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CS 109

Section #5
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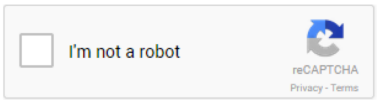
Section 5

Based on the work of many CS109 staffs

1. **Quiz 2 is Coming.** Quiz 2 is less than a week away, and once again, section will be optional. But as your dedicated CS109 staff, we refuse to let the good times in section stop! So we wanted to know, when would work best for you?

2. **ReCaptcha.** Based on browser history, Google believes that there is a 0.2 probability that a particular visitor to a website is a robot. They decide to give the visitor a reCaptcha:



   Google presents the visitor with a 10 mm by 10 mm box. The visitor must click inside the box to show that they are not a robot. You have observed that robots click uniformly in the box. However, the distance location of a human click has X location (mm from the left) and the Y location (mm from the top) distributed as independent normals both with mean $\mu = 5$ and variance $\sigma^2 = 4$.

   a. What is the probability density function of a robot clicking $X = x$ mm from the left of the box and $Y = y$ mm from the top of the box?
   b. What is the probability density function of a human clicking $X = x$ mm from the left of the box and $Y = y$ mm from the top of the box?
   c. The visitor clicks in the box at (x = 6 mm, y = 6 mm). What is Google's new belief that the visitor is a robot?

3. **Binary Tree**: Consider the following function for constructing binary trees:

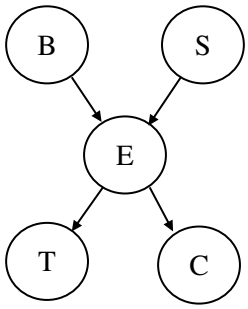
```
struct Node {
    Node *left;
    Node *right;
};

Node *randomTree(float p) {
    if (randomBool(p)) {  // returns true with probability p
        Node *newNode = new Node;
        newNode->left = randomTree(p);
        newNode->right = randomTree(p);
        return newNode;
```

```
        } else {
            return nullptr;
        }
    }
```

The `if` branch is taken with probability p (and the `else` branch with probability $1 - p$). A tree with no nodes is represented by `nullptr`; so a tree node with no left child has `nullptr` for the `left` field (and the same for the right child).

Let X be the number of nodes in a tree returned by `randomTree`. You can assume $0 < p < 0.5$. What is $E[X]$, in terms of p?

4. **Monitoring Satellites**. As a part of CS109's Aeronautics and Space Administration agency, we are in charge of monitoring satellites. Recently, we have had several issues with a satellite; due to our probability expertise, we are in charge of modeling what has happened. As a first step, we decide to model our satellite using a Bayesian Network and binary variables.



Using our background knowledge, we decide to focus on trajectory deviation (T) and communication loss (C). We know that both of these issues are a result of electric system failure (E), which can result from either battery failure (B) or solar panel failure (S). For each variable, let 1 correspond to the failure occurring, and 0 otherwise.

a. What is the probability that everything is working?

b. What is the probability that there is a battery failure, if we have observed trajectory deviation and communication loss?

c. What is the probability that the electric system fails?

d. Let us assume now that we believe that there is an electrical failure and a battery failure. How does that change whether or not we think that there is a solar panel failure? (Conceptual, for fun!)