1) Review relevant concepts
2) Practice problems

The Varince of X is v

Compute $Var(2X)$ in terms of v

$Var(2X)$

$4\,Var(X)$

$4v$

X_1 & X_2 have the same distribution. Variance is also v. Also indep.

Compute $Var(X_1+X_2)$ in terms of v

$Var(X_1+X_2)$

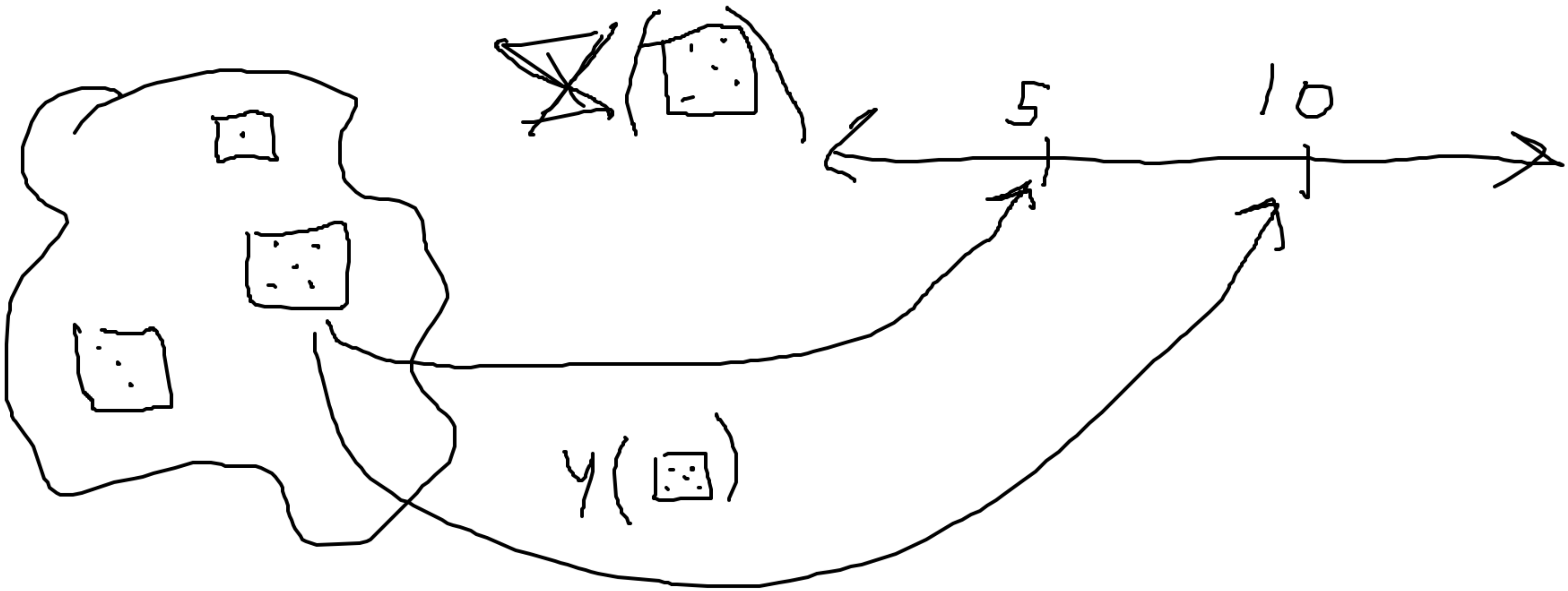
$Var(X_1) + Var(X_2)$

$v + v$

$2v$

Random Variable is a function from sample space to $\mathbb{R}$

Experiment: Dice roll



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

Bayes theorem

Def'n conditional

$$E[X] = \sum_x x\, p(x)$$

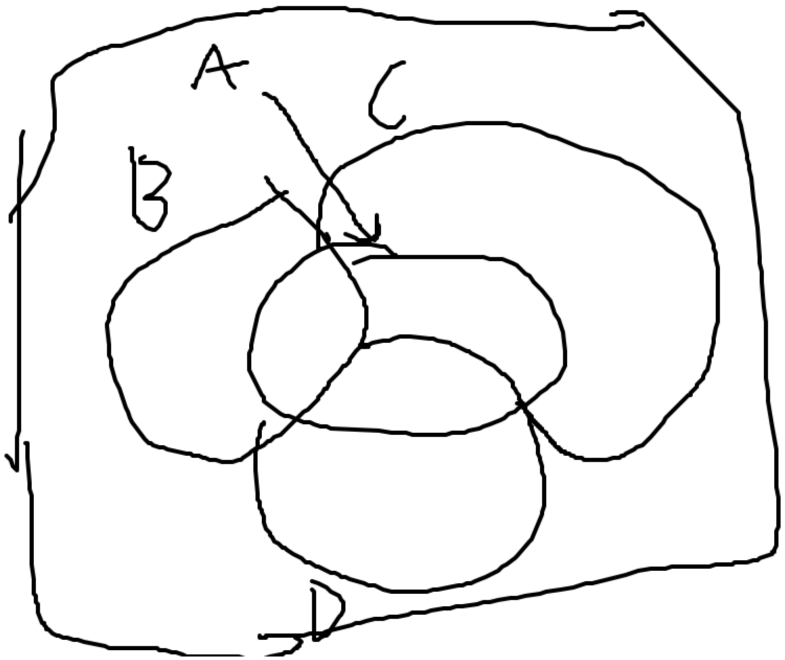
$$y = g(x)$$

$$E[Y] = \sum_y y\, p(y)$$

$$= \sum_x g(x)\, p(x) \leftarrow \text{This is easier}$$

$P(A) = ?$

$P(A) = P(A \cap B) + P(A \cap B^c)$



$$P(A) = P(A \cap B) + P(A \cap C) + P(A \cap D)$$

$$E[X+Y] = E[X] + E[Y]$$

Linearity of expectation

Do not need to be independent

$$E[XY] = E[X]E[Y]$$

MUST be independent

$$Var(X+Y) = Var(X) + Var(Y)$$

$$
\begin{aligned}
P(A \cap B \cap C) &= P(A \mid B \cap C)\, P(B \cap C) \\
&= P(A \mid B \cap C)\, P(B \mid C)\, P(C) \\
&= P(C \mid B \cap A)\, P(B \cap A) \\
&= P(C \mid B \cap A)\, P(B \mid A)\, P(A)
\end{aligned}
$$

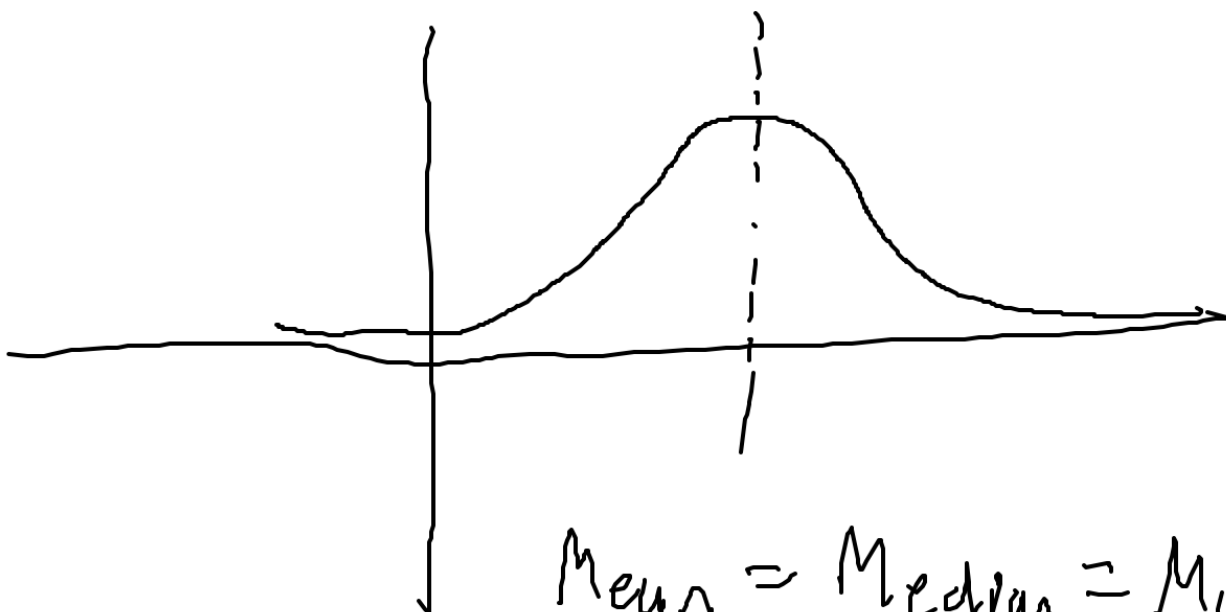
Chain Rule

$X \sim N(\mu, \sigma^2)$

Defined on $(-\infty, \infty)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

hard to integrate



Mean = Median = Mode

$$Z \sim N(0,1)$$

$$X \sim N(\mu_1, \sigma_1^2)$$

$$\frac{X-\mu_1}{\sigma_1} \sim N(0,1) \qquad X \neq Y$$

$$Y \sim N(\mu_2, \sigma_2^2)$$

$$\frac{Y-\mu_2}{\sigma_2} \sim N(0,1)$$

$X \sim Bin(n, p)$

$X \approx Y \sim N(np, np(1-p))$

$P(X \geq 65) \approx P(Y \geq 64.5)$

does not matter if $>$ or $\geq$

$$p_{XY}(x, y) = P(X = x, Y = y) \quad \text{Joint}$$

$$p_X(x) = \sum_y p_{XY}(x, y) \quad \text{Marginal}$$

Given marginals $p_X(x)$ & $p_Y(y)$ is it possible to compute $p_{XY}(x, y)$? False

Given joint $p_{XY}(x, y)$ " " "
" " marginals $p_X(x)$ & $p_Y(y)$? True

Multinomial

$$p(X_1=c_1, X_2=c_2 \ldots) = \binom{n}{c_1, c_2, \ldots} p_1^{c_1} p_2^{c_2} \ldots$$

$$\sum_{i=1}^{m} c_i = n \qquad \sum_{i=1}^{m} p_i = 1$$

Question: Compute a & b, assume X & Y are independent

Joint table

	$X=-3$	$X=9076$
$Y=112$	0.1	0.3
$Y=147$	a	b

$$p_{XY}(x,y) = p_X(x)\,p_Y(y) = \left(\sum_{j \in \mathrm{range}(Y)} p_{XY}(x,j)\right)\left(\sum_{i \in X} p_{X,Y}(i,y)\right)$$

$$0.1 = (0.1+0.3)(0.1+a)$$

$$0.3 = (0.1+0.3)(0.3+b)$$

Combining Independent Random Variables

Convolution

$$P(X+Y=n) = \sum_{k \in Range(X)} P(X=k)\,P(Y=n-k)$$

$X \sim Bin(n_1, p)$

$Y \sim Bin(n_2, p)$

$(X+Y) \sim Bin(n_1+n_2, p)$

MUST be independent

MUST have same p

$X \sim Ber(p)$

$Y \sim Ber(p)$

$X+Y \sim Bin(2, p)$

MUST be independent

$X \sim Poi(\lambda_1)$

Must be Independent

$Y \sim Poi(\lambda_2)$

$X+Y \sim Poi(\lambda_1+\lambda_2)$

$X \sim Geo(p)$

Must be Independent

$Y \sim Geo(p)$

$X+Y \sim NegBin(2, p)$

$$X \sim N(\mu_1, \sigma_1^2)$$

$$Y \sim N(\mu_2, \sigma_2^2)$$

Must be independent

$$X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$$

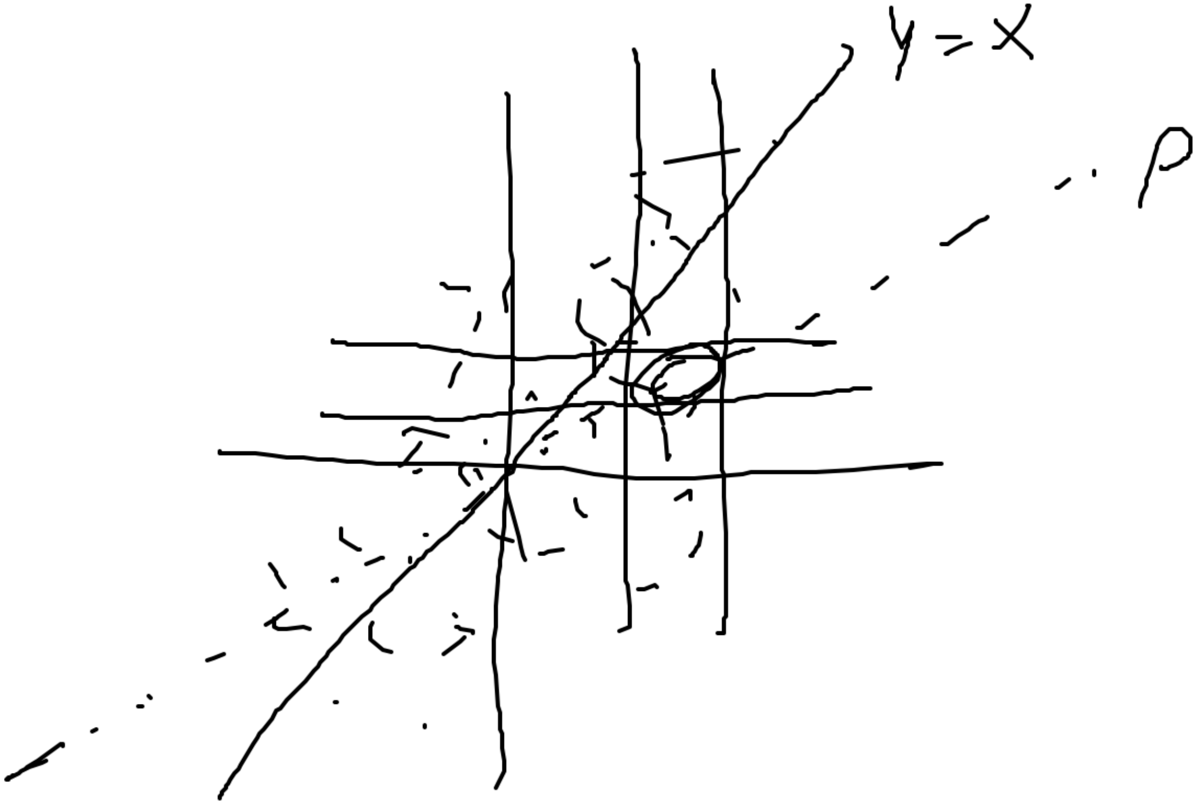
$$\mathrm{Cov}(X,Y) = E[(X-E[X])(Y-E[Y])]$$

$$= E[XY] - E[X]E[Y]$$

Correlation — measures linear relationship

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

$$-1 \leq \rho \leq 1$$


y = x
ρ

$$E[X \mid Y = y] = \sum_x x\, P(X = x \mid Y = y) = \sum_x x\, p_{X|Y}(x|y)$$

$$E[X \mid Y \mid Z]$$

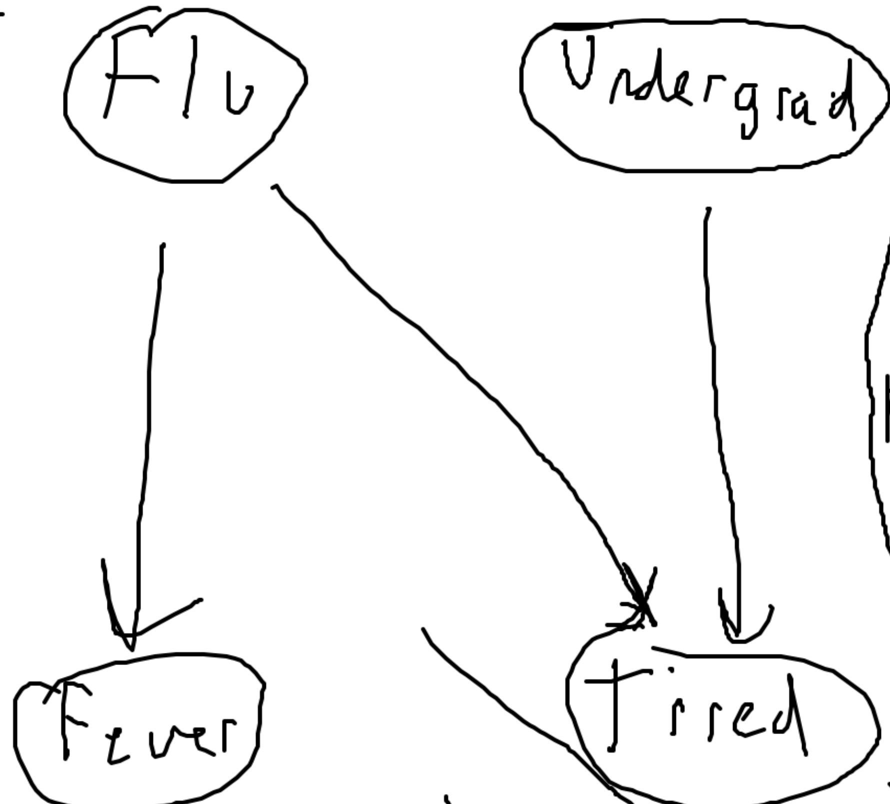
does not make sense

$$E[X] = E\big[E[X|Y]\big] = \sum_{y} P(Y=y)\, E[X|Y=y]$$

Law of Total Probability

Bayesian Networks - each node is a conditional dist. table

$P(Flu=1)=0.1$



$P(F_{ev}=1 \mid Flu=1)=0.9$

$P(F_{ev}=1 \mid Flu=0)=0.05$

$P(Fl=0, F_e=0, U=1, T=1)$

$P(T \mid \underline{Fl}, F_e, \underline{U})\,P(Fl, F_e, U)$

$\widehat{\quad}$

$P(T \mid Fl, U)$ because T & F_e are conditionally independent on parents of T

define as $P(T \mid Fl, U)$

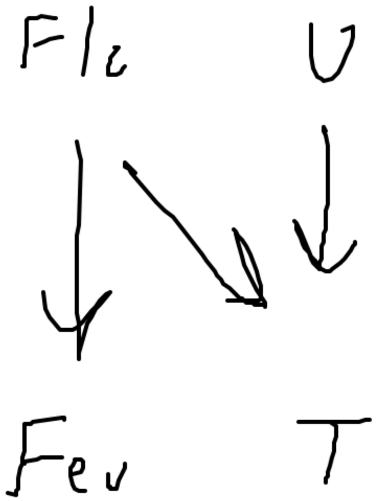
Two things to remember

1) Each variable is conditionally independent of its non-descendents given its parents

$$P(\text{child} \mid \text{Parent}_1, \text{Parent}_2, \underline{\text{non-descendent}})$$

$$= P(\text{child} \mid \text{Parent}_1, \text{Parent}_2)$$

2) Apply chain rule starting with variables with more arrows (good rule of thumb)



$$P(Fl, Fe, U, T)$$

$$= P(T \mid Fl, Fe, U)\, P(Fl, Fe, U)$$

✓ Good

$$= P(Fl \mid Fe, U, T)\, P(Fe, U, T)$$

✗ Bad because no table

Continuous

PDF

$$P(a_1 \le X \le a_2,\ b_1 \le Y \le b_2)$$

$$= \int_{a_1}^{a_2} \int_{b_1}^{b_2} f_{X,Y}(x, y)\, dy\, dx$$

CDF

$$F_{X,Y}(a,b) = \int_{-\infty}^{a} \int_{-\infty}^{b} f_{X,Y}(x,y)\, dy\, dx$$

10 machines of type W

10 " " " X

W_i = usage of W $\sim Poi(4)$

X_i = usage of $X \sim N(5,3)$

Y is usage of select machines

$E[Y] = ?$

$Y = AW + BX$

$E[Y] = E[AW + BX]$

$= E[AW] + E[BX]$

$= E[A]E[W] + E[B]E[X]$

$= (10)(0.2)(4) + (10)(0.2)(5)$

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$$P(Y \geq 20 \mid 3\ W_s, 0\ X_s)$$

$$C = W_1 + W_2 + W_3$$

$$C \sim \text{Poi}(4+4+4)$$

$$P(C \geq 20) = \sum_{i=20}^{\infty} \frac{e^{-12}\, 12^i}{i!}$$

$$= 1 - \sum_{i=1}^{i=19} \frac{e^{-12}\, 12^i}{i!}$$

$$P(Y \geq 20 \mid 3\, X_s,\ 0\, W_s)$$

$$C = X_1 + X_2 + X_3$$

$$C \sim N(5 \cdot 3,\ 3 \cdot 3)$$

$$N(15,\ 9)$$

$$P(C \geq 20) = 1 - P(C < 20)$$

$$= 1 - P\left(\frac{C-15}{3} < \frac{20-15}{3}\right)$$

$$1 - P(Z < 1.67)$$

$$1 - \Phi(1.67)$$

$$= 0.0475$$