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Section #6 Warmup Solutions

Based on the work of many CS109 staffs

1 Lecture 17, 5-18-20: Conditional Joint Distributions II

1. Let $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$. What is μ and σ for $X + Y \sim \mathcal{N}(\mu, \sigma)$?
2. Let $X \sim \text{Uni}(0, 1)$ and $Y \sim \text{Uni}(0, 1)$. What is the PDF for $X + Y$?
3. In general, for two independent random variables X and Y, what is the PDF f of $X + Y$?

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1. $\mu = \mu_1 + \mu_2$ and $\sigma = \sigma_1^2 + \sigma_2^2$. How convenient!
2. $f_{X+Y}(a) = \begin{cases} a & 0 \le a \le 1 \\ 2 - a & 1 \le a \le 2 \\ 0 & \text{otherwise} \end{cases}$
3. $f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a - y) f_y(y) dy$

   It is good to remember these equations, but perhaps another message from lecture that it is difficult to sum random variables. The derivation for Uniform distributions is difficult. And solving for the general random variables is even worse. But we can pick distributions, like the Normal distribution, that are easy to use!

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2 Lecture 18, 5-20-20: Sampling/Bootstrapping

1. Computing the sample mean is similar to the population mean: sum all available points and divide by the number of points. However, sample variance is slightly different from population variance.

   (a) Consider the equation for population variance, and an analogous equation for sample variance.

   $$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 \qquad (1)$$

   $$S_{biased}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \qquad (2)$$

   S_{biased}^2 is a random variable which estimates the constant σ^2. Is $E[S_{biased}^2]$ greater or less than σ^2?

   (b) Write the equation for $S_{unbiased}^2$ (known simply as S^2 in the slides). This is known as *Bessel's correction*.

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1. (a) $E[S_{biased}^2] < \sigma^2$. The intuition is that the spread of a sample of points is generally smaller than the spread of all the points considered together.

   (b) $S_{unbiased}^2 = S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$

3 Lecture 1, 5-22-20: Central Limit Theorem

1. What is the distribution (with name and parameter(s)) of the average of n i.i.d. random variables, X_1, ..., X_n, each with mean μ and variance σ^2?

1. According to the central limit theorem, this can be modeled as $N(\mu, \sigma^2/n)$.