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CS109

Section #2
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# Section #2 Concept Check

## 1 Lecture 5, 1-15-20: Independence

1. Definitions: Cite Bayes' Theorem.
2. True or False. Note that true means *always* true.
   (a) In general, $P(A, B|C) = P(B|C)P(A|B, C)$.
   (b) If $A$ and $B$ are independent, so are $A$ and $B^C$.

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1. Bayes' Theorem: $P(E|F) = \frac{P(F|E)P(E)}{P(F)}$
2. (a) True
   (b) True

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## 2 Lecture 6, 1-17-20: Random Variables and Expectation

1. Definitions:
   (a) If $X$ is a random variable, what is $E[X]$? What is $E[g(X)]$?
   (b) For random variables $X_1, \ldots, X_n$, what is $E[\sum_{i=1}^n X_i]$?
2. True or False: For any random variable $X$, $E[X^2] = E[X]^2$.
3. Short Answer: Let $X$ = the value on one roll of a 6 sided die. Recall that $E[X] = 7/2$. What is $\text{Var}(X)$?

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1. Definitions:
   (a) $E[X] = \sum_x x p_X(x)$ and $E[g(X)] = \sum_x g(x) p_X(x)$.
   (b) $E[\sum_{i=1}^n X_i] = \sum_{i=1}^n E[X_i]$
2. False
3. Remember that $\text{Var}(X) = E[X^2] - E[X]^2$. $E[X^2] = (1^2)\frac{1}{6} + (2^2)\frac{1}{6} + (3^2)\frac{1}{6} + (4^2)\frac{1}{6} + (5^2)\frac{1}{6} + (6^2)\frac{1}{6} = \frac{91}{6}$. Thus, $\text{Var}(X) = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12}$.

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## 3 Lecture 6, 1-22-20: Variance, Bernoulli, Binomial

1. Definitions: PMF for $X \sim Binomial(n, p)$. What is $p_X(k)$?
2. Short Answer: Let $X$ be the number of flips of a coin with $P(head) = p$ up to and including the first head. What is the range of $X$ and $p_X(k)$?

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1. $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
2. Range: $\{1, 2, \ldots\} = \mathbb{N}$. $P(X = k) = (1-p)^{k-1} p$.