Based on handout by Gili Rusak and Alex Tsun

## **Lecture 5: Independence**

- 1. Definitions: Cite Bayes' Theorem.
- 2. True or False. Note that true means always true.
  - (a) In general, P(A, B|C) = P(B|C)P(A|B, C).
  - (b) If A and B are independent, so are A and  $B^C$ .
  - 1. Bayes' Theorem:  $P(E|F) = \frac{P(F|E)P(E)}{P(F)}$
  - 2. (a) True
    - (b) True

## Lecture 6: Random Variables and Expectation

- 1. Definitions:
  - (a) If X is a random variable, what is E[X]? What is E[g(X)]?
  - (b) For random variables  $X_1, \ldots, X_n$ , what is  $E[\sum_{i=1}^n X_i]$ ?
- 2. True or False: For any random variable  $X, E[X^2] = E[X]^2$ .
- 3. Short Answer: Let X = the value on one roll of a 6 sided die. Recall that E[X] = 7/2. What is Var(X)?
  - 1. Definitions:
    - (a)  $E[X] = \sum_{x} x p_X(x)$  and  $E[g(X)] = \sum_{x} g(x) p_X(x)$ .
    - (b)  $E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i]$
  - 2. False
  - 3. Remember that  $\operatorname{Var}(X) = E[X^2] E[X]^2$ .  $E[X^2] = (1^2)\frac{1}{6} + (2^2)\frac{1}{6} + (3^2)\frac{1}{6} + (4^2)\frac{1}{6} + (5^2)\frac{1}{6} + (6^2)\frac{1}{6} = \frac{91}{6}$ . Thus,  $\operatorname{Var}(X) = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12}$ .

## Lecture 7: Variance, Bernoulli, Binomial

- 1. Definitions: PMF for  $X \sim Binomial(n, p)$ . What is  $p_X(k)$ ?
- 2. Short Answer: Let *X* be the number of flips of a coin with P(head) = p up to and including the first head. What is the range of *X* and  $p_X(k)$ ?

- 1.  $P(X = k) = {\binom{n}{k}}p^k(1-p)^{n-k}$
- 2. Range:  $\{1, 2, ...\} = \mathbb{N}$ .  $P(X = k) = (1 p)^{k-1}p$ .