

# 04: Conditional Probability and Bayes

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Lisa Yan

April 13, 2020

# Quick slide reference

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# Conditional Probability

# Dice, our misunderstood friends

Roll two 6-sided dice, yielding values  $D_1$  and  $D_2$ .

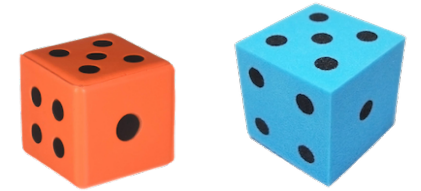
Let  $E$  be event:  $D_1 + D_2 = 4$ .

What is  $P(E)$ ?

$$|S| = 36$$

$$E = \{(1,3), (2,2), (3,1)\}$$

$$P(E) = 3/36 = 1/12$$



Let  $F$  be event:  $D_1 = 2$ .

What is  $P(E, \text{given } F \text{ already observed})$ ?

# Conditional Probability

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The **conditional probability** of  $E$  given  $F$  is the probability that  $E$  occurs given that  $F$  has already occurred. This is known as conditioning on  $F$ .

Written as:	$P(E F)$
Means:	“ $P(E, \text{ given } F \text{ already observed})$ ”
Sample space $\rightarrow$	all possible outcomes consistent with $F$ (i.e. $S \cap F$ )
Event $\rightarrow$	all outcomes in $E$ consistent with $F$ (i.e. $E \cap F$ )

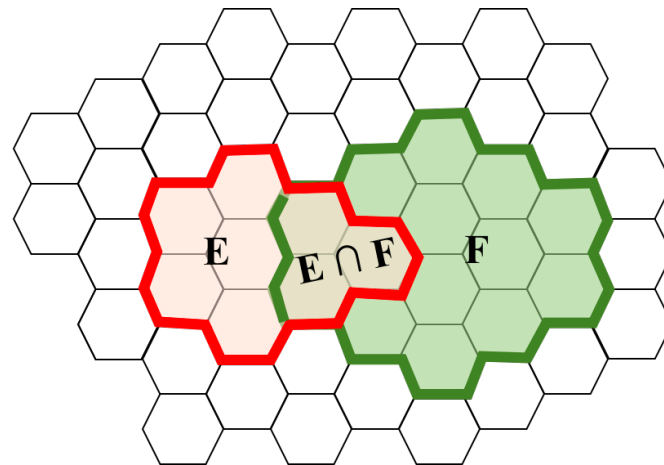
# Conditional Probability, equally likely outcomes

The **conditional probability** of  $E$  given  $F$  is the probability that  $E$  occurs given that  $F$  has already occurred. This is known as conditioning on  $F$ .

With **equally likely outcomes**:

$$P(E|F) = \frac{\# \text{ of outcomes in } E \text{ consistent with } F}{\# \text{ of outcomes in } S \text{ consistent with } F} = \frac{|E \cap F|}{|S \cap F|}$$

$$P(E|F) = \frac{|EF|}{|F|}$$



$$P(E) = \frac{8}{50} \approx 0.16$$

$$P(E|F) = \frac{3}{14} \approx 0.21$$

# Slicing up the spam

$$P(E|F) = \frac{|EF|}{|F|}$$

Equally likely  
outcomes

24 emails are sent, 6 each to 4 users.

- 10 of the 24 emails are spam.
- All possible outcomes are equally likely.

Let  $E$  = user 1 receives  
3 spam emails.

What is  $P(E)$ ?

Let  $F$  = user 2 receives  
6 spam emails.

What is  $P(E|F)$ ?

Let  $G$  = user 3 receives  
5 spam emails.

What is  $P(G|F)$ ?



# Slicing up the spam

$$P(E|F) = \frac{|EF|}{|F|} \quad \text{Equally likely outcomes}$$

24 emails are sent, 6 each to 4 users.

- 10 of the 24 emails are spam.
- All possible outcomes are equally likely.

Let  $E$  = user 1 receives 3 spam emails.

What is  $P(E)$ ?

$$P(E) = \frac{\binom{10}{3} \binom{14}{3}}{\binom{24}{6}} \approx 0.3245$$

Let  $F$  = user 2 receives 6 spam emails.

What is  $P(E|F)$ ?

$$P(E|F) = \frac{\binom{4}{3} \binom{14}{3}}{\binom{18}{6}} \approx 0.0784$$

Let  $G$  = user 3 receives 5 spam emails.

What is  $P(G|F)$ ?

$$P(G|F) = \frac{\binom{4}{5} \binom{14}{1}}{\binom{18}{6}} = 0$$

No way to choose 5 spam from 4 remaining spam emails!



# Conditional probability in general

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General **definition** of conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

The **Chain Rule** (aka Product rule):

$$P(EF) = P(F)P(E|F)$$

These properties hold even when outcomes are not equally likely.

**NETFLIX**

and Learn

# Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$
 Definition of Cond. Probability

Let  $E$  = a user watches Life is Beautiful.

What is  $P(E)$ ?

Equally likely outcomes?

$S = \{\text{watch, not watch}\}$

$E = \{\text{watch}\}$

$P(E) = 1/2$  ?



$$\checkmark P(E) = \lim_{n \rightarrow \infty} \frac{n(E)}{n} \approx \frac{\# \text{ people who have watched movie}}{\# \text{ people on Netflix}}$$

$$= 10,234,231 / 50,923,123 \approx 0.20$$

# Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$
 Definition of  
Cond. Probability

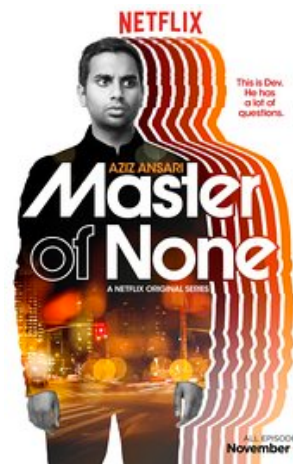
Let  $E$  be the event that a user watches the given movie.



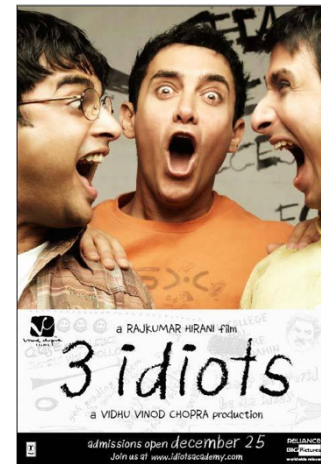
$$P(E) = 0.19$$



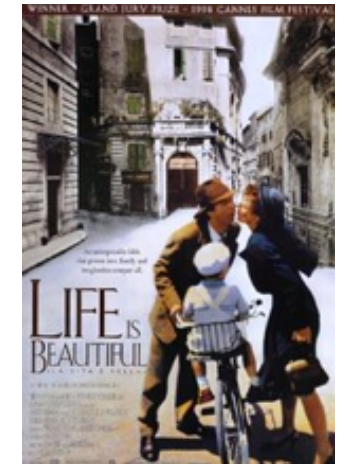
$$P(E) = 0.32$$



$$P(E) = 0.20$$



$$P(E) = 0.09$$



$$P(E) = 0.20$$

# Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)} \quad \text{Definition of Cond. Probability}$$

Let  $E$  = a user watches Life is Beautiful.

Let  $F$  = a user watches Amelie.

What is the probability that a user watches Life is Beautiful, given they watched Amelie?

$$P(E|F)$$

$$\begin{aligned} P(E|F) &= \frac{P(EF)}{P(F)} = \frac{\frac{\# \text{ people who have watched both}}{\# \text{ people on Netflix}}}{\frac{\# \text{ people who have watched Amelie}}{\# \text{ people on Netflix}}} \\ &= \frac{\# \text{ people who have watched both}}{\# \text{ people who have watched Amelie}} \\ &\approx 0.42 \end{aligned}$$

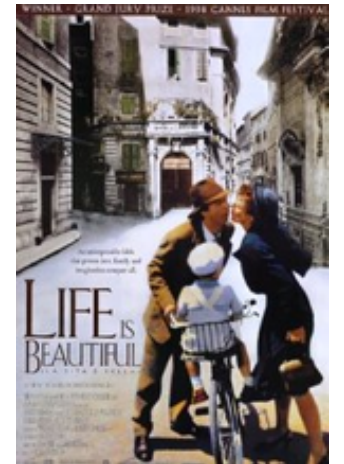
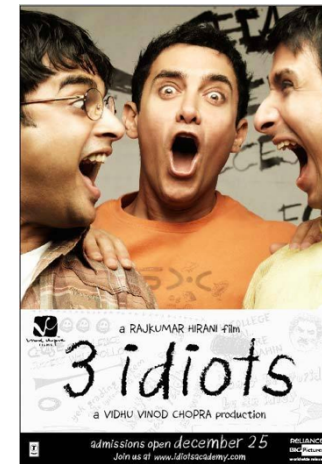
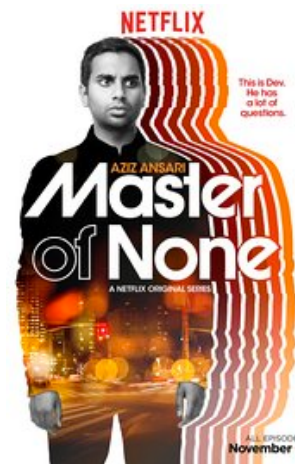


# Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$

Definition of  
Cond. Probability

Let  $E$  be the event that a user watches the given movie.  
Let  $F$  be the event that the same user watches Amelie.



$$P(E) = 0.19$$

$$P(E) = 0.32$$

$$P(E) = 0.20$$

$$P(E) = 0.09$$

$$P(E) = 0.20$$

$$P(E|F) = 0.14$$

$$P(E|F) = 0.35$$

$$P(E|F) = 0.20$$

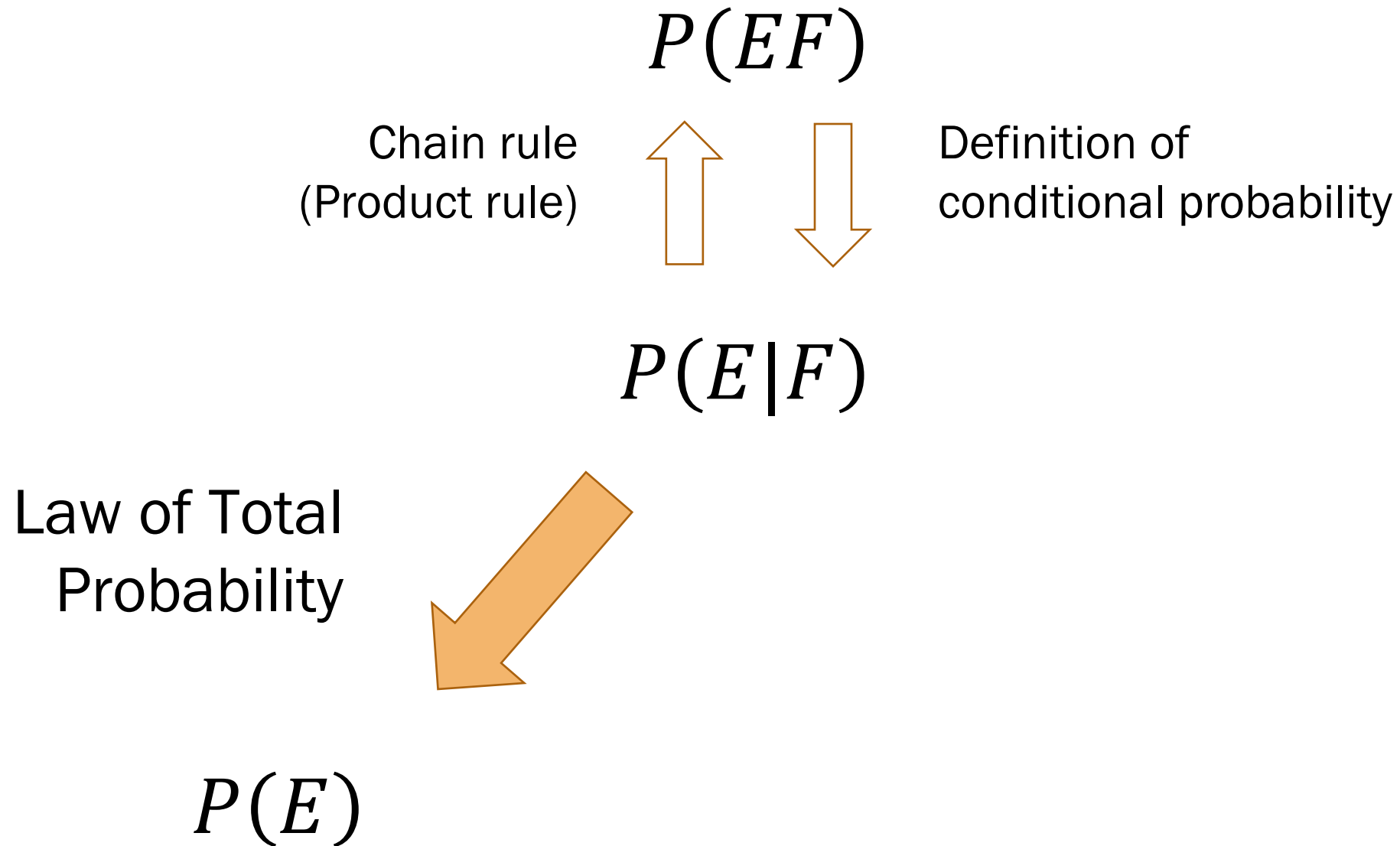
$$P(E|F) = 0.72$$

$$P(E|F) = 0.42$$

# Law of Total Probability

# Today's tasks

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# Law of Total Probability

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Thm Let  $F$  be an event where  $P(F) > 0$ . For any event  $E$ ,

$$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$$

## Proof

1.  $F$  and  $F^C$  are disjoint s.t.  $F \cup F^C = S$  Def. of complement
2.  $E = (EF) \cup (EF^C)$  (see diagram)
3.  $P(E) = P(EF) + P(EF^C)$  Additivity axiom
4.  $P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$  Chain rule (product rule)

Note: disjoint sets by definition are mutually exclusive events

# General Law of Total Probability

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Thm For **mutually exclusive events**  $F_1, F_2, \dots, F_n$   
s.t.  $F_1 \cup F_2 \cup \dots \cup F_n = S$ ,

$$P(E) = \sum_{i=1}^n P(E|F_i)P(F_i)$$

# Finding $P(E)$ from $P(E|F)$

$$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$$

Law of Total Probability

- Flip a fair coin.
- If heads: roll a fair 6-sided die.
- Else: roll a fair 3-sided die.

You win if you roll a 6. What is  $P(\text{winning})$ ?



# Finding $P(E)$ from $P(E|F)$

$$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C) \quad \text{Law of Total Probability}$$

- Flip a fair coin.
- If heads: roll a fair 6-sided die.
- Else: roll a fair 3-sided die.



You win if you roll a 6. What is  $P(\text{winning})$ ?

1. Define events  
& state goal

Let:  $E$ : win,  $F$ : flip heads  
Want:  $P(\text{win})$   
 $= P(E)$

2. Identify known  
probabilities

$$\begin{aligned} P(\text{win} | H) &= P(E|F) = 1/6 \\ P(H) &= P(F) = 1/2 \\ P(\text{win} | T) &= P(E|F^C) = 0 \\ P(T) &= P(F^C) = 1 - 1/2 \end{aligned}$$

3. Solve

$$\begin{aligned} P(E) &= (1/6)(1/2) \\ &\quad + (0)(1/2) \\ &= \frac{1}{12} \approx 0.083 \end{aligned}$$

# Finding $P(E)$ from $P(E|F)$ , an understanding

- Flip a fair coin.
- If heads: roll a fair 6-sided die.
- Else: roll a fair 3-sided die.

You win if you roll a 6. What is  $P(\text{winning})$ ?



## 1. Define events & state goal

Let:  $E$ : win,  $F$ : flip heads

Want:  $P(\text{win})$   
 $= P(E)$

“Probability trees” can help connect your understanding of the experiment with the problem statement.

# Bayes' Theorem

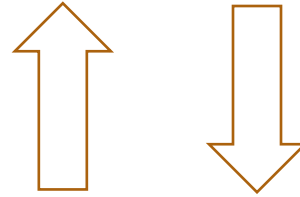
# I

# Today's tasks



Chain rule  
(Product rule)

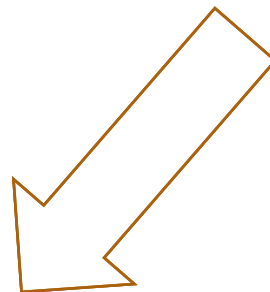
$$P(EF)$$



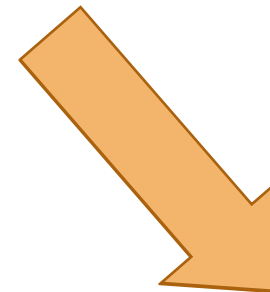
Definition of  
conditional probability

$$P(E|F)$$

Law of Total  
Probability



Bayes'  
Theorem



$$P(E)$$

$$P(F|E)$$

# Thomas Bayes

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Rev. Thomas Bayes (~1701-1761):  
British mathematician and Presbyterian minister

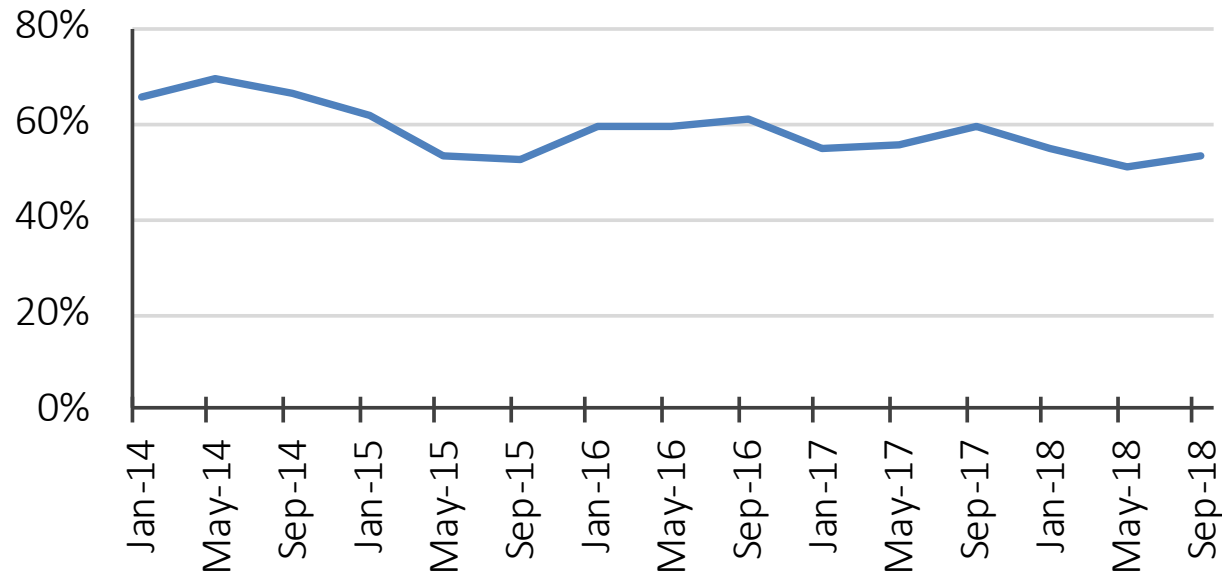


He looked remarkably similar to Charlie Sheen  
(but that's not important right now)



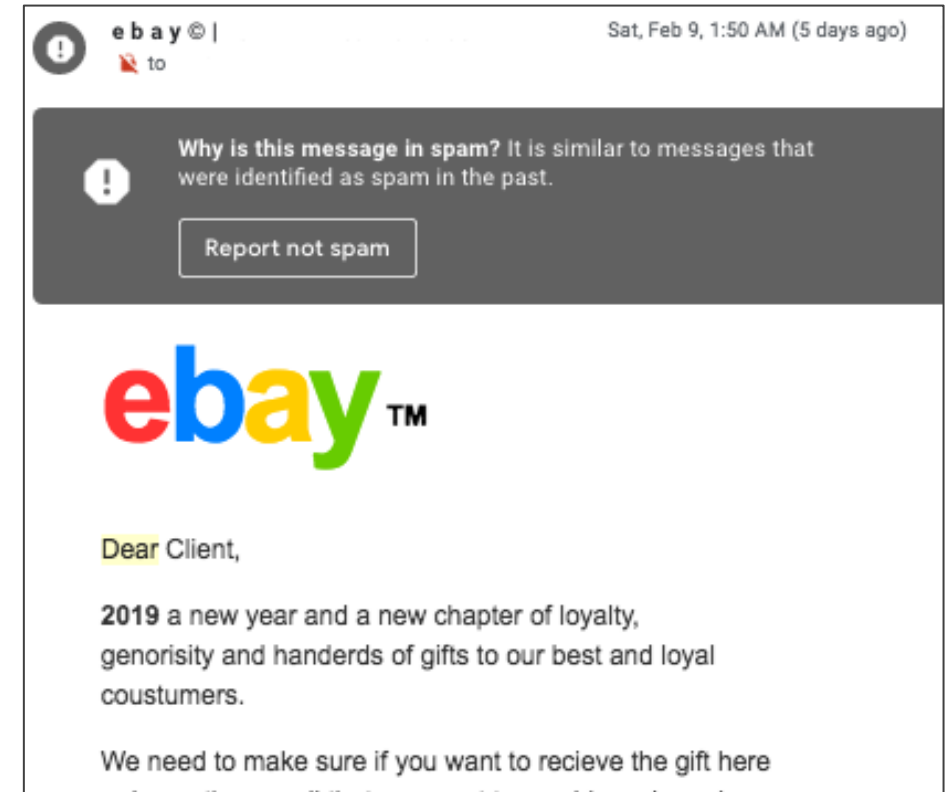
# Detecting spam email

Spam volume as percentage of total email traffic worldwide



We can easily calculate how many spam emails contain “Dear”:

$$P(E|F) = P(\text{“Dear”} \mid \text{Spam email})$$



But what is the probability that an email containing “Dear” is spam?

$$P(F|E) = P(\text{Spam email} \mid \text{“Dear”})$$

(silent drumroll)

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# Bayes' Theorem

$$P(E|F) \Rightarrow P(F|E)$$

Thm For any events  $E$  and  $F$  where  $P(E) > 0$  and  $P(F) > 0$ ,

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Proof

2 steps! See board

Expanded form:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)}$$

Proof

1 more step! See board



# Detecting spam email

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)} \quad \begin{array}{l} \text{Bayes'} \\ \text{Theorem} \end{array}$$

- 60% of all email in 2016 is spam.
- 20% of spam has the word “Dear”
- 1% of non-spam (aka ham) has the word “Dear”

You get an email with the word “Dear” in it.

What is the probability that the email is spam?

1. Define events  
& state goal

2. Identify known  
probabilities

3. Solve

Let:  $E$ : “Dear”,  $F$ : spam

Want:  $P(\text{spam} | \text{“Dear”})$   
 $= P(F|E)$

# Detecting spam email, an understanding

- 60% of all email in 2016 is spam.
- 20% of spam has the word “Dear”
- 1% of non-spam (aka ham) has the word “Dear”

You get an email with the word “Dear” in it.

What is the probability that the email is spam?

## 1. Define events & state goal

Let:  $E$ : “Dear”,  $F$ : spam

Want:  $P(\text{spam} | \text{“Dear”})$   
 $= P(F | E)$

**Note:** You should still know how to use Bayes/ Law of Total Probab., but drawing a probability tree can help you identify which probabilities you have. The branches are determined using the problem setup.

# Bayes' Theorem terminology

- 60% of all email in 2016 is spam.  $P(F)$
- 20% of spam has the word “Dear”  $P(E|F)$
- 1% of non-spam (aka ham) has the word “Dear”  $P(E|F^C)$

You get an email with the word “Dear” in it.

What is the probability that the email is spam? **Want:  $P(F|E)$**

$$\begin{array}{c} \text{posterior} \\ P(F|E) \end{array} = \frac{\begin{array}{c} \text{likelihood} \quad \text{prior} \\ P(E|F)P(F) \end{array}}{\begin{array}{c} P(E) \\ \text{normalization constant} \end{array}}$$

(live)

# 04: Conditional Probability and Bayes

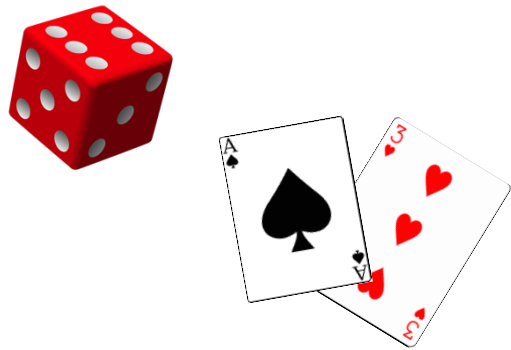
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Lisa Yan

April 13, 2020

# This class going forward

Last week  
Equally likely  
events



$P(E \cap F)$        $P(E \cup F)$   
(counting, combinatorics)

Today and for most of this course  
**Not equally likely events**

$P(E = \text{Evidence} \mid F = \text{Fact})$   
(collected from data)

Bayes'

$P(F = \text{Fact} \mid E = \text{Evidence})$   
(categorize  
a new datapoint)



General **definition** of conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

The **Chain Rule** (aka Product rule):

$$P(EF) = P(F)P(E|F)$$

These properties hold even when outcomes are not equally likely.

# Think, then Breakout Rooms

Then check out the question on the next slide (Slide 35). Post any clarifications here!

<https://us.edstem.org/courses/109/discussion/27277>

Think by yourself: 1 min

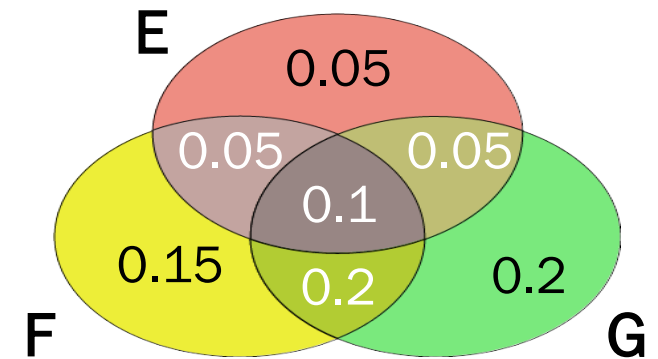
Breakout rooms: 5 min. Introduce yourself!



# Think, then groups

You have a flowering plant.

Let  $E$  = Flowers bloom  
 $F$  = Plant was watered  
 $G$  = Plant got sun



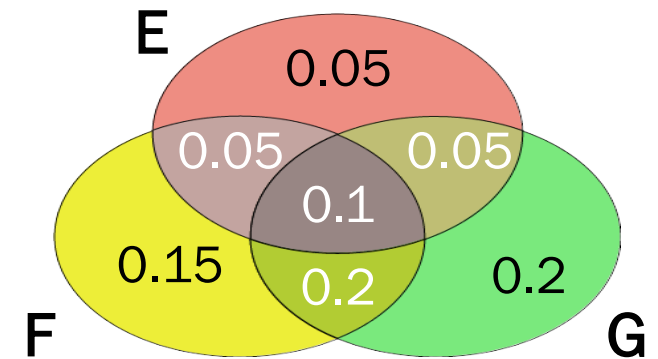
- How would you write
  - the probability that the plant got sun, given that it was watered and flowers bloomed?
  - the probability that the plant got sun and flowers bloomed given that it was watered?
- Using the Venn diagram, compute the above probabilities.
- Chain Rule: Fill in the blanks.
  - $P(GE) = \underline{\hspace{2cm}} \cdot P(E)$
  - $P(GE|F) = P(G|EF) \cdot \underline{\hspace{2cm}}$



# Think, then groups

You have a flowering plant.

Let  $E$  = Flowers bloom  
 $F$  = Plant was watered  
 $G$  = Plant got sun



- How would you write
  - the probability that the plant got sun, given that it was watered and flowers bloomed?
  - the probability that the plant got sun and flowers bloomed given that it was watered?
- Using the Venn diagram, compute the above probabilities.
- Chain Rule: Fill in the blanks.
  - $P(GE) = \underline{\hspace{2cm}} \cdot P(E)$
  - $P(GE|F) = P(G|EF) \cdot \underline{\hspace{2cm}}$

# Bayes' Theorem

## II

# Why is Bayes' so important?



It links **belief** to **evidence** in probability!

$$\overset{\text{posterior}}{P(F|E)} = \frac{\overset{\text{likelihood}}{P(E|F)} \overset{\text{prior}}{P(F)}}{P(E)}$$

*F: Fact*  
*E: Evidence*

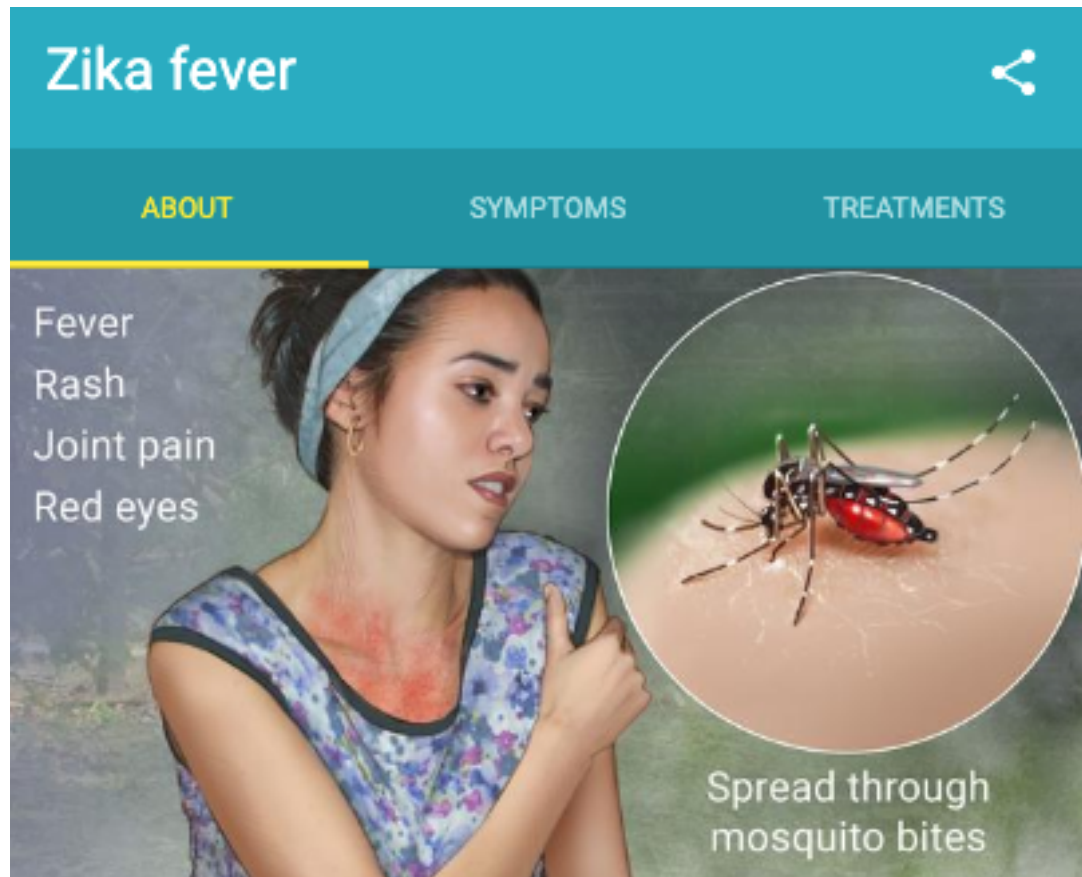
Mathematically:

$$P(E|F) \rightarrow P(F|E)$$

Real-life application:

Given new evidence  $E$ , update belief of fact  $F$   
Prior belief  $\rightarrow$  Posterior belief  
 $P(F) \rightarrow P(F|E)$

# Zika, an autoimmune disease



Ziika Forest, Uganda



Rhesus monkeys

If a test returns positive, what is the likelihood you have the disease?

A disease spread through mosquito bites. Usually no symptoms; worst case paralysis. During pregnancy: may cause birth defects



# Taking tests: Confusion matrix



Fact,  $F$  Has disease  
or  $F^C$  No disease



Evidence,  $E$  Test positive  
or  $E^C$  Test negative

		Fact	
		$F$ , disease +	$F^C$ , disease -
Evidence	$E$ , Test +	True positive $P(E F)$	<b>False positive</b> $P(E F^C)$
	$E^C$ , Test -	<b>False negative</b> $P(E^C F)$	True negative $P(E^C F^C)$

If a test returns positive, what is the likelihood you have the disease?

# Taking tests: Confusion matrix



Fact,  $F$  Has disease  
or  $F^C$  No disease



Evidence,  $E$  Test positive  
or  $E^C$  Test negative

		Fact	
		$F$ , disease +	$F^C$ , disease -
Evidence	$E$ , Test +	True positive $P(E F)$	False positive $P(E F^C)$
	$E^C$ , Test -	False negative $P(E^C F)$	True negative $P(E^C F^C)$

If a test returns positive, what is the likelihood you have the disease?

# Breakout Rooms

Check out the question on the next slide (Slide 43). Post any clarifications here!

<https://us.edstem.org/courses/109/discussion/27277>

Breakout rooms: 5 minutes



# Zika Testing

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)} \quad \begin{array}{l} \text{Bayes'} \\ \text{Theorem} \end{array}$$

- A test is 98% effective at detecting Zika (“true positive”).
- However, the test has a “false positive” rate of 1%.
- 0.5% of the US population has Zika.

What is the likelihood you have Zika if you test positive?

Why would you expect this number?

## 1. Define events & state goal

Let:  $E$  = you test positive  
 $F$  = you actually have  
the disease

Want:

$$\begin{aligned} &P(\text{disease} \mid \text{test+}) \\ &= P(F|E) \end{aligned}$$



# Zika Testing

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)} \quad \text{Bayes' Theorem}$$

- A test is 98% effective at detecting Zika (“true positive”).
- However, the test has a “false positive” rate of 1%.
- 0.5% of the US population has Zika.

What is the likelihood you have Zika if you test positive?

Why would you expect this number?

1. Define events  
& state goal

2. Identify known  
probabilities

3. Solve

Let:  $E$  = you test positive  
 $F$  = you actually have  
the disease

Want:

$$\begin{aligned} &P(\text{disease} \mid \text{test+}) \\ &= P(F|E) \end{aligned}$$

# Bayes' Theorem intuition

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Original question:

What is the likelihood you have Zika if you test positive for the disease?



# Bayes' Theorem intuition

Original question:

What is the likelihood you have Zika if you test positive for the disease?

Interpret

Interpretation:

Of the people who test positive, how many actually have Zika?



# Bayes' Theorem intuition

Original question:

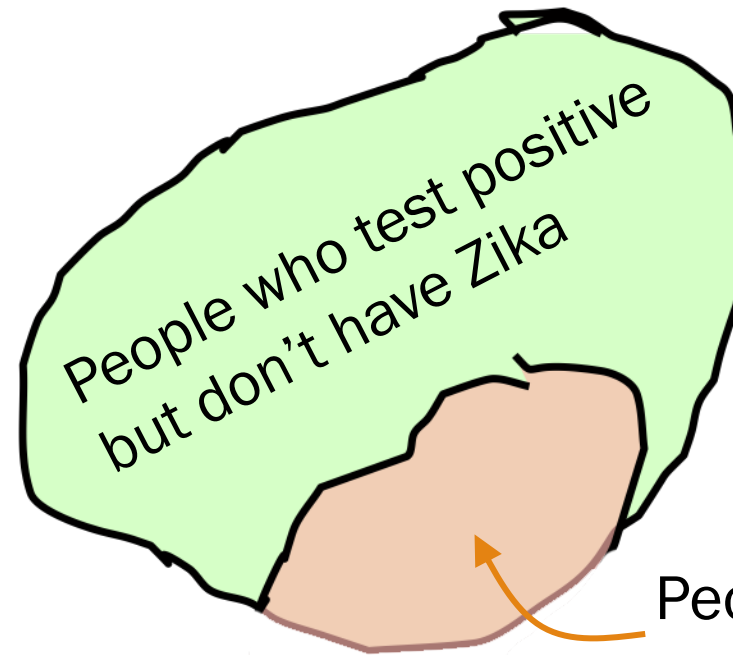
What is the likelihood you have Zika if you test positive for the disease?

Interpret

Interpretation:

Of the people who test positive, how many actually have Zika?

People who test positive



People who test positive and have Zika

The space of facts, conditioned on a positive test result



# Zika Testing

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- A test is 98% effective at detecting Zika (“true positive”).
- However, the test has a “false positive” rate of 1%.
- 0.5% of the US population has Zika.

What is the likelihood you have Zika if you test positive?

Say we have 1000 people:



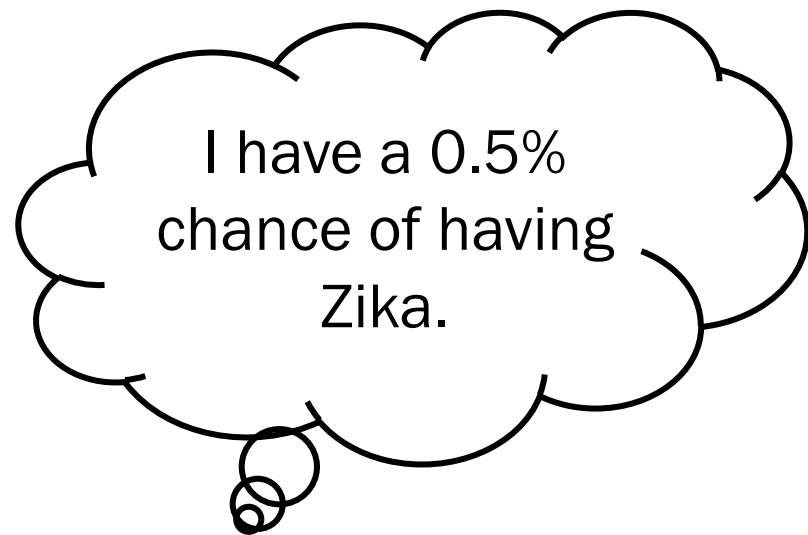
5 have Zika  
and test positive  
985 do not have Zika  
and test negative.  
10 do not have Zika  
and test positive.

$\approx 0.333$

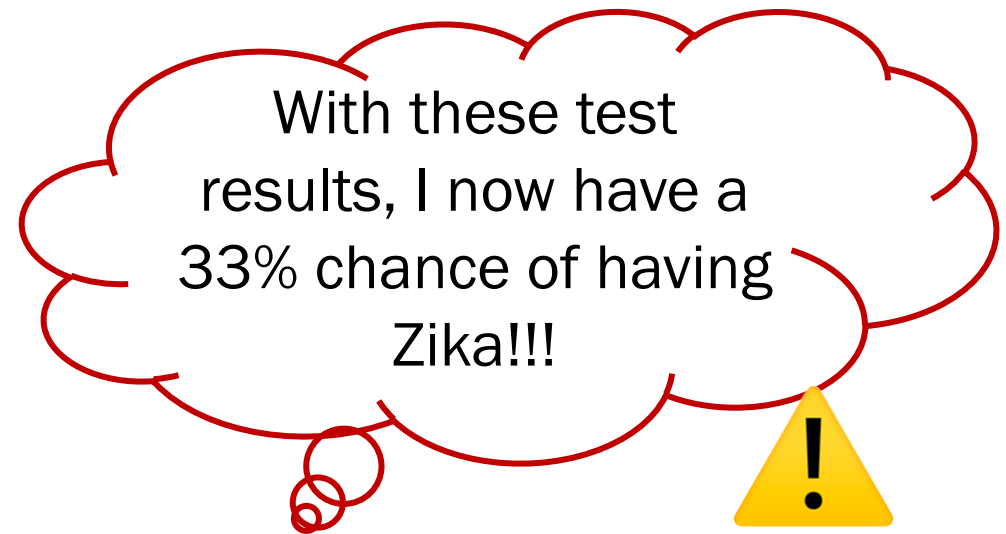
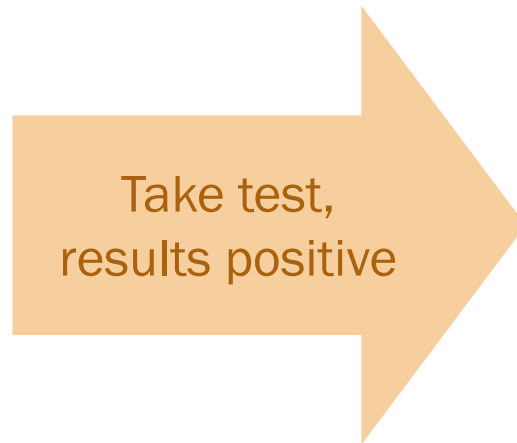
# Update your beliefs with Bayes' Theorem

$E$  = you test positive for Zika

$F$  = you actually have the disease



$P(F)$



$P(F|E)$

# Interlude for jokes/announcements

# Your voices

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Goodness, what are all these concepts on the section sign-up form??  
I know none of them

I have ideas on how to make Ed/OH more accessible to learning!

Lisa's joke on Friday was good

We are all here to learn. By the end of the course, you will look back on this multiple-choice question with fond memories of all the things you learned.

Thank you for your valuable input! We are looking to make these lively, interactive channels of communication. You might see a few changes this week.

You all rock, thank you for making this all worth it

# Topical probability news: Bayes for COVID-19 testing

## Stanford REPORT

Monday, April 13, 2020

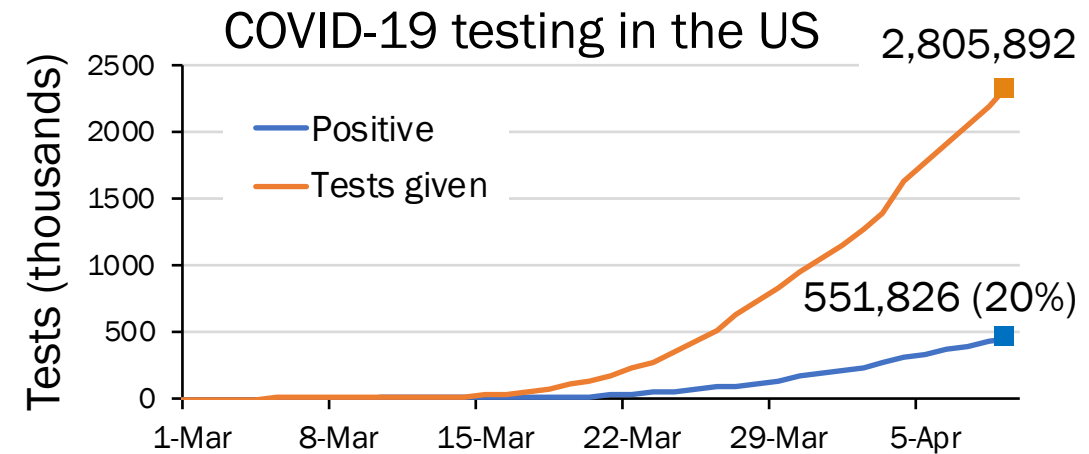
### Testing for novel coronavirus antibodies



Working around the clock, a team of Stanford Medicine scientists developed a test to detect antibodies against the novel coronavirus in blood samples. [Read more.](#)

<https://covidtracking.com/data>

<http://med.stanford.edu/news/all-news/2020/04/stanford-medicine-develops-antibody-test-for-coronavirus.html>



*How representative are today's testing rates?*

*How do we know if a positive test is a true positive or a false positive?*

*Why test if there are errors?*

# Think

Slide 55 is a question to think over by yourself.

We'll go over it together afterwards.

Post any clarifications here!

<https://us.edstem.org/courses/109/discussion/27277>

Think by yourself: 2 minutes



# Why it's still good to get tested

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)} \quad \text{Bayes' Theorem}$$

- A test is 98% effective at detecting Zika (“true positive”).
- However, the test has a “false positive” rate of 1%.
- 0.5% of the US population has Zika.

Let:  $E$  = you test positive  
 $F$  = you actually have the disease

Let:  $E^C$  = you test **negative** for Zika with this test.

	$F$ , disease +	$F^C$ , disease -
$E$ , Test +	True positive $P(E F) = 0.98$	False positive $P(E F^C) = 0.01$

What is  $P(F|E^C)$ ?



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	$F$ , disease +	$F^C$ , disease -
$E$ , Test +	True positive $P(E F) = 0.98$	False positive $P(E F^C) = 0.01$
$E^C$ , Test -	False negative $P(E^C F) = 0.02$	True negative $P(E^C F^C) = 0.99$

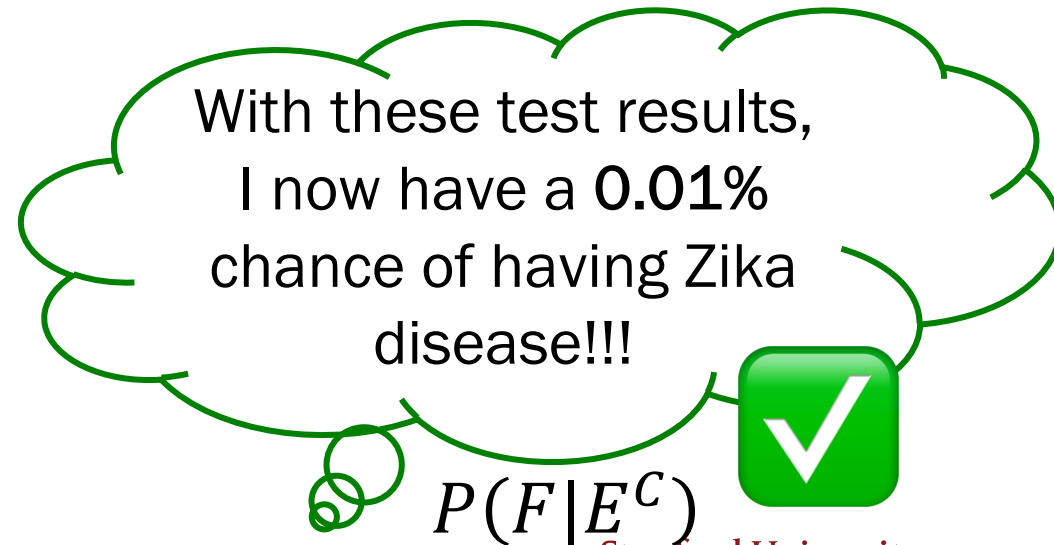
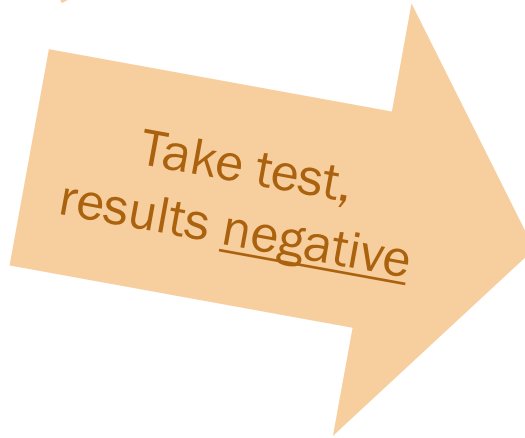
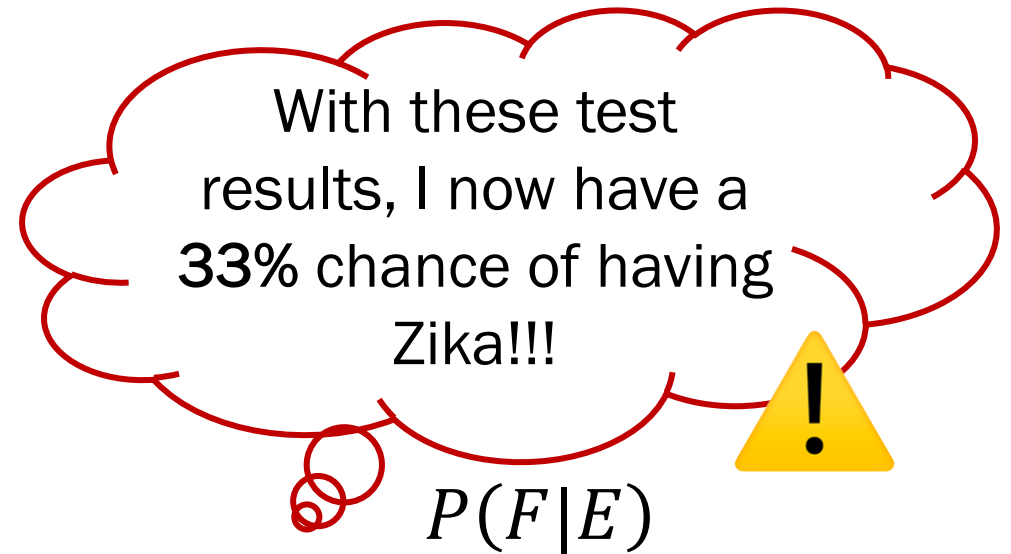
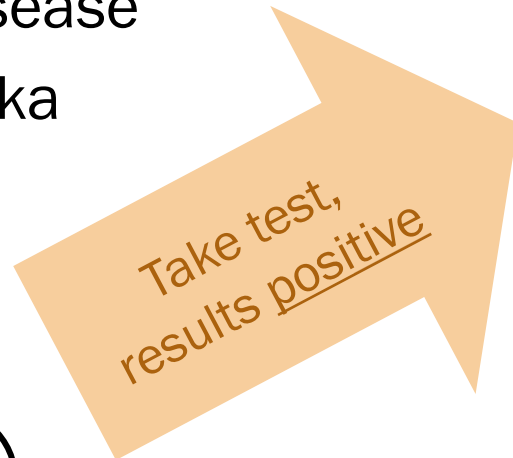
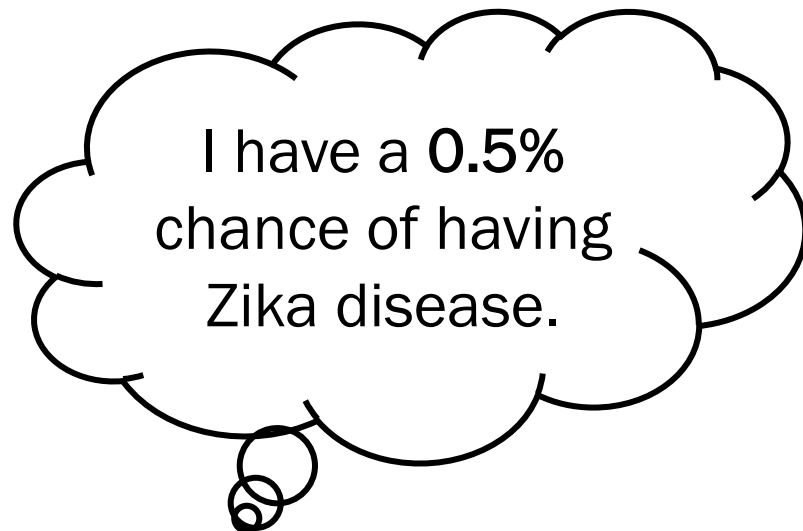
$$P(F|E^C) = \frac{P(E^C|F)P(F)}{P(E^C|F)P(F) + P(E^C|F^C)P(F^C)}$$

# Why it's still good to get tested

$E$  = you test positive for Zika

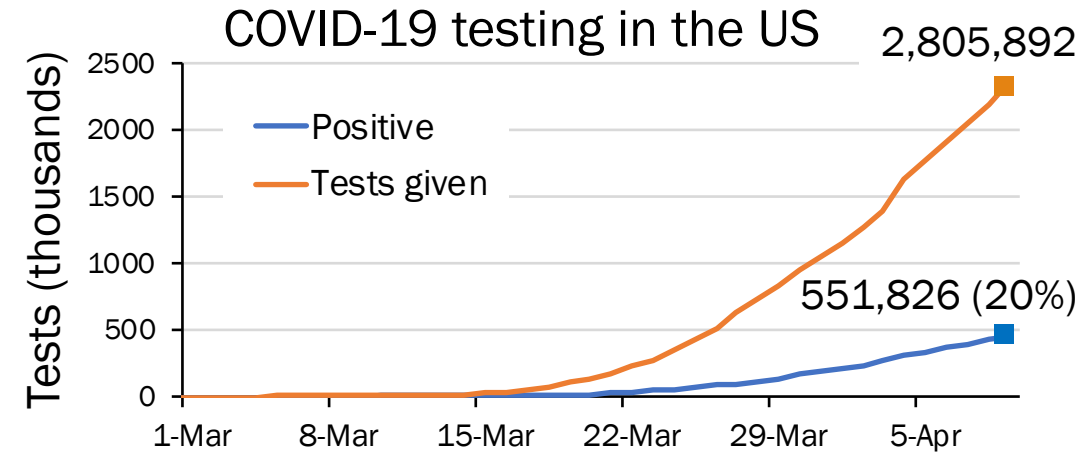
$F$  = you actually have the disease

$E^C$  = you test **negative** for Zika



# Topical probability news: Bayes for COVID-19 testing

- Antibody tests (blood samples) have higher false negative, false positive rates than RT-PCR tests (nasal swab). However, they help explain/identify our body's reaction to the virus.
- The real world has many more “**givens**” (current symptoms, existing medical conditions) that improve our belief **prior** to testing.
- Most importantly, testing gives us a noisy signal of the spread of a disease.



*How representative are today's testing rates?*

*How do we know if a positive test is a true positive or a false positive?*

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# Topical probability news: Sources

US data by state

<https://covidtracking.com/data>

Stanford Medicine (April 13 2020)

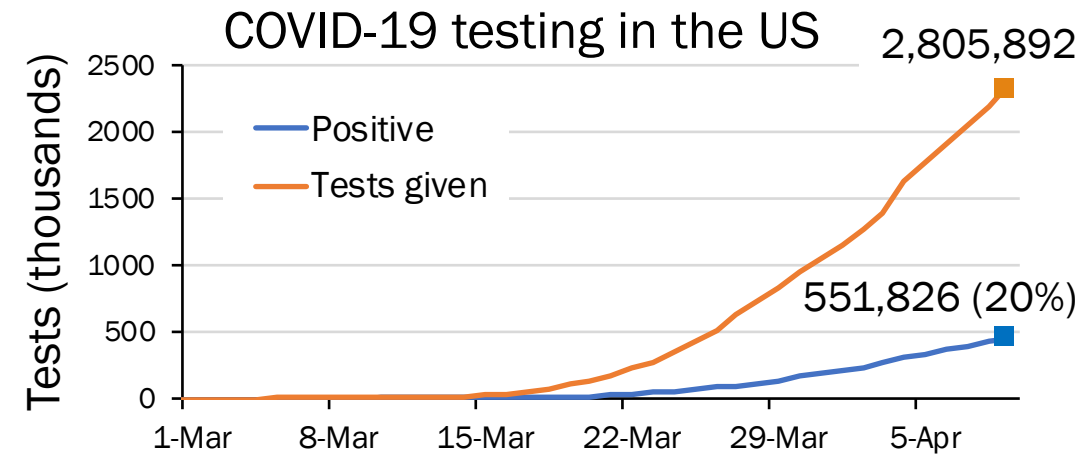
<http://med.stanford.edu/news/all-news/2020/04/stanford-medicine-develops-antibody-test-for-coronavirus.html>

Overview of different testing types

<https://www.globalbiotechinsights.com/articles/20247/the-worldwide-test-for-covid-19>

Compilation of scientific publications on COVID-19

[https://rega.kuleuven.be/if/corona\\_covid-19](https://rega.kuleuven.be/if/corona_covid-19)



# Monty Hall Problem

# Monty Hall Problem and Wayne Brady



# Monty Hall Problem aka Let's Make a Deal

Behind one door is a prize (equally likely to be any door).

Behind the other two doors is nothing

1. We choose a door
2. Host opens 1 of other 2 doors, revealing nothing
3. We are given an option to change to the other door.

Should we switch?

Note: If we don't switch,  $P(\text{win}) = 1/3$  (random)

We are comparing  $P(\text{win})$  and  $P(\text{win} | \text{switch})$ .

Vote here: <http://www.pollev.com/cs109>



Doors A,B,C



# If we switch

Without loss of generality, say we pick A (out of Doors A,B,C).

1/3

1/3

1/3

A = prize

- Host opens B or C
- We switch
- We always lose

$P(\text{win} \mid \text{A prize, picked A, switched}) = 0$

B = prize

- Host must open C
- We switch to B
- We always win

$P(\text{win} \mid \text{B prize, picked A, switched}) = 1$

C = prize

- Host must open B
- We switch to C
- We always win

$P(\text{win} \mid \text{C prize, picked A, switched}) = 1$

$$P(\text{win} \mid \text{picked A, switched}) = 1/3 * 0 + 1/3 * 1 + 1/3 * 1 = 2/3$$

***You should switch.***



# Monty Hall, 1000 envelope version

Start with 1000 envelopes (of which 1 is the prize).

1. You choose 1 envelope.

$$\left\{ \begin{array}{l} \frac{1}{1000} = P(\text{envelope is prize}) \\ \frac{999}{1000} = P(\text{other 999 envelopes have prize}) \end{array} \right.$$

2. I open 998 of remaining 999 (showing they are empty).

$$\begin{aligned} \frac{999}{1000} &= P(998 \text{ empty envelopes had prize}) \\ &\quad + P(\text{last other envelope has prize}) \\ &= P(\text{last other envelope has prize}) \end{aligned}$$

3. Should you switch?

$$\text{No: } P(\text{win without switching}) = \frac{1}{\text{original \# envelopes}}$$

$$\text{Yes: } P(\text{win with new knowledge}) = \frac{\text{original \# envelopes} - 1}{\text{original \# envelopes}}$$