Tim Gianitsos, Alex Tsun
CS109

Section #5 Concept Check

1 Lecture 14, 2-7-20: Conditional Distributions

1. **True or False?**

   (a) Given marginal distributions $p_X(x)$ and $p_Y(y)$, it is possible to compute the joint distribution $p_{X,Y}(x, y)$.

   (b) Given the joint distribution $p_{X,Y}(x, y)$, it is possible to compute marginal distributions $p_X(x)$ and $p_Y(y)$.

   (c) In a conditional distribution table, all the elements add to one.

2. Assume the random variables X and Y are independent. Fill in the values for a and b in the following joint distribution table.

	$X = -3$	$X = 9076$
$Y = 112$	0.1	0.3
$Y = 147$	a	b

1. (a) False. We don't know how dependent X and Y are on each other (or whether they are independent), so we don't know how their probabilities will interact in a joint distribution.

   (b) True. The marginal for a value of X is the sum (or integral for continuous) of Y's across that value.

   (c) False. Only rows (or columns) sum to one. All the elements add to one in a *joint* distribution table (see lecture 14 slide 20).

2. First, observe that the actual values for X and Y (-3, 9076 & 112, 147 respectively) do not factor into our calculations for filling the table. Since X and Y are independent, we know that the row sum times column sum is equal to the cell where they intersect.

$$p_{X,Y}(x, y) = p_X(x)p_Y(y) = \left(\sum_{j \in range(Y)} p_{X,Y}(x, j) \right) * \left(\sum_{i \in range(X)} p_{X,Y}(i, y) \right)$$

From this, it follows that

$$0.1 = (0.1 + 0.3)(0.1 + a)$$

$$0.3 = (0.1 + 0.3)(0.3 + b)$$

Solving each equation for its respective variable results in $a = 0.15$, $b = 0.45$.

2 Lecture 15, 2-10-20: Correlation and Covariance

1. **True or False?** The symbol Cov is covariance, and the symbol ρ is Pearson correlation.

$X \perp Y \implies Cov(X,Y) = 0$	$Var(X + X) = 2Var(X)$
$Cov(X,Y) = 0 \implies X \perp Y$	$X \sim \mathcal{N}(0,1) \wedge Y \sim \mathcal{N}(0,1) \implies \rho(X,Y) = 1$
$Y = X^2 \implies \rho(X,Y) = 1$	$Y = 3X \implies \rho(X,Y) = 3$

1. **True or False?**

True	False ($\ldots = 4Var(X)$)
False (antecedent necessary, not sufficient)	False (don't know how independent X & Y are)
False ($Y = X \implies \ldots$)	False (... = 1)

3 Lecture 16, 2-12-20: Great Expectations

1. **Short Answer**. Let $X \sim Geo(p)$. Use the Law of Total Expectation to prove that $E[X] = 1/p$, by conditioning on whether the first flip is heads or tails.

1. $$E[X] = E[X|H]P(H) + E[X|T]P(T) = 1 \cdot p + (E[1 + X])(1 - p)$$

   Solving yields $E[X] = 1/p$.