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CS 109

Section #4
October 13 - 14, 2020

Section 4 Handout

Based on the work of many CS109 staffs

1 Warmups

1.1 Joint Distributions

1. Given a Normal RV $X \sim N(\mu, \sigma^2)$, how can we compute $P(X \leq x)$ from the standard Normal distribution Z with CDF ϕ?

2. What is a continuity correction and when should we use it?

3. If we have a joint PMF for discrete random variables $p_{X,Y}(x, y)$, how can we compute the marginal PMF $p_X(x)$?

1.2 Independent Random Variables

1. What distribution does the sum of two independent binomial RVs $X + Y$ have, where $X \sim Bin(n_1, p)$ and $Y \sim Bin(n_2, p)$? Include the parameter(s) in your answer. Why is this the case?

2. What distribution does the is of two independent Poisson RVs $X+Y$ have, where $X \sim \text{Poi}(\lambda_1)$ and $Y \sim \text{Poi}(\lambda_2)$? Include the parameter(s) in your answer.

3. If $Cov(X, Y) = 0$, are X and Y independent? Why or why not?

1.3 Joint Random Variables Statistics

1. **True or False?** The symbol Cov is covariance, and the symbol ρ is Pearson correlation.

$X \perp Y \implies Cov(X, Y) = 0$	$Var(X + X) = 2Var(X)$
$Cov(X, Y) = 0 \implies X \perp Y$	$X \sim \mathcal{N}(0, 1) \wedge Y \sim \mathcal{N}(0, 1) \implies \rho(X, Y) = 1$
$Y = X^2 \implies \rho(X, Y) = 1$	$Y = 3X \implies \rho(X, Y) = 3$

2 Problems

2.1 Approximating Normal

Your website has 100 users and each day each user independently has a 20% chance of logging into your website. Use a normal approximation to estimate the probability that more than 21 users log in.

2.2 Are we due for an earthquake?

After the class where we talked about the probability of Earthquakes at Stanford, a student asked a question: “Doesn’t the probability of an earthquake happening change based on the fact that we haven’t had one for a while?" Let’s explore! Recall the USGS rate of earthquakes of magnitude 8+ is λ =0.002 earthquakes per year.

a. What is the probability of no 8+ earthquakes in four years after the 1908 earthquake (recall that earthquakes are exponentially distributed)?

b. What is the probability of no 8+ earthquakes in the 113 years between the 1908 earthquake and four years from now?

c. What is the probability of no 8+ earthquakes in the 113 years between the 1908 earthquake and four years from now *given* that there have been no earthquakes in the last 109 years?

d. Did you notice anything interesting? Would this work for any value of λ?