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CS109

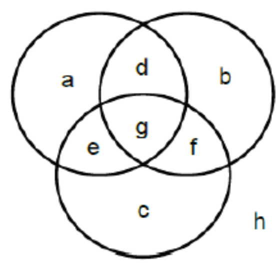
Section #1 Warm-ups

1 Lecture 1, 1-6-20: Counting

The Inclusion Exclusion Principle for three sets is:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Explain why in terms of a venn-diagram.



We want a, b, c, d, e, f, & g to each be accounted for exactly once. If we add A, B, and C together, we properly count a, b, & c exactly once. However we will double count d, e, & f, and triple count g. Removing $A \cap B$, $A \cap C$, & $B \cap C$ will reduce the counts for d, e, & f down to one. However, g went from being counted 3 times too many to being counted zero times. The expression corresponding to g is $A \cap B \cap C$, so we add it in to allow g to have a count of exactly one.

2 Lecture 2, 1-8-20: Permutations and Combinations

Suppose there are 7 blue fish, 4 red fish, and 8 green fish in a large fishing tank. You drop a net into it and end up with 6 fish. What is the probability you get 2 of each color?

$$\frac{\binom{7}{2}\binom{4}{2}\binom{8}{2}}{\binom{19}{6}}$$

3 Lecture 3, 1-10-20: Axioms of Probability

For each of the four statements below, evaluate True or False.

$$P(A|B) + P(A^C|B) = 1 \qquad P(A|B) + P(A|B^C) = 1 \qquad P(A \cap B) + P(A \cap B^C) = 1$$
$$P(A) = 0.4 \wedge P(B) = 0.6 \implies A = B^C$$

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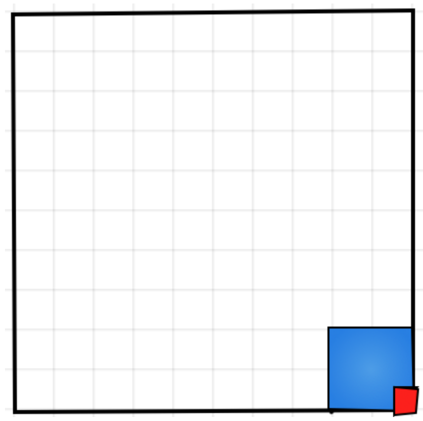
4 Lecture 4, 1-13-20: Conditional Probability and Bayes

Bayes Theorem is $P(H|E) = P(E|H) * P(H)/P(E)$ where H can be thought of as a hypothesis and E as evidence. This equation can be notoriously counter intuitive. Draw a diagram where $P(E|H) = 1$ and $P(E^C|H^C)$ is close to 1, but $P(H|E)$ is still close to 0. How can we interpret this?

Draw the sample space as a square. Draw the (E)vidence as a tiny square inside taking up less than 5% of the sample space. Lastly, draw an even tinier dot for the (H)ypothesis that is completely inside of E. H should be less than 5% of E, (thus less than 0.25% of the sample space).



$P(E|H)$ is 1 because H is inside of E. $P(E^C|H^C)$ is close to 1 because, since both H and E are small compared to the sample space, H^C and E^C will be comparable in size to each other. $P(H|E)$ is close to zero because H is less than 5% of E.

As an example, assume 100% of NBA players are tall, and 99% of non-NBA players are not tall. We still can't infer for certain that a randomly selected tall person is in the NBA.

In general, people assume that $P(H|E) \approx P(E|H)$ without realizing it. Bayes Theorem says $P(H|E) = P(E|H) * P(H)/P(E)$. Therefore this assumption is only true if $P(H)$ / $P(E)$ is close to one - otherwise our intuitions are violated.