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CS109

Section #5
May 13-15, 2020

Section #5 Warmup

Based on the work of many CS109 staffs

1 Lecture 14, 5-6-20: Conditional Expectation

1. **Short Answer**. Let $X \sim Geo(p)$. Use the Law of Total Expectation to prove that $E[X] = 1/p$, by conditioning on whether the first flip is heads or tails.

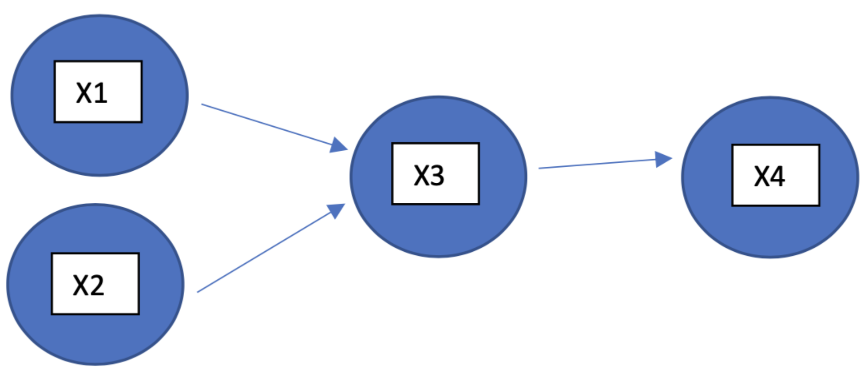
1. $$E[X] = E[X|H]P(H) + E[X|T]P(T) = 1 \cdot p + (E[1+X])(1-p)$$

   Solving yields $E[X] = 1/p$.

2 Lecture 15, 5-8-20: General Inference

Suppose $X_1, \ldots, X_4$ are discrete random variables. We will abuse notation and write $p(x_1, x_2, x_3, x_4)$ to represent $P(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4)$. In your answers, feel free to do the same. For example, $p(x_1, x_3) = P(X_1 = x_1, X_3 = x_3)$. Decompose into four terms, each as simple as possible.

1. If there is no assumption of independence, what is $p(x_1, x_2, x_3, x_4)$?
2. If all variables are assumed independent, what is $p(x_1, x_2, x_3, x_4)$?
3. Assuming the variables follow the Bayesian network structure below, what is $p(x_1, x_2, x_3, x_4)$?



1. $p(x_1)p(x_2|x_1)p(x_3|x_1, x_2)p(x_4|x_1, x_2, x_3)$ (for example)
2. $p(x_1)p(x_2)p(x_3)p(x_4)$
3. $p(x_1)p(x_2)p(x_3|x_1, x_2)p(x_4|x_3)$

3 Lecture 16, 5-16-20: Continuous Joint Distributions I

Consider a continuous joint distribution, (X, Y), where $X \in [0,1]$ is the proportion of the time until the midterm that you actually study for it, and $Y \in [0,1]$ is your percentage score on the exam. Set up but DO NOT EVALUATE any of your answers. Take care in setting up the limits of integration. The joint PDF is:

$$f_{X,Y}(x,y) = \begin{cases} ce^{-(y-x)} & x, y \in [0,1] \text{ and } y \geq x \\ 0 & y < x \end{cases}$$

1. Sketch the joint range for this function and interpret it in English.
2. Write an expression that we could evaluate to find c
3. Write an expression to find the marginal PDF $f_Y(y)$. Carefully define it for all $y \in \mathbb{R}$ (piecewise).

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1. It looks like a triangle, and your score is at least the percentage of time you studied.
2. $$c = \frac{1}{\int_0^1 \int_x^1 e^{-(y-x)} dy dx} = \frac{1}{\int_0^1 \int_0^y e^{-(y-x)} dx dy}$$
3. $f_Y(y) = \int_0^y ce^{-(y-x)} dx$ for $y \in [0,1]$, else 0.