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CS109

Section #4 Concept Check Solutions

1 Lecture 11, 1-31-20: Joint Distributions

1. Given a Normal RV $X \sim N(\mu, \sigma^2)$, how can we compute $P(X \leq x)$ from the standard Normal distribution Z with CDF ϕ?
2. What is a continuity correction and when should we use it?
3. If we have a joint PMF for discrete random variables $p_{X,Y}(x, y)$, how can we compute the marginal PMF $p_X(x)$?

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1. First, we write $\phi((x - \mu)/\sigma)$. We then look up the value we've computed in the Standard Normal Table.
2. Continuity correction is used when a Normal distribution is used to approximate a Binomial. Since a Normal is continuous and Binomial is discrete, we have to use a continuity correction to discretize the Normal. The continuity correction makes it so that the normal variable is evaluated from + or - 0.5 increments from the desired k value.
3. The marginal distribution is $p_X(x) = \sum_y p_{X,Y}(x, y)$

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2 Lecture 12, 2-3-20: Continuous Joint Distributions

1. Consider a continuous joint distribution, (X, Y), where $X \in [0, 1]$ is the proportion of the time until the midterm that you actually study for it, and $Y \in [0, 1]$ is your percentage score on the exam. Set up but DO NOT EVALUATE any of your answers. Take care in setting up the limits of integration. The joint PDF is:

$$f_{X,Y}(x, y) = \begin{cases} ce^{-(y-x)} & x, y \in [0, 1] \text{ and } y \geq x \\ 0 & y < x \end{cases}$$

   (a) Sketch the joint range for this function and interpret it in English.
   (b) Write an expression that we could evaluate to find c
   (c) Write an expression to find the marginal PDF $f_Y(y)$. Carefully define it for all $y \in \mathbb{R}$ (piecewise).
   (d) Write an expression to calculate the $E[Y]$.

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1. (a) It looks like a triangle, and your score is at least the percentage of time you studied.
   (b)
$$c = \frac{1}{\int_0^1 \int_x^1 e^{-(y-x)} dy dx} = \frac{1}{\int_0^1 \int_0^y e^{-(y-x)} dx dy}$$
   (c) $f_Y(y) = \int_0^y ce^{-(y-x)} dx$ for $y \in [0, 1]$, else 0.
   (d) $E[Y] = \int_0^1 y f_Y(y) dy = \int_0^1 \int_0^y cye^{-(y-x)} dx dy$

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3 Lecture 13, 2-5-20: Independent Random Variables

1. What distribution does the sum of two independent binomial RVs $X + Y$ have, where $X \sim Bin(n_1, p)$ and $Y \sim Bin(n_2, p)$? Include the parameter(s) in your answer. Why is this the case?

2. What distribution does the difference of two independent normal RVs $X - Y$ have, where $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\nu, \tau^2)$? Include the parameter(s) in your answer.

3. If $Cov(X, Y) = 0$, are X and Y independent? Why or why not?

1. Binomial; $X + Y \sim Bin(n_1 + n_2, p)$

2. Normal; $X - Y \sim N(\mu - \nu, \sigma^2 + \tau^2)$

3. Not necessarily. Suppose there are three outcomes for X: let X take on values in $\{-1, 0, 1\}$ with equal probability $1/3$. Let $Y = X^2$. Then, $E[XY] = E[X^3] = E[X] = 0$ (since $X^3 = X$) and $E[X] = 0$, so $Cov(X, Y) = E[XY] - E[X]E[Y] = 0 - 0 = 0$ but X and Y are dependent since $P(Y = 1) = 2/3 \neq 1 = P(Y = 1|X = 1)$.