Lisa Yan
CS109

Section #4 Warmup Solutions

Based on the work of Gili Rusak and Alex Tsun

1 Lecture 11, 4-29-20: Joint Distributions

1. Given a Normal RV $X \sim N(\mu, \sigma^2)$, how can we compute $P(X \leq x)$ from the standard Normal distribution Z with CDF ϕ?
2. What is a continuity correction and when should we use it?
3. If we have a joint PMF for discrete random variables $p_{X,Y}(x, y)$, how can we compute the marginal PMF $p_X(x)$?

1. First, we write $\phi((x - \mu)/\sigma)$. We then look up the value we've computed in the Standard Normal Table.
2. Continuity correction is used when a Normal distribution is used to approximate a Binomial. Since a Normal is continuous and Binomial is discrete, we have to use a continuity correction to discretize the Normal. The continuity correction makes it so that the normal variable is evaluated from + or - 0.5 increments from the desired k value.
3. The marginal distribution is $p_X(x) = \sum_y p_{X,Y}(x, y)$

2 Lecture 12, 5-1-20: Independent Random Variables

1. What distribution does the sum of two independent binomial RVs $X + Y$ have, where $X \sim Bin(n_1, p)$ and $Y \sim Bin(n_2, p)$? Include the parameter(s) in your answer. Why is this the case?
2. What distribution does the is of two independent Poisson RVs $X + Y$ have, where $X \sim \text{Poi}(\lambda_1)$ and $Y \sim \text{Poi}(\lambda_2)$? Include the parameter(s) in your answer.
3. If $Cov(X, Y) = 0$, are X and Y independent? Why or why not?

1. Binomial; $X + Y \sim Bin(n_1 + n_2, p)$
2. Poisson; $X + Y \sim \text{Poi}(\lambda_1 + \lambda_2)$
3. Not necessarily. Suppose there are three outcomes for X: let X take on values in $\{-1, 0, 1\}$ with equal probability $1/3$. Let $Y = X^2$. Then, $E[XY] = E[X^3] = E[X] = 0$ (since $X^3 = X$) and $E[X] = 0$, so $Cov(X, Y) = E[XY] - E[X]E[Y] = 0 - 0 = 0$ but X and Y are dependent since $P(Y = 1) = 2/3 \neq 1 = P(Y = 1|X = 1)$.

3 Lecture 13, 5-13-20: Joint Random Variables Statistics

1. **True or False?** The symbol Cov is covariance, and the symbol ρ is Pearson correlation.

$X \perp Y \implies Cov(X, Y) = 0$	$Var(X + X) = 2Var(X)$
$Cov(X, Y) = 0 \implies X \perp Y$	$X \sim \mathcal{N}(0, 1) \wedge Y \sim \mathcal{N}(0, 1) \implies \rho(X, Y) = 1$
$Y = X^2 \implies \rho(X, Y) = 1$	$Y = 3X \implies \rho(X, Y) = 3$

1. **True or False?**

True	False (… = $4Var(X)$)
False (antecedent necessary, not sufficient)	False (don't know how independent X & Y are)
False ($Y = X \implies$ …)	False (... = 1)