple drugs, then each drug works independently (conditioned on the presence or absence of gene G) to cure the infection. If at least one drug cures the patient, then the patient is cured. If all of the drugs fail to cure the patient, then the patient is not cured.

a. (4 points) A patient carrying gene G is given all three drugs. What is the probability that they will be cured?

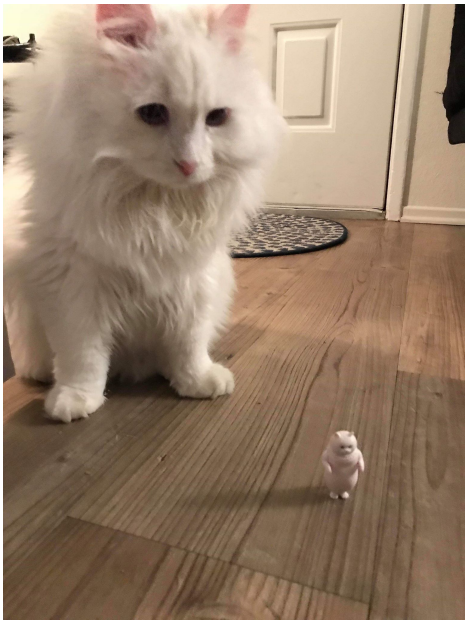
b. (4 points) A random patient is given drug A. What is the probability that they will be cured?

c. (7 points) A patient takes all three drugs and is cured. What is the probability that they carry gene G?

d. (10 points) A patient reports having been cured after taking a single drug but is unsure what drug they took. They also do not know whether they carry gene G. If each of the three drugs is equally likely to be prescribed, then what is the probability that the patient took drug A?

4 Collecting Cat Figurines [35 points]

Tycho is collecting cat figurines. Unfortunately the figurines come in identical boxes, so he does not know which figurine he will get until after he buys it. There are four types of figurines, the Siberian, the Maine Coon, the Ragdoll, and the Burmese, which have respective probabilities p_S, p_M, p_R, and p_B of showing up in a given box.



a. (5 points) If Tycho buys 12 figurines, what is the probability that he will get exactly 3 of each type?

b. Just for this part, assume that $p_S = 0.5$. Tycho buys 100 figurines.

   i. (6 points) If Tycho can sell each Siberian figurine for \$5, what is the variance of the amount of money he can sell the Siberian figurines for?

ii. (8 points) What is the approximate probability that he gets 45 or more Siberians? Give a numerical answer.

c. For the rest of this problem, assume that $p_S = p_M = p_R = p_B = 0.25$. Tycho will continue buying figurines until he has one of each type.

i. (6 points) Say Tycho has one of each figurine except for the Siberian. What is the expected number of boxes he has to buy to complete his collection?

ii. (10 points) If Tycho starts with no figurines, what is the expected number of boxes he has to buy to complete his collection? HINT: break the process down into four steps. Feel free to refer to the solution to part c.i. in your answer even if you did not solve it.

5 Supercomputer Jobs [20 points]

It is believed that the lengths of jobs assigned to supercomputers follows a Lomax Distribution. The Lomax has two parameters, α and λ, and can take on any non-negative value:

If $X \sim \text{Lomax}(\alpha, \lambda)$ it has:

PDF: $$f_X(x) = \begin{cases} \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

CDF: $$F_X(x) = \begin{cases} 1-\left(1+\frac{x}{\lambda}\right)^{-\alpha} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Deep Thought is a supercomputer, and $X \sim Lomax(2,4)$ represents the number of minutes that a random job run on Deep Thought takes to finish. Your answers can include integrals.

a. (3 points) Give an expression for the probability that a random job run on Deep Thought takes greater than 5 and less than 10 minutes to finish. No need to simplify.

b. (7 points) Let's say that a job has been running for 5 minutes but isn't done yet. What is the probability that it lasts at least 5 more minutes (10 or more total)?

c. You decide that it would be simpler to model job lengths with an exponential distribution, $Y \sim Exp(R)$.

i. (5 points) Give an expression for Y's rate parameter, R. Use the Method of Moments.

ii. (5 points) Repeat part b using the assumption that job lengths follow our new distribution, Y. Leave your answer in terms of R. For reference, part b is: *Let's say that a job has been running for 5 minutes but isn't done yet. What is the probability that it lasts at least 5 more minutes (10 or more total)?*

Standard Normal Table

An entry in the table is the area under the curve to the left of z, $P(Z \leq z) = \Phi(z)$.

Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998