# Quick slide reference

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Today’s discussion thread: [https://edstem.org/us/courses/5090/discussion/334921](https://edstem.org/us/courses/5090/discussion/334921)
Lecture with Zoom

- Turn on your camera if you are able, mute your mic in the big room
- Virtual backgrounds are encouraged (classroom appropriate, thank u!)
- Ask questions in chat or use raise hand” feature

Ask your questions in both Ed and Zoom chat
  - Zoom chat: Like raising your hand in the classroom, except with a lower barrier to entry
  - Ed can also be used for asynchronous questions (e.g., if you have Q’s when watching recordings)
  - Persistent copy: Teaching staff and I can answer questions during and after lecture

Join discussion forum here: https://edstem.org/us/join/9TDTm5
Today’s discussion thread: https://edstem.org/us/courses/5090/discussion/334921
Welcome to CS109!
Jerry Cain

I went here from 1987 through 1991 and majored in chemistry.

Then I came here for a PhD in chem, switched to CS

Received MSCS 1998

Lecturer: 25 years today!

Speculation: Have taught more classes than other lecturers.
Why Jerry likes probability

• I majored in chemistry, and my undergraduate research was rooted in surface science and statistical mechanics.

• When I switched to CS as a graduate student here, I focused on CS theory and all the beautiful mathematics that comes with it. Mathematics feeds the soul.

• Probability has revived parts of AI and information theory that were thought to be borderline dead when I was getting my MSCS degree here.

\[
P V = \frac{1}{3} N m v_{\text{rms}}^2, \quad f(v) = 4\pi \left( \frac{m}{2\pi kT} \right)^\frac{3}{2} v^2 e^{-\frac{mv^2}{2kT}} \quad v_{\text{rms}}^2 = \int_0^\infty v^2 f(v) \, dv = 4\pi \left( \frac{m}{2\pi kT} \right)^\frac{3}{2} \int_0^\infty v^4 e^{-\frac{mv^2}{2kT}} \, dv
\]
What makes this quarter important

We are seeing a huge surge in statistics, predictions, and probabilistic models shared through global news, governing bodies, and social media.

Global cases of COVID-19 as of March 27th (JHU)
https://coronavirus.jhu.edu/map.html

National Weather Service Alerts
https://www.weather.gov/

FiveThirtyEight 2020

https://fivethirtyeight.com/

The New York Times 2020

https://www.nytimes.com/

US Politics In Review
https://fivethirtyeight.com/
https://www.nytimes.com/
What makes this quarter important

We are seeing a huge surge in **statistics, predictions, and probabilistic models** shared through global news, governing bodies, and social media.

The challenge of delivering Stanford-class education online reflects our university’s commitment to fostering a **diverse body of students**.
What makes this quarter important

We are seeing a huge surge in **statistics, predictions, and probabilistic models** shared through global news, governing bodies, and social media.

The challenge of delivering Stanford-class education online reflects our university’s commitment to fostering a **diverse body of students**.

The **technological and social innovation** we develop during this time will strongly impact how we approach **truly world-class education**.
What makes this quarter important

We are seeing a huge surge in statistics, predictions, and probabilistic models shared through global news, governing bodies, and social media. The challenge of delivering Stanford-class education online reflects our university’s commitment to fostering a diverse body of students. The technological and social innovation we develop during this time will strongly impact how we approach truly world-class education.

Our teaching goals (at minimum)

To teach you how probability applies to real life
To help you foster and maintain human connections throughout this course
that being said...
What makes this quarter important

Times are still very difficult.
The teaching staff and I realize that this quarter cannot replace an in-person, on-campus experience. Your diverse backgrounds amplify this difference.

All our situations may change.

We’re committed to working through the course together and adapting as a class and as a community. We welcome your suggestions.

Thanks in advance for being patient with the changes we’re trying to make CS109 fulfilling, meaningful, and equitable. Things are looking better for many, but some are as isolated as they were a year ago.
The CS109 teaching team
What about you?
Course mechanics
Course website

http://cs109.stanford.edu/
Prerequisites

**CS106B/X**
- Programming
- Recursion
- Hash tables
- Binary trees

**MATH 51/CME 100**
- Multivariate differentiation
- Multivariate integration
- Basic facility with linear algebra (vectors)

**CS103**
(co-requisite OK)
- Proofs (induction)
- Set theory
- Math maturity

**Important!**
How many units should I take?

Hours per week = Units \times 3

Average about 10 hours / week for assignments

Start Here

Are you an undergrad?

- Yes
  - 5 Units
- No
  - Do you want to take CS109 for fewer units?
    - Yes
      - 3 Units
    - No
      - 4 Units
Companion class: CS109A

- CS109A is an extra 1-unit “Pathfinders” or “ACE” section with additional support, practice, and instruction
- Meets for an additional weekly section and has additional review session
- Entry by application — see course website for details
Lecture format

Before lecture
~30-40 minutes
pre-recorded

Lecture
~40-50 minutes (not 80 minutes)
live (start 11:00am, also recorded)

Concept checks (~15 minutes before lecture)
- Submit infinitely many times before lecture
- Keeps you on pace
- Tests your understanding
- Small part of course grade
# Course components

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Component</th>
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<tbody>
<tr>
<td>10%</td>
<td>Concept Checks</td>
</tr>
<tr>
<td>42%</td>
<td>6 Problem Sets</td>
</tr>
<tr>
<td>42%</td>
<td>3 Quizzes</td>
</tr>
<tr>
<td>6%</td>
<td>Section participation</td>
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</tbody>
</table>
Concept checks

10% Concept Checks
  - Submit before each live lecture
  - Intended to be short check-ins

42% 6 Problem Sets

42% 3 Quizzes

6% Section participation
## Problem Sets

<table>
<thead>
<tr>
<th>Percentage</th>
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<tbody>
<tr>
<td>10%</td>
<td>Concept Checks</td>
<td>Written portion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• LaTeX is optional (but encouraged)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Tutorial on CS109 website</td>
</tr>
<tr>
<td>42%</td>
<td>6 Problem Sets</td>
<td>Coding portion in Python</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Review session #1 Friday 04/02 (time TBA)</td>
</tr>
<tr>
<td>42%</td>
<td>3 Quizzes</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>Section participation</td>
<td>Late policy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Submit by deadline: on-time bonus (~1%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• <strong>Grace period of 1 class day:</strong></td>
</tr>
<tr>
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<td></td>
<td>If you miss the deadline, submitting during the grace period does not earn any on-time bonus</td>
</tr>
<tr>
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<td>• “Pre-granted grace period” on all assignments</td>
</tr>
</tbody>
</table>

Lisa Yan, Chris Piech, Mehran Sahami, and Jerry Cain, CS109, Spring 2021
Quizzes

10%  Concept Checks

42%  6 Problem Sets

42%  3 Quizzes

   - Ideally, 3 hours of individual work + typesetting
   - 47-hour take-home window (between Wed/Fri lectures)
     - Week 4: Wed 4/21 – Fri 4/23
     - Week 7: Wed 5/12 – Fri 5/14
     - Week 10: Wed 6/2 – Fri 6/4

6%  Section participation
## Section participation

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<td>3 Quizzes</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>Section participation</td>
<td>• Section meets Tuesdays (times to be released soon)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Sections start Week 2</td>
</tr>
</tbody>
</table>

More info on [course website](#)
CS109 Contest

- Announced mid-quarter
- Boosts final course grades after letter grade buckets have been determined

Your baseline is CS109, and the sky is the limit.

Previous winning submissions:
- Recidivism Risk: Algorithmic Prediction and Racial Bias
- A Better Way to Reform the Electoral College
- Monte Carlo Tree Search for Tic Tac Toe
- Impact of COVID on Student Interest in Post-Secondary School Education
Stanford Honor Code

Permitted
- Talk to the course staff
- Talk with classmates (cite collaboration)
- Look up general material online

NOT permitted:
- Copy answers:
  - from classmates
  - from former students
  - from previous quarters
- Copy answers from the internet
- Ask for answers on the internet
  Besides, these are usually incorrect

Updated after lecture:
Cite all references aside from instructors, staff, lecture slides/notes, and the optional Ross textbook.
Why you should take CS109
Traditional View of Probability
CS view of probability

http://www.site.com

But wait... There’s MORE!!
Machine Learning
= Machine (compute power)
+ Probability
+ Data
Machine Learning Algorithm

Data → Build a *probabilistic model* → Do one thing
Classification

Chihuahua or muffin?
Classification: Where is this useful?

A machine learning algorithm performs **better than** the best dermatologists.

Developed in 2017 at Stanford.


Lisa Yan, Chris Piech, Mehran Sahami, and Jerry Cain, CS109, Spring 2021
Classification: Image tagging
Decision-making: The last remaining board game
Natural language processing

Augmented reality machine translation on Google Translate

Voice assistants: voice to text to answer
Probability is *more* than just machine learning.
Probability and medicine

How do COVID-19 testing rates in a region correlate with the actual spread of the disease?

Predicted Hospital Resource Use in United States (IHME)
https://covid19.healthdata.org/projections
Probability and art
Probability and climate
Probabilistic analysis of algorithms
Probability in practice
Probability at your fingertips
Probability and philosophy
We’ll get there!
Probability is not always intuitive.
Disease testing

A patient takes a virus test that returns positive. What is the probability that they have the virus?

- 0.03% of people have the virus
- Test has 99% positive rate for people with the virus
- Test has 7% positive rate for people without the virus

Correct answer: 0.42%
Probability = Important + Needs Studying
Counting I
What is Counting?

An experiment in probability:

Counting: How many possible outcomes can occur from performing this experiment?
What is Counting?

Roll: 6 \{1, 2, 3, 4, 5, 6\}

Roll even only: 3 \{2, 4, 6\}

Roll: 36

\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}
Sum Rule of Counting

If the outcome of an experiment can be either from

Set \( A \), where \( |A| = m \),

or Set \( B \), where \( |B| = n \),

where \( A \cap B = \emptyset \),

Then the number of outcomes of the experiment is

\[ |A| + |B| = m + n. \]
Product Rule of Counting

If an experiment has two parts, where

The first part’s outcomes are from Set $A$, where $|A| = m$, and

the second part’s outcomes are from Set $B$, where $|B| = n$,

Then the number of outcomes of the experiment is

$|A||B| = mn$. 

Two-step experiment

\[ \rightarrow A \rightarrow B \]
Lecture with **zoom**

**Breakout Rooms for working through lecture exercises**

- We may incorporate some of these during lecture
- You are **always welcome** to exit breakout rooms if you are more comfortable staying in the main room
Breakout Rooms

Introduce yourself!

Then check out the question on Slide 57. Post any clarifications here!

https://edstem.org/us/courses/5090/discussion/334921

Breakout Room time: 5 minutes
Again, you can decline our invitation or return to main room at any time.
Let’s try it out

Sum Rule, Product Rule, or something else? How many outcomes?

1. Video streaming application
   • Your application has distributed servers in 2 locations (SJ: 100, Boston: 50).
   • If a web request is routed to a server, how large is the set of servers it can get routed to?

2. Dice
   • How many possible outcomes are there from rolling two 6-sided dice?

3. Strings
   • How many different orderings of letters are possible for the string BOBA?
Let’s try it out

Sum Rule, Product Rule, or something else? How many outcomes?

1. Video streaming application
   • Your application has distributed servers in 2 locations (SJ: 100, Boston: 50).
   • If a web request is routed to a server, how large is the set of servers it can get routed to?

2. Dice
   • How many possible outcomes are there from rolling two 6-sided dice?

3. Strings
   • How many different orderings of letters are possible for the string BOBA?
For next time

• Watch pre-recorded lectures for today (Monday 3/29) and Wednesday 3/31
• Complete one concept check that covers both lectures

http://cs109.stanford.edu/
Questions?
Counting II

Gradescope quiz, blank slide deck, etc.  
(Available Monday 3/29 evening PT)  
http://cs109.stanford.edu/
**TOP DEFINITION**

**kick it up a notch**

To make things more intense, exciting, or interesting.

(introduced by chef Emeril Lagasse in reference to spicing up his recipes.)
Inclusion-Exclusion Principle

If the outcome of an experiment can be either from

Set $A$ or set $B$, where $A$ and $B$ may overlap,

Then the total number of outcomes of the experiment is

$$|A \cup B| = |A| + |B| - |A \cap B|.$$
Transmitting bytes over a network

An 8-bit string is sent over a network.
• The receiver only accepts strings that either start with 01 or end with 10.

How many 8-bit strings will the receiver accept?

Define

\( A \) : 8-bit strings starting with 01
\( B \) : 8-bit strings ending with 10
Transmitting bytes over a network

An 8-bit string is sent over a network.

- The receiver only accepts strings that either start with 01 or end with 10.

How many 8-bit strings will the receiver accept?

Define

A : 8-bit strings starting with 01
B : 8-bit strings ending with 10
General Principle of Counting

If an experiment has \( r \) steps, such that

Step \( i \) has \( n_i \) outcomes for all \( i = 1, \ldots, r \),

Then the number of outcomes of the experiment is

\[
\prod_{i=1}^{r} n_i.
\]

Multi-step experiment

Product Rule of Counting:

A special case

\[ 1 \rightarrow 2 \rightarrow \ldots \]
License plates

How many CA license plates are possible if...

(pre-1982)

(present day)
License plates

How many CA license plates are possible if...

(pre-1982)

(present day)

Lisa Yan, Chris Piech, Mehran Sahami, and Jerry Cain, CS109, Spring 2021
Pigeonhole Principle

Gradescope quiz, blank slide deck, etc.
http://cs109.stanford.edu/
## Floors and ceilings

<table>
<thead>
<tr>
<th>Floor function</th>
<th>Ceiling function</th>
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</thead>
<tbody>
<tr>
<td>([x])</td>
<td>([x])</td>
</tr>
<tr>
<td>The largest integer (\leq x)</td>
<td>The smallest integer (\geq x)</td>
</tr>
</tbody>
</table>

Check it out:

- \([1/2]\)
- \([2.9]\)
- \([8.0]\)
- \([−1/2]\)

- \([1/2]\)
- \([2.9]\)
- \([8.0]\)
- \([−1/2]\)
Pigeonhole Principle

For positive integers \( m \) and \( n \), if \( m \) objects are placed in \( n \) buckets, then at least one bucket must contain at least \( \lfloor m/n \rfloor \) objects.

Example:

\[ m \text{ objects} = 10 \text{ pigeons} \]
\[ n \text{ buckets} = 9 \text{ pigeonholes} \]

At least one pigeonhole must contain \( \lfloor m/n \rfloor = 2 \) pigeons.

Bounds: an important part of CS109

Lisa Yan, Chris Piech, Mehran Sahami, and Jerry Cain, CS109, Spring 2021
Balls and urns

$n$ balls

$r$ urns (buckets)
Balls and urns  Hash Tables and strings

Consider a hash table with 100 buckets.
950 strings are hashed and added to the table.

1. Is it guaranteed that at least one bucket contains \textit{at least} 10 entries?

2. Is it guaranteed that at least one bucket contains \textit{at least} 11 entries?

3. Is it possible to have a bucket with \textit{no entries}?
Balls and urns  Hash Tables and strings

Consider a hash table with 100 buckets.  
950 strings are hashed and added to the table.  

\[ n = 100 \]
\[ m = 950 \]

1. Is it guaranteed that at least one bucket contains \textit{at least} 10 entries?  
   Yes

2. Is it guaranteed that at least one bucket contains \textit{at least} 11 entries?  
   No

3. Is it possible to have a bucket with \textit{no entries}?  
   Sure
Permutations I

Gradescope quiz, blank slide deck, etc.  
http://cs109.stanford.edu/
Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of six distinct numbers?
Sort $n$ indistinct objects
Sort $n$ distinct objects

Ayesha  Tim  Irina  Joey  Waddie
Sort $n$ distinct objects

Steps:
1. Choose 1$^{st}$ can 5 options
2. Choose 2$^{nd}$ can 4 options
   ...
5. Choose 5$^{th}$ can 1 option

Total = $5 \times 4 \times 3 \times 2 \times 1$
      = 120
Permutations

A permutation is an ordered arrangement of objects.

The number of unique orderings (permutations) of $n$ distinct objects is

$$n! = n \times (n - 1) \times (n - 2) \times \cdots \times 2 \times 1.$$
Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of six distinct numbers?

Total = 6!

= 720 passcodes
Unique 6-digit passcodes with five smudges

How many unique 6-digit passcodes are possible if a phone password uses each of five distinct numbers?