

CSE 109: Probability for Computer Scientists

**Instructor:** Alex Tsun, Tim Gianitsos

**Lecture Topics:** 4.3 The Normal RV, 4.4 Transforming Continuous RVs

[Tags: PSet3 Q4ab, The Normal RV]

1. Suppose the time that Java takes to sort a 1,000,000 length array is approximately  $J \sim \mathcal{N}(\mu = 46, \sigma^2 = 6^2)$  milliseconds (ms), since it uses the (randomized) QuickSort Algorithm.
  - a. Python initially implements a (deterministic) MergeSort Algorithm, and it always finishes in  $P = 49$  ms. What is the probability that Java sorts a single 1,000,000 length array faster than Python does? Show your work and give your answer rounded to 4 decimal places.
  - b. Python attempts to implement QuickSort as well, but did it less efficiently. Its runtime is approximately  $P \sim \mathcal{N}(\mu = 55, \sigma^2 = 8^2)$ . What is the probability that Java sorts a single 1,000,000 length array faster than Python does? Show your work and give your answer rounded to 4 decimal places.
  - c. The remaining parts are left for you ☺.

[Tags: Transforming Continuous RVs]

2. Suppose  $X \sim \text{Exp}\left(\lambda = \frac{1}{2}\right)$  is the waiting time in hours until your pizza delivery arrives, and suppose we decide to tip  $Y = g(X) = \frac{24}{X+1}$  dollars.
  - a. What is the range, PDF, and CDF of  $X$ ? Hint: You can look this up.
  - b. What is the range  $\Omega_Y$ ?
  - c. Find  $F_Y(y)$  using the CDF method, then find  $f_Y(y)$  afterwards.
  - d. Find  $f_Y(y)$  using the explicit formula, after verifying the monotonicity and invertibility criteria.
  - e. Set up integrals for  $E[Y]$  in two ways: one with LOTUS and  $f_X(x)$ , and one with  $f_Y(y)$ . Explicitly define your limits of integration and the integrand so that one could enter your integral into WolframAlpha.