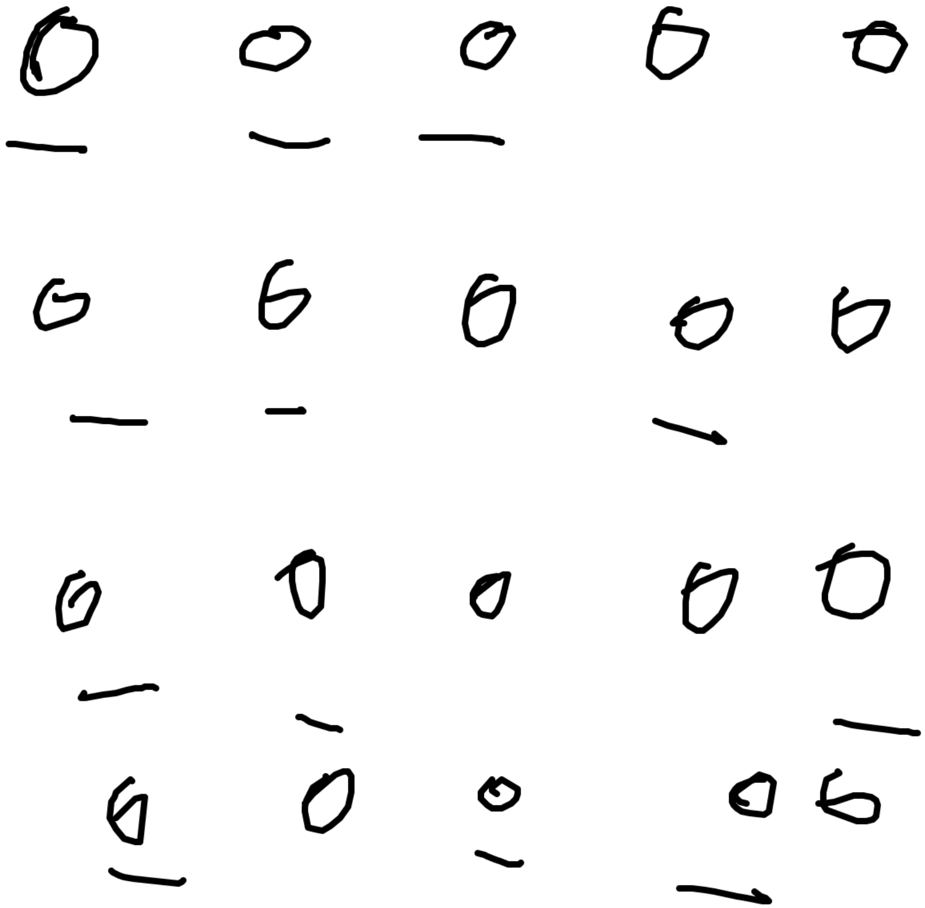
1) Review relevant concepts
2) Practice midterm questions
3) With remaining time, warm ups

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\frac{n!}{n!\,(n-r)!} = \;''$$

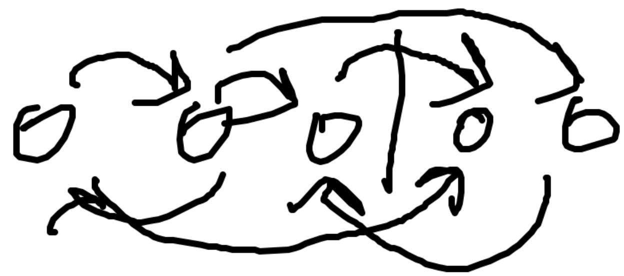


$_nC_r = \binom{n}{r}$

$_nP_r = \frac{n!}{(n-r)!}$

divide by

3! 2!



Counting

Choose k

Distinct

$\binom{n}{k}$ 1 group

$\binom{n}{a, b, c \ldots}$ n groups

$\binom{6}{3, 2, 1}$

Counting

(previous slide)

Put objects in r buckets

Distinct

r^n

o o o o o

1 3

2

Indistinct

$$\binom{n+r-1}{r-1}$$

o|o o|o|o

||o o o o o

o o o||o o|

Question:

2^n

Count all subsets of n elements

$0 = 1$

$1 = 2$

$2 = 4$

$3 = 8$

2^n

$$\sum_{i=0}^{n} \binom{n}{i}$$

$$2^n$$

| 0 0 0 0 0

0 | 0 0 0 0

0 0 | 0 0 0

in subset   out of subset

Probability

Static

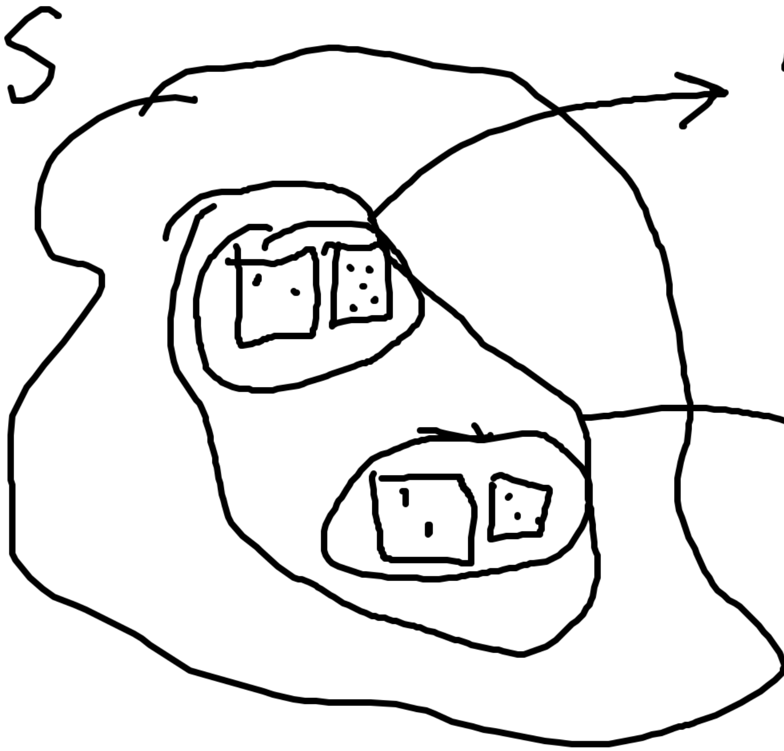
No time or ordering

No causality

$$\frac{|E|}{|S|} = \frac{\text{Count ways of occurrence}}{\text{All ways}}$$

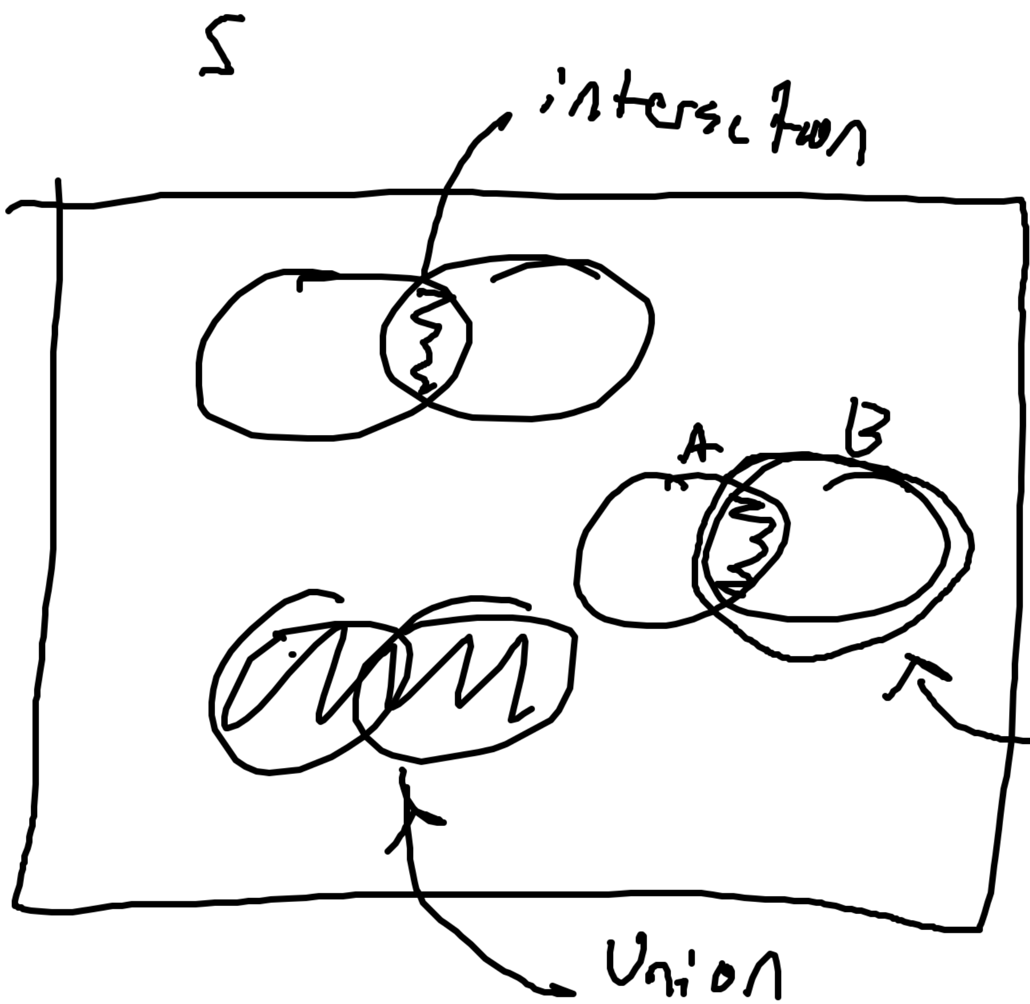
Only holds when all outcomes have equal probability

Experiment: roll two dice



"outcome" → element of sample space

"event" → subset of sample space

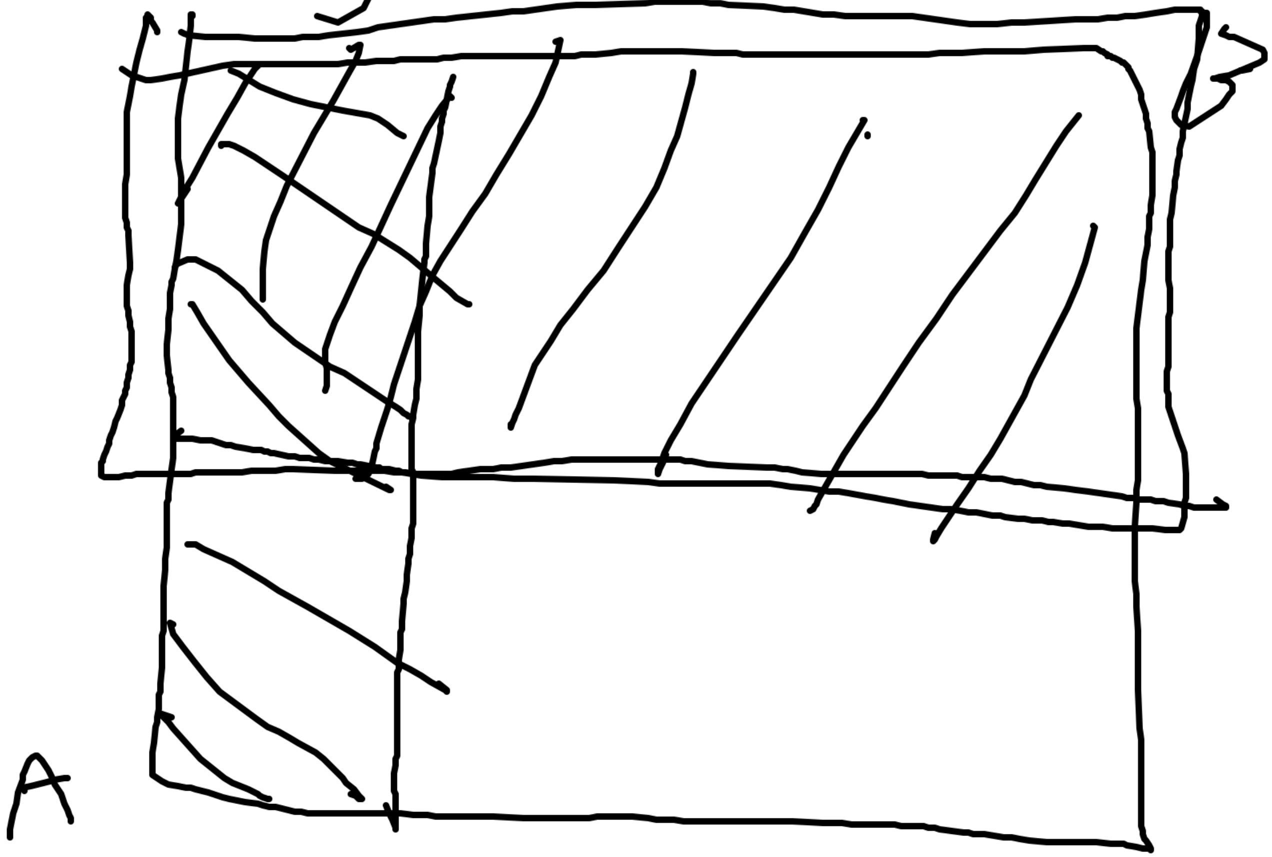


Probability → area in this diagram

$P(A|B) \approx \frac{1}{5}$

New "universe"

Drawing Independance → difficult to draw with more than 2 events



$$P(A \cap B) = P(A)P(B)$$

$$P(A) = \frac{1}{4}$$

Events

A, B, C

Pairwise independence

A ind B

B ind C

" "

for every pair

Joint independence

A ind $(B \cap C)$

B ind $(C \cap A)$

"

Joint Independence implies pairwise, but pairwise does not imply Joint ☆

Q: King has 1 sibling

P(sibling is girl)

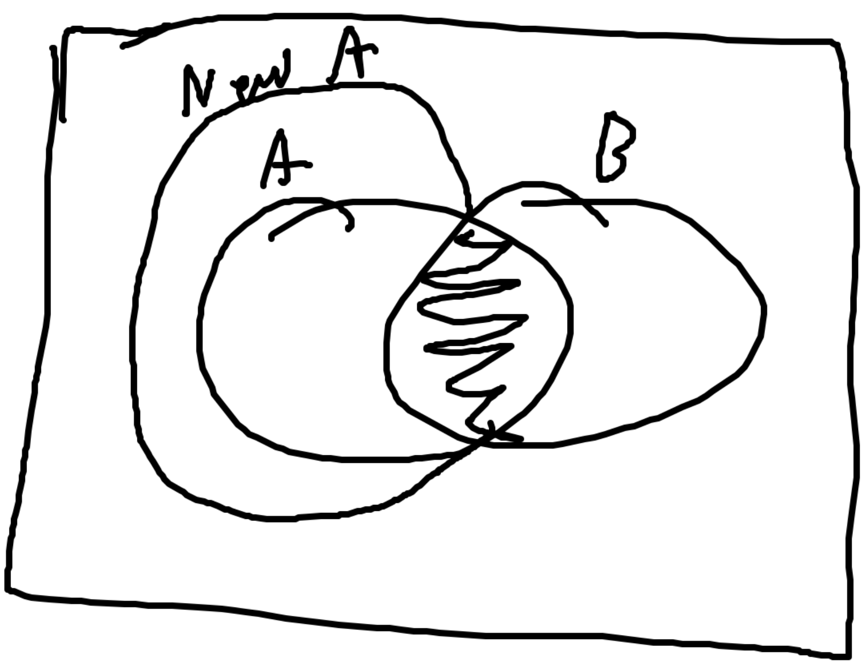
a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{1}{3}$

d. $\frac{2}{3}$

	B	G
B	BB	GB
G	BG	GG



$P(A|B) = \frac{1}{3}$

Lets say A should be bigger in new model

$P(A|B) > \frac{1}{3}$?

Not tive

Bayes theorem $\rightarrow$ tells us how to update beliefs

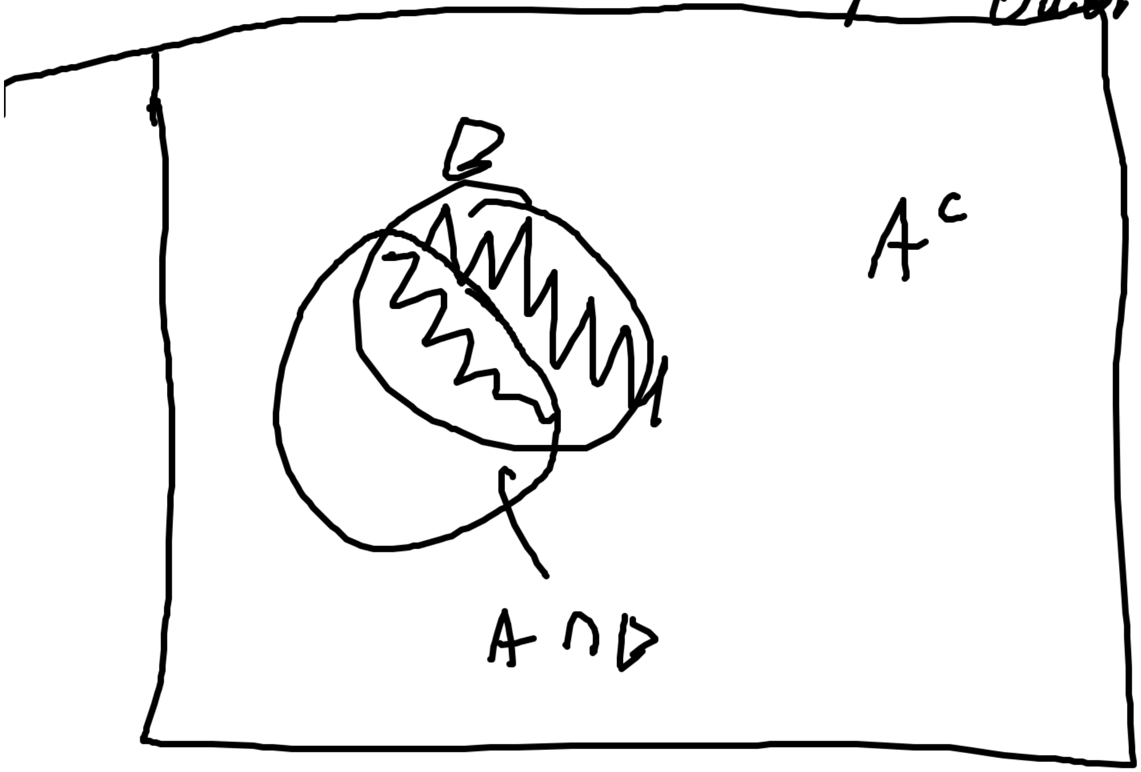
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$= \frac{P(B|A)\,P(A)}{P(B \cap A) + P(B \cap A^c)}$$

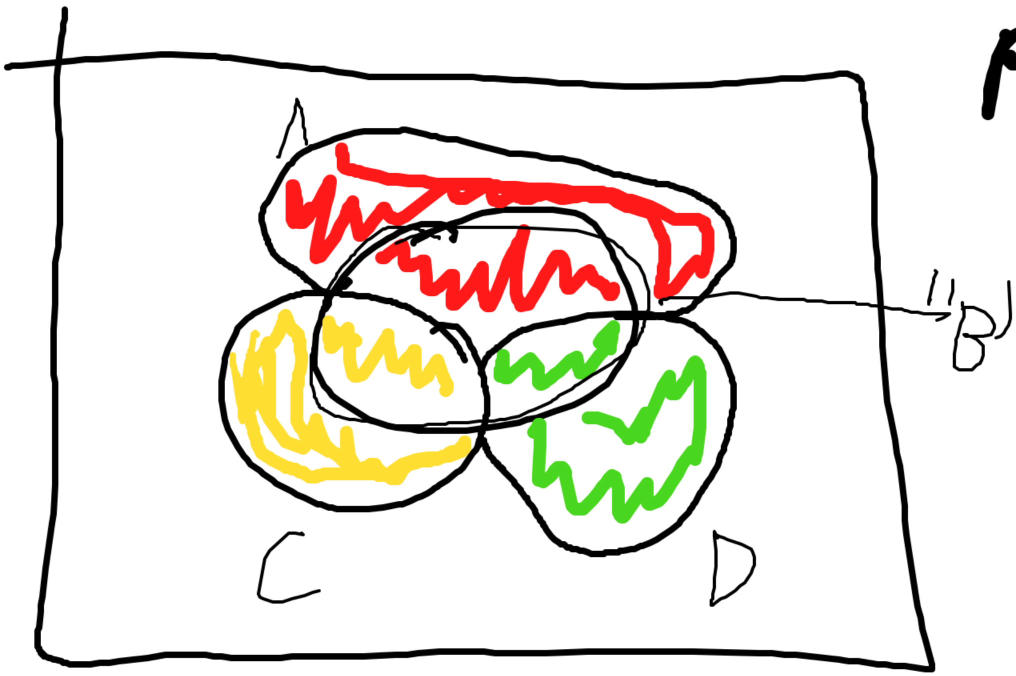
$$\rightarrow P(B|A)P(A) + P(B|A^c)P(A^c)$$

Law of total probability w/ Complement

Want "B"



LoTP with "shards"



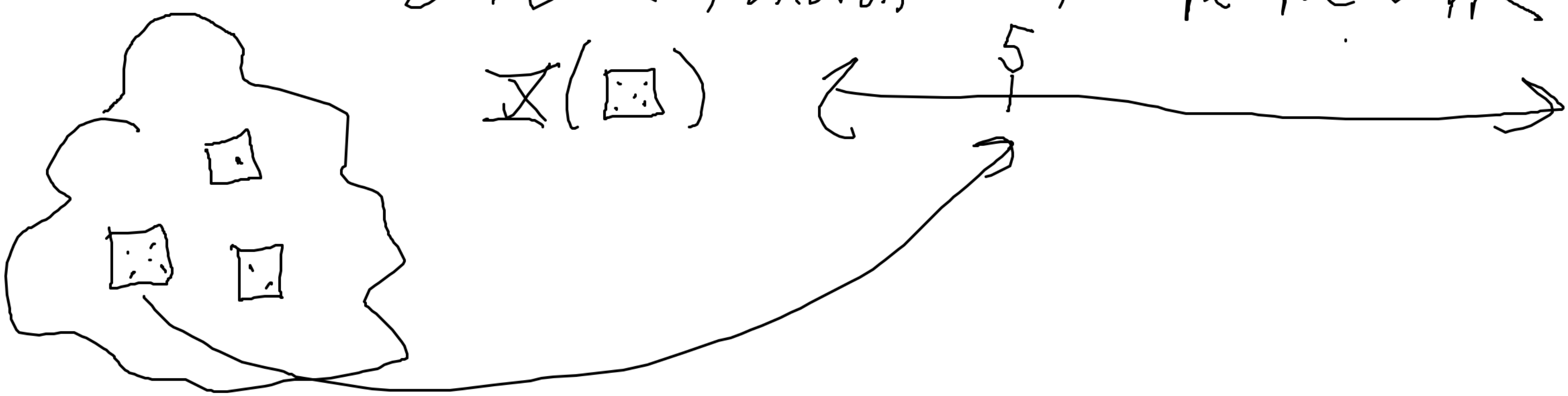
$$P(B) = P(A \cap B) + P(C \cap B) + P(D \cap B)$$

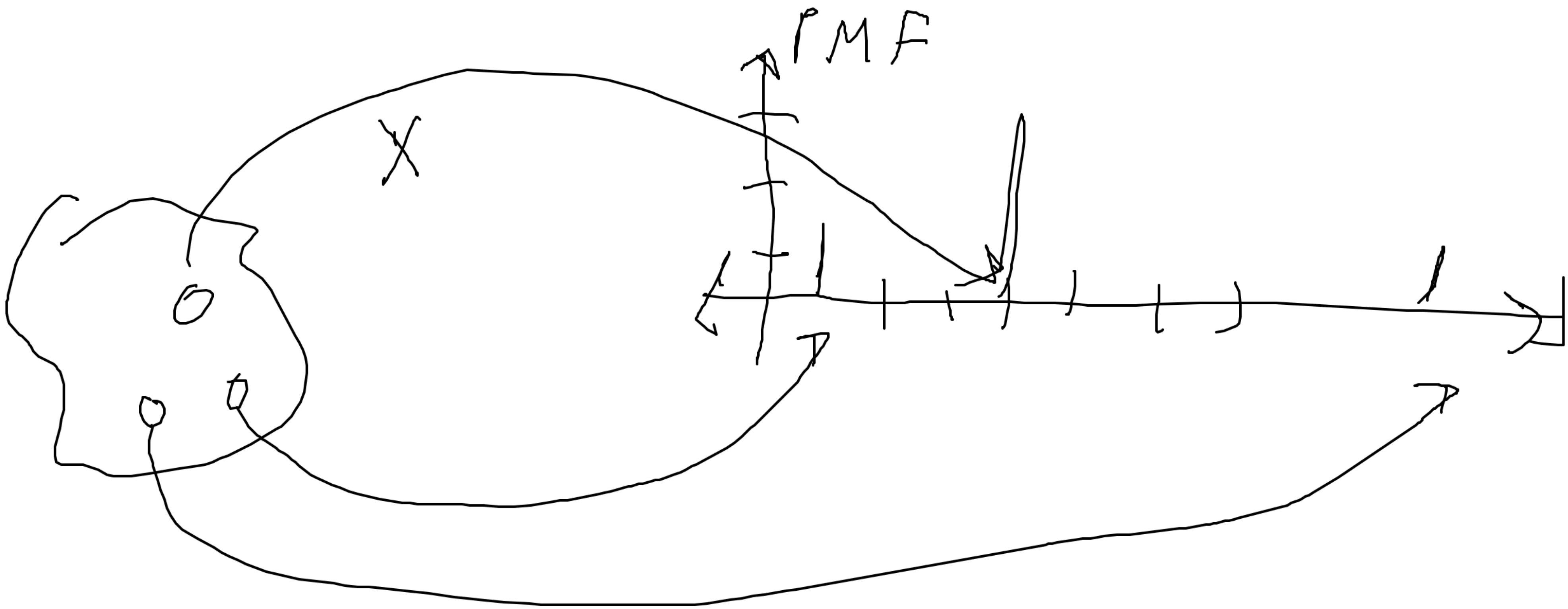
$X = \#$ successful trials

or

$X =$ sum two die

Random Variable is a Function from sample space to $\mathbb{R}$




PMF
X

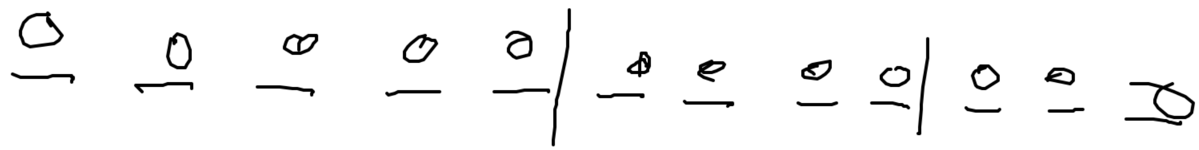
12 fruits

5 apples

4 Mandarin

3 Persimmons

$$\binom{12}{5,4,3} = \frac{12!}{5!\,4!\,3!}$$



$$\binom{12}{7,5}$$

$$\binom{12}{7}$$

$$\binom{12}{5,4,3}$$

bottom must add to top

Larry & Serg want same fruit

$$\binom{10}{\underline{3},4,3} + \binom{10}{5,\underline{2},3} + \binom{10}{5,2,1}$$

Two remain in bag

$$\binom{10}{3,4,3} + \binom{10}{5,2,3} + \binom{10}{5,4,1} + \binom{10}{4,3,3} + \binom{10}{4,4,2} + \binom{10}{5,3,2}$$

$$P(X=i) = \frac{\frac{1}{i}}{\sum_{n=1}^{N} \frac{1}{n}}$$

$N = \#$ songs $= 30$ mil

a)

$$P(X=10) = \frac{\frac{1}{10}}{\sum \frac{1}{\lambda}}$$

b) 1 billion $\rightarrow$ # trials

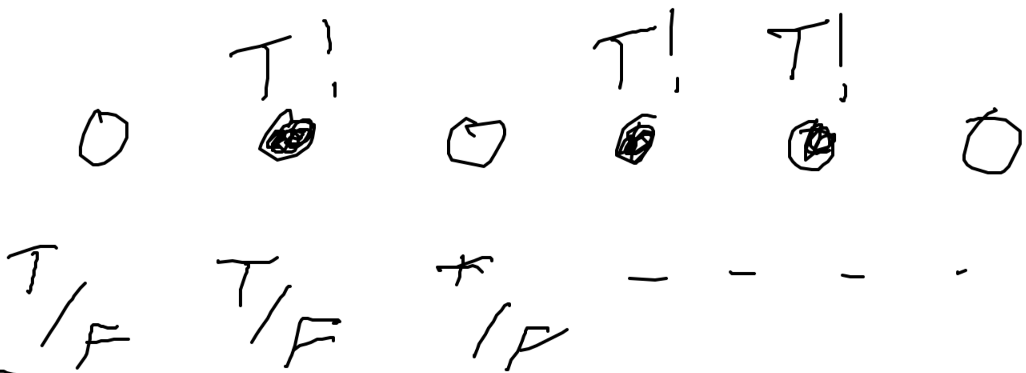
$$Y \sim Bin(\overset{n}{\#\text{trials}}, \overset{p}{\text{probability}})$$

$$W \sim Pois(\#\text{trials} * \text{probability})$$

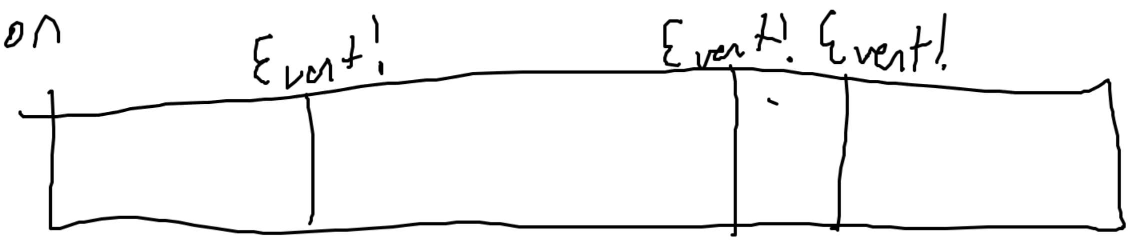
$$P(Y > 10^8) \approx P(W > 10^8)$$

$$= 1 - \sum_{i=0}^{10^8} e^{-np} \frac{(np)^2}{i!}$$

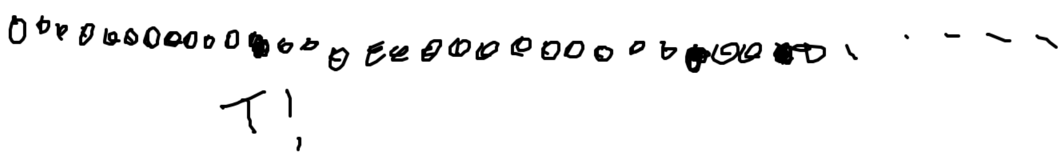
Binomial



Poisson



Bnoil



$$\boxed{\begin{aligned} P(B \mid A) &= 1.00 \\ P(B^c \mid A^c) &= 0.99 \end{aligned}}$$

A = image has cancer

B = model says "cancer"

$$P(A \mid B) \stackrel{?}{=} \text{small?}$$

intuition

reality

$$P(A \mid B) \neq P(B \mid A)$$

$$P(A \mid B) = P(B \mid A)\,\frac{P(A)}{P(B)}$$