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CS109

Section #1 Analytic Probability

Based on a handout by previous CS109 CAs Tim Gianitsos, Alex Tsun

Overview of Section Materials

The warmup questions provided will help students practice concepts introduced in lectures. The section problems are meant to apply these concepts in more complex scenarios similar to what you will see in problem sets and quizzes.

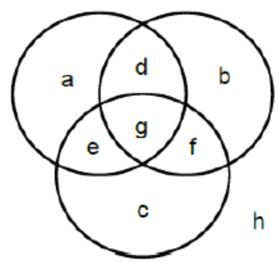
1 Warmups

1.1 Lecture 1: Counting

The Inclusion Exclusion Principle for three sets is:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Explain why in terms of a venn-diagram.



1.2 Lecture 2 Warmup: Permutations and Combinations

Suppose there are 7 blue fish, 4 red fish, and 8 green fish in a large fishing tank. You drop a net into it and end up with 6 fish. What is the probability you get 2 of each color?

1.3 Lecture 3 Warmup: Axioms of Probability

For each of the four statements below, evaluate True or False.

$$P(A|B) + P(A^C|B) = 1 \qquad P(A|B) + P(A|B^C) = 1 \qquad P(A \cap B) + P(A \cap B^C) = 1$$
$$P(A) = 0.4 \wedge P(B) = 0.6 \implies A = B^C$$

1.4 Lecture 4 Warmup: Conditional Probability and Bayes

Bayes Theorem is $P(H|E) = P(E|H) * P(H)/P(E)$ where H can be thought of as a hypothesis and E as evidence. This equation can be notoriously counter intuitive. Draw a diagram where $P(E|H) = 1$ and $P(E^C|H^C)$ is close to 1, but $P(H|E)$ is still close to 0. How can we interpret this?

2 Problems

2.1 Lecture 1 Generative Processes: The Birthday Problem

Preamble: When solving a counting problem, it can often be useful to come up with a generative process, a series of steps that “generates" examples. A correct generative process to count the elements of set A will (1) *generate every element of A* and (2) *not generate any element of A more than once*. If our process has the added property that (3) *any given step always has the same number of possible outcomes*, then we can use the product rule of counting.

Example: Say we want to count the number of ways to roll two (distinct) dice where one die is even and one die is odd. Our process could be: (1) choose a number for the first die, (2) choose a number of opposite parity for the second die. Since the first step has 6 options and the second step has 3 options regardless of the outcome of the first step, the number of possibilities is $6 * 3 = 18$.

Problem: Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we’ll ignore leap years).

a. What is the probability that of the n people in class, at least two people share the same birthday?

b. What is the probability that this class contains exactly one pair of people who share a birthday?

2.2 Lecture 2 Permutations and Combinations: Flipping Coins

Preamble: One thing that students often find tricky when learning combinatorics is how to figure out when a problem involves permutations and when it involves combinations. Naturally, we will look at a problem that can be solved with both approaches. Pay attention to what parts of your solution represent distinct objects and what parts represent indistinct objects.

Problem: We flip a fair coin n times, hoping (for some reason) to get k heads.

a. How many ways are there to get exactly k heads? Characterize your answer as a *permutation* of H’s and T’s.

b. For what x and y is your answer to part a equal to $\binom{x}{y}$? Why does this *combination* make sense as an answer?

c. What is the probability that we get exactly k heads?

2.3 Lecture 4 Bayes Rule: Song Identification

Preamble: In this class, seeing a problem written in English can often throw you off of its scent. In this problem, we will practice translating a problem from English to equations and then applying Bayes Rule, which you learned this week.

Problem: Shazam is an application which can predict what song is playing. Based on the frequency of requests it’s been getting these days, Shazam has found that:

- 80% of songs are Hold Up by Beyonce
- 20% of songs are Can’t Get Used to Losing You by Andy Williams

When a request is made, Shazam receives an audio sample that it uses to update its belief. From one particular audio sample, S, Shazam estimates that:

- S would have a 50% chance of appearing if Hold Up were playing.
- S would have a 90% chance of appearing if Can't Get Used to Losing You were playing.

What is the updated probability that the song is Hold Up given the audio sample heard? HINT: Define variables and write all of the information we have given to you in terms of those variables.