9.

a. Using Maximum Likelihood Estimators, we obtain the following parameters for the conditional distributions of X_1, X_2, and X_3:

$P(X_1 \mid Y = 0) \sim \text{Uni}(0.1, 0.7)$
$P(X_2 \mid Y = 0) \sim \text{Uni}(0.4. 0.8)$
$P(X_3 \mid Y = 0) \sim \text{Uni}(0.1, 0.6)$

$P(X_1 \mid Y = 1) \sim \text{Uni}(0.5, 0.9)$
$P(X_2 \mid Y = 1) \sim \text{Uni}(0.2, 0.7)$
$P(X_3 \mid Y = 1) \sim \text{Uni}(0.4, 0.8)$

b. We want to compute P(Y = 0 | test instance *i*) / P(Y = 1 | test instance *i*), and if this is greater than 1, we predict Y = 0 and otherwise we predict Y = 1.

Note that: P(Y = 0 | test instance *i*) / P(Y = 1 | test instance *i*)

$$= \frac{P(Y=0,\mathbf{X})}{P(\mathbf{X})} \bigg/ \frac{P(Y=1,\mathbf{X})}{P(\mathbf{X})} = \frac{P(Y=0,\mathbf{X})}{P(Y=1,\mathbf{X})}$$

$$= P(\mathbf{X} \mid Y = 0)\, P(Y = 0) \,/\, P(\mathbf{X} \mid Y = 1)\, P(Y = 1)$$

Using the Naive Bayes assumption, we have:

$P(\mathbf{X} \mid Y = 0)\, P(Y = 0) \,/\, P(\mathbf{X} \mid Y = 1)\, P(Y = 1)$

$= P(X_1 \mid Y{=}0)\, P(X_2 \mid Y{=}0)\, P(X_3 \mid Y{=}0)\, P(Y{=}0) \,/\, P(X_1 \mid Y{=}1)\, P(X_2 \mid Y{=}1)\, P(X_3 \mid Y{=}1)\, P(Y{=}1)$

Here are the predictions for Y we make for each of the test instances:

P(Y = 0 | test instance 1)/P(Y = 1 | test instance 1)

$= (1/0.6)(1/0.4)(1/0.5)(4/8)/(1/0.4)(1/0.5)(1/0.4)(4/8) = (5/3)(5/2)(2)/(5/2)(2)(5/2) = 2/3$

Since this is < 1, we **classify test instance 1 as class Y = 1**

P(Y = 0 | test instance 2)/P(Y = 1 | test instance 2)

$= (1/0.6)(1/0.4)(0)(4/8)/(1/0.4)(1/0.5)(1/0.4)(4/8) = (0)/(5/2)(2)(5/2) = 0$

Since this is < 1, we **classify test instance 2 as class Y = 1**

P(Y = 0 | test instance 3)/P(Y = 1 | test instance 3)

$= (1/0.6)(1/0.4)(1/0.5)(4/8)/(1/0.4)(1/0.5)(0)(4/8) = (5/3)(5/2)(2)/(0) = \infty$

Since this is > 1, we **classify test instance 3 as class Y = 0**